

Properties of DFT

Aim:

Verify following properties of DFT using Matlab/Scilab.

1. Linearity Property
2. Parseval's Theorem
3. Convolution Property
4. Multiplication Property

Theory:

1. Linearity Property

The linearity property of the DFT states that if you have two sequences $x_1[n]$ and $x_2[n]$, and their corresponding DFTs are $X_1[k]$ and $X_2[k]$, then for any scalar a and b :

$$\text{DFT}\{a \cdot x_1[n] + b \cdot x_2[n]\} = a \cdot \text{DFT}\{x_1[n]\} + b \cdot \text{DFT}\{x_2[n]\}$$

2. Parseval's Theorem

Parseval's theorem states that the total energy of a signal in the time domain is equal to the total energy in the frequency domain. For a sequence $x[n]$ and its DFT $X[k]$:

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

3. Convolution Property

The convolution property of the DFT states that the circular convolution of two sequences in the time domain is equivalent to the element-wise multiplication of their DFTs in the frequency domain:

$$\text{DFT}\{x_1[n] \odot x_2[n]\} = \text{DFT}\{x_1[n]\} \cdot \text{DFT}\{x_2[n]\}$$

4. Multiplication Property

The multiplication property of DFT states that pointwise multiplication in the time domain corresponds to circular convolution in the frequency domain:

$$\text{DFT}\{x_1[n] \cdot x_2[n]\} = \frac{1}{N} \text{DFT}\{x_1[n]\} \odot \text{DFT}\{x_2[n]\}$$

Program:

1. Linearity Property

```
clc;
```

```

clear all;
close all;
x=input("enter first sequence");
h=input("enter sequence sequence:");
lx=length(x);
lh=length(h);
if lx>lh
    h=[h zeros(1,lx-lh)]
else
    x=[x zeros(1,lh-lx)]
end
a=input("enter value of 'a':");
b=input("enter value of 'b':");
lhs=fft((a.*x)+(b.*h));
rhs=a.*fft(x)+b.*fft(h);
disp('LHS');
disp(lhs);
disp('RHS');
disp(rhs);
if lhs==rhs
    disp('Linearity property verified');
else
    disp('Linearity property not verified');
end

```

2. Parseval's Theorem

```

clc;
clear all;
close all;
x=input("enter first sequence:");

```

```

h=input("enter second sequence:");
N=max(length(x),length(h));
xn=[x zeros(1,N-length(x))];
hn=[h zeros(1,N-length(h))];
lhs=sum(xn.*conj(hn));
rhs=sum(fft(xn).*conj(fft(hn)))/N;
disp('LHS');
disp(lhs);
disp('RHS');
disp(rhs);
if lhs==rhs
    disp("Parseval's Theorem verified");
else
    disp("Parseval's Theorem not verified");
end

```

3.Convolution Property

```

clc;
clear all;
close all;
x=input("enter first sequence");
h=input("enter sequence sequence:");
N=max(length(x), length(h));
xn=[x zeros(N-length(x))];
hn=[h zeros(N-length(h))];
Xn=fft(xn);
Hn=fft(hn);
lhs=cconv(xn,hn,N);
rhs=ifft(Xn.*Hn);
disp('LHS');

```

```

disp(lhs);
disp('RHS');
disp(rhs);
if lhs==rhs
    disp('Circular Convolution verified')
else
    disp('Circular Convolution not verified');
end

```

4. Multiplication Property

```

clc;
clear all;
close all;
x=input("enter first sequence");
h=input("enter sequence sequence:");
N=max(length(x), length(h));
xn=[x zeros(N-length(x))];
hn=[h zeros(N-length(h))];
lhs=fft(xn.*hn);
Xn=fft(xn);
Hn=fft(hn);
rhs=(cconv(Xn,Hn,N))/N;
disp('LHS');
disp(lhs);
disp('RHS');
disp(rhs);
if lhs==rhs
    disp('Multiplication property verified');
else
    disp('Multiplication property not verified');
end

```

Result:

Performed and verified the following properties of DFT:

- 1.Linear Property
- 2.Parseval's Theorem
- 3.Convolution Property
- 4.Multiplication Property

Observation:

1. Linearity Property

enter first sequence[1 2 3 4]

enter sequence sequence:[1 1 1 1]

x =

1 2 3 4

enter value of 'a':2

enter value of 'b':3

LHS

$32.0000 + 0.0000i$ $-4.0000 + 4.0000i$ $-4.0000 + 0.0000i$ $-4.0000 - 4.0000i$

RHS

$32.0000 + 0.0000i$ $-4.0000 + 4.0000i$ $-4.0000 + 0.0000i$ $-4.0000 - 4.0000i$

Linearity property verified

2. Parseval's Theorem

enter first sequence:[1 2 3 4]

enter second sequence:[1 1 1 1]

LHS

10

RHS

10

Parseval's Theorem verified

3.Convolution Property

enter first sequence[1 2 3 4]

enter sequence sequence:[1 1 1 1]

LHS

10 10 10 10

RHS

10 10 10 10

Circular Convolution verified

4. Multiplication Property

enter first sequence[1 2 3 4]

enter sequence sequence:[1 1 1 1]

LHS

Columns 1 through 3

$10.0000 + 0.0000i$ $-2.0000 + 2.0000i$ $-2.0000 + 0.0000i$

Column 4

$-2.0000 - 2.0000i$

RHS

Columns 1 through 3

$10.0000 + 0.0000i$ $-2.0000 + 2.0000i$ $-2.0000 + 0.0000i$

Column 4

$-2.0000 - 2.0000i$

Multiplication property verified