4. From Ptolemy's theorem on quadrilaterals, we deduce that if P_1, P_2, P_3, P_4, in cyclic order, are the feet of the perpendiculars from the focus O of a conic, on four arbitrary tangents, then

$$\frac{\overline{P_1 P_3}}{\overline{OP_1} \cdot \overline{OP_3}} \cdot \frac{\overline{P_2 P_4}}{\overline{OP_2} \cdot \overline{OP_4}} = \frac{\overline{P_2 P_3}}{\overline{OP_2} \cdot \overline{OP_3}} \cdot \frac{\overline{P_1 P_4}}{\overline{OP_1} \cdot \overline{OP_4}} + \frac{\overline{P_1 P_2}}{\overline{OP_1} \cdot \overline{OP_2}} \cdot \frac{\overline{P_3 P_4}}{\overline{OP_3} \cdot \overline{OP_4}} .$$

Clearing of fractions we have an equation which proves that P_1, P_2, P_3, P_4 are either collinear or concyclic. Hence, the pedal of a conic with respect to a focus is a line or a circle. (It is a line if the conic is a parabola.)

VI. A NOTE ON THE HINDU-ARABIC NUMERALS

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In Professor Smith's *History of Mathematics* (vol. II, p. 64) and in the work on the *Hindu-Arabic Numerals* written by him and Professor Karpinski, due credit is given to the evidence of the Hindu origin of these forms. In the latter work, however, it appears (p. 41) that the authors entertain the belief that, although the place value was invented in the sixth century A.D., "not until a considerably later period did it become well known."

It is certain, however, that the decimal system of numeration was familiar to the Vedic Hindus in almost the same form as at present. There are references in Indian literature which prove conclusively that the idea of place value was well known in India in the fifth century A.D., if not a century or two earlier. Fortunately, too, the references are not merely epigraphical, nor are they of doubtful value. One such reference is found in Śankarāchārya's commentary on the Brahma-Sūtra, and also in Vyāsa's commentary on the Yoga-Sūtra of Patañjali. Commenting on Chapter II, Pāda 2, Sūtra 17, Śankarāchārya writes:

"As though the line[1] is one and the same, but being placed in different positions, it indicates the ideas denoted by the different words 'one,' 'ten,' 'hundred,' 'thousand,' etc., so · · · ".

The translation is literal, the text is simple and explicit on the subject of place value, and it cannot, considering the context, be interpreted otherwise.

The passage in the commentary on the Yoga-Sūtra is almost identical and has been translated by Professor Wood as follows[2]:

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[1] See footnote 3.

[2] *Vide* the *Yoga-System of Patañjali*, translated by Professor J. H. Wood (*Harvard Oriental Series*) p. 216. In the footnote Professor Wood has noted the significance of this passage as indicative of the knowledge of place value amongst the ancient Hindus. *Vide* also Sarkar, *Hindu Achievement in Exact Science* (London).

"Thus the same stroke[1] is termed one in the unit-place and ten in the ten's place and a hundred in the hundred's place."

This reference affords, therefore, the most conclusive evidence that both Vyāsa and Śankarāchārya were acquainted with the significance of the place value.

Now, Śankarāchārya was not a mathematician; he was a triumphant Hindu religious reformer. He travelled on foot throughout the greater part of India, giving a new interpretation to the Ancient Śāstras, especially the Brahma-Sūtra, challenging the savants to philosophical controversies, defeating them, and bringing them into his fold. He wrote his commentary on the Brahma-Sūtra to help his preaching, so it may be safely assumed without any fear of controversion that the idea of place value was generally known to the élite of his time. If this were not the case, and if the subject did not form a part of the elementary training of his boyhood, how could a philosopher like Śankarāchārya, who took up the life of a secluded ascetic at the early age of eight, not becoming a religious reformer and preacher until a later period, know of one of the greatest of human inventions in a branch of knowledge so different from his own? It is enjoined in Kautilya's Arthaśāstra (book I, chapter V), a book written in the beginning of the fourth century B.C., that "having undergone the ceremony of tonsure" (that is, at about the age of five), "the student shall learn the alphabet (lipi) and arithmetic" (sankhyanam, literally the science of numerals). The usual rule in such cases is to assume that the idea in question must have originated about a century or two previous to the epoch of the document in which it appears.

The question then arises as to the time in which Śankarāchārya lived, a matter upon which scholars differ widely. Orthodox Hindu writers, following the chronology preserved in the archives of the Muṭhas (monasteries) founded by Śankarāchārya, believed that he flourished in the beginning of the first century B.C. Amongst other oriental scholars, however, there are some (like Telang) who are of opinion that he lived in the latter part of the sixth century A.D., while others (like Pathak, followed by Max Müller, and Wilson) put him in the eighth century and still others place him in the seventh century. It is also noteworthy that amidst the latter group Tilak and Ghose have come to nearly concurrent dates by proceeding on different lines. The date of Vyāsa's commentary on the Yoga-Sūtra of Patañjali cannot be later than the sixth century A.D. So in any case it seems safe to assume that the place value was well known in India in the fifth century A.D., and probably earlier.

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[1] The original sanskrit text in either case is 'rekhā' which may mean 'line,' as also 'numeral'. I have translated it as 'line' on the idea that the first numeral was denoted by a simple line, a stroke, about the same age as is found in ancient inscriptions. Prof. Wood translates it by 'stroke.' It may be further noted that the word 'rekhā' comes from the same root as the word 'lekhā' which means 'writing.'