



## Editorial to the special issue on high order fuzzy sets

High order fuzzy sets can be used to build intelligent systems for achieving different goals in real-world applications. High order fuzzy sets include interval type-2 fuzzy sets, generalized fuzzy sets and other generalizations of traditional type-1 fuzzy sets. High order fuzzy sets have been theoretically proposed to model more closely the uncertainty present in real-world problems. This special issue consists of eight papers that consider models of high order fuzzy sets for different types of problems. The eight papers, of this special issue, describe methods and applications of high order fuzzy sets to real-world problems and can be considered a significant contribution to the field of fuzzy sets and systems.

The first paper, “The Collapsing Method of Defuzzification for Discretised Interval Type-2 Fuzzy Sets”, by Sarah Greenfield et al. proposes a new approach for defuzzification of interval type-2 fuzzy sets. The collapsing method converts an interval type-2 fuzzy set into a type-1 *representative embedded set* (RES), whose defuzzified values closely approximates that of the type-2 set. As a type-1 set, the RES can then be defuzzified straightforwardly. The novel *Representative Embedded Set Approximation* (RESA), to which the method is inextricably linked, is expounded, stated and proved within this paper. It is presented in two forms: (1) Simple RESA: This approximation deals with the most simple interval FOU, in which a vertical slice is discretised into two points. (2) Interval RESA: This approximation concerns the case in which a vertical slice is discretised into two or more points. The collapsing method (Simple RESA version) was tested for accuracy and speed, with excellent results on both criteria. The collapsing method proved more accurate than the Karnik–Mendel Iterative Procedure (KMIP) for an asymmetric test set. For both a symmetric and an asymmetric test set, the collapsing method outperformed the KMIP in relation to speed.

The second paper, “Ranking L–R fuzzy number based on deviation degree”, by Zhong-Xing Wang et al. proposes a novel approach to ranking fuzzy numbers based on the left and right deviation degree (L–R deviation degree). In the approach, the maximal and minimal reference sets are defined to measure L–R deviation degree of fuzzy number, and then the transfer coefficient is defined to measure the relative variation of L–R deviation degree of fuzzy number. Furthermore, the ranking index value is obtained based on the L–R deviation degree and relative variation of fuzzy numbers. Additionally, to compare the proposed approach with the existing approaches, five numerical examples are used. The comparative results illustrate that the approach proposed in this paper is simpler and better.

The third paper, “A Hybrid Approach for Image Recognition Combining Type-2 Fuzzy Logic, Modular Neural Networks and the Sugeno Integral”, by Olivia Mendoza et al. provides a hybrid approach for image recognition combining type-2 fuzzy logic, modular neural networks and the Sugeno integral. Interval Type-2 Fuzzy Inference Systems are used to perform edge detection and to calculate fuzzy densities for the decision process. A type-2 fuzzy system is used for edge detection, which is a preprocessing applied to the training data for better use in the neural networks. Another type-2 fuzzy system calculates the fuzzy densities necessary for the Sugeno integral, which is used to integrate results of the neural network modules. In this case, fuzzy logic is shown to be a good methodology to improve the results of a neural system facilitating the representation of the human perception. A comparative study is also made to verify that the proposed approach is better than existing approaches and improves the performance over type-1 fuzzy logic.

The fourth paper, “Interval type-2 fuzzy membership function generation methods for pattern recognition”, by Byung-In Choi and Frank Chung-Hoon Rhee, provides methods for generating interval type-2 membership functions. Type-2 fuzzy sets (T2 FSs) have been shown to manage uncertainty more effectively than T1 fuzzy sets (T1 FSs) in several areas of engineering. However, computing with T2 FSs can require undesirably large amount of computations since it involves numerous embedded T2 FSs. To reduce the complexity, interval type-2 fuzzy sets (IT2 FSs) can be used, since the secondary memberships are all equal to one. In this paper, three novel interval type-2 fuzzy membership function (IT2 FMF) generation methods are proposed. The methods are based on heuristics, histograms, and interval type-2 fuzzy C-means. The performance of the methods is evaluated by applying them to back-propagation neural networks (BPNNs). Experimental results for several data sets are given to show the effectiveness of the proposed membership assignments.

The fifth paper, “Type-2 Fuzzy Inference Systems as Integration Methods in Modular Neural Networks for Multimodal Biometry and its Optimization with Genetic Algorithms”, by Denisse Hidalgo et al. describes a comparative study between

fuzzy inference systems as methods of integration in modular neural networks for multimodal biometry. These methods of integration are based on techniques of type-1 fuzzy logic and type-2 fuzzy logic. Also, the fuzzy systems are optimized with simple genetic algorithms with the goal of having optimized versions of both types of fuzzy systems. First, the use of type-1 fuzzy logic and later the approach with type-2 fuzzy logic are considered. The fuzzy systems were developed using genetic algorithms to handle fuzzy inference systems with different membership functions, like the triangular, trapezoidal and Gaussian; since these algorithms can generate fuzzy systems automatically. Then the response integration of the modular neural network was tested with the optimized fuzzy systems of integration. The comparative study of the type-1 and type-2 fuzzy inference systems was made to observe the behavior of the two different integration methods for modular neural networks for multimodal biometry.

The sixth paper, “Hybrid Learning for Interval Type-2 Fuzzy Logic Systems Based on Orthogonal Least-Squares and Back Propagation Methods”, by Gerardo Maximiliano Mendez and M. de los Angeles Hernandez presents a novel learning methodology based on a hybrid algorithm for interval type-2 fuzzy logic systems. Since only the back-propagation method has been proposed in the literature for the tuning of both the antecedent and the consequent parameters of type-2 fuzzy logic systems, a hybrid learning algorithm has been developed. The hybrid method uses a recursive orthogonal least-squares method for tuning the consequent parameters and the back-propagation method for tuning the antecedent parameters. Systems were tested for three types of inputs: (a) interval singleton, (b) interval type-1 non-singleton, and (c) interval type-2 non-singleton. Experiments were carried out on the application of hybrid interval type-2 fuzzy logic systems for prediction of the scale breaker entry temperature in a real hot strip mill for three different types of coil. The results proved the feasibility of the systems developed here for scale breaker entry temperature prediction. Comparison with type-1 fuzzy logic systems shows that hybrid-learning interval type-2 fuzzy logic systems provide improved performance under the conditions tested.

The seventh paper, “Optimization of Interval Type-2 Fuzzy Logic Controllers for a Perturbed Autonomous Wheeled Mobile Robot Using Genetic Algorithms”, by Ricardo Martinez et al. describes a tracking controller for the dynamic model of a unicycle mobile robot by integrating a kinematic and a torque controller based on interval type-2 fuzzy logic theory and genetic algorithms. Computer simulations are presented confirming the performance of the tracking controller and its application to different navigation problems. The results of the interval type-2 fuzzy logic controller are shown to outperform the type-1 fuzzy logic controller.

The eighth paper, “A Hybrid Learning Algorithm for a Class of Interval Type-2 Fuzzy Neural Networks”, by Juan R. Castro et al., describes type-2 fuzzy neural system models with a detailed mathematical derivation of the learning algorithms. In this paper, a class of Interval Type-2 Fuzzy Neural Networks (IT2FNN) is proposed, which is functionally equivalent to interval type-2 fuzzy inference systems. The computational process envisioned for fuzzy-neural systems is as follows: it starts with the development of the “Interval Type-2 Fuzzy Neuron”, which is based on biological neural morphologies, followed by the learning mechanisms. It is described how to decompose the parameter set such that the hybrid learning rule of adaptive networks can be applied to the IT2FNN architecture for the Takagi-Sugeno-Kang reasoning.

In conclusion, it can be stated that all of the papers, in this special issue, make an important contribution to the state of the art in the field of fuzzy sets and systems and also to the areas of application, such as pattern recognition, intelligent control, manufacturing and robotics.

Oscar Castillo  
Patricia Melin

*Division of Graduate Studies and Research,  
Tijuana Institute of Technology,  
22500 Tijuana, Mexico*

*E-mail address: [ocastillo@hafsamx.org](mailto:ocastillo@hafsamx.org) (Oscar Castillo)*