2. Construct DFSA to accept the following languages.
   a. $\{a, b\}^+$
   b. $\{a, b\}^*$
   c. $\{(ab)^n | n \geq 1\}$
   d. $\{a^n b^m c^p | n, m, p \geq 1\}$
   e. $\{w | w \in \{a, b\}^*$, w has an even number of a's and even number of b's$\}$
3. Find a DFSA for each of the following languages.
   a. $L = \{w | w$ is a binary string containing both substrings 010 and 101$\}$.
   b. For any fixed integer $k \geq 0$, $L = \{0^i w 1^i | w$ is a binary string and $0 \leq i \leq k\}$.
4. Suppose we restrict DFSA's so that they have at most one accepting state. Can any regular language L be recognized by this restricted form of DFSA? Justify your answer.
5. Design deterministic finite automata for each of the following sets:
   a. the set of all strings in $\{1, 2, 3\}^*$ containing 231 as substring.
   b. the set of strings $x \in \{0, 1\}^*$ such that $\#_0(x)$ is even and $\#_1(x)$ is a multiple of three.
   c. the set of strings in $(a)^*$ whose length is divisible by either 2 or 7.
6. Let $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Consider the base 10 numbers formed by strings from Σ^*. 15 represents fifteen, 408 represents four hundred, and eight and so on. Let $L = \{x \in \Sigma^* |$ the numbers represented by x is exactly divisible by 7$\} = \{\in, 0, 00, 000, \ldots, 7, 07, 007, \ldots, 14, 21, 28, 35, \ldots\}$. Find a DFSA that accepts L.
7. Let $\Sigma = \{a, b\}$. Consider the language consisting of all words that have neither consecutive a's nor consecutive b's. Draw a DFSA that accepts this language.
8. Let $\Sigma = \{a, b, c\}$. Let $L = \{x \in \{a, b, c\}^* || x| = 0 \bmod 5\}$. Draw a DFSA that accepts L.
9. Let $\Sigma = \{a, b, c\}$. Consider the language consisting of words that begin and end with different letters. Draw a DFSA that accepts this language.
10. Let $\Sigma = \{a, b, c\}$.
    a. Draw a DFSA that rejects all words for which the last two letters match.
    b. Draw a DFSA that rejects all words for which the first two letters match.
11. Suppose we alter the definition of a DFSA so that once the automaton leaves its start state, it cannot return to its start state. For such a DFSA, if an input w causes it to take a transition from a state p to its start state q_0, where $p \neq q_0$, then the DFSA immediately halts and rejects w. Call this new type of DFSA as no-return-DFSA.

Prove that the class of languages recognized by no-return-DFSA is the class of regular languages.

12. Construct a NFSA for each of the languages given in problem no. 1,3,5,6,7,8,9.
13. Given a non-deterministic finite automaton M without ε-transitions, show that it is possible to construct a NFSA with ε-transitions M' such that:

    a. M' has exactly one start state and one end state

    b. $L(M) = L(M')$.
14. What is the language accepted by the NFSA M (Figure 3.50)?

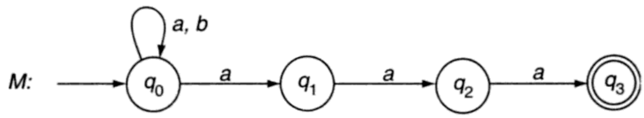

Figure 3.50. State diagram for Exercise 14

15. Let $\Sigma = \{a, b\}$. Find a NFSA for each of the following languages:

    a. $\{x|x$ contains an even number of a's$\}$.

    b. $\{x|x$ contains an odd number of b's$\}$.

    c. $\{x|x$ contains an even number of a's and an odd number of b's$\}$

    d. $\{x|x$ contains an even number of a's or an odd number of b's$\}$
16. Suppose we alter the definition of an NFSA so that we now identify two types of states in its state set Q: the good states $G \subseteq Q$, and the bad states $B \subseteq Q$ where $G \cap B = \phi$. (Note that a state in Q may be neither good nor bad, but no state is both good and bad). The automata accepts input w if, considering all possible computations on w, some computation ends in G and no computation ends in B. Call this new type of NFSA as a good-bad NFSA.

    Prove that the class of languages recognized by good-bad NFSAs is the class of regular languages.
17. In the questions below, given a language L we describe how to form a new language from the strings in L. Prove that if L is regular then the new languages are regular by constructing an NFSA for the new languages.

    a. $skip(L) = \{xy|xcy \in L$, where x and y are strings, c is a letter$\}$

    b. $suffix(L) = \{y|xy \in L, x, y$ are strings$\}$.
18. Convert the following two NFSA to equivalent DFSA using subset construction method (Figure 3.51).

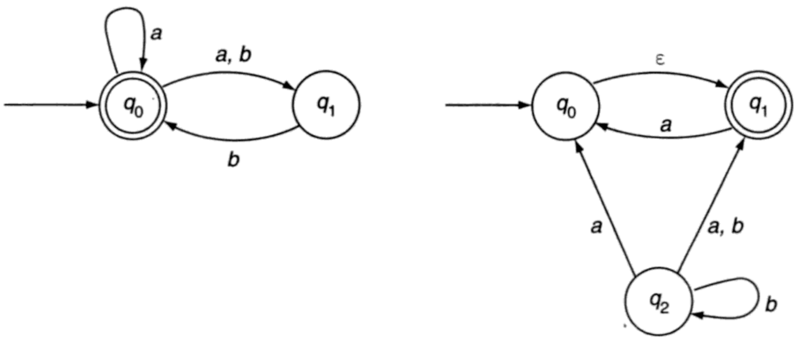

Figure 3.51. State diagrams for Exercise 18

19. Prove the following identities for regular expressions r, s and t. Here $r = s$ means $L(r) = L(s)$.

    a. $r + s = s + r$

    b. $\phi^* = \varepsilon$

    c. $(r^*s^*)^* = (r + s)^*$

    d. $(rs)^* r = r(sr)^*$

20. Construct a NFSA accepting the language denoted by the following regular expressions. Also convert the NFSA to an equivalent DFSA.

    a. $a^*ba^*ab^*$

    b. $a^*bb^*(a + b)ab^*$

    c. $b((aab^* + a^4)b)^*a$

21. Given the following DFSAs, construct equivalent regular expressions (Figure 3.52).

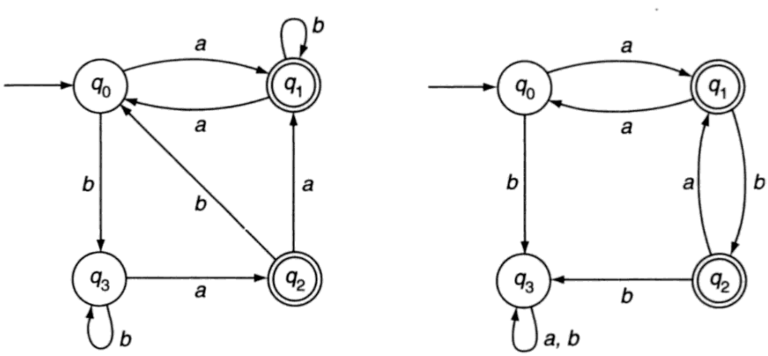

Figure 3.52. State diagrams for Exercise 21

22. Give an NFSA with four states equivalent to the regular expression $(01 + 011 + 0111)^*$. Convert this automaton to an equivalent DFSA using subset construction.

23. Give regular expressions that describe each of the following languages, which are over the alphabet $\{0, 1\}$. Explain how you constructed your regular expressions.

    a. $\{w|\ w$ contains substrings 010 and 101$\}$

    b. $\{w|w$ does not contain substring 0110$\}$

    c. $\{w|\ w$ has an even number of 0's and an even number of 1's$\}$

    d. $\{w|w$ has the same number of occurrences of 10 and 01$\}$

    Note that in (a) and (d) occurrences can overlap.

24. For each of the following languages, give two strings that are members and two strings that are not members – a total of four strings for each part. Let $\Sigma = \{a, b\}$ in all parts.

    a. a^*b^*

    b. $a(ba)^*b$

    c. $a^* \cup b^*$

    d. $(aaa)^*$

    e. $\Sigma^*a\Sigma^*b\Sigma^*a\Sigma^*$

    f. $aba \cup bab$

    g. $(\Sigma \cup a)b$

    h. $(a \cup ba \cup bb)\ \Sigma^*$

25. Let $\Sigma = \{a, b\}$. Give (if possible) a regular expression that describes the set of all even-length words in Σ^*.

26. Give examples of sets that demonstrate the following inequalities. Here, r_1, r_2, r_3 are regular expressions.

    a. $r_1 + \in \neq r_1$

    b. $r_1 \cdot r_2 \neq r_2 \cdot r_1$

    c. $r_1 \cdot r_1 \neq r_1$

    d. $r_1 + (r_2 \cdot r_3) \neq (r_1 + r_2) \cdot (r_1 + r_3)$

    e. $(r_1 \cdot r_2)^* \neq (r^*_1 \cdot r^*_2)^*$

27. Find examples of sets that show the following expressions maybe equal under some conditions. Here, r_1, r_2, r_3 are regular expressions.

    a. $r_1 + \in = r_1$

    b. $r_1 \cdot r_2 = r_2 \cdot r_1$ (even if $r_1 \neq r_2$)

    c. $r_1 \cdot r_1 = r_1$

    d. $r_1 + (r_2 \cdot r_3) = (r_1 + r_2) \cdot (r_1 + r_3)$ (even if $r_1 \neq r_2$ and $r_3 = r_1$)