



# Improving Scalability of Bundle Adjustment

Partha Ghosh

Master Thesis

Supervised by: Thomas Schneider, Marcin Dymczyk

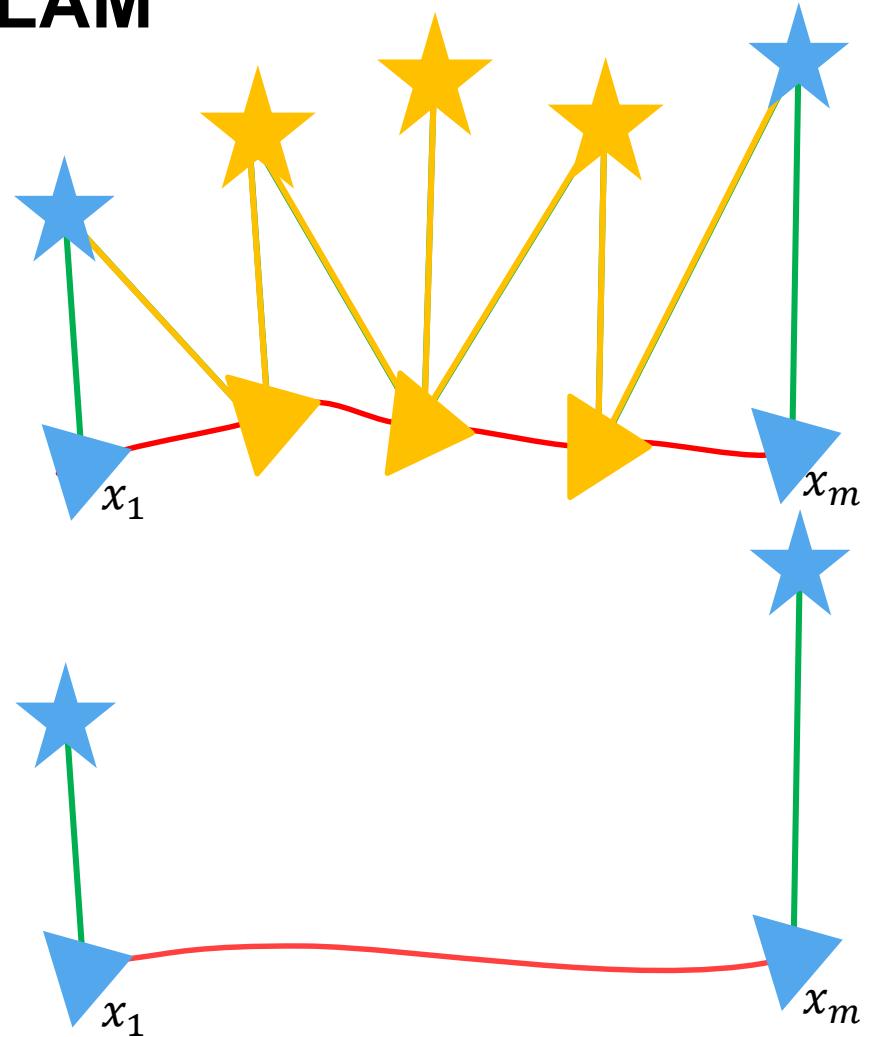
# Agenda

- Bundle Adjustment
  - Motivation
  - Keyframing
- CKLAM
  - Formulation
  - Sparsity
  - Results
- R-CKLAM
  - Formulation
  - Results

# Bundle Adjustment in SLAM

- $\operatorname{argmin}_{x,f}(\sum C_{IMU} + \sum C_{visual})$

- Exclude redundant information

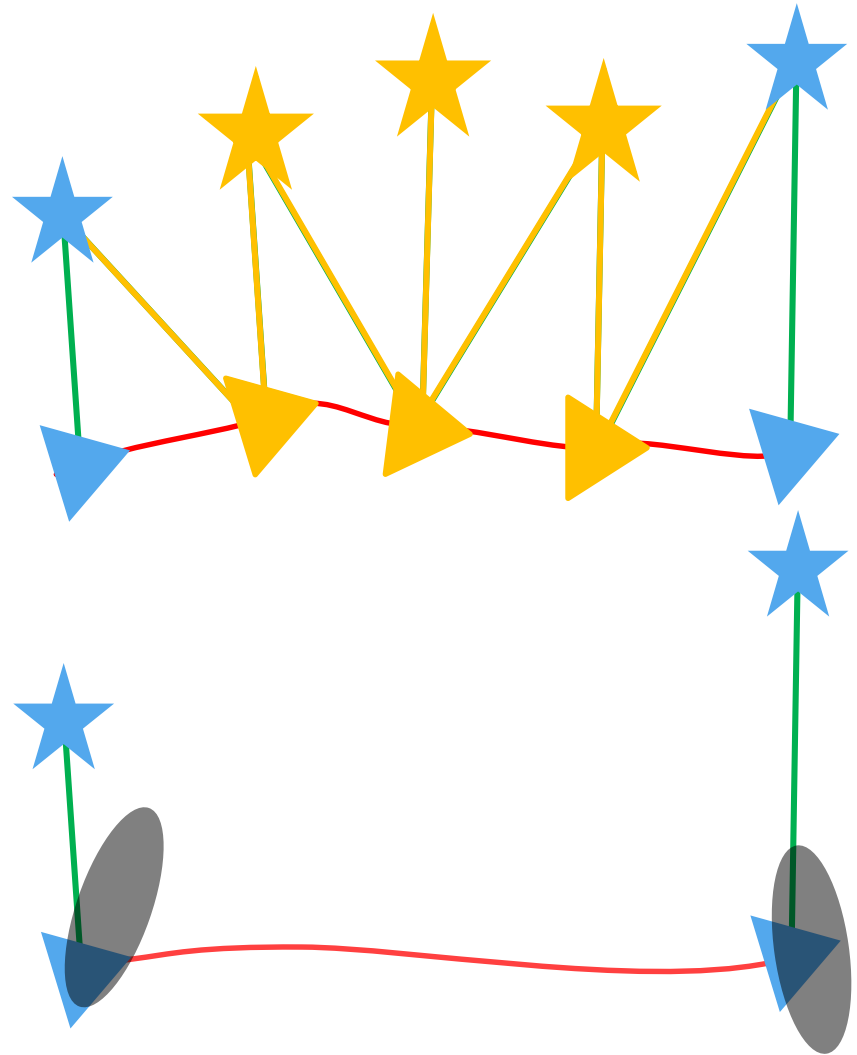


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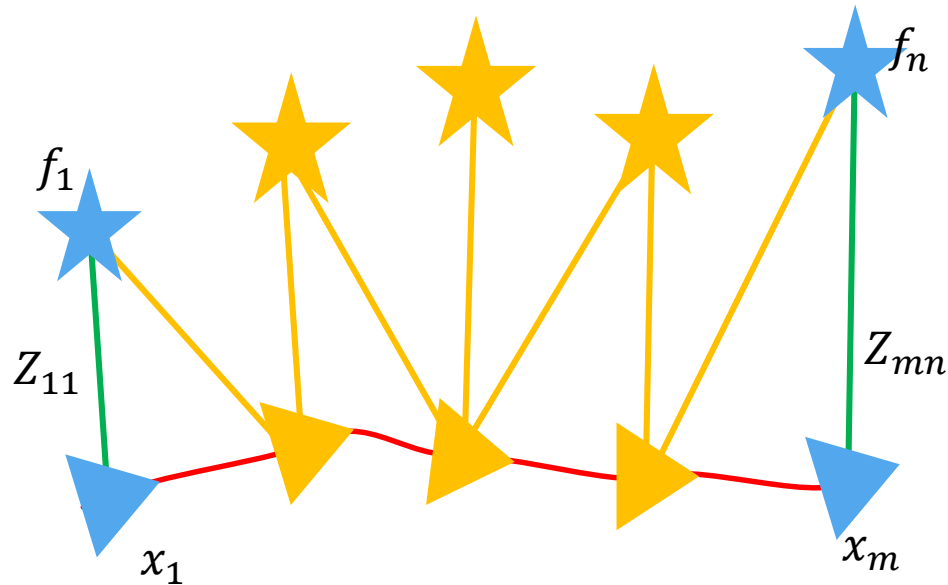
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# Constrained Keyframe-Based Localization and Mapping (CKLAM)

- Removal of Non-keyframes → Loss of information
- CKLAM → Preserve information as much as possible



# CKLAM Formulation



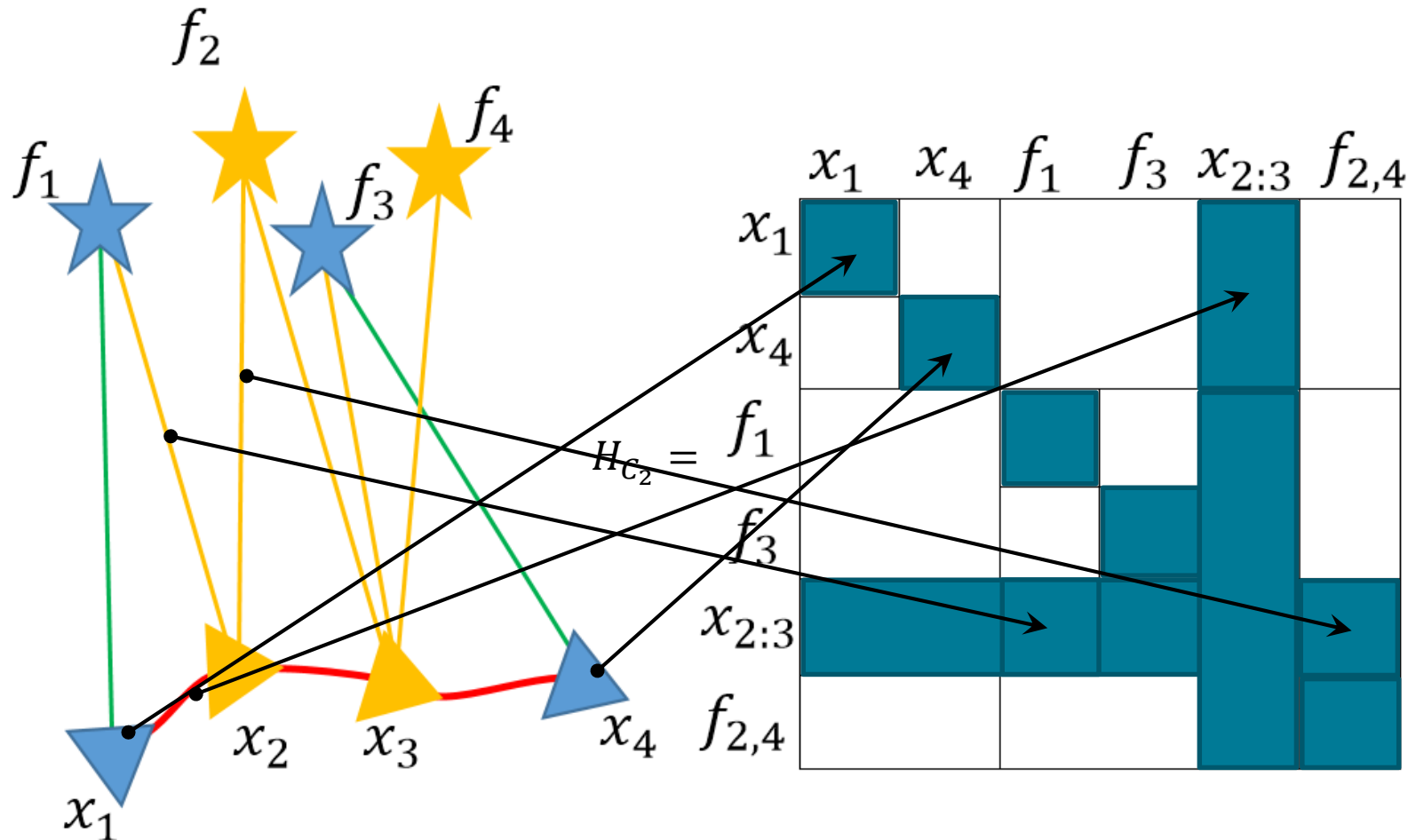
- $C = C_P(x_1; \hat{x}_{1|1}) + C_0(x_1, f_1; Z_{11}) + C_0(x_m, f_n; Z_{mn}) + \sum_{i=0}^3 C_M(x_{i+1}, x_i; u_i) + \sum_Z C_o(x_i, f_j; z_{ij})$
- Quadratize cost  $C_2 = f(a_2)$  and project on  $C_1 = f(a_1)$

[1]

# CKLAM Objective Simplified

- Have  $C_2 = f(a_1, a_2)$  need  $C_{new} = f_{new}(a_1)$ 
  - Such that  $\operatorname{argmin}_{a_1, a_2}(C_2) \approx \operatorname{argmin}_{a_1}(C_{new})$
- Quadratzation
  - $C_2 = \frac{1}{2} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}^T H \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + g^T \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \alpha$
- $C_{NEW} = a_1^T H_{new} a_1 + g_{new}$   
where  $H_{new} = (H_{11} - H_{12}H_{22}^{-1}H_{21})$  and  $g_{new} = (g_1 - H_{11}H_{22}^{-1}g_2)$

# Hessian Matrix for Bundle Adjustment

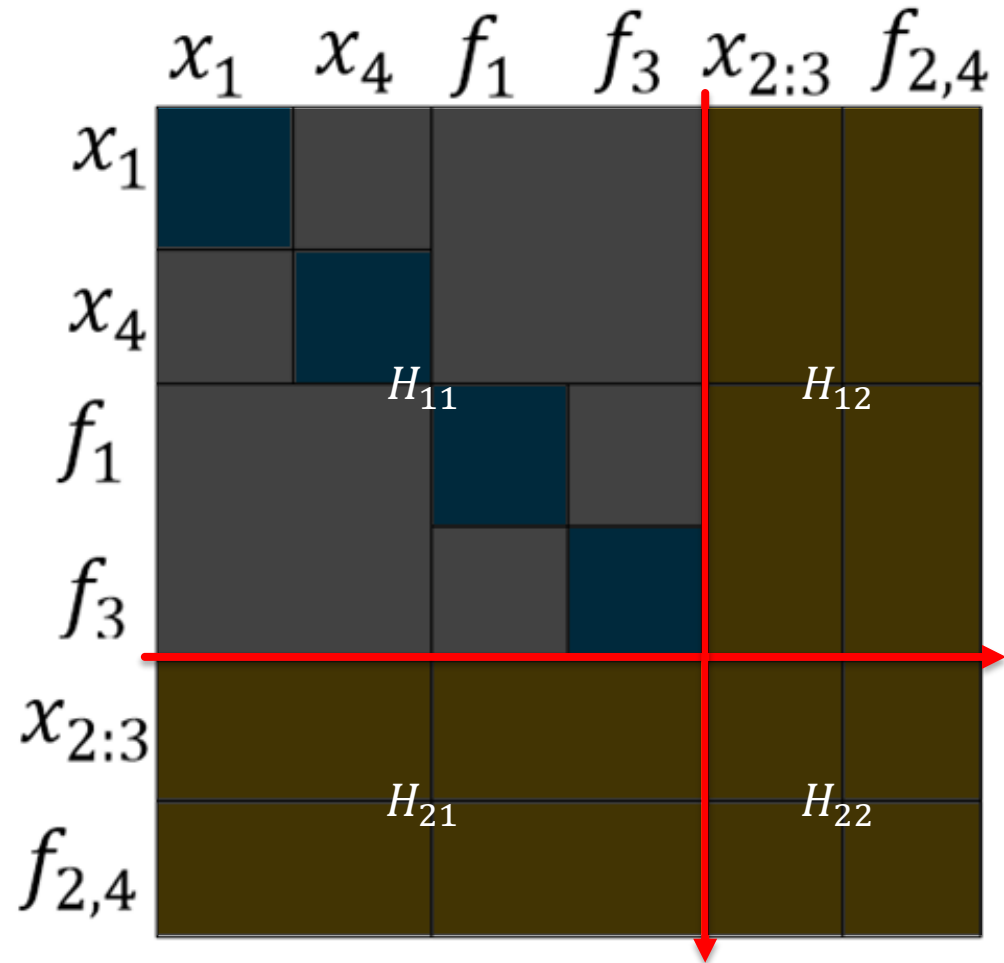




# CKLAM Sparsity

- Cross terms between  $f_1$  and  $f_5$  are created

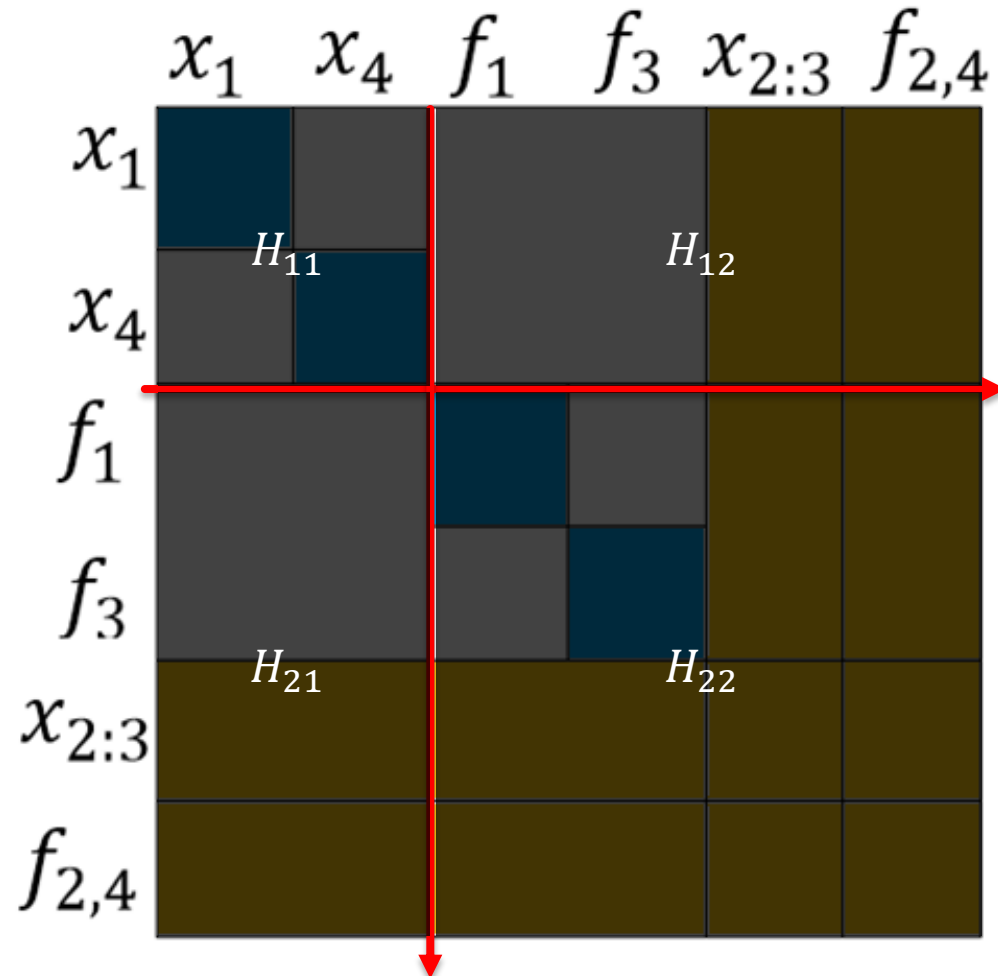
$$H_{C_2} =$$



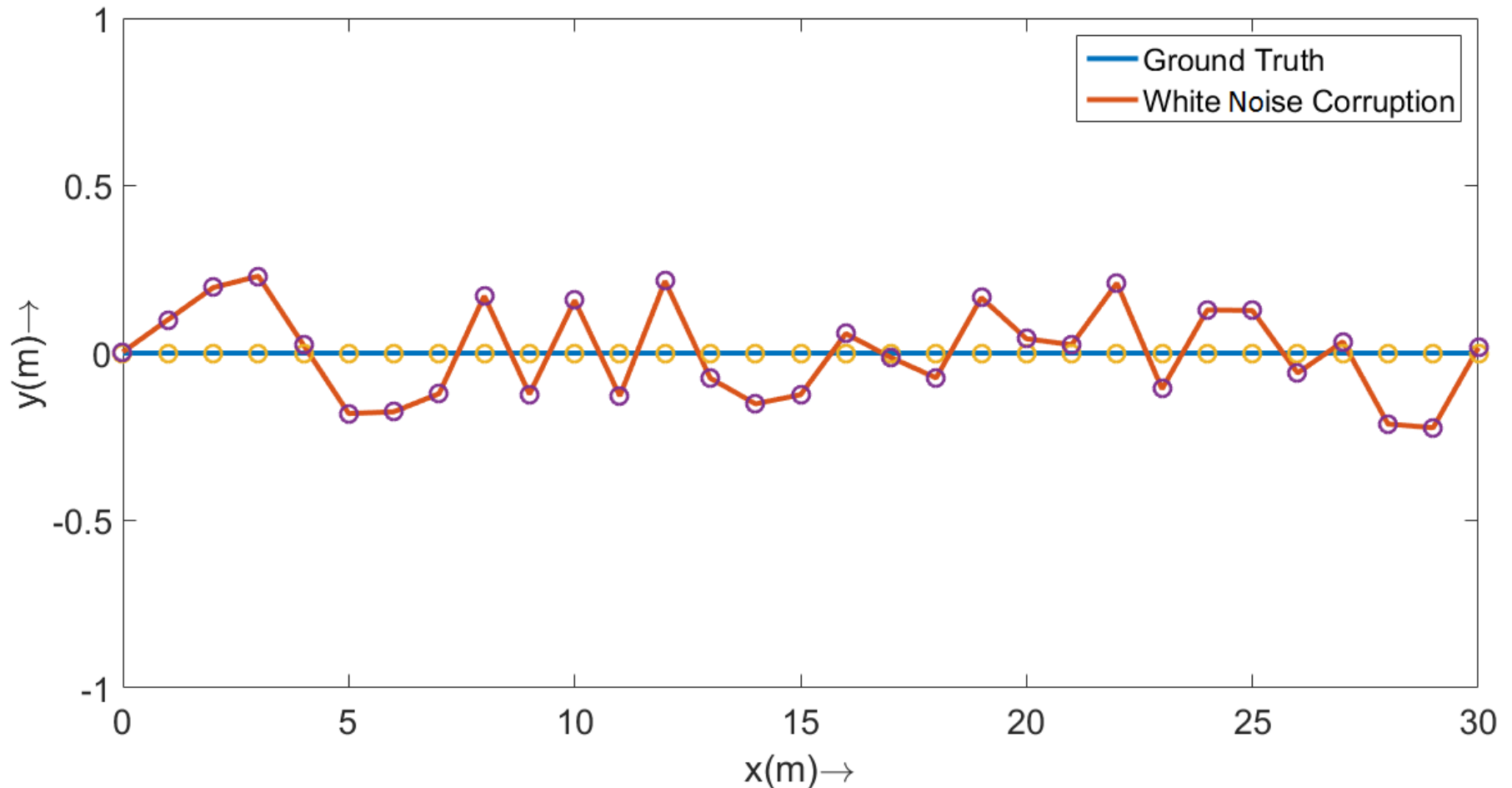
# CKLAM Sparsity

- No cross dependency between  $f_1$  and  $f_5$

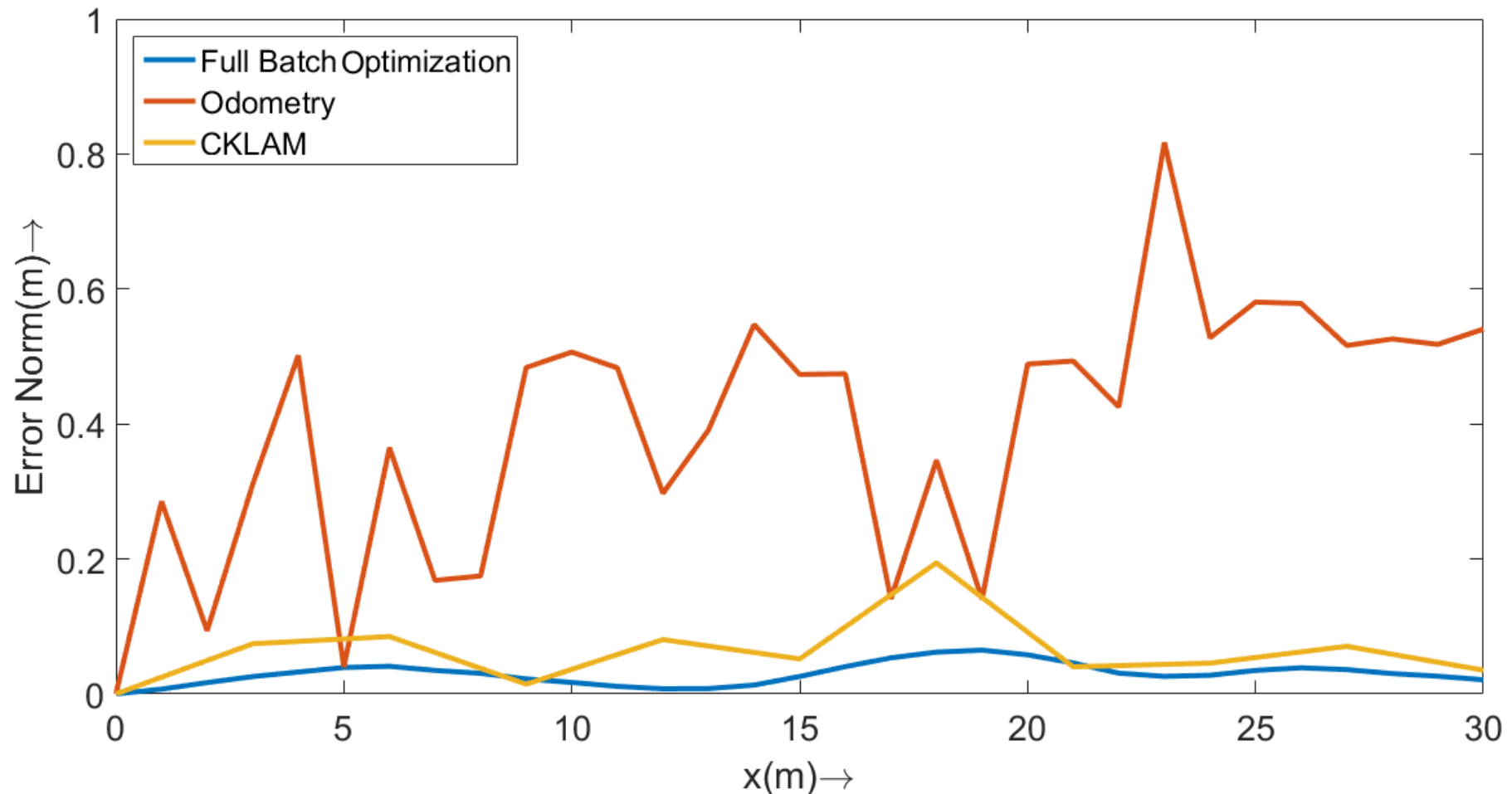
$$H_{C_2} =$$



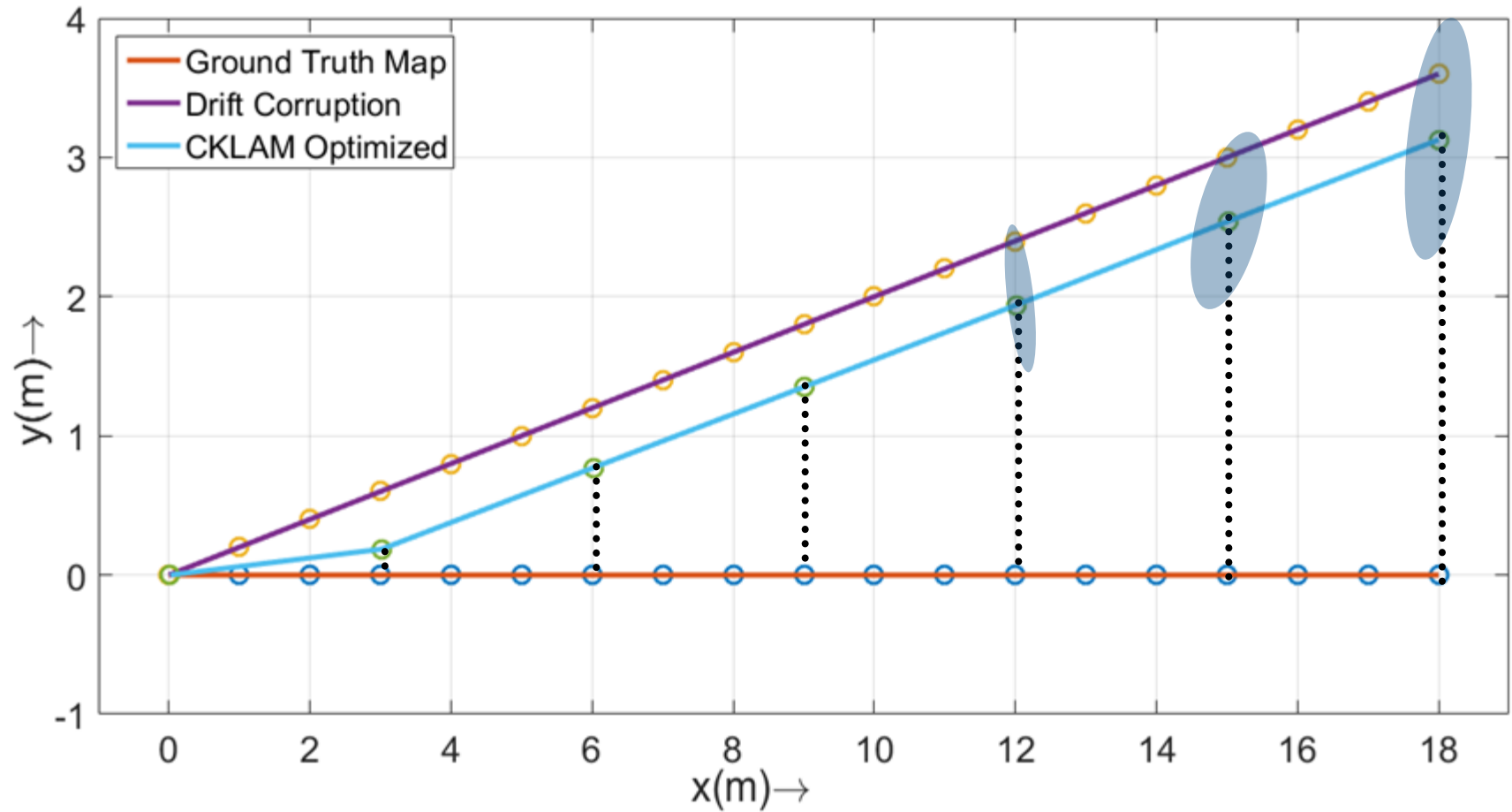
# CKLAM Results (Simulated Map)



# CKLAM vs Bundle Adjustment



# CKLAM Results



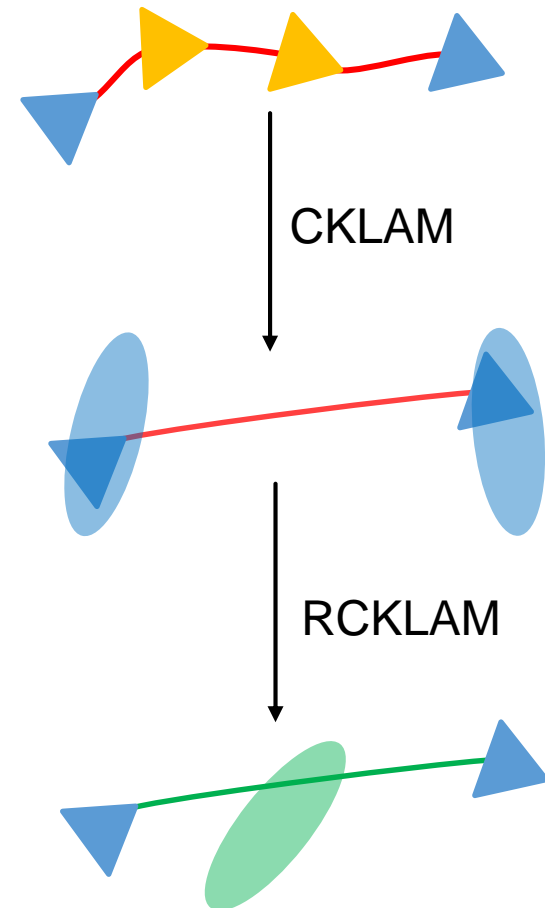
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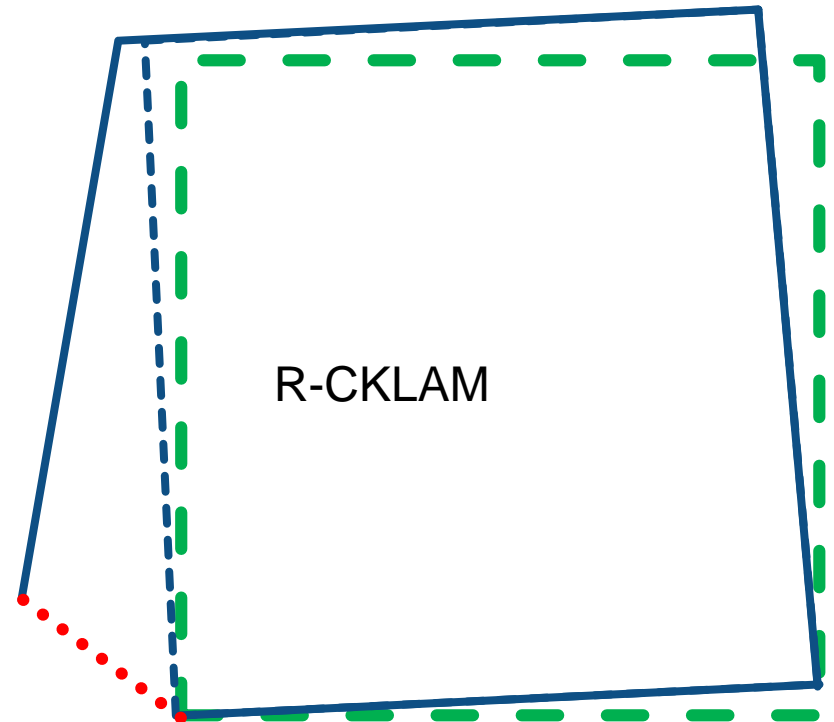
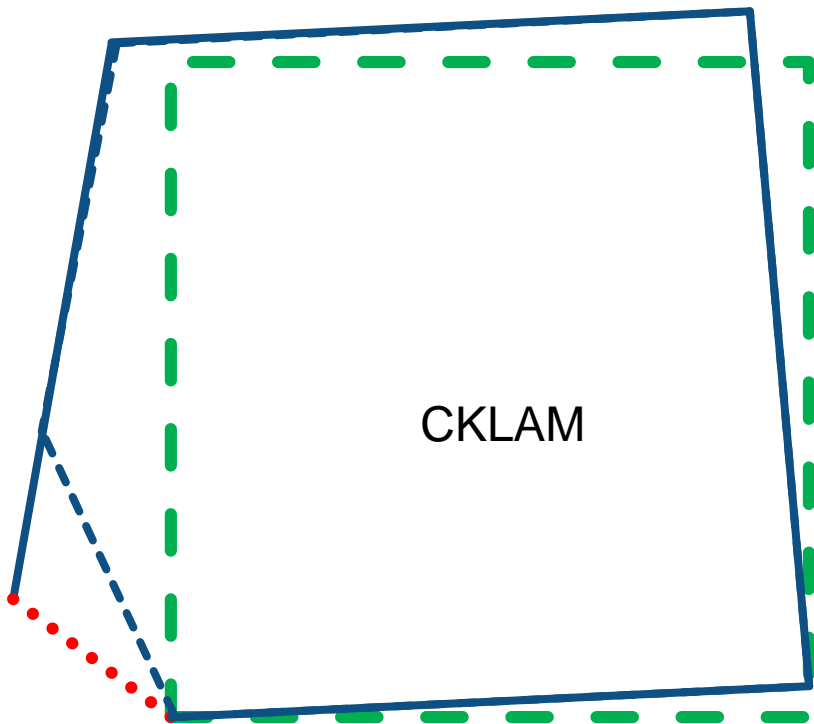
# Relative CKLAM (RCKLAM) Motivation

- Modeling integrated error as absolute Gaussian is unnatural
- No substantial changes of individual vertices
- Hard to move local information to global frame

Model uncertainty of relative pose between consecutive key frames



# R-CKLAM vs CKLAM





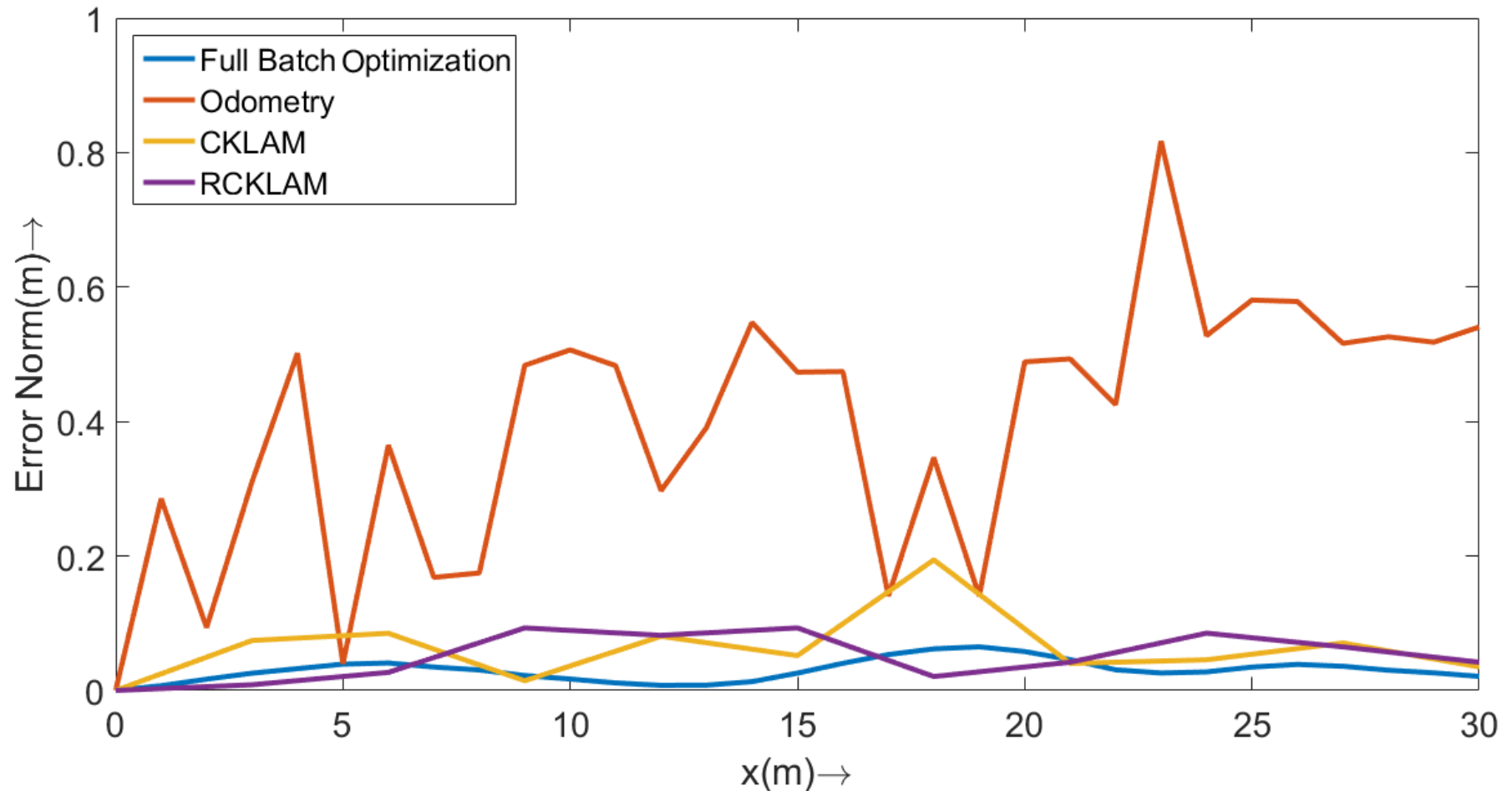
# RCKLAM Formulation

- $C_{CKLAM} = f(T_{GB_1}, T_{GB_2}) \rightarrow C_{RCKLAM} = g(T_{B_1B_2})$ 
  - $T_{B_1B_2} = T_{GB_1}^{-1} T_{GB_2}$

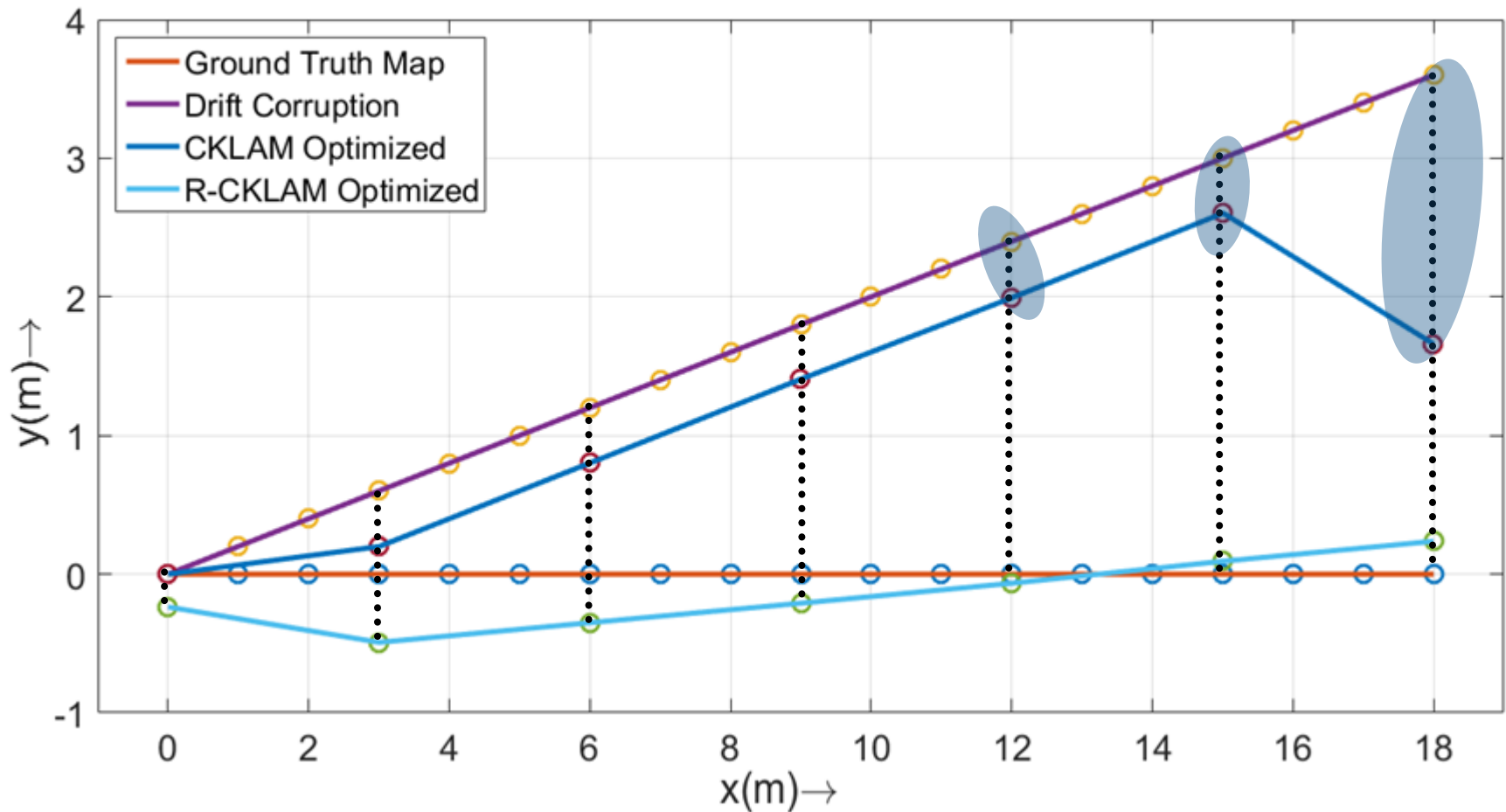
- Linearize  $T_{B_1B_2}$

- $H = \begin{bmatrix} J_1 & J_2 \\ 0 & 1 \end{bmatrix}^T H' \begin{bmatrix} J_1 & J_2 \\ 0 & 1 \end{bmatrix}$

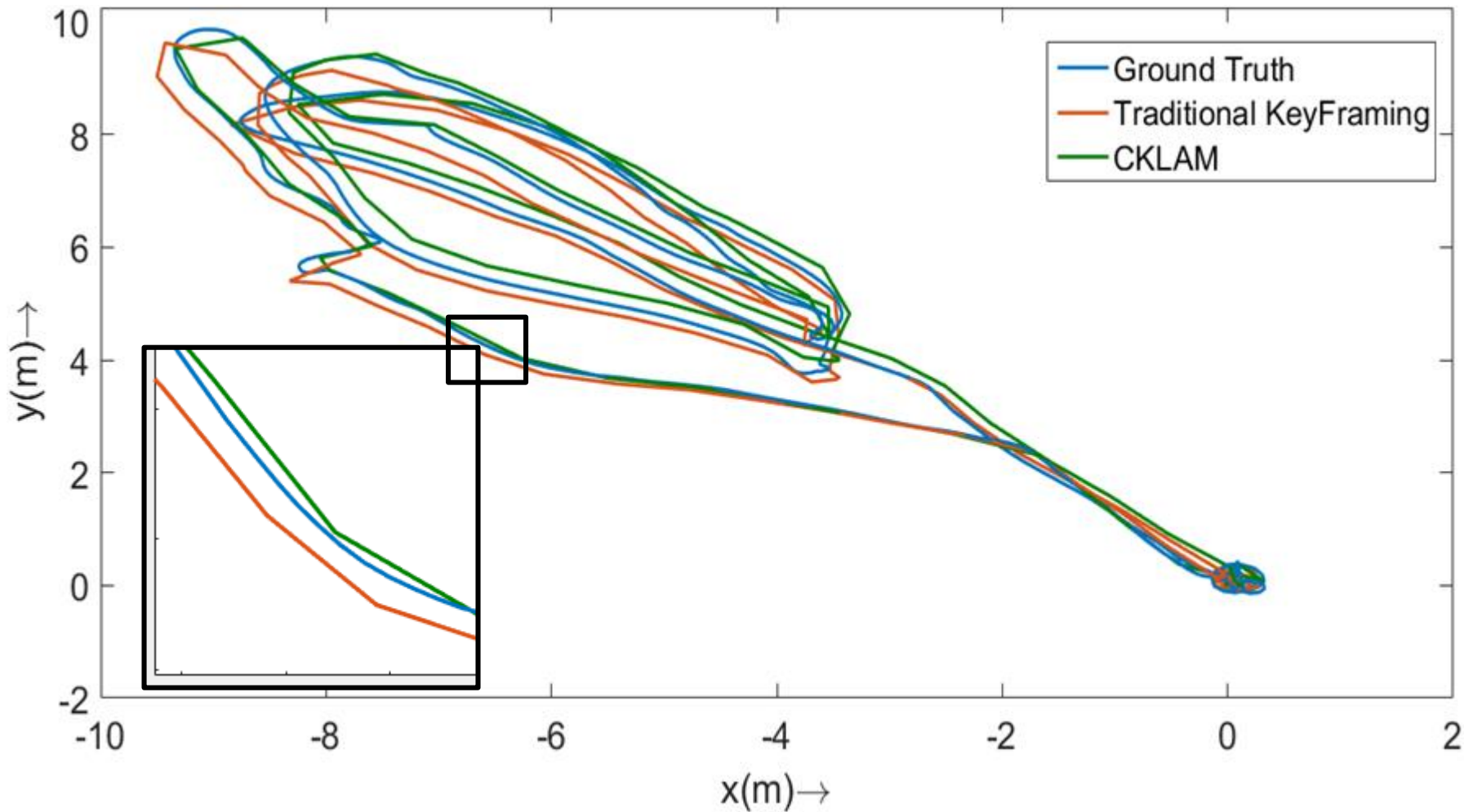
# RCKLAM vs CKLAM (Simulated Map)



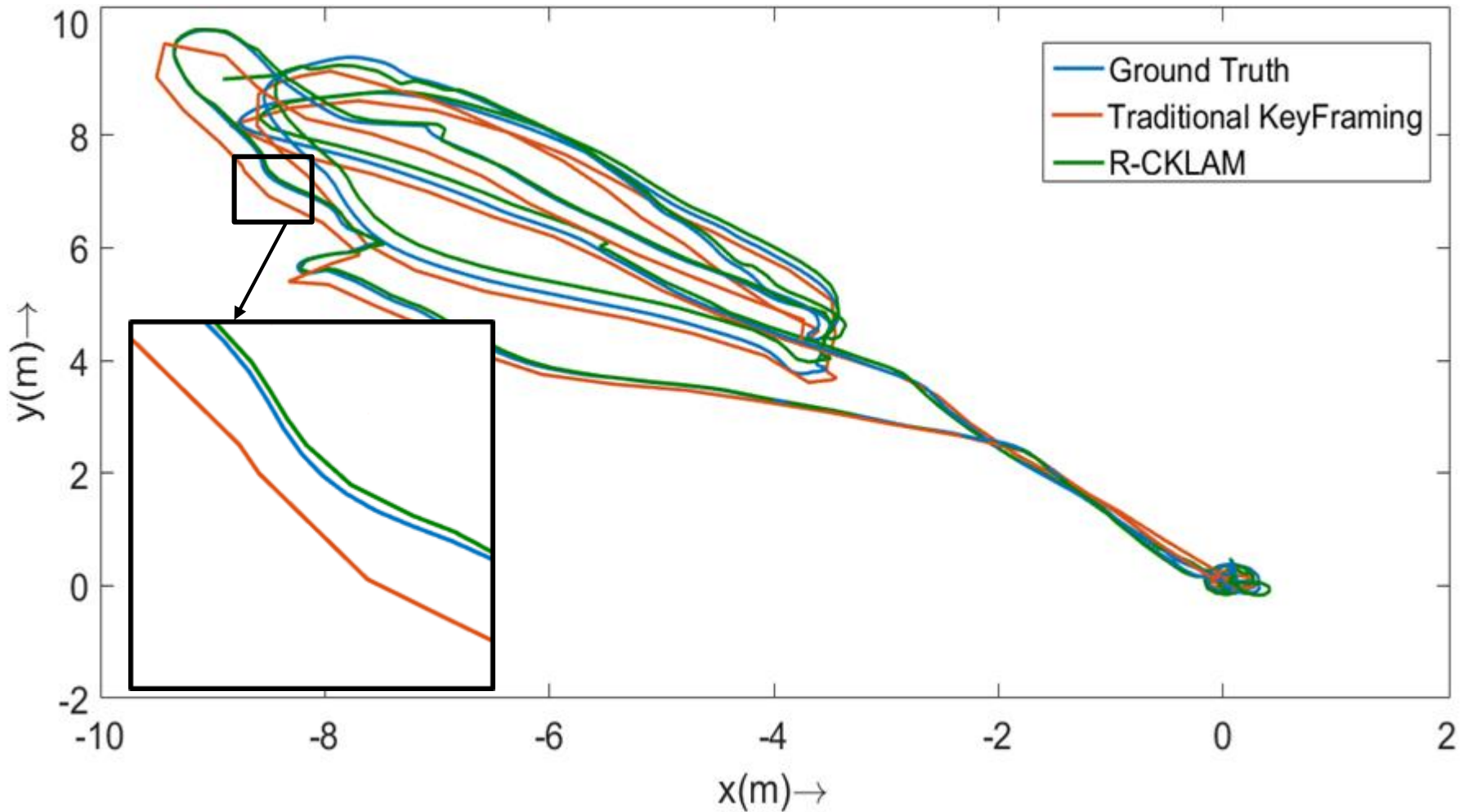
# RCKLAM and Loop-closure



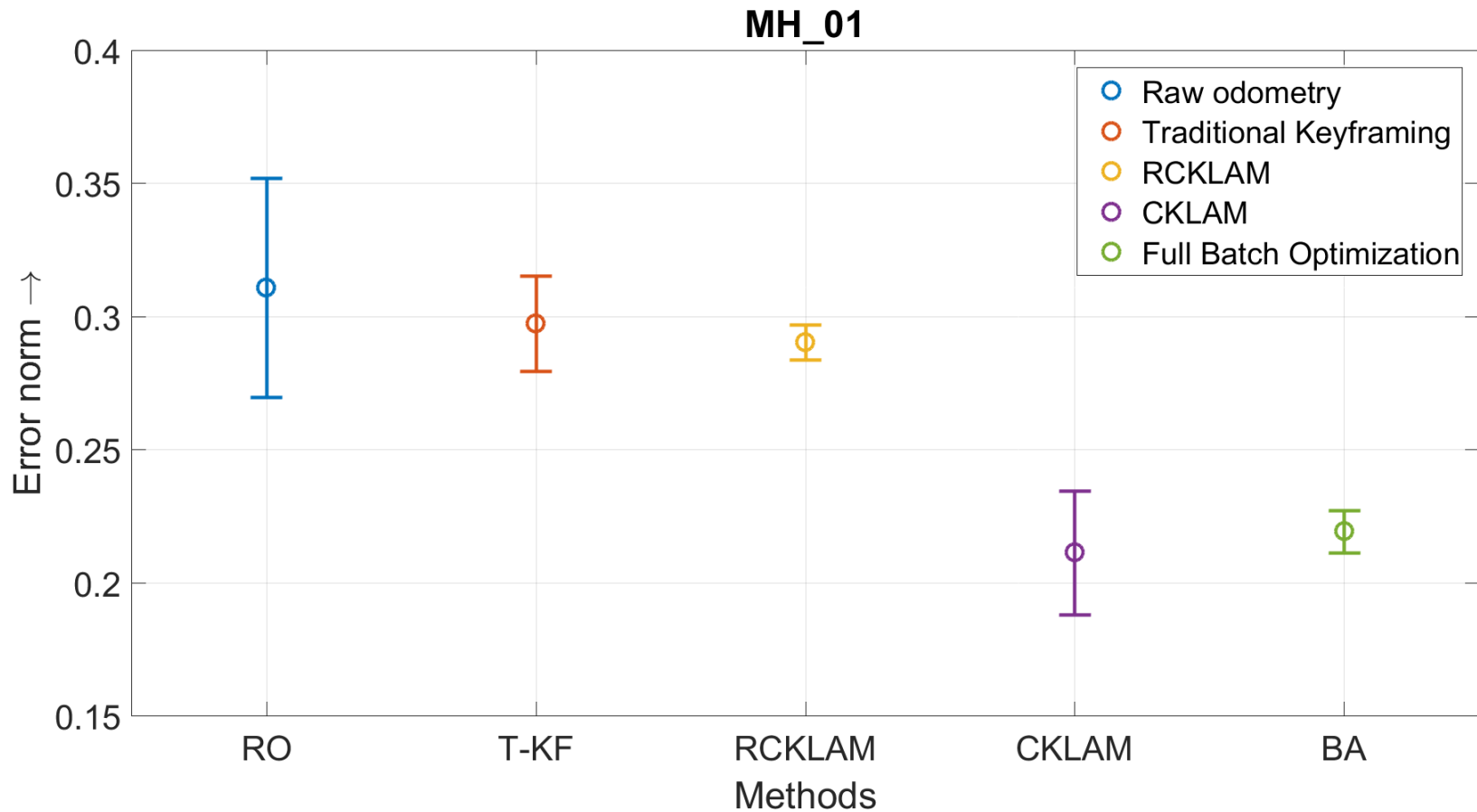
# Evaluation on Euroc Datasets



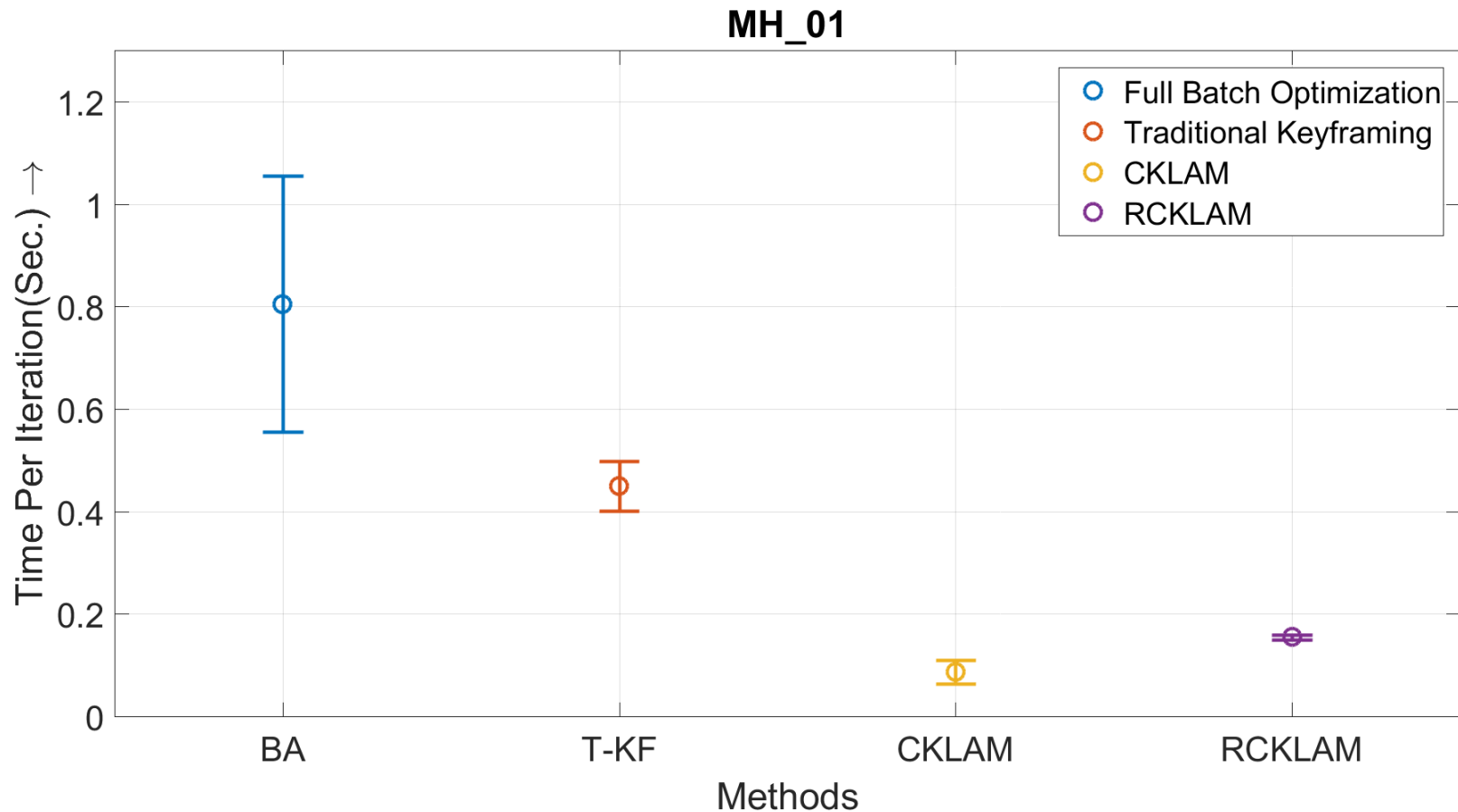
# Evaluation on Euroc Datasets



# Results Real Maps



# Computational time advantage (Only Optimization time)

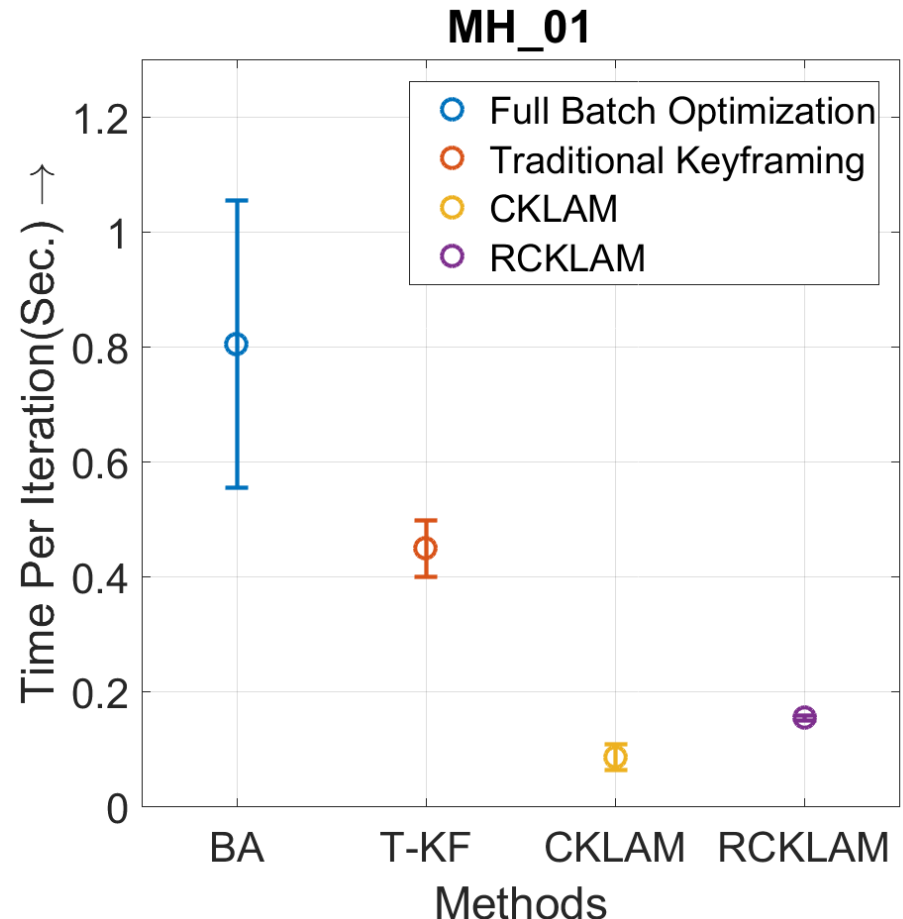


## Conclusion

- A new method **R-CKLAM** has been proposed
- Flexibility significantly improved
- Advantages with loop closure shown

## Future Work

- Building online versions of CKLAM and RCKLAM
- Keyframe selection coupled with CKLAM

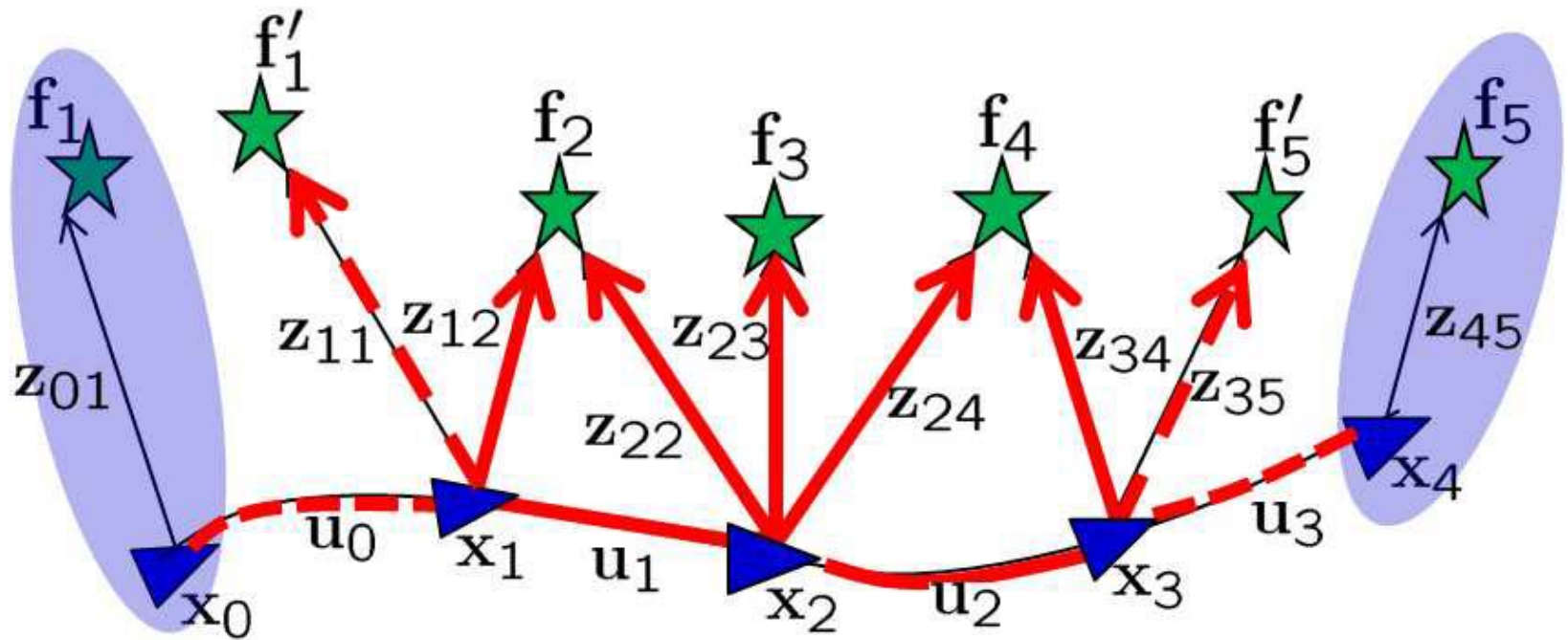




# References

- 1. Esha D. Nerurkar , Kejian J. Wu , and Stergios I. Roumeliotis , “C-KLAM: Constrained Keyframe-Based Localization and Mapping”, In IEEE International Conference on Robotics and Automation Workshop on Long-term Autonomy, 2013.
- 2. G. Klein and D. Murray. Parallel tracking and mapping for small AR workspaces. In Proc. of the IEEE and ACM International Symposium on Mixed and Augmented Reality, pages 225–234, Nara, Japan, Nov. 13–16 2007.
- 3. J. Folkesson and H. Christensen, “Graphical slam - a self-correcting map,” in Proc. of the IEEE International Conference on Robotics and Automation, New Orleans, LA, Apr. 26 – May 1, 2004, pp. 383–390.

# CKLAM Marginalization



[1]

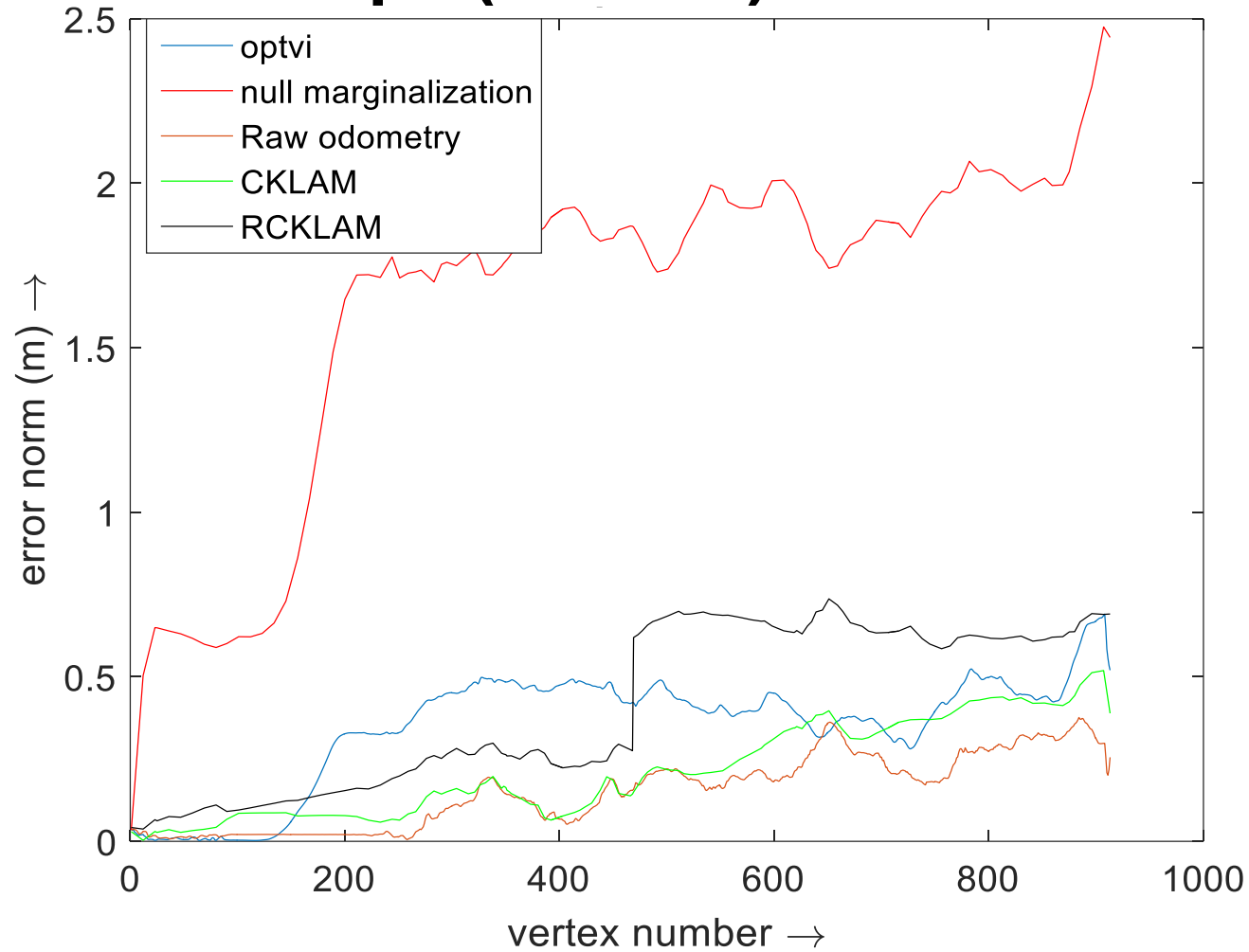
# RCKLAM Formulation

- $C_{CKLAM} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \alpha$
- $C_{RCKLAM} = \begin{bmatrix} f(x_2, x_1) \\ x_2 \end{bmatrix}^T \begin{bmatrix} H'_{11} & H'_{12} \\ H'_{21} & H'_{22} \end{bmatrix} \begin{bmatrix} f(x_2, x_1) \\ x_2 \end{bmatrix} + \begin{bmatrix} g'_1 \\ g'_2 \end{bmatrix}^T \begin{bmatrix} f(x_2, x_1) \\ x_2 \end{bmatrix} + \alpha$
- $\begin{bmatrix} x_2 - x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$  then  $H = \begin{bmatrix} J_1 & J_2 \\ 0 & 1 \end{bmatrix}^T H' \begin{bmatrix} J_1 & J_2 \\ 0 & 1 \end{bmatrix}$

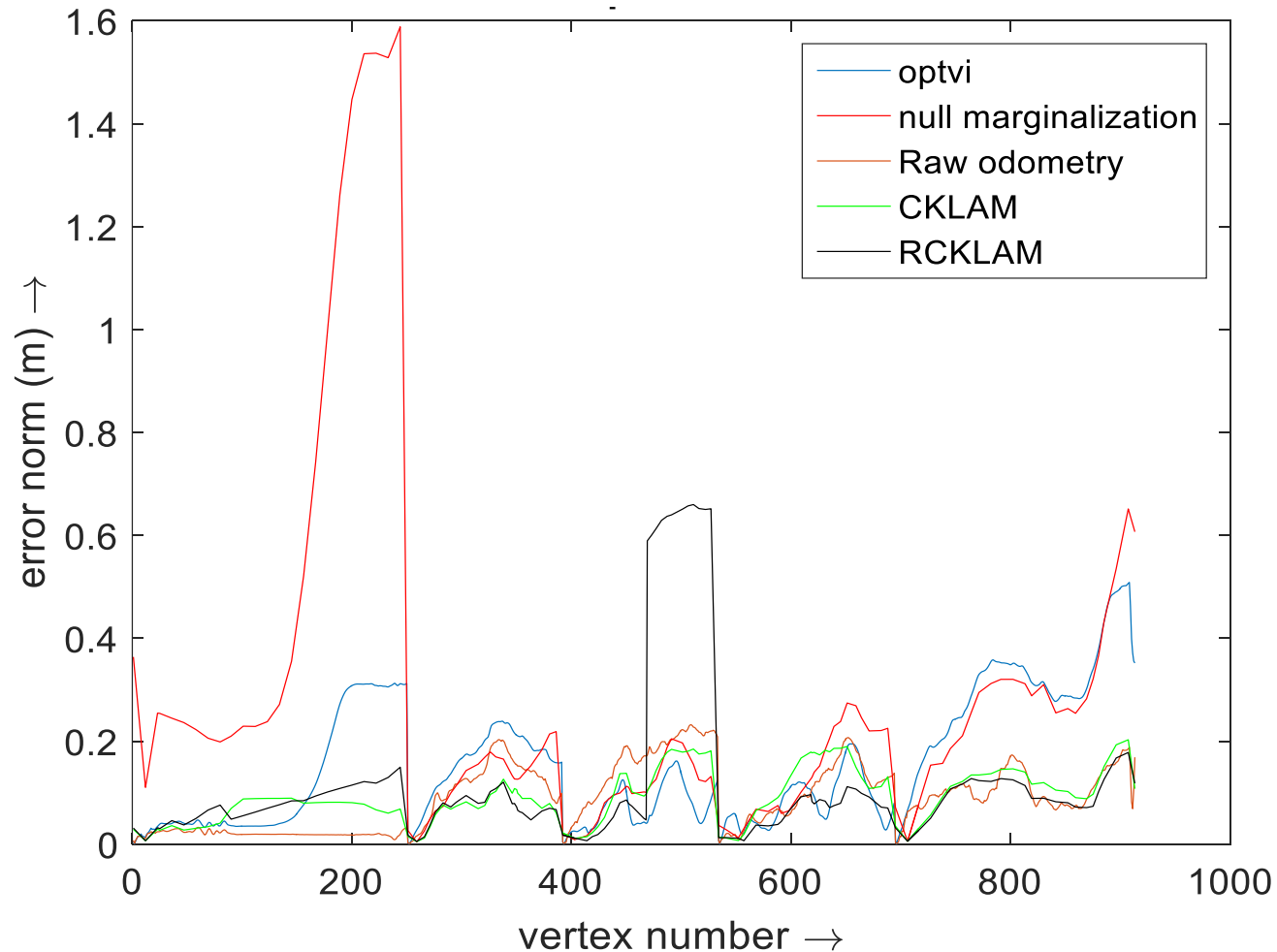
# RCKLAM Formulation

- Reformulate  $C = \frac{1}{2}X^T HX + g^T X + \alpha$  as  $C = (AX + b)^T (AX + b)$
- Linearize  $f = f(x_{10}, x_{20}) + \frac{\partial f}{\partial x_1} (x_1 - x_{10}) + \frac{\partial f}{\partial x_2} (x_2 - x_{20})$
- Need  $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b = A' \begin{bmatrix} f(x_2, x_1) \\ x_2 \end{bmatrix} + b'$  - replace linearization of  $f$  and reorder
- $A' = A \begin{bmatrix} J_1 & J_2 \\ 0 & 1 \end{bmatrix}^{-1}$  and  $b' = b - A' \begin{bmatrix} \alpha_1 \\ 0 \end{bmatrix}$  where  $\alpha_1 = f(x_{10}, x_{20}) - J_1 x_{10} - J_2 x_{20}$  with  $J_1 = \frac{\partial f}{\partial x_1}$  and  $J_2 = \frac{\partial f}{\partial x_2}$

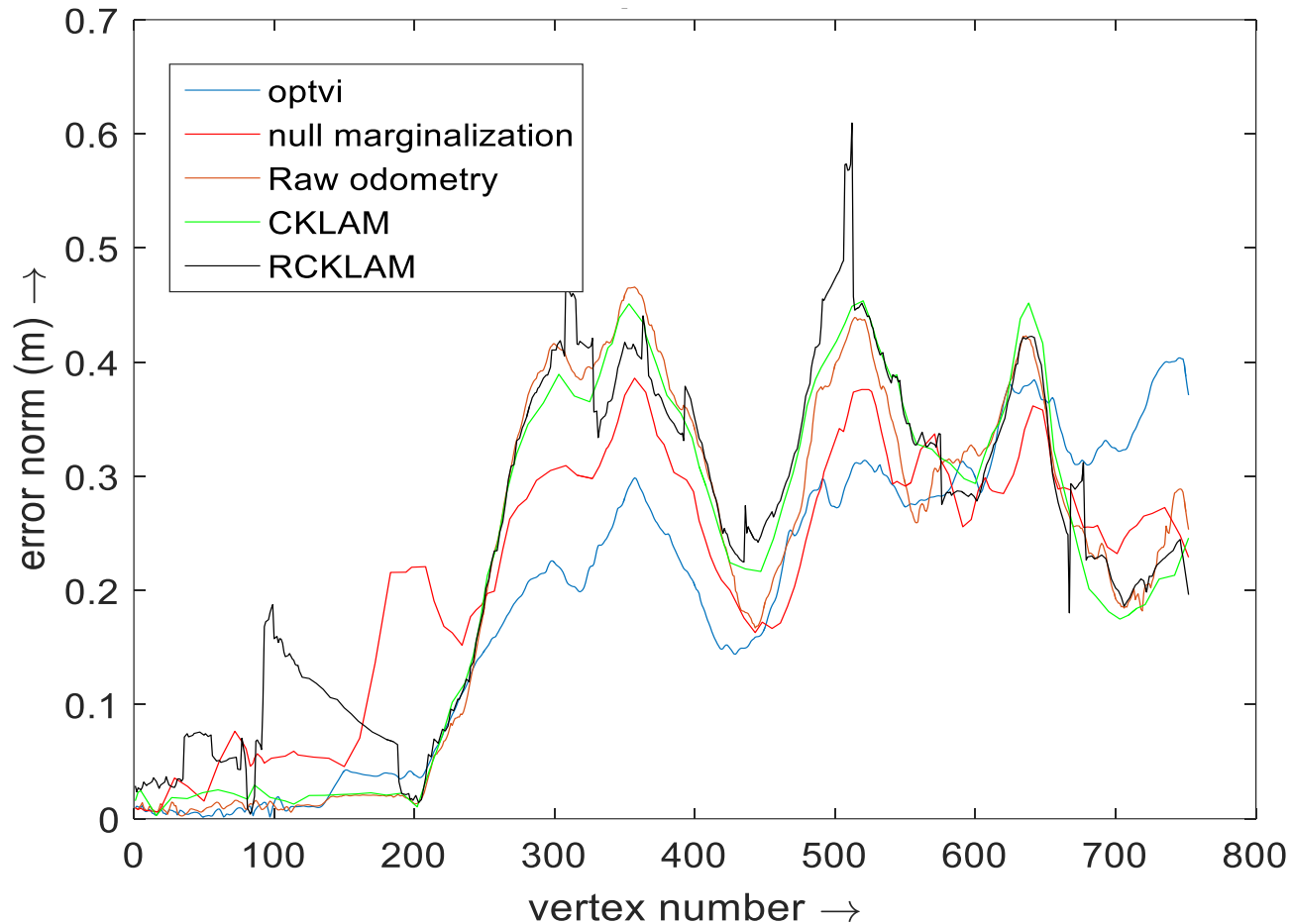
# Results Real Maps (MH - 01)



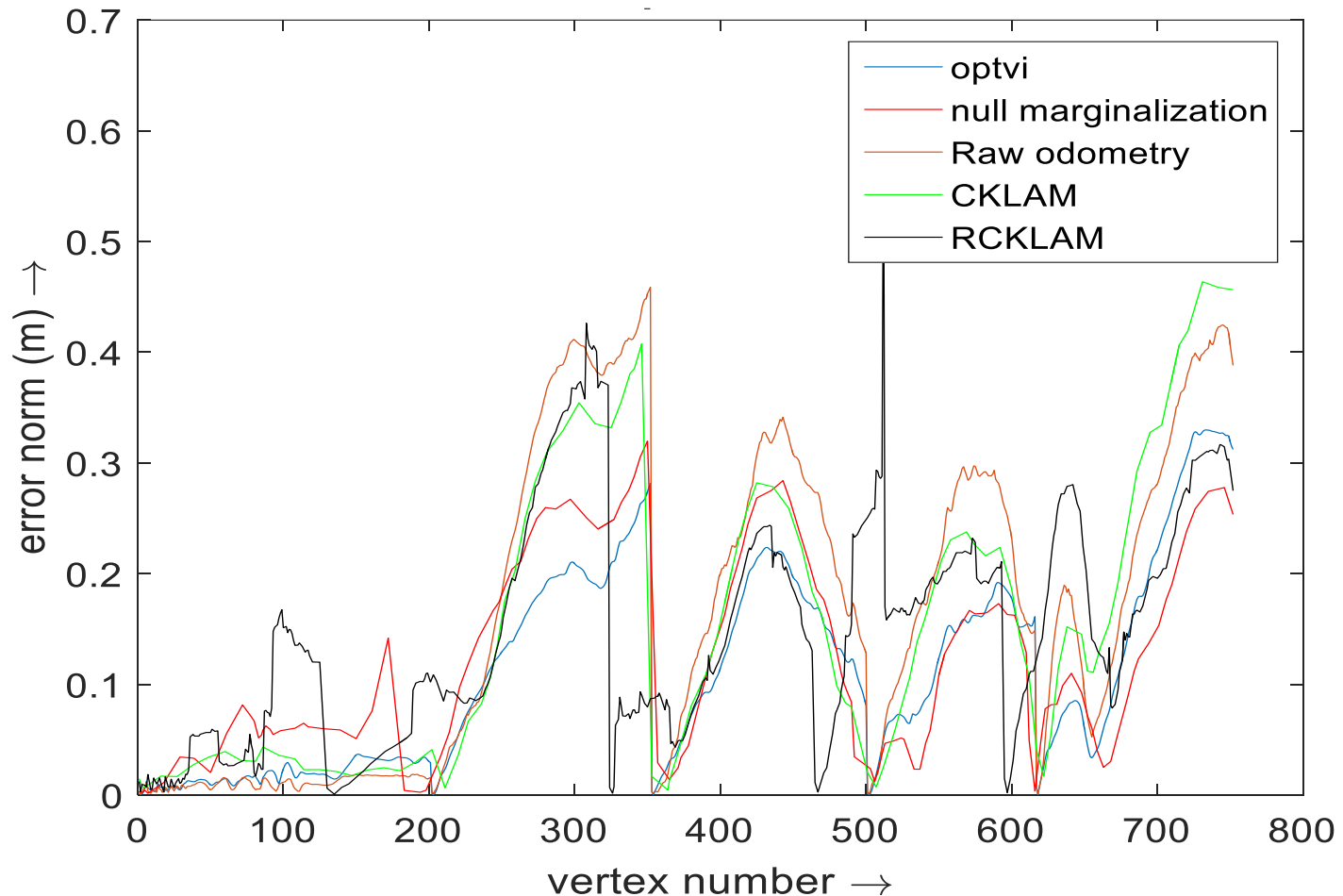
# Results Real Maps (MH - 01)



## Results Real Maps (MH - 02)

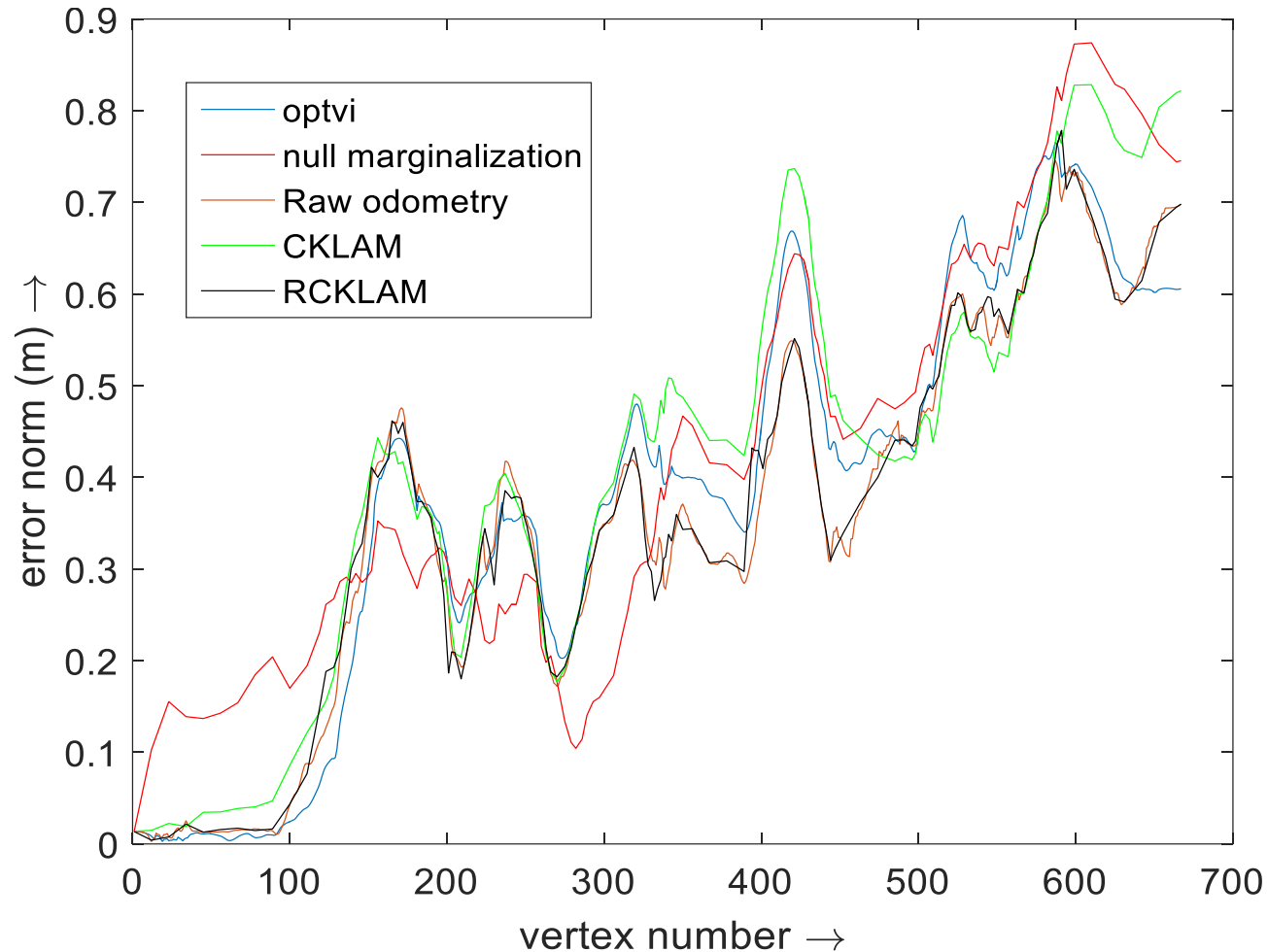


## Results Real Maps (MH - 02)

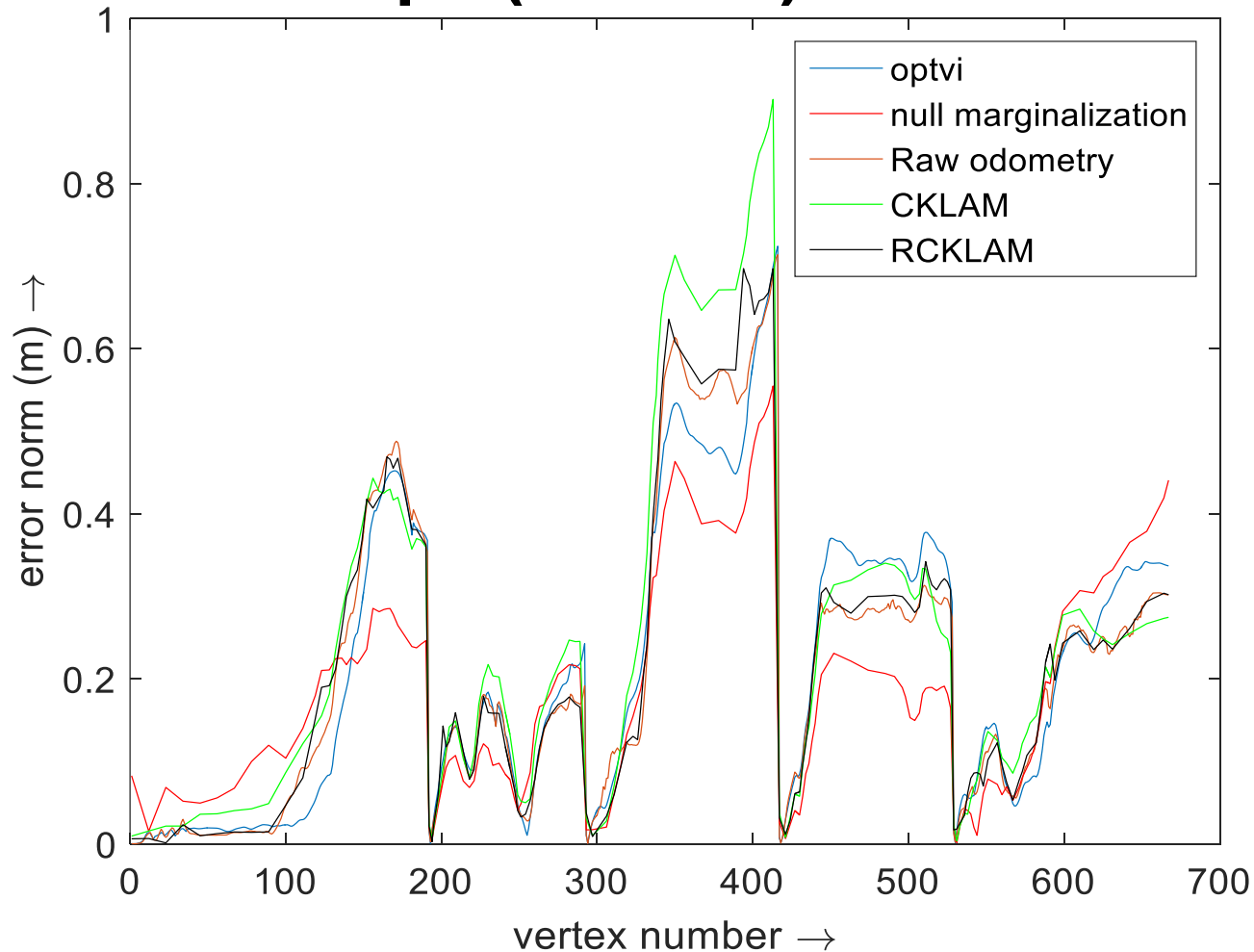




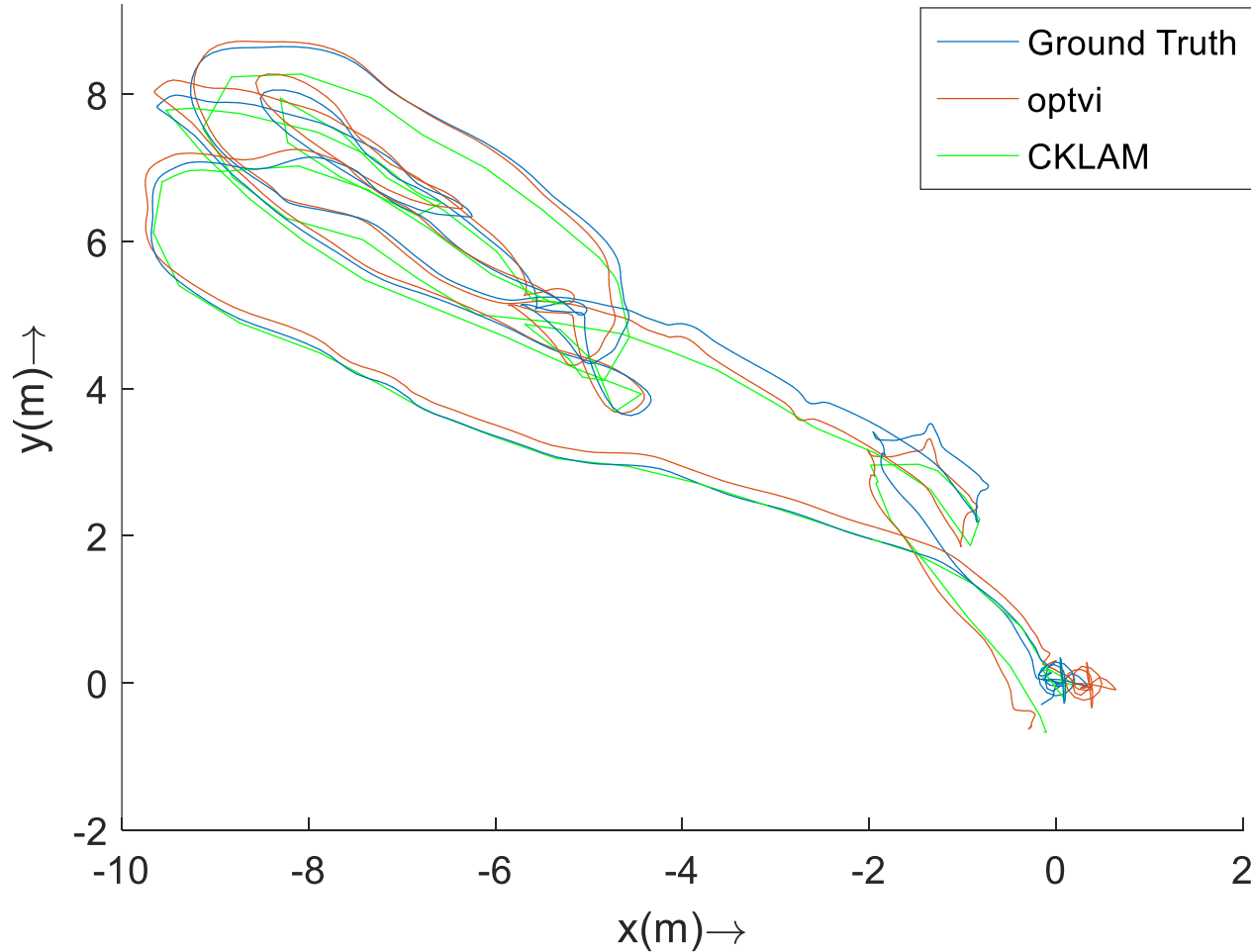
## Results Real Maps (MH - 03)



## Results Real Maps (MH - 03)



# Results Real Maps (MH - 01)



# Results Real Maps (MH - 01)

