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# Ranking L–R fuzzy number based on deviation degree

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ABSTRACT

This paper proposed a novel approach to ranking fuzzy numbers based on the left and right deviation degree (*L–R* deviation degree). In the approach, the maximal and minimal reference sets are defined to measure *L–R* deviation degree of fuzzy number, and then the transfer coefficient is defined to measure the relative variation of *L–R* deviation degree of fuzzy number. Furthermore, the ranking index value is obtained based on the *L–R* deviation degree and relative variation of fuzzy numbers. Additionally, to compare the proposed approach with the existing approaches, five numerical examples are used. The comparative results illustrate that the approach proposed in this paper is simpler and better.


## 1. Introduction

In decision analysis under the fuzzy environment, ranking fuzzy numbers is a very important decision-making procedure [12,27]. *L–R* fuzzy number as the most general form of fuzzy number has been used extensively, as Ref. [19] pointed out. A key issue in operationalzing fuzzy set theory is how to compare fuzzy numbers. Various approaches have been developed for raking fuzzy numbers [1,2,5,14,16,21,29,30]. Some of these ranking approaches have compared and reviewed by Bortolan and Degani [3]. Recently, Chen and Hwang [5] thoroughly reviewed the existing approaches to ranking fuzzy numbers and pointed out that some illogical conditions arose among them. Moreover, Wang and Kerre [22] proposed some axioms as reasonable properties to determine the rationality of a fuzzy number ranking approach and systematically compared a wide array of the existing fuzzy number ranking methods. Almost each approach, however, has pitfalls in some aspect, such as inconsistency with human intuition, indiscrimination, and difficulty of interpretation. So far, none of them is commonly accepted.

In the existing research, the commonly used technique is firstly to construct proper maps to transform fuzzy numbers into real numbers. These real numbers are then compared. Herein, in approaches [1,2,9–11,16,18–21,28–31], a fuzzy number is mapped to a real number based on the area measurement. In approaches [6–8,17], $\alpha$-cut set and decision-maker's preference are used to construct ranking function. On the other hand, another commonly used technique is the centroid-based fuzzy number ranking approach. Yager [26] proposed a centroid index ranking approach with weighting function. Nevertheless, as Cheng [4] pointed out, only $x$ value on the horizontal axis for each fuzzy number is calculated Yager's approach. When two fuzzy numbers have the same $x$ value on the horizontal axis and different $y$ value on the vertical axis, it failed. Consequently, Cheng [4] proposed a centroid index ranking approach, where the distance of the centroid point of each fuzzy number and original point are calculated to improve Yager's approach. Lee and Li [15] conducted the comparison of fuzzy

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numbers, in which the mean and standard deviation values based on the uniform and proportional probability distributions are used to rank fuzzy numbers. Also, Cheng [4] defined the coefficient variance (CV) index, i.e., CV = $\sigma$(standard error)/$|\mu|$(mean), $\mu \neq 0$, $\sigma > 0$, to improve Lee and Li's ranking approach. However, distance index usually contradicts the CV index both given by Cheng on ranking fuzzy numbers (see Examples 1 and 2 in Section 4 of this paper). To overcome these above limitations, Chu and Tsao [9] proposed an approach to ranking fuzzy number with the area between the centroid and original points. But their approach still has some drawbacks (see the after-mention details in Example 3). Furthermore, Wang et al. [24] found that the centroid formulae for ranking fuzzy numbers provided by Cheng [4] is incorrect and leads to some misapplications in Ref. [9]. They corrected the centroid formulae for ranking fuzzy numbers and justified them from the viewpoint of analytical geometry. After that, Wang and Lee [23] made a revision on ranking fuzzy numbers with an area between the centroid and original points to improve Chu and Tsao's approach [9]. Though some improvements are made, Wang and Lee's approach cannot still differentiate two fuzzy number with the same centroid point (see Example 3). Recently, Wang et al. [25] proposed an approach to ranking fuzzy numbers based on lexicographic screening procedure and summarized some limitations of the existing methods (for more details, see Table 2 in Ref. [25]). However, Wang et al.'s approach can not differentiate these kinds of fuzzy numbers as shown in Example 3 in Section 4 of this paper, as most of the existing approaches do. The ranking results are counter-intuition.

To overcome the limitations of the existing studies and simplify the computational procedures, in this paper, an approach to ranking $L$–$R$ fuzzy numbers based on deviation degree is proposed. In the approach, the maximal and minimal reference sets are constructed to measure the $L$–$R$ deviation degree of fuzzy number. Then, the transfer coefficient is defined to measure the relative variation of the $L$–$R$ deviation degree of fuzzy number. Furthermore, ranking index value is obtained based on $L$–$R$ deviation degree and the transfer coefficient. Finally, fuzzy numbers are ranked based on the derived ranking index values.

The rest of this paper is organized as follows. Section 2 introduces basic concepts and definitions of fuzzy numbers. Section 3 proposes an approach to ranking $L$–$R$ fuzzy numbers based on deviation degree. Section 4 presents five numerical examples to illustrate the advantages of the proposed approach. Lastly, Section 5 summarizes and highlights the main features of this paper.

## 2. Preliminaries

For the convenience of analysis, some basic concepts and definitions on fuzzy number are needed. They are stated as follows [8,25].

Let $X$ be a universe set. A fuzzy subset $A$ of $X$ is defined with a membership function $\mu_A(x)$ that maps each element $x$ in $A$ to a real number in the interval $[0,1]$. The function value of $\mu_A(x)$ signifies the grade of membership of $x$ in $A$. When $\mu_A(x)$ is large, its grade of membership of $x$ in $A$ is strong [13,32].

**Definition 1.** A $L$–$R$ fuzzy number $A = (m,n,\alpha,\beta)_{LR}$, $m \leqslant n$ and $\alpha$, $\beta \geqslant 0$, is defined as follows (see [25]):

$$\mu_A(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & \text{if } -\infty < x < m, \\ 1, & \text{if } m \leqslant x \leqslant n, \\ R\left(\frac{x-n}{\beta}\right), & \text{if } n < x < +\infty, \end{cases} \tag{1}$$

where $\alpha$ and $\beta$ are the left-hand and the right-hand spreads, respectively. In the closed interval $[m,n]$, the membership function is equal to 1. $L\left(\frac{m-x}{\alpha}\right)$ and $R\left(\frac{x-n}{\beta}\right)$ are non-increasing functions with $L(0) = 1$ and $R(0) = 1$, respectively. For convenience, they are, respectively, denoted as $\mu_{A^L}(x)$ and $\mu_{A^R}(x)$. It needs to point out that when $L\left(\frac{m-x}{\alpha}\right)$ and $R\left(\frac{x-n}{\beta}\right)$ are linear functions and $m < n$, fuzzy number $A$ denotes trapezoidal fuzzy number. When $L\left(\frac{m-x}{\alpha}\right)$ and $R\left(\frac{x-n}{\beta}\right)$ are linear functions and $m = n$, fuzzy number $A$ denotes triangular fuzzy number.

**Definition 2.** For fuzzy set $A$, the support set of $A$ is defined as follows (see Ref. [8]):

$$S(A) = \{x \in R | \mu_A(x) > 0\}.$$

## 3. The proposed approach to raking fuzzy numbers

In this section, firstly, the minimal and maximal reference sets are defined. Then, the $L$–$R$ deviation degree is defined based on the minimal and maximal reference sets. Moreover, the transfer coefficient is constructed to measure the relative variation of $L$–$R$ deviation degree of fuzzy number. Additionally, we will summarize the algorithm of the proposed approach for ranking fuzzy numbers.

**Definition 3.** For an arbitrary group of $L$–$R$ fuzzy numbers, $A_1, A_2, \ldots, A_n$, let $x_{\min}$ and $x_{\max}$ be the infimum and supremum of support set of these fuzzy numbers, respectively. Then, $A_{\min}$ and $A_{\max}$ are the minimal reference set and maximal reference set (the minimal and maximal reference sets, thereafter) of these fuzzy numbers, respectively, and their membership functions is given by

$$\mu_{A_{\min}}(x) = \begin{cases} \frac{x_{\max}-x}{x_{\max}-x_{\min}}, & \text{if } x \in S, \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

and

$$\mu_{A_{\max}}(x) = \begin{cases} \frac{x-x_{\min}}{x_{\max}-x_{\min}}, & \text{if } x \in S, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where $S$ is the support set of these fuzzy numbers, i.e., $S = \bigcup_{i=1}^{n} S(A_i)$.

**Definition 4.** For a group of $L$–$R$ fuzzy numbers, $A_1, A_2, \ldots, A_n$, let $A_{\max}$ and $A_{\min}$ be the maximal and minimal reference sets of these fuzzy numbers, respectively. Then, the left deviation degree and right deviation degree of $A_i$ ($i \in \{1, 2, \ldots, n\}$) are defined as follows (see Fig. 1):

$$d_i^L = \int_{x_{\min}}^{a_i} (\mu_{A_{\min}}(x) - \mu_{A_i^L}(x))\mathrm{d}x \tag{4}$$

and

$$d_i^R = \int_{b_i}^{x_{\max}} (\mu_{A_{\max}}(x) - \mu_{A_i^R}(x))\mathrm{d}x, \tag{5}$$

where $a_i$ is the abscissa of the crossover point of $\mu_{A_i^L}(x)$ and $\mu_{A_{\min}}(x)$, and $b_i$ is that of $\mu_{A_{\max}}(x)$ and $\mu_{A_i^R}(x)$, $i = 1, 2, \ldots, n$.

**Definition 5** [20]. For a $L$–$R$ fuzzy number $A_i = (m_i, n_i, \alpha_i, \beta_i)_{LR}$, its expectation value of centroid is defined as follows:

$$M_i = \frac{\int_{m_i-\alpha_i}^{n_i+\beta_i} x\mu_{A_i}(x)\mathrm{d}x}{\int_{m_i-\alpha_i}^{n_i+\beta_i} \mu_{A_i}(x)\mathrm{d}x}. \tag{6}$$

For an arbitrary group of $L$–$R$ fuzzy numbers, $A_1, A_2, \ldots, A_n$, if a fuzzy number is closer to the maximal reference set and is father away from the minimal reference set, then the fuzzy number is larger. In other words, the larger the left deviation degree and the smaller the right deviation degree are, the larger the fuzzy number is. The relative closeness coefficient of a fuzzy number with respect to the minimal and maximal reference sets is given by

$$d_i^* = \frac{d_i^L}{1 + d_i^R}, \quad i = 1, 2, \ldots, n, \tag{7}$$

where the larger $d_i^*$ is, the larger the corresponding fuzzy number $A_i$ is.

In order to reflect the relative variation of $L$–$R$ deviation degree of fuzzy number, we employ the expectation value of centroid of fuzzy number to construct the transfer coefficient. For fuzzy numbers, $A_1, A_2, \ldots, A_n$, the transfer coefficient of $A_i$ ($i \in \{1, 2, \ldots, n\}$) is given by

$$\lambda_i = \frac{M_i - M_{\min}}{M_{\max} - M_{\min}}, \tag{8}$$

where $M_{\max} = \max\{M_1, M_2, \ldots, M_n\}$, $M_{\min} = \min\{M_1, M_2, \ldots, M_n\}$ and $M_{\max} \neq M_{\min}$.

Based on Eqs. (7) and (8), the ranking index value of fuzzy number $A_i$ ($i \in \{1, 2, \ldots, n\}$) is given by

$$d_i = \begin{cases} \frac{d_i^L \lambda_i}{1 + d_i^R(1-\lambda_i)}, & M_{\max} \neq M_{\min}, \quad i = 1, 2, \ldots, n, \\ \frac{d_i^L}{1 + d_i^R}, & M_{\max} = M_{\min}, \quad i = 1, 2, \ldots, n. \end{cases} \tag{9}$$

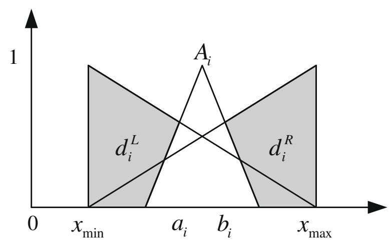


**Fig. 1.** The left and right deviation degrees of $A_i$.

For any two $L$–$R$ fuzzy numbers, $A_i$ and $A_j$, the ranking order is determined based on the following rules:

(1) $A_i \succ A_j$, if and only if $d_i > d_j$.
(2) $A_i \prec A_j$, if and only if $d_i < d_j$.
(3) $A_i \sim A_j$, if and only if $d_i = d_j$.

Assume that there are $n$ different $L$–$R$ fuzzy numbers, $A_1, A_2, \ldots, A_n$. $S$ is the support set of these fuzzy numbers. Let $A_i$, $A_j$ and $A_k$ be any three arbitrary $L$–$R$ fuzzy numbers, $i \neq j \neq k$ and $1 \leqslant i,j,k \leqslant n$. Then, there are the following properties (refer to Ref. [22]):

(1) If $A_i \succ A_j$ and $A_j \succ A_k$, then $A_i \succ A_k$.
(2) If $A_i \succ A_j$, then $A_j \succ A_i$ cannot hold.
(3) If $\inf S(A_i) \geqslant \sup S(A_j)$, then $A_i \succ A_j$.
(4) If $A_i \succsim A_j$ and $A_j \succsim {}_{\sim} A_i$, then $A_i \sim A_j$.
(5) If$A_i \succsim A_j$, then $A_i' \succsim A_j'$, where $A_i' = (m_i + k, n_i + k, \alpha_i, \beta_i)_{LR}$ and $A_j' = (m_j + k, n_j + k, \alpha_j, \beta_j)_{LR}$ are defined on $S$, and $k$ is a real number.

In sum, the algorithm for raking $n$ $L$–$R$ fuzzy numbers, $A_1, A_2, \ldots, A_n$, based on deviation degree is given as follows:

*Step 1*. According to Eqs. (2) and (3), the membership functions of minimal and maximal reference sets, $\mu_{A_{\min}}(x)$ and $\mu_{A_{\max}}(x)$, are determined.
*Step 2*. Through Eqs. (4) and (5), left deviation degree $d_i^L$ and right deviation degree $d_i^R$ of each $L$–$R$ fuzzy number are obtained, respectively.
*Step 3*. According to Eq. (7), relative closeness coefficient $d_i^*$ is obtained.
*Step 4*. By Eqs. (6) and (8), transfer coefficient $\lambda_i$ is obtained.
*Step 5*. According to Eq. (9), the ranking index value $d_i$ is obtained.
*Step 6*. Based on the ranking index values $d_1, d_2, \ldots, d_n$, the ranking order of a group of $L$–$R$ fuzzy numbers $A_1, A_2, \ldots, A_n$ are determined.

## 4. Numerical examples

In this section, five numerical examples are used to illustrate the proposed approach to ranking $L$–$R$ fuzzy numbers.

**Example 1.** Consider the data used in Ref. [9], i.e., two fuzzy numbers, $A_1 = (2,2,0.1,0.1)_{LR}$ and $A_2 = (3,3,0.9,1)_{LR}$, as shown in Fig. 2.

In order to rank the two fuzzy numbers $A_1$ and $A_2$, according to Eqs. (2)–(9), the ranking index values are obtained, i.e., $d_1 = 0$ and $d_2 = 0.5714$. Therefore, the ranking order of fuzzy numbers is $A_2 \succ A_1$. From Fig. 2, it can be seen that the result obtained by our approach is consistent with human intuition. However, by the CV index proposed by Cheng [4], the ranking order is $A_1 \succ A_2$, which is unreasonable.

**Example 2.** Consider the data used in Ref. [1], i.e., the three fuzzy numbers, $A_1 = (6,6,1,1)_{LR}$, $A_2 = (6,6,0.1,1)_{LR}$ and $A_3 = (6,6,0,1)_{LR}$, as shown in Fig. 3.

According to Eqs. (2)–(9), the ranking index values are obtained, i.e., $d_1 = 0$, $d_2 = 0.5047$ and $d_3 = 0.75$. Accordingly, the ranking order of fuzzy numbers is $A_3 \succ A_2 \succ A_1$. However, by Chu and Tsao's approach [9], the ranking order is $A_2 \succ A_3 \succ A_1$. Meanwhile, using CV index proposed by Cheng [4], the raking order is $A_1 \succ A_2 \succ A_3$. From Fig. 3, it is easy to see that the ranking results obtained by the existing approaches [4,9] are unreasonable and are not consistent with human intuition. On the

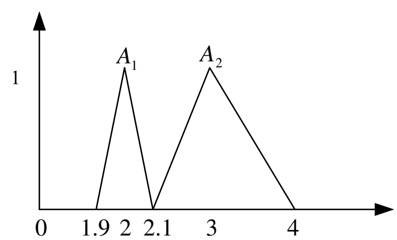


**Fig. 2.** Fuzzy numbers in Example 1.

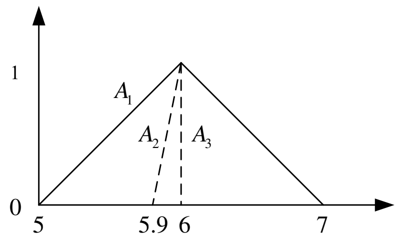


**Fig. 3.** Fuzzy numbers in Example 2.

**Table 1**
Comparative results of Example 2

| Fuzzy number | The proposed method | Sign distance $p = 1$ | Sign distance $p = 2$ | Chu–Tsao | Cheng distance | CV index |
|---|---|---|---|---|---|---|
| $A_1$ | 0.25 | 6.12 | 8.52 | 3 | 6.021 | 0.028 |
| $A_2$ | 0.5339 | 12.45 | 8.82 | 3.126 | 6.349 | 0.0098 |
| $A_3$ | 0.5625 | 12.5 | 8.85 | 3.085 | 6.3519 | 0.0089 |
| Results | $A_3 \succ A_2 \succ A_1$ | $A_3 \succ A_2 \succ A_1$ | $A_3 \succ A_2 \succ A_1$ | $A_2 \succ A_3 \succ A_1$ | $A_3 \succ A_2 \succ A_1$ | $A_1 \succ A_2 \succ A_3$ |

other hand, by Abbasbandy and Asady's approach [1], the ranking result is $A_3 \succ A_2 \succ A_1$, which is the same as the one obtained by our approach. However, our approach is simpler in the computation procedure. Based on the analysis results from Ref. [1], the ranking results using our approach and other approaches are listed in Table 1.

**Example 3.** Consider the data used in Ref. [7], i.e., two fuzzy numbers, $A_1 = (6,6,3,3)_{LR}$ and $A_2 = (6,6,1,1)_{LR}$, as shown in Fig. 4.

Through the proposed approach in this paper, the ranking index values can be obtained as $d_1 = 0.1429$ and $d_2 = 0.1567$. Then, the ranking order of fuzzy numbers is $A_2 \succ A_1$. Because fuzzy numbers $A_1$ and $A_2$ have the same mode and symmetric spread, most of existing approaches fail. For instance, by Abbasbandy and Asady's approach [1], different ranking orders are obtained when different index values ($p$) are taken. When $p = 1$ and $p = 2$, the ranking order of fuzzy numbers is $A_1 \sim A_2$ and $A_1 \succ A_2$, respectively. Meanwhile, using the approaches in Refs. [2,9,23,30], the ranking order is same, i.e., $A_1 \sim A_2$. Nevertheless, inconsistent results produced when distance index and CV index of Cheng's approach [4] are respectively used. Moreover, the ranking order obtained by Wang et al.'s approach [25] is $A_1 \succ A_2$. Tran–Duckein [21] obtains conflict results when $D_{\max}$ and $D_{\min}$ are respectively used. Additionally, by the approaches provided in Refs. [18,19,29], different ranking order is obtained when different indices of optimism are taken. However, decision makers prefer to the result $A_2 \succ A_1$ intuitionally.

**Example 4.** Considering the two sets from Yao and Wu [30], i.e.,

Set 1 (see Fig. 5): $A_1 = (0.5,0.5,0.1,0.5)_{LR}$, $A_2 = (0.7,0.7,0.3,0.3)_{LR}$ and $A_3 = (0.9,0.9,0.5,0.1)_{LR}$;
Set 2 (see Fig. 6): $A_1 = (0.4,0.7,0.1,0.2)_{LR}$, $A_2 = (0.7,0.7,0.4,0.2)_{LR}$ and $A_3 = (0.7,0.7,0.2,0.2)_{LR}$.

By the approach proposed in this paper, the ranking index values of Set 1 can be obtained as $d_1 = 0$, $d_2 = 0.0476$ and $d_3 = 0.1364$. Then, the ranking order of fuzzy numbers is $A_3 \succ A_2 \succ A_1$. As to Set 2, the ranking index values are $d_1 = 0$, $d_2 = 0.0246$ and $d_3 = 0.2$. The ranking order is $A_3 \succ A_2 \succ A_1$. Based on the analysis results from Refs. [1,2], the computation results using our approach and other ones are given in Table 2. With regard to Set 1, by the CV index approach in [4], the ranking order is $A_1 \succ A_3 \succ A_2$, which is counter-intuition (see Fig. 5). Meanwhile, if use the approaches in Refs. [3,9,21,30] and the distance index approach provided in Ref. [4], the ranking order is $A_3 \succ A_2 \succ A_1$. Although the ranking result is the same as the one by our approach, the computation procedure of ours is simpler than others. Meanwhile, by the approach

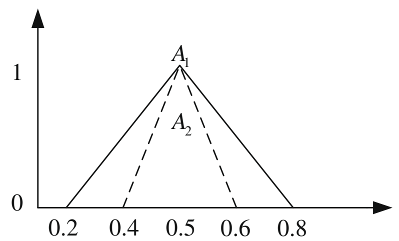


**Fig. 4.** Fuzzy numbers in Example 3.

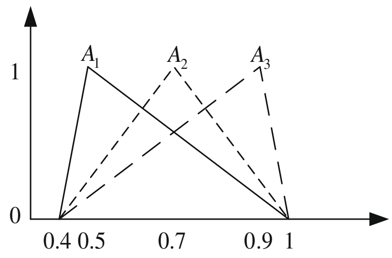


**Fig. 5.** Fuzzy numbers in Example 4.

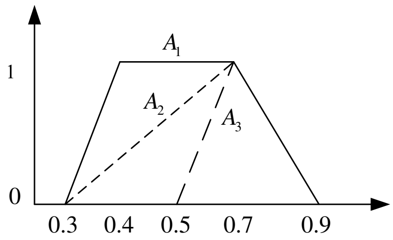


**Fig. 6.** Fuzzy numbers in Example 4.

**Table 2**
Comparative results of Example 4

| Author | Fuzzy number | | | | | |
|---|---|---|---|---|---|---|
| | Set 1 | | | Set 2 | | |
| | $A_1$ | $A_2$ | $A_3$ | $A_1$ | $A_2$ | $A_3$ |
| Bortolan–Degani | 0.30 | 0.33 | 0.44 | 0.27 | 0.27 | 0.37 |
| | $A_3 \succ A_2 \succ A_1$ | | | $A_3 \succ A_2 \sim A_1$ | | |
| Chu–Tsao | 0.2999 | 0.350 | 0.3993 | 0.2847 | 0.32478 | 0.350 |
| Results | $A_3 \succ A_2 \succ A_1$ | | | $A_3 \succ A_2 \succ A_1$ | | |
| Yao–Wu | 0.6 | 0.7 | 0.8 | 0.575 | 0.65 | 0.7 |
| Results | $A_3 \succ A_2 \succ A_1$ | | | $A_3 \succ A_2 \succ A_1$ | | |
| Sign distance ($p = 1$) | 1.2 | 1.4 | 1.6 | 1.15 | 1.3 | 1.4 |
| Results | $A_3 \succ A_2 \succ A_1$ | | | $A_3 \succ A_2 \succ A_1$ | | |
| Sign distance ($p = 2$) | 0.8869 | 1.0194 | 1.1605 | 0.8756 | 0.9522 | 1.0033 |
| Results | $A_3 \succ A_2 \succ A_1$ | | | $A_3 \succ A_2 \succ A_1$ | | |
| Cheng distance | 0.79 | 0.8602 | 0.9268 | 0.7577 | 0.8149 | 0.8602 |
| Results | $A_3 \succ A_2 \succ A_1$ | | | $A_3 \succ A_2 \succ A_1$ | | |
| Cheng CV uniform distribution | 0.0272 | 0.0214 | 0.0225 | 0.0328 | 0.0246 | 0.0095 |
| Results | $A_1 \succ A_3 \succ A_2$ | | | $A_1 \succ A_2 \succ A_3$ | | |
| Cheng CV proportional distribution | 0.0183 | 0.0128 | 0.0137 | 0.026 | 0.0146 | 0.0057 |
| Results | $A_1 \succ A_3 \succ A_2$ | | | $A_1 \succ A_2 \succ A_3$ | | |
| Asady–Zendehnam distance | 0.6 | 0.7 | 0.9 | 0.575 | 0.65 | 0.7 |
| Results | $A_3 \succ A_2 \succ A_1$ | | | $A_3 \succ A_2 \succ A_1$ | | |

provided in Ref. [17], the ranking order varies when different confidential optimistic level is adopted. In Set 2, by the approaches proposed in Ref. [3], the ranking order is $A_3 \succ A_2 \sim A_1$. By the CV index approach in Ref. [4], the ranking order is $A_1 \succ A_2 \succ A_3$. Through Fig. 6, it is easy to see that neither of them is consistent with human intuition.

**Example 5.** Consider two fuzzy numbers from Ref. [16], i.e., $A_1 = (2,2,1,3)_{LR}$ and $A_2 = (2,2,1,2)_{LR}$, as shown in Fig. 7. The membership function of $A_1$ is as follows:

$$\mu_{A_1}(x) = \begin{cases} x - 1, & \text{if } x \in [1,2], \\ \frac{(5-x)}{3}, & \text{if } x \in [2,5], \\ 0, & \text{otherwise}. \end{cases}$$

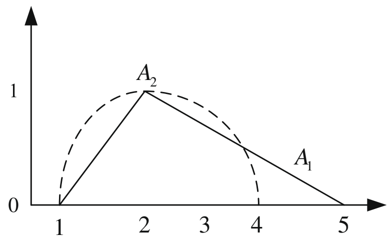


**Fig. 7.** Fuzzy numbers in Example 5.

The membership functions of $A_2$ is as follows:

$$\mu_{A_2}(x) = \begin{cases} [1-(x-2)^2]^{1/2}, & \text{if } x \in [1,2], \\ \left[1-\frac{1}{4}(x-2)^2\right]^{1/2}, & \text{if } x \in [2,4], \\ 0, & \text{otherwise}. \end{cases}$$

According to Eqs. (2) and (3), we can, respectively, obtain the membership function of $A_{\min}$ and $A_{\max}$ as follows:

$$\mu_{A_{\min}}(x) = \begin{cases} \frac{1}{4}(5-x), & \text{if } x \in [1,5], \\ 0, \text{otherwise} \end{cases}$$

and

$$\mu_{A_{\max}}(x) = \begin{cases} \frac{1}{4}(x-1), & \text{if } x \in [1,5], \\ 0, & \text{otherwise}. \end{cases}$$

According to Eqs. (4) and (5), we can obtain the left and right deviation degrees of two fuzzy numbers $A_1$ and $A_2$, i.e., $d_1^L = 0.4$, $d_1^R = 0.8572$, $d_2^L = 0.16251$, $d_2^R = 1.5571$. By Eqs. (6)–(9), we can obtain the ranking index values as $d_1 = 0.2154$ and $d_2 = 0$. Accordingly, the ranking order of fuzzy numbers is $A_1 \succ A_2$. Meanwhile, if the approach in Ref. [9] is used, the ranking order is consistent with the one by ours. But our approach is simpler in calculation procedure than the approach in Ref. [9]. If Lious and Wang's approach is used, different ranking order can be obtained when different optimistic indices are adopted. Moreover, if the approach in Ref. [10] is used, the ranking order is $A_1 \prec A_2$. From Fig. 7, we can conclude that $A_1 \succ A_2$ is more consistent with human intuition.

## 5. Summary and conclusion

This paper presents a new approach for ranking *L–R* fuzzy numbers. In the approach, the *L–R* deviation degree and transfer coefficient of fuzzy number are defined. Then, the ranking index value is obtained based on the *L–R* deviation degree and transfer coefficient. The examples given in this paper illustrate that the proposed approach has the distinct characteristics. Comparing with the exiting approaches, it is reasonable and efficient. Additionally, the proposed approach can provide decision makers with a new alternative to rank fuzzy numbers. It enriches the theories and methods for ranking fuzzy numbers.

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