B. VAN NOOTEN

BINARY NUMBERS IN INDIAN ANTIQUITY

I. PRELIMINARY

Binary numbers have in recent time become indispensible for the workings of the digital computer since they allow the representation of any whole number in terms of two markers: "on" (1) and "off" (0). I have found good reason to believe that the rudiments of binary calculation were discovered in India well in advance of their discovery by the German philosopher Gottfried Leibniz in 1695.

Historians of Indian mathematics have for many decades recognized the contributions of the Indian mathematicians from as early as the time that the handbooks for constructing Vedic altars were composed (5th century B.C. ?). The better known contributions date from the period of the schools of Ujjain, Kusumapura and Mysore, from the fifth century A.D. until the eleventh. The discovery of the binary number may have escaped attention because its formulation is not contained in any of the strictly mathematical treatises of the Indian tradition.[1] Instead, I discovered it is an entirely different branch of science, the *chandaḥśāstra*, or "science of verse meters".

II. THE SANSKRIT METRICAL TRADITION

Piṅgala

The Vedic tradition ascribed a great, almost mystical significance to the meters of the sacrificial chants. Careful studies were made not only of the meters of the chant, but also of its language, prosody, proper place and proper time of recitation. The methodology developed to study and analyze meters became a respected field of study from a very early time onward. In this tradition the earliest comprehensive treatise on Vedic and Sanskrit meters that has been preserved is the *Chandaḥśāstra* by Piṅgala. Though most of the work is purely descrip-

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tive and is devoted to sorting out and classifying the meters according to their structure, in its eighth chapter occur a few brief statements that purport to establish a more general theory for dealing with the classification of meter. These statements, or *sūtras* treat of the classification of metrical feet in a manner that suggests that Piṅgala was aware of the binary number. They have not been the subject of any special study in the West since the edition and translation of the *Chandaḥśāstra* by Albrecht Weber in 1863. They form the subject of this paper and I hope to show that in a devious and unexpected manner Piṅgala has succeeded in introducing the binary number as a means for classifying metrical patterns.

Piṅgala's *Chandaḥśāstra* fits into the literary genre of sūtras, or series of aphorismic statements that are memorized and serve as *aides-mémoires* for a more complete theoretical exposition that is usually supplied by an explanatory commentary. The *Chandaḥśāstra* itself consists of some 310 brief sūtras divided over eight books and it treats of the structure and nomenclature of meters. The main commentary on the *Chandaḥśāstra* is Halāyudha's 13th century *Mṛtasaṃjīvinī*. From an earlier date (8th century ?) we have Kedāra's *Vṛttaratnākara*, an independent work based on Piṅgala, but dealing with non-vedic meters only.

The text-critical problems associated with the *Chandaḥśāstra* are similar to those of most ancient Indian literary works. We do not know who the author Piṅgala was, we do not know where he lived, when his work was composed and finally, whether the work going by his name was really all his, or a product of his school, or a conglomerate of text fragments assembled at one time and thenceforth transmitted under his name. Part of the evidence of its date is internal, part external. The text-critical work has been done almost exclusively by Albrecht Weber.

The main evidence for Piṅgala's date is external: his treatise is mentioned by the commentator Śabara on Mīmāṃsāsūtra I.1.5 who has been assigned to the 4th century A.D.[2] The treatise as we have it now is probably a composite. However, the passage where the binary system is developed is to all likelihood part of the original work,[3] and not an addition by a later metrist. The internal evidence of the treatise does not militate against a date before the 2nd century A.D. since the composition of the sūtras follows the pattern of the older unversified sūtras, such as those contained in Pāṇini's *Aṣṭādhyāyī*.[4] It is not possible, on objective grounds, to decide whether Piṅgala's treatise preceded or followed Pāṇini, nor is is possible to prove that Piṅgala's work existed before the third century A.D.

Piṅgala's Classification of Meters

A Sanskrit meter consists of verse feet which are composed of syllables which are prosodically either light (*laghu-*) or heavy (*guru-*). A light syllable (which I will represent as ◡), consists of a short vowel followed by at most one consonant and any other syllable is heavy (–). A verse meter usually consists of a set of four quarter verses (*pādas*) with the same number of syllables each. The majority of Vedic verses have either 8-syllabic, or 11-syllabic, or 12-syllabic quarter verses. Within each quarter verse the succession of *laghu* and *guru* syllables varies within predictable limits. The metrist's task is to discover the parameters within which that variation takes place, to classify the meters and to organize them into larger categories.

The question may be raised why earlier Sanskritists and mathematicians have failed to pay attention to the binary theory of classification that Piṅgala proposed. The main reason is that this theory was one of two alternative solutions to the problem of the classification of meters. One solution, which will not be discussed here, is to divide each meter mechanically into units of three syllables then assign names to the meters on the basis of the combinations of triplets. This has become the accepted method of metrical analysis in India and has superseded every other classificational systems that may have been devised earlier or later.[5]

But in addition, Piṅgala experimented with another, less arbitrary and more universal means for inventorizing the meters, one that is of interest here. Instead of giving names to the meters he constructs a *prastāra*, a "bed", or matrix, in which the *laghus* and *gurus* are listed horizontally. Before I continue with the relevant passages from the Sanskrit texts, three remarks are in order: One, most of the metrists regard the value of the first and the last syllable of a verse line as indifferent to the definition of its meter. They can always be either short or long and are exempt from analysis. As a result, an eight-syllable verse is analyzed as if it had six syllables only. Two, the table they aim for starts with the number 1, not zero. This is of importance in the conversion of the binary number. Three, the Hindu scientific and arithemetical formulations, though precise and unambiguous, are at times difficult to translate without adding explanatory phrases. Therefore, I have also translated remarks from the more explicit commentaries.

The device of the *prastāra* has to be visualized as an actual table written on a board, or in the dust on the ground.[6] Each horizontal line of the table stands for a line of verse represented as a succession of *laghu* and *guru* syllables. Every possible combination of *laghus* and *gurus* is spelled out for a particular meter. Hence there will be separate *prastāra* for 8-syllabic, for 11-syllabic and 12-syllabic meters. The first line in each will consist of all *gurus*, the last line of all *laghus*.

The two questions Piṅgala sets out to answer are: A. Is it possible to give a numeric value to each line in a given *prastāra* so that we could give a unique value to every metrical quarter verse with a corresponding succession of *laghu* and *guru* syllables? B. Suppose we wish to find the representation of a meter of which we know the rank number in the *prastāra* only, how do we find that representation? Piṅgala is obviously not looking for an arbitrary numeration, such as counting the lines in the *prastāra* from top to bottom and assigning successive numbers. Instead, he produces mathematical formulas which define the position of the verse meter within the *prastāra* unambiguously. In the following paragraphs I have given the relevant Sanskrit text followed by its translation. The reason for being this explicit is that many of these texts have not been translated and some are rather difficult to locate. Piṅgala's own formulations are very brief.

A. *Finding the decimal equivalent of a metrical pattern*

The rule for constructing the *prastāra* is given in *ChŚ*. 8.23:[7]

ekottarakramaśaḥ, pūrvapṛktā lasaṃkhyā

"In an order of one addition, the *la* is united with the previous one".

The metrist Kedāra (8th century A.D.?) expands on this cryptic statement as follows:

(a) pāde sarvagurāv ādyāl laghuṁ nyasya guror adhaḥ
(b) yathopari tathā śeṣaṃ bhūyaḥ kuryād amuṃ vidhim
(c) ūne dadyād gurūn eva yāvat sarvalaghur bhavet
(d) prastāro 'yaṃ samākhyātaś chandovicitivedibhiḥ

(a) "In a line consisting entirely of gurus, starting from the beginning, having placed a laghu underneath a guru,

(b) The rest remaining as [in the row] above, again and again one applies this rule.

(c) One should place the gurus on one [place] less until [the row] consists entirely of laghus.

(d) This has been defined as a *prastāra* 'matrix', by the experts in *chandoviciti*, 'metric analysis'."

The rules for generating the *prastāra*, therefore, involve the following procedure:

One starts with writing down a row of *guru* for as many syllables as the verse meter requires. For example, a four-syllabic verse will require:

— — — —

Next a new row of *gurus* is started, but underneath the first *guru* in the row above, a *laghu* is written instead. The remainder of the row remains unchanged:

— — — —
∪ — — —

On the next row again *gurus* are written until the first *guru* of the row above it is reached. Then a *laghu* is written. The remainder of the row remains unchanged:

— — — —
∪ — — —
— ∪ — —

In the next row, again, a *laghu* is written underneath the first *guru* and the rest is copied from the row above it:

— — — —
∪ — — —
— ∪ — —
∪ ∪ — —,

and so the procedure continues:

— — — —
∪ — — —
— ∪ — —
∪ ∪ — —
— — ∪ —
∪ — ∪ —
— ∪ ∪ —
∪ ∪ ∪ —
. . .
∪ ∪ ∪ ∪

The last line consists entirely of *laghus* so that the rule will fail. Then the *prastāra* is complete.

Table I shows the complete *prastāra* for an eight-syllabic verse. As usual, its first and last syllables are ignored. Kedāra continues:

(5ab) uddiṣṭaṃ dviguṇān upary ādyāt samālikhet

(5cd) laghusthāne tu ye 'ṅkāḥ syus taiḥ saikair miśritair bhavet

(5ab) "[Starting] from the beginning one should write numerals doubled above [the verse feet],

(5cd) However, the numerals which would be above a short foot, by those numerals combined together and augmented by one the [number of the meter] would be indicated."

We find a fuller explanation of this statement in a subcommentary, the *Tātparyaṭīkā* of the commentator Trivikrama (12th century):[8]

> tatrā 'yam udāharaṇapradarśanāyā 'lāpaḥ: ∪ ∪ — ∪ — ∪ ∪ —; idaṃ vyaktisvarūpam anuṣṭubhi jātau kathitaṃ bhavatī 'ty ukte idam īdṛśam eva kṣitau praviralaṃ dhriyate. tasmād upari dviguṇān "ādyād upary aṅkān samālikhet" iti vacanāt pratyakṣaraṃ krameṇa ekasmād ārabhya dviguṇān aṅkān samālikhed, yathā:

"Here is a formula to show an example: ∪ ∪ — ∪ — ∪ ∪ —. If the question is raised: 'What number does this particular pattern in the anuṣṭubh class [of meters] have?', this very same [pattern] is retained separately on the board. Next, because of the statement that one

TABLE I
The Prastāra for six syllables (Weber 1863)

1. 1.2.4.8.16.32 ——————	17. 1.2.4.8.16.32 ————∪—	33. 1.2.4.8.16.32 —————∪	49. 1.2.4.8.16.32. ————∪∪
2. ∪—————	18. ∪———∪—	34. ∪————∪	50. ∪———∪∪
3. —∪————	19. —∪——∪—	35. —∪———∪	51. —∪——∪∪
4. ∪∪————	20. ∪∪——∪—	36. ∪∪———∪	52. ∪∪——∪∪
5. ——∪———	21. ——∪—∪—	37. ——∪——∪	53. ——∪—∪∪
6. ∪—∪———	22. ∪—∪—∪—	38. ∪—∪——∪	54. ∪—∪—∪∪
7. —∪∪———	23. —∪∪—∪—	39. —∪∪——∪	55. —∪∪—∪∪
8. ∪∪∪———	24. ∪∪∪—∪—	40. ∪∪∪——∪	56. ∪∪∪—∪∪
9. ———∪——	25. ———∪∪—	41. ———∪—∪	57. ———∪∪∪
10. ∪——∪——	26. ∪——∪∪—	42. ∪——∪—∪	58. ∪——∪∪∪
11. —∪—∪——	27. —∪—∪∪—	43. —∪—∪—∪	59. —∪—∪∪∪
12. ∪∪—∪——	28. ∪∪—∪∪—	44. ∪∪—∪—∪	60. ∪∪—∪∪∪
13. ——∪∪——	29. ——∪∪∪—	45. ——∪∪—∪	61. ——∪∪∪∪
14. ∪—∪∪——	30. ∪—∪∪∪—	46. ∪—∪∪—∪	62. ∪—∪∪∪∪
15. —∪∪∪——	31. —∪∪∪∪—	47. —∪∪∪—∪	63. —∪∪∪∪∪
16. ∪∪∪∪——	32. ∪∪∪∪∪—	48. ∪∪∪∪—∪	64. ∪∪∪∪∪∪

should write doubled figures above it, one writes numbers as follows, one after the other, beginning with one, syllable by syllable:

1 2 4 8 16 32 64 128”
∪ ∪ — ∪ — ∪ ∪ —

Trivikrama continues:

> tatrā ’ṅkāḥ taiḥ saikair miśritair bhavet iti vacanād etad vyaktau laghustham ekaṃ dvāv aṣṭau dvātriṃśac catuḥṣaṣṭiṃ ca saṃgṛhṇīyāt. tatraikaṃ dhruvakarūpaṃ ca kṣipet. etasmin miśreṇa jāto rāśiḥ 108.

“Next, since it is stated that the numerals above a *laghu* in the scheme have to be combined and augmented by one, we would get: one, two, eight, thirty-two, sixty-four. Then we should add one to the sum obtained. In this [case] through the combination is obtained the value 108.”

1 +2 +8 +32 +64 = 107. Add one: 108.
∪ ∪ — ∪ — ∪ ∪ —

So the rank number of the metrical pattern in the *prastāra* is 108.

Through this procedure the metrical pattern is interpreted as a number. Each syllable is assigned a numerical value based on its position in the meter. The first syllable has the value 1 and each subsequent syllable has a positional value twice that of its left neighbor. In effect, this scheme is exactly equivalent to a system of numerical notation where the positional values of the digits increase as the powers of 2, from 2^0 to 2^n, which we call the binary notation. The difference between Piṅgala's system and the one current in the West is that the Indian system placed the higher positional value to the right of the lower, whereas in the West we find the lowest value on the right. The *laghu* in Piṅgala's system serves to indicate that the position in the number is significant, the equivalent of our notation 1. The *guru* means that the position is ignored, our 0. The Western representation of this number would be: 1 1 0 1 0 1 1.

In this way Piṅgala has shown that a metrical pattern can be regarded as a binary number. As a further illustration, let us find the rank number of an eight-syllabic Vedic *gāyatrī* verse:

tát savitúr varéṇ*i*aṃ

whose metrical representation is:

$\underline{\cup}\ \cup\ \cup\ -\ \cup\ -\ \cup\ \underline{\cup}.$

We list the six central syllables in a vertical column (1) and start a new column (2) where each *laghu* ($\cup$) is given the value 1 and each *guru* (–) is left without a numerical value. Column (3) begins with 1 and the numbers in the column below it are the doubled value of the preceding number. Next, in column (4), the values in column (2) are multiplied with those in column (3):

(1) verse foot	(2) numerical value	(3) binary base	(4) product (2) × (3)
$\cup$ laghu	1	1	1
$\cup$ laghu	1	2	2
– guru		4	
$\cup$ laghu	1	8	8
– guru		16	
$\cup$ laghu	1	32	32

The numbers in column (4) are added up and the total, 43, is the decimal value of the binary number represented in column (2). To get the rank number of the metrical pattern in the *prastāra* we add 1 to the total: 43 + 1 = 44.

Next, the converse process, that of determining the binary equivalent of a decimal number is given.

B. *Finding the binary equivalent of a decimal number*

The process of converting a decimal number to a binary number is formulated as one of finding the metrical pattern if one knows its (decimal) rank number and the metrical representation has been "destroyed" (*naṣṭa-*), presumably a reference to the brief useful life of a diagram written in sand.

Piṅgala, (*Ch.Ś.* 8.24—25):[9]

l ardhe, saike g

"A *laghu* in case of a half; in the case of an additional 1, a *guru*"

Halāyudha explaining Piṅgala states:

> l ardhe
> yadai 'vaṃ vijijñāset: gāyatryāṃ samavṛttaṃ ṣaṣṭhaṃ kīdṛśam, iti tadā tam eva ṣaṭsaṅkhyāviśeṣam ardhayeta. tasminn ardhīkṛte laghur eko lakṣyate, sa bhūmau vinyāsyaḥ. idānīm avaśiṣṭā trisaṅkhyāviṣamatvād ardhayituṃ na śakyate. tata kiṃ pratipattavyam ity āha:
> saike g
> taṃ pūrvalabdhāl lakārāt paraṃ sthāpayet. tato dvisaṅkhyā 'vaśiṣyate. punas tām ardhayeta, tataś cai 'kalakāraṃ dadyāt. tataś cai 'kasaṅkhyā 'vaśiṣyate. tatra tāvat sai 'ke g iti lakṣaṇam āvartanīyaṃ yāvad vṛttākṣarāṇi ṣaṭ pūryante. evaṃ saṅkhyāntare 'pi yojyam.

"If [the metrist] is concerned to determine which pattern in the the six-syllable *gāyatrī*, for instance, is the sixth, provided all are of the same length, then that numeral six should be halved. When this has been halved, one obtains one *laghu* which is written separately on the ground (*bhūmau*). Now, since the remaining number three, because of its odd-ness cannot be halved, how should one proceed?" (Quotes:) 'A *laghu* in case of a half; in the case of an additional 1, a *guru*.'

"To the odd number he adds a 1 and halves again. In this way he gets a single *guru* which he places beside the *laghu*. The remaining three are halved again and he gets a *guru*. To the remaining 1 he adds a 1, gets a *guru*, which he places beside the *laghu* he got earlier. So the number 2 remains. He halves that again and adds another *laghu*. This continues until the six-syllable gāyatrī is full. In the same way one can proceed with any other number."

The procedure outlined here involves a series of divisions by 2 of the rank number of the meter. It speaks for itself: the initial number is halved and if the quotient is an even number, a *laghu* is written in a separate table. If the quotient is an odd number, a *guru* is written in the same separate table, one is added to the quotient and the halving continues.

To give an example, if the decimal number one has is 54 and one wishes to find the metrical pattern that represents it, one proceeds as follows:

Number of pattern desired: 54	Write:
divide by 2, → 27:	*laghu*
add 1, divide by 2, → 14:	*guru*
divide by 2, → 7	*laghu*
add 1, divide by 2, → 4:	*guru*
divide by 2, → 2	*laghu*
divide by 2, → 1	*laghu*

The last column reads from top to, bottom:
laghu guru laghu guru laghu laghu, or:
∪ — ∪ — ∪ ∪

A glance at the earlier described *prastāra* (Table I) shows that, indeed, the metrical pattern we have obtained is no. 54. In this way Piṅgala solves the problem of reconstructing a metrical pattern if only the rank number is known. The procedure is equivalent to that of finding the binary representation of a decimal number, except that the rank number in the *prastāra* is one higher than the value of the binary number that represents it. To obtain the true binary equivalent, one has to be subtracted: $54 - 1 = 53$. In the Western notation the binary representation of 53 is 1 1 0 1 0 1. This representation follows easily from the *prastāra* by reading the metrical pattern: ∪ — ∪ — ∪ ∪ from right to left and substituting 1 for ∪ and 0 for —.

III. HISTORICAL IMPORTANCE

The importance of Piṅgala's discovery is twofold: it implies that he was aware of a place-value system of numerical notation and it also shows that he was working with a numerical base other than base 10. In the following paragraphs I propose to determine whether Piṅgala's discovery sprang from a known tradition, or whether he was the originator of his theory.

Piṅgala's notation represents two separate historical developments of numerical systems: that of the place-value system of numeration and that of the recognition of numerical systems not based on powers of ten. Ifrah[10] adduces evidence to show that four world civilizations have invented a place-value system: the Babylonians, the Indians, the Chinese with their rod numerals and entirely independently, the Mayans of the New World.[11] Was India influenced by the other Old-World cultures?

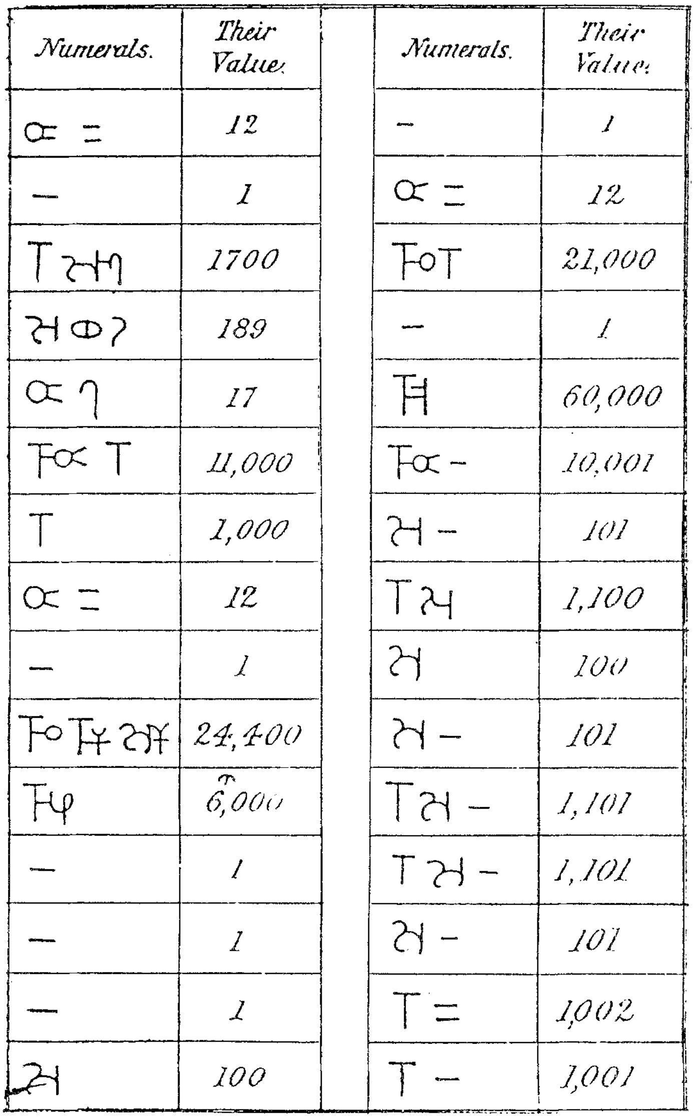
The Babylonian place value system was sexagesimal, or based on the number 60. Its influence on India is found in the later astronomical works, but not in early India. The Chinese discovered a good decimal place value system during the Han Dynasaty (3d century B.C.),[12] but there is no evidence to show that the Indians knew of this system. Within India itself evidence for systems of numbering and counting can come from two sources: one, the counting system of the language itself and two, the representation of numbers by numerals. We shall now briefly examine that evidence by looking at the sources: the ancient Indic languages, the older Indian literature and the inscriptions.

Sanskrit, being an Indo-European language, has inherited a system of counting in tens. The numbers from 1 through 10 have arbitrary names, but the numeration continues systematically by tens from twenty through ninety. The word for "ten" in the enumeration of numerals is followed by expressions for 10 + 1, 10 + 2, etc. The numerals 20, 30, etc. are variations on a compound °*śati*+, or *ṭi*+. The hundreds again are based on a new name, *śatam*, and so are thousands and one hundred thousand. The counting system of a language tends to influence the manner in which numbers are written.[13] At Piṅgala's time the prevailing language was Sanskrit or one of its derivatives and so people counted in tens.

History in India begins with the Vedic civilization where in the *Taittirīya Samhitā* of the Black Yajurveda (1000 B.C.) we have the first written indication of a numeration system based on the 10. Different ways of counting are presented, not with figures, but with the names of the numbers:

> thousand . . . ten thousand, one hundred thousand, ten hundred thousand, ten million, one hundred million, ten thousand million, one hundred thousand million, ten hundred thousand million.
>
> (*TS*. vii.2.19—20, Keith 1914)

The assumption underlying the decimal notation is that reading from left to right, within a decimal number each digit represents a value equal to one-tenth of its left neighbor. This process is first formally defined by the mathematician Āryabhaṭa I (b. 476 A.D.) who writes that "the value of a place [of a number] is ten times that of the [preceding] place: (*sthānāt sthānaṃ daśaguṇaṃ syāt*.[14] Later still, in approximately the 7th century A.D., we find this principle repeated in a different form in Vyāsa's commentary on Patañjali's Yogasūtra no. 3.13: *yathai' kā rekhā śatasthāne śataṃ daśasthāne daśai 'kā cai 'kasthāne* '. . . just as one and the same numeral (*rekhā*) in the 100-position is 100, in the 10-position is ten and is 1 in the 1-position'.[15] Though these remarks postdate Piṅgala's presumed date, they are not ever contradicted by statements in the literature that might imply that a non-decimal place value system was in use. The writing system of ancient India is also evidence for the prevalence of the decimal place value system. From a few centuries before Piṅgala's time we have numerals contained in the inscriptions of Nāneghāṭ on the western scarp of the Deccan Plateau, north and east of Bombay. They belong to the Brāhmī alphabet which is written from left to right (see Figure 1). In these numbers the digit with the highest value is written on the left, and the lowest number to its right, e.g., 12 is written as 10 2, or "O = =" and 17 as "O = ?".[16] The numeral is often, but not necessarily, preceded by words spelling out the number. From the fact that independent symbols existed for "10", "20", "80", "100", etc. we can infer that the counting system underlying these numerals was founded



Numerals.	Their Value.	Numerals.	Their Value.
	12		1
	1		12
	1700		21,000
	189		1
	17		60,000
	11,000		10,001
	1,000		101
	12		1,100
	1		100
	24,400		101
	6,000		1,101
	1		1,101
	1		101
	1		1,002
	100		1,001

Fig. 1. Indian numerals from Piṅgala's time. (Nâneghâṭ, ca. 1st century. B.C.).

on a decimal base. Of course, it is not a pure place-value because in such a system there would be no need for designing separate symbols for 10, 50, 100, etc. But none of the numerals suggest that another system, such as a duodecimal or sexagesimal was in use. The same is true for all the subsequent numbers of dates in Indian history. To our knowledge, Piṅgala was not exposed to a tradition of marking numerals in a base other than 10.[17]

To summarize the evidence for the state of mathematics in Piṅgala's time, we can be reasonably certain that his counting system was predicated on a base of ten, but there is no proof that the the place-value system of notation was used. Piṅgala's *Chandaḥśāstra* itself does not provide us with representations of numerals. The manuscripts of the commentaries do write them, but these postdate the invention of the zero. Although the word for "zero" (*śūnya*) does occur in the *Chandaḥśāstra*, it is always spelled out and not represented by a symbol.

It remains for us to determine in how far the procedures that Piṅgala prescribed for converting a verse meter into a decimal number, and vice versa, are equivalent to those required to convert a binary number into a decimal number. A system of notation that simply uses "Off" and "On", or "+" and "–" as markers of a place value does not necessarily produce a binary system. In fact, you have no more than people who count up to two have. But if we have a system that uses two symbols, "+" and "–" in such a way that every string of "+"s and "–"s has a unique decimal equivalent and we are shown how to derive this decimal equivalent and also how to convert any decimal number into a string of "+" 's and "–" 's with a unique numerical value, then indeed we do have a binary system. Piṅgala has done at least this much and as I will show next, his representation of numbers as members of a *prastāra* bears close comparison with the method Leibniz followed in his discovery of the binary number.

IV. EUROPE AND INDIA

The discovery of the binary notation of numbers in Europe was the accomplishment of the German philosopher Gottfried Leibniz (1646–1716) at the end of the 17th century.[18] In a letter which spells out the first description of his discovery, he writes:

> . . . ich sehe, dass sich aus dieser Schreibart der Zahlen wunderliche Vorteil ergeben [werden], die hernach auch in der gemeinen Rechnung zu statten kommen werden . . .[19]

The hastily sketched table (Figure 2) makes it clear how Leibniz derived the values of binary numbers. He writes 15 rows each beginning with a 1. In each subsequent row he writes an additional "0" after the "1". To the right of the last column of each line he draws a vertical line and to the right of this line writes a column of numbers 1 2 4 8, etc. Each of these numbers corresponds to the binary value represented by the preceding row of "1" 's and "0" 's. This table can be reconciled with the *prastāra* model (p. 38) as follows: The column of powers of 2 in both diagrams are exactly equivalent, but the rank numbers given to the metrical patterns in the *prastāra* are systematically one more than their binary equivalents. Piṅgala's *prastāra* for 6 syllables (Table I) ends with the pattern ⏑ ⏑ ⏑ ⏑ ⏑ ⏑, binary 63, to which the metrist assigns the number 64. It is one short of Leibniz's 7-digit binary number 1000000, which is listed before 64 in his table. The pattern – – – – – – ⏑, which would conform exactly to Leibniz's 1000000, would be the next entry, #65 starting a 7-syllable *prastāra*. The difference in numbering is due to the fact that the rank numbers in a *prastāra* are shifted up one decimal value.

To find an explanation for this difference we can conjecture that the metrist may have added the value of 1 to the value of the binary number because of practical considerations. If he had used the binary number pure and simple, then the first metrical pattern in the *prastāra* would have had the rank number 0. The metrist may have been reluctant to do so, because it runs counter to the intuitive method of counting. The fact is that the construction of the *prastāra* requires the first row to consist of all *gurus*, or zeroes in the Western system. One may conjecture that a metrist would be reluctant to assign the rank number "0" to the first member of a class of metrical patterns, even though that procedure would better conform to the *prastāra* theory. He would be more inclined to give it the rank number "1" with the result that every subsequent rank number will have to be raised by one. The metrical *prastāra* obviously had a very practical purpose, that of classifying meters, and so we should regard this *prastāra* procedure

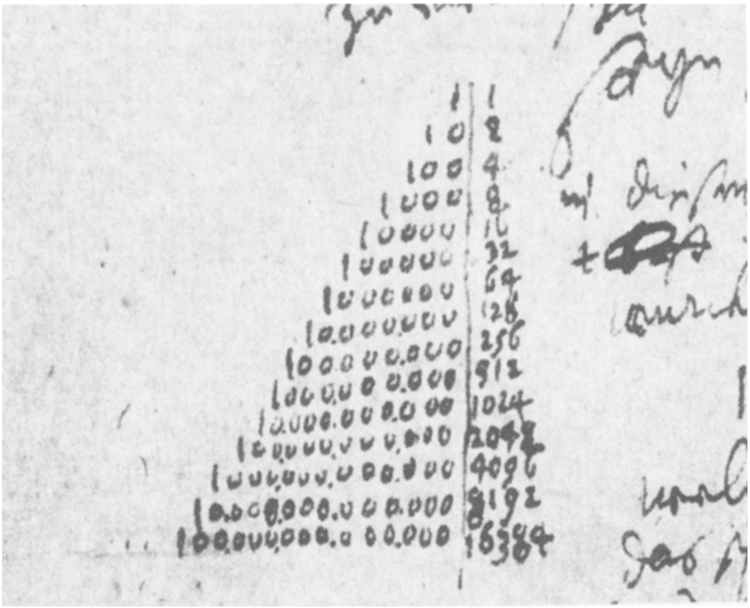


Fig. 2. Leibniz' diagram of binary numbers. (Couturat, 1903, p. 284)

as a practical application of a more general mathematical theory of binary numberals to the science of metrics. Further evidence in support of this supposition is the fact that in some of the theoretical expositions the first and last syllables of the meter are not ignored (see Trivikrama on Kedāra 2.238, above p. 7). These expositions, in other words, treat the metrical pattern of a verse as an abstract unit of calculation, while the practical metrists leave the first and last syllables out of consideration. The first and last syllables have no practical value for the classification of meters and their inclusion would unnecessarily have increased the number of lines in a *prastāra*. So just as concessions were made in this respect to the rigorous application of the theory of binary numbers for a practical purpose, so, we may conjecture, is the demand that the table begin with the rank number 1.

The most striking difference between Leibniz's system and Piṅgala's is the order in which the powers of 2 are arranged. In the binary number of Leibniz's design the low digit is on the right, the high value on the left. It is the precursor of the Western system of binary notation. In Piṅgala's arrangement (Table 1) the lower value is on the left. This fundamental difference is primary evidence of the originality of both discoveries.

Leibniz's letter referred to above is, indeed, the earliest published account of his discovery. However, Leibniz's notes from 20 years earlier (1677—78) contain a fragment which shows that already at that time he was experimenting with the binary notation which he calls "Progressio Dyadica", or "Characteristica bimalis":

"... perfectior est characteristica numerorum bimalis quam decimalis, vel alia quaecunque ...".[20]

I mention this fragment, for several years later Leibniz became intrigued with the Chinese hexagram depictions of Fu Hsi (Fohy) in the *Book of Changes* (*I Ching*) which he interpreted as binary numbers (Figure 3). The letter where he expresses this opinion is included in Loosen and Vonessen (1968), pp. 126—131. The fragment quoted earlier makes it obvious that he had discovered the binary number prior to learning of the Chinese text. In fact, however, the *I Ching*, unlike Piṅgala, does not contain evidence for the use of the binary number. Chinese culture was the great discovery of the Enlightenment and so Leibniz looked there for expressions of rational thought and not to India which might have had more to offer. Sanskrit texts were simply unavailable to scholars of the 17th century. India was the discovery of the Romantic Period and became, for better or worse, identified as the land of religion. Since Leibniz could not have known Piṅgala's work we have another instance here of a mathematical principle known to Asian scholars and rediscovered by the West, in this case one millennium and a half later.

In summary, I have tried to show that Piṅgala used a binary calculation to classify metrical verses as early as the second or third century A.D. He also knew how to convert that binary notation to a decimal notation and vice versa. We know of no sources from which he could have drawn his inspiration, so he may well have been the originator of the system. Piṅgala leaves no record of further applications of his discovery, but it is of great interest to realize that for many centuries, in fact down to the present day, this knowledge was available to and preserved by Sanskrit students of metrics. Unlike the

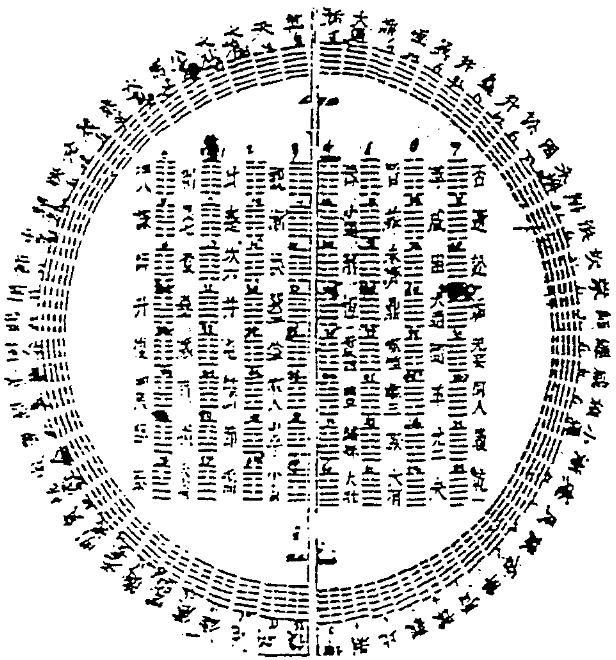


Fig. 3. The "Prior to Heaven Hexagram Order", Hsien-t'ien tzu-hsü (D. E. Mungello: 1977, *Leibniz and Confucianism, the search for accord*, p. 52).

case of the great linguistic discoveries of the Indians which directly influenced and inspired Western linguistics, the discovery of the theory of binary numbers has so far gone unrecorded in the annals of the West[21].

NOTES

[1] See, for instance, H. L. Resnikoff and R. O. Wall, *Mathematics in Civilization*, Dover Publ. 1984 (2nd ed.), who correctly credits the Babylonians with inventing the place-value system of numerals, but is silent about the Indian contributions. The *Āryabhaṭīya*, in fact, does have a chapter on metrics, but it confines itself to the theory of permutations.

[2] Śabara was known to the astronomer Varāhamihira whose date we know fairly accurately. The date of 200 B.C. that Datta and Singh (1935, p. 75) assign to Piṅgala is puzzling. They do not justify the date, nor quote authorities confirming it.

[3] Jacobi (1933) is of this opinion ("... zweifellos echt", p. 138) pointing out that discussions of the *prastāra* are found also in the *Bhāratanāṭyam* XIV (approx. 4th

century A.D.) and agreements can be found with earlier passages in Piṅgala's work. The passage preceding it (VIII.2—19) is absent from all the Ṛg-recensions of the piṅgalaśāstra, and from many of the Yajus manuscripts and is, therefore, suspect (Weber, 1863, p. 414).

[4] We recognize the use of *adhikāras*, such as "*chandas*" (1.1), the *anuvṛttis*, the use of "*śeṣe*" (2.12) to include unnamed contexts, the use of the ablative to indicate context-after and the locative for the context-before. In brief, enough similarities exist to show that the descriptive techniques used by Piṅgala and the grammarians were similar.

[5] The theory of analysis of meter by gaṇas can be found in Weber (1863) and in Appendix A to Apte (1959), Vol. III, after p. 1755.

[6] One of the names of mathematics in ancient India was "Science of Dust", since figures and diagrams were drawn in the sand.

[7] Weber (1863), p. 429; *Vṛttaratnākara* 6.2, Bhārati (1908), p. 90; Sharma *et al.* (1969), p. 306.

[8] Sharma *et al.* (1969), pp. 316f.

[9] *Chandaḥśāstra* p. 192.

[10] Ifrah (1985), Chapters 26—27.

[11] Seidenberg (1986).

[12] Ifrah (1985), p. 398.

[13] This correspondence is not necessary and not always observed. The Jains, for instance, have a separate numeral for 400, but no special name for it. Conversely, many different terms can be used to express the values of the first nine numerals of Sanskrit. See Sircar (1965), pp. 248—247.

[14] Āryabhaṭīya Gaṇita, vs. 2, Shankar (1976).

[15] Yogasūtra, pp. 131—132 and Woods (1913), pp. 215—216.

[16] See Bhagavānlāl Indrājī (1876), pp. 404—406.

[17] The location of this inscription on a monument at the head of a mountain pass on a busy trade route connecting the Arabian Sea to the Indian hinterland, may well indicate that the system of notation was similar to a system adopted by traders and merchants. These traders were likely to have used an abacus for their calculations which may have played a role in spreading the decimal system. (Bühler, *Indian Palaeography*, § 35B). The oldest abacus, or calculating board known to us was invented in China during the Han dynasty (Ifrah *op. cit.*, pp. 118—119), or between 206 B.C. and A.D. 221. In the latter period of this dynasty Buddhist monks began to travel from China to India, so that the presence of an abacus in Nāneghāṭ at that time should not surprise us. However, there is no clear proof of the use of an abacus in India from this early a period.

[18] My thanks are to Professor Benson Mates of the Department of Philosophy, Berkeley, for referring me to the relevant passages of Leibniz.

[19] Loosen and Vonessen (1968), p. 23.

[20] Couturat (1903), p. 284.

[21] I thank Professor J. F. Staal, Professor of Philosophy and Sanskrit, University of California, Berkeley, for his very helpful comments on this paper.

BIBLIOGRAPHY

Apte, V. S.: 1959, *The Practical Sanskrit-English Dictionary*. 3 vols. Poona: Prasad Prakashan.

Bag, A. K.: 1979, *Mathematics in Ancient and Medieval India*. Varanasi: Chaukhambha Orientalia.

Bhārati, Srī Rāmacandra (ed.): 1908, *Vṛttaratnākara by Pandit Kedārabhaṭṭa, with its Commentary Vṛttaratnākarapancikā*. Bombay: Nirṇaya-Sāgar Press.

Bodas, Rajaram Shastri (ed.): 1917, *The Yogasūtras of Patañjali with the scholium of Vyāsa*. Bombay Sanskrit and Prakrit Series No. XLVI. Bombay: Government Central Press: 131-132.

Bühler, J. Georg (1904), "Indian Paleography", [trl. from the German by J. F. Fleet]. *Indian Antiquary* 33. Appendix.: 1—102.

Closs, Michael P.: 1986, *Native American Mathematics*. Austin: University of Texas Press.

Couturat, Louis (ed.): 1903, *Opuscules et fragments inédits de Leibniz, extraits des manuscrits de la Bibliothèque royale de Hanovre par Louis Couturat* . . . Paris: Alcan.

Datta, Bibhutibhusan and Avadesh N. Singh (1935), *History of Indian Mathematics*. 2 vols; repr. 1 vol. 1962. Bombay: Asian Publishing House.

Gokhale, Sh. L.: 1966, *Indian Numerals*. Poona: Deccan College Silver Jubilee Series No. 43. Charts nos. 4. 6. 14. 20.

Ifrah, Georges: 1985. *From One to Zero*. New York: Viking Penguin Inc.

Indraji, Bhagavānlāl: 1876, "On Ancient Nágarí Numeration; from an Inscription at Nánegháṭ". *Journal of the Bombay Branch of the Royal Asiatic Society* 12: 404—406.

Jacobi, Hermann: 1933, "Über die ältesten indischen Metriker und ihr Werk." *Indian Linguistics: Grierson Comm. Volume*. Lahore: 131—141.

Keith, Arthur Berriedale: 1914, *The Veda of the Black Yajus School*. Cambridge: Harvard Oriental Series 19 Part II: 350—351.

Loosen, Penate and Franz Vonessen (ed.): 1968, *Gottfried Wilhelm Leibniz. Zwei Briefe über das Binäre Zahlensystem und die Chinesische Philosophie*. Stuttgart: Belser-Presse.

Seidenberg, A.: 1986, "The Zero in the Mayan Numerical Notation," in Michael Closs, 1986, pp. 371—386.

Shankar, Kripa (ed., trans.): 1976, *Aryabhatiya of Aryabhata*. New Delhi: the Indian National Science Academy.

Sharma, Aryendra, K. Deshpande and D. G. Padhye (edd.): 1969, Vṛttaratnākara of Śrī Kedāra Bhaṭṭa. Hyderabad: Osmania University.

Sircar, D. C.: 1965, *Indian Epigraphy*. Delhi: Motilal Banarsidass.

Velankar, H. D.: [1949], "[Kedāraviracito Vṛttaratnākaraḥ] in *Jayadāman*. Bombay: Haritosha Samiti, pp. 71—93.

Weber, Albrecht: 1863, *Über die Metrik der Inder*. Indische Studien No. 8. Berlin.

Woods, James H. (trans.): 1914, *The Yoga-system of Patañjali*, Cambridge: Harvard Oriental Series 17.