The Indian Sciences

CHAPTER 16

The Science of Language

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The assertions and propositions of many Indian sciences are formulated by means of brief expressions called *sūtra*, literally "thread" and often translated as "rule." The earliest *sūtras* are the *Śrauta Sūtras* of the science of ritual (*kalpa*); the most perfect *sūtras* are those of Pāṇini's grammar of Sanskrit. Both sciences include, along with the notion of rule, those of *paribhāṣā* or "metarule," that is, a rule about rules; and of *rule order*, a requirement often formulated by means of a metarule of the form "Rule A precedes Rule B." A simple example from ritual is that the rules about lighting the ritual fire precede those that describe how oblations are made into it.

Some of the early discoveries of ritualists and grammarians anticipate rules or techniques in modern linguistics, logic, mathematics or computer science. An example is the notion of *default.* The ritual *Āpastamba Śrauta Sūtra* (24.1.23–6) singles out default options for oblations, priests, and implements. It specifies that the default *oblation* is clarified butter; the default *priest*, the Adhvaryu; and the default *implement*, the *juhū* ladle. There are degrees of default: when the *juhū* is already used, and no other implement is specified, the oblation is made with the help of the *sruva.* The notion of multiple default echoes or is echoed by Pāṇini's metarule 2.3.1 which deals with *kāraka* or syntactic relations: "(the following rules apply) when it [i.e., the *kāraka*] is not (already) expressed."

Using metarules and rule order, the ritual and grammatical systems express *recursiveness*, that is, they describe an infinite domain of facts with the help of a finite number of rules. Pāṇini's commentator Patañjali compares the expressions of language to rituals of infinite duration and explains (by telling a story about Bṛhaspati, Professor of the Gods, trying to teach Sanskrit to Indra) why an attempt to enumerate all the forms of language can never reach an end. He concludes that "a work containing rules has to be composed."

Rules can only operate on units that are clearly demarcated. This requirement was met in the oral transmission of the Vedas: for Vedic mantras are recited in one breath, the caesura at the end being marked by a pause or the particle *iti*. Vedic Brahmins continue to the present day to recite *mantras* in this manner. Ritualists and reciters generally do not know – and need not know – their *meaning*; but they know their precise *form* along with their accents and modes of recitation which incorporate a good measure of linguistic analysis.

For the Vedas to be orally transmitted, its sentences were analysed "word-for-word." This is not easy because Sanskrit possesses *sandhi* or "junction" (literally "putting together"). Sandhi is common in spoken English but uncommon in writing where, for example: the indefinite article "a" is replaced by "an" when a *vowel* follows so that we have: "*a* book" but "*an* apple." (The rule could be formulated the other way round depending on what is taken to be the default option: "an" is replaced by "a" when a *consonant* follows.) In Sanskrit, *sandhi* is all-pervasive, for example:

> *orvaprā amartyā nivato devyudvataḥ* (*Rigveda* 10.127.2)
>
> "The immortal Goddess has pervaded wide space, depths and heights"

The CONTINUOUS RECITATION of this *mantra* is called *saṃhitā-pāṭha* (the word for "continuous", *saṃhitā*, is related to *sandhi*, and *pāṭha* means "recitation"). The WORD-FOR-WORD ANALYSIS, called *padapāṭha* (from *pada* "word"), is:

> *ā* / *uru* / *aprāḥ* / *amartyā* / *ni-vataḥ* / *devī* / *ut-vataḥ* //
>
> ("per / wide / vaded / immortal / depths / goddess / heights")

This analysis goes deeper than *words*: the "pre-verb" *ā* is separated from the verb *aprāḥ* (in the translation: "per" from "vaded"). Later rules express *sandhi* transformations, e.g.:

ā + *uru* > *oru*

uru + *aprāḥ* > *urvaprāḥ*,

where the arrow ">" may be read as: "has to be replaced by", "changes into," or "becomes."

Generalizations of such rules are the *sūtras* with which the linguistic study of language or "grammar" (*vyākaraṇa*) began. I shall subdivide its history into four parts or periods: (1) PADAPĀṬHA or "word-for-word" analysis; (2) PRĀTIŚĀKHYA or treatises that formalize these procedures; (3) PĀṆINI; and (4) LATER SCHOOLS.

1 Padapāṭha

The first system we know of is Śākalya's *Padapāṭha* of the Rigveda but it contains ideas and techniques that may go back to an older period (also preserved in the

Iranian *Avesta*). The Padapāṭha "word-for-word" analysis separated words and other elements, insuring their proper pronunciation, including accents (which I omit from our discussion) and orally fixed the corpus or "text" thus contributing to *canon formation* during the late Vedic period in eastern India (Videha around 600–500 BC).

There are several stages in the development of this analysis which depict a gradual extension or generalization, i.e., scientific progress. At first the separation between words or stems and suffixes was marked by a brief pause in the recitation as in the above example from *Rigveda* 10.127.2 or in *ṛṣi-bhiḥ*, "by the seers." This method was extended to nominal compounds: *saptaputram* > *sapta-putram* "seven sons." It was called *avagraha*, "separation" and we express it by a hyphen though no writing was known in India at this period. Since "separation" was used in each analysis only once, a problem arose: many compounds consist of more than two members.

Śākalya's solution was inspired by semantics: *daśapramatīm* is analyzed as *daśa-pramatīm* "ten protectors" (i.e., fingers), not *daśapra-matīm* which would correspond to the equally meaningless "tenpro-tectors." More complex cases were taken care of with the help of two other methods of analysis: *pragraha*, "marking," e.g., *saptaputram iti*; and *parigraha*, "marking-and-separation," e.g., *saptaputram iti sapta-putram*.

The Padapāṭha is an analysis of the *Saṃhitāpāṭha* or "continuous recitation," but by isolating words from each other, it facilitated the opposite of what was intended: the *forgetting* of single words. Special patterns of repetition or *vikṛtis* were constructed to minimize this risk. The question arises whether these procedures were *prescriptive* or *descriptive* – a relevant question for the historian of science since science is presumably descriptive. How can we find out?

If we represent the words of the Saṃhitāpāṭha as: 1 2 3 4 . . . and the Padapāṭha as: 1 / 2 / 3 / 4 / . . . , the next two *vikṛti* variations may be represented as follows:

> *Kramapāṭha* "Step-by-step recitation": 1 2 / 2 3 / 3 4 / . . .
>
> *Jaṭāpāṭha* "Plaited recitation": 1 2 / 2 1 / 1 2 / 2 3 / 3 2 / 2 3 / . . .

The *Kramapāṭha* does not introduce any new *sandhi* combinations: all its expressions belong to the Vedic Saṃhitā and are *ārṣa*, i.e., they were used by the "seers" (*ṛṣi*); but the *Jaṭāpāṭha* (called after plaited or matted hair) introduces "2 1" and "3 2" which are reversals of the original order: they introduce something that was not in the Veda and therefore "not from the seers" (*anārṣa*).

The *Kramapāṭha* of the first words of our first example of Sanskrit *sandhi* is straightforward:

> *ā* + *uru* / *uru* + *aprāḥ* / *aprāḥ* + *amartyā* > *oru* / *urvaprāḥ* / *aprā amartyā* / . . .

The *Jaṭāpāṭha* is plaited. It begins: *ā* + *uru* + *uru* + *ā* + *ā* + *uru* which becomes: *orvurvoru* – a real tangle. Worse, it contains an element that does not come from the Saṃhitā of the Vedic seers: *uru* + *ā* > *urvā*. However, *urvā* is not an artificial *pre*scription because the reciters *de*scribe the regular form that would result from *uru* followed by *ā* in their ordinary speech, that is, the contemporary Sanskrit they spoke and were familiar with. The form *urvā*, which happens to be absent from the finite corpus of the Vedas, is *grammatical*: it belongs to the infinitely many expressions of language that a native speaker is able to produce. The so-called *anārṣa* forms, then, which do not come from the Vedas, are Sanskrit and *descriptive*. They are, in fact, the precursors of Pāṇini's description of a form of the spoken language that he regarded as exemplary.

2 Prātiśākhya

The practice of the Padapāṭha was formalized in the *Prātiśākhya* compositions, in principle one for each *śākhā* or "Vedic school." The earlier name *pārṣada* suggests that they belonged to a community (*pariṣad*) which orally transmitted and discussed the contents of their *śākhā*. The relationship between *continuous recitation* and *word-for-word* analysis is expressed ambiguously by the Sanskrit sentence *padaprakṛtiḥ saṃhitā* which may be taken to mean: "The Saṃhitā is the base of the Pada" or: "The Saṃhitā has the Pada as its base." The *Prātiśākhya* generally adopt the second interpretation, puzzling from a historical perspective, but signalling a return to the original composers who put words together in their inspired speech just as any user of language puts words together when speaking or writing. This illustrates that Indian grammarians regarded language as *unchanging*, an erroneous assumption which led, however, to the linguistically productive *synchronic* analysis that was advocated in Europe more than two millennia later by Ferdinand de Saussure.

The *Prātiśākhyas* are early because of their structure and function, not because of the forms that survive and may have been influenced by Pāṇini or other grammarians. Their original aim was completeness. Whitney noted on a section of *Taittirīya Prātiśākhya* that he could not discover any case of a retroflex nasal arising in the *Taittirīya Saṃhitā* from a dental nasal in the *Taittirīya Padapāṭha* "that was not duly provided for." Weber used the term "complete" (*vollständig*) and a century later, Surya Kanta called the *Rik-Prātiśākhya* "*entirely* free from all oversights." To be complete is only possible when we deal with a *finite* corpus of *utterances*, (copies of) unique events in space and time such as the extent of a Vedic *śākhā*. A living language consists of *sentences* which cannot be enumerated because they are infinite in number as Patañjali knew. Following Suryakanta, we postulate that the *Prātiśākhya*s originally produced exhaustive listings (*gaṇas*) of examples, showing patterns of linguistic structure, and proceeded only later to generalizations explained by *sūtra* rules. This dichotomy of ideal types does not always survive in the texts as they have come down to us because these have been influenced by Pāṇini or other grammarians.

The *Prātiśākhyas* introduced the metalinguistic use of case-endings, at first the Nominative and Accusative as formulated by the metarule:"the expression 'this (Nom.) that (Acc.)' means 'becoming-that' with reference to the sound which stands nearest to it" *(Rik Prātiśākhya* 1.14); and later simply as: "a change is expressed as an Accusative" (*Vājasaneyi Prātiśākhya* 1.133).

The Padapāṭha discovered the differences between sentences, words, stems, pre- and suffixes, roots, etc., but the Prātiśākhya added an almost perfect analysis of the sounds of language into vowels, consonants, semi-vowels, stops, dentals, velars, nasals, etc. They placed these sounds in a two-dimensional configuration, developed from the square or *varga of* five-by-five series (also called *varga*) of syllables that begin with a stop followed by a short *a*:

ka	*kha*	*ga*	*gha*	*ṅa*	
ca	*cha*	*ja*	*jha*	*ña*	
ṭa	*ṭha*	*ḍa*	*ḍha*	*ṇa*	
ta	*tha*	*da*	*dha*	*na*	
pa	*pha*	*ba*	*bha*	*ma*	(1)

These sounds are marked in our modern transcription by diacritics which we need because our alphabet cannot express all the sounds of Sanskrit. It is not necessary for the reader to know how exactly they are pronounced (but see below); however, we need to know that all the vowels *a* in the syllables of the table are *short* (like the first *a* of *aprāḥ*). A bar above a vowel (like the second *a* of *aprāḥ*) indicates that it is *long*, that is, lasts twice as long as the short.

The 5 × 5 *varga* "square" was extended and completed with fricatives or sibilants, semi-vowels and vowels. One should not look upon the resulting inventory as the *beginning* of linguistics or compare it to our haphazard "ABC's" to which there is no rhyme or reason. Like Mendelejev's Periodic System of Elements, the *varga* system was the result of centuries of analysis. In the course of that development the basic concepts of phonology were discovered and defined.

In the *mental* representation of a such a square, it does not matter what is its spatial orientation and whether *varga* denotes a row or a column. The configuration remains the same when the square is rotated around its diagonal. Proximity and distance are also the same whatever the direction. Since two directions are met with, this shows that these squares were composed and transmitted orally. Renou and Filliozat (1953: 668) had already made a more significant general observation: "One is forced to observe in this context that a Semitic type of writing would have hindered phonetic studies if it had existed at the time in India, because it would have provided a model of analysis of the sounds of language that was practical but not scientific." The Indian science of language, in other words, did not originate *in spite of* the absence of writing but *because* of it.

3 Pāṇini

The "Eight Chapters" (*Aṣṭādhyāyī*) of Pāṇini's grammar of Sanskrit (early to mid fourth century BC) consist of rules, metarules, and defining rules. In the area of phonology, they are sometimes similar or almost the same as some Prātiśākhya rules, e.g., the definition of "homorganic":

> (Homorganic is) having the same place, producing organ, and effort of articulation in the mouth. (2)

Here "place" refers to "throat," "palate," "teeth," etc., "producing organ" means "tip of the tongue," "rolling back the tip of the tongue," "tip of the teeth," "middle of the jaw," etc. "Effort of articulation" refers to "closed," "semi-closed," "open," etc. – all Prātiśākhya concepts. Pāṇini (or earlier grammarians eclipsed by him) added to the study of phonology that of morphology, syntax and semantics. Pāṇini then freed himself from the notion of a finite corpus like that of the Vedic mantras, and began to treat Sanskrit as a creative and infinite *energeia* in the sense in which that Greek concept is now associated with von Humboldt or Chomsky.

Pāṇini started his grammar with a new classification of sounds, the result of an at first sight surprising overhaul of the Prātiśākhya classification. He replaced the two-dimensional *varga* system by a linear sequence, later called the *Śivasūtra:*

> a i u Ṇ / ṛ ḷ K / e o Ṅ / ai au C /ha ya va ra Ṭ / la Ṇ / ña ma ṅa ṇa na M / jha bha Ñ / gha ḍha dha Ṣ / ja ba ga ḍa da Ś / kha pha cha ṭha tha ca ṭa ta V / ka pa Y / śa ṣa sa R / ha L // (3)

The sounds I have expressed here by small letters are the sounds of the Sanskrit object-language. They consist of vowels and consonants (including semivowels), the latter followed by a short *a*. Capitals are part of the metalanguage: they refer to metalinguistic markers to which I will return. The five vowels with which (3) starts may be long or short (*ṛ* and *ḷ* in their short form sound approximately like *ry* in "crystal" and *li* in "Clinton").

Pāṇini used short vowels (*a*, etc.) to refer to both short and long vowels (*a* and *ā*, etc.) because of an important generalization that he would otherwise miss. It happens to be a fact about Sanskrit (and some other languages) that many rules of grammar that apply to vowels, apply to them whether they are long or short. For example, *a* + *a*, *a* + *ā*, *ā* + *a* and *ā* + *ā*, all become long *ā* (e.g., *atra* + *agni* > *atrāgni* "here, Agni!", *vā* + *agni* > *vāgni* "or Agni," etc.).

To express this by four rules would not only be unwieldy; it would be unnatural for it would fail to express a generalization that captures a feature of the language. Pāṇini, therefore, uses *in his metalanguage* a single vowel to express the short and long forms of *the object language.* This enables him to express by a single rule of the form "*a* + *a* > *ā*", all four combinations of long and short. He generalizes further, because this lenghtening applies to the other vowels: e.g., the *i* in *dadhi* + *indra* > *dadhīndra* ("milk, Indra!"). We shall see in a moment how he expressed these facts.

What happens to grammatical rules that apply to a short or long vowel only? Pāṇini marks them with *T* in accordance with metarule (4):

A vowel followed by *T* denotes its own length. (4)

This means: "aT" denotes *a*, "āT" denotes *ā*, "uT" denotes *u*, etc.

Kātyāyana, another early grammarian, objected to (4), that "long" and "short" are merely habits of speech: some people speak fast and others slow. Patañjali retorted that this objection pertains to *dhvani*, "speech sound," not to the subject matter of linguistics which is *sphoṭa*, the meaningful unit of expression. Patañjali is right because, in Sanskrit, as in many other languages, the difference between short and long may affect meaning. Patañjali's concept of *sphoṭa* is concerned with *competence* of the language; speakers' habits such as rapid or slow speech belong to the psychological domain of *performance*. This is one of several cases where rules of Pāṇini's grammar have logical, psychological, or philosophical implications.

We are ready to return to the metalinguistic "indicatory markers" indicated by the capitals "Ṇ," "K," "Ṅ," etc. in (3). Their use is explained by metarule (5):

An initial sound joined to a final (indicatory) sound (denotes the intervening sounds as well). (5)

Thus, aṆ denotes "*a i u*," aK denotes "*a i u ṛ ḷ*," iK denotes "*i u ṛ ḷ*," yaṆ denotes "*ya va ra la*," and aC denotes vowels. The expression aṆ may denote "*a i u*" or the class of all vowels and semivowels because the indicatory marker "Ṇ" occurs twice as the reader will have noted. I shall not discuss whether this is a flaw or a particular merit of the system but Patañjali and other commentators pay plenty of attention to such problems. They do not assume that Pāṇini was perfect and reject what he said if they find that it contradicts the data or does not work.

Pāṇini needs the notations aṆ, aK, yaṆ etc. as elements of his artificial language, because phonology requires many combinations of sounds that cannot be simply or naturally expressed in the *varga* system of (1) or in any form of (3) without metalinguistic markers. The notations of (5) enable him to express how two vowels combine into a single long one as follows:

aK is lengthened when followed by a homorganic vowel. (6)

Another rule is needed to express the fact that the vowels *i*, *u*, *ṛ* and *ḷ* are replaced by the semivowels *y*, *v*, *r* and *l*, respectively, when followed by a heterorganic (i.e., nonhomorganic) vowel: e.g., *dadhi* + *atra* > *dadhyatra* ("milk here!"), *madhu* + *atra* > *madhvatra* ("honey here!"), etc. (compare the contrast in English between *penniless* and *penny arcade*). Using (5), Pāṇini's first step may have been to formulate this fact as: "iK is replaced by yaṆ when a heterorganic aC follows." But "heterorganic" is omitted by default because (5) already took care of the homorganic cases.

Why does *dadhi* + *atra* becomes *dadhyatra* and not **dadhvatra*? I have expressed it by using the English term "respectively." Pāṇini uses a metarule:

Reference to elements of the same number is in the same order. (7)

Therefore, *i* > *y*, *u* > *v*, etc., but not, e.g., *i* > *v*. The rule we need may now be expressed as:

iK is replaced by yaṆ when aC follows. (8)

Expressions such as these are extremely common. Most phonological rules, and many others, are of the form:

After A, in the place of B, substitute C, when D follows. (9)

Pāṇini expresses this formula and captures this generalization about language by introducing an artificial expression that makes metalinguistic use of the cases of the Sanskrit of his object language, a method inspired by Sanskrit usage and introduced by the Prātiśākhyas as we have seen. Pāṇini's starting point is the subject of the rule, i.e., the element which is substituted and therefore expressed by the Nominative Case. The metalinguistic uses of three other Cases are laid down by three metarules which refer to the Genitive, Locative, and Ablative Cases:

The Genitive case ending is used for that in the place of which (something is substituted);

when something is referred to by the Locative ending, (the substitute appears) in the place of the preceding element;

when (something is referred to) by the Ablative ending, (the substitute appears in the place) of the following. (10)

Applying these rules, Pāṇini formulates (9) as:

A + Ablative ending, B + Genitive ending, C + Nominative ending, D + Locative ending. (11)

Applying this to the sandhi rule we have been discussing, where there is no restriction on the left so that there is no need for the Ablative ending, we arrive at:

iK + Genitive, yaṆ + Nominative, aC + Locative,

which in Sanskrit becomes:

ikaḥ yaṇ aci

to which sandhi is applied, producing the actual rule as it occurs in Pāṇini's grammar:

iko yaṇ aci. (12)

As it happens, a language consists of sentences, not just of words. From the perspective of the history of science, therefore, Pāṇini's step is momentous because it resulted not just in the creation of artificial expressions but of an artificial metalanguage. Rule (12) is not merely an artificial expression but an element of an artificial language like the artificial languages of algebra that came into being much later through Arab and European efforts. Pāṇini demonstrates in passing that artificial languages need not be written.

Patañjali refers to Pāṇini's first step of generalization by declaring: "this science pertains to all the Vedas" or "this science is a Parṣad = Prātiśākhya for all the Vedas" (*sarvavedapāriṣadam idam śāstram*). Actually, Pāṇini's Vedic rules are haphazard and incomplete while his rules for the spoken language are almost perfect, if not in syntax, at least in phonology and morphology. Claims like these sound like cheap commercials but are substantiated by comparing his grammar with linguistic usage as we know it. It is sometimes said that "Pāṇini was, of course, aided in his analysis by the extraordinary clarity of the Sanskrit language," but John Brough observed: "We are apt to overlook the possibility that this structure might not have seemed so clear and obvious to us if Pāṇini had not analysed it for us."

The perfection of Pāṇini's grammar is not only due to its high degree of formalization. His science is as empirical as it is formal. The *locus classicus* on the importance of empirical description is the laconic expression *lokataḥ* "on account of (the usage) of the people." "The people" are the native speakers as illustrated by Patañjali: "He who needs a pot for some purpose goes to the house of a potter and says: 'You make a pot. I need a pot for some purpose.' No one who wants to use words goes to the house of a grammarian and says: 'You make words. I want to use them.' (On the contrary,) having brought something to mind, without further ado, he uses words."

4 Later Schools

Patañjali's "Great Commentary" of around 150 BC seems to have been followed by a lull of several centuries. In the fifth century AD, the great grammatical philosopher Bhartṛhari broke the silence by composing an original subcommentary on Patañjali. By that time, Jainas and Buddhists had also begun to use Sanskrit for their canonic writings and this resulted in several new grammars, at first practical manuals, but increasingly systematical and scientific treatises. Devanandin and Candragomin, Jaina and Buddhist grammarians, respectively, also of the fifth century, adopted Pāṇini's system and metalanguage and sought to simplify or abbreviate his expressions, sometimes introducing innovations. Candragomin refers to Middle-Indic forms that had been used in earlier Buddhist writings as *ārṣa*, "belonging to the seers," just as the Prātiśākhya authors had done with respect to Vedic *mantras*.

A later development of Sanskrit grammar benefited from the "new logic" (*navya nyāya*) that had originated in Bengal with Gangeśopādhyāya (thirteenth century). This development continued, at least, through the eighteenth century when Nāgojī Bhaṭṭa wrote a treatise on Pāṇini's metarules and philosophical works in which he combines Bhartṛhari with the Indian philosophy of language – a discipline not included in the present sketch.

Grammatical concepts and techniques from the Sanskrit tradition influenced the early (second century BC?) grammar of Tamil *Tolkāppiyam* which is, in other respects, very different in outlook and structure. Sanskrit models were followed more closely in later Dravidian grammars as well as grammars of Persian, Marathi, and other languages. Tibetan grammars were inspired by the Sanskrit tradition but it took almost 500 years (from the ninth to the fourteenth century AD) before they fully captured the power and sophistication of the Indian originals.

The Indian grammatical tradition influenced not only a few grammarians but much of Asian civilization. We have seen that the first scientific classification of the sounds of language, that of the *varga* sysem of the Prātiśākhyas, was due to an *oral* analysis. It is not surprising that this classification was taken into account when the first Indian scripts evolved, but it went much further and served, for millennia to come, as a sound foundation for most of the numerous scripts and writing systems of south, southeast and east Asia – from Kharosthi, Khotanese, Tibetan, Nepali, and all the modern scripts of India (except the Urdu/Persian) to Sinhalese, Burmese, Khmer, Thai, Javanese, and Balinese. In south and southeast Asia, the *shapes* of earlier Indian syllables inspired some of these inventions, but it is the *system* of classification that was of enduring significance wherever it became known. In east Asia, the bastion of Chinese characters could not adapt it; but in Japan it led to the creation of the *hiragana* and *katakana* syllabaries during the Heian period (794–1185), and in Korea it inspired the world's most perfect script, *han'gul*, developed in 1444 by a committee of scholars appointed by the emperor Sejong. All these Asian scripts are a far cry from the haphazard jumble of the "ABC" and the countless spelling problems that result from it in English and other modern languages that use the alphabet.

The Indian Science of Language influenced modern linguistics primarily through Franz Bopp (1791–1867) who was inspired by the Sanskrit grammar of Charles Wilkins of 1807, based in turn upon Pāṇini. But Bopp did not use *rules* and the celebrated nineteenth-century sound laws were discovered by others. That rules could be formal had been discovered by Aristotle but remained confined to logic. The equations of algebra, another formal science from Asia, were restricted to mathematics and the natural sciences. That linguistics could be a formal science was perceived, or at least envisaged, by de Saussure, who predicted in 1894, that the expressions of linguistics "will be algebraic or will not be." Leonard Bloomfield was familiar with Pāṇini and used ordered rules once (*Menomini Morphonemics* of 1939). Formal rules were used extensively by Noam Chomsky and his school, and finally replaced by more abstract principles.

In classical India, "the arrangement of forms and speculation on forms reveal most clearly their content – they ultimately are the content itself." I translate this phrase from Charles Malamoud who adopted it from Louis Renou, the foremost twentieth-century scholar of ancient Indian civilization. Attention to form is a characteristic of all science and even the Mīmāṃsā system of ritual philosophy declares: "when a visible result is possible, it is improper to postulate an invisible one" (*Mīmāṃsā Paribhāṣā*).

To the modern age, the most important contribution of the Sanskrit grammatical tradition is its construction of linguistics as a formal science. The study of language, the most characteristic feature of the human animal, is as formal as that of the so-called natural sciences. The need of an artificial, formal language shows that natural language is unable to express adequately the structures not only of the nonhuman universe but also those of human language itself. When linguistics is an exact science, the distinction between human and natural sciences falls to the ground and it becomes far-fetched to assert that humans stand isolated in the universe – a postulate that has never been popular in India.

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CHAPTER 17

Indian Mathematics

Takao Hayashi

I Vedic Mathematics

I.1 Vedas

Since the *Vedas* are religious texts produced by poets, we cannot expect in them enough information for systematically describing the mathematical knowledge of those times. We can only gather scattered terms for whole numbers, for basic fractions, and for simple geometric figures.

The *Ṛgveda* contains a number of numerical expressions. The Vedic poets were particularly fond of three and seven as holy numbers, and often used their multiples such as 3×7, 33 (= 3×11), 3×50, 3×60, 3×70, $3 \times 7 \times 70$, and 333 (= 3×111). It has been argued that the number of gods, 3339, mentioned in *Ṛgveda* 3.3.9, is the sum of three numbers, namely, 33, 303, and 3003. Although it has not been proved that the Vedic Indians had a place-value notation of numbers, this summation itself must have not been difficult for them since their numeration system was basically decimal. They used the words *eka*, *daśa*, *śata*, *sahasra*, and *ayuta*, for 1, 10, 10^2, 10^3, and 10^4. For multiples of 10^3 and of 10^4 they often used expressions based respectively on 10^2 and on 10^3 as well; for example, $2 \times 10^3 = 20$ (*víṃśati*) $\times\ 10^2$ (*śata*), $3 \times 10^4 = 30$ (*triṃśat*) $\times\ 10^3$ (*sahasra*), etc. The words for "hundred" and "thousand" are sometimes employed in the sense of "a number of." Thus, Indra is said to have destroyed a hundred old fortresses of Śambara and slain a hundred thousand (*śataṃ sahasraṃ*) brave men of Varcin (ibid. 2.14.6). The number sixty seems to have had a certain weight with the Vedic poets since we come across expressions like "a thousand and sixty (1060) cows" (ibid. 1.126.3), "sixty thousand and ninety-nine (60099) kings" (*ibid.* 1.53.9), and "sixty thousand *ayuta* ($60 \times 10^3 \times 10^4 = 6 \times 10^8$) of horses" (*ibid.* 8.46.22), but its significance is not known.

There are also references in the *Ṛgveda* to the basic unit-fractions 1/2 (*ardha* or "a half"), 1/4 (*pāda* or "a foot," from the 4 "feet" of a quadruped), 1/8 (*śapha* or "a hoof," from the 8 "hooves" of a cow), and 1/16 (*kalā* or "a digit," from the 16 "digits" of the moon).

The Vedic people seem not to have felt much difficulty in performing the addition, subtraction, and, perhaps, multiplication of whole numbers, but division appears to have troubled them when it left a remainder, for only Indra and Viṣṇu are said to have succeeded in dividing a thousand cows into three. According to the *Śatapathabrāhmaṇa* (3.3.1.13, 4.5.8.1), they correctly obtained 333 for the quotient with the remainder 1. This story is believed to be implicitly alluded to already in *Ṛgveda* 6.69.8.

By the time of, at latest, the *Yajurvedasaṃhitā*, the Indians had extended their list of the names of powers of ten up to 10^{12}. In *Vājasaneyisaṃhitā* 17.2, they are used for counting the numbers of bricks. In *Taittirīyasaṃhitā* 7.2.11–20, they constitute part of the sacrificial formulas (*mantras*) to be uttered on the occasion of an *annahoma* or "food-oblation rite," which is performed at a certain stage in the *aśvamedha* or "horse-sacrifice rite." In the *annahoma*, a priest makes a series of oblations of ghee, honey, rice, barleycorns, etc. to the fire (*agni*) called *āhavanīya* through the night until sunrise, while uttering *mantras* in which a unit formula, "the dative case of a numeral + *svāhā* (hail to)," is repeated.

The first *mantra* reads: "Hail to one, hail to two, hail to three, . . . , hail to nineteen, hail to twenty, hail to twenty-nine, hail to thirty-nine, . . . , hail to ninety-nine, hail to one hundred, hail to two hundred, hail to all (*sarva*)." The numbers that actually occur in this *mantra* are:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 29,
39, 49, 59, 69, 79, 89, 99, 100, 200;

but this seems to be an abbreviation of a series of natural numbers from 1 to 200 or more. According to the traditional interpretation, "one" represents Prajāpati ("lord of creatures" or creator), the rest being all things in the world which have evolved from him. The *mantras* that follow consist of arithmetical progressions such as odd numbers, even numbers, multiples of 4, etc., some of which are accompanied by an additional number either at the end or at the beginning. Then comes the last *mantra*, which contains the decimal names from *śata* (10^2) to *parārdha* (10^{12}): "Hail to *śata*, hail to *sahasra*, hail to *ayuta*, . . . , hail to *parārdha*, hail to the dawn (*uṣas*), hail to the twilight (*vyuṣṭi*), hail to the one which is going to rise (*udeṣyat*), hail to the one which is rising (*udyat*), hail to the one which has just risen (*udita*), hail to the heaven (*svarga*), hail to the world (*loka*), hail to all." The four phrases beginning with "hail to the dawn" are uttered immediately before the sunrise, and the four beginning with "hail to the one which has just risen" immediately after that. Some regard the last seven terms, *uṣas* to *loka*, as the names of 10^{13} to 10^{19}, but there is no support for this conjecture.

It has been conjectured that zero is indicated by the word *kṣudra* (lit. petty, trifling) in the *Atharvaveda* (19.22.6 = 19.23.21), and a negative number by *anṛca* (lit. without a hymn) (*ibid.* 19.23.22). But this conjecture still awaits a proof.

It is probable that the Pythagorean Theorem (the sum of the areas of the squares on the two sides of a right-angled triangle is equal to that on its hypotenuse), which was explicitly stated and utilized by the authors of the *Śulbasūtras*, had already been known at latest in the times of the Brāhmaṇas.

The *Śatapathabrāhmaṇa* (10.2.3.18) speaks of successive increases of the areas of the fire altars (*agni*): the basic fire altar, called the "seven-fold *agni*," has the area of seven and one half square *puruṣas* (one *puruṣa* or "man" is the height of the sacrificer with his hands stretched upwards), and it is increased, in subsequent rites performed by the same sacrificer, by one square *puruṣa* each time up to the "one hundread and one-fold *agni*." The augmentation of area was made most probably by means of the Pythagorean Theorem.

The inverse case of the Pythagorean Theorem (if the sum of the areas of the squares on two sides of a triangle is equal to that on the remaining side, then the first two sides contain a right angle) is also possibly referred to in the *Śatapathabrāhmaṇa.*

The *Vājasaneyisaṃhitā* (30.20) and the *Taittirīyabrāhmaṇa* (3.4.15) include the word *gaṇaka* ("a calculator") in their lists of people to be sacrificed on the occasion of the *puruṣa-medha* ("human-sacrifice rite"). The *gaṇaka* in the former list, who is sacrificed for the divinity of a sea animal, is usually taken to be an astrologer, but this interpretation seems not to be decisive at all since an astrologer called *nakṣatradarśa* ("a star-gazer"), who is sacrificed for the divinity of wisdom, is listed prior to it in the same list (30.10). The *gaṇaka* in the latter list, who is sacrificed together with a *vīṇā* player for the divinity of songs, is presumably related with music.

I.2 Śulbasūtras

Śulbasūtras, which constitute part of, or appendices to, the *Śrautasūtras*, are basically practical manuals meant for the preparation of the sites of śrauta rituals, the main topic being the construction of fire altars with burned bricks. A sacrificer (*yajamāna*) had to strictly follow the regulations affecting the sizes and shapes of the sites and altars, because otherwise he was supposed to lose the merit to be attained through the specific ritual he was performing. Hence follows the connection between rituals and geometry. But it would be wrong to suppose only practical geometry like the measurement of land in the *Śulbasūtras.* The geometry conceived by the śu1ba mathematicians already has theoretical aspects as will be seen below.

The two oldest *Śulbasūtras*, those of Āpastamba and Baudhāyana, can be each divided into two parts. The first part treats geometry in general terms. The topics dealt with in this geometry are certainly related to the construction of ritual sites, but it contains few words which indicate rituals. The second part treats the construction of ritual sites including various altars. It is mostly devoted to the description of various fire-altars (*agni*) to be employed in the *agni-cayana* ("fire-altar-construction rite"). For the construction of a standard *agni*, the following requirements had to be satisfied: (1) an *agni* consists of five layers of 200 bricks each with the total height equal to the knee-height of the sacrificer; (2) the odd-numbered and the even-numbered layers each have one and the same arrangement of bricks, and no brick should coincide with the one above or below it; and (3) an *agni* should occupy an area of seven and one half square *puruṣas.* The area of an *agni* was increased by one square *puruṣa* each time the same rite was repeated by the same sacrificer.

The topics dealt with in the geometric portion of the *Śulbasūtras* of Āpastamba (abbr. A) and Baudhāyana (B) are as follows.

Linear measures (B 1.3–21; A provides them in later chapters at need).
Construction of geometric figures (A 1.2–3, 1.7–2.3; B 1.22–44, 46–7).
Relationships between the diagonal and the side of a rectangle (oblong and square).

Pythagorean Theorem (A 1.4–5, B 1.45, 48–9).
Computation of the diagonal of a square (A 1.6, B 1.61–2).

Sum and difference of two squares (A 2.4–6, B 1.50–1).
Equi-area transformation of geometric figures (A 2.7, 3.1–3, cf. 12.5, 12.9, 15.9; B 1.52–60).
Relationship between the area and the side of a square (A 3.4–10).

The tools used for drawing figures in the *Śulbasūtras* are a rope called *rajju* or *śulba* and pegs or posts called *śaṅku*. A bamboo rod is sometimes used instead of a rope. By means of these tools, one can draw a straight line, cut out a line segment having any desired length, and draw a circle or an arc with any desired radius.

No specific rules are given for the drawing of a line, a circle, and an arc; these can be easily obtained by a rope and pegs. The main problem for the śulba geometers, who were required to construct geometric figures like a square, a rectangle, and a trapezium, was how to draw a line orthogonal to, or parallel to, a given line.

The most important mathematical motif of the *Śulbasūtras* is the area. The core of the Śulba mathematics is concerned with the religious requirement that one should construct altars in varoius shapes with a given area, a requirement which seems to have originated from agriculture where a harvest greatly depends on the area of the land (*bhū* or *bhūmi*).

The *Śulbasūtras* contain the earliest extant verbal expression of the Pythagorean Theorem in the world, although it had already been known to the Old Babylonians. It is stated in exactly the same words by Āpastamba (1.4), Baudhāyana (1.48) and Kātyāyana (2.7): "The diagonal rope (*akṣṇayā-rajju*) of an oblong produces both which the flank (*pārśvamānī*) and the horizontal (*tiryaṅmānī*) ⟨ropes⟩ produce separately." What "the ropes produce" are not explicitly mentioned, but are no doubt the square areas constructed on them.

Highlights of the śulba mathematics are transformations of geometric figures with their areas kept unchanged. This theme was originally related with the practical requirement of drawing altars in various shapes with a given area, but the Śulba makers seem to have taken a step forward and tried to treat the theme with a theoretical perspective in mind.

Baudhāyana deals with seven transformations. Five out of them are concerned with transformations from a square to a circle, a rectangle, an isosceles trapezium, an isosceles triangle, and a rhombus, while the other two are those from a rectangle and a circle to a square. All these, except the last one, can be put into the following scheme for constructing various figures.

$$\text{a rectangle} \rightarrow \text{a square} \rightarrow \text{a circle, etc.}$$

This scheme consists of three steps. Given an area A:

1 Construct a rectangle having the area A.
2 Transform it into a square having the area A.
3 Transform it into the desired figure (a circle, etc.) having the area A.

For example, in order to draw a circle with the area A, one first constructs a rectangle with the two sides a and A/a for any rational number a, and then transforms the rectangle into a square and the square obtained into a circle. In fact, for the śulba mathematicians, this was the most natural way of constructing a circle with a given area, since they did not know the formula, $A = \pi r^2$, from which we would compute r to draw a circle with the radius r.

An apparent gap between śulba mathematics and later Indian mathematics used to puzzle scholars, and this made some suppose Western influence upon the latter. But now we know some links between the two.

The word, *karaṇī*, used in later mathematics to denote a square number and the square root of a number, originated from its use in the *Śulbasūtras.*

In order to draw a line measured by $\sqrt{n}$, Brahmagupta constructs an isosceles triangle, whose base and two sides are measured respectively by $(n/m - m)$ and by $(n/m + m)/2$, where m is any optional number. Its perpendicular is the line to be obtained (*Brāhmasphuṭasiddhānta* 18.37). This is a generalization of Kātyāyana's rule (6.7), where $m = 1$.

The square nature or an indeterminate equation of the second degree of the type, $Px^2 + t = y^2$, which was to be investigated in detail by Brahmagupta and others, presumably has its origin in the śulba mathematicians' investigation of the diagonal of a square or $\sqrt{2}$.

Two root-approximation formulas used in Jaina works and in the *Bakhshālī Manuscript*, seem to have been obtained by algebraically (or numerically) interpreting *Śulbasūtras* geometric rules for the rectangle-squaring transformation.

Āryabhaṭa I's terminology for series, *citi*, etc., too, is probably related to the *Śulbasūtras*, whose main theme was the piling (*citi*) of bricks for Vedic altars.

II Gaṇita (Mathematics)

II.1 Up to the 5th century – prologue

It is but natural that one who calculates most in a society is called a calculator. Before the introduction and spread of horoscopic astrology and mathematical astronomy to the Indian subcontinent, the occupation that, in Indian society, required calculations most seems to have been that of the accountant, since he was called either *gaṇaka* or *saṃkhyāyaka*, both meaning "a calculator." In the *Mahābhārata* (2.5.62) the sage Nārada recommends to the king Yudhiṣṭhira that he make his calculator (*gaṇaka*) and scribe (*lekhaka*) report to him the revenue and expenditure every morning. Kauṭilya's *Arthaśāstra* (1.19.9), too, refers to the same daily task of a king. The salary of a king's calculator and scribe is 500 *paṇas* each, while the highest salary, 48,000 *paṇas*, is paid to a minister, a prince, etc., and the lowest, 60 *paṇas*, to a servant who takes care of animals (*ibid.* 5.3.3–17). The superintendents of governmental departments are said to be assisted by five persons, namely, calculators (*saṃkhyāyakas*), scribes (*lekhakas*), inspectors of coins (*rūpa-darśakas*), receivers of balances (*nīvī-grāhakas*), and supervisors (*uttarādhyakṣas*) (ibid. 2.9.28). According to a later law book, the *Bṛhaspatismṛti* (1.1.81–90), a court consists of ten elements including a *gaṇaka*, who calculates money and assets, and a *lekhaka*, who writes sentences.

The 107th story, *Gaṇakamoggallānasutta*, of the *Majjhimanikāya* narrates a discourse of the Buddha with a brāhmaṇa *gaṇaka* named Moggallāna, from which we know: (1) that a *gaṇaka* lived on calculation (*gaṇanā*), (2) that a *gaṇaka* took live-in pupils (*antevāsins*) and taught them calculation (*saṅkhāna*), and (3) that a *gaṇaka* first taught his pupils to count from one to one hundred. The *gaṇanā* and the *saṅkhāna* in this story seem to mean respectively calculation (or mathematics) in general and a relatively elementary skill of computation beginning with the counting of numbers.

According to the *Arthaśāstra* (1.2–5), a prince learns *lipi* (writing) and *saṃkhyāna* after his hairdressing rite, and then, after his initiation rite, learns the four disciplines (*vidyās*), namely, philosophy, the *Vedas* with related fields, practical knowledge like agriculture and commerce, and politics. This *saṃkhyāna* is perhaps as elementary as the *saṅkhāna* of Moggallāna, although a calculator in general is called not a *gaṇaka* but a *saṃkhyāyaka* in the *Arthaśāstra.*

In the *Mahābhārata* (3.70), the king Ṛtuparṇa is proud of his ability in *saṃkhyāna* in addition to that in dice, when he correctly estimates, without counting, the number of nuts, 2095, on two branches of a Vibhītaka tree (*Terminalia bellerica*). This *saṃkhyāna*, therefore, contains a sort of statistical estimate of the quantities of nuts, crops, etc. The *saṅkhāna* of the Jaina canonical text, *Ṭhāṇaṃga*

(747), consists of ten topics, which presumably cover the entire mathematics known to the Indians of those days. They are: basic computation (*parikamma*), procedure or applied mathematics (*vavahāra*), rope (*rajju*), quantity (*rāsi*), reduction of fractions (*kalāsavanna*), "as many as" (*jāvaṃtāvati*), square (*vagga*), cube (*ghana*), square of square (*vaggavagga*), and choice (*vikappa*, combinatorics). The Jainas played an important role in the making of Indian mathematics.

It was, however, neither *gaṇanā* nor *saṃkhyāna* but *gaṇita* that was used later as the most general term for mathematical science. Āryabhaṭa I, in the first verse of his *Āryabhaṭīya* (AD 499 or a little later), enumerates the three subjects to be dealt with in its subsequent chapters, namely, *gaṇita*, *kāla-kriyā* (time-reckoning), and *gola* (spherics).

The Vedic numerals continued to be used in later Hindu society as well, while the Buddhists and the Jainas each developed their own systems of numerals for numbers greater than a thousand.

Apart from the Indus script, the earliest extant script in India is the one called Karoṣṭhī of, probably, Aramaic origin. Its use was restricted to north-western India and central Asia from the fourth century BC to the fourth century AD. At nearly the same time, in the Aśokan edicts, appeared another script called Brāhmī which was to become the origin of many varieties of south and south-east Asian scripts. Its relationship to the former is not certain. These scripts had their own numerical symbols, but in both systems particular symbols were used not only for units but also for tens, hundreds, thousands, etc., and some of them were made by the principle of addition, and others by the principle of multiplication. The numeral systems in both scripts were therefore not based on a place-value system, and the Brāhmī, non-place-value, numerals continued to be used in epigraphy even after the place-value system was introduced in daily calculations and in mathematical literature in the early centuries of the Christian era.

The oldest datable evidence of the decimal place-value system in India is found in the *Yavanajātaka* (AD 269/270) of Sphujidhvaja, a book on astrology. It is not certain if the decimal place-value notation in India had a symbol for zero from the beginning. We have to keep it in mind that, historically, not all place-value notations were accompanied by a zero symbol as the sexagesimal notation of the Old Babylonians proves. There is no reference to zero in the *Yavanajātaka.* In Piṅgala's *Chandaḥsūtra* (8.28–31), a work on Sanskrit prosody, zero (*śūnya*) and two (*dvi*) are used as signs for the multiplication by two and for the squaring, respectively, in the computation of powers of two, which occurs in the right-hand side of the equation, $2 + 2^2 + 2^3 + \ldots + 2^n = 2 \times 2^n - 2$. The date of the work is controversial: some ascribe it to ca. 200 BC but others to the third century AD or later. The oldest datable evidence of zero as a symbol as well as of that as a number are found in Varāhamihira's *Pañcasiddhāntikā* (ca. AD 550).

II.2 The fifth to sixth centuries – beginning

Āryabhaṭa I is so called to distinguish him from another astronomer of the same name, the author of the *Mahāsiddhānta*, who is called Āryabhaṭa II.

Āryabhaṭa I was one of the most influential mathematicians and astronomers in India through his two works, *Āryabhaṭīya* and *Āryabhaṭasiddhānta.* The latter work is not extant but has been referred to by many later scholars. The work mainly influenced northwestern India. It is also known to have had some influence upon the Sasanian and Islamic astronomy. The *Āryabhaṭīya*, on the other hand, mainly influenced south India.

Āryabhaṭa I was born in AD 476. This is known from his own statement in the *Āryabhaṭīya* (3.10): "When sixty of sixty years and three quarters of the Yuga had passed, then twenty-three years had passed here from my birth." That is, he was 23 years old in AD 499 (= 3600 – 3101, since the last quarter, called Kaliyuga, of the current Yuga began in 3102 BC). The year mentioned here, AD 499, is usually taken to refer to the date of the composition of the *Āryabhaṭīya.*

The *Āryabhaṭīya* is divided into four "quarters" (*pādas*), that is,

1 Quarter in Ten Gīti verses,
2 Quarter for Mathematics (*gaṇita*, in 33 Āryā verses),
3 Quarter for Time-Reckoning (*kālakriyā*, in 25 Āryā verses), and
4 Quarter for Spherics (*gola*, in 50 Āryā verses).

Chapter 1 consists of 13 verses. The first verse contains the author's salute to God Brahmā and refers to the three fields to be treated in the next three chapters. The second verse defines an alphabetical notation of numbers. The next ten verses (the words, "ten Gīti verses," in the title of this chapter refer to this part) contain tables of astronomical parameters and of sine-differences expressed in that notation. For example, the number of the sun's revolutions in a *yuga*, 4320000, is expressed as *khyughṛ* $= (2 + 30) \times 10^4 + 4 \times 10^6$.

Chapter 2 of the *Āryabhaṭīya* is the oldest extant mathematical text in Sanskrit after the *Śulbasūtras.* Although the chapter does not have a clear division into sections, it may be divided into four parts:

i. Rules for basic computations (vv. 2–5),
ii. Rules for geometric figures (vv. 6–18),
iii. Rules for both figures and quantities (vv. 19–24), and
iv. Rules for numerical quantities (vv. 25–33).

Āryabhaṭa gives the names of the first 10 decimal places, and says: "⟨Each⟩ place shall be ten times greater than ⟨the previous⟩ place."

Problems of proportion were solved by means of the *trairāśika* or "the ⟨computation⟩ related to three quantities." The *trairāśika* was not only indispensable for astronomy but also essential for monetary economy. The first of the seven examples for the *trairāśika* supplied by the commentator, Bhāskara I, is this: "Five *palas* of sandal-wood were bought by me for nine *rūpaka*s. Then, how much of sandal-wood can be obtained for only one *rūpaka*?" The *pala* and the *rūpaka* are units of weight and money, respectively.

The last rule provides a general solution called *kuṭṭaka* or "pulverizer" to an indeterminate system of linear equations of the following type: "When a certain unknown integer, N, is divided by a set of integers, $\{a_1, a_2, \ldots, a_n\}$, one by one, the remainders are $\{r_1, r_2, \ldots, r_r\}$. What is that number, N?"

Varāhamihira is one of the most famous authorities on astrology in India. He flourished in Avantī (modern Ujjain) in the sixth century AD.

He divided the "astral science" (*jyotiḥśāstra*) into three major "branches" (*skandhas*), namely, (1) mathematics including mathematical astronomy (*gaṇita* or *tantra*), (2) horoscopic astrology (*horā*), and (3) natural astrology or divination in general (*saṃhitā*) (*Bṛhatsaṃhitā* 1.8–9, 2.2, 2.19), and wrote several books each in the second and third branches. His only work in the first branch, *Pañcasiddhāntikā*, is a compendium of the texts of five earlier astronomical schools, and no work on mathematics proper is known to have been written by him, but his works are important from the view point of the history of mathematics as well.

In the *Pañcasiddhāntikā*, zero occurs as a number, that is, the object of mathematical operations like addition, subtraction, etc. For example, he states the mean daily velocity of the sun in each of the 12 zodiacal signs beginning with Aries as follows: "The daily velocity of the sun is in order 60 ⟨minutes⟩ minus 3, 3, 3, 3, 2, 1; plus 1, 1, 1, 1; and minus 0, 1" (*Pañcasiddhāntikā* 3.17).

Presumably, the existence of both the zero symbol for vacant places in the place-value notation and calculations by this notation brought about the concept of zero as a number, because we cannot calculate, for instance, $15 + 20 = 35$ without the rule, $5 + 0 = 5$. This is not the case with an abacus where no symbol exists for vacant places.

He expressly stated the "graphic procedure" for constructing a sine table, a method which had been only alluded to by Āryabhaṭa I, and gave a sine table for the radius, $R = 120$ (ibid. 4.1–15).

In a chapter on the combination of perfume of his work on divination, *Bṛhatsaṃhitā* (76.22), he provided a rule for calculating the number of combinations, ${}_nC_r$, when r things are taken at a time from n things, and a method called "spread by token" (*loṣṭaka-prastāra*) for enumerating all the possible combinations correctly. In the same chapter he gave the correct numbers, 84 and 1820, of ${}_9C_3$ and ${}_{16}C_4$ respectively, but he mistakenly regarded $4! \times 4 \times {}_{16}C_4 = 174720$ as the number of possible combinations when 4 are taken at a time out of 16 ingredients for perfumes, in the ratio, 1 : 2 : 3 : 4 (ibid. 76.13–21); the correct number should be ${}_{16}P_4 = 16 \times 15 \times 14 \times 13 = 43680$.

In the same chapter he utilized a magic square of order four in order to prescribe the perfumes called "good for all purposes." It consists of the four rows: 2, 3, 5, 8; 5, 8, 2, 3; 4, 1, 7, 6; and 7, 6, 4, 1 (ibid. 76.23–6). This is the oldest magic square in India. It is irregular because it is made not of the numbers 1 to 16 but of two sets of the numbers 1 to 8, but it is "pan-diagonal" in the sense that not only the two main diagonals but also all the "broken" diagonals each amounts to the magic constant, 18 in this case. Presumably, Varāhamihira first constructed a regular magic square with the numbers 1 to 16, and then modified it by subtracting 8 each from the numbers greater than 8 to arrive at his irregular square.

II.3 The seventh to eighth centuries – restructuring

The seventh century saw a restructuring of Indian mathematics. It was divided into two major fields, that is, arithmetic with mensuration and algebra, which were later called *pāṭī-gaṇita* ("mathematics of algorithms") and *bīja-gaṇita* ("mathematics of seeds"), respectively. The "seeds" here means algebraic equations (*samīkaraṇa*, lit. "to make ⟨both sides⟩ equal"), which, just like seeds of plants, have "the potentiality to generate" the solutions of mathematical problems. A work for *pāṭī-gaṇita* usually consists of two categories of rules often accompanied by examples, that is, *parikarman* (basic operations) and *vyavahāra* (practical or applied mathematics).

Brahmagupta included two chapters corresponding to these two fields in his astronomical work, *Brāhmasphuṭasiddhānta* (AD 628), and began to use the word *gaṇaka* ("calculator") in the sense of one who knows mathematical astronomy.

According to his own words, Brahmagupta, son of Jiṣṇu, wrote the *Brāhmasphuṭasiddhānta* in 25 chapters at the age of 30 in Śaka 550 = AD 628. This means that he was born in AD 598. He was still active at the age of 67 (AD 665), when he composed another work on astronomy, the *Khaṇḍakhādyaka.*

In the *Brāhmasphuṭasiddhānta*, five chapters are particularly concerned with mathematics. They are: chap. 12, "Mathematics," on arithmetic and mensuration, chap. 18, "pulverizer," on algebra (the title has simply been taken from the name of the first topic in the chapter), chap. 19, "Knowledge about Gnomon and Shadow," on measurements of shadows and lights, chap. 20, Answer to <the Problems of> Piling of Meters" (*chandaścityuttara*, on combinatorics concerning prosody, and a small section called "sine-production" (*jyā-utpatti*) in chap. 21 ("Spherics") on trigonometry.

It is in chap. 12 that he gives his famous theorem on the diagonals of a cyclic quadrilateral. In chap. 18, he prescribes rules for surds, negative quantities, zero, and unknown quantities, and provides rules called *varga-prakṛti* ("square nature") for quadratic indeterminate equations of the type, $Px^2 + t = y^2$.

Bhāskara I flourished in the first half of the seventh century in Saurāṣṭra, perhaps in Valabhī near modern Bhāvnagar, and composed three works as the expositions of the teachings of Āryabhaṭa I, "based on the continuity of tradition" (*sampradāya-avicchedāt*). They are, in chronological order, *Karmanibandha* ("Treatise on ⟨Astronomical⟩ Computation") alias *Mahābhāskarīya* ("Large ⟨Book⟩ of Bhāskara"), a prose commentary on the *Āryabhaṭīya* (written in Śaka 551 = AD 629), and an abridged version of the first work also called *Karmanibandha* alias *Laghubhāskarīya* ("Small ⟨Book⟩ of Bhāskara").

Particularly important for the history of Indian mathematics is the second work, that is, the commentary on the *Āryabhaṭīya*, which provides valuable information on, among other things, mathematical procedures and expressions of his time.

A vacant place (*kha*) in the decimal place-value notation was indicated by a small circle (*bindu*, lit. "a dot"), which was also put on the right shoulder of a negative number. A fraction was expressed, just like our fractions, by placing the numerator above the denominator (without a bar between them), and the fractional part of a number was put below its integer part. Thus, for example, the diameter of the reference circle of Āryabhaṭa's sine table whose circumference is 21600 is

$$\begin{array}{l} 6875 \\ 625 \\ 1309 \end{array}$$

since Āryabhaṭa's value of π is 62832/20000, and the circumference of the earth, whose diameter is 1050 *yojanas* according to Āryabhaṭa, is

$$\begin{array}{c} 3299 \\ 8^{\circ} \\ 25 \end{array}$$

($= 3299 - \frac{8}{25}$) *yojanas*. The value of the linear measure, *yojanas*, varies from time to time and from place to place. Āryabhaṭa equates it with 8000 *nṛs* ("men"), where 1 *nṛ* = 4 *hastas* (cubits) = 96 *aṅgulas* ("fingers").

In geometry, Bhāskara I probably knew a proof of the Pythagorean Theorem, because he provides a square figure (figure 17.1) divided into 8 equal right-

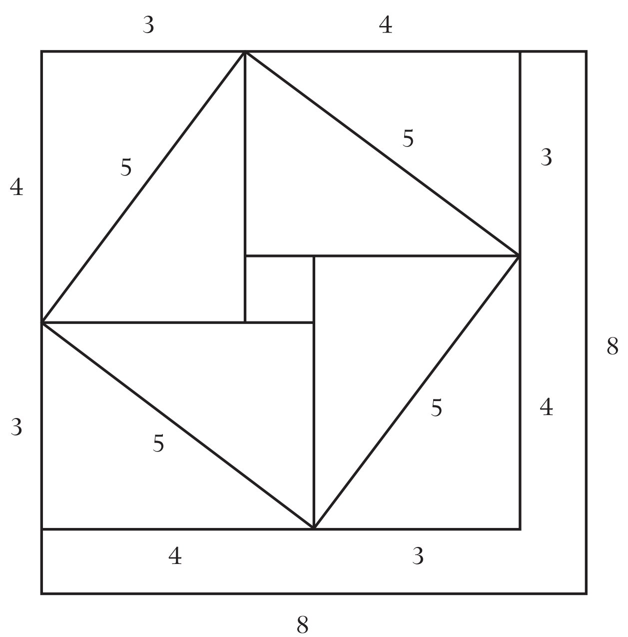

Figure 17.1: Diagram by Bhāskara I, probably used for a proof of the Pythagorean Theorem.

angled triangles and a central square, while discussing the validity of Āryabhaṭa's statement, "A square (*varga*) is an equi-quadri-lateral ⟨figure⟩" (*Āryabhaṭīya* 2.3). Bhāskara's square demonstrates that $4 \times (ab/2) + (a - b)^2 = c^2$, from which he would easily obtain the equation, $a^2 + b^2 = c^2$, numerically.

The *Bakhshālī Manuscript*, whose original title is not known, is the oldest extant manuscript in Indian mathematics. It was unearthed in a deteriorated condition at Bakhshālī near Peshawar (now in Pakistan) in 1881 and is now preserved in the Bodleian Library at Oxford University. The extant portion of the manuscript consists of 70 fragmentary leaves. It is written on birchbark with the earlier type of the Śāradā script, which was used in the northwestern part of India from the eighth to the twelfth centuries AD. The language is Sanskrit but it has largely been influenced by the vernacular(s) of those regions.

The work is a loose compilation of mathematical rules and examples collected from different works. They are written in verse, and the examples are solved in prose commentaries on them and are often given verifications of the answers.

The dates so far proposed for the Bakhshālī work vary from the third to the twelfth centuries AD, but a recently made comparative study has shown many similarities, particularly in the style of exposition and terminology, between the Bakhshālī work and Bhāskara I's commentary on the *Āryabhaṭīya.* This seems to indicate that both works belong to the nearly same period, although this does not deny the possibility that some of the rules and examples in the Bakhshālī work date from anterior periods.

The rules that occur in the extant portion of the Bakhshālī work are: (1) arithmetical operations such as addition, etc.; (2) general rules applicable to different kinds of problems such as the rule of three, *regula falsi*, etc.; (3) rules for purely numerical problems such as algebraic equations and arithmetical progressions; (4) rules for problems of money such as buying and selling, etc.; (5) rules for problems of travelers such as equations of journeys, etc.; and (6) rules for geometric problems such as the volume of an irregular solid. The Bakhshālī work employs a decimal place value notation with a dot for zero.

Śrīdhara, who flourished between Brahmagupta and Govindasvāmin, is one of the earliest mathematicians who wrote separate treatises for the two major fields, *pāṭī-gaṇita* and *bīja-gaṇtia*, although his work on the latter is known only from a quotation. He included many new topics such as combinations of the six tastes, the hundred fowls problem, the cistern problem, etc., in his *Pāṭīgaṇita.*

II.4 The ninth to fourteenth centuries – later developments

A follower of the Āryabhaṭa school of mathematics and astronomy, Govindasvāmin flourished in the first half of the ninth century in Kerala. His commentaries on Bhāskara I's *Mahābhāskarīya* and on the latter half of Parāśara's *Horāśāstra* (between 600 and 750) are extant, but three works of his, *Govindakṛti* on astronomy, *Govindapaddhati* on astrology, and *Gaṇitamukha* on mathematics, are known only from references and quotations by later writers.

Govindasvāmin shows a keen interest in the logical foundations of the rule of three. He provides a definition, with a detailed explanation, of the four terms (*pramāṇa*, *pramāṇa-phala*, *icchā*, and *ichhā-phala*) of the rule of three in his commentary on the *Mahābhāskarīya* (1.7), and, in three verses cited by Śaṅkara, compares these four terms to the constituent parts of the inference (*anumāna*) of Indian logicians. A typical inference according to them is as follows: "That mountain has fire because of its having smoke. That which has smoke has fire, like a kitchen." According to Govinda, the four terms of the rule of three correspond in order to the smoke and fire in the kitchen and the same two in the mountain, and thus the rule of three can be regarded as an inference.

A Jaina mathematician, Mahāvīra wrote a book for *pāṭī-gaṇita* entitled *Gaṇitasārasaṃgraha* during the reign of King Amoghavarṣa (ca. 814/815–80). The work is quite voluminous and comprises more than 1130 verses for rules and examples. He seems to be the first in India who admitted two solutions of a quadratic equation.

Bhāskara II was born in AD 1114 to a family which produced a number of scholars and literary men before and after him. He lived in Vijjaḍaviḍa at the foot of the Sahya mountain situated at the northern end of the Western Ghats, and completed his main work, *Siddhāntaśiromaṇi*, when he was 36 years old (AD 1150). He also wrote an astronomical manual, *Karaṇakutūhara*, in AD 1183, and a commentary (date unknown) on Lalla's *Śiṣyadhīvṛddhidatantra.* The *Siddhāntaśiromaṇi* consists of four parts. Two of them, *Līlāvatī* and *Bījagaṇita*, are on mathematics, and the other two, *Grahagaṇitādhyāya* and *Golādhyāya*, on astronomy. These four parts were often regarded as independent works.

The most popular among them was the *Līlāvatī*, which is a well organized textbook of *pāṭī-gaṇita* written in a plain and elegant Sanskrit. It circulated all over India and was commented upon in Sanskrit and in north Indian languages (such as Marāṭhī and Gujarātī) by a number of persons and translated not only into Indian languages of the north and of the south (such as Kaṇṇaḍa and Telugu) but also several times into Persian.

Bhāskara II included a whole theory of the pulverizer in the *Līlāvatī*, a book of *pāṭī*. This was possible because it required neither algebraic symbolism nor "intelligence" (*mati*) which were essential for *bīja-gaṇita* or algebra according to Bhāskara II.

The last chapter, "the nets of digits," deals with permutations of numerical figures. The last problem, for example, reads as follows: "How many varieties of numbers are there with digits placed in five places when their sum is thirteen? It should be told, if you know."

The *Bījagaṇita* is the culmination of Indian algebra. Bhāskara II's main contribution to algebra is his treatment of various types of equations of order two or more (up to six) with more than one unknown. The equations of higher orders are solved by reducing them to quadratic equations. In his solutions, the square nature and pulverizer, as well as the "elimination of the middle term" (which is the name given to the solution procedure of quadratic equations), play important roles.

Nārāyaṇa Paṇḍita wrote, in AD 1356, a book for *pāṭī-gaṇita*, *Gaṇitakaumudī*, in which he developed, among other things, theories of factorization, of partitioning, of combinatorics, and of magic squares. His work for *bīja-gaṇita*, *Bījagaṇitāvataṃsa*, seems, judging from its extant portion, to have been modeled after Bhāskara II's *Bījagaṇita.*

II.5 The fifteenth to seventeenth centuries – a new wave in the south

South India in this period produced many talented mathematicians and astronomers. Particularly important is the academic lineage headed by Mādhava of Saṅgamagrāma (fl. 1380/1420), which is often called the Mādhava school.

Mādhava, a resident of Saṅgamagrāma near Cochin in Kerala, was one of the most brilliant mathematicians in the world. His name is, and will be, remembered for his discovery of a power series expansion of π at least and perhaps also of those of trigonometric functions such as sine, cosine, versed sine, and arctangent. The verses that state these series are found not in his extant astronomical works but in the works of his successors.

In his commentary on the *Līlāvatī*, Śaṅkara explicitly ascribes two methods for calculating the circumference of a circle to Mādhava. Śaṅkara also cites Mādhava's verse which expresses, in the word-numeral notation (*Bhūtasaṃkhyā*), an approximation to π correct to 11 decimal places, $2827433388233/(9 \cdot 10^{11})$.

Nīlakaṇṭha Somayāji, son of Jātavedas, was born ca. 14 June 1444 in Kuṇḍapura near Tirur, Kerala, and studied under Dāmodara, son of Parameśvara, at Ālattūr, Kerala.

He wrote an elaborate commentary on the *Āryabhaṭīya* in about 1510. It shows his great talent in mathematics as well as in astronomy. To cite a few examples, he rediscovered the correct meaning of Āryabhaṭa's rule for sine-differences. He expressly states the incommensurability of the diameter and the circumference of a circle, although whether he has proved it or not is not known. He cites and proves Mādhava's formulas for interpolation in the sine table, and for the sum and difference of sines. He died after 1542.

Indian mathematics thus made unique, remarkable progress up to the sixteenth century. It was only in the 1720's that Jagannātha (1652–1744), at the request of his patron, Jai Singh Sawai (1688–1744), produced the first Sanskrit version of Euclid's *Elements* under the title *Rekhā-gaṇita* (Mathematics of Lines) from the Arabic version of Naṣīr al-Dīn al-Ṭūsī.

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CHAPTER 18

Calendar, Astrology, and Astronomy

Michio Yano

Another title of this chapter could be *jyotiḥśāstra*, or "science of heavenly bodies." Sometimes mathematics (*gaṇitaśāstra*) is regarded as a part of *jyotiḥśāstra*, but an independent chapter is given to mathematics in this book (see chapter 17). Thus, what is touched on in this chapter is *jyotiḥśāstra* minus mathematics proper. The remainder can be expressed in the three words in the title of this chapter. Topics are limited to those which would be useful for the students of Hinduism.

1 Vedāṅga Calendar

Even taking into account the refined taste of Vedic poets who refrained from describing natural phenomena in a direct manner, observational records of heavenly phenomena are scarce in the Vedic *saṃhitā* literature. Of course the poets were interested in the sky as nature, but they were less eager to engage in mathematical formulation of the periodic changes in the starry heaven. So there is nothing systematic in the *saṃhitā* texts that can be called mathematical astronomy. What we find in them is the hymns to the Sun and the Moon, and *nakṣatras*. It is even doubtful whether they knew the five planets as such (*grahas* in later texts), namely, as a special class of stars which are distinguished from the fixed stars.

It was as one of the six auxiliary branches (*aṅga*) for the pursuit of Vedic rituals that the earliest astronomical knowledge of ancient India was systematically described. This branch was called *jyotiṣavedāṅga*. The word *jyotiṣa* comes from *jyotiḥ*, "light (in the sky), luminary." The text of the *jyotiṣavedāṅga* survived in two recensions: that of the *Ṛgveda*, which is older and ascribed to a Lagadha belonging to the fifth century BC, and that of the *Yajurveda*, which belongs to somewhat later period (Pingree 1981: 10). The two recensions, consisting of 36 and 44 verses respectively, have many verses in common.

The following quotation is from the Ṛgvedic recension:

> The Vedas advanced for the sake of sacrificial rituals. And the sacrificial rituals were prescribed in the order of time. Therefore one who has known this science of prescription of time, namely, astronomy, has known sacrificial rituals. (Ārca-jyautisam 33, in Dvivedin 1907)

This verse gave a strong motivation to those who devoted themselves in the study of astronomy. Thus, for instance, Bhāskara II, the author of the *Līlāvatī* and one of the most popular astronomer-mathematicians in India, wrote in the middle of the twelfth century:

> First of all the Vedas advanced through the activities of sacrificial rituals, while sacrificial rituals are said to be dependent on time. Since knowledge of time is from this science, therefore astronomy is said to be a branch of Vedic studies. (Siddhāntaśiromaṇi; Grahagaṇitādhāya 1.1.9)

The main purpose of the *jyotiṣavedāṅga* was the preparation of a calendar in order to fix the date of sacrificial rituals. Nothing is written on planets. The calendar described here represents the earliest stage of Indian calendrical tradition. Almost all the important elements which characterize the Indian calendar are already found in this literature. The main feature of this calendar is the five-year cycle which is clearly stated as:

> A year is 366 days, 6 seasons, 2 *ayanas*, and 12 solar months. This, multiplied by five, is a *yuga*. (Yājuṣajyautiṣam 28cd, in Dvivedin 1907)
>
> (The number of) *sāvana* months, lunar months, sidereal months (in a *yuga*) are 61, 62, and 67 (respectively). A *sāvana* [month] has 30 days. A solar year is a turn of the stars. (Yājuṣajyautiṣam 31)

This statement can be tabulated as follows. Items in [ ] are not explicitly stated.

1 year	= 366 days	= 6 seasons	
	= 2 *ayanas*	= 12 solar months	
1 *sāvana* month		=30 days	
5 years	= 60 solar months		[= 1,800 solar days]
	= 61 *sāvana* months		[= 1,830 *sāvana* days]
	= 62 synodic months		[= 1,860 *tithis*]
	= 67 sidereal months		

This system requires two intercalary months (*adhimāsas*) in every five years, in other words, one intercalation in every two and a half years. This is clearly mentioned in Kauṭilya's *Arthaśāstra* (2.20.66). We can interpret the word *sāvana* here as equivalent to the modern adjective "civil", although there were different meanings of this word and this verse has been subject to different interpretations.[1]

With our interpretation it follows that

62 synodic months = 1,830 days, i.e., 1 synodic month ≈ 29.515 days.

Since this value is not very accurate (the modern value is 29.530589 days), the ancient Indian ritualists should have put an adjustment rather frequently – addition of one day is necessary about every 64 months – in order to keep the relation of the date of rituals and the lunar phases. But there are many textual evidences which show the use of this five-year *yuga*, for instance, the *Arthaśāstra* as mentioned above, the medical text *Suśrutasaṃhitā*, the Buddhist text *Śārdūlakarṇāvadāna*, and the Jain text *Sūryapannatti*. It was only after the introduction of Hellenistic astronomy that the calendrical values were improved to a useful degree.

2 *Nakṣatra* and Lunar Astrology

2.1 Two systems of nakṣatra

Sometime in the later Vedic period the meaning of the word *nakṣatra* shifted from its original sense of "star" in general to its narrower sense, namely, one of the 28 or 27 groups of stars which were regarded as the lunar stations along the ecliptic. The earliest list of *nakṣatras* in this sense is found in the *Taittirīyasaṃhitā* (Macdonell and Keith 1912: 409–31). The names of the *nakṣatras* show some variations and their grammatical forms do not always agree. In table 18.1 the most commonly used forms are given. When 28 *nakṣatras* are counted, Abhijit is put between Uttarāṣāḍhā and Śravaṇa. The most important difference between the 28 *nakṣatra* system and the 27 *nakṣatra* system is that in the latter each *nakṣatra* occupies an equal space of 13°20′ while in the former the distance is irregular. When, therefore, we find a *nakṣatra* name in Sanskrit texts, we should know which of the two system is intended. Since similar systems of unevenly spaced 28 lunar mansions are found in Chinese and Islamic astronomy, there have been disputes concerning their origin. In my view, the Chinese and Indian systems are independent while the Islamic *manāzil al-qamar* (stations of the moon) should have been influenced by the Indian *nakṣatras*.

The identification of stars comprising each *nakṣatra* poses even more difficult problems, but some secure identification can be made: for example, Kṛttikā is a group of stars corresponding to Pleiades, and Citrā is α Virginis (= Spica).

In the earlier Sanskrit texts *nakṣatras* are counted from Kṛttikā, while in the later period Aśvinī (β and γ Arietis) became the first mansion. The shift of the starting point of the *nakṣatra* system can be ascribed to the precession of the equinoxes. The time when Pleiades was near the vernal equinox is about

2300 BC. This is one of the reasons that some people claim the high antiquity and originality of Indian astronomy. There is also an attempt at deciphering some of the Indus scripts as the names of *nakṣatra*. Such attempts are not utterly impossible, but we must remember the difference between the *nakṣatra* as a well-defined coordinate system and the *nakṣatra* as a star or a group of stars in general.

According to the *jyotiṣavedāṅga* the sun's northern course (*uttarāyaṇa*) begins at its entry into the first point of the *nakṣatra* Dhaniṣṭhā, which, therefore, was regarded as the winter solstice. Since the *jyotiṣavedāṅga* used the equally spaced 27 *nakṣatra* system, it turns out that the vernal equinox was assumed to be at Bharaṇī 10°, which is 23°20′ distant from the first point of Aśvinī which, in the later period, was equated with the first point (*meṣādi*, i.e. "the beginning of Aries") of the ecliptic longitude. If this difference were accepted as the amount of the precession, and if we could admit that the accurate observation was made in the *jyotiṣavedāṅga* period, the text might be dated about 1,600 years before the time when Indian *nakṣatra* coordinate system was fixed. This would put the date of the *jyotiṣavedāṅga* in about 1300 BC. But here I should remind the reader of what I already said about Kṛttikā. What we can say about the date is only relative.

The *nakṣatra* system played a very important role in some aspects of Indian culture, i.e., calendar making, astrology and ritual.

2.2 **Nakṣatra** ***in calendar and astrology***

The lunar month was named after the *nakṣatra* where the full moon is located. Thus, for instance, Caitra is the month during which the full moon is stationed in Citrā. The relation of the *nakṣatras* and the month names is shown in table 18.1. This naming system must have been one of the reasons that the sidereal (*nirayaṇa*) position of the Sun and the Moon was more important than their tropical (*sāyana*) position. In this way the relation of month names to seasons could not but be sacrificed by the effect of the precession of equinoxes.

The *nakṣatras* played the central role in the earlier period of Indian astrology, namely, the period before the introduction of horoscopic astrology from the west. Predictions were made according to the *nakṣatras* where the moon is located. To each *nakṣatra* a deity from the Indian pantheon was assigned, for example, Agni to Kṛttikā and Prajāpati to Rohiṇī, etc. (see table 18.1). Thus *nakṣatra* worship became an important part of Indian rituals. Since little attention was paid to the position of planets nor to the solar position, I would call this type of astrology "lunar astrology" in contrast to horoscopic astrology where the planetary position against the background of the zodiacal signs is more important. Among the texts preserving this old lunar astrology are the *Śārdūlakarṇāvadāna* and the *Sūryapannatti* which we have mentioned above, and the *Pariśiṣṭa* of the *Atharvaveda*.

Table 18.1: Nakṣatras

No.	*Nakṣatra*	*Deity*	*Month name*
1	Kṛttikā	Agni	Kārttika
2	Rohiṇī	Prajāpati	
3	Mṛgaśīrṣa	Soma	Mārgaśira
4	Ārdrā	Rudra	
5	Punarvasu	Aditi	
6	Puṣya	Bṛhaspati	Pauṣa
7	Āśleṣā	Sarpa	
8	Maghā	Pitaras	Māgha
9	Pūrvaphālgunī	Bhaga	
10	Uttaraphālgunī	Aryaman	Phālguna
11	Hasta	Āditya	
12	Citrā	Tvaṣṭṛ	Caitra
13	Svāti	Vāyu	
14	Viśākhā	Indrāgnī	Vaiśākha
15	Anurādhā	Mitra	
16	Jyeṣṭhā	Indra	Jyaiṣṭha
17	Mūla	Nirṛti	
18	Pūrvāṣāḍhā	Toya	Āṣāḍha
19	Uttarāṣāḍhā	Viśvadeva	
	(Abhijit)	Brahmā	
20	Śravaṇa	Viṣṇu	Śrāvaṇa
21	Dhaniṣṭhā	Vasu	
22	Śatabhiṣaj	Varuṇa	
23	Pūrvabhadrapadā	Ajapāda	Bhādrapada
24	Uttarabhadrapadā	Ahirbudhnya	
25	Revatī	Pūṣan	
26	Aśvinī	Aśvin	Āśvina
27	Bharaṇī	Yama	

3 Nine *Grahas*

3.1 Grahas *in medical texts*

Most Hindu temples have a shrine which contains the images of *navagraha*, i.e. nine planetary gods. How and when Indian people began worshiping planets, after being indifferent to them for a very long time, is an interesting but difficult question.

The word *graha* is used in medical texts referring to the demons which possess children and cause mental diseases. For instance, there is a chapter devoted to "nine *grahas*" in the *Suśrutasaṃhitā*, which was compiled sometime around the third to fourth century AD. The nine *grahas* enumerated in this context have nothing to do with planets. Whether the coincidence of the number nine is accidental or not is yet to be investigated. That the *graha* in some other contexts in medical texts can be properly interpreted as a heavenly body complicates our problem. In one passage of the *Carakasaṃhitā* (3.3.4), *graha* is mentioned along with *nakṣatra*, which means that *graha* here is one of the heavenly bodies. It should be remembered that in this context the Sun and the Moon are separately enumerated and thus they are excluded from the group of *grahas*.

In three passages of the *Suśrutasaṃhitā* (1.6.19, 1.32.4, and 6.39.266cd–267ab) *grahas* as heavenly bodies are mentioned. Especially deserving attention is the second passage, because here we find the word *graha* along with *horā* in the enumeration of bad omens (*nimittas*) leading to a patient's death. The word *horā* is a phonetic transcription of the Greek word ὥρα which denotes (1) a spatial unit of half a zodiacal sign or (2) a temporal unit of the 24th part of a day, or (3) the first astrological place (ascendant). In Sanskrit texts on astrology this word became one of the basic technical terms.

This brief summary of *grahas* in medical texts would suffice to reveal the complicated nature of Sanskrit texts. One and the same word was used in different meanings according to the context, and new ideas were sometimes incorporated into the older text. Even if we limit our topic to the *grahas* as heavenly bodies, they are subject to a variety of meanings, as will be shown below.

3.2 **Grahas** *as planets*

Semantic change The concept of *graha* (etymologically meaning "one which seizes") as a heavenly body experienced at least the following stages of development.

1 A demon which eclipses the Sun and the Moon was called *svarbhānu* and, probably, *graha*.
2 The demon got the name Rāhu and, somewhat later, the tail of the truncated Rāhu was called Ketu.
3 Five planets were regarded as *grahas*.
4 The Sun and the Moon joined the *grahas*, and a group of seven or nine *grahas* was established, though without fixed order.
5 The week-day order of the seven *grahas* and the concept of the nine *grahas* were established.

This process shows an interesting semantic change where the Sun and the Moon, which were originally considered as "one which is seized" finally turned to be "seizer" (*graha*). The chronological order of the stages 2 and 3 is difficult to decide.

Eclipse demon Since a solar or lunar eclipse is one of the most conspicuous heavenly phenomena, it is difficult to imagine that it escaped attention of the

Vedic poets. In fact a demon called Svarbhānu in the *Ṛgveda* 5.40.5 seems to have been regarded as the cause of eclipses. This is the only occurrence of *svarbhānu* in the *Ṛgveda* and there is no evidence that this demon was identified as *graha*. In epics, however, *svarbhānu* is explicitly called "*graha*" (*Mahābhārata* 6.13.40ab and *Rāmāyaṇa* 3.22.11cd).

Old references to planets In one passage of the *Atharvaveda* a *graha* appears as an eclipse demon. The oldest text which mentions Rāhu as an eclipse demon is the *Chandogya-Upaniṣad* (8.13). The *Maitrāyaṇī-Upaniṣad* (7.6) enumerates Rāhu and Ketu along with Saturn (*śani*) as one of the semi-deities. But the date of these passages is problematical.

It is only after the period of Greek settlement in Bactria (third century BC) that explicit references to planets are attested in Sanskrit texts.

The *Arthaśāstra* is one of the oldest texts which clearly mentions Jupiter and Venus by the name Bṛhaspati and Śukra, respectively:

> Its ascertainment (is made) from the position, motion and impregnation of Jupiter, from the rising, setting and movements of Venus and from the modification in the nature of the Sun. From the Sun (is known) the successful sprouting of seeds, from Jupiter the formation of stalks in the crops, from Venus rain. (*Arthaśāstra* 2.24.7–8)

The data of the text has not been well established. Probably its oldest part was composed about two hundred years before Christ. It is evident that this passage concerns weather prognostics. Pingree (1981) regards such prognostics as of Babylonian origin.

Nine grahas without fixed order The *Gārgyajyotiṣa* (between BC and AD)[2] arranges the nine *grahas* in a strange order: Moon, Rāhu, Jupiter, Venus, Ketu, Saturn, Mars, Mercury, and Sun. Here Ketu (almost always in plural form) is not yet the tail of Rāhu but comets. The great epic *Mahābhārata* abounds in the enumeration of *grahas*,[3] but the order of the enumeration is not fixed, nor is the weekday order attestable. Ketu is not always included in the group of *grahas*. Often it is called *dhūmaketu* ("smoke-bannered") and no reference to the single "Ketu" without similar modification is found in the *Mahābhārata*. This evidence shows that there was a period in India when all the nine *grahas* were known but the order was not yet fixed.

To this period belongs the *Śārdūlakarṇāvadāna*. The passage which concerns us runs as follows:

> Now, oh Puṣkarasārin, I will talk about *grahas*. Hear about them. They are Venus, Jupiter, Saturn Mercury, Mars, Sun, and the Lord of the stars (Moon). (Mukhopadhyaya 1954: 53)

The *Modengqie Jing*, one of the Chinese translations of the *Śārdūlakarṇāvadāna*, adds in this context Rāhu and comets (or a comet) to the seven luminaries and regards "nine" *grahas* as making one group. What is more interesting is that the week-day order of seven *grahas* is attested in a passage of this Chinese translation (Taisho Daizôkyô, vol. 21: 410). It is quite puzzling to find the week-day order here, because the Chinese translation is said to have been made in the early third century AD. This was the time when the notion of the week-day was just introduced into India. In order to settle this puzzle we must hypothesize either (1) that the Chinese translators got some new information directly from the west or (2) that this passage was inserted in a somewhat later period.

Week-day order of grahas It was only after the transmission of Hellenistic astrology that the order of planets in India was fixed in that of the seven-day week. This order is the outcome of the combination of the Greek cosmological idea of concentric spheres and the Egyptian belief of the planetary gods presiding over the 24 hours (ὥρα mentioned above). In order to get the present order of week-day, the concentric spheres must be arranged in the order of Saturn, Jupiter, Mars, the Sun, Venus, Mercury, and the Moon. This order was known sometime in the second century BC. But the evidence of the earliest use of the week-day belongs to a considerably later period.

The first evidence of the introduction of Greek astrology in India is the *Yavanajātaka*. The text was translated into Sanskrit prose in AD 149/150 and it was versified in AD 269/270 by a Sphujidhvaja. Only the verse version is extant (Pingree 1978). Sphujidhvaja's version enumerates seven planets on many occasions, but it is only towards the end of the work (chapter 77) that the week-day order is attestable. This order does not seem to have been widely spread in that period in India. Neither Rāhu nor Ketu appears in this text. About a quarter century later, however, Mīnarāja in his *Vṛddhayavanajātaka* (about AD 300–25) describes planets in the week-day order, together with Rāhu, although he does not mention Ketu (Pingree 1976). Varāhamihira (mid-sixth century) does not regard Ketu as the tail of Rāhu but as comets.

The oldest Indian inscription which gives a date with a week-day is that of Thursday, 21 July, AD 484 (Fleet 1877: 80–4). The first astronomical text which defines the week-day is the *Āryabhaṭīya* of Āryabhaṭa (born AD 476). His definition is:

> These seven Lords of *horā* beginning with Saturn are (more and more) speedy in this order (of concentric spheres). Every fourth by the order of swiftness is the Lord of the day (which begins) with the sunrise. (ABh. 3.16)

What we can safely say is, therefore, that the week-day and the order of the days of the week gradually became known to Indian people at the end of the third century and it became wide spread about a century later.

Thus all the passages in Sanskrit texts which describe planets in the week-day order should be dated later than the end of the third century.

Graha worship section of the Gṛhyasūtras Many variations of the order of the nine *grahas* are found in the section of the rite of worshiping *grahas* (*grahayajña*) in the *Gṛhyasūtras*. Thus the time in which this rite originated belongs to the fourth stage mentioned above. Even in one and the same text we can find different orders of enumeration. In some texts the seven planets are arranged in the order of Sun, Mars, Moon, Mercury, Jupiter, Venus, and Saturn. Some texts presuppose the week-day order using the word *krameṇa* or *kramāt* ("by the order (of week-day)"). Such texts or at least this part of such texts must be considered as belonging to the later period.[4] Old elements are of course preserved and repeatedly appear in the later texts, so we cannot use such elements as the means of fixing the lower limit of the date of a document, while new elements can surely serve as the criterion to judge the upper limit of the text, or at least the part of the text which contains them.

4 Classical Period

4.1 New aspects

Zodiacal signs A great change of the *jyotiḥśāstra* resulted from the introduction of Hellenistic astrology and astronomy into India. The most remarkable element in this was the important role played by the seven planets. Other new elements transmitted to India were the twelve zodiacal signs beginning with Aries and the twelve astrological places beginning with the ascendant. The first point of Meṣa (a translation of the Greek word corresponding to Aries) was equated with the first point of the *nakṣatra* Aśvinī. With the relation 27 *nakṣatras* = 12 zodiacal signs (i.e., 9/4 *nakṣatras* = 1 zodiacal sign), one zodiacal sign was equated to "nine quarters" of a *nakṣatra*.

Referring system The Sanskrit names for zodiacal signs are translations of the Greek words and in some texts we find phonetic transliterations from Greek. However, there is a remarkable difference between the western zodiacal signs and Indian signs (called *rāśi*). In Indian astronomy the precession (*ayana*) of equinoxes was not taken into account and the initial point of the ecliptic coordinates was fixed sometime about AD 285.[5] This is the so-called *nirayaṇa* system. In the course of time, therefore, the true vernal equinox (V.E. in figure 18.1), moving backward, separated from the initial point (*meṣādi*) of the *nirayaṇa* longitude. The amount of this difference, called *ayanāṃśa*, has accumulated to some 23.5 degrees in modern times. This is the reason why the day of

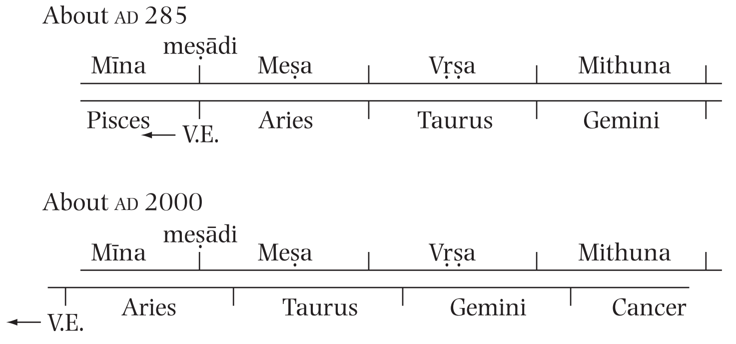

Figure 18.1: Precession of equinoxes.

Table 18.2

Saṃkrānti	*Present date*	*Lunar month*
Meṣa (Aries)	April 14	Caitra
Vṛṣa (Taurus)	May 14	Vaiśākha
Mithuna (Gemini)	June 15	Jyaiṣṭha
Karkaṭa (Cancer)	July 16	Āṣāḍha
Siṃha (Leo)	August 16	Śrāvaṇa
Kanyā (Virgo)	September 17	Bhādrapada
Tulā (Libra)	October 17	Āśvina
Vṛścika (Scorpion)	November 16	Kārttika
Dhanus (Sagittarius)	December 15	Mārgaśira
Makara (Capricorn)	January 14	Pauṣa
Kumbha (Aquarius)	February 13	Māgha
Mīna (Pisces)	March 14	Phālguna

Meṣasaṃkrānti, which was originally equivalent to the vernal equinox, now falls on around April 14. This also explains the date of Makarasaṃkrānti, the festival of winter solstice, falling on around January 14.

4.2 Calendar system

Month names The traditional Indian calendar is essentially luni-solar, although the solar calendar is also used additionally in some regions. In order to keep the relation of the lunar months with the change of seasons, the lunar month name was related to the solar month. First of all the solar month was determined by the "entry" (*saṃkrānti* or *saṃkramaṇa*) of the sun into each of the zodiacal signs. The relation between *saṃkrānti* and the lunar month names is shown in table 18.2. Thus, for example, the lunar month Caitra is defined as the month which contains the Meṣasaṃkrānti and Vaiśākha is the month which contains the

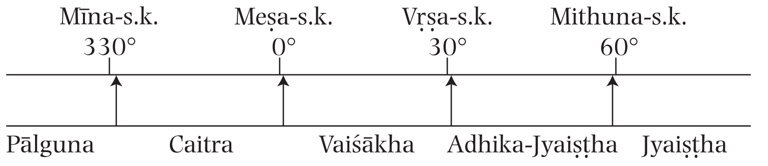

Figure 18.2: *Adhimāsa* (amānta system).

Vṛṣasaṃkrānti, etc. For the sake of the reader's convenience the approximate date of *saṃkrāntis* in the modern calendar is given in the second column.

Amānta and Pūrṇimānta One should remember that there are two different systems of naming the month, i.e. *amānta* ("new moon ending") and *pūrṇimānta* ("full moon ending"). In the bright half month (*śukla-pakṣa*) nothing is different, but in the dark half month (*kṛṣṇna-pakṣa*), the *pūrṇimānta* month name is ahead of the *amānta* month name by one. Roughly speaking, sough Indian calendars follow the *amānta* method, while in north India the *pūrṇimānta* system has been used since ancient times.

Intercalary month (adhimāsa) Since a lunar month (synodic month) is a little shorter than a solar month, sometimes there occurs a lunar month which does not contain any *saṃkrānti*. Such an additional month (*adhimāsa*) was called by the name of the following month prefixed by *adhika-*. In the example of figure 18.2 the month after Vaiśākha is called Adhika-Jyaiṣṭha. An *adhimāsa* was regarded as inauspicious and no religious ceremony was performed during this month. Thus it is also called *malamāsa* ("impure month").

Omitted month (kṣayamāsa) In very rare cases there occurs a month which contains two *saṃkrāntis*. In such cases the second *saṃkrānti* does not contribute to the naming of the month and it is omitted as *kṣayamāsa*. When such a case happens there are inevitably two *adhimāsas*, one before and the other after the *kṣayamāsa*. In such years the first *adhimāsa* is called *saṃsarpa* and the rituals can be performed as usual. The month containing two *saṃkrāntis* is called *aṃhaspati*, while the second *adhimāsa* is the *adhimāsa* proper when rituals are not performed (see Kane 1962: vol. 5, p. 671).

The possibility of *kṣayamāsa* was first mentioned by Bhāskara II in his *Siddhāntaśiromaṇi*. He correctly remarks that a *kṣayamāsa* usually occurs in every 141 years and sometimes after the interval of 19 years. According to him, *kṣayamāsa* is possible only during the three months beginning with Kārttika. In figure 18.3 I have shown the case where Pauṣa is omitted. The *kṣayamāsa* shows the theoretical nature of the Indian calendar.

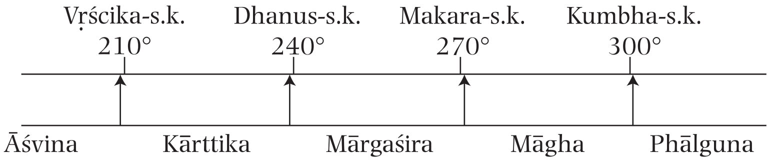

Figure 18.3: *Kṣayamāsa* (amānta system).

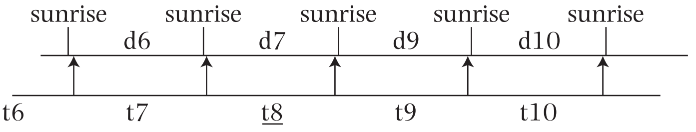

Figure 18.4: *Kṣayadina.*

Omitted day (kṣayadina) Since the average length of a synodic month is about 29.5 days, a month in a lunar calendar consists of either 29 days or 30 days. In the Indian calendar the concept of *tithi* plays an important role in order to determine whether a month contains 29 days or 30 days.

In the earlier period, a *tithi* was simply defined as one-thirtieth of a lunar month, but later it was more exactly defined as the period of time in which the increment of the lunar longitude over the solar longitude becomes 12 degrees. In both definitions one month consists inevitably of 30 *tithis*. A month is divided into two halves (*pakṣas*), a bright half (*śuklapakṣa*) and a dark half (*kṛṣṇapakṣa*) each having 15 *tithis*. Since the sunrise is the beginning of a civil day, the ordinal number of a civil day in a half month is determined by the *tithi* which is current at the sunrise. For instance, as in figure 18.4, if the sixth *tithi* (t6) is current at a sunrise, the day is called "sixth day" (d6). In this example, however, the next day (d7) contains two ends of *tithi* (t7 and t8) and at the sunrise of the following day the ninth tithi (t9) is current. Thus the eighth *tithi* does not contribute to the naming of the civil day. The eighth day (d8), therefore, is omitted from this half month. This omitted day is called *kṣayadina*.

Additional day (adhikadina) Quite opposite to the omitted days, sometimes it happens that a *tithi* contains two sunrises. In such case the same date is repeated and the second one is called *adhikadina.* In the example of figure 18.5 the eighth day (d8) of this half month is repeated.

Pañcāṅga The traditional Indian calendar is called *pañcāṅga*, i.e., "that which is consisting of five elements." The five elements are *vāra* (days of week), *tithi*, *nakṣatra*, *karaṇa*, and *yoga*. The *karaṇa* is a time unit of half of *tithi*, and the

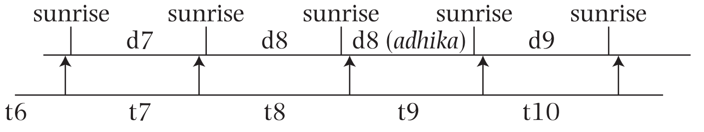

Figure 18.5: *Adhikadina*.

yoga is based on the "sum" of the longitude of the Sun and the Moon. The present author has written a computer program for *pañcāṅga* based on the *Sūryasiddhānta*. The program is available at my web page (http://www.kyoto-su.ac.jp/~yanom/).

4.3 Mathematical astronomy

Sanskritization The planetary astrology of Hellenistic origin gave a strong motivation for Indian people to learn Greek astronomy. How they got access to Greek astronomical texts and which texts were studied are not known yet. When the first Sanskrit astronomical text based on Greek astronomy appeared it was already in a well established form. The text was the *Āryabhaṭīya* of Āryabhaṭa (born in AD 476). During the interval of 300 years between the first *Yavanajātaka* and the *Āryabhaṭīya* Indian astronomers must have been occupied with the task of Indianizing and Sanskritizing Greek astronomy. The astronomical texts of this period were all lost, but fortunately we have Varāhamihira's *Pañcasiddhāntikā* (Neugebauer and Pingree 1970) where the five astronomical schools which were known in his time were summarized. They are: the *Paitāhamasiddhānta*, the *Vasiṣṭhasiddhānta*, the *Romakasiddhānta*, the *Pauliśasiddhānta*, and the *Sūryasiddhānta*. Varāhamihira evaluates them by the following words:

> The Pauliśa is accurate, that which was pronounced by Romaka is near it; the Sāvitra (i.e. the Sūrya) is more accurate; the remaining two have strayed far away (from the truth). (PS 1.3; Neugebauer and Pingree 1970: 27)

In fact the *Paitāmahasiddhānta*, belonging to the *vedāṅga* tradition, was obsolete already in Varāhamihira's time. What characterizes the *Vasiṣṭhasiddhānta* is period relations and linear zigzag functions which are probably of Babylonian origin. No geometrical method is used in these two older schools of astronomy. The *Pauliśa* and *Romaka*, which are highly regarded, are of western origin as the names suggest. These two texts were, according to Varāhamihira's own statement, commented upon by a Lāṭadeva. The author of the *Sūryasiddhānta*, the most accurate of the five *siddhāntas*, is not known. This text is different from the later and more popular *Sūryasiddhānta*.

Table 18.3

Fixed stars	R_s	1,582,237,828
Solar years	Y	4,320,000
Civil days	$D = R_s - Y$	1,577,917,828
Sidereal months	R_m	57,753,336
Synodic months	$M = R_m - Y$	53,433,336
Intercalary months	$A = M - 12Y$	1,593,336
Tithis	$T = 30 \times M$	1,603,000,080
Omitted days	$K = T - D$	25,082,580

Pre-Ptolemaic system What is to be stressed here is that the Greek astronomy transmitted to India was not the system of Ptolemy who was active in the middle of the second century in Alexandria, but that of certain schools belonging to the earlier period. This can be shown from several technical aspects: for example, the effect of the evection (second anomaly) in lunar motion, the equant point in the planetary model, and the spherical trigonometry, which were first used by Ptolemy, are all absent in Indian astronomy.

It is interesting that Indian astronomy preserved some older elements of Greek astronomy which disappeared in their homeland. One of the good examples in this respect is trigonometry. It was by tracking backward along this line of transmission that the first chord table ascribed to Hipparchus (fl. 150 BC) was recovered by G. J. Toomer (Toomer 1973: 6–28) from an Indian sine table. Toomer showed that some numerical values ascribed to Hipparchus in Ptolemy's *Almagest* could be explained by hypothesizing the use of this reconstructed table. It does not follow, however, that the Indian astronomers were only uncritical receivers of Greek astronomy. Rather, they introduced the foreign elements in a very limited time through a very small number of texts, and after this initial stage of introduction all the developments were made by themselves without foreign influence.

With the introduction of Greek astronomy, Indian astronomical constants were greatly improved. After Āryabhaṭa the constants were given as the rotations in a *mahāyuga* (4,320,000 years) or in a *kalpa* (4,320,000,000 years).

The number of civil days (D) in a *mahāyuga* is the difference between the number of the rotations of fixed stars (R_s) and those of the Sun (Y). Similarly, the number of lunar months (M) is the difference between the Moon's rotations (R_m) and those of the Sun. As we have seen in the *jyotiṣavedāṅga*, the difference between the number of lunar months and that of solar months ($12Y$) is the number of intercalary months (A). In the same way the difference between the number of *tithis* (T) and that of civil days is the number of omitted days (K).

Let us give a set of these numbers according to the later *Sūryasiddhānta*, belonging to about the eighth or ninth century. From table 18.3 we can get the length of a solar (i.e. sidereal) year and that of a synodic month:

$$\text{solar year} = \frac{D}{Y} = 365.258757,$$

$$\text{synodic month} = \frac{D}{M} = 29.530588.$$

These numbers are good enough to prepare a calendar. This is why there still survive some traditional calendars which are based on the *Sūryasiddhānta* and which give almost the same results as my computer program mentioned above.

Kali yuga epoch Since the mean positions of the Sun and Moon and the planets are the function of time, one should assume a certain time as the initial point of calculation. The class of astronomical literature called *karaṇa* is characterized by the use of an epoch which is not very far back from the time of the text, while the *siddhānta* texts employed an epoch in a very remote past where all the planets were assumed to be in mean conjunction at the starting point of the ecliptic coordinates. The epoch was sought by means of indeterminate equations of the first degree – at a certain time true positions were observed and they were converted into mean positions, while the mean motions of all the planets had been somehow known. The epoch arrived at by this method was midnight of February 17/18 in 3102 BC according to the midnight (*ārdharātrika*) school, and the sunrise of February 18 (Friday) of the same year according to the sunrise (*audayika*) school. That the epoch was very accurately chosen can be demonstrated by the fact that even today we can get a fairly good calendar using this epoch and the constants of the *siddhānta* texts.

Planetary theory The numbers for the rotations of planets are also given in astronomical texts. They vary slightly according to the schools (Pingree 1981: 15). Let us give here (table 18.4) those from the later *Sūryasiddhānta*.

Table 18.4

Rotations in a Mahāyuga	
Fixed stars	1,582,237,828
Saturn	146,580
Jupiter	364,212
Mars	2,296,832
Sun	4,320,000
Venus's *śīghra*	7,022,364
Mercury's *śīghra*	17,937,076
Moon	57,753,336
Moon's apogee	488,203
Moon's node	–232,246

The "rotations of the fixed stars" in this table is the "rotations of the earth on its axis" according to *Āryabhaṭa*, because he thought that the earth was moving while the stars were fixed. But no Indian astronomers followed his idea until Nīlakaṇṭha of the sixteenth century. The different behaviors of the outer planets (Saturn, Jupiter, and Mars) and the inner planets (Venus and Mercury) were known to Indian astronomers. The numbers given for the outer planets are their sidereal rotations and those of their *śīghra* (literally "swift one") are equal to that of the Sun. Those numbers for the inner planets are the rotations of the *śīghra*, while their sidereal rotations are equal to that of the Sun. From modern astronomical point of view, therefore, the *śīghra* can be interpreted as the mean Sun. Since, however, ancient Indian planetary theory was geocentric, they regarded the *śīghra* as rotating on the geocentric orbit of the planet and constantly drawing the planet in its direction.

After computing the longitude of the mean planets as the function of time since epoch, the true position was obtained by means of the eccentric-epicyclic theory. Here is a remarkable difference from Ptolemy's theory. Ptolemy combined two effects which cause the irregular motions of planets, namely, that of eccentricity and that of anomaly, and put them together in a single geometric model. In doing so he had to introduce a controversial point, which was later called the "equant," outside the center of the eccentric circle. In India, on the other hand, the two effects were kept separate and no unified model was conceived. One was called the *manda* (literally "slow one") epicycle which explains, in modern words, the combined effects of the eccentricities of the Sun and the planet, the other was called the *sīghra* epicycle as mentioned above. The effects of these two elements for each planet were separately tabulated. The procedures of using the two tables in order to get the final equation (*antyaphala*) show some variations depending on the school. Thus we can say that Indian planetary theory is not totally geometrical. Once they introduced geometrical models from the west, they developed their own functional method.

It is worth mentioning that Āryabhaṭa's school survived in south India, especially in Kerala, and was revived as the Mādhava school in the fourteenth century. The culmination of this school is Nīlakaṇṭha (1444 to after 1542), who tried to combine the two effects in a single geometrical model, and the result was quite similar to the partial heliocentric model of Tycho Brahe.

Notes

1 I follow Dr. Ohashi's interpretation. Cf. Yukio Ohashi, 1993.

2 *Yugapurāṇa*, a part of the *Gārgya-jyotiṣa*, was edited and translated by John E. Mitchiner, Calcutta 1986.

3 MBh.2.11.20; 3.3.19; 6.3.11–17; 13.151.12 etc. Thanks are due to my friend, Prof. M. Tokunaga by whom the whole text of the *Mahābhārata* has been digitalized.

4 Some other examples are: *Rāmāyana* 1.17 (Rāma's horoscope), *Mudrārākṣasa* 4.19 (horoscopic prediction), and a part of the *Atharvaveda-pariśiṣṭa*.

5 The time when the longitude of α Virginis was 180°. Cf. Chatterjee 1998.

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CHAPTER 19

The Science of Medicine

Dominik Wujastyk

The Beginnings of Medical Science

Indian medicine, as a systematic and scholarly tradition, begins historically with the appearance of the great medical encyclopedias of Caraka, Suśruta and Bhela about two thousand years ago.[1] These are the oldest Indian medical texts we have, and also the most influential. Just as Pāṇini's famous linguistic study of Sanskrit leaps into the historical record fully formed, like the Buddha from Queen Maya's side, so the medical encyclopedias too emerge with a learned medical tradition in an almost fully articulated form.

The antecedents

In the case of Pāṇini, we do have some preceding literature, which shows us traditional Indian linguistics in its childhood, so to speak, notably the *Nirukta* of Yāska, as well as the various *śikṣā* and *prātiśākhya* texts. But in the case of medicine far less precursory material has survived. Early medical texts which are now known only by name include the *Jatūkarṇatantra*, the *Hārītasaṃhitā*, the *Parāśarasaṃhitā*, and the *Kharanādasaṃhitā*, all of which apparently existed at the time of Śivadāsa who commented on the *Carakasaṃhitā* in the fifteenth century. Other lost works include the *Viśvāmitrasaṃhitā*, the *Atrisaṃhitā*, the *Kapilatantra*, and the *Gautamatantra* (Roy 1986: 157–9 and Meulenbeld 1999–2002: Ia.145–79, 369–71, 689–99). But even before these specialist treatises on medicine, there is a certain amount of material on the history of medicine which can be recovered from earlier, chiefly religious, texts.

Medicine in Vedic times It is often claimed that āyurveda evolved organically from the medical traditions discernible in Vedic literature. The respected scholar Mira Roy, for example, draws attention to several areas of apparent continuity between the Vedic concepts, especially from the *Atharvaveda*, and the āyurvedic compendia (1986:6.155f.). One of the examples she cites is the fact that five vital breaths are mentioned in both the *Atharvaveda* and the *Carakasaṃhitā* (AV 10.2.13, Ca.sū.12.8).

But on closer examination, all of these supposed parallels break down. Thus, it is true that Caraka's *Compendium* does have a discourse on the five vital breaths. This discourse is put into the mouth of a scholar called Vāyorvida ("he who knows about air"), who presents his theory as a cornerstone of physiology. As soon as he finishes his description, another scholar, Marīci, disputes his statement impatiently, saying (Ca.sū.12.9):

> Even if this is so, what is its general relevance to the purpose of this discussion or with knowledge of medical science? This is a discussion on the subject of medical science!

Vāyorvida tries to defend his point of view briefly, but without introducing any new ideas, and Mārica proceeds to put forward his own view that fire (*agni*) is the cornerstone of medicine. This too is superseded by the sage Kāpya with yet another view that *soma* is the cornerstone, and so the discussion continues. The conclusion presented by the chairman of the debate, Punarvasu Ātreya, is that while he regrets contradicting anyone, health ultimately comes down to a balance of the three humors (*doṣas*) (Ca.sū.12.13).

All we can really deduce from these passages is that a doctrine of five breaths existed at the time of the composition of the medical encyclopedias. Of course this is well known: the five breaths are already discussed in the much earlier literature of the Upaniṣads and Brāhmaṇas. But although the doctrine of the breaths is mentioned in the early medical texts, it does not become an important part of medical thought or practice until the composition of a much later work called the *Āyurvedasūtra*. This synthetic work, probably written in the early seventeenth century, tries for the first time to combine doctrines from āyurveda and a form of tantric yoga (Meulenbeld 1999–2002: IIa.499 ff.).

Roy herself finally concludes that in spite of some superficial similarities,

> Āyurveda, which incorporates different traditions [from the Veda], has a distinct place alongside of the Vedas. . . . Although glorified as an appendage of Vedic literature, Āyurveda as such is not mentioned there. (1986:6.156)

Roy points out that although a later Vedic text, the *Ṛgvedaprātiśākhya* (16.54), refers to a medical treatise called *Good Medicine* (*subheṣaja*), it is the *Mahābhārata* that first refers to medicine as a science of eight parts (*cikitsāyāṃ aṣṭāṅgāyāṃ* 2.50.80), and uses the word "āyurveda" as the name of the science of medicine (12.28.44, 12.328.9, 12.330.22).

The *Compendium* of Caraka contains a passage in which the physician is advised on how to respond, when pressed by questioners on the subject of which Veda his science belongs to (Ca.sū.30.21). He should answer that he is devoted to the *Atharvaveda* because that Veda prescribes rituals and prayers to enhance and prolong life, and this is the purpose of medicine too. The context suggests that this passage should be read as a slightly knowing suggestion, in which the physician is being advised to claim allegiance to a Veda because his interlocutor requires it of him, and as part of a didactic strategy, rather than for any more fundamental reason connected with real historical connections. It is tempting to read Roy's arguments above, and others like them, as adhering to exactly this ancient recommendation.

If āyurveda does not derive from Vedic medical traditions, what then are its antecedents?

This has been one of the most outstanding problems for the history of āyurveda for most of the last century. One serious suggestion which has recurred in the literature on āyurvedic history is that some of the the innovative doctrines of āyurveda were taken from Greek physicians in Gandhāra. Jean Filliozat tested this idea in his book on classical Indian medicine, and indeed found parallels between Indian and Greek thought, especially regarding the doctrines of breath (Skt. *prāṇa*, Grk. *pneuma*) (Filliozat 1964). But Indian medical literature has no loan-words from Greek, and is in this respect quite different from the Indian astral sciences (*jyotiḥśāstra*) which have borrowed many items of Greek vocabulary. There are philologically puzzling words in āyurveda, for example *jentāka*, meaning a steam bath or sauna. This is almost certainly not a Sanskrit word in origin, but it is not from the Greek either, and its origin has not yet been traced. In fact, Michio Yano has, as reported elsewhere in this volume, discovered one Greek word in the early Sanskrit medical corpus. The word *horā* (ὥρα) occurs in Suśruta's *Compendium* (Su.sū.32.4) in a passage listing omens which foretell the death of a patient. If the patient's zodiacal sign (*horā*) has burning lights or meteors in it, the patient is doomed. This proves that the compiler of this part of the text was already aware of the Hellenistic astrology that became available in India during the second century CE. But this makes it even more striking that not one Greek loanword for a medical term appears in Sanskrit medical literature. Indian physicians almost certainly had the opportunity to imbibe Greek medical ideas, but apparently no motive.

Until recently, few other serious ideas had been mooted for the origin of āyurveda. The conjecture that āyurveda embodies traditions that somehow came from the Indus valley civilization is tempting, of course, but impossible to establish. Scholars working within a traditional framework have tended not to engage with the problem, because of the strong traditional belief that āyurveda is indeed a continuation of medicine from the Vedic *saṃhitās*. Many texts on the history of āyurveda, even written by contemporary scholars, start by repeating the mythological accounts given in the beginning of the saṃhitās in which medicine is passed from the gods to the humans through a chain of divine beings and spiritual teachers. Such scholars seem unable or unwilling to see such an account for what it is, a common frame for initiating any orthodox śāstra, which occurs in variant forms at the beginning of a number of other major texts, such as the *Bṛhajjātaka*, and in various places in purāṇic literature (Pollock 1985; Zysk 1999).

Accounts of origins cast as historical discourses can be considered as having two dimensions: a horizontal and a vertical, rather as Ferdinand de Saussure divided linguistic study into orthogonal diachronic and synchronic dimensions. The horizontal dimension is that of mundane time: history in this dimension is a narrative of the events of past times. The vertical dimension measures closeness to God: the history of this dimension is the account of how the present manifest situation has evolved, or descended, from an original, pristine world of absolute unity. When at the start of a Sanskrit text we are told by the author, as so often happens, that the work once consisted of millions of verses, but was handed from the original omniscient sages to human scholars only in abbreviated form, we must understand that we are dealing with vertical history. This is the story of how knowledge – which is essentially of God – has come to us mere mortals. Such a spiritual narrative is not to be confused or conflated with horizontal history, although the narrative may be cast in the language of past tenses and linear teacher–pupil descent. What we are being told is how the present work is an imperfect reflection of divine omniscience, a mirror – and many Sanskrit texts are called "Mirrors" of this or that subject – of what is known in heaven. So when, at the start of the foundational texts of Sanskrit medicine, we are told of the passage of medical knowledge from the gods to ancient sages such as Dhanvantari and Ātreya, and thence to other humans such as Agniveśa and Suśruta, to Caraka and Nāgārjuna, we do not necessarily need to try to grasp all these figures as historical personages in the horizontal dimension. We are in the presence, rather, of a kind of *apologia*, an explanation of how something which was (past tense!) perfect, is now presented, brought into the present, in the blemished, mundane form of a textbook. It is an account of how knowledge which was once privileged is now commonly accessible.

It was Debiprasad Chattopadhyaya who first began to grapple with the sociology of Indian medical history in his fascinating book *Science and Society in Ancient India* (Calcutta, 1977). In that text he presented strong arguments for considering the early medical encyclopedias to be nonreligious, empirically oriented works which had undergone a secondary process of "Hinduization," in order to make them into works acceptable to a Hindu brahmin elite. Chattopadhyaya, writing from a Communist perspective on Indian history, had his own motivations for discovering materialist and empirical traditions wherever possible in Indian intellectual history, and this probably biased many readers against accepting his conclusions about the history of Indian medicine. In the case of āyurveda, however, there is much to commend his arguments. But even Chattopadhyaya was not able to suggest where this empirical tradition came from.

Medicine in the Buddhist community Evidence for the beginnings of a systematic science of medicine in India appears first in the literature of the earliest Buddhists, with many medical tales being recounted in the *Tripiṭaka*. The Buddha instructed his monks to care for each other in sickness, since they had abandoned the social structures which would have provided them with treatment if they had not left their families to become monks.

> You, O bhikkhus, have neither a mother nor a father who could nurse you. If, O bhikkhus, you do not nurse one another, who, then, will nurse you? Whoever, O bhikkhus, would nurse me, he should nurse the sick. (*Mahāvagga* 8.26.3, cited in Zysk 1998: 41)

The earliest Buddhist monks seem to have concentrated on providing medical help only for each other, but before long the lay community started to request help from the monks. Zysk (1998) has collected evidence to show that early Buddhist monasteries included infirmaries and had standing instructions to aid all those who were sick, not only monks.

Buddhist monks thus seem to have taken an active attitude to their own health and that of their lay supporters. This attitude may have been encouraged by the many medical epithets and turns of phrase attributed to the Buddha in the recorded sermons. In his parables he often used images such as "removing the arrows of suffering." One of the forms in which the Buddha has been revered since at least the first century CE is as the "Medicine Buddha" (*bhaiṣajyaguru*), and there is even a sūtra devoted to him under this name (Zysk 1998: 62).

Zysk's research into the medical materials recoverable from the Buddhist canon has revealed close similarities with the classical Sanskrit sources on medicine. It now seems almost certain that the foundations of classical āyurveda were being laid at the time of early Buddhism in the Buddhist and other ascetic communities.

In the centuries of Buddhist missionary expansion, Indian medical doctrines were carried across the Himalayas into Central Asia and beyond, as well as into Sri Lanka. The rare manuscripts that have survived from this diaspora, such as the Bower Manuscripts, share a common character: they are practical handbooks, manuals listing ailments and explaining the herbs and compounds that should be administered to cure them (Wujastyk 2001: ch. 4). There is little theory, little explanation, little philosophy. In this they differ from the classical compendia of āyurveda.

It is also possible that some important authors of Sanskrit medical texts, such as the famous Vāgbhaṭa, were Buddhists.

The Medical Body

The medical system which evolved from this ascetic milieu contained a sophisticated set of doctrines, supported by close observation and long experience of treating patients.

The body to which Indian medicine addresses itself is the physical body as understood to the senses and to empirical examination. In particular, āyurveda knows no *cakras*, nor the spinal conduits of breath (*prāṇa*) known from tantric literature. The concept of the *cakras* has today entered public consciousness world-wide, and is widely viewed as an ancient and immutable element of the Indian world view. This view needs to be qualified in two directions. First, the idea of the *cakras* is a relatively recent development in Indian tantric thought. It is datable only to the tenth century CE, making its appearance in texts such as the *Kubjikāmatatantra* and the *Mālinīvijayottaratantra* (Heilijgers-Seelen 1990). Secondly, the *cakras* make no appearance whatsoever in āyurveda. Notwithstanding the contemporary growth of various forms of massage and therapy focussed on the *cakras*, there is no such theme in the classical Sanskrit literature on medicine. The *cakras* really are an idea specific to *tantra* and yoga, and it is not until relatively recent times that this idea has been synthesized with medical thought and practice.

With a customary Indian interest in itemization (Smith 1994), the āyurvedic literature is keen to enumerate the receptacles, ligatures, conduits, orifices, and tissues which can be found in the human body. The *Śārṅgadharasaṃhitā* (ca. 1300) offers a fairly standard and clearly-presented version of such a list (Wujastyk 2001: 322–8). There are: 7 receptacles (*āśaya*); 7 body tissues (*dhātu*); 7 impurities of the body tissues (*dhātumala*); 7 subsidiary body tissues (*upadhātu*); 7 membranes (*tvac*); 3 humors (*doṣa*); 900 sinews (*snāyu*); 210 ligaments (*sandhi*); 300 bones (*asthi*); 107 lethal points (*marman*); 700 ducts (*sirā*); 24 pipes (*dhamanī*); 500 muscles (*māṃsapeśī*); 20 extra ones for women; 16 tendons (*kaṇḍarā*); 10 orifices of the male body; 13 orifices of the female body. Although these items may not in all cases be organs in the modern biomedical sense (Zimmermann 1983), there is a definite sense that āyurveda views the body as a locus of medical organs and processes which would be recognizable in general terms to a modern anatomist. After making his own much earlier enumeration of anatomical parts, Caraka noted, perhaps wistfully, that (Ca.śā.7.17),

> The parts of the body cannot, however, be counted because they are divided into tiny atoms (*paramāṇu*), and these are too numerous, too minute, and beyond perception. The cause of the conjunction and separation of these tiny atoms is wind (*vāyu*) and an innate disposition to action (*karmasvabhāva*).

This demonstrates an acute sense of the limits of possible scientific investigation, but at the same time contains fascinating and plausible suggestions about the nature of these "tiny atoms." Throughout medical and scientific discourse in Sanskrit, "wind" often appears in contexts which would, in early European scientific discourse, require the word "force."

The metabolic process

The central process of the body is digestion. The Sanskrit words for the processes of digestion (*pācana, dīpana*) all imply "cooking" or "burning." And the digestive force itself is simply called the "fire" (*agni*), or "fire in the belly" (*jāṭharāgni*). Once food has been eaten and cooked by this digestive fire, it turns into the first of the seven "body tissues" (*dhātu*), namely chyme or chyle (*rasa*), the pulpy juice to which food is reduced in the stomach. Then the other principle of heat in the body, choler (*pitta*), goes to work and the chyle is transformed into the next body tissue in the chain, blood. Blood transforms into flesh, and similarly the remaining tissues, fat, bone, and marrow; are converted one into the next, until the seventh and highest essence of the body is generated: semen. This, of course, suggests a purely male view of the body, and āyurveda's picture of women's metabolism includes no obvious equivalent to semen: the evolution of the chain of body tissues does not seem to fit the substances in a woman's body. One passage in Suśruta's *Compendium* locates menstrual blood in the place of semen; another seems to suggest a certain degree of homology between male semen and female breast-milk. Yet another passage suggests that two women having intercourse may "somehow" (*kathaṃcana*) produce semen (Su.ni.10.18–23ab, Su.śā.2.47). Āyurveda understands conception as the union of male semen and female menstrual blood (there is no concept of "ovum"). It is the woman's blood discharged during menstruation, but retained during pregnancy (when it is transformed into breast-milk), which joins with male semen and goes towards building a child's body.

Suśruta's *Compendium* gives the time scale for this principle metabolic process (Su.sū.14.10–16). The nutritive juice (*rasa*) spends about 108 hours in each of the body tissues. Thus, it takes a lunar month for the nutritive juice to become semen, or menstrual blood. The total time spent in metabolizing is 648 hours. In a curious and interesting verse, Suśruta notes that, "This nutritive juice (*rasa*) flows throughout the whole body like a tiny particle, in a manner similar to the propagation of sound, light, and water."[2] However, this is not the normal āyurvedic conception of how fluids are transported around the body. How then is the irrigation of the body – a metaphor used by Suśruta – carried out?

Fluids and their conduits

The types of fluid in the āyurvedic body include blood (*rakta*), milk, semen, breath (*prāṇa*), the juice of digested food (*rasa*), and the humors wind (*vāta*), bile (*pitta*), and phlegm (*kapha*).

These fluids are transported from place to place by three principle types of conduit: ducts (*sirā*), pipes (*dhamanī*), and tubes (*srotas*). Given the importance of this system of fluid distribution to the āyurvedic physiology, surprisingly little work has been done on clarifying what these conduits do, and how they are explained in āyurvedic theory (exceptions include Dasgùpta 1969: ii.l3 and Kutumbiah 1999: ch. 2).

Ducts (*sirā*) According to the *Suśrutasaṃhitā*, the function of the 700 ducts is to carry wind, bile, phlegm, and blood around the body, starting from their origin in the navel. In a vivid pair of metaphors, one agricultural and one botanical, Suśruta's text describes the ducts as follows (Su.śā.7.3):

> As a garden or a field is irrigated by water-carrying canals, and each part receives nourishment, so the ducts provide nutrition to the body by means of their contraction and dilation. Their branches are just like the veins on a leaf.

A point of special interest is that the ducts are colored according to what they carry: those carrying wind are yellowish brown (*aruṇa*), those carrying bile are dark blue, those carrying phlegm are white, and those carrying blood are red (Su.śā.7.18). It seems likely that these distinctions are based on the observation of different-colored vessels under the surface of the skin. In yet another simile, Suśruta likens the distribution of these ducts from the umbilical center through the body to the spokes radiating from the center of a wheel (Su.śā.7.7).

Pipes (*dhamanī*) There are said to be 24 pipes in the body (Su.śā.9). Like the ducts, they originate in the navel. From there, 10 go up, 10 down, and 4 sideways.

Those which go up from the navel support the body by carrying particular items (*viśeṣa*) such as sound, touch, vision, taste, smell, out-breath (*praśvāsa*), in-breath (*ucchvāsa*), yawning, sneezing, laughter, speech, crying, etc. These 10 pipes go from the navel to the heart and there each one divides into 3 branches, thus producing 30 pipes. Ten of these are devoted to carrying the humors, wind, bile, and phlegm, as well as blood and nutritive fluid (two pipes for each substance). Eight more carry sense impressions: sound, form, taste, and smell (again, two pipes each). Two pipes are used for speech (*bhāṣā*), two for making sound (*ghoṣa*), two for sleeping, and two more for waking up. Two pipes carry tears. Two pipes connected to the breasts carry women's breast-milk; curiously, in men the same two pipes are said to carry semen from the breasts.

Those pipes which go down from the navel carry substances such as wind, urine, feces, semen, and menstrual blood. In between the receptacles of raw and digested food, the pipes divide into three branches, as before. The first 10 pipes have the same functions as the first 10 upward pipes. The next two carry food to the intestines, and another two carry water. Two carry urine to the bladder. Two generate and transport semen, and two make it ejaculate. In women, the same four pipes carry and discharge menstrual blood. Two pipes are connected to the intestines and function in defecation. The remaining 8 pipes supply sweat to the horizontal pipes.

Each of the four pipes which run sideways are said to subdivide hundreds of thousands of times, holding the body together in a network. Their ends are connected to the hair follicles, and through these sweat is carried out and nutritive juice is carried in. This is how massage oils, showers, and ointments can move through the skin and affect the body internally. They are also the means by which pleasant and unpleasant sensations of touch are experienced.

Tubes (*srotas*) According to Suśruta, there are initially 22 tubes in the body, 2 for each of 11 substances. Two of the tubes (*srotas*) carry breath (*prāṇa*), and are joined to the heart and the pipes (*dhamanī*) which carry nutritive juice. Two more carry food, and are joined to the food-carrying pipes and the stomach. Two carry water and are joined to the palate and the lung (*kloman*). Two carry nutritive juice and are joined to the same places as those carrying breath. Two carry blood, and are joined to the liver, the spleen, and the pipes which carry blood. Two carry flesh, and are joined to the ligaments, skin, and pipes which carry blood. Two carry fat and are joined to the waist (*kaṭī*) and the kidneys. Two carry urine and are joined to the bladder and penis. Two carry feces and are joined to the receptacle of digested food and the rectum. Two carry semen and are joined to the breasts and testicles. Two carry menstrual blood and are joined to the womb and the pipes which carry menstrual blood. (There is no suggestion that these last pairs are specific to either gender.) Caraka adds three more categories of tube: two carrying bone, two carrying marrow (completing the set of 7 basic body elements (*dhātu*)), and two carrying sweat. He omits menstrual blood. Like the horizontal pipes, the tubes in the body divide and subdivide into innumerable tiny branches.

In contrast to the ducts and pipes, the description of these tubes is embedded in a discourse of injury, and the symptoms arising from damage to them are listed.

Suśruta records the existence of an ancient disagreement amongst physicians as to whether the pipes, ducts, and tubes are really separate types of vessel, and in particular whether there is a significant difference between pipes (*dhamanī*) and tubes (*srotas*). He argues that there is indeed a difference between these three types of vessel: they look different, have different connections, and different functions. The authoritative tradition of medical science also asserts their difference. It is merely because of their close proximity, similarity, and small size that they are conflated. Caraka also testifies to contemporary debates about the nature of these vessels; he records – and rejects – an extreme view that the human body consists only of a conglomeration of tubes.

Diagnosis

Another disagreement in the early medical tradition concerns the methods of diagnosis. Caraka uses the traditional scheme of the three "epistemological standards" (*pramāṇa*) as the basis for his diagnostic scheme. Diseases are discovered by means of the combined application of authoritative testimony, direct perception, and inference. The tradition of medical learning and science counts as authority. Direct perception means examining the patient using all the senses, although Caraka is distinctly squeamish about the sense of taste, and offers several ways of avoiding the need to taste the patient. Finally, inference is used to deduce the state of nonvisible features of the patient's body and functioning.

Using a simpler approach, Suśruta first records the tradition that there are three methods a physician should use to examine a patient: touching, looking, and questioning. But he then argues that a doctor has five senses, and that he should use all of them when examining a patient. For some reason, this common-sense view did not prevail in later medical textbooks, nor did Caraka's complex system. Later medical tradition normally reproduces Suśruta's triple-examination method.

Pulse

Debate and questioning on the topic of diagnosis probably continued, for by the late fifteenth century a new set of diagnostic methods had emerged as standard, the "examination of the eight bases" (*aṣṭasthānaparīkṣā*): pulse, urine, eyes, face, tongue, faeces, voice, and skin. These methods are first mentioned as a fixed set in the *Jvaratimirabhāskara* of the Mewari physician Cāmuṇḍa (fl. ca. 1474–1538; Meulenbeld 1999–2002: IIa.165), and become a standard in later medical textbooks.

The diagnosis of disease by pulse first appears in Sanskrit in the fourteenth-century *Compendium* of Śārṅgadhara (Wujastyk 2001: ch. 7). He begins by describing the pipe (*dhamanī*) on the hand at the base of the thumb as "an indicator of life," and notes that an expert can tell the well-being or ill health of the body by its behavior. He then connects various humoral conditions with different movements felt in the tube (*nāḍī*). Thus, inflamed wind feels like the movement of a leech or a snake; inflamed bile feels like the gait of a sparrow-hawk, crow or frog; inflamed phlegm feels like the gait of a swan or pigeon. The tube is also characterized as feeling weak or strong, cold or hot, firm or sluggish (Wujastyk 2001: 318).

In Śārṅgadhara's text, and until the advent of influences from European medicine, the understanding and use of pulse is closely tied to prognostication techniques. The ability to foretell the course of a patient's illness has formed a part of āyurvedic medicine from the earliest times. Caraka, for example, devotes a section of his *Compendium*, the *Indriyasthāna*, to the various signs by which a doctor can read the impending death of a patient. Thus, a patient who is about to die is called blossomed (*puṣpita*), partly because of the metaphor of a flower inevitably preceding a fruit, and partly because a dying person may produce unusual and unexpected smells, including the smell of various flowers. In looking for signs of death, the physician is advised to feel the patients body for temperature, perspiration, and resilience. He should also look for changes in the breathing and in the pulsations at the nape of the neck (Ca.ni.3.6). Thus, when the examination of the pulse appears in āyurveda, it fits well into a preceding tradition of prognostication. In a medical tradition which does not know of the pumping function of the heart or of the circulation of the blood, one has to ask what the physicians thought they were feeling in the pulse (cf. Kuriyama 1999). The position of the first historical description of āyurvedic pulse lore, in Śārṅgadhara's text, immediately precedes his sections on the interpretation of omens and dreams. This context sheds important light on how this new diagnostic technique was understood.[3]

Disease Etiology

The question of disease etiology in *āyurveda* is of great interest, and is far more sophisticated than the simple idea that "disease is an imbalance of the humors," although this statement is certainly part of the classical tradition. One of the central etiological ideas in *āyurveda* is the "abrogation of wisdom" (*prajñāparādha*), the idea that we fall ill through actions that follow lapses of judgment.[4] This "judgment" (*prajñā*) consists of the combined work of three mental faculties: intelligence (*dhī*), will-power (*dhṛti*), and memory (*smṛti*).[5] As an example of impaired intelligence, the classical authors cite errors such as mistaking something permanent as temporary, or something harmful as helpful, etc. Poor will-power would be exemplified by a lack of self-control in the face of sensual enjoyments which are unhealthy. Faulty memory is exemplified when a person's mind becomes so confused by passion or darkness, that they cease to be able to see things as they really are, and they cannot remember what should be remembered. The concept of memory is expanded elsewhere in Caraka's *Compendium* into a full-blown doctrine of yogic self-remembering, strongly reminiscent of the Buddhist mindfulness (*sati*).[6] Erroneous mental processes are likely to lead a person to engage in several types of faulty action. The person may misuse or abuse their senses, body, speech, or mind in various ways, and this abuse leads to sickness.

A related cause of illness is the suppression of natural urges. Urges related to urine or feces, semen, wind, nausea, sneezing, clearing the throat, and yawning should always be obeyed, without hesitation. So should the urgings of hunger and thirst, tears, sleep, or the panting induced by exertion. The suppression of any of these natural urges can lead to disease and is another example of a lapse of good judgment. Of course, bad urges, such as to impetuous or dishonorable deeds, should be suppressed, and this applies also to extreme feelings of negative emotion, the vocal expression of hatred or criticism, or physical violence.

Yet another disease etiology is the operation of karma: diseases afflict people due abrogations of their good judgment in the past. In the medical texts, the workings of karma are described in more detail than is usual. The karma one created oneself during a previous embodiment shows itself in the present as good or bad luck. Added to that is the further karma one creates in the present lifetime. These two kinds of karma may be graded according to strength or weakness: karma can be low, medium, or superior. A combination of the superior kinds of the two karma types gives rise to a long and happy lifetime. A combination of the low ones brings about a short and miserable life, and a combination of medium karmas is expected to result in an average lifespan.[7] The literature of "the ripening of deeds" (*karmavipāka*) develops these ideas, sometimes in great detail, with personal case histories exemplifying diseases and their karmic antecedents (Pingree 1997; Wujastyk 1999).

Demonic interference and possession was viewed as another valid cause of illness. Women and children are particularly vulnerable to such possession, which is also often presented as a punishment for bad deeds (Wujastyk 1999). Disease contagion is not a standard feature of the āyurvedic understanding of how illness arises (Zysk 2000; Das 2000), but interestingly a form of spirit-contagion is described in Kāśyapa's *Compendium*, in which a demon (*graha*) which has taken up abode in one unfortunate person may be transferred to another by means of touch (Wujastyk 2001: ch. 5).

Therapy

Āyurveda recommends a wide range of therapeutic techniques, including herbal drugs, massage, sauna, exercise, diet (including the use of meat broths and other non-vegetarian and alcoholic tonics), blood-letting (including leeching), simple psychotherapy, and surgery. One important group of five therapies (*pañcakarman*) became established early. According to Caraka, these were: emetics, purgation, two types of enema, and nasal catharsis. Suśruta replaced one of the enema treatments with bloodletting. Other authors introduced sweating and massage, as well as other therapies, into what became historically an increasingly important and elaborate complex of treatments.

Almost every other therapeutic application in āyurveda is preceded by a standard regime of oiling and sweating. "Oiling" usually consists of taking oils or fats by mouth, often with food. But it can also consist of oil enemas, nasal drops, bodily anointing, gargling, or the application of oils to the head, eyes, or ears. "Sweating" can mean warming the body by any of a range of methods: with a hot cloth, a warm metal plate, or the hands, the application of hot poultices, taking a traditional steam sauna, or the pouring of infusions of herbs and meats over the patient from a kettle. These preliminaries help to open the channels in the patient's body and to liquefy the humors which have been causing blockages, enabling them either to flow out of the body through the digestive tract, or to return to their proper locations in the body.

Surgery

The discussion of surgery in early āyurveda is most highly developed in the *Compendium* of Suśruta. There are many chapters here on such topics as the training of the surgeon, the preparation and maintenance of a wide range of scalpels, probes, pincers, and other surgical tools, and the diagnosis of medical problems which are to be treated specifically by surgery. Elaborate and varied surgical techniques are described, including perineal lithotomy, ophthalmological couching for cataract, the reduction of dislocations, the lancing of boils, the piercing of earlobes, the removal of obstructions and foreign bodies of all kinds from the flesh and orifices, rhinoplasty and the repair of hare-lip, and the suturing of wounds (Mukhopādhyāya 1913; Majno 1975; Wujastyk 2001: ch. 3). Suśruta's surgical chapters are justly famous. Why such an extraordinarily advanced school of surgery should have arisen so early in India, and why its work should have been recorded in Sanskrit, remain unanswered questions. The vibrant tradition evidenced by Suśruta's text did not survive as part of professional medical practice, although isolated techniques such as cataract couching did continue to be performed by barber-surgeons in a tradition apparently unsupported by a learned literature or formal training.

Materia medica

A large part of the āyurvedic literature, including general works, monographs, and dictionaries, is devoted to herbal medicine and materia medica generally. Several thousand plants are known and described in terms of a pharmacological typology based on flavorings (six types), potency (usually two: hot and cold), post-digestive flavorings (usually three), and pragmatic efficacy (used when the effect of a medicine is not adequately defined by the earlier categories). This typology is keyed to the system of humors and other physiological categories as expressed through the vocabulary of pathology. The system of humors functions in medicine in somewhat the same manner as the "case function" (*kāraka*) system in Pāṇinian grammar. Just as the six case functions provide the grammarian with a set of categories though which the urge to express a meaning (*vivakṣā*) can be related to morphological units of grammar, so the three medical humors provide a set of mediating categories through which diseases can be related to herbal medicines.

Rules of interpretation

There are certain rules of interpretation (*paribhāṣā*) which are applied when using herbal medicines, and these exemplify the important notion of "default values" which Frits Staal has highlighted elsewhere in this volume in the context of ritual and grammar. Thus, unless otherwise stated, the time of any action is dawn, the part of a plant is the root, the quantity of substances is equal, the container is made of clay, the liquid is water, and the oil is from sesame. By default, herbs should be fresh, not dried, and fresh herbs should be used in double the specified measure (Wujastyk 2001: ch. 7). There are many other standard defaults which are silently applied in medical situations, including a set of more than 30 subtle and interesting rules called "the logic of the system" (*tantrayukti*) which are to be used when interpreting medical statements (Su.ut.65, Ca.si.12.41–48).[8]

Medical Philosophy

Several modern authors have written about the interesting philosophical passages which occur in the early medical literature, especially in Caraka's *Compendium* (e.g., Dasgupta 1969: ii.13; Larson 1993). Caraka's use of Sāṃkhya and Vaiśeṣika concepts is of particular interest: his extensive treatment of the theory and practice of formal argument (Ca.vi.8) led Dasgupta to argue that the medical literature preserved perhaps the earliest stratum of Nyāya thought. Less attention has been paid to Caraka's version of the Yoga system (Ca.śā.1 esp. 137 ff.). Comba (2001) has shown that this chapter of Caraka's work cites several passages from the *Vaiśeṣikasūtra*. For Caraka, yoga and liberation (*mokṣa*) are both states in which all sensations (*vedanā*) cease. In liberation, however, this cessation is complete, while in yoga it is a goal. Quoting from the *Vaiśeṣikasūtras*, Caraka asserts that yoga arises when the mind is concentrated steadily on the self; in that state, the contact between the self and the sense organs, etc., ceases to exist, and several special powers arise. These are the standard eight siddhis of yoga and Indian magic. Caraka then focuses on the concept of mindfulness or remembering, in particular the memory of reality (*tattvasmṛti*), which both gives rise to a serious and soteriologically oriented lifestyle, and is produced by it. The full emergence of this special kind of memory (*smṛti*) results in freedom from suffering. At this point, Caraka presents his own unique eightfold path of yoga, which is quite different from the classical scheme of Patañjali. The path is aimed at developing memory, and consists of the following eight elements: understanding causes, forms (*nimittarūpagrahaṇa*), similarity (*sādṛśya*), and difference (*viparyaya*); adherence to purity (*sattvānubandha*), practice (*abhyāsa*), the yoga of knowledge (*jñānayoga*), and repeated listening (*punaḥśruta*). The mindfulness of reality (*tattvasmṛti*) produced by these eight practices leads to the identification of the self with *brahman*.

The Wider Influence of Āyurveda

Classical Indian medicine, āyurveda, has exerted a long and pervasive influence on other indigenous traditions in India, as well as on those of foreign countries. The fields of dharmaśāstra, arthaśāstra, tantra, alchemy, kāmaśāstra, and other sciences were all influenced by āyurveda in varying degrees. Āyurvedic treatises, such as the toxicological tract which is embedded in Suśruta's *Kalpasthāna*, became famous in Arabic translations from a very early period (Wujastyk 2001: 123). The Tibetan translation movement from the eighth century onwards resulted in many āyurvedic works becoming an integral part of the Tibetan healing tradition, and āyuvedic manuscripts recovered from the oasis towns of the Taklamakan desert testify to its importance in Central Asia. The Persian *Kitāb Firdaws al-ḥikma* by ʿAlī ibn-Sahl aṭ-Ṭabarī, written in 850, included a detailed account of āyurveda, based on already existing Persian and Arabic translations of the āyurvedic classics. The great Muslim physician Muḥammad ibn-Zakariyyā" ar-Rāzī (d. 925) frequently cited Arabic translations of Caraka (Ullmann 1978: 19). Later, through the works of da Orta (1563), van Rheede (1678–1703) and Linnaeus (1748, 1753), āyurvedic traditions exerted an important and lasting influence on the development of botanical science in Europe (Grove 1995: ch. 2). During the twentieth century, āyurveda has been supported at the national level in post-independence India, with hospitals, colleges, clinics, and a thriving āyurvedic pharmaceutical industry. And a process of globalization – similar to that which took place earlier with yoga – has begun to occur also with āyurveda. As might be expected, āyurveda "in diaspora" is changing and adapting, as it moves from its premodern role as the only learned medicine available to the population to a new position as one part of a portfolio of alternative and complementary therapies offered alongside modern biomedicine.

Notes

1 Abbreviations used in this chapter: Ah. = *Aṣṭāṅgahṛdayasaṃhitā* (Kuṃṭe et al. 1995), Ca. = *Carakasaṃhitā* (Ācārya 1981), Su. = *Suśrutasaṃhitā* (Ācārya 1992). All translations are my own unless otherwise stated.
2 Su.sū.14.16: *sa śabdārcirjalasantānavad aṇunā viśeṣeṇānudhāvaty evaṃ śarīraṃ kevalam*.
3 I am grateful to Anupam Goenka, with whom these ideas were discussed and developed (Goenka 2001).
4 For accessible introductions to the concept of *prajñāparādha*, see Dasgupta (1969: II, 415–18 *et passim*) and Weiss (1980).
5 Ca.śā.1.98–109.
6 Ca.śā.1.137–55. Cf. Thera 1996.
7 Ca.Vi.3.29.
8 These same rules also appear in the *Arthaśāstra*.

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