ON THE SOCIOLOGY OF MATHEMATICS

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DEFINING THE PROBLEM

THE sociology of mathematics concerns itself with the influence of forms of social organization on the origin and growth of mathematical conceptions and methods, and the rôle of mathematics as part of the social and economic structure of a period.

The primitive forms of society, the oriental, the Graeco-Roman, the medieval feudal, the early capitalistic, the modern capitalistic and the socialist forms of society have all influenced in their various ways the acquisition of mathematical knowledge and have been in their turn subjected to its influence. There are many subsidiary problems, centering about groups in each society, all suggesting questions to be resolved. What has been the rôle of the merchant, the engineer, the surveyor, the astronomer, the navigator, the administrator, the artist, the philosopher and the professional mathematician? Why, for instance, does mathematical research flourish in one form of society, remain stagnant in another, and decay or fail to appear in a third?

Very little investigation of these problems has, as yet, been undertaken. Few persons will deny that sociological factors have influenced the development of exact sciences. Herodotus' account of the origin of geometry in the necessity of redividing the land in Egypt after the yearly inundations is a classical affirmation of this thesis. It is customary to point out that Greek rationalistic reasoning took its origin, understandably enough, in the Ionian merchant towns of Asia Minor. Leonardo Pisano's interest in the cultivation of arithmetic and algebra is always related to his professional interests as a travelled merchant. Nobody denies, in modern days, the strong connection between the expansion of electrical and aircraft industry and applied mathematics. Old and modern textbooks of trigonometry show in examples and pictures the decisive influence of astronomy, navigation and surveying on the development of the subject.

This type of correlation requires more precise definition before it is subjected to any specific analysis. Here is an example of the way in which the origin of Ionian mathematics is usually presented:

> About the seventh century B.C. an active commercial intercourse sprang up between Greece and Egypt. Naturally there arose an interchange of ideas as well as of merchandise. Greeks, thirsting for knowledge, sought the Egyptian priests for instruction. Egyptian ideas were thus transplanted across the sea and there stimulated Greek thought, directed it into new lines, and gave it a basis to work upon.[1]

This may be called a sociological approach—of a kind. But closer observation shows that it is formulated in vague generalities indicating no real sociological understanding. There had been commercial intercourse between many nations and Egypt. Why should the Greeks have formed an exception in achieving remarkable results? Travelling merchants had flourished in Egypt at least as early as 5000 B.C., and commercial intercourse can be dated back into paleolithic times. The only explanation offered is the avid "thirst for knowledge" felt by the Greeks. Whence this thirst? Not all merchants experience it. The Babylonians, upon their arrival in Sumerian lands, either as merchants or as conquerors, were sufficiently eager for knowledge to absorb the whole Sumerian science, yet no science of the Greek type resulted. They continued and improved the ancient methods. The difference in the case of the Greeks must be explained by more penetrating methods than a reference to mercantile activity and a thirst for knowledge. The sociologist can contribute materially by showing in which way Greek society differed radically from all previous and all later forms of social structure. Something entirely new must have occurred, and an attempt must therefore be made to define it, first in social terms, and then in terms of scientific thought.

The rise of mathematics in ancient Ionia presents, then, the following problem. How is it possible that a group of Greeks, mainly of mercantile class, not only became interested in the science of the ancient Orient, but approached it from an entirely original point of view? And furthermore, how is it possible that only some

[1] F. Cajori, *History of Mathematics* (2nd ed., New York, 1930), p. 15.

of the problems raised by these Ionians received full, if onesided, attention, and others remained dormant until modern times?

The merchant has had an influence on mathematics, but only on certain parts of it. He is interested in computation and needs a sound elementary arithmetic. If he is a travelling man, his interests go out toward some geometry sufficient for geography and the knowledge of the motion of sun and stars. His wit is sharpened by competition and the possibility of individual profit, although this may not always lead to the search for profounder truth in mathematics. Sometimes he may help science by playing the Maecenas. To establish the existence of mercantile relations is not of itself sufficient to prove the assured growth of mathematics beyond a certain elementary level.[2]

The institution of slavery poses a similar problem in causation. There is a widely accepted theory that the nature of Graeco-Roman society, and with it that of its peculiar type of science, was determined chiefly by its slave character. The decay of the system of slavery is thus made a main factor in the decline of science. There is a reasonable foundation for this theory, but it must be thoroughly qualified. Ancient oriental society also knew slavery, but its science had an entirely different character from that of Greece and Rome. Its slavery showed little decline and the political and cultural expansion, contraction and stagnation of the oriental forms of society did not necessarily correspond with developments in slavery or in science. A primary distinction is to be made between Graeco-Roman slavery for production and oriental slavery for luxury or for public works.[3] A second distinction is that slavery in Greece was to be found mainly in industry, and Roman slavery not only in industry, but in agriculture as well. We submit that further analysis of these forms of slavery and their development can yield a new under-

[2] This is borne out by books which point out the influence of the merchant on mathematics, as D. E. Smith, *History of Mathematics*, 2 vols. (Boston, 1923), where even on an elementary level the stimulating influence of other factors is taken into account, notably that of astronomy. See for the influence of the merchant. D. E. Smith, "Mathematical Problems in Relation to the History of Economics and Commerce," *American Mathematical Monthly*, XXIV (1917), p. 221-223, or W. Sombart, *Der Bourgeois* (München-Leipzig, 1920), p. 164-169.

[3] E. Meyer, "Sklaverei im Altertum" (1898), in *Kleine Schriften* (1910), p. 189-190; also K. Wittfogel, *Wirtschaft und Gesellschaft Chinas*, I (Leipzig, 1931), p. 393; "Die Theorie der orientalischen Gesellschaft," *Zeitschr. für Sozialforschung*, VII (1930), p. 96.

standing of the scientific advances and limitations of each period, and of the relative place in the history of science of men like Ahmes, the scribe of the Egyptian Papyrus Rhind; Euclid, the Alexandrinian of the early period; Plotinus, the Neoplatonist; or Alkhwarizmi, the Mohammedan algebrist. We must lean here on work already done, mostly in related fields, by men like Farrington and Winspear for Graeco-Roman antiquity, and Wittfogel for the ancient orient.[4]

We should not, on the other hand, stretch our sociological analysis too far, and forget a very obvious thing: namely, that a scientific process often follows the courses suggested by its internal logical structure, the frame built by results already obtained. The student writes his thesis because the professor wants him to clear up an obscure point in his own work. We are likely to find such a situation often within a relatively stable social and cultural frame. An example is furnished by ancient Egyptian, Babylonian and Chinese mathematics, once its pattern was well established. Here we find, sometimes for many centuries, actual stagnation, and when progress is made it is obtained by discoveries directly dependent on the previously existing knowledge and technique. Another example is the development of algebra in sixteenth century Italy. After Tartaglia and Cardan, working as direct pupils of certain Bolognese professors, had solved the third degree equation before an admiring public, Cardan's pupil, Ferrari, showed in 1545, how to solve the biquadratic equation. This is an example of the discovery of an objective relation, the mathematical kinship of two kinds of equations, inside the framework of a particular social structure. The continuity of this structure allowed the further discovery of more related mathematical relations. In Cardan's work a real number had appeared as the sum of two complex ("sophistical") numbers. This led the Bolognese professor Bombelli to the publication, in 1572, of a systematic investigation of the algebra of imaginaries. The history of mathematics, as described in our usual textbooks, concentrates on just such discoveries, tracing the discovery of new results almost exclusively to internal logical development with relative neglect of sociological factors. Yet, even in cases where the purely

[4] B. Farrington, "Vesalius on the Ruin of Ancient Medicine," *The Modern Quarterly* (1938), p. 23-28; A Winspear, *The Genesis of Plato's Thought* (New York, 1940), p. 348; K. Wittfogel, *l.c.*

mathematical factor is dominant, sociological factors do exist. They are apparent in the purely social fact that talented men of the time go in for mathematics, are encouraged to teach, to publish and to foster disciples. In other periods, there may be as many men of talent, but their genius is led into other channels, some of which we now consider as sidetracks.[5]

CONDITIONS FOR THE OPERATION OF SOCIOLOGICAL FACTORS

In periods of profound social upheaval, of transition, or in periods in which peoples learn from other peoples of a widely varied social and cultural background, the sociological factor may give the most important clue to the understanding of the changes in mathematical knowledge. It is a matter of record that the fundamental change in Chinese mathematics from the oriental stage into western technique and scholarship was due to the Jesuit missionaries who entered China in the sixteenth century on the wave of European capitalist expansion. Let us pass to a case where the sociological factors have not so well been established. The so-called Hindu-Arabic number system, a decimal position system with ten tokens, including a token for zero, was introduced into Mesopotamia, probably from India, in the time of the Sassanian kings, perhaps in the seventh century.[6] In the thirteenth century and later it penetrated into Europe and thus it became our present number system. The old explanation for its success was rather simple. It assumed that the intrinsic superiority of this system eventually impressed itself by its own weight upon practical minds to such an extent that all other number systems were discarded. It is true that the Hindu-Arabic system is far superior to the Roman system or to the ancient abacus. Less convincing is the argument that it was superior to

[5] "There were perhaps as many men of genius in the Middle Ages as now; at least, my survey gives that impression, which would be confirmed, I am sure, by statistical inquiry. If these medieval scientists did not make a good showing, we may ask what men like Gauss, Faraday or Claude Bernard would have done if they had been born in the eighth or ninth century." G. Sarton, *Introduction to the History of Science*, I (Baltimore, 1927), p. 20.

[6] The data are still incomplete. This number system, according to some investigators, may well have been known to Alexandrian Greeks. See D. E. Smith and L. Karpinski, *Hindu-Arabic Numerals* (Boston, London, 1911), and F. Cajori, *l.c.*, p. 88-90. It became widely used after the eighth century under the Arabs.

the system used by the Greeks. This system, with twenty-seven tokens, taken from the alphabet, had no position method and no token for zero, and seems at first to us rather cumbersome. It loses much of its clumsiness, however, for anyone who has ever tried it. After a little practice, it is seen that this system is as easy for the purpose of elementary arithmetic, inclusive of fractions, as the Hindu-Arabic system.[7] For a merchant or a surveyor of the Near Orient it made little difference whether he used the Hindu or the Greek system.[8] We conclude that there must have been extra-mathematical reasons for the victory of one system over the other. There are several indications that such reasons may actually be found in the history of Sassanian and Abbasid times, when there was widespread prejudice among the orientals against everything, a hatred based on a difference in economic position. Our first extant reference to the ten Hindu numerals outside of India is found in the writings of Bishop Severus Sebokht (662 A.D.), who mentions these numerals for the express purpose of showing that the Greeks did not have a monopoly of culture.[9] There existed under the early Abbasid khalifs in Bagdad a mathematical school which seems to have refused intentionally to accept lessons from the Greeks and turned for inspiration to the ancient Jewish and Babylonian sources.[10] To this school belonged Alkhwarizmi, the father of our algebra. This hatred of Greek influence and domination was one of the chief causes for the easy victories gained by the rising Islam in Syria.[11]

We find it again easy to understand that the Hindu-Arabic system gained a victory over the abacus and the Roman numerals among the rising European merchant class of the thirteenth and

[7] This has been observed, *e.g.* by J. G. Smyly, "The Employment of the Alphabet in Greek Logistics," *Recueil offert à J. Nicole* (Genève, 1905), p. 515-530, who defends his thesis against the older point of view of Hankel, Gow and Cantor. See also T. L. Heath, *A Manual of Greek Mathematics* (Oxford, 1931), p. 28 and 31, who calls the Greek system for elementary operations "little less convenient than ours."

[8] In order to appreciate the value of the Greek system, we must keep in mind that the arithmetical meaning of the Greek letters was as familiar to the Greeks as the meaning of 0, 1, 2, etc. is to us.

[9] The reference in D. E. Smith, *History of Mathematics,* 1, p. 166-167.

[10] See S. Gandz, "The Mishnat ha Middot, the First Hebrew Geometry of about 150 C.E. and the Geometry of Muhammed ibn Musa Al-Khowarizmi," *Quellen und Studien zur Geschichte der Mathematik A* 2 (1936).

[11] P. K. Hitti, *History of the Arabs* (London, 1937), p. 153.

sixteenth centuries. It was indeed technically superior. It is, however, often forgotten that the ancient Greek system was still in existence, used in influential Constantinople and certainly known to the Venetians, who were masters of Constantinople from 1203-1261. Why then did the Italian merchants select the Arabic system? The answer to this question does not seem to have been given, but some facts may be suggestive. "Arabic civilization was tangible enough in Sicily and Spain, not only in the Moorish part of it, but in places like Toledo, which were again in Christian hands; the Arabic speaking people encircled the Mediterranean Sea and dominated the Asiatic trade."[12] The Arabic system seems to have appeared in Italy first in Florence and Pisa, which had closer connections with the Arabic world than with the Greek and which were in endless commercial rivalry with Venice.[13]

In present-day society technology influences the development of mathematics either directly, by placing technical problems capable of mathematical treatment before the expert, or indirectly through physics, chemistry and other natural sciences. An attempt has been made to discriminate between "applied" and "pure" mathematics, the "applied" part being directly related to technology, and thus showing more clearly its social background. This distinction, which only dates back to the nineteenth century, becomes every year less popular with the gradual discovery of the elementary dialectical law that all applied mathematics may as well be pure, and all pure mathematics may be applied. Even the "purest" type of mathematics is not without its sociological stain. Modern aircraft design, communication engineering, the new work in plasticity, and many other fields of engineering require extremely refined mathematical tools which are only partly discussed in our purely theoretical texts. Fields such as harmonic analysis, tensor calculus, real and complex integration, group theory, mostly developed without any special relation to technological applicability turn out to be abstractions of some very concrete relationships in nature and useful guides for control of industry. I believe that modern mathematicians generally discover the social background of their science rather through these

12 G. Sarton, *Introduction to the History of Science,* II (Baltimore, 1931), p. 6.

13 There were also other numerical tokens in use, of yet dubious origins. See G. Falco, "Un indovinello paleografico," *Bolletino Storico-bibliografico Subalpino,* XXXVII (1935).

conscious relations to technology than through a study of history.[14]

It is, however, important to notice that this fundamental rôle of technology is typical only for modern industrial society. It was almost entirely unknown in the ancient Orient and in Graeco-Roman society. There was some indirect influence through navigation and architecture, but we do not hear of the mathematical computation of a technical structure until capitalist times (with some exceptions, as some of Archimedes' structures). We have no evidence that the huge technical works of the Chinese, the Babylonians, the Romans required more than the most elementary type of mathematics. There certainly was an influence of technology on exact science, as testified by the many writers on mechanics, but the application never got beyond elementary beginnings. Vitruvius, in his *Architecture,* pays high tribute to mathematics, but theory and practice are divorced. His eighth book describes practical means to convey water to houses and cities, deals with leaden pipes, earthen tubes and channels, and can be read without any mathematical training. His ninth book deals with methods of doubling the area of a square, the theorem of Pythagoras, Archimedes' law of hydrostatics and the duplication of a cube, but seems to be without much bearing on architecture. The names of inventors and technicians of antiquity (Ctesibios, Philon, Frontinus) have no place or hardly a place in the history of mathematics, not even for their indirect influence, the only important exception being Archimedes and perhaps also Heron.

With the advent of modern capitalism, technology begins to play an important part in the development of mathematics. The development of mechanical appliances in renaissance times is, as H. Grossmann has analyzed, the immediate source of modern mechanics as a science.[15] Mechanics now advanced beyond the narrow limits of antiquity and became the source of inspiration for the mathema-

[14] See T. C. Fry, "Industrial Mathematics," in *Research—A National Resource,* II (Dec., 1940), Report of the National Research Council to the National Research Planning Board (Washington, 1941), p. 268-288. Dr. Fry quotes a stimulating remark of Dr. H. M. Evjen: "Higher mathematics, of course, means simply those branches of the science which have not as yet found a wide field of application and hence have not as yet, so to speak, emerged from obscurity. It is therefore a temporal and subjective term" (p. 273). The remark is of interest as an oversimplification of the sociological approach.

[15] H. Grossmann, "Die gesellschaftlichen Grundlagen der mechanistischen Philosophie und die Manufaktur," *Zeitschr. für Sozialforschung,* IV (1935), p. 161-231.

ticians. In this first stage it inspired the philosophers and led to their belief in the mathematical structure of the universe. Many mathematicians were inventors themselves, as Descartes, Pascal, Newton and Leibniz. For Leibniz, the study of exact science was part of his search for the general method of invention and discovery.

Application of the new tools to problems of technology came only gradually. In the eighteenth century, Euler and the Bernoullis applied their mathematics to ship and canal building. Most influence of technology remained indirect, even after the Industrial Revolution, through the intermediary rôle of mechanics and physics. The present direct influence of technology on mathematics begins in the twentieth century with the whole transformation of the rôle of science in industry.

From this brief sketch we may conclude that a study of the influence of technology on mathematics might be very instructive. One thing stands out. The technological foundation of a form of society, its buildings, aqueducts, tools, ships, its inventive genius, its technical skill, is not of itself an indicator of the size and the tendencies of the mathematics cultivated in this society. There may be influence, of the one upon the other, but this influence may differ in quantity and in quality. We must see the technological structure of a society not as an isolated thing, but in its full relationship with many other aspects of the society such as the existence of slavery, the extent of mercantile intercourse, the influence of class struggles, the stability of the social forms and the amount of leisure.

A startling series of new conceptions is hardly ever the result of a development of mathematical ideas in isolated independence. Sociological factors play some sort of rôle, it is clear. Yet new scientific conceptions need not be born in the very midst of struggles, but perhaps rather on the periphery. The stimuli operating for advancement take effect slowly and require, not only a certain maturity, but also quietude. There are some obvious exceptions like the case of Thales, the legendary father of Greek mathematics, who is supposed to have been himself a militant member of his class. Descartes enlisted in the Dutch army, at that time one of the spearheads of the bourgeois revolution, and participated in the Thirty Years War, but soon discovered that most great ideas are born in quietude. Galois, the young French genius of two centuries later, participated actively in the July revolution, and soon afterwards found his death. More typical is the original work done by men in peaceful settings, not too far away from the centers of social economic activity, like Newton at Cambridge, Euler at Berlin or Poincaré at Paris. The first mathematical research institute was founded, not in Athens with its sharp class struggles, but in Alexandria where economic activity was carried on in relatively peaceful surroundings. Sometimes the geographical distance between social activity and mathematical inventiveness becomes very large, the most extreme case being the discovery of non-Euclidean geometries in Hungary and in Russia during the early decades of the nineteenth century. Both discoveries were inspired by the genius of Gauss, who himself led an uneventful life in a provincial town of semi-feudal Germany. It would be a great mistake, however, to see in these remarkable cases of semi-detachment of social progress and scientific activity a proof of the independence of mathematical invention. Gauss belongs, with Lessing before him and with his older contemporary Goethe, to the great German awakening of the late eighteenth and the early nineteenth century, which Franz Mehring has traced to its bourgeois roots.[16] The relation of Gauss' work to that of the mathematicians of revolutionary France is intimate to such an extent that Felix Klein has drawn up a long comparative table between the achievements of Gauss and those of Legendre, one of the French mathematicians in whom we can study the influence of the French Revolution on the development of mathematics. Gauss, compared to Legendre, always dug deeper, but Göttingen gave more leisure than Paris.[17] Working on ideas which would dominate modern mathematics, Gauss remained in person rather the type of the eighteenth century.

MATHEMATICS IN RELATION TO OTHER DISCIPLINES

Felix Klein, in comparing great mathematicians of the nineteenth century, has remarked that creative mathematicians may have very different outlooks on life, and yet little of their particular out-

[16] For instance in *Die Lessing-Legende* (4ᵉ Aufl., Stuttgart, 1913).

[17] Felix Klein, *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, I (Berlin, 1926), p. 60-61.

look appears in their work.[18] He came to this conclusion by comparing the work of Catholics like Weierstrass and Cauchy, Protestants like Riemann, and Jews like Jacobi, and in the relatively small period with which he deals and the relatively small range of religious views, his statement is not incorrect. It constitutes a warning to us when we try to generalize. Philosophies and religions reflect certain objective social relationships, perhaps those of a bygone age, and as such may carry elements stimulating or discouraging to mathematical work. In Greek society, mathematics was born and fostered in the sharpest clash of philosophies. Materialists and idealists struggled for the interpretation of mathematical conceptions. The first clear statements of the problems of the infinitesimal came from Zeno of Elea in an attempt to harass either the Pythagoreans or the Atomists. When the dust of the battlefields cleared, the strategic mathematical positions were held by the idealist philosophers of the Platonic school with their emphasis on logical structure and sterility of application.[19] Euclid, who was in the Platonic tradition, embodied this point of view in his "Elements." In so far as these "Elements" still form, in essence and in spirit, the backbone of our ordinary plane and solid geometry, Plato's influence is still felt in our present-day instruction. The ordinary type of American high school texts on geometry combines a maximum of logical structure with a minimum of applications to modern science and to modern life. Here the obstinate persistence of a certain type of mathematics in instruction goes back to social and cultural conditions of long ago.

Another example of a philosophical school which profoundly

[18] *Ibid.*, p. 71: "This short survey affirms the experience of every study of man, namely that in questions of *Weltanschauung* the gifts of the intellect are not decisive. The disposition of heart and will, the influences of education, the experiences, the whole impact of the exterior world and of one's own nature are active in its formulation Again: "There is often an opinion prevailing among the public that mathematicians and natural scientists must have a tendency for liberal, and even radical opinions, in accordance with their unprejudiced logically keen way of thinking. One look at history teaches that this opinion is not at all in agreement with the facts. Our science has outstanding representatives in all camps and parties."

[19] For the interpretation of Zeno's statements see F. Cajori, "The History of Zeno's Arguments in Motion," *American Mathematical Monthly*, XXII (1915). A guide to the understanding of the social meaning of mathematics in ancient Greece is Winspear, *op. cit.*, and M. A. Dinnik, *Outline of the History of Philosophy of Classical Greece* (Moscow, 1936, in Russian). We disagree, however, with Winspear's interpretation of Zeno's paradoxes as mere sophistry; they were remarkable documents of mathematical philosophy, and as such their constructive influence has been enormous.

influenced mathematical creation is the materialism of Descartes and his followers, on which we have already touched before. One of the main tenets of this school was the mathematical nature of the universe and the view of mathematics as the key to the control of nature. This creed was shared by many men who could not follow the Cartesians in their materialism, like Leibniz, and it accounts for the popularity of mathematics in progressive circles of the seventeenth and eighteenth centuries. And, to give an example of recent times, dialectical materialism, far more than any of the past philosophies, has acted as a stimulant on the study of the exact sciences in the U.S.S.R., flourishing under the impact of socialist construction.

There are also cases of philosophical attitudes which have discouraged the study of mathematics. The gradual decay of Roman slave society focused the attention of many logically trained minds on the salvation of the soul rather than on questions concerning control of number and space. This explains why the Church Fathers, with all their intellectual endowment, devoted their attention to dialectical exercises of a theological kind, neglecting natural science and mathematics. All the well-known mathematicians of this period, from Ptolemy to Proclus, were heathen, and one of them, Hypatia, died an anti-Christian martyr. They hardly contributed new results, and some of them, like Proclus, concentrated also on ethical problems.

We may go nearer home to see the effects of a philosophy, of perhaps simply a mental attitude, on the pursuit of mathematical studies. Whence the pride among so many high school and college graduates that mathematics means absolutely nothing to them? Whence, at the same time, the timid wish among some others that they might know more about it, in the way of Henry Adams? [20] How is it that even among cultured men and women of our age, there is a firm belief that mathematics is fixed, finished for all time, and the mathematician a repeater of the lore of the past? This attitude of mind is destructive for further progress of exact science as a

[20] "At best he would never have been a mathematician; at worst he would never have cared to be one; but he needed to read mathematics, like any other universal language, and he never reached the alphabet," *The Education of Henry Adams* (Boston: Houghton Mifflin, 1927), p. 60. See also p. 449. Henry Adams' yearning has the flavor of a bygone century—in agreement with the whole character he gives himself throughout.

tool for social reconstruction. Its roots must be determined by a study of the rôle of mathematics in modern society as a whole, not only in its schools or industries.

These widely sundered instances will suffice to show that there is room for a sociology of mathematics. They also show that a careful analysis of the nature of social structures is needed before an attempt is made to interpret its influence on the status of exact science. Superficiality only engenders disappointment and makes such work look ludicrous. The tremendous advances in economic history, during the last decades, have made the task at any rate potentially possible. The history of technology, also an important factor in a sociological approach, as we have seen, is nevertheless still in a rather unsatisfactory state.[21] The lack can be made good without too much difficulty with the abundant documentation at our disposal. Fortunately the periods in which the mathematician is interested are usually also the periods most keenly studied by others —the civilizations of China, Babylonia, ancient Egypt, classical Greece, the Roman Empire, Europe under feudalism and after its disintegration, and modern capitalism. An exception seems to be the early world of Islam, and India before the Mohammedans, concerning which sociological information is extremely unsatisfactory.

We finish with a final warning. We should always be aware that a mathematical discovery, a state of mind regarding mathematics, a system of teaching is never explained by one cause alone. Life is complex and even the most modest and the most subtle act reflects in some way or another an infinity of aspects of the real world. We cannot say that one particular factor was responsible for a particular occurrence or mental state. We must discover the way all factors—sociological, logical, artistic, and personal—have played a rôle in the case under investigation, never forgetting, however, that man is a social being even when he worries about the straight lines on hypercones in seven dimensional space.

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[21] One of the first writers to point out the crucial significance of a historical study of technology was Marx, see *Capital* 1 (Modern Library ed.), ch. 25, section 1, p. 406.