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Design of extended dissipativity state estimation for generalized neural networks with mixed time-varying delay signals

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ABSTRACT

This paper investigates the issue of extended dissipativity state estimation of generalized neural networks (GNNs) with mixed time-varying delay signals. The integral terms in the time derivative of the Lyapunov–Krasovskii functionals (LKFs) are estimated by the famous Jensen's inequality, reciprocally convex combination (RCC) approach together with the Wirtinger double integral inequality (WDII) technique. In addition, in order to estimate the double integral terms in the derivative of the LKF, a new integral inequality is proposed. As a result, a new delay-dependent criterion is derived under which the estimated error system is extended dissipative. The concept of extended dissipativity state estimation can be applied to deal with the $\mathcal{L}_2-\mathcal{L}_\infty$ state estimation, $\mathcal{H}_\infty$ state estimation, passivity state estimation, mixed $\mathcal{H}_\infty$ and passivity state estimation, $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma$-dissipativity state estimation of GNNs by choosing the weighting matrices. The advantage of the proposed method is demonstrated by five numerical examples, among them one example was supported by real-life application of the benchmark problem that is associated with reasonable issues in the sense of an extended dissipativity performance.

1. Introduction

In recent years, a fast development has been made in the investigation of neural networks (NNs), because they have been widely applied in various sciences and engineering fields, such as image processing, fault diagnosis, reproducing moving pictures, signal processing, industrial automation and pattern recognition [5,27,42]. All these applications are closely depends on the dynamical behaviors of the underlying NNs. As we know, delay systems usually have complicated structures which lead to complex dynamics [7,8,18,36]. The effects of time delay on system dynamics are bilateral, namely, it may not only

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cause the degradation of performance or some undesired dynamics of a system but also, inversely, make an unstable system stable or a system possess certain desired performance which is not possessed before. Because of the finite switching speed of the amplifiers, time delay, which may result in complex dynamic behaviors, often occurs in NNs. However, it is desirable in some engineering applications (e.g., speed detection of moving objects and processing of moving images) that the time delay is intentionally taken into account. Furthermore, the time delay is always encountered because the NNs are frequently implemented by all kinds of hardware circuits-digital or integrated circuits. In general, simple NNs with a minimum number of neurons can be approximately expressed by a delayed feedback model with discrete delay. However, a neural network has a quantity of parallel pathways with various axon sizes and lengths, and the signal propagation is may be distributed during a certain time period [5]. Accordingly, the distributed delays should be incorporated into the neural system. In addition, the existence of time delays into the NNs is often one of the main sources to cause instability, divergence, oscillation, chaos or other poor performance. Therefore, the study of dynamical behaviors of NNs including time-delay has become an important research topic in recent years. For this purpose, recently, a great number of significant subjects have been investigated on the dynamical behaviors of NNs including time delays, such as dissipativity analysis [27], passivity analysis [16], control and filter problems [48] and state estimation problem [44]. Therefore, NNs including time delays have received considerable attention in the past several years and many successful investigations have been developed on the topic (see., [1]–[6]).

The concept of dissipativity theory was introduced by Willems [42] in 1972, then this theory has received extensive attention, because which gives primary structure for synthesis and analysis in several domains of control engineering problems, which are frequently represented by an input-output relationship, and it plays an important role in many control problems, such as $\mathcal{L}_2 - \mathcal{L}_\infty$ performance [1], passivity problem [33], $\mathcal{H}_\infty$ control problem [34], and dissipativity problem [27]. Therefore, the dissipativity theory has attracted more attention from the scholars, because it does not only covers the passivity and $\mathcal{H}_\infty$ performance [41] but also perform a good adjustable control performance in many engineering applications, such as power converters and chemical process control. Recently, $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity performance is proposed in [43]; however, those criteria in [43] does not cover the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance. To meet this demand, initially Zhang et al. [48] proposed general performance index is called an extended dissipativity concept to study the dynamical behaviors filtering on Markovian jump delay systems, which unifies all these performance indices. Furthermore, recently, Lee et al. [14] developed the concept of extended dissipativity theory for continuous-time delayed NNs by composing a tuning parameter to divide the neuron activation function into two situations. After that, Feng and Zheng. [6] further developed the concept of extended dissipativity theory for discrete-time delayed NNs by using the RCC approach with the forward difference of triple-summable terms. Accordingly, many studies have been developed to study the extended dissipativity analysis for NNs including time delays in recent years (see., [6,14,44]).

On the other hand, recently, according to the states of the neurons, NNs can be classified into static NNs (SNNs) or local-field NNs (LFNNs). Therefore, it can be well founded that most of the researchers have focused on the dynamical behaviors of LFNNs [14] and SNNs [43] in separate sense. However, it is mentioned that these two classes of NNs are not always equivalent; thus, it is possible to combine them into unified models called generalized neural networks (GNNs) to another by some valid assumptions. However, these assumptions cannot hold forever in many applications. For this reason, the combined system model has been taken into consideration is more fashionable in recent years (see., [22,25,26,31]), and it is named as GNNs. In this research area, a little work has been done on the dynamical behaviors of GNNs with time delays (see., [22,25,26,31]). Recently, the state estimation of NNs has been an important issue [15]. In general, a neural network is a highly interconnected network with a large number of neurons. As a result, most of the NNs are large-scale and complex networks, so it would be hard and expensive (or even impossible) to completely acquire the state information of all neurons. As pointed out in [40], the neuron states are not often completely available in the network output, it is often the case that only partial information about the states of the nodes are available in the network outputs. Therefore, in order to understand the network behavior better, it is important to estimate the neuron states through available measurement outputs. In many real-life situations, one needs to know the information about the neuron states and then use the estimated neuron states to achieve certain objectives such as system modeling and state feedback control. In this research area, recently various approaches have been explored in the issue of state estimation problem for NNs by many authors (see., [2–4,15,20,21,28,34,40,41,46,49]).

The exponential state estimation for Markov jumping NNs including mixed time-delays have been studied in [49]. The robust state estimation for uncertain stochastic BAM networks with time delays has been investigated in [3]. Furthermore, the authors in [28] discussed the state estimation for NNs of neutral type with time-delay. The robust state estimation problem for uncertain NNs was also discussed in [2,21]. The authors in [20] have focused on the $\mathcal{H}_\infty$ state estimation of NNs with mixed time delays. Recently, in [34] authors proposed an improved results on $\mathcal{H}_\infty$ state estimation of SNNs with interval time delays. Recently, Wen et al. [41] established new results on $\mathcal{H}_\infty$ state estimation of SNNs with time-varying delays. More recently, utilizing a novel approach to study neuronal state estimation for NNs with additive time-varying delay components, and this was explored by Zhang et al. [46]. However, those state estimation results in [2–4,15,20,21,28,34,40,41,46,49] does not cover the passivity, $\mathcal{L}_2 - \mathcal{L}_\infty$, mixed $\mathcal{H}_\infty$ and passivity, and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity performance. However, until now, there has been no attempt has been developed to give a unified framework analysis for state estimation of generalized neural network (GNN) model. Therefore, to meet this demand, a major contribution of this paper is to fill such a gap by making the first attempt to discuss the extended dissipativity state estimation problem for GNNs, which covers simultaneously $\mathcal{L}_2 - \mathcal{L}_\infty$, $\mathcal{H}_\infty$, passivity, mixed $\mathcal{H}_\infty$ and passivity, and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity state estimation of LFNNs and SNNs. This partly motivates our first interest to study on this issue.

Moreover, in recent years, many researchers have focused on the development of various analysis techniques to reduce possible conservatism of the delayed systems. To obtain this, much progress has been devoted to obtain a maximum admissible upper bounds (MAUBs) of the delayed network systems would be stable via various method, such as augmented LKFs method [25], RCC approach [27], WDII approach [29], integral inequality technique (IIT) [31], Wirtinger single integral inequality (WSII) technique [32], and free-weighting-matrix (FWM) method [50]. For instance, by implementing IIT technique, the stability analysis for NNs including time-delays were extensively proposed by Manivannan et al. (see [25,31]), and proposed criteria was successfully applied to a four-tank benchmark problem in showing feasibility on a realistic model. Very recently, Xiao et al. [44] proposed an extended dissipative state estimation problem for memristive NNs including time delays by implementing a nonsmooth analysis approaches together with LKFs. More recently, some sufficient conditions for the existence of a desired filter design are given to ensure that the filtering error system is strictly exponentially passive with desired disturbance attenuation which was established by Shi et al. [33], where a convex optimization algorithm for the filter design has been developed. In a fuzzy based environment, Lu et al. [24], proposed a fuzzy based neural network speed estimation technique for induction motor speed sensorless control, where the steepest descent algorithm is used to adjust the fuzzy neural network parameters to minimize the error between the error rotor flux and estimated rotor flux. However, there has been no attempt can be considered in the literature based on the WDII approach combined with the RCC technique for GNNs. Therefore, the highlights and major contributions of this paper lies in the following standpoints:

- As a first attempt, the WDII technique is combined together with the RCC approach. As a result, a new integral inequality is proposed in terms of Lemma 2.5, which is introduced based on the Lemmas 2.3 and 2.4.
- The extended dissipativity state estimation, concept is proposed for the first time to study dynamical behaviors of the generalized neural systems, which covers all these performances, such as $\mathcal{L}_2-\mathcal{L}_\infty$ state estimation, $\mathcal{H}_\infty$ state estimation, passivity state estimation, mixed $\mathcal{H}_\infty$ and passivity state estimation, and $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma$-dissipativity state estimation of GNNs.
- A simple LMI conditions are acquired to discuss the extended dissipativity state estimation issue for the generalized neural network model. Subsequently, based on these conditions, the unknown desired state estimation gain matrix is determined.
- To show the real-life application, the quadruple tank process system is considered in this paper in terms of an NN model, to showing the feasibility and useful, realistic model for obtaining more information to design the state estimator on a benchmark problem, which is realized in the sense of an extended dissipativity performance.

The remainder of this paper is structured as follows: Some of the existing results are discussed related to our problem in Section 2. The unified NN model is formulated and some assumptions, definition, and lemmas are presented in Section 3. In Section 4, we derive a new extended dissipativity state estimation criteria for GNN model is extended dissipative. In Section 5, an interesting numerical simulation studies are sponsored. Finally, some conclusions and future work directions are made in Section 6.

Notation. The superscripts T and -1 stand for matrix transposition and matrix inverse, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathcal{L}_2$ denotes the space of square integrable vector functions on $[0,+\infty)$ with norm $\left(\int_0^\infty \|\cdot\|^2 dt\right)^{\frac{1}{2}}$, where $\|\cdot\|$ denotes the Euclidean vector norm $\mathbb{R}^{n\times n}$ denotes the set of all $n\times n$ real matrices; $P>0$ means that P is positive definite. In symmetric block matrices, we use an asterisk ✠ to represent a term that is induced by symmetry and diag $\{\cdot\}$ stands for a block-diagonal matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Literature survey

Recently, the state estimation problem for delayed NNs has been an ongoing research issue and attracted considerable attention owing to its potential applications in system modeling, signal processing, and so on. When designing an NN model to handle complicated nonlinear problems, it often has a great number of neurons with tremendous connectivities between them. In such large-scale NNs, it is difficult and expensive (even may be impossible) to completely obtain the information of the neuron states from network measurements. On the other hand, for example, the authors in [12] proposed a recurrent NN to design an unknown nonlinear system. The neuron states were then utilized to design a suitable control law. That is, in many practical situations, one needs to know the neuron states and then make use of them to achieve certain objectives, such as state feedback control and system modeling. Based on the above discussion, the state estimation problem for delayed NNs is thus of great importance and practical significance from the science and engineering point of view. In recent years, many authors have been studied the issues of state estimation problems and some interesting results have been explored in the literature (see [2–5,10,13,15,17,19–21,23,28,34,37–41,44–47,49]). Specifically, among these references, the delay-dependent robust asymptotic state estimation of fuzzy NNs with a mixed time delay signal was studied using the Takagi–Sugeno fuzzy model representation in [2]. In [15] the problem of $\mathcal{H}_\infty$ state estimation for SNNs with time delay signal is investigated. The problem of an exponential state estimation for Markovian jumping NNs with time delay signal was successfully investigated by Zhang and Yu [49]. By developing the sampled-data control together with a stochastically varying sampling interval to design the state estimation problem for an NNs with time delays, and using a novel approach that divides the bounding of the activation function into two subintervals and this was demonstrated by Lee et al. [13]. Recently, Lakshmanan et al.

[20] has investigated the $\mathcal{H}_\infty$ state estimation of NNs with a mixed time-varying delay signals by constructing a suitable LKFs with triple integral terms and using the Jensen's inequality technique and LMI framework analysis, where the activation functions are assumed to satisfy sector-like nonlinearity condition. The issues of non-fragile observer based design for an NNs with mixed time delay signals and MJPs are considered in [37], by implementing the RCC technique, LKFs approach, and stochastic theory. The sampled-data state estimation for jumped systems was discussed via discontinuous Lyapunov approach in [17]. Recently, Zhang et al. [47] has explored the problems of synchronization and state estimation for a class of discrete-time hierarchical hybrid NNs with time delay signals. A novel $\mathcal{H}_\infty$ state estimation design for SNNs with interval time-varying delay signals was proposed in [41]. The $\mathcal{H}_\infty$ state estimation problem for a class of stochastic NNs with MJPs and leakage delay is investigated by Li et al. [19]. Recently, the robust $\mathcal{H}_\infty$ state estimation problem for a class of memristive recurrent NNs with stochastic time delays in [23], where the stochastic time-delays under consideration are governed by a Bernoulli-distributed stochastic sequence. Recently, by implementing a novel technique together with LKFs to investigate the neuronal state estimation for NNs with additive time delay signals in [46]. By applying nonsmooth analysis approach and newly constructed LKFs to investigate the state estimation problems for memristor-based NNs with time delays, which was examined by Xiao et al. [44]. Currently, the non-fragile state estimation problem is investigated for a class of continuous NNs including time delays and nonlinear perturbations in [45]. Very recently, event-based finite-time state estimation problem for a class of discrete-time stochastic NNs with mixed discrete and distributed time delays have been successfully investigated by Wang et al. [38]. More recently, by using the Taylor series and LMI technique to address the state estimation problem for recurrent NNs with unknown delays both in state equation and the output equation (see [39]). Therefore, it is worth pointing out that all these results are investigated for the LFNNs or SNNs. On the other hand, recently the GNN model has attracted much attention and taken into consideration is more comfortable from the researchers because this dynamic model unifies the LFNNs and SNNs (see [22,25,26,31]). However, until now, the research in the field of unified system model has only focused on either stability or dissipativity performance analysis. Therefore, in the existing literature, the design of extended dissipativity state estimation for GNNs with mixed time delay signals has not been adequately addressed in the literature yet. Therefore, the main purpose of this paper is to fill such a gap by making the first attempt to consider the design of extended dissipativity state estimation problem for GNNs with mixed time delay signals and investigate it widely. By proposing a novel LKFs and estimating their derivative by the WDII technique, RCC approach, and newly developed integral inequality (Lemma 2.5), several sufficient conditions are obtained in terms of LMIs that ensures the generalized error system is extended dissipative.

3. Problem statement and preliminaries

Consider the following GNNs with mixed time-varying delay signals:

$$\begin{aligned}
\dot{x}(t) &= -\mathcal{A}x(t) + \mathcal{W}_0 f(\mathcal{W}_3 x(t)) + \mathcal{W}_1 f(\mathcal{W}_3 x(t-h(t))) + \mathcal{W}_2 \int_{t-\tau(t)}^{t} f(\mathcal{W}_3(x(s)))ds + \mathcal{J} + \mathcal{B}_1\omega(t),\\
y(t) &= \mathcal{C}x(t) + \mathcal{D}x(t-h(t)) + \mathcal{B}_2\omega(t),\\
z(t) &= \mathcal{H}x(t),\\
x(t) &= \phi(t) \quad \forall\, t \in \big[-h^*, 0\big], \quad \delta^* = \max\big\{h_U, \bar{\tau}\big\}
\end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector of the NN associated with n neurons; $z(t) \in \mathbb{R}^q$ to be estimated, which is a linear combination of the states; $\omega(t) \in \mathbb{R}^m$ is the noise input belonging to $\mathcal{L}_2[0, +\infty)$; $\mathcal{A} = \mathrm{diag}\big\{\mathfrak{a}_1, \mathfrak{a}_2, \ldots, \mathfrak{a}_n\big\}$ is a positive diagonal matrix with each $\mathfrak{a}_i > 0$; $\mathcal{W}_0$, $\mathcal{W}_1$, $\mathcal{W}_2$, and $\mathcal{W}_3$ represents the connection weight matrix between neurons; $\mathcal{C}$, $\mathcal{D}$, $\mathcal{B}_1$, $\mathcal{B}_2$, and $\mathcal{H}$ are known real constant matrices with compatible dimensions; $f(\mathcal{W}_3 x(t)) = [f_1(\mathcal{W}_3 x_1(t)), f_2(\mathcal{W}_3 x_2(t)), \ldots, f_n(\mathcal{W}_3 x_n(t))]^T \in \mathbb{R}^n$ denotes the neuron activation function and $\mathcal{J} = [\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n]^T \in \mathbb{R}^n$ is constant external input vector; $\phi(t) \in \mathcal{C}[-2h^*, 0]; \mathbb{R}^n)$ is a real valued continuous initial condition on $[-2h^*, 0]$. The time-delay signals $h(t)$ and $\tau(t)$ being time-varying and satisfies the following conditions:

$$0 \le h_L \le h(t) \le h_U, \quad \dot{h}(t) \le \mu, \quad 0 \le \tau(t) \le \bar{\tau}, \tag{2}$$

where h_L, h_U, μ and $\bar{\tau}$ are known real constants.

Assumption A1 ([27]). Each $f_i(\cdot)$ of GNN (1) is continuous and bounded, and there exists constants k_i^- and k_i^+ such that

$$k_i^- \le \frac{f_i(\mathcal{W}_{3i}\Psi_1) - f_i(\mathcal{W}_{3i}\Psi_2)}{\mathcal{W}_{3i}\Psi_1 - \mathcal{W}_{3i}\Psi_2} \le k_i^+, \tag{3}$$

where k_i^- and k_i^+ are known real constants and $\Psi_1, \Psi_2 \in \mathbb{R}$, $\Psi_1 \neq \Psi_2$.

We consider the following state estimator for the GNN (1):

$$\begin{aligned}
\dot{\widehat{x}}(t) &= -\mathcal{A}x(t) + \mathcal{W}_0 f(\mathcal{W}_3\widehat{x}(t)) + \mathcal{W}_1 f(\mathcal{W}_3\widehat{x}(t-h(t)))\\
&\qquad + \mathcal{W}_2 \int_{t-\tau(t)}^{t} f(\mathcal{W}_3(\widehat{x}(s)))ds + \mathcal{J} + \mathcal{K}[y(t) - \widehat{y}(t)],\\
\widehat{y}(t) &= \mathcal{C}\widehat{x}(t) + \mathcal{D}\widehat{x}(t-h(t)) + \mathcal{B}_2\omega(t),
\end{aligned}$$

$$\widehat{z}(t) = \mathcal{H}\widehat{x}(t),$$
$$\widehat{x}(0) = \phi(0), \tag{4}$$

where $\widehat{x}(t) \in \mathbb{R}^n$ denotes the estimated state signal, $\widehat{z}(t) \in \mathbb{R}^q$ denotes the estimated measurements of $z(t)$ and $\mathcal{K}$ is the estimator gain matrix to be determined. Define the error signal as $e(t) = x(t) - \widehat{x}(t)$ and the output signal as $\bar{z}(t) = z(t) - \widehat{z}(t)$. Then the error system can be represented as follows:

$$\begin{aligned}\dot{e}(t) &= -(\mathcal{A} + \mathcal{KC})e(t) - \mathcal{KD}e(t - h(t)) + \mathcal{W}_0 g(\mathcal{W}_3 e(t)) + \mathcal{W}_1 g(\mathcal{W}_3 e(t - h(t)))\\ &\quad + \mathcal{W}_2 \int_{t-\tau(t)}^{t} g(\mathcal{W}_3(e(s)))ds + (\mathcal{B}_1 - \mathcal{KB}_2)\omega(t),\\ \tilde{y}(t) &= \mathcal{C}e(t) + \mathcal{D}e(t - h(t)) + \mathcal{B}_2\omega(t),\\ \bar{z}(t) &= \mathcal{H}e(t),\end{aligned} \tag{5}$$

where $e(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^T \in \mathbb{R}^n$ is the state vector of the transformed system and $g(\mathcal{W}_3 e(\cdot)) = [g_1(\mathcal{W}_3 e_1(\cdot)), g_2(\mathcal{W}_2 e_2(\cdot)), \ldots, g_n(\mathcal{W}_3 e_n(\cdot))]^T$, $g(\mathcal{W}_3 e(\cdot)) = f(\mathcal{W}_3(\cdot) + \widehat{x}(\cdot)) - f(\mathcal{W}_3\widehat{x}(\cdot))$. It follows from Assumption (A1) that the neuron activation function satisfies

$$k_i^- \le \frac{g_i(\Psi)}{\Psi} \le k_i^+, \tag{6}$$

where $\Psi \in \mathbb{R}$, $\Psi \neq 0$. Before we proceed further, the following assumption, definition, and lemmas are essential to obtain the main results.

Assumption A2. For given real symmetric matrices Π_1, Π_2, Π_3, and Π_4 the following conditions are satisfied:

1. $\Pi_1 \le 0, \Pi_3 > 0$, and $\Pi_4 \ge 0$;
2. $(\| \Pi_1 \| + \| \Pi_2 \|)\cdot \| \Pi_4 \| = 0$;
3. $\| \mathcal{B}_2 \| \| \Pi_4 \| = 0$;
4. $\mathcal{B}_2^T \Pi_1 \mathcal{B}_2 + \mathcal{B}_2^T \Pi_2 + \Pi_2^T \mathcal{B}_2 + \Pi_3 > 0$.

Definition 2.1 ([44]). For given matrices Π_1, Π_2, Π_3, and Π_4 satisfying Assumption (A2), the error system (5) is said to be extended dissipative if for any $t_f \ge 0$ with the zero initial state, all $\omega(t) \in \mathcal{L}_2[0, +\infty)$, the following relation is holds:

$$\int_0^{t_f} J(t)dt \ge \sup_{0 \le t \le t_f} \tilde{y}^T(t)\Pi_4\tilde{y}(t), \tag{7}$$

where $J(t) = \tilde{y}^T(t)\Pi_1\tilde{y}(t) + 2\tilde{y}^T(t)\Pi_2\omega(t) + \omega^T(t)\Pi_3\omega(t)$.

Remark 2.1. The relation (7) implies that the new performance index that contains a more general solution by choosing the weighting matrices Π_i, $i = 1, 2, 3, 4$. (i.e).,

- If taking $\Pi_1 = 0$, $\Pi_2 = 0$, $\Pi_3 = \gamma^2 I$, and $\Pi_4 = I$, then, the relation in (7) represents $\mathcal{L}_2 - \mathcal{L}_\infty$ performance;
- If taking $\Pi_1 = -I$, $\Pi_2 = 0$, $\Pi_3 = \gamma^2 I$, and $\Pi_4 = 0$, then, the relation in (7) yields $\mathcal{H}_\infty$ performance;
- If taking $\Pi_1 = 0$, $\Pi_2 = I$, $\Pi_3 = \gamma I$, and $\Pi_4 = 0$, then, the relation in (7) degenerates the passivity performance;
- If taking $\Pi_1 = -\gamma^{-1}\alpha I$, $\Pi_2 = (1 - \alpha)I$ $(0 \le \alpha \le 1)$, $\Pi_3 = \gamma I$, and $\Pi_4 = 0$, then, the relation in (7) becomes the mixed $\mathcal{H}_\infty$ and passivity performance;
- If taking $\Pi_1 = \mathcal{Q}$, $\Pi_2 = \mathcal{S}$, $\Pi_3 = \mathcal{R} - \gamma I$, and $\Pi_4 = 0$, then, the relation in (7) leads to $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity performance.

Lemma 2.1 ([9]). *For any matrix $\mathbb{M} \in \mathbb{R}^{n\times n}$ and $\mathbb{M} = \mathbb{M}^T > 0$ and given scalars $\eta > 0$, the vector function is $\psi : [0, \eta] \to \mathbb{R}^n$ such that the following relation holds:*

$$-\eta \int_0^{\eta} \psi^T(s)\mathbb{M}\psi(s)ds \le -\Big(\int_0^{\eta} \psi(s)ds\Big)^T \mathbb{M}\Big(\int_0^{\eta} \psi(s)ds\Big).$$

Lemma 2.2 ([9]). *For any matrix $\mathbb{M} \in \mathbb{R}^{n\times n}$, $\mathbb{M} = \mathbb{M}^T > 0$, a scalar function $\eta := \eta(t) > 0$, and a vector valued function $\dot{\psi} : [-\eta, 0] \to \mathbb{R}^n$ such that the following relation holds:*

$$-\eta \int_{t-\eta}^{t} \dot{\psi}^T(s)\mathbb{M}\dot{\psi}(s)ds \le \begin{bmatrix}\psi(t)\\ \psi(t-\eta)\end{bmatrix}^T \begin{bmatrix}-\mathbb{M} & \mathbb{M}\\ \maltese & -\mathbb{M}\end{bmatrix} \begin{bmatrix}\psi(t)\\ \psi(t-\eta)\end{bmatrix}.$$

Lemma 2.3 ([30]). *For any vectors Ω_1, Ω_2, constant matrices P, Q, and real scalars α, $\beta \ge 0$ satisfying that $\begin{bmatrix}P & Q\\ \maltese & P\end{bmatrix} \ge 0$ and $\alpha + \beta = 1$, then the following relation holds:*

$$-\frac{1}{\alpha}\Omega_1^T P\Omega_1 - \frac{1}{\beta}\Omega_2^T P\Omega_2 \le -\begin{bmatrix}\Omega_1\\ \Omega_2\end{bmatrix}^T \begin{bmatrix}P & Q\\ \maltese & P\end{bmatrix} \begin{bmatrix}\Omega_1\\ \Omega_2\end{bmatrix}.$$

Lemma 2.4 ([29]). *Let $\mathbb{M} > 0$ be any constant matrix, and for given scalars a and b with $a < b$, the following relation is well defined for any differentiable function ψ in $[a, b] \to \mathbb{R}^n$:*

$$-\frac{b^2 - a^2}{2}\int_{-a}^{-b}\int_{t+\beta}^{t}\dot{\psi}^T(s)\mathbb{M}\dot{\psi}(s)dsd\beta \leq -\Omega_1^T\mathbb{M}\Omega_1 - 2\Omega_2^T\mathbb{M}\Omega_2,$$

where

$$\Omega_1 = (b-a)\psi(t) - \int_{t-a}^{t-b}\psi(s)ds, \quad \Omega_2 = -\frac{(b-a)}{2}\psi(t) - \int_{t-a}^{t-b}\psi(s)ds + \frac{3}{b-a}\int_{-a}^{-b}\int_{t+\beta}^{t}\psi(s)dsd\beta.$$

Lemma 2.5 (See Appendix A for a proof). *For given matrix $T > 0$, h_m, h_M, and any compatible dimension matrix $\mathcal{M}$, which satisfy $\begin{bmatrix}\overline{T} & \mathcal{M}\\ \maltese & \overline{T}\end{bmatrix} \geq 0$, where $\overline{T} = \begin{bmatrix}T & 0\\ \maltese & 2T\end{bmatrix}$, and $h_m \leq h(t) \leq h_M$. Then, the following relation holds for all continuously differentiable function $\psi(t)$:*

$$-\frac{h_M^2 - h_m^2}{2}\int_{-h_M}^{-h_m}\int_{t+\beta}^{t}\dot{\psi}^T(s)T\dot{\psi}(s)dsd\beta \leq -\xi^T(t)\begin{bmatrix}\overline{T} & \mathcal{M}\\ \maltese & \overline{T}\end{bmatrix}\xi(t),$$

where

$$\begin{aligned}
\xi(t) &= [\xi_1^T(t) \quad \xi_2^T(t) \quad \xi_3^T(t) \quad \xi_4^T(t)]^T,\\
\xi_1(t) &= (h(t) - h_m)\psi(t) - \int_{-h(t)}^{-h_m}\psi(s)ds,\\
\xi_2(t) &= -\frac{(h(t) - h_m)}{2}\psi(t) - \int_{-h(t)}^{-h_m}\psi(s)ds + \frac{3}{h(t) - h_m}\int_{-h(t)}^{-h_m}\int_{t+\beta}^{t}\psi(s)dsd\beta,\\
\xi_3(t) &= (h_M - h(t))\psi(t) - \int_{-h_M}^{-h(t)}\psi(s)ds,\\
\xi_4(t) &= -\frac{(h_M - h(t))}{2}\psi(t) - \int_{-h_M}^{-h(t)}\psi(s)ds + \frac{3}{h_M - h(t)}\int_{-h_M}^{-h(t)}\int_{t+\beta}^{t}\psi(s)dsd\beta.
\end{aligned}$$

Remark 2.2. For stability problems an improved stability criterion with less conservatives is mainly achieved by the choice of choosing LKFs and estimating their derivative via an improved technique. To obtain this, recently many techniques have been developed for the delayed systems, such as the Jensen's inequality technique [9], WDII technique [29], IIT technique [31], WSII technique [32], FMB inequality [50], and so on. On the other hand, the RCC approach was used to obtain good information as well as significant improvements in the upper bounds of the delayed systems. However, until now, no attempt has been developed for the WDII technique combined together with the RCC approach. Thus, we have established, an attempt in terms of Lemma 2.5, is employed for handling double integral terms in LKFs. Therefore, a new integral inequality is proposed in this paper, which is introduced based on the improved integral inequality [29] and the RCC approach [30]. Therefore, Lemma 2.5 may induce new features as well as good information on the stability performance of delayed system, which is the main contribution of this paper.

Lemma 2.6 ([35]). *Let $\xi \in \mathbb{R}^n$, $\Xi = \Xi^T \in \mathbb{R}^{n\times n}$ such that rank$(B) < n$. The following statements are identical*

(i) $\xi^T\Xi\xi < 0, \quad \forall B\xi = 0, \quad \xi \neq 0,$
(ii) $B^{\perp T}\Xi B^{\perp} < 0$, *where $B^{\perp}$ is a right orthogonal complement of B.*

4. Main results

This section is dedicated to displaying a new methodology to discuss the extended dissipativity state estimation issue for the error system (5). For clarity, we represent the matrix and vector notations, some reconstructed elements as follows: we define $e_p = \begin{bmatrix}0_{n\times(p-1)n} & I_n & 0_{n\times(17-p)n}\end{bmatrix}$, $p = 1, 2, \ldots, 17$ and $\widehat{e}_q = \begin{bmatrix}0_{n\times(q-1)n} & I_n & 0_{n\times(12-q)n}\end{bmatrix}$, $q = 1, 2, \ldots, 12$. The other notations are defined as

$$\begin{aligned}
\zeta(t) = \Big[&e^T(t) \quad e^T(t-h_L) \quad e^T(t-h(t)) \quad e^T(t-h_U) \quad g^T(\mathcal{W}_3e(t)) \quad g^T(\mathcal{W}_3e(t-h_1)) \quad g^T(\mathcal{W}_3e(t-h(t)))\\
&g^T(\mathcal{W}_3e(t-h_U)) \quad \int_{t-h_L}^{t}e^T(s)ds \quad \int_{t-h(t)}^{t-h_L}e^T(s)ds \quad \int_{t-h_U}^{t-h(t)}e^T(s)ds \quad \int_{-h_L}^{0}\int_{t+\beta}^{t}e^T(s)dsd\beta\\
&\int_{-h(t)}^{-h_L}\int_{t+\beta}^{t}e^T(s)dsd\beta \quad \int_{-h_U}^{-h(t)}\int_{t+\beta}^{t}e^T(s)dsd\beta \quad \int_{t-\tau(t)}^{t}g^T(\mathcal{W}_3e(s))ds \quad \dot{e}^T(t) \quad \omega^T(t)\Big]^T,
\end{aligned}$$

$$\widehat{\zeta}(t) = \Big[e^T(t) \quad e^T(t-h(t)) \quad e^T(t-h) \quad g^T(\mathcal{W}_3e(t)) \quad g^T(\mathcal{W}_3e(t-h(t))) \quad \int_{t-h(t)}^{t}e^T(s)ds \quad \int_{t-h}^{t-h(t)}e^T(s)ds$$

$$\int_{-h(t)}^{0}\int_{t+\beta}^{t} e^T(s)dsd\beta \quad \int_{-h}^{-h(t)}\int_{t+\beta}^{t} e^T(s)dsd\beta \quad \int_{t-\tau(t)}^{t} g^T(\mathcal{W}_3 e(s))ds \quad \dot{e}^T(t) \quad \omega^T(t)\Big]^T,$$

$K_1 = \text{diag}\{k_1^-, k_2^-, \dots k_n^-\}$, $K_2 = \text{diag}\{k_1^+, k_2^+, \dots k_n^+\}$, $\Theta_1 = \text{diag}\{\theta_{11}, \theta_{12}, \dots, \theta_{1n}\}$, $\Theta_2 = \text{diag}\{\theta_{21}, \theta_{22}, \dots, \theta_{2n}\}$, $\Theta_3 = \text{diag}\{\theta_{31}, \theta_{32}, \dots, \theta_{3n}\}$.

Theorem 4.1 (See Appendix B for a proof). *Under Assumption (A2), for given scalar $0 < \varrho < 1$, h_L, h_U, $\overline{\tau}$ and μ, then the error system (5) is extended dissipative for any time-varying delay signal function $h(t)$ and $\tau(t)$ satisfying (2), if there exist symmetric matrices $P > 0 \in \mathbb{R}^{n\times n}$, $\mathcal{Q}_i > 0 \in \mathbb{R}^{n\times n}$, $i = 1, 2, 3$, R_1^{11}, R_1^{12}, R_1^{22}, R_2^{11}, R_2^{12}, $R_2^{22} > 0 \in \mathbb{R}^{n\times n}$, $S_i > 0 \in \mathbb{R}^{n\times n}$, $i = 1, 2$, $T > 0 \in \mathbb{R}^{n\times n}$, $\mathcal{M}, \mathcal{N} > 0 \in \mathbb{R}^{n\times n}$, $\mathcal{O} > 0 \in \mathbb{R}^{2n\times 2n}$, diagonal matrices $\Theta_i > 0$, $i = 1, 2, 3$, and any compatible dimension matrix N, such that the following LMIs hold:*

$$\begin{bmatrix} R_2^{11} & \mathcal{M} \\ \maltese & R_2^{11} \end{bmatrix} \geq 0, \quad \begin{bmatrix} R_2^{22} & \mathcal{N} \\ \maltese & R_2^{22} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \overline{S}_2 & \mathcal{O} \\ \maltese & \overline{S}_2 \end{bmatrix} \geq 0, \tag{8}$$

$$\Sigma = \begin{bmatrix} \varrho P - \mathcal{C}^T \Pi_4 \mathcal{C} & -\mathcal{C}^T \Pi_4 \mathcal{D} \\ \maltese & (1-\varrho)P - \mathcal{D}^T \Pi_4 \mathcal{D} \end{bmatrix} > 0, \tag{9}$$

$$\left(\Gamma^\perp\right)^T \ \Xi \ \left(\Gamma^\perp\right) \leq 0, \tag{10}$$

where

$$\Gamma = \Big[-(\mathcal{A}+\mathcal{K}\mathcal{C}) \quad 0 \quad -\mathcal{K}\mathcal{D} \quad 0 \quad \mathcal{W}_0 \quad 0 \quad \mathcal{W}_1 \quad \underbrace{0\dots\dots\dots0}_{7\ times} \quad \mathcal{W}_2 \quad 0 \quad (\mathcal{B}_1 - \mathcal{K}\mathcal{B}_2)\Big],$$

$$\Xi = \Xi_1 + \Xi_2 + \Xi_3 + \Xi_4 + \Xi_5 + \Xi_6 + \Xi_7 + \Xi_8 + \Xi_9,$$

$$\Xi_1 = 2e_1 P e_{16}^T,$$

$$\Xi_2 = 2\big[e_5 - K_1\mathcal{W}_3 e_1\big]H_1 e_{16}^T + 2\big[K_2\mathcal{W}_3 e_1 - e_5\big]H_2 e_{16}^T,$$

$$\begin{aligned}\Xi_3 = {} & [e_1 \quad e_5](\mathcal{Q}_1 + \mathcal{Q}_2 + \mathcal{Q}_3)[e_1 \quad e_5]^T \\ & -(1-\mu)[e_3 \quad e_7]\mathcal{Q}_1[e_3 \quad e_7] - [e_2 \quad e_6]\mathcal{Q}_2[e_2 \quad e_6]^T - [e_4 \quad e_8]\mathcal{Q}_3[e_4 \quad e_8],\end{aligned}$$

$$\begin{aligned}\Xi_4 = {} & h_L^2\big[e_1 R_1^{11} e_1^T + 2e_1 R_1^{12} e_{16}^T + e_{16} R_1^{22} e_{16}^T\big] + (h_U - h_L)^2\big[e_1 R_2^{11} e_1^T + 2e_1 R_2^{12} e_{16} + e_{16} R_2^{22} e_{16}^T\big] \\ & - h_L\big[e_1 R_1^{12} e_1^T - e_2 R_1^{12} e_2^T\big] - (h_U - h_L)\big[e_2 R_2^{12} e_1^T - e_4 R_2^{12} e_4^T\big] \\ & - e_9 R_1^{11} e_9^T - [e_1 - e_2]R_1^{22}[e_1 - e_2]^T - \begin{bmatrix} e_{10} \\ e_{11} \end{bmatrix}\begin{bmatrix} R_2^{11} & \mathcal{M} \\ \maltese & R_2^{11} \end{bmatrix}\begin{bmatrix} e_{10} \\ e_{11} \end{bmatrix}^T \\ & - \begin{bmatrix} e_2 - e_3 \\ e_3 - e_4 \end{bmatrix}\begin{bmatrix} R_2^{22} & \mathcal{N} \\ \maltese & R_2^{22} \end{bmatrix}\begin{bmatrix} e_2 - e_3 \\ e_3 - e_4 \end{bmatrix}^T,\end{aligned}$$

$$\begin{aligned}\Xi_5 = {} & e_{16}\left(\frac{h_L^4}{4}S_1 + \frac{(h_U^2 - h_L^2)^2}{4}S_2\right)e_{16}^T \\ & - \begin{bmatrix} h_L e_1 - e_9 \\ -\frac{h_L}{2}e_1 - e_9 + \frac{3}{h_L}e_{12} \end{bmatrix}\begin{bmatrix} S_1 & 0 \\ \maltese & 2S_1 \end{bmatrix}\begin{bmatrix} h_L e_1 - e_9 \\ -\frac{h_L}{2}e_1 - e_9 + \frac{3}{h_L}e_{12} \end{bmatrix}^T - \chi \begin{bmatrix} \overline{S}_2 & \mathcal{O} \\ \maltese & \overline{S}_2 \end{bmatrix}\chi^T,\end{aligned}$$

$$\chi = \big[\chi_1(t) \quad \chi_2(t)\big],$$

$$\chi_1(t) = \left[(h(t) - h_L)e_1 - e_{10} \quad -\frac{(h(t) - h_L)}{2}e_1 - e_{10} + \frac{3}{h(t) - h_L}e_{13}\right],$$

$$\chi_2(t) = \left[(h_U - h(t))e_1 - e_{11} \quad -\frac{(h_U - h(t))}{2}e_1 - e_{11} + \frac{3}{h_U - h(t)}e_{14}\right],$$

$$\Xi_6 = e_5 \overline{\tau}^2 T e_5^T - e_{15} T e_{15}^T,$$

$$\Xi_7 = \big[e_1 + \eta e_{16}\big]N\big[-(\mathcal{A} + \mathcal{K}\mathcal{C})e_1 - \mathcal{K}\mathcal{D}e_3 + \mathcal{W}_0 e_5 + \mathcal{W}_1 e_7 + \mathcal{W}_2 e_{15} + (\mathcal{B}_1 - \mathcal{K}\mathcal{B}_2)e_{17} - e_{16}\big]^T,$$

$$\Xi_8 = \left[e_5 - K_2\mathcal{W}_3e_1\right]\Theta_1\left[K_1\mathcal{W}_3e_1 - e_5\right]^T + \left[e_7 - K_2\mathcal{W}_3e_3\right]\Theta_2\left[K_1\mathcal{W}_3e_3 - e_7\right]^T$$
$$+ \left[e_5 - e_7 - K_2\mathcal{W}_3e_1 + K_2\mathcal{W}_3e_3\right]\Theta_3\left[K_1\mathcal{W}_3e_1 - K_1\mathcal{W}_3e_3 - e_5 + e_7\right]^T,$$
$$\Xi_9 = \left[\mathcal{C}e_1 + \mathcal{D}e_3 + \mathcal{B}_2e_{17}\right]\Pi_1\left[\mathcal{C}e_1 + \mathcal{D}e_3 + \mathcal{B}_2e_{17}\right]^T + 2\left[\mathcal{C}e_1 + \mathcal{D}e_3 + \mathcal{B}_2e_{17}\right]\Pi_2e_{17}^T + e_{17}\Pi_3e_{17}^T,$$
$$\bar{S}_2 = \begin{bmatrix} S_2 & 0 \\ \maltese & 2S_2 \end{bmatrix}.$$

Moreover, the estimator gain matrix $\mathcal{K}$ of the state estimator (5) can be designed as $\mathcal{K} = N^{-1}Y$.

Remark 4.1. When the discrete time-delay signal $0 \le h(t) \le h$, that is, the lower bound of the time delay signal is 0, we define the LKF as follows:

$$\mathbb{V}(e_t) = \sum_{n=1}^{6}\mathbb{V}_n(e_t), \tag{11}$$

where

$$\mathbb{V}_3(e_t) = \int_{t-h(t)}^{t}\eta^T(s)\mathcal{Q}_1\eta(s)ds,$$
$$\mathbb{V}_4(e_t) = h\int_{-h}^{0}\int_{t+\beta}^{t}\begin{bmatrix} e(s) \\ \dot{e}(s)\end{bmatrix}^T\begin{bmatrix} R_2^{11} & R_2^{12} \\ \maltese & R_2^{22}\end{bmatrix}\begin{bmatrix} e(s) \\ \dot{e}(s)\end{bmatrix}dsd\beta,$$
$$\mathbb{V}_5(e_t) = \frac{h^2}{2}\int_{-h}^{0}\int_{\beta}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)S_2\dot{e}(s)dsd\theta d\beta,$$

and remaining $\mathbb{V}_1(e_t)$, $\mathbb{V}_2(e_t)$, $\mathbb{V}_6(e_t)$, and $\eta(t)$ are the same as defined in Theorem 4.1. By a similar method to that employed in Theorem 4.1, we can obtain the following Corollary 4.1.

Corollary 4.1. *Under Assumption (A2), for given scalar $0 < \varrho < 1$, h, $\overline{\tau}$ and μ, then the error system (5) is extended dissipative for any time-varying delay signal function $h(t)$ and $\tau(t)$, if there exist symmetric matrices $P > 0 \in \mathbb{R}^{n\times n}$, $\mathcal{Q}_1 > 0 \in \mathbb{R}^{n\times n}$, R_2^{11}, R_2^{12}, $R_2^{22} > 0 \in \mathbb{R}^{n\times n}$, $S_2 > 0 \in \mathbb{R}^{n\times n}$, $T > 0 \in \mathbb{R}^{n\times n}$, $\mathcal{M}, \mathcal{N} > 0 \in \mathbb{R}^{n\times n}$, $\mathcal{O} > 0 \in \mathbb{R}^{2n\times 2n}$, diagonal matrices $\Theta_i > 0$, $i = 1, 2, 3$, and any compatible dimension matrix N, such that (8) and (9) are satisfied and the following LMI holds:*

$$\left(\widehat{\Gamma}^{\perp}\right)^T \widehat{\Xi}\left(\widehat{\Gamma}^{\perp}\right) \le 0, \tag{12}$$

where

$$\widehat{\Gamma} = \left[-(\mathcal{A} + \mathcal{K}\mathcal{C}) \quad -\mathcal{K}\mathcal{D} \quad 0 \quad \mathcal{W}_0 \quad \mathcal{W}_1 \quad \underbrace{0\ldots\ldots\ldots0}_{4\ times} \quad \mathcal{W}_2 \quad 0 \quad (\mathcal{B}_1 - \mathcal{K}\mathcal{B}_2)\right],$$
$$\widehat{\Xi} = \widehat{\Xi}_1 + \widehat{\Xi}_2 + \widehat{\Xi}_3 + \widehat{\Xi}_4 + \widehat{\Xi}_5 + \widehat{\Xi}_6 + \widehat{\Xi}_7 + \widehat{\Xi}_8 + \widehat{\Xi}_9,$$
$$\widehat{\Xi}_1 = 2\widehat{e}_1P\widehat{e}_{11}^T,$$
$$\widehat{\Xi}_2 = 2\left[\widehat{e}_4 - K_1\mathcal{W}_3\widehat{e}_1\right]H_1\widehat{e}_{11}^T + 2\left[K_2\mathcal{W}_3\widehat{e}_1 - \widehat{e}_4\right]H_2\widehat{e}_{11}^T,$$
$$\widehat{\Xi}_3 = [\widehat{e}_1 \quad \widehat{e}_4]\mathcal{Q}_1[\widehat{e}_1 \quad \widehat{e}_4]^T - (1-\mu)[\widehat{e}_4 \quad \widehat{e}_5]\mathcal{Q}_1[\widehat{e}_4 \quad \widehat{e}_5]^T,$$
$$\widehat{\Xi}_4 = h^2\left[\widehat{e}_1R_2^{11}\widehat{e}_1^T + 2\widehat{e}_1R_2^{12}\widehat{e}_{11} + \widehat{e}_{11}R_2^{22}\widehat{e}_{11}^T\right] - (h(t) - h)\left[\widehat{e}_5R_2^{12}\widehat{e}_5^T - \widehat{e}_5R_2^{12}\widehat{e}_5^T\right]$$
$$-\begin{bmatrix}\widehat{e}_6 \\ \widehat{e}_7\end{bmatrix}\begin{bmatrix} R_2^{11} & \mathcal{M} \\ \maltese & R_2^{11}\end{bmatrix}\begin{bmatrix}\widehat{e}_6 \\ \widehat{e}_7\end{bmatrix}^T - \begin{bmatrix}\widehat{e}_1 - \widehat{e}_2 \\ \widehat{e}_2 - \widehat{e}_3\end{bmatrix}^T\begin{bmatrix} R_2^{22} & \mathcal{N} \\ \maltese & R_2^{22}\end{bmatrix}\begin{bmatrix}\widehat{e}_1 - \widehat{e}_2 \\ \widehat{e}_2 - \widehat{e}_3\end{bmatrix}^T,$$
$$\widehat{\Xi}_5 = \widehat{e}_{11}\left(\frac{(h^2(t) - h^2)^2}{4}S_2\right)\widehat{e}_{11}^T - \chi\begin{bmatrix}\bar{S}_2 & \mathcal{O} \\ \maltese & \bar{S}_2\end{bmatrix}\chi^T,$$
$$\chi = \left[\chi_1(t) \quad \chi_2(t)\right],$$
$$\chi_1(t) = \left[h(t)\widehat{e}_1 - \widehat{e}_6 \quad -\frac{h(t)}{2}\widehat{e}_1 - \widehat{e}_6 + \frac{3}{h(t)}\widehat{e}_8\right],$$
$$\chi_2(t) = \left[(h - h(t))\widehat{e}_1 - \widehat{e}_7 \quad -\frac{(h - h(t))}{2}\widehat{e}_1 - \widehat{e}_7 + \frac{3}{h - h(t)}\widehat{e}_9\right],$$

$$\widehat{\Xi}_6 = \widehat{e}_4\overline{\tau}^2T\widehat{e}_4^T - \widehat{e}_{10}T\widehat{e}_{10}^T,$$

$$\widehat{\Xi}_7 = \big[\widehat{e}_1 + \eta\widehat{e}_{11}\big]N\big[-(\mathcal{A}+\mathcal{K}\mathcal{C})\widehat{e}_1 - \mathcal{K}\mathcal{D}\widehat{e}_2 + \mathcal{W}_0\widehat{e}_4 + \mathcal{W}_1\widehat{e}_5 + \mathcal{W}_2\widehat{e}_{10} + (\mathcal{B}_1 - \mathcal{K}\mathcal{B}_2)\widehat{e}_{12} - \widehat{e}_{11}\big]^T,$$

$$\widehat{\Xi}_8 = \big[\widehat{e}_4 - K_2\mathcal{W}_3\widehat{e}_1\big]\Theta_1\big[K_1\mathcal{W}_3\widehat{e}_1 - \widehat{e}_4\big]^T + \big[\widehat{e}_5 - K_2\mathcal{W}_3\widehat{e}_2\big]\Theta_2\big[K_1\mathcal{W}_3\widehat{e}_2 - \widehat{e}_5\big]^T$$
$$+\big[\widehat{e}_4 - \widehat{e}_5 - K_2\mathcal{W}_3\widehat{e}_1 + K_2\mathcal{W}_3\widehat{e}_2\big]\Theta_3\big[K_1\mathcal{W}_3\widehat{e}_1 - K_1\mathcal{W}_3\widehat{e}_2 - \widehat{e}_4 + \widehat{e}_5\big]^T,$$

$$\widehat{\Xi}_9 = \big[\mathcal{C}\widehat{e}_1 + \mathcal{D}\widehat{e}_2 + \mathcal{B}_2\widehat{e}_{12}\big]\Pi_1\big[\mathcal{C}\widehat{e}_1 + \mathcal{D}\widehat{e}_2 + \mathcal{B}_2\widehat{e}_{12}\big]^T + 2\big[\mathcal{C}\widehat{e}_1 + \mathcal{D}\widehat{e}_2 + \mathcal{B}_2\widehat{e}_{12}\big]\Pi_2\widehat{e}_{12}^T + \widehat{e}_{12}\Pi_3\widehat{e}_{12}^T,$$

$$\bar{S}_2 = \begin{bmatrix} S_2 & 0 \\ \maltese & 2S_2 \end{bmatrix}.$$

Moreover, the estimator gain matrix $\mathcal{K}$ of the state estimator (5) can be designed as $\mathcal{K} = N^{-1}Y$.

Algorithm 4.1. Step 1: Input an appropriate matrix parameters for the NN model (1) and (4) together with the error system (5).

Step 2: For a given μ, choose the given scalars $0 < \varrho < 1$, h_L, h_U, and $\overline{\tau}$ accordingly satisfy the relation (2).

Step 3: Find a feasible solution to the LMIs in (8)–(12) for given positive scalars ϱ, h_L, h_U, $\overline{\tau}$, and μ. If the inequalities in (8)–(12) is feasible and update the upper bound of h_U and $\overline{\tau}$, then go to Step 4; otherwise, go to Step 1 and Step 2 for resetting the matrix parameters and given scalars.

Step 4: Calculate the state estimator gain matrix $\mathcal{K}$ from the feasible solution of the LMIs in Theorem 4.1.

Remark 4.2. It is pointed out that, the model GNN (1) includes LFNNs and SNNs, and it can be simply modified to each of them by the choice of $\mathcal{W}_0$, $\mathcal{W}_1$, $\mathcal{W}_2$ and $\mathcal{W}_3$. Now we consider the following SNN recently studied in [34,41], which is a special case of GNN model (1), when $\mathcal{W}_0 = 0$, $\mathcal{W}_1 = I$, and $\mathcal{W}_2 = 0$:

$$\begin{aligned} \dot{x}(t) &= -\mathcal{A}x(t) + f(\mathcal{W}_3x(t-h(t))) + \mathcal{J} + \mathcal{B}_1\omega(t), \\ y(t) &= \mathcal{C}x(t) + \mathcal{D}x(t-h(t)) + \mathcal{B}_2\omega(t), \\ z(t) &= \mathcal{H}x(t). \end{aligned} \tag{13}$$

Remark 4.3. Very recently, the WDII approach can be developed to reduce the conservatism as much as possible for the delayed systems (see [29]), which was first proposed by Park et al. Inspired by the idea of [29], the WDII technique was applied in this paper to bound the double integral terms, such as $-\frac{h_L^2}{2}\int_{-h_L}^{0}\int_{t+\beta}^{t}\dot{e}^T(s)S_1\dot{e}(s)dsd\beta$ and $-\frac{h_U^2-h_L^2}{2}\int_{-h_U}^{-h_L}\int_{t+\beta}^{t}\dot{e}^T(s)S_2\dot{e}(s)dsd\beta$; as a result, the WDII technique has provided a tighter upper bound analysis approach for such LKFs. The method proposed in [34], [41], and [46] has used simply the WSII technique, free-matrix-based technique, and extended RCC approach respectively, when estimating the time derivative of the such LKFs. Therefore, we mentioned that, in this paper we used the WDII technique together with the RCC approach when estimating their derivative as shown in (32), which is more effective to improve the extended dissipativity state estimation performance of GNNs.

Remark 4.4. It is worthy noting that combining of the WDII with RCC approach can get less conservative criteria that contains the following three aspects information such as $e(t)$, $\int_{t-h}^{t}e(s)ds$ or $\int_{t-h(t)}^{t}e(s)ds$ and $\int_{-h}^{0}\int_{t+u}^{t}e(s)dsd\beta$ or $\int_{-h(t)}^{0}\int_{t+u}^{t}e(s)dsd\beta$. In this paper, two tighter integral inequalities (Lemmas 2.4 and 2.5) are effectively applied in the corresponding integral terms to obtain better performances.

Remark 4.5. It is mentioned that, the criteria established in [34], [41], and [46], the condition, $k_i^- \le \frac{g_i(\Psi)}{\Psi} \le k_i^+$ with $\Psi_2 = 0$, which is a special case of $k_i^- \le \frac{g_i(\Psi_1)-g_i(\Psi_2)}{\Psi_1-\Psi_2} \le k_i^+$, $i = 1, 2, \ldots, n$, was usually employed to reduce the possible conservatism. It is mentioned that, the Assumption (A1) used in this paper not only consideration of the term such as $k_i^- \le \frac{g_i(\mathcal{W}_{3i}e(t))}{\mathcal{W}_{3i}e(t)} \le k_i^+$ and $k_i^- \le \frac{g_i(\mathcal{W}_{3i}e(t-h(t)))}{\mathcal{W}_{3i}e(t-h(t))} \le k_i^+$, but also the term $k_i^- \le \frac{g_i(\mathcal{W}_{3i}e(t))-g_i(\mathcal{W}_{3i}e(t-h(t)))}{\mathcal{W}_{3i}e(t))-\mathcal{W}_{3i}e(t-h(t))} \le k_i^+$ have been considered in (38). Therefore, more information on the neuron activation functions have been utilized in this paper to discusses the extended dissipativity state estimation of GNNs, which helped us to reduce the conservatism more effectively than [34], [41], and [46].

Remark 4.6. Up to now, many researchers have studied state estimation for LFNNs and SNNs (see., [2–4,15,20,21,28,34,40,41,46,49]) based on different kinds of analysis concepts such as robust state estimation or $\mathcal{H}_\infty$ state estimation. However, almost all of the existing works in mentioned above have solved it separately. In this paper, the proposed system model and analysis, concept are more general than the previous investigations. Moreover, in this paper, the state estimation problem is considered for GNNs based on a unified framework analysis which covers more dynamic analysis, such as $\mathcal{L}_2 - \mathcal{L}_\infty$ performance, $\mathcal{H}_\infty$ performance, passivity performance, mixed $\mathcal{H}_\infty$ and passivity performance, and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity performance.

Remark 4.7. It is noteworthy that, In many industrial process, the dynamic behaviors are generally complex and nonlinear, and their real mathematical models are always difficult to obtain. How to design the filter for unknown systems using input/output data has become one main focus of research. In this regard, recently, Shi et al. [33] proposed extensively the problem of exponential passive filtering for a class of stochastic neutral-type NNs with both semi-Markovian jump parameters and mixed time delays. The semi-Markovian switching signal has been simultaneously introduced to reflect the complex hybrid nature existing in the NNs. In addition, in order to showing practical application of proposed methodology on a synthetic oscillatory network of transcriptional regulator model were considered, which has been used to model repressilators, where it is assumed that the switching between different modes can be governed by a semi-Markov chain. Likewise, Lu et al. [24] explored a fuzzy based neural network speed estimation method for induction motor speed sensorless control. In [24], the proposed control scheme is easy to implement in a practical world and obtained experimental results are shown to confirm that the proposed fuzzy NN speed estimation can provide good performance over a wide speed range for induction motor speed control. Moreover, the experimental results proved that the proposed fuzzy NN speed estimation method is practical and the performance is great. However, the proposed results in [24] and [33] have dealt with usual stability and passivity concept. Hence, the analysis technique and system model proposed in this paper deserve much attention to fill the such a demands more effectively. Therefore, it should be mentioned that, the proposed techniques in [24] and [33] can be extendable to the generalized dissipativity concept, which unifies the $\mathcal{H}_\infty$ performance, passive performance, mixed $\mathcal{H}_\infty$ and passive performance, dissipative performance, and $\mathcal{L}_2-\mathcal{L}_\infty$ performance. Therefore, in our future work, it will be one of our main research topic to explore the practical realization and applications of the dynamical problems.

5. Numerical examples

This section provides five numerical examples to demonstrate the usefulness of the developed method:

Example 5.1. Consider the error system described in (5) with the following matrix parameters:

$$\mathcal{A}=\begin{bmatrix}2.5 & 0\\ 0 & 1.75\end{bmatrix},\ \mathcal{W}_0=\begin{bmatrix}1.2 & 0.6\\ -0.5 & 1.6\end{bmatrix},\ \mathcal{W}_1=\begin{bmatrix}-0.9 & 0.6\\ -0.4 & 0.7\end{bmatrix},\ \mathcal{W}_2=\begin{bmatrix}0.4 & 1.1\\ -0.3 & 0.9\end{bmatrix},\ \mathcal{W}_3=\begin{bmatrix}0.6 & -1.5\\ 0.6 & 0.2\end{bmatrix},$$
$$\mathcal{B}_1=\begin{bmatrix}-0.4\\ 0.4\end{bmatrix},\ \mathcal{B}_2=0.4,\ \mathcal{C}=\begin{bmatrix}-1.4 & 0\end{bmatrix},\ \mathcal{D}=\begin{bmatrix}0.6 & 0.1\end{bmatrix},\ \mathcal{H}=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}.$$

Here, the neuron activation function is taken as $g(e)=\tanh(e)$ with $k_1^-=k_2^-=0$ and $k_1^+=k_2^+=1$. The time-varying delay signals is taken as $h(t)=0.7+0.7\cos(t)$ which means $h_U=1.4$ with $h_L=0.5,\ \mu=0.5,$ and $\eta=0.25$. Also, the distributed time-varying delay signal is chosen as $0\le\tau(t)\le\overline{\tau}=0.65$. Furthermore, the noise distraction is taken as:

$$\omega(t)=0.01e^{-0.0005t}\sin(0.0002t),\ t\ge 0.$$

By setting the values of the free matrices Π_1, Π_2, Π_3 and Π_4 and solving the LMIs in (8) to (12) with the above parameters, the corresponding state estimation is drawn as following:

(i) $\mathcal{L}_2-\mathcal{L}_\infty$ **state estimation:** Let $\Pi_1=0,\ \Pi_2=0,\ \Pi_3=\gamma^2 I$ and $\Pi_4=I$, the extended dissipative state estimation becomes $\mathcal{L}_2-\mathcal{L}_\infty$ state estimation. By solving the LMIs in Theorem 4.1, the estimator gain matrix is obtained as:

$$\mathcal{K}=\begin{bmatrix}-0.5271\\ -0.2453\end{bmatrix}.$$

Simultaneously, the relative simulation is shown to verify the acquired consequence from Figs. 1 to 2. Under the random initial conditions, Fig. 1 describe the corresponding state trajectories of the neuron states and their estimation. From Figs. 1 to 2, it is clear to see that the error system (5) with the distraction $\omega(t)$ can really keep internally stable. Therefore, we conclude that, given the above parameter values, the error system (5) exhibits $\mathcal{L}_2-\mathcal{L}_\infty$ performance.

(ii) $\mathcal{H}_\infty$ **state estimation:** If we take $\Pi_1=-I,\ \Pi_2=0,\ \Pi_3=\gamma^2 I$ and $\Pi_4=0$, the extended dissipativity state estimation is reduced to the H_∞ state estimation. By solving the LMIs in Theorem 4.1, the estimator gain matrix is obtained as:

$$\mathcal{K}=\begin{bmatrix}-0.4123\\ -0.1875\end{bmatrix}.$$

Likewise, the corresponding simulation is illustrated to verify the obtained consequence from Figs. 3 to 4. Fig. 3 represents the corresponding state trajectories and their estimations under the random initial conditions, respectively. Under randomly chosen initial conditions, Fig. 4 describes the corresponding error state trajectories because of the effect of the estimation gain matrix. From Figs. 3 to 4, it is confirmed that the error system (5) with the distraction $\omega(t)$ can really keep internally stable. Furthermore, the error system (5) behaves $\mathcal{H}_\infty$ performance under the above parameter values.

(iii) **Passivity state estimation:** When $\Pi_1=0,\ \Pi_2=I,\ \Pi_3=\gamma^2 I$ and $\Pi_4=0$, the extended dissipativity state estimation is converted into the passivity state estimation. By solving the LMIs in Theorem 4.1, the corresponding estimation gain matrix is given by

$$\mathcal{K}=\begin{bmatrix}-0.9237\\ -0.4138\end{bmatrix}.$$

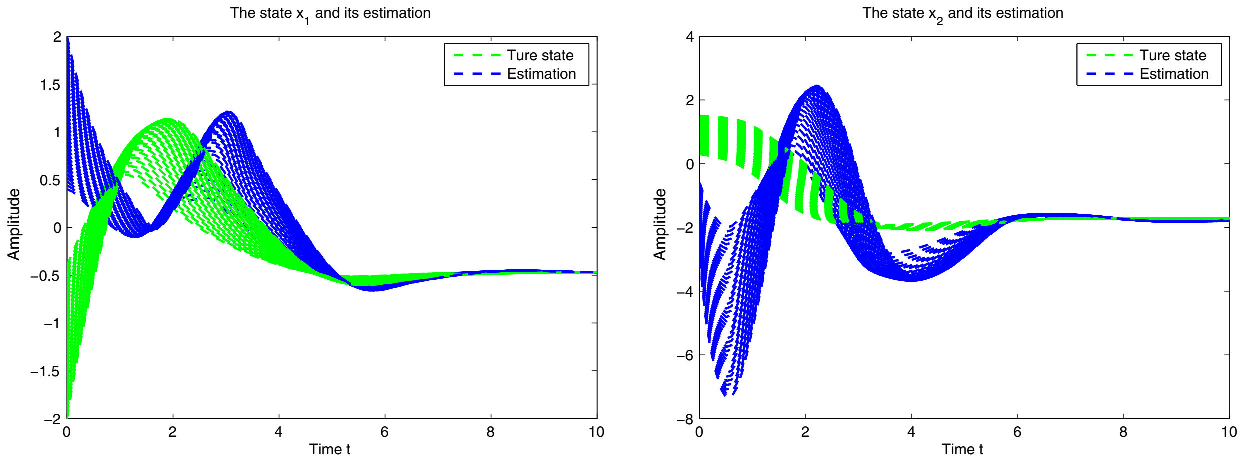

Fig. 1. State trajectories of the neuron states and their estimation based on $\mathcal{L}_2 - \mathcal{L}_\infty$ analysis.

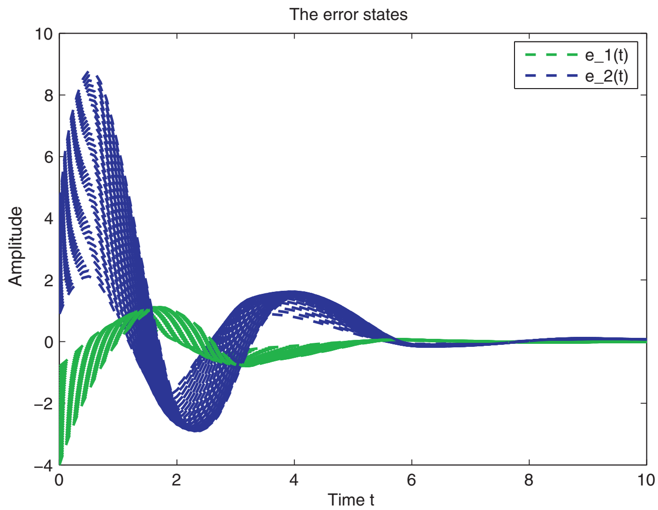

Fig. 2. The error states based on $\mathcal{L}_2 - \mathcal{L}_\infty$ analysis.

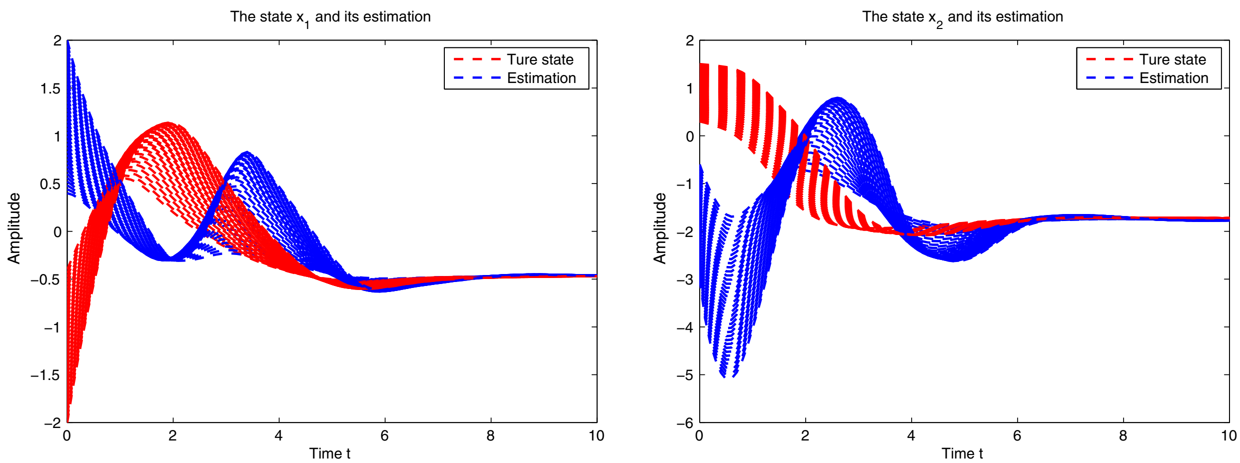

Fig. 3. State trajectories of the neuron states and their estimation based on $\mathcal{H}_\infty$ analysis.

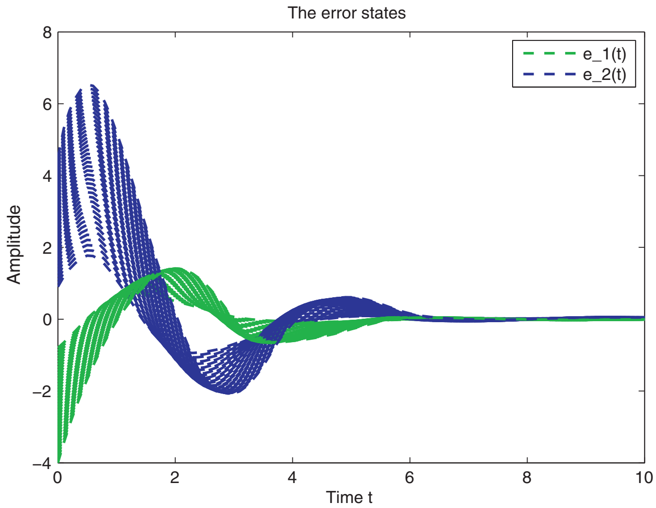

Fig. 4. The error states based on $\mathcal{H}_\infty$ analysis.

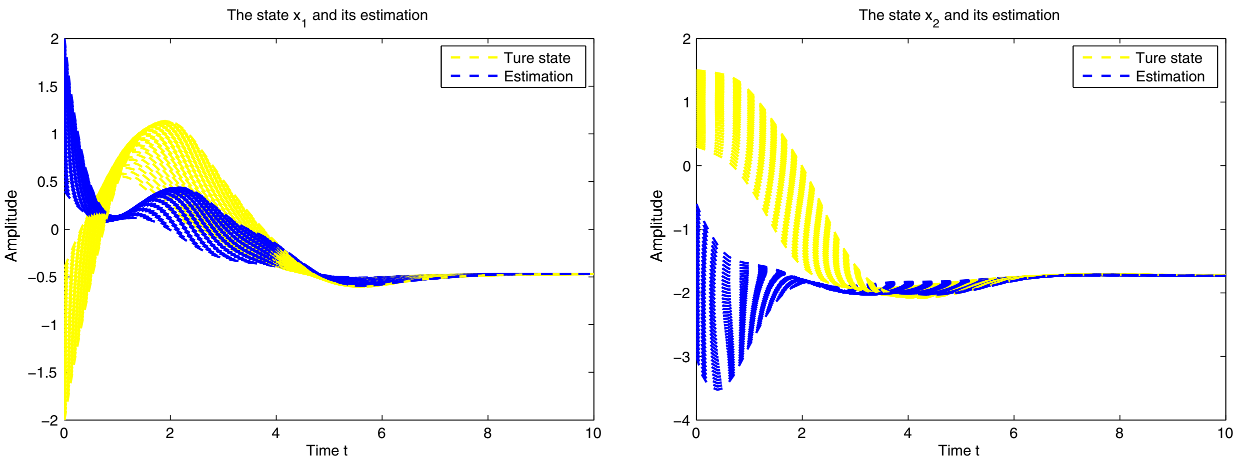

Fig. 5. State trajectories of the neuron states and their estimation based on passivity analysis.

Simultaneously, the relative simulation is drawn to demonstrate the acquired result from Figs. 5 to 6. Under the randomly chosen initial conditions, Fig. 5 describes the corresponding state trajectories and their estimations. Owe to the role of the estimation gain matrix, Fig. 6 represents the corresponding error state trajectories with random initial conditions. From 5 to 6, it is easy to see that the error system (5) with the distraction $\omega(t)$ can really keep internally stable. In conclusion, given the above parameter values, the error system (5) exhibits passive performance.

(iv) **Mixed $\mathcal{H}_\infty$ and passivity state estimation:** If we take $\Pi_1 = -\gamma^{-1}\alpha I,\ \Pi_2 = (1-\alpha)I,\ (0 \le \alpha \le 1),\ \Pi_3 = \gamma I$ and $\Pi_4 = 0$ with $\alpha = 0.5$, the extended dissipativity state estimation is reduced to the mixed $\mathcal{H}_\infty$ and passivity state estimation. By solving the LMIs in Theorem 4.1, the estimator gain matrix is obtained as:

$$\mathcal{K} = \begin{bmatrix} -0.6103 \\ -2.5170 \end{bmatrix}.$$

Likewise, the corresponding simulation is illustrated to verify the obtained consequence from Figs. 7 to 8. Fig. 7 represents the corresponding state trajectories and their estimations under the random initial conditions, respectively. Under randomly chosen initial conditions, Fig. 8 describes the corresponding error state trajectories because of the effect of the estimation gain matrix. From Figs. 7 to 8, it is confirmed that the error system (5) with the distraction $\omega(t)$ can really keep internally stable. Furthermore, the error system (5) behaves mixed $\mathcal{H}_\infty$ and passivity performance under the above parameter values.

(v) $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$**-dissipativity state estimation:** When we take $\Pi_1 = \mathcal{Q},\ \Pi_2 = \mathcal{S},\ \Pi_3 = \mathcal{R} - \gamma I,\ \Pi_4 = 0,\ \mathcal{Q} = I,\ \mathcal{S} = \begin{bmatrix} 1 & 0 \\ 0.3 & 1 \end{bmatrix}$ and $\mathcal{R} = 3I$, the extended dissipativity state estimation is reduced to the $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity state

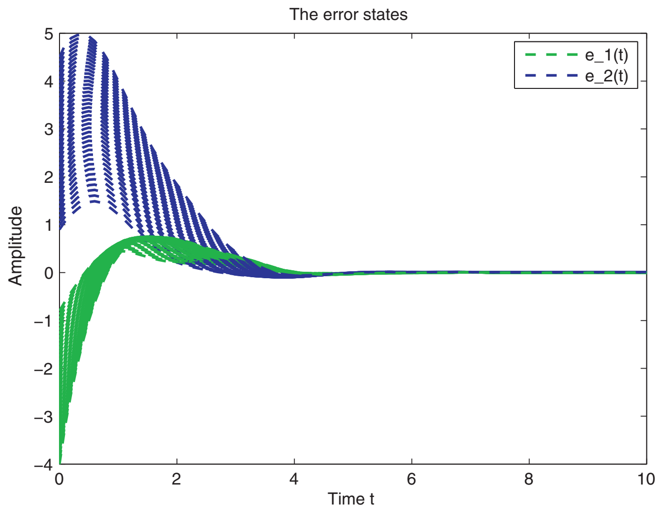

Fig. 6. The error states based on passivity analysis.

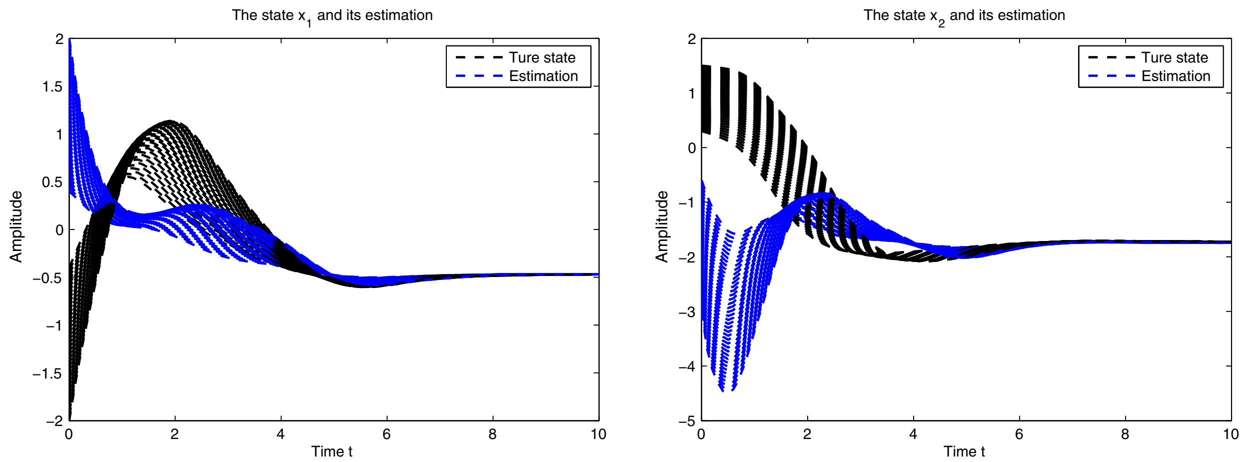

Fig. 7. State trajectories of the neuron states and their estimation based on mixed $\mathcal{H}_\infty$ and Passivity analysis.

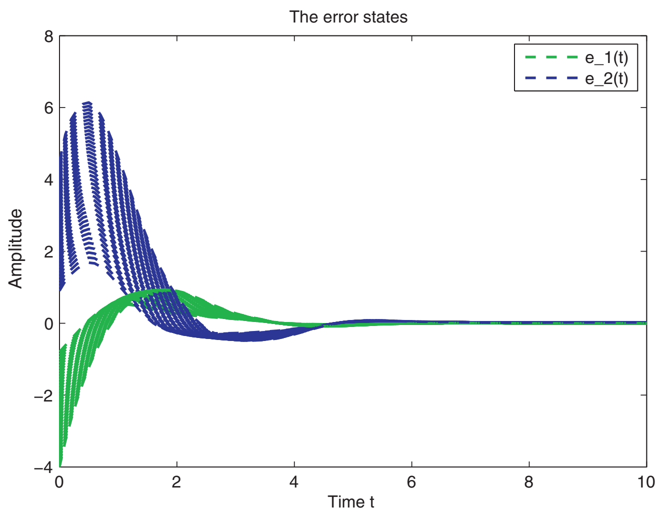

Fig. 8. The error states based on mixed $\mathcal{H}_\infty$ and Passivity analysis.

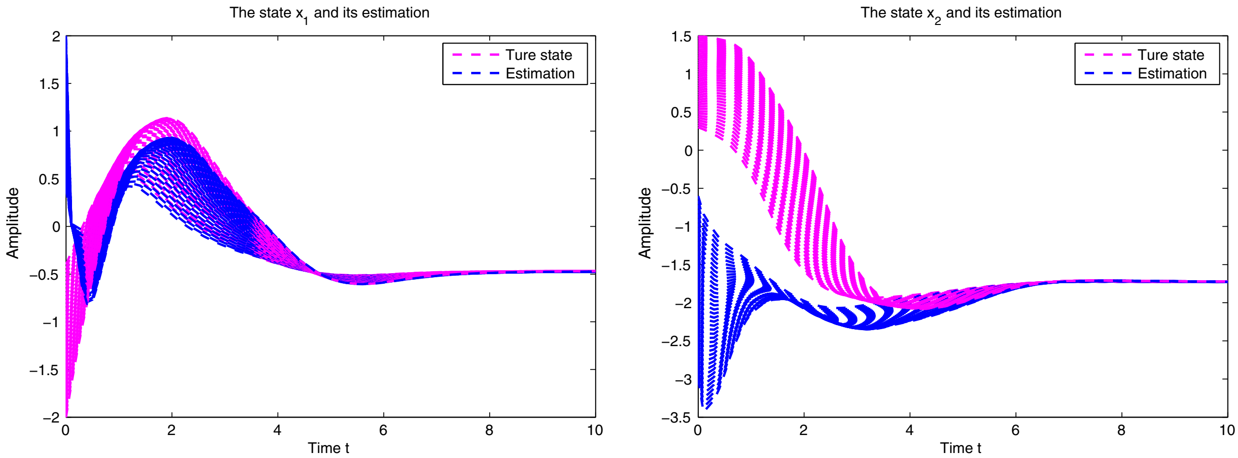

Fig. 9. State trajectories of the neuron states and their estimation based on $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity analysis.

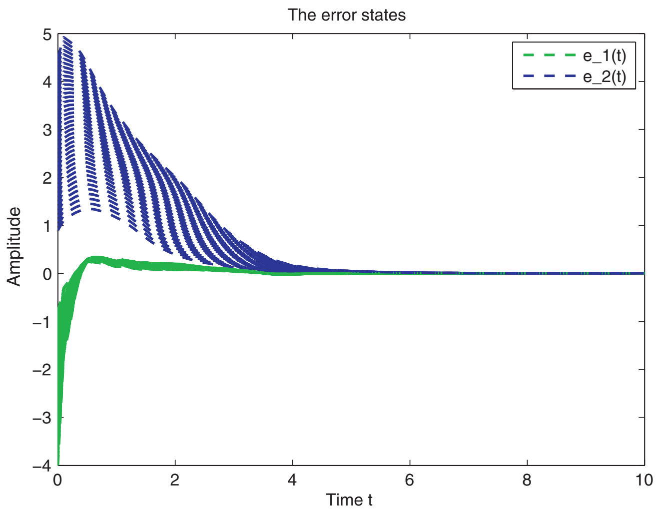

Fig. 10. The error states based on $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity analysis.

estimation. By solving the LMIs in Theorem 4.1, the corresponding estimation gain matrix is obtained as:

$$\mathcal{K} = \begin{bmatrix} -0.7818 \\ -1.1026 \end{bmatrix}.$$

Simultaneously, the relative simulation is demonstrated to verify the acquired result from Figs. 9 to 10. Fig. 9 represents the corresponding state trajectories and their estimations under the random initial conditions. Under the random initial conditions, Fig. 10 describes the corresponding error state trajectories on account of the effect of the estimation gain matrix. From Figs. 9 to 10, it is not difficult to see that the error system (5) with distraction $\omega(t)$ can really keep internally stable. Furthermore, the error system (5) behaves $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity performance under the above parameter values.

Example 5.2. In this example, we will show the real-life application of the developed result in a practical model-called, the quadruple-tank process system (QTPS), as displayed in Fig. 11. Thus, artificial NNs can be considered in the form of a practical model (QTPS). Therefore, the entire physical model is administered by a delay differential equation of the experimental problem, which was first inspected by Johansson [11]. The setup consists of four interconnected water tanks, two water pumps, and two valves. Our main target is to control the water level in the two lower tanks using the 2 pumps. Here, the process inputs are the voltages (υ_1 and υ_2) to the water pumps, pump 1 and pump 2, whereas the outputs are evaluated by water levels of Tank 1 and Tank 2. Tank 1 and Tank 2 are placed below Tank 3 and Tank 4 to take in water flow by the gravitational force. Therefore, the QTPS can clearly use the NN model, which has attracted a great deal of research attention in recent years (see [25,31]). Recently, Lee et al. [13] and Vembarasan et al. [37] proposed the state-space equation of the QTPS. Therefore, the QTPS can be rewritten to the form of an LFNNs (14) if we setting $\mathcal{W}_3 = I$, $\mathcal{W}_2 = \mathcal{D} = 0$ in (5), as

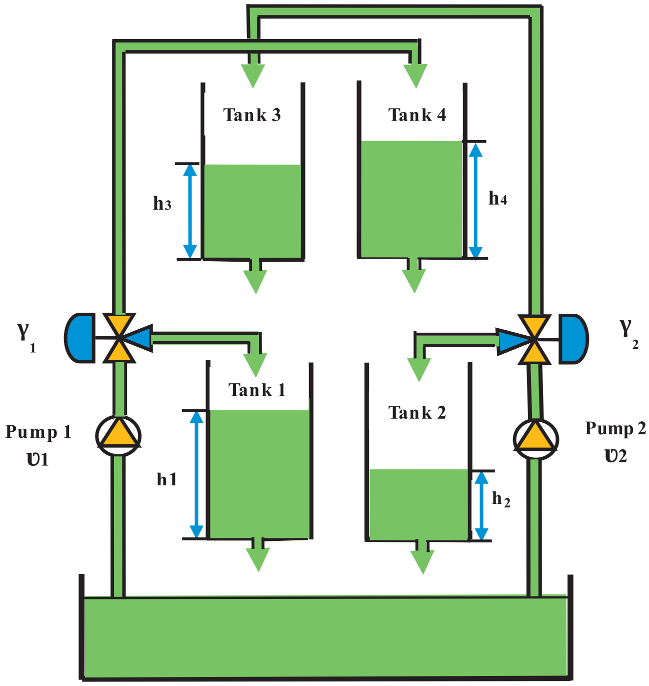

Fig. 11. Schematic representation of the QTPS. Source: Johansson [11].

follows:

$$\begin{aligned}\dot{e}(t) &= -(\mathcal{A}+\mathcal{K}\mathcal{C})e(t)+\mathcal{W}_0 g(e(t))+\mathcal{W}_1 g(e(t-h(t)))+(\mathcal{B}_1-\mathcal{K}\mathcal{B}_2)\omega(t),\\ \tilde{y}(t) &= \mathcal{C}e(t)+\mathcal{B}_2\omega(t),\\ \bar{z}(t) &= \mathcal{H}e(t),\end{aligned} \tag{14}$$

where

$$\mathcal{A}=\begin{bmatrix}0.0021 & 0 & -0.0424 & 0\\ 0 & 0.0021 & 0 & -0.0424\\ 0 & 0 & 0.0424 & 0\\ 0 & 0 & 0 & 0.0424\end{bmatrix},\quad \mathcal{W}_0=\begin{bmatrix}-0.0060 & -0.0065 & -0.0029 & -0.0077\\ -0.0063 & -0.0051 & -0.0073 & -0.0025\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix},$$

$$\mathcal{W}_1=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -0.0143 & -0.0114 & -0.0165 & -0.0056\\ -0.0119 & -0.0131 & -0.0059 & -0.0154\end{bmatrix},\quad \mathcal{B}_1=\begin{bmatrix}1\\1\\1\\1\end{bmatrix},\quad \mathcal{B}_2=\begin{bmatrix}1\\1\\1\\1\end{bmatrix},\quad \mathcal{C}=\mathcal{H}=I_4.$$

Here, the activation function satisfies Assumption (A1) with $k_i^-=-0.05$, $k_i^+=0.05$, $i=1,\dots,4$, and thus we can get the parameters as $K_1=-0.05I$, $K_2=0.05I$. The activation function $g(e(t))=0.01\big(|e(t)+1|-|e(t)-1|\big)$. In addition, the time-varying delay signals is taken as $h(t)=0.9+0.9\cos(t)$ which means $h_U=1.8$ with $h_L=0.6$, $\mu=0.5$, and $\eta=0.75$. Furthermore, the noise disturbance is taken as $\omega(t)=0.001e^{-0.0003t}\sin(0.00001t)$, $t\geq 0$. Originally, the water levels of the two lower tanks are the only accessible and usable information in the QTPS, whereas the water levels of the two upper tanks are not. Supposing, the water in the upper tanks overflow, no action can be taken. Hence, there is a strong need to know the water level of the upper tanks, which is one of the methods for obtaining the information to design the state estimator. Moreover, this paper mainly focuses on transporting time delay signals between valves and tanks being interval time-varying. Therefore, in the sense of practical applications, it deserves that the QTPS should be applied to the proposed state estimation method based on the extended dissipativity performance. However, until now, this has not been explored (see [13,16,25,31,37]). Accordingly, the main objective of this example is to generalize our results on the extended dissipativity state estimation of the QTPS is determined, which unifies all the famous performances and this was realized under the state estimation control parameter, such as $\mathcal{L}_2-\mathcal{L}_\infty$ state estimation, $\mathcal{H}_\infty$ state estimation, passivity state estimation, mixed $\mathcal{H}_\infty$ and passivity state estimation, and $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma$-dissipativity state estimation performances. By the choice of choosing free matrices Π_1, Π_2, Π_3, and Π_4 and solving the LMIs in (8) to (12) with the above parameters, the corresponding state estimation is drawn as following:

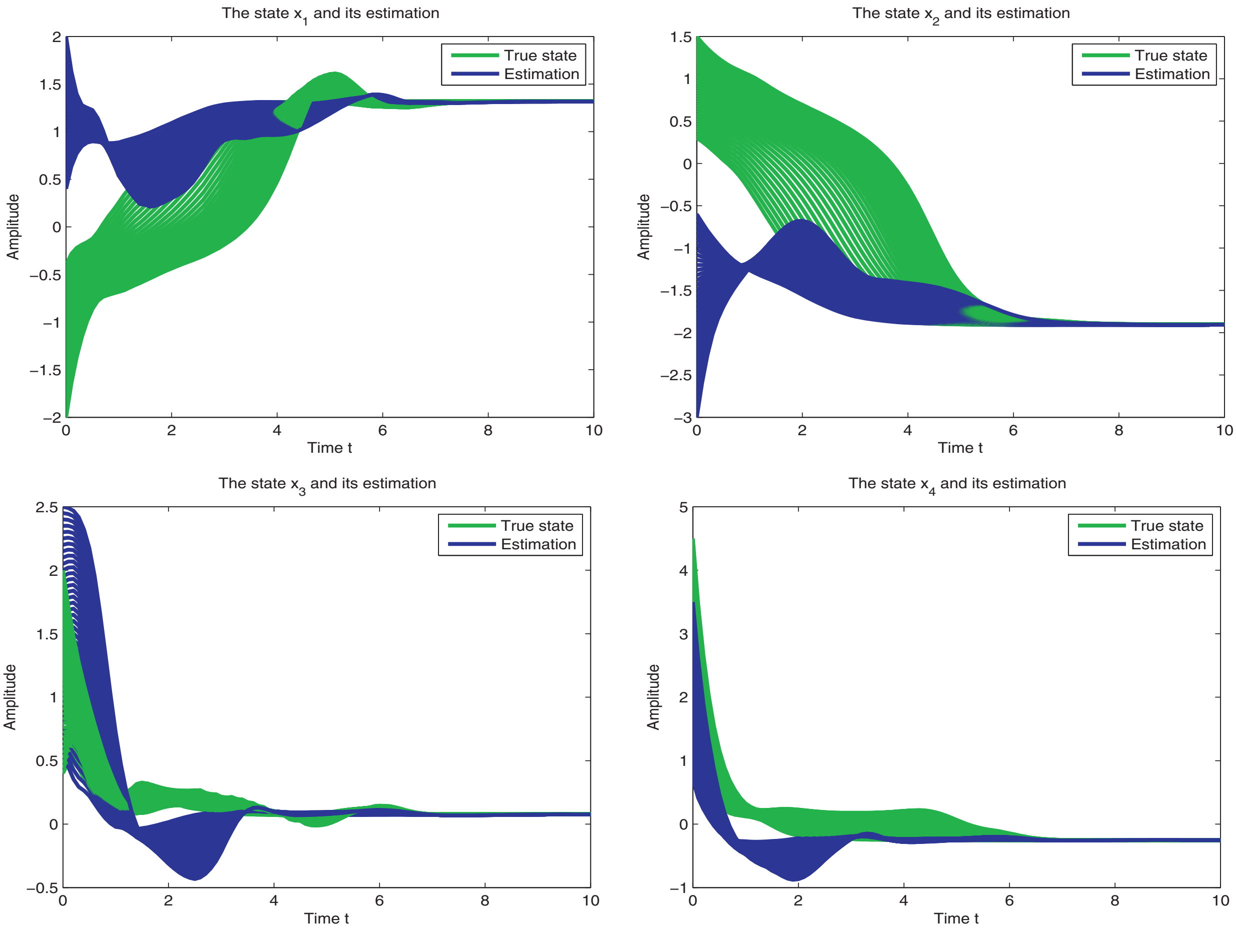

Fig. 12. State trajectories of the neuron states and their estimation of the QTPS based on $\mathcal{L}_2 - \mathcal{L}_\infty$ analysis.

(i) $\mathcal{L}_2 - \mathcal{L}_\infty$ **state estimation of the QTPS:** If we take $\Pi_1 = 0$, $\Pi_2 = 0$, $\Pi_3 = \gamma^2 I$ and $\Pi_4 = I$, the extended dissipative state estimation becomes $\mathcal{L}_2 - \mathcal{L}_\infty$ state estimation. By solving the LMIs in Theorem 4.1, the estimator gain matrix is obtained as:

$$\mathcal{K} = \begin{bmatrix} 2.6673 & -0.5103 & -0.0894 & -0.0068 \\ -0.0103 & 2.6687 & -0.0065 & -0.0898 \\ -0.0887 & -0.0056 & 1.5105 & -0.0006 \\ -0.0068 & -0.0891 & -0.0012 & 2.5100 \end{bmatrix}.$$

Simultaneously, the relative simulation is shown to verify the acquired consequence from Figs. 12 to 13. Under the random initial conditions, Fig. 12 describe the corresponding error state trajectories of the neuron states and their estimation. From Figs. 12 to 13, it is clear to see that the error system (5) with the distraction $\omega(t)$ can really keep internally stable. Therefore, we conclude that, given the above parameter values, the error system (5) exhibits $\mathcal{L}_2 - \mathcal{L}_\infty$ performance of the QTPS.

(ii) $\mathcal{H}_\infty$ **state estimation of the QTPS:** If we take $\Pi_1 = -I$, $\Pi_2 = 0$, $\Pi_3 = \gamma^2 I$ and $\Pi_4 = 0$, the extended dissipativity state estimation is reduced to the $\mathcal{H}_\infty$ state estimation. By solving the LMIs in Theorem 4.1, the estimator gain matrix is obtained as:

$$\mathcal{K} = \begin{bmatrix} 1.7243 & -0.5362 & -0.1239 & -1.2001 \\ -0.0023 & 1.0057 & -0.0152 & -0.0324 \\ -0.0089 & -0.2351 & 1.2617 & -0.0128 \\ -0.1352 & 1.9423 & 0.4251 & 1.7524 \end{bmatrix}.$$

Likewise, the corresponding simulation is illustrated to verify the obtained consequence from Figs. 14 to 15. Fig. 14 represents the corresponding state trajectories and their estimations under the random initial conditions, respectively. Under randomly chosen initial conditions, Fig. 15 describes the corresponding error state trajectories because of the effect of the estimation gain matrix. From Figs. 14 to 15, it is confirmed that the error system (5) with

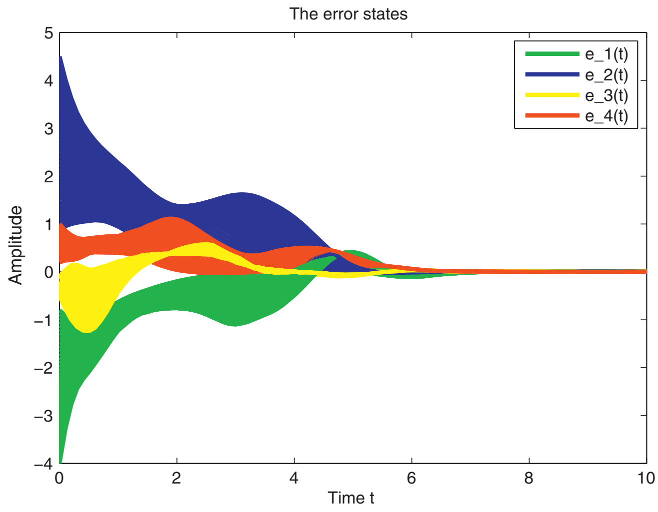

Fig. 13. The error states of the QTPS based on $\mathcal{L}_2 - \mathcal{L}_\infty$ analysis.

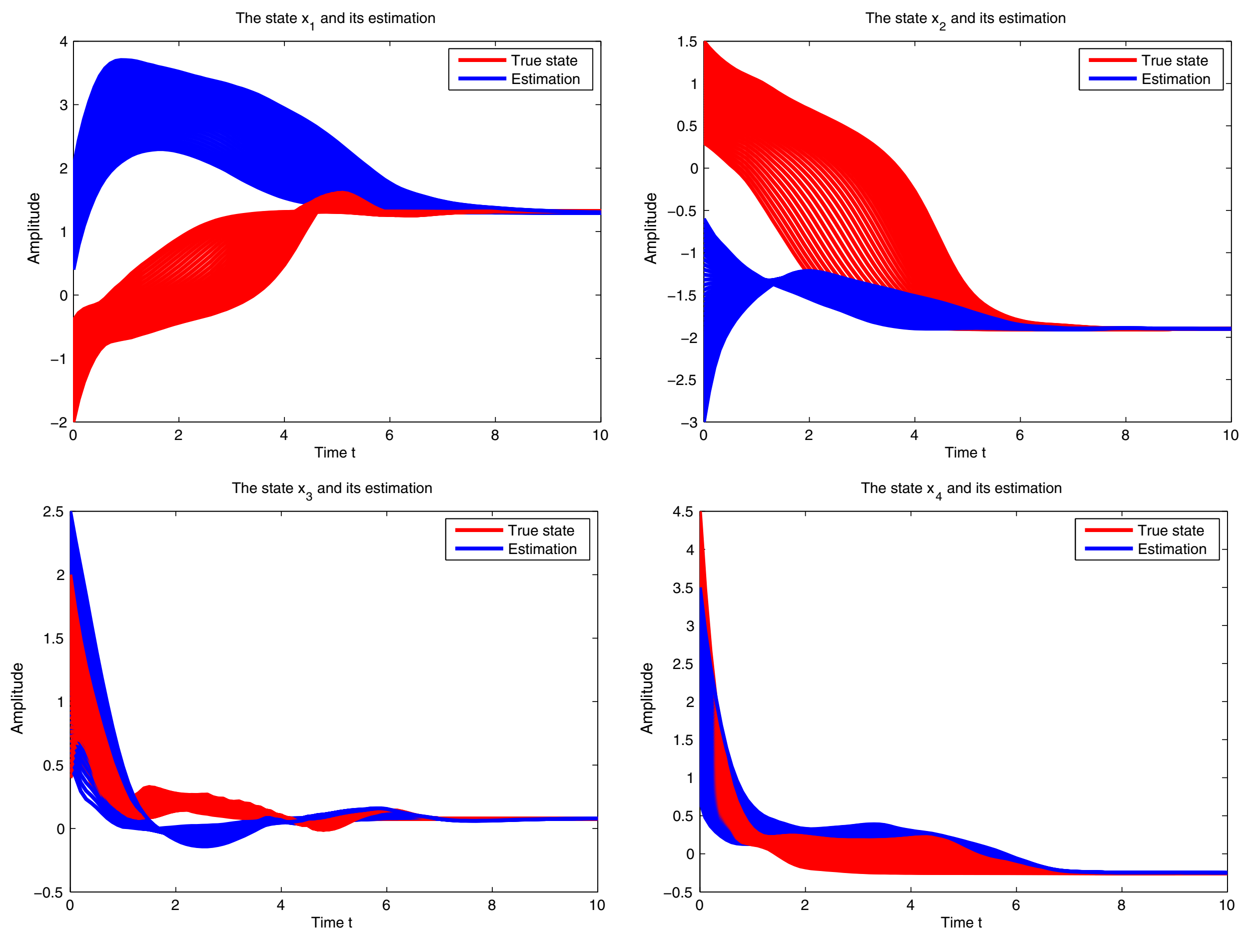

Fig. 14. State trajectories of the neuron states and their estimation of the QTPS based on $\mathcal{H}_\infty$ analysis.

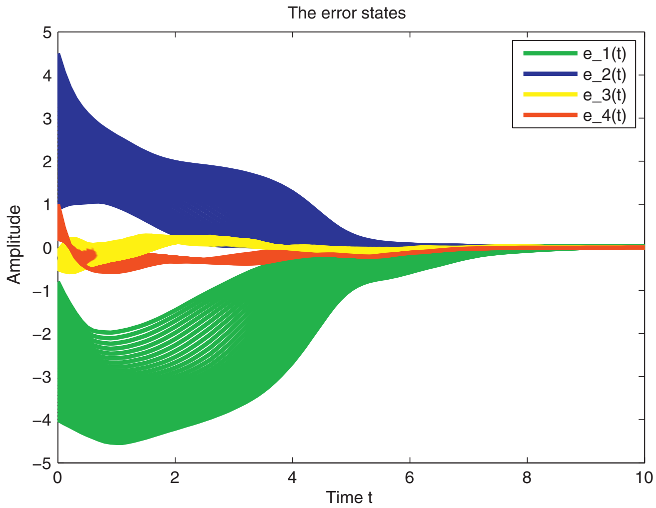

Fig. 15. The error states of the QTPS based on $\mathcal{H}_\infty$ analysis.

the distraction $\omega(t)$ can really keep internally stable. Furthermore, the error system (5) behaves $\mathcal{H}_\infty$ performance of the QTPS under the above parameter values.

(iii) **Passivity state estimation of the QTPS:** When $\Pi_1 = 0$, $\Pi_2 = I$, $\Pi_3 = \gamma^2 I$ and $\Pi_4 = 0$, the extended dissipativity state estimation is converted into the passivity state estimation. By solving the LMIs in Theorem 4.1, the corresponding estimation gain matrix is given by

$$\mathcal{K} = \begin{bmatrix} 0.9834 & -0.7435 & -0.3324 & -0.8865 \\ -0.0253 & 0.7810 & -0.2516 & -0.4562 \\ -0.0945 & -0.4351 & 0.5324 & 0.4231 \\ 0.2351 & 2.0032 & 1.0432 & 2.0341 \end{bmatrix}.$$

Simultaneously, the relative simulation is drawn to demonstrate the acquired result from Figs. 16 to 17. Under randomly chosen initial conditions, Fig. 16 describes the corresponding state trajectories and their estimations, respectively. Owe to the role of the estimation gain matrix, Fig. 17 represents the corresponding error state trajectories with the random initial conditions. From Figs. 16 to 17, it is easy to see that the error system (5) with the distraction $\omega(t)$ can really keep internally stable. In conclusion, given the above parameter values, the error system (5) exhibits passive performance of the QTPS.

(ii) **Mixed $\mathcal{H}_\infty$ and passivity state estimation of the QTPS:** If we take $\Pi_1 = -\gamma^{-1}\alpha I$, $\Pi_2 = (1-\alpha)I$, $(0 \le \alpha \le 1)$, $\Pi_3 = \gamma I$ and $\Pi_4 = 0$ with $\alpha = 0.5$, the extended dissipativity state estimation is reduced to the mixed $\mathcal{H}_\infty$ and passivity state estimation. By solving the LMIs in Theorem 4.1, the estimator gain matrix is obtained as:

$$\mathcal{K} = \begin{bmatrix} 1.9950 & 0.6342 & -0.7835 & -1.7421 \\ 0.1003 & 1.4067 & -0.3241 & -0.1172 \\ 0.1284 & -0.8901 & 1.5682 & -0.8701 \\ -1.1352 & 1.7682 & 1.0042 & 1.9853 \end{bmatrix}.$$

Likewise, the corresponding simulation is illustrated to verify the obtained consequence from Figs. 18 to 19. Fig. 18 represents the corresponding state trajectories and their estimations under the random initial conditions, respectively. Under randomly chosen initial conditions, Fig. 19 describes the corresponding error state trajectories because of the effect of the estimation gain matrix. From Figs. 18 to 19, it is confirmed that the error system (5) with the distraction $\omega(t)$ can really keep internally stable. Furthermore, the error system (5) behaves mixed $\mathcal{H}_\infty$ and pssive performance of the QTPS under the above parameter values.

(iv) $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$**-dissipativity state estimation of the QTPS:** When we take $\Pi_1 = \mathcal{Q}$, $\Pi_2 = \mathcal{S}$, $\Pi_3 = \mathcal{R} - \gamma I$, $\Pi_4 = 0$, with $\mathcal{Q} = -I_4$, $\mathcal{S} = I$, and $\mathcal{R} = 3I$, the extended dissipativity state estimation is reduced to the $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity state estimation. By solving the LMIs in Theorem 4.1, the corresponding estimation gain matrix is obtained as:

$$\mathcal{K} = \begin{bmatrix} 3.3242 & 0.6423 & -0.7521 & 0.5342 \\ -0.1352 & 0.5673 & 0.4352 & -0.6781 \\ -0.2516 & 0.7631 & 0.7842 & -0.7681 \\ 0.4561 & -1.0032 & 0.9321 & 1.9842 \end{bmatrix}.$$

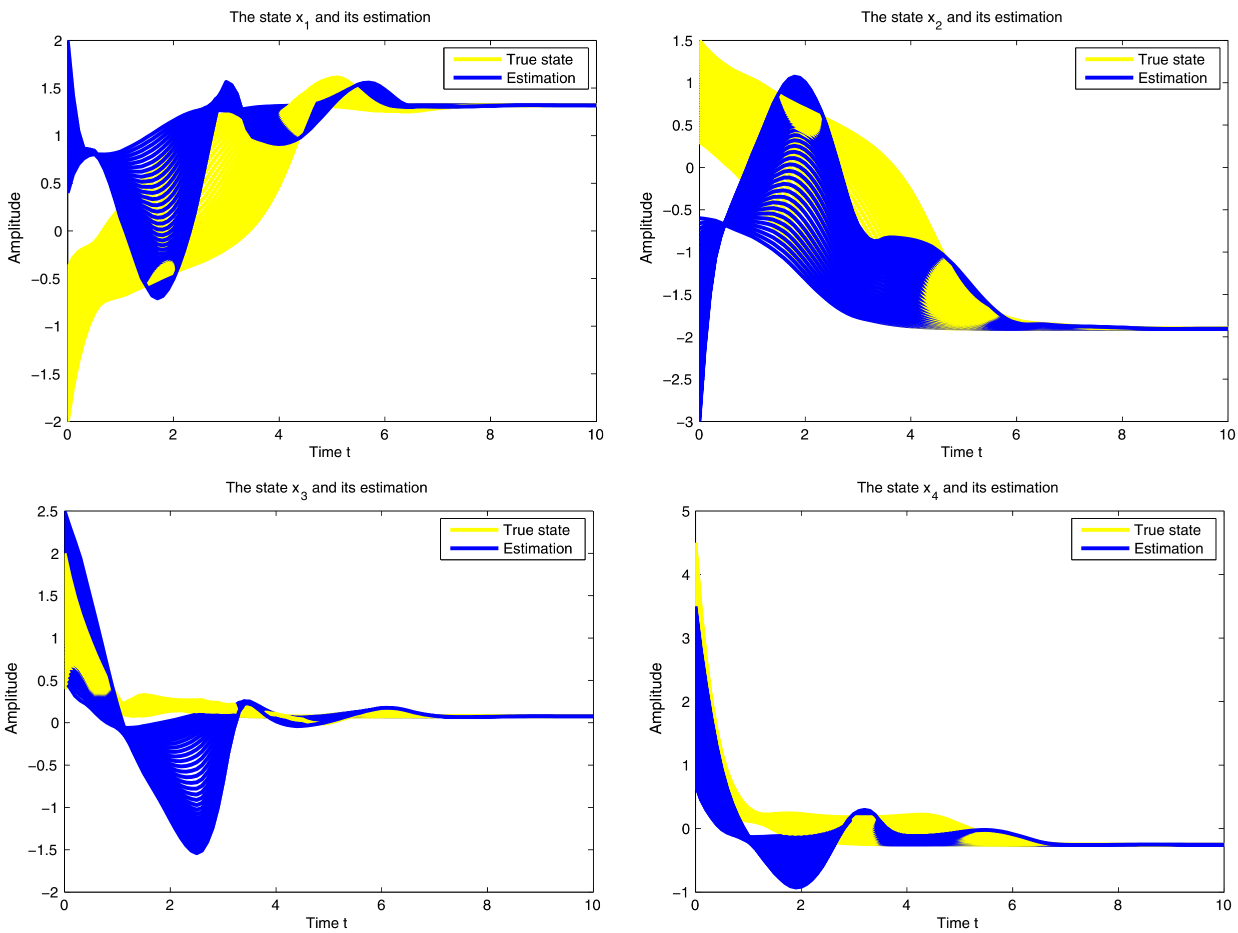

Fig. 16. State trajectories of the neuron states and their estimation of the QTPS based on passivity analysis.

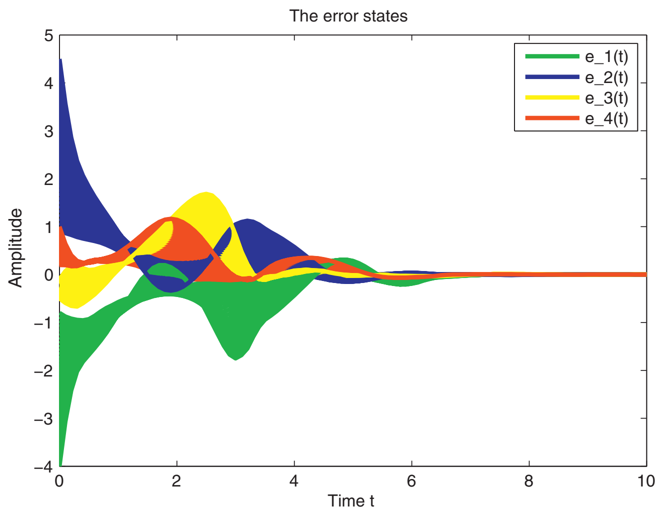

Fig. 17. The error states of the QTPS based on passivity analysis.

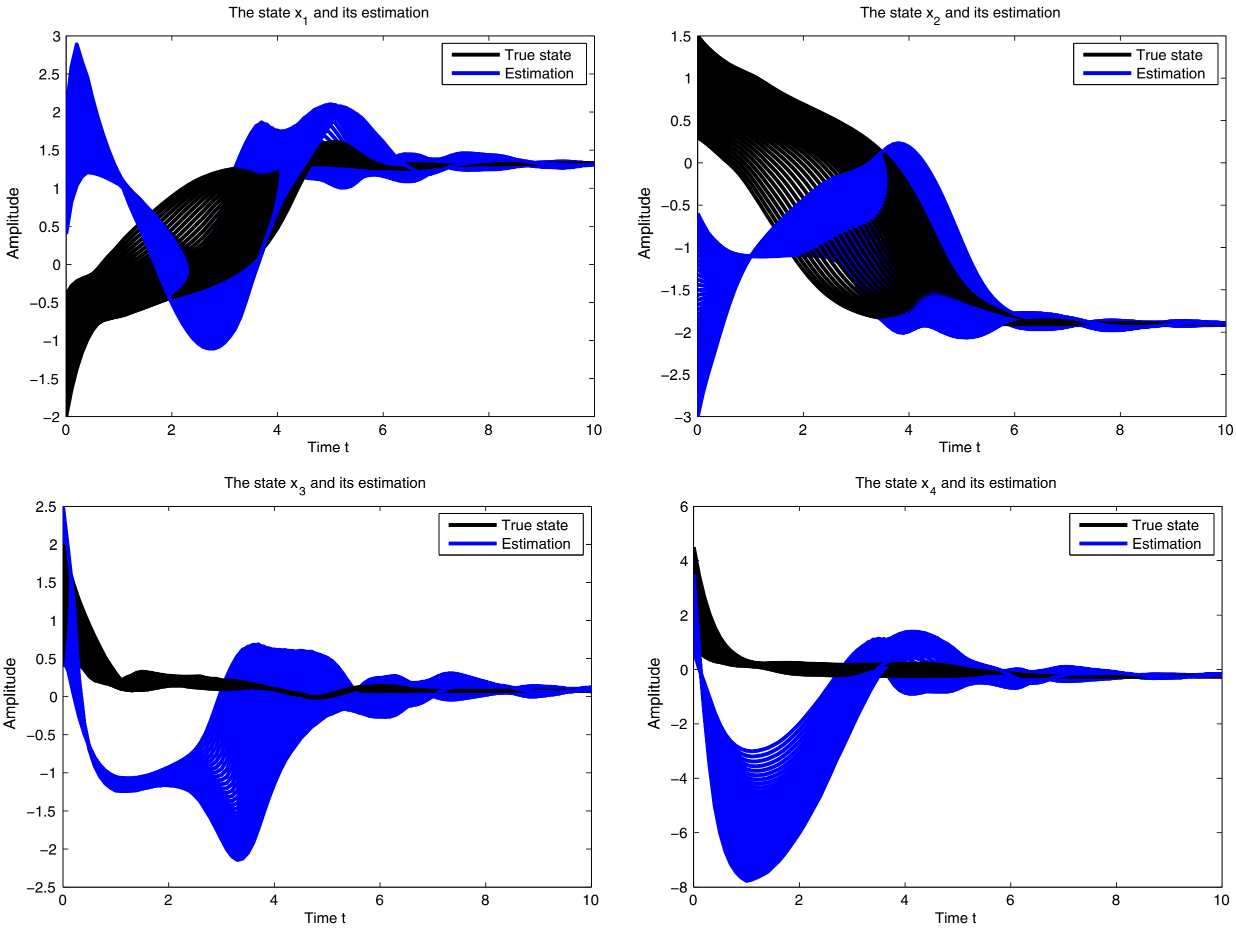

Fig. 18. State trajectories of the neuron states and their estimation of the QTPS based on mixed $\mathcal{H}_\infty$ and passivity analysis.

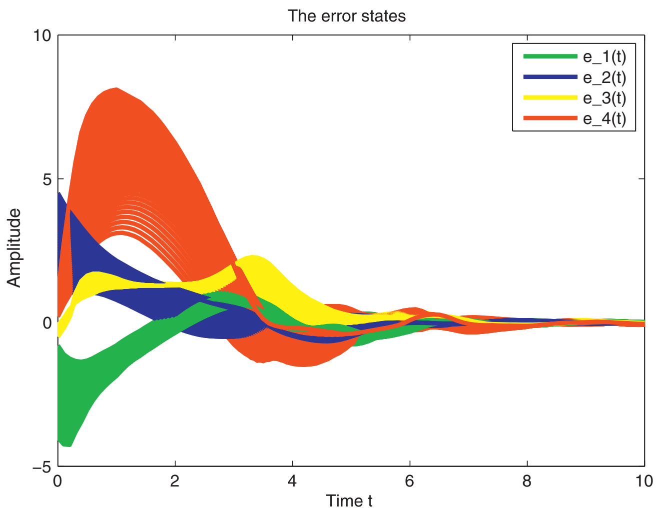

Fig. 19. The error states of the QTPS based on mixed $\mathcal{H}_\infty$ and passivity analysis.

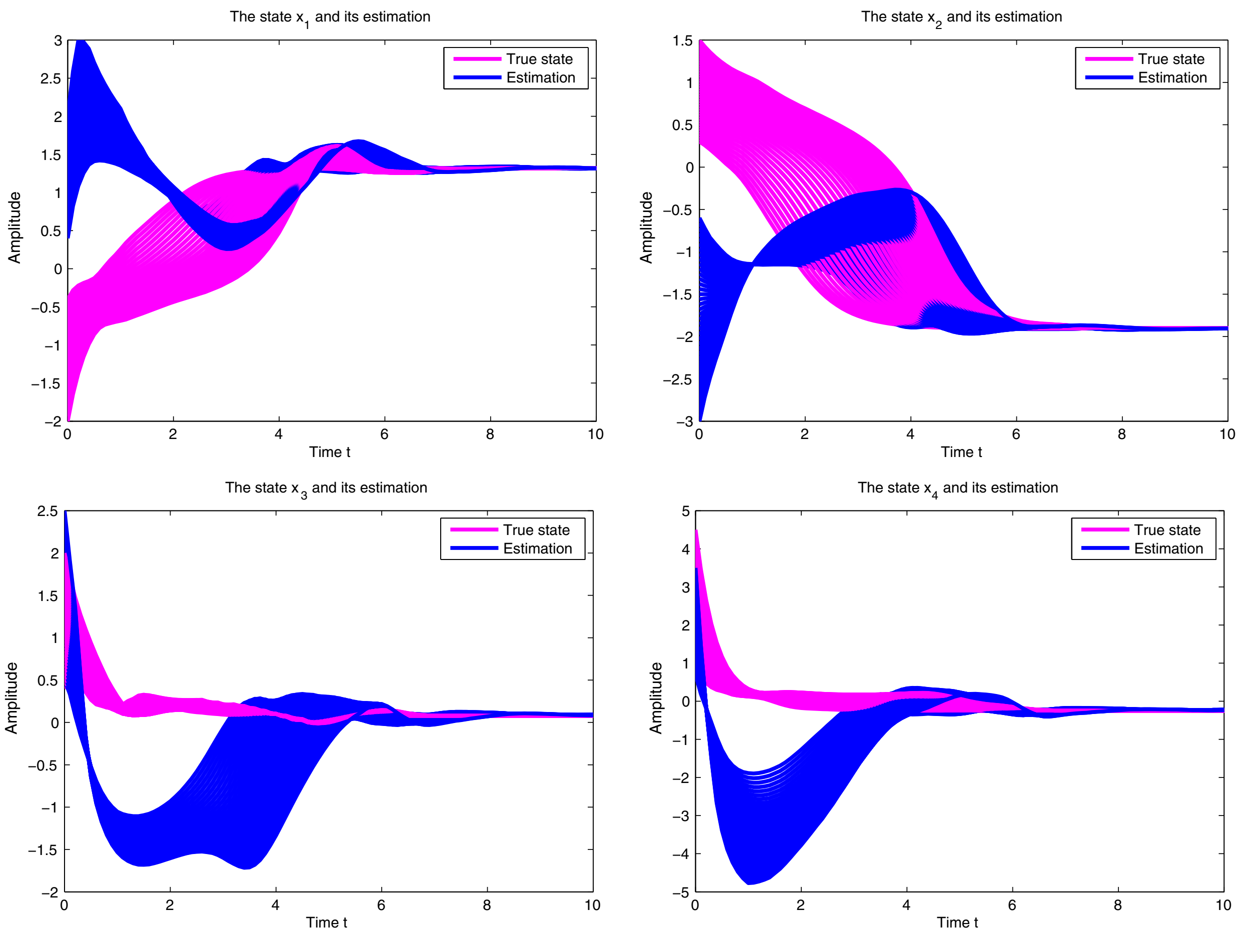

Fig. 20. State trajectories of the neuron states and their estimation of the QTPS based on $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity analysis.

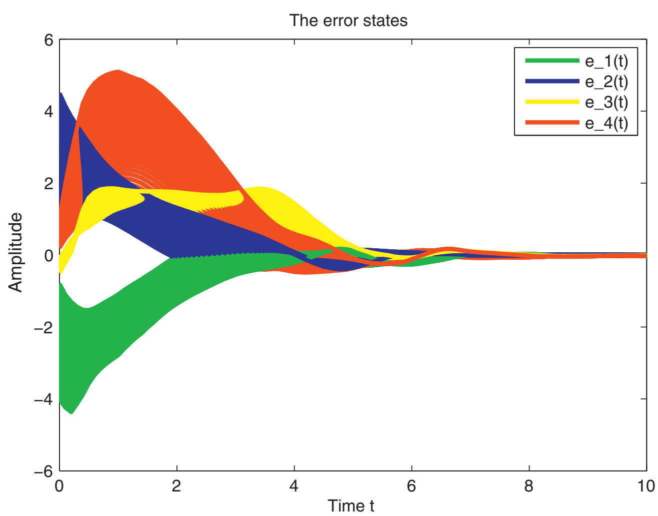

Fig. 21. The error states of the QTPS based on $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity analysis.

Table 1
The minimum $\mathcal{H}_\infty$ performance γ with various (h_L, h_U, μ) in Example 5.3.

(h_L, h_U, μ)	(0.2,0.5,0.5)	(0.3,0.7,0.6)	(0.4,0.8,0.7)	(0.5,0.9,1.2)
[34]	0.2080	0.2466	0.2507	0.2555
Theorem 4.1	0.1539	0.1692	0.1783	0.1896

Simultaneously, the relative simulation is demonstrated to verify the acquired result from Figs. 20 to 21. Fig. 20 represents the corresponding state trajectories and their estimations under random initial conditions, respectively. Under the same initial conditions, Fig. 21 describes the corresponding error state trajectories on account of the effect of the estimation gain matrix. From Fig. 20 to 21, it is not difficult to see that the error system (5) with distraction $\omega(t)$ can really keep internally stable. Furthermore, the error system (5) behaves $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma$-dissipativity performance of the QTPS under the above parameter values.

Remark 5.1. It is very interesting that, Example 5.2 shows the modified version of an LFNN model to represent the QTPS, which considers the transporting time-delay signal as a process variable. This model selected, to demonstrate the proposed method can be applied to the real-life applications of an LFNN model in the sense of state estimation method based on the extended dissipativity performance. In recent years, the QTPS has attracted much attention from the scholars, because it can be easy to use and realize the applications in our day-to-day living. Until now, the research of the QTPS has focused on designing the stabilizing controller (see [13,16,25,31,37]). For instance, in the sense of practical applications, Lee et al. [13] have applied sampled data controller in their estimation method. Recently, Vembarasan et al. [37] have considered the non-fragile observer based design method to estimate the upper tank levels of the QTPS. More recently, time-delayed coupling should be taken into account to showing applicability on a QTPS, which was explored by Manivannan et al. (see [16,25,31]). Howbeit, no related works to suggest unifying framework analysis to study the dynamical behaviors of the QTPS in the sense of the state estimation method. To meet this demand, this work gives a positive answer and the simulation studies are investigated under the state estimation method based on the extended dissipativity performance, which covers all these famous performances such as $\mathcal{L}_2-\mathcal{L}_\infty$ state estimation, $\mathcal{H}_\infty$ state estimation, passivity state estimation, mixed $\mathcal{H}_\infty$ and passivity state estimation, and $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma$ dissipativity state estimation of the QTPS. Therefore, the proposed results in this paper are more effective and general than the previous works.

Example 5.3. Consider the static NN (13) with the following matrix parameters as discussed in [34]:

$$\mathcal{A}=\begin{bmatrix}7.0214 & 0\\ 0 & 7.4367\end{bmatrix},\ \mathcal{W}_3=\begin{bmatrix}-6.4993 & -12.0257\\ -0.6867 & 5.6614\end{bmatrix},\ \mathcal{H}=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix},\ \mathcal{B}_1=\begin{bmatrix}0.2\\ 0.2\end{bmatrix},\ \mathcal{B}_2=-0.1,$$
$$\mathcal{J}=\begin{bmatrix}0\\ 0\end{bmatrix},\ \mathcal{C}=\begin{bmatrix}1 & 0\end{bmatrix},\ \mathcal{D}=\begin{bmatrix}2 & 0\end{bmatrix},$$

and $K_1=0$, $K_2=0.5I$, $\eta=1$. The NN described in this example is an static NN. Here, we take this example for the purpose of comparison with the $\mathcal{H}_\infty$ state estimation criteria in [34]. To do this, if we take the free matrices $\Pi_1=-I$, $\Pi_2=0$, $\Pi_3=\gamma^2 I$ and $\Pi_4=0$, the extended dissipativity state estimation is reduced to the $\mathcal{H}_\infty$ state estimation performance. Then, by applying the LMIs in Theorem 4.1 of this paper we obtain the minimum $\mathcal{H}_\infty$ performance index γ for various h_L, h_U, and μ of this paper and the result of [34] are listed in Table 1. From this table, we can observe that our results given this paper are far better than those of [34]. Thus, it is clear to show that our approach is more effective than the one given in [34].

Example 5.4. Consider the static NN (13) with the following matrix parameters as discussed in [41]:

$$\mathcal{A}=\begin{bmatrix}0.96 & 0 & 0\\ 0 & 0.8 & 0\\ 0 & 0 & 1.48\end{bmatrix},\ \mathcal{W}_3=\begin{bmatrix}0.5 & 0.3 & -0.36\\ 0.1 & 0.12 & 0.5\\ -0.42 & 0.78 & 0.9\end{bmatrix},\ \mathcal{H}=\begin{bmatrix}1 & 1 & 0\\ 1 & 0 & -1\\ 0 & 1 & 1\end{bmatrix},\ \mathcal{B}_1=\begin{bmatrix}0.1 & 0 & 0\\ 0.2 & 0 & 0\\ 0.1 & 0 & 0\end{bmatrix},$$
$$\mathcal{B}_2=\begin{bmatrix}-0.1 & 0 & 0\end{bmatrix},\ \mathcal{J}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix},\ \mathcal{C}=\begin{bmatrix}1 & 0 & -2\end{bmatrix},\ \mathcal{D}=\begin{bmatrix}0.5 & 0 & -1\end{bmatrix},$$

and $K_1=0$, $K_2=I$, and $\eta=1$. The NN addressed in this example is an static NN. Here, we take this example for the intention of comparison with [41]. To achieve this, if we setting the free matrices $\Pi_1=-I$, $\Pi_2=0$, $\Pi_3=\gamma^2 I$ and $\Pi_4=0$, the extended dissipativity state estimation is converted to the $\mathcal{H}_\infty$ state estimation. Then, by applying the LMIs in Corollary 4.1 of this paper we obtain the minimum $\mathcal{H}_\infty$ performance index γ for different h and μ are listed in Table 2. From Table 2, we can observe that our results given this paper are far better than those of [41]. Therefore, it is clear that, the proposed estimation method is much better than those one given in [41].

Example 5.5. Consider the error system (5) subject to (1) and (4). For the purpose of comparing those results in [46], we take $\mathcal{C}=\mathcal{H}=I$, $\mathcal{W}_2=\mathcal{D}=\omega(t)=0$, then Theorem 4.1 delivers a stability criterion for the error system (5) with an interval time delay signal. If we setting $\mathcal{K}=0$, it can be used to check the stability of the NN (1) with matrix parameters as follows:

Table 2
The minimum $\mathcal{H}_\infty$ performance γ with various (h, μ) in Example 5.4.

(h, μ)	(0.8,0.4)	(0.9,0.7)	(1.1,0.5)
[41]	0.7175	0.9916	0.7655
Corollary 4.1	0.5768	0.7011	0.6328

Table 3
The maximum upper bounds of h_U for $h_L = 1$ and various μ in Example 5.5.

Method	$\mu = 0.1$	$\mu = 0.5$	$\mu = 0.9$
Corollary 4.1 [46]	2.5043	1.7457	1.3700
Theorem 4.1	2.7884	2.4325	2.1347

$$\mathcal{A} = \text{diag}\{1.2769, 0.6231, 0.9230, 0.4480\},$$

$$\mathcal{W}_0 = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix}, \quad \mathcal{W}_1 = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$k_1^- = -0.4$, $k_2^- = 0.1$, $k_3^- = 0$, $k_4^- = -0.3$, $k_1^+ = 0.1137$, $k_2^+ = 0.1279$, $k_3^+ = 0.7994$, and $k_4^+ = 0.2368$. Therefore, the NN considered in this example is an LFNN. For this example, we applying Theorem 4.1 and calculate the maximum upper bounds h_U for $h_L = 1$ and various μ are listed in Table 3. Hence, it is clear from the table, the criteria developed in this paper is less conservative and more effective than [46], which is greatly benefited from the introduction of the Lemma 2.5.

6. Conclusions

In this paper, the problem of the extended dissipativity state estimation has been investigated for GNNs with mixed time delay signals. By proposing a novel LKFs and estimating their derivative by the WDII technique, RCC approach, and newly developed integral inequality (Lemma 2.5). A new extended dissipativity state estimation criterion is established in terms of LMIs. It has been shown that a desired state estimation gain matrix can be constructed with the given LMIs. Numerical simulations are given to show the effectiveness and usefulness of the proposed approach with a real-life benchmark problem that is associated with reasonable issues in the sense of an extended dissipativity analysis. The results are also compared with the existing methods to show the less conservativeness. Hence, the proposed results and methods can be extendable to many glorious dynamical models, such as control based filtering problems [48], Markovian jump NNs [19], event-triggering based state estimation problem [38], a memristor-based NNs [44], non-fragile state estimation problem [45], and synchronization based state estimation problem [47]. This will appear in the near future.

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Appendix A. Proof of Lemma 2.5

According to Lemma 2.4, we have

$$\begin{aligned}
& -\frac{h_M^2 - h_m^2}{2} \int_{-h_M}^{-h_m} \int_{t+\beta}^{t} \dot{\psi}^T(s) T \dot{\psi}(s) ds d\beta \\
= & -\frac{h_M^2 - h_m^2}{2} \int_{-h(t)}^{-h_m} \int_{t+\beta}^{t} \dot{\psi}^T(s) T \dot{\psi}(s) ds d\beta \\
& -\frac{h_M^2 - h_m^2}{2} \int_{-h_M}^{-h(t)} \int_{t+\beta}^{t} \dot{\psi}^T(s) T \dot{\psi}(s) ds d\beta \\
\leq & -\left[\frac{h_M^2 - h_m^2}{h^2(t) - h_m^2}\right] \left\{ \xi_1^T(t) T \xi_1(t) + 2\xi_2^T(t) T \xi_2(t) \right\} \\
& -\left[\frac{h_M^2 - h_m^2}{h_M^2 - h^2(t)}\right] \left\{ \xi_3^T(t) T \xi_3(t) + 2\xi_4^T(t) T \xi_4(t) \right\}
\end{aligned}$$

$$
\begin{aligned}
&= -\left[\frac{h_M^2-h_m^2}{h^2(t)-h_m^2}\right]\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix} - \left[\frac{h_M^2-h_m^2}{h_M^2-h^2(t)}\right]\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}\\
&= -\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix} - \frac{h_M^2-h^2(t)}{h^2(t)-h_m^2}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}\\
&\quad -\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix} - \frac{h^2(t)-h_m^2}{h_M^2-h^2(t)}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}.
\end{aligned}
\tag{15}
$$

If $\begin{bmatrix}\overline{T} & \mathcal{M}\\ * & \overline{T}\end{bmatrix} > 0$, the following relation is satisfied by the RCC method [30]:

$$
\begin{bmatrix}\sqrt{\frac{h_M^2-h^2(t)}{h^2(t)-h_m^2}}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}\\ -\sqrt{\frac{h^2(t)-h_m^2}{h_M^2-h^2(t)}}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}\end{bmatrix}^T\begin{bmatrix}\overline{T} & \mathcal{M}\\ \maltese & \overline{T}\end{bmatrix}\begin{bmatrix}\sqrt{\frac{h_M^2-h^2(t)}{h^2(t)-h_m^2}}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}\\ -\sqrt{\frac{h^2(t)-h_m^2}{h_M^2-h^2(t)}}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}\end{bmatrix} \geq 0,
\tag{16}
$$

which implies

$$
\begin{aligned}
&-\frac{h_M^2-h^2(t)}{h^2(t)-h_m^2}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix} - \frac{h^2(t)-h_m^2}{h_M^2-h^2(t)}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix} \leq -\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}^T\mathcal{M}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}\\
&-\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}^T\mathcal{M}^T\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}.
\end{aligned}
\tag{17}
$$

Then, we can get from (15) and (17) that

$$
\begin{aligned}
-\frac{h_M^2-h_m^2}{2}\int_{-h_M}^{-h_m}\int_{t+\beta}^{t}\dot{\psi}^T(s)T\dot{\psi}(s)dsd\beta &\leq -\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix} - \begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}^T\overline{T}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}\\
&\quad -\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}^T\mathcal{M}\begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix} - \begin{bmatrix}\xi_3(t)\\ \xi_4(t)\end{bmatrix}^T\mathcal{M}^T\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}\\
&= -\xi^T(t)\begin{bmatrix}\overline{T} & \mathcal{M}\\ \maltese & \overline{T}\end{bmatrix}\xi(t).
\end{aligned}
\tag{18}
$$

It should be notable that when $h^2(t) = h_m^2$ or $h^2(t) = h_M^2$, we have $\xi_1(t) = \xi_2(t) = 0$ or $\xi_3(t) = \xi_4(t) = 0$, respectively, and thus (18) still holds. This completes the proof. ■

Appendix B. Proof of Theorem 4.1

Choose the LKF candidate as follows:

$$
\mathbb{V}(e_t) = \sum_{n=1}^{6}\mathbb{V}_n(e_t),
\tag{19}
$$

where

$$
\begin{aligned}
\mathbb{V}_1(e_t) &= e^T(t)Pe(t),\\
\mathbb{V}_2(e_t) &= 2\sum_{i=1}^{n}\int_0^{\mathcal{W}_{3i}e(t)}\Big[h_{1i}\big(g_i(s)-k_i^-(s)\big)+h_{2i}\big(k_i^+(s)-g_i(s)\big)\Big]ds,\\
\mathbb{V}_3(e_t) &= \int_{t-h(t)}^{t}\eta^T(s)\mathcal{Q}_1\eta(s)ds + \int_{t-h_L}^{t}\eta^T(s)\mathcal{Q}_2\eta(s)ds + \int_{t-h_U}^{t}\eta^T(s)\mathcal{Q}_3\eta(s)ds,\\
\mathbb{V}_4(e_t) &= h_L\int_{-h_L}^{0}\int_{t+\beta}^{t}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}^T\begin{bmatrix}R_1^{11} & R_1^{12}\\ \maltese & R_1^{22}\end{bmatrix}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}dsd\beta\\
&\quad +(h_U-h_L)\int_{-h_U}^{-h_L}\int_{t+\beta}^{t}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}^T\begin{bmatrix}R_2^{11} & R_2^{12}\\ \maltese & R_2^{22}\end{bmatrix}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}dsd\beta,\\
\mathbb{V}_5(e_t) &= \frac{h_L^2}{2}\int_{-h_L}^{0}\int_{\beta}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)S_1\dot{e}(s)dsd\theta d\beta + \frac{h_U^2-h_L^2}{2}\int_{-h_U}^{-h_L}\int_{\beta}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)S_2\dot{e}(s)dsd\theta d\beta,\\
\mathbb{V}_6(e_t) &= \overline{\tau}\int_{-\overline{\tau}}^{0}\int_{t+\beta}^{t}g^T(\mathcal{W}_3(e(s)))Tg(\mathcal{W}_3(e(s)))dsd\beta,
\end{aligned}
$$

with $\eta(t)=\left[e^T(t) \quad g^T(\mathcal{W}_3 e(t))\right]^T$.

Taking the time derivative of $\mathbb{V}(e_t)$ along the trajectory of system (5), it gives that

$$\dot{\mathbb{V}}(e_t) = \sum_{n=1}^{6} \dot{\mathbb{V}}_n(e_t), \tag{20}$$

where

$$\begin{aligned}\dot{\mathbb{V}}_1(e_t) &= 2e^T(t)P\dot{e}(t)\\ &= \zeta^T(t)\Xi_1\zeta(t),\end{aligned} \tag{21}$$

$$\begin{aligned}\dot{\mathbb{V}}_2(e_t) &= 2\left[g(\mathcal{W}_3e(t)) - K_1\mathcal{W}_3e(t)\right]^T H_1\dot{e}(t) + 2\left[K_2\mathcal{W}_3e(t) - g(\mathcal{W}_3e(t))\right]^T H_2\dot{e}(t)\\ &= \zeta^T(t)\Xi_2\zeta(t),\end{aligned} \tag{22}$$

$$\begin{aligned}\dot{\mathbb{V}}_3(e_t) &\le \eta^T(t)(\mathcal{Q}_1+\mathcal{Q}_2+\mathcal{Q}_3)\eta(t) - (1-\mu)\eta^T(t-h(t))\mathcal{Q}_1\eta(t-h(t))\\ &\quad -\eta^T(t-h_L)\mathcal{Q}_2\eta(t-h_L) - \eta^T(t-h_U)\mathcal{Q}_3\eta(t-h_U)\\ &= \zeta^T(t)\Xi_3\zeta(t),\end{aligned} \tag{23}$$

$$\begin{aligned}\dot{\mathbb{V}}_4(e_t) &= h_L^2\left(e^T(t)R_1^{11}e(t) + 2e^T(t)R_1^{12}\dot{e}(t) + \dot{e}^T(t)R_1^{22}\dot{e}(t)\right) + (h_U-h_L)\left(e^T(t)R_2^{11}e(t)\right.\\ &\quad \left.+2e^T(t)R_2^{12}\dot{e}(t) + \dot{e}^T(t)R_2^{22}\dot{e}(t)\right) - h_L\left(e^T(t)R_1^{12}e(t) - e^T(t-h_L)R_1^{12}e(t-h_L)\right.\\ &\quad -(h_U-h_L)\left(e^T(t-h_L)R_2^{12}e(t-h_L) - e^T(t-h_U)R_2^{12}e(t-h_U)\right)\\ &\quad -h_L\int_{t-h_L}^{t} e^T(s)R_1^{11}e(s)ds - h_L\int_{t-h_L}^{t}\dot{e}^T(s)R_1^{22}\dot{e}(s)ds\\ &\quad -(h_U-h_L)\int_{t-h_U}^{t-h_L} e^T(s)R_2^{11}e(s)ds - (h_U-h_L)\int_{t-h_U}^{t-h_L}\dot{e}^T(s)R_2^{22}\dot{e}(s)ds.\end{aligned} \tag{24}$$

By using Lemma 2.1, it follows that

$$-h_L\int_{t-h_L}^{t} e^T(s)R_1^{11}e(s)ds \le -\left(\int_{t-h_L}^{t} e(s)ds\right)^T R_1^{11}\left(\int_{t-h_L}^{t} e(s)ds\right). \tag{25}$$

By using Lemma 2.2, it follows that

$$\begin{aligned}-h_L\int_{t-h_L}^{t}\dot{e}^T(s)R_1^{22}\dot{e}(s)ds &\le -\left(\int_{t-h_L}^{t}\dot{e}(s)ds\right)^T R_1^{22}\left(\int_{t-h_L}^{t}\dot{e}(s)ds\right)\\ &\le -\Big[e(t)-e(t-h_L)\Big]^T R_1^{22}\Big[e(t)-e(t-h_L)\Big].\end{aligned} \tag{26}$$

By using Lemmas 2.1 and 2.3, we obtain

$$-(h_U-h_L)\int_{t-h_U}^{t-h_L} e^T(s)R_2^{11}e(s)ds \le -\begin{bmatrix}\int_{t-h(t)}^{t-h_L} e(s)ds\\ \int_{t-h_U}^{t-h(t)} e(s)ds\end{bmatrix}^T \begin{bmatrix}R_2^{11} & \mathcal{M}\\ \maltese & R_2^{11}\end{bmatrix}\begin{bmatrix}\int_{t-h(t)}^{t-h_L} e(s)ds\\ \int_{t-h_U}^{t-h(t)} e(s)ds\end{bmatrix}. \tag{27}$$

By using Lemmas 2.2 and 2.3, we obtain

$$\begin{aligned}-(h_U-h_L)\int_{t-h_U}^{t-h_L}\dot{e}^T(s)R_2^{22}\dot{e}(s)ds &\le -\begin{bmatrix}e(t-h_L)-e(t-h(t))\\ e(t-h(t))-e(t-h_U)\end{bmatrix}^T\begin{bmatrix}R_2^{22} & \mathcal{N}\\ \maltese & R_2^{22}\end{bmatrix}\\ &\quad\times\begin{bmatrix}e(t-h_L)-e(t-h(t))\\ e(t-h(t))-e(t-h_U)\end{bmatrix}.\end{aligned} \tag{28}$$

From (24)–(28), it is concluded that

$$\dot{\mathbb{V}}_4(e_t) = \zeta^T(t)\Xi_4\zeta(t), \tag{29}$$

$$\begin{aligned}\dot{\mathbb{V}}_5(e_t) &= \dot{e}^T(t)\left\{\left(\frac{h_L^2}{2}\right)^2 S_1 + \left(\frac{h_U^2-h_L^2}{2}\right)^2 S_2\right\}\dot{e}(t) - \frac{h_L^2}{2}\int_{-h_L}^{0}\int_{t+\beta}^{t}\dot{e}^T(s)S_1\dot{e}(s)dsd\beta\\ &\quad -\frac{h_U^2-h_L^2}{2}\int_{-h_U}^{-h_L}\int_{t+\beta}^{t}\dot{e}^T(s)S_2\dot{e}(s)dsd\beta.\end{aligned} \tag{30}$$

To obtain new bounds we apply Lemma 2.4, we have

$$\begin{aligned}
&-\frac{h_L^2}{2}\int_{-h_L}^{0}\int_{t+\beta}^{t}\dot{e}^T(s)S_1\dot{e}(s)dsd\beta\\
&\leq-\begin{bmatrix} h_Le(t)-\int_{t-h_L}^{t}e(s)ds\\ -\frac{h_L}{2}e(t)-\int_{t-h_L}^{t}e(s)ds+\frac{3}{h_L}\int_{-h_L}^{0}\int_{t+\beta}^{t}e(s)dsd\beta\end{bmatrix}^T\begin{bmatrix}S_1 & 0\\ \maltese & 2S_1\end{bmatrix}\\
&\times\begin{bmatrix} h_Le(t)-\int_{t-h_L}^{t}e(s)ds\\ -\frac{h_L}{2}e(t)-\int_{t-h_L}^{t}e(s)ds+\frac{3}{h_L}\int_{-h_L}^{0}\int_{t+\beta}^{t}e(s)dsd\beta\end{bmatrix}.
\end{aligned}\tag{31}$$

By adopting Lemma 2.5, we have

$$\begin{aligned}
&-\frac{h_U^2-h_L^2}{2}\int_{-h_U}^{-h_L}\int_{t+\beta}^{t}\dot{e}^T(s)S_2\dot{e}(s)dsd\beta\\
&\leq-\frac{h^2(t)-h_L^2}{2}\int_{-h(t)}^{-h_L}\int_{t+\beta}^{t}\dot{e}^T(s)S_2\dot{e}(s)dsd\beta-\frac{h_U^2-h^2(t)}{2}\int_{-h_U}^{-h(t)}\int_{t+\beta}^{t}\dot{e}^T(s)S_2\dot{e}(s)dsd\beta\\
&\leq-\xi^T(t)\begin{bmatrix}\bar{S}_2 & \mathcal{M}\\ \maltese & \bar{S}_2\end{bmatrix}\xi(t),
\end{aligned}\tag{32}$$

where

$\xi(t)=[\xi_1^T(t)\quad \xi_2^T(t)\quad \xi_3^T(t)\quad \xi_4^T(t)]^T$,

$\xi_1(t)=(h(t)-h_L)e(t)-\int_{-h(t)}^{-h_L}e(s)ds$,

$\xi_2(t)=-\frac{(h(t)-h_L)}{2}e(t)-\int_{-h(t)}^{-h_L}e(s)ds+\frac{3}{h(t)-h_L}\int_{-h(t)}^{-h_L}\int_{t+\beta}^{t}e(s)dsd\beta$,

$\xi_3(t)=(h_U-h(t))e(t)-\int_{-h_U}^{-h(t)}e(s)ds$,

$\xi_4(t)=-\frac{(h_U-h(th))}{2}e(t)-\int_{-h_U}^{-h(t)}e(s)ds+\frac{3}{h_U-h(t)}\int_{-h_U}^{-h(t)}\int_{t+\beta}^{t}e(s)dsd\beta$.

From (30)–(32), it is concluded that

$$\dot{\mathbb{V}}_5(e_t)=\zeta^T(t)\Xi_5\zeta(t),\tag{33}$$

$$\dot{\mathbb{V}}_6(e_t)=\overline{\tau}^2g^T(\mathcal{W}_3e(t))Tg(\mathcal{W}_3e(t))-\overline{\tau}\int_{t-\overline{\tau}}^{t}g^T(\mathcal{W}_3(e(s)))Tg(\mathcal{W}_3(e(s)))ds.\tag{34}$$

Using Lemma 2.1, we have

$$-\overline{\tau}\int_{t-\overline{\tau}}^{t}g^T(\mathcal{W}_3(e(s)))Tg(\mathcal{W}_3(e(s)))ds\leq-\Big(\int_{t-\tau(t)}^{t}g(\mathcal{W}_3e(s))ds\Big)^TT\Big(\int_{t-\tau(t)}^{t}g(\mathcal{W}_3e(s))ds\Big).\tag{35}$$

From (34) and (35), it is concluded that

$$\dot{\mathbb{V}}_6(e_t)=\zeta^T(t)\Xi_6\zeta(t).\tag{36}$$

Furthermore, for any matrix N with compatible dimensions and scalar η, the following equation holds:

$$\begin{aligned}
0&=2\Big[e^T(t)+\eta\dot{e}^T(t)\Big]N\Big[-(\mathcal{A}+\mathcal{K}\mathcal{C})e(t)-\mathcal{K}\mathcal{D}e(t-h(t))+\mathcal{W}_0g(\mathcal{W}_3e(t))\\
&\quad+\mathcal{W}_1g(\mathcal{W}_3e(t-h(t)))+\mathcal{W}_2\int_{t-\tau(t)}^{t}g(\mathcal{W}_3(e(s)))ds+(\mathcal{B}_1-\mathcal{K}\mathcal{B}_2)\omega(t)-\dot{e}(t)\Big]\\
&=2\zeta^T(t)\Xi_7\zeta(t).
\end{aligned}\tag{37}$$

From Assumption (A1), it can be obtained that for any $\theta_{1i}\geq0,\theta_{2i}\geq0,\theta_{3i}\geq0,\ i=1,2,\ldots,n$.

$$\begin{aligned}
0&\leq\Big\{\Big[g_i(\mathcal{W}_{3i}e(t))-k_i^-\mathcal{W}_{3i}e(t)\Big]\Theta_{1i}\Big[k_i^+\mathcal{W}_{3i}e(t)-g_i(\mathcal{W}_{3i}e(t))\Big]\\
&\quad+\Big[g_i(\mathcal{W}_{3i}e(t-h(t)))-k_i^-\mathcal{W}_{3i}e(t-h(t))\Big]\Theta_{2i}\Big[k_i^+\mathcal{W}_{3i}e(t-h(t))-g_i(\mathcal{W}_{3i}e(t-h(t)))\Big]\\
&\quad+\Big[g_i(\mathcal{W}_{3i}e(t)-g_i(\mathcal{W}_{3i}e(t-h(t)))-k_i^-\mathcal{W}_{3i}(e(t)-e(t-h(t)))\Big]\Theta_{3i}\\
&\quad\times\Big[k_i^+\mathcal{W}_{3i}(e(t)-e(t-h(t)))-g_i(\mathcal{W}_{3i}e(t))+g_i(\mathcal{W}_{3i}e(t-h(t)))\Big]\Big\}\\
&=\zeta^T(t)\Xi_8\zeta(t).
\end{aligned}\tag{38}$$

Now combining from (19) to (38), we obtain

$$\dot{\mathbb{V}}(e_t) \le \zeta^T(t)\sum_{i=1}^{8}(\Xi_i)\zeta(t), \tag{39}$$

Now, we define the performance index

$$\begin{aligned} J(t) &= \tilde{y}^T(t)\Pi_1\tilde{y}(t) + 2\tilde{y}^T(t)\Pi_2\omega(t) + \omega^T(t)\Pi_3\omega(t) \\ &= \big[\mathcal{C}e(t) + \mathcal{D}e(t-h(t)) + \mathcal{B}_2\omega(t)\big]^T\Pi_1\big[\mathcal{C}e(t) + \mathcal{D}e(t-h(t)) + \mathcal{B}_2\omega(t)\big] \\ &\quad +2\big[\mathcal{C}e(t) + \mathcal{D}e(t-h(t)) + \mathcal{B}_2\omega(t)\big]^T\Pi_2 e_{17} + e_{17}^T\Pi_3 e_{17} \\ &= -\zeta^T(t)\Xi_9\zeta(t). \end{aligned} \tag{40}$$

Then, it follows that

$$\dot{\mathbb{V}}(e_t) - J(t) \le \zeta^T(t)\sum_{i=1}^{9}(\Xi_i)\zeta(t) = \zeta^T(t)\Xi\zeta(t) \le 0. \tag{41}$$

where Ξ is declared in Theorem 4.1. It is clear that if (8)–(12) are hold, then by the Lyapunov stability theory we can conclude that $\dot{\mathbb{V}}(e_t) - J(t) \le 0$. Then, applying Lemma 2.6, $(\Gamma^{\perp})^T\,\Xi\,(\Gamma^{\perp}) \le 0$ is equivalent to $\zeta^T(t)\Xi\zeta(t) \le 0$ with $\Gamma\zeta(t) = 0$. Then, it follows from (41) that, there exists a sufficiently small scalar $c > 0$, such that

$$\dot{\mathbb{V}}(e_t) - J(t) < -c \mid e(t) \mid^2 . \tag{42}$$

For any $t_f > 0$, integrating (42) from 0 to t_f gives

$$\int_0^{t_f}\big[\dot{\mathbb{V}}(e_t) - J(t)\big]dt \le 0, \tag{43}$$

which implies that

$$\int_0^{t_f} J(t)dt \ge \mathbb{V}(t_f, e_{t_f}) - \mathbb{V}(0) \ge 0. \tag{44}$$

Assuming the zero initial condition yields

$$\int_0^{t_f} J(t)dt \ge \mathbb{V}(e_{t_f}) \ge e^T(t_f)Pe(t_f). \tag{45}$$

To prove the error system (5) is extended dissipative, Definition 2.1 should be satisfied. To satisfy (41), the following inequality should be hold:

$$\int_0^{t_f} J(t)dt \ge \sup_{0\le t\le t_f}\big(\tilde{y}^T(s)\Pi_4\tilde{y}(s)\big). \tag{46}$$

According to Assumption (A2), on one hand, the extended dissipativity criterion can describe the $\mathcal{H}_\infty$ performance, the strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity condition and the passivity while $\Pi_4 = 0$; on the other hand, it can express the $\mathcal{L}_2 - \mathcal{L}_\infty$ performance while $\Pi_4 > 0$. Thus, the next discussion should be divided into two cases.

(i) While $\Pi_4 = 0$,

$$\int_0^{t_f} J(t)dt \ge 0 = \sup_{0\le t\le t_f}\big(\tilde{y}^T(s)\Pi_4\tilde{y}(s)\big), \tag{47}$$

holds for any $t_f \ge 0$, then (45) holds. Therefore, the error system (5) can be deduced to perform $\mathcal{H}_\infty$, $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity and the passivity performance.

(ii) While $\Pi_4 > 0$, due to Assumption (A2), it implies that matrices $\Pi_1 = 0$, $\Pi_2 = 0$, $\Pi_3 > 0$, $\mathcal{B}_2 = 0$ and $J(t) = \omega^T(t)\Pi_3\omega(t) \ge 0$ hold. Then for any $0 \le t \le t_f$ and $0 \le t - h(t) \le t_f$, (45) implies that $\int_0^{t_f} J(t)dt \ge \int_0^{t} J(t)dt \ge \mathbb{V}(e_t) \ge e^T(t)Pe(t)$ and $\int_0^{t_f} J(t)dt \ge \int_0^{t-h(t)} J(t)dt \ge \mathbb{V}(t-h(t), e_t - h(t)) \ge e^T(t-h(t))Pe(t-h(t))$, respectively, while for $t - h(t) \le 0$, it can be proved that $e^T(t-h(t))Pe(t-h(t)) \le \| P \| \mid x(t-h(t)) \mid^2 \le \| P \| \sup_{-h^* \le s \le 0} \mid \phi(s) \mid^2 \le 0 \le \int_0^{t_f} J(t)dt$. Thus, there exists a scalar $0 < \varrho < 1$ such that

$$\int_0^{t_f} J(t)dt \ge \varrho e^T(t)Pe(t) + (1-\varrho)e^T(t-h(t))Pe(t-h(t)). \tag{48}$$

According to (9), it is easy to obtain

$$\begin{aligned} \tilde{y}^T(s)\Pi_4\tilde{y}(s) &= -\begin{bmatrix} e(t) \\ e(t-h(t)) \end{bmatrix}^T \Sigma \begin{bmatrix} e(t) \\ e(t-h(t)) \end{bmatrix} + \varrho e^T(t)Pe(t) + (1-\varrho)e^T(t-h(t))Pe(t-h(t)) \\ &\le \varrho e^T(t)Pe(t) + (1-\varrho)e^T(t-h(t))Pe(t-h(t)). \end{aligned} \tag{49}$$

According to (48)–(49), it is clear that (46) holds for any t satisfying $0 \le t \le t_f$. This completes the proof. ■

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