Information Sciences 424 (2018) 224–234

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Exponential stability and $\mathcal{L}_1$-gain analysis for positive time-delay Markovian jump systems with switching transition rates subject to average dwell time

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A R T I C L E I N F O

Article history:
Received 23 November 2016
Revised 25 September 2017
Accepted 3 October 2017
Available online 3 October 2017

Keywords:
Exponential stability
Switching transition rates
Linear co-positive Lyapunov function
Average dwell time

A B S T R A C T

The paper deals with the problems of exponential stability and $\mathcal{L}_1$-gain analysis for positive time-delay Markovian jump systems (MJSs) with switching transition rates. Another set of useful regime-switching model is given, which extends fixed transition rates to time-varying transition rates. By resorting to the linear co-positive Lyapunov function and average dwell time, sufficient conditions for exponential stability are proposed in terms of standard linear programming. Based on the obtained results, $\mathcal{L}_1$-gain performance is analyzed. Finally, an example is proposed to illustrate the validity of the main results.

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1. Introduction

In practical systems, there exists a special class of dynamic systems whose state variables and output signals remain nonnegative whenever both the initial condition and the input signal are nonnegative. This unordinary kind of systems, usually denoted as positive systems [8,12], also known as nonnegative systems, have extensive applications including industrial engineering [3] and communication networks [31]. Over the past decade, positive systems have received ever-increasing research interest and many interesting properties have been unraveled (see e.g., [1,4,6,10,23,33,36,45]). For example, the l_1-induced norm has been constructed and the existence of l_1 state feedback controller has been solved by using an iterative convex optimization approach in [4]. For continuous-time positive switched linear systems [33], sufficient conditions for asymptotic stability under appropriate switching have been given by constructing the joint linear co-positive Lyapunov function. For positive Takagi-Sugeno fuzzy nonlinear systems [45], some less conservative conditions for stability have been proposed and a constrained fuzzy tracking controller has been designed to ensure the tracking performance and positivity of closed-loop system.

At the same time, time delay [48,49] is very common in practical dynamical systems, such as manufacturing systems, networked control systems, economic systems, biological systems, telecommunication and chemical systems, etc. It has been shown that time delay is usually an important factor of instability or poor performance in a dynamical system. Recently,

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https://doi.org/10.1016/j.ins.2017.10.008
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many significant results on stability, stabilization, and performance analysis for positive systems with time delay can be found (see e.g., [7,11,22,30,37,38,43]). For example, for positive two-dimensional Takagi-Sugeno fuzzy systems with state delays [7], the l_1-gain performance analysis has been studied. For fuzzy positive systems with time-varying delays [11], the $\mathcal{L}_-$ fault detection filter and multi-objective $\mathcal{L}_-/\mathcal{L}_1$ fault detection filter design problems have been addressed. By constructing the Lyapunov–Krasovskii functional, an absolute exponential $\mathcal{L}_1$ controller for positive switched nonlinear time-delay systems has been designed in [38]. The $\mathcal{H}_\infty$ dynamic output feedback controller gains for positive systems with multiple delays have been obtained by use of cone complementarity linearization techniques in [43].

On the other hand, as a special class of hybrid systems, MJSs have some advantages of describing dynamic systems subject to sudden changes between subsystems caused by external causes, such as networked control systems, manufacturing systems, fault-detection systems, and system theory (see e.g., [13–19,26–29,32,34,35,41,42,44]). To mention a few, for uncertain time-delay systems with Markovian switching parameters [13], some new sufficient conditions for delay-dependent stochastic stability and stabilization with $\mathcal{H}_\infty$ performance have been established in terms of linear matrix inequalities. In [14], a sliding-mode approach has been proposed for the exponential $\mathcal{H}_\infty$ synchronization problem of master-slave time-delay systems with nonlinear uncertainties and Markovian jumping parameters. In [28], for continuous-time Markovian jump linear systems with deficient transition descriptions, the authors have developed an input-output approach to the delay-dependent stability analysis and $\mathcal{H}_\infty$ controller synthesis. For continuous-time Markovian jump linear systems with time-varying delay and partially accessible mode information [34], by utilizing the model transformation idea and the scaled small gain theorem, the underlying full-order and reduced-order $\mathcal{H}_\infty$ filtering synthesis problems have been formulated. For discrete-time Markovian jump nonlinear systems with time-delays represented by Takagi–Sugeno model [44], sufficient conditions in the form of linear matrix inequalities have been presented for stochastic finite-time $\mathcal{H}_\infty$ stabilization via observer-based fuzzy state feedback. However, many results about positive MJSs [2,9,20,21,24,25,39,40,46] have been obtained in view of time-invariant transition rates. In fact, different from those uncertain or partly known transition rates, an extension to Markovian jump model with time-varying transition rates can provide another set of useful regime-switching model. In such model, MJSs with switching transition rates have some advantages over that with fixed transition rates in terms of flexibility [47]. Recently, some papers concerning MJSs with switching transition rates [5,47] have been published.

This paper will deal with the issues of exponential stability and $\mathcal{L}_1$-gain analysis for positive MJSs with time-varying delay and switching transition rates. It is necessary to point out the differences between the present work and the existing relative works [2,9,20,21,24,25,37,39,40,46]. First, the switching law in [37] followed asynchronous switching law with mode-dependent average dwell time while the switching law in this paper follows the Markovian process with time-varying property. Second, the literatures [2,9,20,21,24,25,39,40,46] mainly investigated the positive MJSs with time-invariant transition rates while the transition rates in this paper are time-varying. Third, the literatures [2,9,24,39,46] mainly investigated the positive MJSs without time delay while the model in this paper considers time delay. The problems of exponential stability and $\mathcal{L}_1$-gain analysis for positive MJSs with time-varying delay and switching transition rates are theoretically challenging, difficult, and open all the time. More specifically, compared with positive time-delay MJSs with time-invariant transition rates, it is very difficult to handle positive time-delay MJSs with time-varying transition rates. Moreover, one needs to consider not only stability of dynamic systems governed by the Markovian process, but also the constrained positivity. In addition, the dynamical behaviors are affected by the interaction among Markovian process, time delay, switching transition rates, disturbance input, and positivity, which are very complicated. These above aspects motivate our current research work. The main contributions are given as follows: (i) By resorting to the linear co-positive Lyapunov function and average dwell time, sufficient conditions for exponential stability are proposed; (ii) The disturbance attenuation performance (the $\mathcal{L}_1$-gain performance) is analyzed; (iii) Additionally, due to the quantity of virus mutation treatment model satisfying the nonnegative property and abrupt change of genetic mutation, modeling virus mutation treatment model as positive MJSs shows the validity of the main algorithms.

Notation. $A\succeq$ ($\preceq 0$, $\succ$, $\prec$) means all entries of matrix A are nonnegative (non-positive, positive, negative); $A\succ B$ ($A\succeq B$) means $A-B\succ 0$ ($A-B\succeq 0$). $\mathbb{R}$ ($\mathbb{R}_+$) denotes the set of all real (positive real) numbers; $\mathbb{R}^n$ ($\mathbb{R}^n_+$) stands for n-dimensional real (positive) vector space. The vector 1-norm is denoted by $||x||_1=\sum_{k=1}^n |x_k|$, where x_k is the kth element of $x\in\mathbb{R}^n$; $A=[a_{ij}]_{n\times n}$ is a Metzler matrix if $a_{ij}\geq 0$, $i\neq j$. Symbol $E\{\cdot\}$ stands for the mathematical expectation. $\mathbf{1}_n$ means the all-ones vector in $\mathbb{R}^n$.

2. Problem statement and preliminaries

Consider the positive time-delay MJSs as follows:

$$\begin{aligned} \dot{x}(t) &= A(g_t)x(t)+A_d(g_t)x(t-d(t))+G(g_t)w(t),\\ z(t) &= C(g_t)x(t)+D(g_t)w(t),\\ x(t_0+\theta) &= \varphi(\theta), \forall\theta\in[-d,0], \end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$, $w(t)\in\mathbb{R}^l$, and $z(t)\in\mathbb{R}^q$ are, respectively, the state vector, the disturbance input, and the estimated output; $\varphi(\theta)$ is the initial condition; g_t stands for a time-homogeneous Markovian process and takes values in $S_1=\{1,2,\ldots,N\}$ with transition rates as:

$$P\{g_{t+\Delta t}=j|g_t=i\}=\begin{cases}\pi_{ij}^{(r_t)}\Delta t+o(\Delta t), & i\neq j,\\ 1+\pi_{ij}^{(r_t)}\Delta t+o(\Delta t), & i=j,\end{cases} \tag{2}$$

where $\Delta t\geq 0$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t)=0$ and $\pi_{ij}^{(r_t)}\geq 0$, for $i\neq j$ and $\sum_{j=1,i\neq j}^{N}\pi_{ij}^{(r_t)}=-\pi_{ii}^{(r_t)}$.

The time-varying delay $d(t)$ satisfies

$$0<d(t)\leq d,\dot{d}(t)\leq h,$$

where d and h are known real constant scalars.

In addition, the process $\{g_t,\ t\geq 0\}$ describes the time-varying characters of transition rates. Through $r_t=\alpha$, we can see that the transition rates are time-varying. Moreover, $r_t\in S_2$ means that r_t is a piecewise constant function of time, which takes values in a finite set $S_2=\{1,2,\dots,M\}$. For simplicity, for $g_t=i\in S_1$, $A(g_t)$, $A_d(g_t)$, $G(g_t)$, $C(g_t)$, and $D(g_t)$ are, respectively, denoted as A_i, A_{di}, G_i, C_i, and D_i.

Remark 1. Compared with some similar works [2,9,20,21,24,25,39,40,46], the new design algorithms have many advantages. First, compared with the time-invariant transition rates [2,9,20,21,24,25,39,40,46], the transition rates in this paper are time-varying and more suitable to describe positive MJSs since the assumption for time-invariant transition rates is not natural in some applications and the resulting transition rates may vary throughout the running time of the modeled control system. Second, in the switching law, the high-level switching signal complies with the average dwell time switching constraint greater than or equal to T_a^* and the low-level switching follows stochastic Markovian switching law. It should be noted that when $M=1$, MJSs (1) reduce to a homogeneous MJSs [2,9,20,21,24,25,39,40,46]. Thus, the homogeneous MJSs can be regarded as a special case of MJSs with time-varying transition rates which are more general and complex than homogeneous MJSs. Thirdly, compared with the literatures [20,21,40], we propose sufficient conditions for exponential stability with $\mathcal{L}_1$-gain performance that depends on the bound of time-varying delay. Finally, an appropriate linear co-positive Lyapunov function is chosen and the desired disturbance rejection performance is obtained.

Definition 1. [8,12] System (1) is said to be positive if, for any initial condition $\varphi(\theta)\succeq 0$, there hold $x(t)\succeq 0$ and $z(t)\succeq 0$, $t>0$.

Lemma 1. [8,12] *Consider the following positive systems:*

$$\begin{aligned}\dot{x}(t)&=Ax(t)+A_dx(t-d(t))+Gw(t),\\ z(t)&=Cx(t)+Dw(t),\\ x(t_0+\theta)&=\varphi(\theta),\forall\theta\in[-d,0].\end{aligned} \tag{3}$$

System (3) is positive if and only if A is Metzler matrix, $A_d\succeq 0$, $G\succeq 0$, $C\succeq 0$, *and* $D\succeq 0$.

Definition 2. [36] System (1) is said to be exponentially stable if the following inequality holds

$$E\{\|x(t)\|\}\leq\tilde{\alpha}e^{-\tilde{\beta}(t-t_0)}\|x(t_0)\|,\forall t\geq t_0, \tag{4}$$

for constants $\tilde{\alpha}>0$, $\tilde{\beta}>0$, where $\|x(t_0)\|=sup_{-d\leq\theta\leq 0}\|\varphi(\theta)\|$.

Definition 3. [36] For a switching signal r_t and any $t>t_0$, let $N_r(t_0,\ t)$ be the switching numbers of r_t over the interval $(t_0,\ t)$. For some $T_a>0$ and $N_0\geq 0$, if the inequality $N_r(t_0,t)\leq N_0+\frac{t-t_0}{T_a}$ holds, then the positive constant T_a is called an average dwell time and N_0 is called a chatting bound.

As commonly used in the literature, for convenience, we choose $N_0=0$ in this paper.

Definition 4. [36] For $\gamma>0$, system (1) is said to have a weighted $\mathcal{L}_1$-gain, if the following conditions hold

(i) System (1) is exponentially stable when $w(t)=0$;
(ii) The following inequality holds

$$E\left\{\int_{t_0}^{\infty}e^{-\lambda(t-t_0)}||z(t)||_1dt\right\}\leq\gamma E\left\{\int_{t_0}^{\infty}||w(t)||_1dt\right\},$$

when $\varphi(\theta)=0$ and $w(t)\neq 0$.

In this paper, sufficient conditions are proposed such that system (1) with switching transition rates is exponentially stable with $\mathcal{L}_1$-gain performance.

3. Main results

In this section, we will consider the problems of exponential stability analysis and $\mathcal{L}_1$-gain performance analysis for system (1).

3.1. Exponential stability analysis

Theorem 1. *For given scalars $\lambda > 0$ and $\mu > 1$, system (1) with $w(t) = 0$ is exponentially stable, if there exists a set of vectors $\nu_{\alpha,i}$, $\sigma_{1\alpha,i}$, $\sigma_{2\alpha,i}$, $\sigma_{1\alpha}$, $\sigma_{2\alpha}$, and $\rho_{\alpha,i} \in \mathbb{R}^n_+$, such that the following inequalities hold*

$$A_i^T \nu_{\alpha,i} + \lambda \nu_{\alpha,i} + \sigma_{1\alpha,i} + \sigma_{2\alpha,i} + d\sigma_{1\alpha} + d\sigma_{2\alpha} + \rho_{\alpha,i} + \sum_{j=1}^{N} \pi_{ij}^{\alpha} \nu_{\alpha,j} \prec 0, \tag{5}$$

$$A_{di}^T \nu_{\alpha,i} - (1-h)e^{-\lambda d}\sigma_{1\alpha,i} - \rho_{\alpha,i} \prec 0, \tag{6}$$

$$\sum_{j=1}^{N} \pi_{ij}^{\alpha} \sigma_{1\alpha,j} \preceq \sigma_{1\alpha}, \sum_{j=1}^{N} \pi_{ij}^{\alpha} \sigma_{2\alpha,j} \preceq \sigma_{2\alpha}, \tag{7}$$

$$\nu_{\alpha,i} \preceq \mu \nu_{\beta,j}, \sigma_{1\alpha,i} \preceq \mu \sigma_{1\beta,j}, \sigma_{2\alpha,i} \preceq \mu \sigma_{2\beta,j}, \sigma_{1\alpha} \preceq \mu \sigma_{1\beta}, \sigma_{2\alpha} \preceq \mu \sigma_{2\beta}, \forall \alpha, \beta \in S_2, \forall i, j \in S_1, \alpha \neq \beta, \tag{8}$$

with the average dwell time satisfying

$$T_a > T_a^* = \frac{\ln \mu}{\lambda}. \tag{9}$$

Proof. The mode-dependent integral terms in Lyapunov function could provide more freedom and overcome the conservativeness of mode-independent Lyapunov function. For system (1), choose the linear co-positive Lyapunov function candidate as

$$\begin{aligned} V(x(t), t, \alpha, i) = {} & x^T(t)\nu_{\alpha,i} + \int_{t-d(t)}^{t} e^{\lambda(s-t)} x^T(s)\sigma_{1\alpha,i} ds + \int_{t-d}^{t} x^T(s) e^{\lambda(s-t)} \sigma_{2\alpha,i} ds \\ & + \int_{-d}^{0}\int_{t+\theta}^{t} e^{\lambda(s-t)} x^T(s)(\sigma_{1\alpha} + \sigma_{2\alpha}) ds d\theta, \end{aligned} \tag{10}$$

where $\nu_{\alpha,i}$, $\sigma_{1\alpha,i}$, $\sigma_{2\alpha,i}$, $\sigma_{1\alpha}$, $\sigma_{2\alpha} \in \mathbb{R}^n_+$ and

$$\sum_{j=1}^{N} \pi_{ij}^{\alpha} \sigma_{1\alpha,j} \preceq \sigma_{1\alpha}, \sum_{j=1}^{N} \pi_{ij}^{\alpha} \sigma_{2\alpha,j} \preceq \sigma_{2\alpha}.$$

Along the trajectory of system (1), one has

$$\begin{aligned} & \Gamma V(x(t), t, \alpha, i) \\ = {} & x^T(t)\left[A_i^T \nu_{\alpha,i} + \lambda \nu_{\alpha,i} + \sigma_{1\alpha,i} + \sigma_{2\alpha,i} + d\sigma_{1\alpha} + d\sigma_{2\alpha} + \sum_{j=1}^{N} \pi_{ij}^{\alpha} \nu_{\alpha,j}\right] \\ & + x^T(t-d(t))A_{di}^T \nu_{\alpha,i} - (1-\dot{d}(t))e^{-\lambda d(t)} x^T(t-d(t))\sigma_{1\alpha,i} \\ & + \sum_{j=1}^{N} \pi_{ij}^{\alpha} \int_{t-d(t)}^{t} e^{\lambda(s-t)} x^T(s)\sigma_{1\alpha,j} ds - \int_{t-d}^{t} e^{\lambda(s-t)} x^T(s)(\sigma_{1\alpha} + \sigma_{2\alpha}) ds \\ & - e^{-\lambda d} x^T(t-d)\sigma_{2\alpha,i} + \sum_{j=1}^{N} \pi_{ij}^{\alpha} \int_{t-d}^{t} e^{\lambda(s-t)} x^T(s)\sigma_{2\alpha,j} ds - \lambda V(x(t), t, \alpha, i), \end{aligned}$$

where Γ is the weak infinitesimal operator.

Using Leibniz–Newton formula, one can obtain

$$\int_{t-d(t)}^{t} \dot{x}(s) ds = x(t) - x(t-d(t)), \tag{11}$$

and

$$\int_{t-d(t)}^{t} \dot{x}(s) ds = \int_{t-d(t)}^{t} (A_i x(s) + A_{di} x(s-d(s))) ds. \tag{12}$$

Then, for any vectors $\rho_{\alpha,i} \in \mathbb{R}_+^n$, according to the inequalities (11) and (12), we obtain

$$[x(t) - x(t-d(t)) - \int_{t-d(t)}^{t} (A_i x(s) + A_{di} x(s-d(s)))ds]^T \rho_{\alpha,i} = 0. \tag{13}$$

According to $-\int_{t-d}^{t} e^{\lambda(s-t)} x^T(s)\sigma_{1\alpha} ds \le -\int_{t-d(t)}^{t} e^{\lambda(s-t)} x^T(s)\sigma_{1\alpha} ds$, we have

$$\begin{aligned}
&\Gamma V(x(t), t, \alpha, i) \\
&\le x^T(t)\left[A_i^T \nu_{\alpha,i} + \lambda \nu_{\alpha,i} + \sigma_{1\alpha,i} + \sigma_{2\alpha,i} + d\sigma_{1\alpha} + d\sigma_{2\alpha} + \rho_{\alpha,i} + \sum_{j=1}^{N} \pi_{ij}^{\alpha} \nu_{\alpha,j}\right] \\
&\quad + x^T(t-d(t))\left[A_{di}^T \nu_{\alpha,i} - (1-h)e^{-\lambda d}\sigma_{1\alpha,i} - \rho_{\alpha,i}\right] - e^{-\lambda d} x^T(t-d)\sigma_{2\alpha,i} \\
&\quad - \left[\int_{t-d(t)}^{t} (A_i x(s) + A_{di} x(s-d(s)))ds\right]^T \rho_{\alpha,i} - \lambda V(x(t), t, \alpha, i).
\end{aligned} \tag{14}$$

From the inequalities (5)–(7), one has

$$\Gamma V(x(t), t, \alpha, i) \le -\lambda V(x(t), t, \alpha, i). \tag{15}$$

Multiplying (15) by $e^{-\lambda t}$ and integrating both sides of (15) from t_k to t yield

$$V(x(t), t, r_t, g_t) \le e^{-\lambda(t-t_k)} V(x(t_k), t_k, r_{t_k}, g_{t_k}). \tag{16}$$

Combining conditions (8) and (16) leads to

$$V(x(t), t, r_t, g_t) \le \mu e^{-\lambda(t-t_k)} V(x(t_k^-), t_k^-, r_{t_k^-}, g_{t_k^-}). \tag{17}$$

Therefore, following (17) and making the recursion from t_{k-1} to t_k, t_{k-2} to t_{k-1}, $\cdots$, until t_0, finally, we can get

$$V(x(t), t, r_t, g_t) \le e^{-(\lambda - \frac{\ln \mu}{T_a})(t-t_0)} V(x_0, t_0, r_0, g_0). \tag{18}$$

Considering the definition of $V(x(t), t, \alpha, i)$, and denoting

$$\begin{aligned}
\varepsilon_1 &= min_{(\alpha,i,p)\in S_2\times S_1\times S_3}\{\nu_{p,\alpha,i}\},\ \varepsilon_2 = max_{(\alpha,i,p)\in S_2\times S_1\times S_3}\{\nu_{p,\alpha,i}\}, \\
\varepsilon_3 &= max_{(\alpha,i,p)\in S_2\times S_1\times S_3}\{\sigma_{p,1\alpha,i}\},\ \varepsilon_4 = max_{(\alpha,i,p)\in S_2\times S_1\times S_3}\{\sigma_{p,2\alpha,i}\}, \\
\varepsilon_5 &= max_{(\alpha,p)\in S_2\times S_3}\{\sigma_{p,1\alpha}\},\ \varepsilon_6 = max_{(\alpha,p)\in S_2\times S_3}\{\sigma_{p,2\alpha}\},\ S_3 = \{1, 2, \cdots, n\},
\end{aligned}$$

the following inequalities hold

$$\begin{aligned}
&V(x(t), t, r_t, g_t) \ge \varepsilon_1 \|x(t)\|, \\
&V(x(t), t, r_t, g_t) \le \epsilon_2 \|x(t_0)\| + \epsilon_3 \int_{t_0-d(t)}^{t} e^{\lambda(s-t_0)} \|x(s)\| ds + \epsilon_4 \int_{-d}^{0}\int_{t_0+\theta}^{t_0} e^{\lambda(s-t_0)} \|x(s)\| ds \\
&\qquad + \epsilon_5 \int_{t_0-d}^{t_0} e^{\lambda(s-t_0)} \|x(s)\| ds + \epsilon_6 \int_{-d}^{0}\int_{t_0+\theta}^{t_0} e^{\lambda(s-t_0)} \|x(s)\| ds.
\end{aligned} \tag{19}$$

Combining (18) and (19) gives rise to

$$\|x(t)\| \le \tilde{\alpha} e^{-\tilde{\beta}(t-t_0)} \|x(t_0)\|, \tag{20}$$

where

$$\tilde{\alpha} = \frac{\epsilon_2 + \epsilon_3 d e^{-\lambda d} + \frac{1}{2}\epsilon_4 d^2 e^{-\lambda d} + \epsilon_5 d e^{-\lambda d} + \frac{1}{2}\epsilon_6 d^2 e^{-\lambda d}}{\epsilon_1},\ \tilde{\beta} = \left(\lambda - \frac{\ln \mu}{T_a}\right).$$

Finally, if the inequalities (5)–(8) hold, then system (1) is exponentially stable with switching signal satisfying the average dwell time constraint (9).

This completes the proof. □

Remark 2. The constraints $\sum_{j=1}^{N} \pi_{ij}^{\alpha}\sigma_{1\alpha,j} \preceq \sigma_{1\alpha}$ and $\sum_{j=1}^{N} \pi_{ij}^{\alpha}\sigma_{2\alpha,j} \preceq \sigma_{2\alpha}$ can be helpful to eliminate the following integral terms $\sum_{j=1}^{N} \pi_{ij}^{\alpha} \int_{t-d(t)}^{t} e^{\lambda(s-t)} x^T(s)\sigma_{1\alpha,j} ds$, $-\int_{t-d}^{t} e^{\lambda(s-t)} x^T(s)(\sigma_{1\alpha} + \sigma_{2\alpha})ds$, $\sum_{j=1}^{N} \pi_{ij}^{\alpha} \int_{t-d}^{t} e^{\lambda(s-t)} x^T(s)\sigma_{2\alpha,j} ds$ in the proof of Theorem 1.

Remark 3. Generally speaking, for MJSs with time-varying delay, the Lyapunov function is frequently chosen as follows:

$$V(x(t),t,\alpha,i)=x^T(t)\nu_{\alpha,i}+\int_{t-d(t)}^{t}e^{\lambda(s-t)}x^T(s)\sigma_{1\alpha}ds+\int_{-d}^{0}\int_{t+\theta}^{t}e^{\lambda(s-t)}x^T(s)\sigma_{1\alpha}dsd\theta.$$

The parameters in integral term of the above equality are independent of Markovian process, which may lead to some conservativeness. Here, an appropriate mode-dependent co-positive type Lyapunov function (10) is constructed and the parameters in integral term are dependent of Markovian process, which may reduce some conservativeness.

Remark 4. In Theorem 1, due to the existence of switching signal, the Markovian process is piecewise integrated. Moreover, from conditions (17) and (18), it is clear that the Lyapunov function is allowed to increase not only in the same mode, but also at the switching instants, which may reduce some conservativeness.

3.2. $\mathcal{L}_1$-gain performance analysis.

Theorem 2. *For given scalars $\lambda > 0$ and $\mu > 1$, system (1) is exponentially stable with $\mathcal{L}_1$-gain performance, if there exists a set of vectors $\nu_{\alpha,\,i}$, $\sigma_{1\alpha,\,i}$, $\sigma_{2\alpha,\,i}$, $\sigma_{1\alpha}$, $\sigma_{2\alpha}$, and $\rho_{\alpha,i}\in\mathbb{R}^n_+$, such that the following inequalities hold*

$$A_i^T\nu_{\alpha,i}+\lambda\nu_{\alpha,i}+\sigma_{1\alpha,i}+\sigma_{2\alpha,i}+d\sigma_{1\alpha}+d\sigma_{2\alpha}+\rho_{\alpha,i}+\sum_{j=1}^{N}\pi_{ij}^{\alpha}\nu_{\alpha,j}+C_i^T\mathbf{1}\prec 0, \tag{21}$$

$$G_i^T\nu_{\alpha,i}+D_i^T\mathbf{1}-\gamma\mathbf{1}\prec 0, \tag{22}$$

$$A_{di}^T\nu_{\alpha,i}-(1-h)e^{-\lambda d}\sigma_{1\alpha,i}-\rho_{\alpha,i}\prec 0, \tag{23}$$

$$\sum_{j=1}^{N}\pi_{ij}^{\alpha}\sigma_{1\alpha,j}\preceq\sigma_{1\alpha},\ \sum_{j=1}^{N}\pi_{ij}^{\alpha}\sigma_{2\alpha,j}\preceq\sigma_{2\alpha}, \tag{24}$$

$$\nu_{\alpha,i}\preceq\mu\nu_{\beta,j},\sigma_{1\alpha,i}\preceq\mu\sigma_{1\beta,j},\sigma_{2\alpha,i}\preceq\mu\sigma_{2\beta,j},\sigma_{1\alpha}\preceq\mu\sigma_{1\beta},\sigma_{2\alpha}\preceq\mu\sigma_{2\beta},\forall\alpha,\beta\in S_2,\forall i,j\in S_1,\alpha\neq\beta, \tag{25}$$

with the average dwell time satisfying (9).

Proof. According to (21), we can get (5), which means that system (1) ($w(t)=0$) is exponentially stable.

When $w(t)\neq 0$, considering Lyapunov function (10) gives rise to

$$\begin{aligned}
&\Gamma V(x(t),t,\alpha,i)+\|z(t)\|_{L_1}-\gamma\|w(t)\|_{L_1}\\
\le&x^T(t)\Big[A_i^T\nu_{\alpha,i}+\lambda\nu_{\alpha,i}+\sigma_{1\alpha,i}+\sigma_{2\alpha,i}+d\sigma_{1\alpha}+d\sigma_{2\alpha}+\rho_{\alpha,i}+C_i^T\mathbf{1}+\sum_{j=1}^{N}\pi_{ij}^{\alpha}\nu_{\alpha,j}\Big]\\
&+x^T(t-d(t))\Big[A_{di}^T\nu_{\alpha,i}-(1-h)e^{-\lambda d}\sigma_{1\alpha,i}-\rho_{\alpha,i}\Big]-e^{-\lambda d}x^T(t-d)\sigma_{2\alpha,i}\\
&-\Big[\int_{t-d(t)}^{t}(A_ix(s)+A_{di}x(s-d(s))+G_iw(s))ds\Big]^T\rho_{\alpha,i}\\
&+w^T(t)(G_i^T\nu_{\alpha,i}+D_i^T\mathbf{1}-\gamma\mathbf{1})-\lambda V(x(t),t,\alpha,i).
\end{aligned} \tag{26}$$

From $t\in[t_k,t_{k+1})$, the inequalities (21)–(25) imply

$$V(x(t),t,r_t,g_t)\le e^{-\lambda(t-t_k)}V(x(t_k),t_k,r_{t_k},g_{t_k})-E\left\{\int_{t_k}^{t}e^{-\lambda(t-s)}\Pi(s)ds\right\}, \tag{27}$$

where $\Pi(s)=||z(s)||_1-\gamma||w(s)||_1$.

Combining (8) leads to

$$\begin{aligned}
&E\{V(x(t),t,r_t,g_t)\}\le\mu e^{-\lambda(t-t_k)}E\{V\{(x(t_k^-),t_k^-,r_{t_k^-},g_{t_k^-})\}\}-E\left\{\int_{t_k}^{t}e^{-\lambda(t-s)}\Pi(s)ds\right\}\\
\le&\mu^ke^{-\lambda(t-t_0)}V\{(x_0,t_0,r_0,g_0)\}-\mu^kE\left\{\int_{t_0}^{t_1}e^{-\lambda(t-s)}\Pi(s)ds\right\}\\
&-\mu^{k-1}E\left\{\int_{t_1}^{t_2}e^{-\lambda(t-s)}\Pi(s)ds\right\}-\cdots-\mu^0E\left\{\int_{t_k}^{t}e^{-\lambda(t-s)}\Pi(s)ds\right\}.
\end{aligned} \tag{28}$$

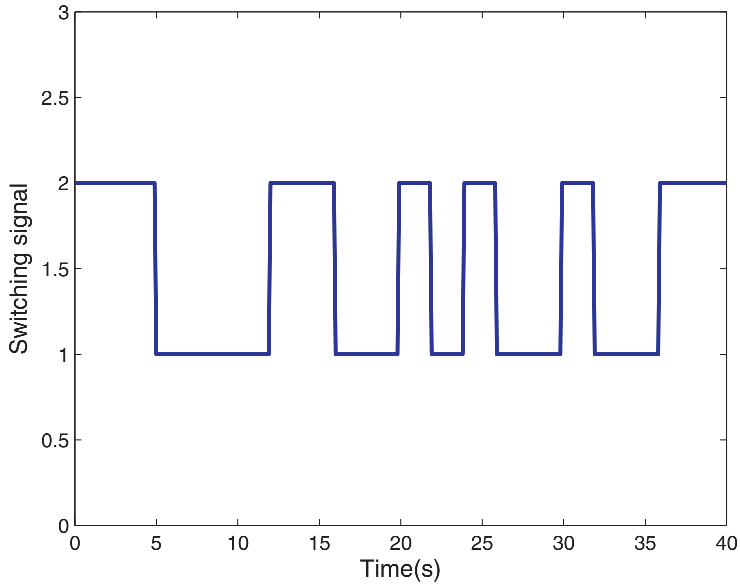

Fig. 1. Switching signal.

Under zero initial condition, we get

$$-E\left\{\int_{t_k}^{t} e^{-\lambda(t-s)+N_\sigma(s,t)\ln\,\mu}\Pi(s)ds\right\} \geq 0. \tag{29}$$

Multiplying both sides of (29) by $N_\sigma(t_0,\, t)\ln\mu$ yields

$$E\left\{\int_{t_0}^{t} e^{-\lambda(t-s)-N_\sigma(t_0,s)\ln\,\mu}||z(s)||_1 ds\right\} \leq E\left\{\gamma\int_{t_0}^{t} e^{-\lambda(t-s)-N_\sigma(t_0,s)\ln\,\mu}||w(s)||_1 ds\right\}. \tag{30}$$

Noticing that $N_\sigma(t_0, s) \leq (s-t_0)/T_a$ and (9), we have $N_\sigma(t_0, s)\ln\,\mu \leq \lambda(s-t_0)$, which means that

$$E\left\{\int_{t_0}^{t} e^{-\lambda(t-s)-\lambda(s-t_0)}||z(s)||_1 ds\right\} \leq E\left\{\gamma\int_{t_0}^{t} e^{-\lambda(t-s)}||w(s)||_1 ds\right\}. \tag{31}$$

Integrating both sides of (31) from t_0 to ∞ leads to $E\left\{\int_{t_0}^{\infty} e^{-\lambda(t-t_0)}||z(t)||_1 dt\right\} \leq \gamma E\left\{\int_{t_0}^{\infty}||w(t)||_1 dt\right\}$.

This completes the proof. □

Remark 5. For given scalars $\lambda > 0$ and $\mu > 1$, we can take γ as the optimized variable to obtain the minimum $\mathcal{L}_1$-gain value that conditions (21)–(25) hold. Then, the optimization problem can be described as follows:

$$\begin{array}{ll} \min & \gamma \\ & \nu_{\alpha,i}, \sigma_{1\alpha,i}, \sigma_{2\alpha,i}, \sigma_{1\alpha}, \sigma_{2\alpha}, \rho_{\alpha,i} \in \mathbb{R}_+^n, \lambda > 0, \mu > 1 \\ s.t. & \textit{Inequalities}\ (21)-(25). \end{array} \tag{32}$$

Remark 6. Although the Lyapunov function is complex and the computational complexity increases, sufficient conditions in the form of linear programming have been proposed to reduce the conservativeness in optimization problem (32). The example about mathematical model of virus mutation treatment demonstrates the validity of the main results.

Remark 7. The model parametric uncertainties are not considered in this paper. As the proposed conditions are based on linear programming, they can be easily extended to positive time-delay MJSs with model parametric uncertainties.

4. Numerical example

Diseases are more or less associated with one person's life. A good body can help us to avoid the disease. On the contrary, a bad body often bothers us and seriously affects the quality of our life. The virus mutation model system is an important system in medical area. Thus, how to maintain the stability of the virus mutation model is a topic worth studying.

Many physical systems in the actual operation of the process may be subject to random parameters change, which will lead to a sudden change of the parameters or structures of the system. MJSs play a great part in analysis of the virus mutation treatment model. Due to the quantity of virus mutation treatment model satisfying the nonnegative property,

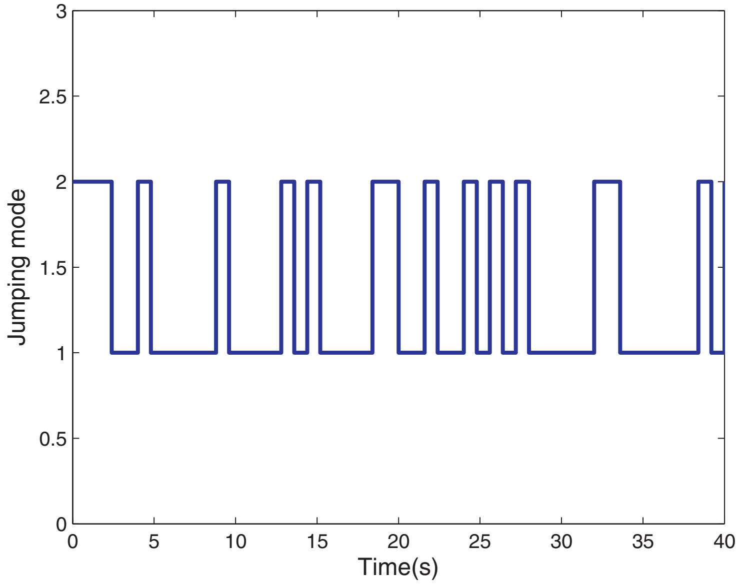

Fig. 2. Jumping mode.

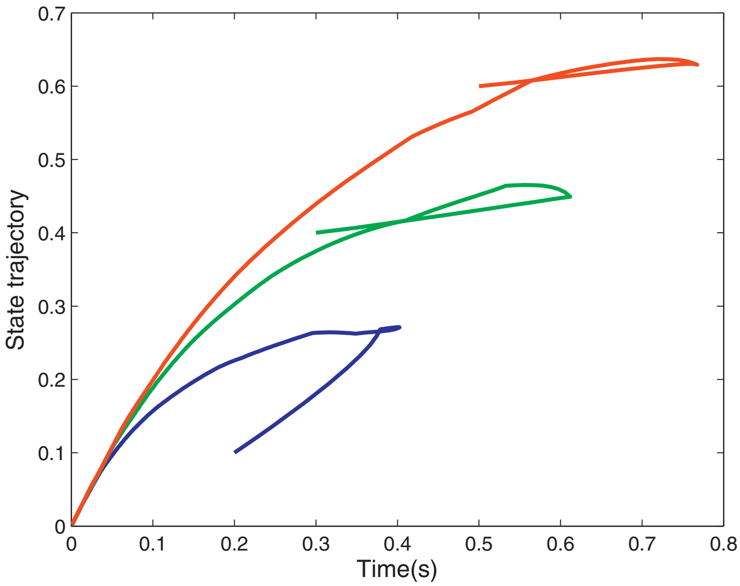

Fig. 3. State trajectory.

abrupt change of genetic mutation, and the time-varying characteristics of the transition rates, the virus mutation treatment model can be described by positive MJSs with switching transition rates.

Consider the mathematical model of virus mutation treatment form [10]:

$$\dot{x}(t) = (R_i - \delta I + \zeta M)x(t) + G_i w(t), z(t) = C_i x(t) + D_i w(t), \tag{33}$$

where $x(t)$ indicates two different viral genotypes; $w(t)$ and $z(t)$ are the disturbance input and the estimated output; $i = 1, 2$ means a Markovian process; ζ, δ, and $M = [M_{mn}]$ represent the mutation rate, the death or decay rate, and the system matrices; $M_{mn} \in \{0, 1\}$ means the genetic connections between genotypes. The parameters are given as follows:

$$R_1 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.25 \end{bmatrix}, R_2 = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.26 \end{bmatrix}, M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, G_1 = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, G_2 = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1.0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1.0 & 0 \end{bmatrix}, D_1 = \begin{bmatrix} 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.2 \end{bmatrix}, \delta = 0.68, \zeta = 0.001.$$

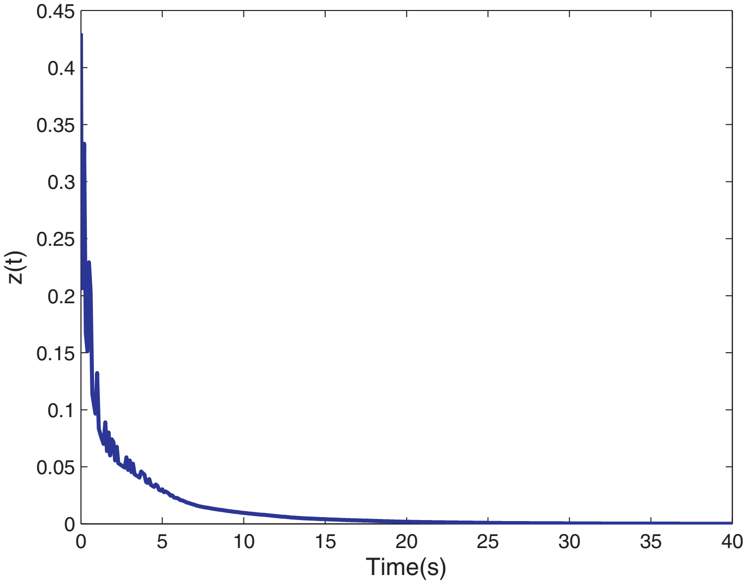

Fig. 4. Estimated output.

We assume that there exists the time-varying delay in the mathematical model of virus mutation treatment. Then, the other parameters are given as follows:

$$A_{d1}=\begin{bmatrix}0.2 & 0.1\\ 0.1 & 0.2\end{bmatrix}, A_{d2}=\begin{bmatrix}0.2 & 0.3\\ 0.1 & 0.1\end{bmatrix}, d(t)=1.2(1-cos(t)), d=2.4, h=1.2.$$

The transition rate matrices are given as follows:

$$\prod^1=\begin{bmatrix}-1.2 & 1.2\\ 2 & -2\end{bmatrix}, \prod^2=\begin{bmatrix}-0.8 & 0.8\\ 0.5 & -0.5\end{bmatrix}.$$

Let $\lambda=0.1019$, $\mu=1.49$, $w(t)=e^{-t}(2-sin(t))$. Solving optimization problem (32) results in the corresponding parameters as follows:

$$\begin{aligned}
&\nu_{11}=\begin{bmatrix}6.6583\\ 13.6691\end{bmatrix}, \nu_{12}=\begin{bmatrix}6.7052\\ 13.7375\end{bmatrix}, \nu_{21}=\begin{bmatrix}6.5475\\ 13.9734\end{bmatrix}, \nu_{22}=\begin{bmatrix}6.4849\\ 13.9107\end{bmatrix},\\
&\sigma_{111}=\begin{bmatrix}0.1471\\ 0.2305\end{bmatrix}, \sigma_{112}=\begin{bmatrix}0.1549\\ 0.2424\end{bmatrix}, \sigma_{121}=\begin{bmatrix}0.1566\\ 0.2481\end{bmatrix}, \sigma_{122}=\begin{bmatrix}0.1377\\ 0.2307\end{bmatrix},\\
&\sigma_{211}=\begin{bmatrix}0.1790\\ 0.2730\end{bmatrix}, \sigma_{212}=\begin{bmatrix}0.1872\\ 0.2852\end{bmatrix}, \sigma_{221}=\begin{bmatrix}0.1891\\ 0.2922\end{bmatrix}, \sigma_{222}=\begin{bmatrix}0.1687\\ 0.2725\end{bmatrix},\\
&\sigma_{11}=\begin{bmatrix}0.0414\\ 0.0674\end{bmatrix}, \sigma_{12}=\begin{bmatrix}0.0338\\ 0.0725\end{bmatrix}, \sigma_{21}=\begin{bmatrix}0.0415\\ 0.0675\end{bmatrix}, \sigma_{22}=\begin{bmatrix}0.0339\\ 0.0725\end{bmatrix},\\
&\rho_{11}=\begin{bmatrix}2.7714\\ 3.5022\end{bmatrix}, \rho_{12}=\begin{bmatrix}2.8260\\ 3.5359\end{bmatrix}, \rho_{21}=\begin{bmatrix}2.8084\\ 3.6142\end{bmatrix}, \rho_{22}=\begin{bmatrix}2.7276\\ 3.4543\end{bmatrix},\\
&\gamma=0.2231.
\end{aligned}$$

Then, according to (9), we can get $T_a^*=3.9134$ and choose the average dwell time $T_a=4$.

Figs. 1–4 stand for switching signal r_t, jumping mode g_t, state trajectory $x(t)$ with initial conditions $x(0)=\begin{bmatrix}0.2 & 0.1\end{bmatrix}^T$, $x(0)=\begin{bmatrix}0.3 & 0.4\end{bmatrix}^T$, $x(0)=\begin{bmatrix}0.5 & 0.6\end{bmatrix}^T$, and estimated output $z(t)$ with initial condition $x(0)=\begin{bmatrix}0.3 & 0.4\end{bmatrix}^T$. These figures demonstrate that our results are effective.

Remark 8. If the Lyapunov function is chosen as $V(x(t),t,\alpha,i)=x^T(t)\nu_{\alpha,i}+\int_{t-d(t)}^{t}e^{\lambda(s-t)}x^T(s)\sigma_{1\alpha}ds+\int_{-d}^{0}\int_{t+\theta}^{t}e^{\lambda(s-t)}x^T(s)\sigma_{1\alpha}dsd\theta$, we can get $\gamma=1.3432$, which means that sufficient conditions in this paper can effectively suppress the disturbance and reduce some conservativeness. Although the computational complexity increases, the smaller parameter γ can be obtained.

5. Conclusion

This study has concerned with exponential stability and $\mathcal{L}_1$-gain analysis for positive time-delay MJSs with switching transition rates subject to average dwell time approach. By using a linear co-positive Lyapunov function, sufficient conditions for exponential stability with $\mathcal{L}_1$-gain performance are proposed. As the proposed conditions are based on linear programming, they can be easily extended to other problems for the underlying systems, such as 2D systems, MJSs with model parametric uncertainties, fuzzy control, fault detection filter design, etc.

Acknowledgments

This work of W. Qi is supported by National Natural Science Foundation of China under Grant No. 61703231 and 61503184, Natural Science Foundation of Shandong under Grant No. ZR2017QF001, and Postdoctoral Science Foundation of China under Grant No. 2017M612235. This work of Y. Kao is supported by National Natural Science Foundation of China under Grant No. 61473097. This work of X. Gao is supported by National Natural Science Foundation of China under Grant No. 61573088. Also, the work of J.H. Park was supported by Basic Science Research Programs through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (Grant number NRF-2017R1A2B2004671) and in part by the Natural Science Foundation of CQ under Grant CSTC 2014J-CYJA40004 and the Natural Science Foundation of China under Grant 11471061.

References

[1] V.S. Bokharaie, O. Mason, On delay-independent stability of a class of nonlinear positive time-delay systems, IEEE Trans. Autom. Control 59 (7) (2014) 1974–1977.
[2] P. Bolzern, P. Colaneri, G. Nicolao, Stochastic stability of positive markov jump linear systems, Automatica 50 (4) (2014) 1181–1187.
[3] L. Caccetta, V.G. Rumchev, A positive linear discrete-time model of capacity planning and its controllability properties, Math. Comput. Model. 40 (1–2) (2004) 217–226.
[4] X.M. Chen, L. James, P. Li, Z. Shu, l_1-induced norm and controller synthesis of positive systems, Automatica 49 (5) (2013) 1377–1385.
[5] J. Cheng, H. Zhu, S.M. Zhong, F.X. Zheng, K.B. Shi, Finite-time boundedness of a class of discrete-time Markovian jump systems with piecewise-constant transition probabilities subject to average dwell time switching, Can. J. Phys. 92 (2) (2014) 93–102.
[6] J.G. Dong, Stability of switched positive nonlinear systems, Int. J. Robust Nonlinear Control 26 (14) (2016) 3118–3129.
[7] Z.X. Duan, Z.R. Xiang, H.R. Karimi, Stability and l_1-gain analysis for positive 2d T-S fuzzy state-delayed systems in the second FM model, Neurocomputing 142 (2014) 209–215.
[8] L. Farina, S. Rinaldi, Positive Linear Systems: Theory and Applications, Wiley, New York, 2000.
[9] Y.F. Guo, Stabilization of positive Markov jump systems, J. Franklin Inst. 353 (14) (2016) 3428–3440.
[10] E. Hernandez-Varga, P. Colaneri, R. Middleton, F. Blanchini, Discrete-time control for switched positive systems with application to mitigating viral escape, Int. J. Robust Nonlinear Control 21 (10) (2011) 1093–1111.
[11] S.P. Huang, Z.Z. Xiang, H.R. Karimi, Mixed $\mathcal{L}_{-}/\mathcal{L}_1$ fault detection filter design for fuzzy positive linear systems with time-varying delays, IET Control Theor. Appl. 8 (12) (2014) 1023–1031.
[12] T. Kaczorek, Positive 1D and 2D Systems, Springer, London, 2002.
[13] H.R. Karimi, Robust delay-dependent control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters, IEEE Trans. Circuits Syst. I, Reg. Papers 58 (8) (2011) 1910–1923.
[14] H.R. Karimi, A sliding mode approach to $\mathcal{H}_\infty$ synchronization of master-slave time-delay systems with Markovian jumping parameters and nonlinear uncertainties, J. Franklin Inst. 349 (4) (2012) 1480–1496.
[15] F.B. Li, P. Shi, L.G. Wu, M. Basin, C.C. Lim, Quantized control design for cognitive radio networks modeled as nonlinear semi-Markovian jump systems, IEEE Trans. Ind. Electron. 62 (4) (2015) 2330–2340.
[16] F.B. Li, P. Shi, C.C. Lim, L.G. Wu, Fault detection filtering for nonhomogeneous Markovian jump systems via fuzzy approach, journalIEEE Trans. Fuzzy Syst. https://doi.org/10.1109/TFUZZ.2016.2641022, in press, 2016.
[17] F.B. Li, L.G. Wu, P. Shi, C.C. Lim, State estimation and sliding-mode control for semi-Markovian jump systems with mismatched uncertainties, Automatica 51 (2015) 385–393.
[18] H.Y. Li, H.G. Gao, P. Shi, X.D. Zhao, Fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach, Automatica 50 (7) (2014) 1825–1834.
[19] H.Y. Li, P. Shi, D.Y. Yao, L.G. Wu, Observer-based adaptive sliding mode control of nonlinear Markovian jump systems, Automatica 64 (2016) 133–142.
[20] S. Li, Z.R. Xiang, Stochastic stability analysis and $\mathcal{L}_\infty$-gain controller design for positive Markov jump systems with time-varying delays, Nonlinear Anal. 22 (2016) 31–42.
[21] S. Li, Z.R. Xiang, H. Lin, H.R. Karimi, State estimation on positive Markovian jump systems with time-varying delay and uncertain transition probabilities, Inf. Sci. 369 (2016) 251–266.
[22] J. Liu, J. Lian, Y. Zhuang, Robust stability for switched positive systems with D-perturbation and time-varying delay, Inf. Sci. 369 (2016) 522–531.
[23] J.J. Liu, K.J. Zhang, H.K. Wei, Robust stability of positive switched systems with dwell time, Int. J. Syst. Sci. 47 (11) (2016) 2553–2562.
[24] W.H. Qi, X.W. Gao, State feedback controller design for singular positive Markovian jump systems with partly known transition rates, Appl. Math. Lett. 46 (2015) 111–116.
[25] W.H. Qi, X.W. Gao, $\mathcal{L}_1$ Control for positive Markovian jump systems with time-varying delays and partly known transition rates, Circuits, Syst. Singal Process. 34 (8) (2015) 2711–2716.
[26] W.H. Qi, J.H. Park, J. Cheng, Y.G. Kao, Robust stabilization for nonlinear time-delay semi-Markovian jump systems via sliding mode control, IET Control Theor. Appl. 11 (10) (2017) 1504–1513.
[27] W.H. Qi, J.H. Park, J. Cheng, Y.G. Kao, X.W. Gao, Anti-windup design for stochastic Markovian switching systems with mode-dependent time-varying delays and saturation nonlinearity, Nonlinear Anal. 26 (2017) 201–211.
[28] J.B. Qiu, Y.L. Wei, H.R. Karimi, New approach to delay-dependent $\mathcal{H}_\infty$ control for continuous-time Markovian jump systems with time-varying delay and deficient transition descriptions, J. Franklin Inst. 352 (1) (2015) 189–215.
[29] H. Shen, Z.G. Wu, J.H. Park, Reliable mixed passive and $\mathcal{H}_\infty$ filtering for semi-Markov jump systems with randomly occurring uncertainties and sensor failures, Int. J. Robust Nonlinear Control 25 (17) (2015) 3231–3251.
[30] J. Shen, J. Lam, $\mathcal{L}_\infty$-gain analysis for positive systems with distributed delays, Automatica 50 (1) (2014) 175–179.
[31] R. Shorten, F. Wirth, D. Leith, A positive systems model of TCP-like congestion control: asymptotic results, IEEE/ACM Trans. Network 14 (3) (2006) 616–629.

[32] P. Shi, F.B. Li, L.G. Wu, C.C. Lim, Neural network-based passive filtering for delayed neutral-type semi-Markovian jump systems, IEEE Trans. Neural Network 28 (9) (2017) 2101–2114.
[33] Y.G. Sun, Stability analysis of positive switched systems via joint linear copositive lyapunov functions, Nonlinear Anal. 19 (2016) 146–152.
[34] Y.L. Wei, J.B. Qiu, H.R. Karimi, M. Wang, A new design of $\mathcal{H}_\infty$ filtering for continuous-time Markovian jump systems with time-varying delay and partially accessible mode information, Signal Process. 93 (9) (2013) 2392–2407.
[35] T.B. Wu, F.B. Li, C.H. Yang, W.H. Gui, Event-based fault detection filtering for complex networked jump systems, IEEE/ASME Trans. Mech. https://doi.org/10.1109/TMECH.2017.2707389, in press, 2017.
[36] M. Xiang, Z.R. Xiang, Stability, $\mathcal{L}_1$-gain and control synthesis for positive switched systems with time-varying delay, Nonlinear Anal. 9 (2013) 9–17.
[37] M. Xiang, Z.R. Xiang, H.R. Karimi, Asynchronous $\mathcal{L}_1$ control of delayed switched positive systems with mode-dependent average dwell time, Inf. Sci. 278 (2014) 703–714.
[38] J.F. Zhang, X.D. Zhao, X.S. Cai, Absolute exponential $\mathcal{L}_1$-gain analysis and synthesis of switched nonlinear positive systems with time-varying delay, Appl. Math. Comput. 284 (2016) 24–36.
[39] J.F. Zhang, Z.Z. Han, F.B. Zhu, Stochastic stability and stabilization of positive systems with Markovian jump parameters, Nonlinear Anal. 12 (2014) 147–155.
[40] J.F. Zhang, X.D. Zhao, F.B. Zhu, Z.Z. Han, $\mathcal{L}_1/l_1$-gain analysis and synthesis of Markovian jump positive systems with time delay, ISA Trans. 63 (2016) 93–102.
[41] L.X. Zhang, Y. Leng, P. Colaneri, Stability and stabilization of discrete-time semi-Markov jump linear systems via semi-Markov kernel approach, IEEE Trans. Autom. Control 61 (2) (2016) 503–508.
[42] L.X. Zhang, Y.Z. Zhu, P. Shi, Y.X. Zhao, Resilient asynchronous $\mathcal{H}_\infty$ filtering for Markov jump neural networks with unideal measurements and multiplicative noises, IEEE Trans. Cybern. 45 (12) (2015) 2840–2852.
[43] Q.L. Zhang, Y.M. Zhang, B.Z. Du, T. Tanaka, $\mathcal{H}_\infty$ control via dynamic output feedback for positive systems with multiple delays, IET Control Theor. Appl. 9 (17) (2015) 2574–2580.
[44] Y.Q. Zhang, P. Shi, N.S. Kiong, H.R. Karimi, Observer-based finite-time fuzzy $\mathcal{H}_\infty$ control for discrete-time systems with stochastic jumps and time-delays, Signal Process. 97 (2014) 252–261.
[45] X.L. Zheng, X.Y. Wang, Y.F. Yin, L.L. Hu, Stability analysis and constrained fuzzy tracking control of positive nonlinear systems, Nonlinear Dynam. 83 (4) (2016) 2509–2522.
[46] S.Q. Zhu, Q.L. Han, C.H. Zhang, l_1-gain performance analysis and positive filter design for positive discrete-time Markov jump linear systems: a linear programming approach, Automatica 50 (8) (2014) 2098–2107.
[47] Q.S. Zhong, J. Cheng, Y.Q. Zhao, J.H. Ma, B. Huang, Finite-time $\mathcal{H}_\infty$ filtering for a class of discrete-time Markovian jump systems with switching transition probabilities subject to average dwell time switching, Appl. Math. Comput. 225 (2013) 278–294.
[48] G.D. Zong, R.H. Wang, W.X. Zheng, L.L. Hou, Finite time stabilization for a class of switched time-delay systems under asynchronous switching, Appl. Math. Comput. 219 (11) (2013) 5757–5771.
[49] G.D. Zong, R.H. Wang, W.X. Zheng, L.L. Hou, Finite-time $\mathcal{H}_\infty$ control for discrete-time switched nonlinear systems with time delay, Int. J. Robust Nonlinear Control 25 (6) (2015) 914–936.