Information Sciences 424 (2018) 301–316

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

ELECTRE methods in prioritized MCDM environment

Xiaohan Yu*, Suojuan Zhang, Xianglin Liao, Xiuli Qi

College of Command Information Systems, PLA University of Science and Technology, Nanjing Jiangsu 210007, China

ARTICLE INFO

Article history:
Received 30 November 2016
Revised 18 September 2017
Accepted 20 September 2017
Available online 27 September 2017

Keywords:
ELECTRE
Multicriteria decision making
Prioritized MCDM
Prioritization

ABSTRACT

In most cases, ELECTRE methods just apply to traditional multicriteria decision making (MCDM) problems with independent criteria. However, there exist more or less interdependences among criteria in actual situations. A special case is MCDM with prioritizations among criteria, called prioritized MCDM. In recent years, how to deal with MCDM problems in the environment of prioritized criteria becomes hot topic increasingly. Lots of existing methods, including PROMETHEE, are modified for the prioritized MCDM, but the advantages of ELECTRE have not been exploited to the prioritized MCDM. In my opinion, design specific ELECTRE methods for prioritized MCDM problems will benefit the developments of both ELECTRE and prioritized MCDM. In this paper, we firstly reformulate the expressions of concordance and discordance indices for use in the MCDM problems with dependent criteria, especially prioritized criteria, based on the concepts of fuzzy measures and digraphs respectively. After replacing the fuzzy measure by a prioritized measure, we successfully validate the concordance of an assertion under prioritized criteria. Furthermore, we design an approach to validate the discordance of the assertion based on a digraph constructed by the criteria and their prioritizations. Finally, we design three procedures to construct outranking relations in the prioritized MCDM environment according to the ideas of ELECTRE-I, ELECTRE-Iv, and ELECTRE-IS respectively. It is meaningful of this paper to provide a new idea to solve the prioritized MCDM problems and widen the application scope of ELECTRE methods by means of the reformulations of concordance and discordance.

1. Introduction

Since it was introduced in mid-1960s, multicriteria decision making (MCDM) has been a hot topic in decision making and systems engineering [9,14,22], and been proven as a useful tool due to its broad applications in a number of practical problems, such as energy planning [8,21,34], supply-chain selection [11,32,33], risk management [17,23,31], water-resources management [4,15,35], and so on. The family of ELECTRE methods, proposed by B. Roy in 1965, is famous and efficient to solve MCDM problems. The original ideas of ELECTRE methods were first merely published as a research report in 1966 [1] (the notorious Note de Travail 49 de la SEMA). Shortly after its appearance, ELECTRE I was found to be successful when applied to a vast range of fields [3]. Two further versions known as ELECTRE-Iv [18] and ELECTRE-IS [28] appeared subsequently. ELECTRE-Iv took into account the notion of a veto threshold, and ELECTRE-IS was used to modeling situations in which the data was imperfect. These are the versions of ELECTRE methods for choice problematic. But how to establish an adequate system of ranking for several actions/alternatives? This led to the birth of ELECTRE-II: a method for dealing with

* Corresponding author.
E-mail address: yua2006@126.com (X. Yu).

https://doi.org/10.1016/j.ins.2017.09.061

the problem of ranking actions from the best option to the worst [10]. Just a few years later a new method for ranking actions was devised: ELECTRE-III [27]. The main new ideas introduced by this method were the use of pseudo-criteria and fuzzy binary outranking relations. Another ELECTRE method, known as ELECTRE-IV [12], made it possible to rank actions without using the relative criteria importance coefficients. This method was equipped with an embedded outranking relations framework. However, in the late seventies a new technique of sorting actions into predefined and ordered categories was proposed. The first sorting method is ELECTRE-A. After being extended and improved, this method was developed into a simpler and more general method: ELECTRE-TRI [46].

However, up to now most of the ELECTRE methods have just focused on the MCDM problems with independent criteria. Actually, there widely exist interdependences among criteria, due to the complexity of MCDM problems in our daily lives. A possible kind of interdependences among criteria can be prioritizations [5,6,40–42,47–50]. As stated by Yager [40,41], a typical example can be the relationship between the criteria of *safety* and *cost* in the case of selecting a bicycle for child. We usually do not allow a loss in *safety* to be compensated by a benefit in *cost*, i.e., any tradeoff between *safety* and *cost* is unacceptable in this case. Simply speaking, there is prioritization between the criteria of *safety* and *cost*, and *safety* has a higher priority than *cost*. Such a kind of MCDM problems with prioritizations among criteria are called prioritized MCDM ones. Recently, the research about the prioritized MCDM problems has focused on generating or devising weights associated with criteria for common aggregation operators (such as ordered weighted averaging (OWA) operator [20,39], triangular norms and conorms [16], Choquet integral [30], etc.) according to the prioritizations on the basis of consensus, in which the importance weights associated with the criteria with lower priorities are related to the satisfactions of those with higher priorities. For example, Yager [40] introduced an OWA prioritized criteria aggregation, in which the weight of a criterion is determined by the original OWA weighting vector together with the satisfactions of the criteria with higher priorities. Yager [41] designed several approaches to derive the overall satisfactions associated with the higher prioritized hierarchies (by means of the min operator, the OWA operator, etc.), based on which the weight of each hierarchy can be calculated and then a prioritized “anding” operator and a prioritized “oring” operator were introduced based on triangular norms and conorms respectively. The prioritized multicriteria aggregation problems were solved with strictly ordered prioritizations on the basis of the OWA operator by Yager [42]. A monotonic set measure [19] was used to describe the prioritizations by Yager et al. [43], and then an integral type aggregation (Choquet integral) was used to aggregate the evaluation values of criteria. Then, Yan et al. [44] proposed a prioritized weighted aggregation operator based on the OWA operator along with triangular norms, and furthermore, considering the decision maker’s requirements toward the higher priority hierarchy, a benchmark based approach was designed to induce the priority weight for each priority hierarchy. On the basis of existing work, Yu and Xu [47] introduced prioritized aggregation operators into intuitionistic fuzzy environment.

Yu et al. [48,50] have discovered an imperfectness of the prioritized aggregation operators based methods: such methods are inapplicable in a special situation that the evaluation values of all alternatives with respect to the criterion with the highest priority are close and do not satisfy the requirement of the decision maker. For example, suppose the *safeties* of two bicycles are identical and cannot reach the requirement of the consumer, then the overall evaluations of the two bicycles (the results of prioritized aggregations) are the same, and we cannot give out an effective advice which bicycle shall be chosen, even though the cheaper one shall be selected intuitively. In order to overcome such a drawback, Yu et al. [50] introduced one outranking method, PROMETHEE, into the prioritized MCDM problems. In such a case, we can select the best action/alternative by comparing the actions in pairs. Subsequently, Chen and Xu [5] proposed a PROMETHEE method for prioritized MCDM problems with weakly ordered prioritizations among criteria. Up to now, the existing work has introduced outranking methods into prioritized MCDM problems, but just PROMETHEE methods. How to use ELECTRE, the most famous outranking methods, to deal with prioritized MCDM problems, has not been developed. It is valuable to introduce ELECTRE into prioritized MCDM problems. On one hand, we can enrich the solutions of the prioritized MCDM problems. On the other hand, we extended the application domains of ELECTRE methods through generalizing the construction of outranking relations. Therefore, we try to develop some ELECTRE methods for prioritized MCDM problems in this paper.

The rest of the paper is organized as follows. We firstly introduce basic concepts of ELECTRE and prioritized MCDM in Sections 2 and 3, respectively. Then we design approaches to validate the concordance and discordance in the environment of prioritized MCDM based on the ideas of ELECTRE-I, ELECTRE-Iv and ELECTRE-IS in Sections 4 and 5, respectively. In Section 6, we describe how to construct outranking relations for prioritized criteria based on ELECTRE-I, ELECTRE-Iv and ELECTRE-IS, respectively. At length, the conclusions are given in Section 8.

2. ELECTRE

2.1. Outranking relation

In a multicriteria decision making (MCDM) problem with a set of m actions, $X = \{x_1, x_2, \ldots, x_m\}$, and a set of n criteria, $C = \{c_1, c_2, \ldots, c_n\}$, we always want to select the best action from X through evaluating or ranking all actions under the n criteria. Kaliszewski et al. [13] formulated the underlying model for MCDM as a vector optimization problem:

$$\begin{array}{ll} \max & \boldsymbol{c}(x) \\ s.t. & x \in X \end{array} \tag{1}$$

where $\boldsymbol{c}$: $X \to [0,\ 1]^n$, $\boldsymbol{c}(x) = (c_1(x), c_2(x), ..., c_n(x))$ are satisfactions of x under respective criteria. In the viewpoint of Kaliszewski et al. [13], an action x^* is picked up from X as the best one, then $\nexists x \in X \backslash \{x^*\}$ such that $x \triangleright x^*$, where "$\triangleright$" is the symbol of dominance relation. $x \triangleright x^*$ means x is dominating x^*, or x^* is dominated by x, i.e., $c_i(x) \geq c_i(x^*)$ $(i = 1, 2, ..., n)$ and $c_j(x) > c_j(x^*)$ for at least one $j \in \{1,\ 2,\ ...,\ n\}$. Generally speaking, the dominance relation is too strict to make the decision outcome converged, because there usually are lots of actions which cannot be dominated by others. A weaker relation, called outranking relation [25], can cover the above shortage of the dominance relation.

The foundation of the outranking relation relies on decision makers are willing to take the risk of approve one action is superior to another. Roy [25] gave the following definition.

Definition 1 [25]. According to the given decision information, $\forall x_j,\ x_k \in X$, if and only if decision makers or analysts have reasons to believe x_j is superior or indifferent to x_k, then there exists outranking relation between x_j and x_k, denoted as $x_j \mathcal{S} x_k$, meaning x_j is at least as good as x_k.

Any outranking relation must obey two properties:

a) (*weak transitivity*) $\forall x_j,\ x_k,\ x_l \in X$,

$$\left.\begin{matrix} x_j \mathcal{S} x_k \\ x_k \trianglerighteq x_l \end{matrix}\right\} \Rightarrow x_j \mathcal{S} x_l$$

and

$$\left.\begin{matrix} x_j \trianglerighteq x_k \\ x_k \mathcal{S} x_l \end{matrix}\right\} \Rightarrow x_j \mathcal{S} x_l$$

b) (*reflexivity*) $\forall x \in X \Rightarrow x \mathcal{S} x$, i.e., any action is outranking itself.

According to the two properties, we can easily deduce another property, $x_j \trianglerighteq x_k \Rightarrow x_j \mathcal{S} x_k$ for $x_j,\ x_k \in X$. In other word, the dominance relation is stricter than the outranking relation.

Up to now, there have been many methods for MCDM problems based on multiform outranking relations, called outranking methods as a common name. The most common outranking methods are ELECTRE (ELimination Et Choix Traduisant la REalité) [24], and PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) [2]. Different outranking methods rely on their distinct designs of outranking relations. Subsequently, we will discuss the outranking relations for ELECTRE-I and its evolved versions in detail.

2.2. ELECTRE

For respective purposes, there are three kinds of ELECTRE methods: ELECTRE-I [24], ELECTRE-Iv [18] and ELECTRE-IS [28] for choice problems, ELECTRE-II [10], ELECTRE-III [27] and ELECTRE-IV [12] for ranking problems, and ELECTRE-TRI [46] for sorting problems. In this paper, we focus on choice problems, and discuss how to deal with prioritized MCDM problems base on ELECTRE-I and its evolved versions.

ELECTRE-I, the origin of ELECTRE methodologies, was design by Roy [25] in order to deal with choice problems. Outranking methods, including ELECTRE-I, usually involve two steps [26]. First, the actions/alternatives are compared pairwise in order to build outranking relations. In the second step, outranking relations are exploited in order to provide a recommendation to decision makers. In this paper, we mainly discuss the first step of ELECTRE-I and its evolved versions.

For ELECTRE-I and its evolved versions, the construction of an outranking relation is based on two major concepts:

1) *Concordance:* For an outranking $x_j \mathcal{S} x_k$ to be validated, a sufficient majority of criteria should be in favor of this assertion.
2) *Non-discordance:* When the concordance condition holds, none of the criteria in the minority should oppose too strongly to the assertion $x_j \mathcal{S} x_k$.

On one hand, the strength of the concordance coalition must be powerful enough to support the assertion, $x_j \mathcal{S} x_k$. ELECTRE-I defined a concordance index to measure the strength of the concordance coalition.

Suppose there are a group of weights, $w_1,\ w_2,\ ...,\ w_n$, associated to respective criteria with $\sum_{i \in N} w_i = 1$ $(N = \{1, 2, ..., n\})$, and a given concordance level, α, whose value generally falls within the range $[0.5, 1 - \min_{i \in N} w_i]$. For each pair of actions $x_j,\ x_k \in X$, we can construct three sets of indices below:

$$N_{jk}^{+} = N^{+}(x_j, x_k) = \{i \in N | c_i(x_j) > c_i(x_k)\} \tag{2}$$

$$N_{jk}^{=} = N^{=}(x_j, x_k) = \{i \in N | c_i(x_j) = c_i(x_k)\} \tag{3}$$

$$N_{jk}^{-} = N^{-}(x_j, x_k) = \{i \in N | c_i(x_j) < c_i(x_k)\} \tag{4}$$

The concordance index can be then defined:

$$I_{jk} = \sum_{i \in N_{jk}^{+}} w_i + \sum_{i \in N_{jk}^{=}} w_i \tag{5}$$

If the concordance index is not less than the concordance level, i.e., $I_{jk} \geq \alpha$, it means the assertion $x_j \mathcal{S} x_k$ is validated by the concordance.

On the other hand, no discordance against the assertion "x_j is at least as good as x_k" may occur. The discordance is measured by a discordance index as follows:

$$D_{jk} = \max_{i \in N} \left\{ \frac{c_i(x_k) - c_i(x_j)}{\nu} \right\} \tag{6}$$

where ν denotes a given veto threshold. If $D_{jk} \geq 1$, the assertion $x_j \mathcal{S} x_k$ is no longer valid; otherwise, discordant exerts no power. If the assertion $x_j \mathcal{S} x_k$ passes both the concordance validation and discordance validation, i.e., $I_{jk} \geq \alpha$ and $D_{jk} < 1$, x_j will outrank x_k and the outranking relation $x_j \mathcal{S} x_k$ exists.

As evolved versions, ELECTRE-Iv and ELECTRE-IS have the same procedures with ELECTRE-I. We must valid the concordance and non-discordance conditions for the two versions. However, little changes occur when moving from ELECTRE-I to ELETRE-Iv or ELECTRE-IS.

Concerning ELECTRE-Iv, the only difference being the non-discordance condition, called no veto condition, which stated as follows:

$$c_i(x_j) + \nu_i(c_i(x_j)) \geq c_i(x_k), \quad \forall i \in N \tag{7}$$

where ν_i is called veto thresholds that can be attributed to certain criterions $c_i \in C$. Moreover, the discordance index can be computed by

$$D_{jk} = \max_{i \in N} \left\{ \frac{c_i(x_k) - c_i(x_j)}{\nu_i(c_i(x_j))} \right\} \tag{8}$$

If D_{jk} in (8) is less than 1, the discordance will lose its efficacy against the assertion.

The mainly novelty of ELECTRE-IS is the use of pseudo-criteria instead of true-criteria. Both concordance and no veto conditions change in ELECTRE-IS. Firstly, two sets of indices are built:

$$N_{jk}^{S} = N^{S}(x_j, x_k) = \left\{ i \in N | c_i(x_j) + q_i(c_i(x_j)) \geq c_i(x_k) \right\} \tag{9}$$

and

$$N_{jk}^{Q} = N^{Q}(x_j, x_k) = \left\{ i \in N | c_i(x_j) + q_i(c_i(x_j)) < c_i(x_k) \leq c_i(x_j) + p_i(c_i(x_j)) \right\} \tag{10}$$

Then the concordance index will be

$$I_{jk} = \sum_{i \in N_{jk}^{S}} w_i + \sum_{i \in N_{jk}^{Q}} \varphi_{ijk} w_i \tag{11}$$

where we set $0 \leq q_i(c_i(x_j)) \leq p_i(c_i(x_j))$ and

$$\varphi_{ijk} = \frac{c_i(x_j) + p_i(c_i(x_j)) - c_i(x_k)}{p_i(c_i(x_j)) - q_i(c_i(x_j))} \tag{12}$$

In this case, the no veto condition can be stated as

$$c_i(x_j) + \nu_i(c_i(x_j)) \geq c_i(x_k) + q_i(c_i(x_k))\eta_i \tag{13}$$

where

$$\eta_i = \frac{1 - I_{jk} - w_i}{1 - \alpha - w_i} \tag{14}$$

where α is the given concordance level mentioned above, and w_i are the weights of criteria. Similarly to (6) and (8), we also can compute the discordance index according to (13):

$$D_{jk} = \max_{i \in N} \left\{ \frac{c_i(x_k) - c_i(x_j) + q_i(c_i(x_k))\eta_i}{\nu_i(c_i(x_j))} \right\} \tag{15}$$

For each pair of actions in X, a binary relation can be determined by the above validations of concordance and discordance, and only one of the following four situations may occur:

1) $x_j \mathcal{S} x_k$ and not $x_k \mathcal{S} x_j$, i.e., $x_j \mathcal{P} x_k$ (x_j is strictly preferred to x_k);
2) $x_k \mathcal{S} x_j$ and not $x_j \mathcal{S} x_k$, i.e., $x_k \mathcal{P} x_j$ (x_k is strictly preferred to x_j);
3) $x_j \mathcal{S} x_k$ and $x_k \mathcal{S} x_j$, i.e., $x_j \mathcal{I} x_k$ (x_j is indifferent to x_k);
4) Not $x_j \mathcal{S} x_k$ and not $x_k \mathcal{S} x_j$, i.e., $x_j \mathcal{R} x_k$ (x_j is incomparable to x_k).

After confirming all pairwise relations of actions in X, we then complete the first step of ELECTRE-I and its evolved versions, i.e., the construction of outranking relations on X.

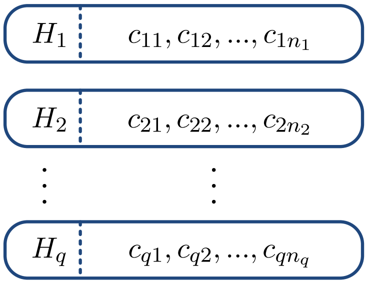

Fig. 1. Prioritized hierarchies of criteria.

3. Prioritized multicriteria decision making

In recent years, a new kind of relationships among criteria, called prioritizations, have been introduced in some literatures [5,6,40–44,47–50]. Assume that a set of criteria $C = \{c_1, \ldots, c_n\}$ are partitioned into q distinct prioritized hierarchies, $H = \{H_1, H_2, \ldots, H_q\}$, such that $H_k \succ H_l$ if $k < l$, where $H_k = \{c_{k1}, c_{k2}, \ldots, c_{kn_k}\}$ is a subset of C, and "$\succ$" means "prior to". In this case, the criteria in H_k have higher priorities than those in H_l if $k < l$, and $H_1 \succ H_2 \succ \ldots \succ H_q$. We also suppose that there is no identical element for any two hierarchies, i.e., $H_k \bigcap H_l = \emptyset$ for $k \neq l \in \{1, 2, \ldots, q\}$ ($\emptyset$ means a null set), and $C = \bigcup_{k=1}^{q} H_k$, then $n = \sum_{k=1}^{q} n_k$. In a multicriteria decision making (MCDM) problem, if there are prioritizations among criteria, we call it a prioritized MCDM problem. The prioritized hierarchy structure of C can be portrayed as in Fig. 1, where each row means corresponding prioritized hierarchy, the name of a prioritized hierarchy is in the front of the row, and which criteria are contained in the prioritized hierarchy is shown in the body of the row.

According to Yager [40,41], the prioritizations can be classified into two cases: 1) *strictly ordered prioritizations*, if the prioritizations among the criteria are a strict linear ordering, i.e., each prioritized hierarchy has only one criterion, and the number of prioritized hierarchies is n; otherwise and 2) the prioritizations are called *weakly ordered ones*.

After analyzing the prioritizations in depth, Yager [43] and Chen et al. [6] introduced a special fuzzy measure, called prioritized measure, to convey the prioritizations.

We assume C is a finite set, with power set 2^C, then a fuzzy measure on C can be defined as follows:

Definition 2 [30]. A *fuzzy measure* on C is a function μ: $2^C \rightarrow [0, 1]$, satisfying: 1) $\mu(\emptyset) = 0$, $\mu(C) = 1$; 2) $\mu(B_1) \leq \mu(B_2)$ if $B_1 \subseteq B_2 \subseteq C$.

Especially, if we have $B_1 \bigcup B_2 = B_3 \Rightarrow \mu(B_1) + \mu(B_2) = \mu(B_3)$ for $\forall B_1, B_2, B_3 \subseteq C$, the fuzzy measure is then called *probability measure* [19]. The fuzzy measures can reflect the interdependences among criteria in detail, through measuring the importance weights of coalitions of criteria.

In 2012, Yager [43] introduced the concept of prioritized measure, in order to measure the prioritizations among criteria quantitatively. Motivated by Yager [43], Chen et al. [6] proposed a strict definition of the prioritized measure:

Definition 3 [6]. A *prioritized measure* on C is a function μ: $2^C \rightarrow [0, 1]$, satisfying:

1) $\mu(\emptyset) = 0$, $\mu(C) = 1$;
2) $\mu(B_1) \leq \mu(B_2)$ if $B_1 \subseteq B_2 \subseteq C$;
3) Let $B = \{c_{k_1}, c_{k_2}, \ldots, c_{k_l}\}$ and $B' = \{c_{k'_1}, c_{k'_2}, \ldots, c_{k'_l}\}$ be two subsets of C, where $c_{k_i} \succ c_{k_j}$ and $c_{k'_i} \succ c_{k'_j}$ for $i < j \in \{1, 2, \ldots, l\}$. If $c_{k_1} \succ c_{k'_1}$, $c_{k_2} \succ c_{k'_2}, \ldots, c_{k_l} \succ c_{k'_l}$, then $\mu(B) \geq \mu(B')$ ("$\succ$" denotes "prior to or similar to" in this paper).

The 3rd condition means that the coalition of the higher prioritized criteria is more important than those of the lower prioritized ones. In the situation of strictly ordered prioritizations among criteria, how to calculate the prioritized measure is proposed by Chen et al. [6]. Suppose there exist strictly ordered prioritizations, $c_1 \succ c_2 \succ \ldots \succ c_n$, in the criteria set C, together with respective weights $w_1, w_2, \ldots, w_n$ then for any subset $A = \{c_{l_1}, c_{l_2}, \ldots, c_{l_s}\} \subseteq C$ $(l_1 < l_2 < \ldots < l_s)$, the prioritized measure of A can be calculated by

$$\mu(A) = \sum_{i=1}^{s} w_{l_i} \cdot f_i(l_i) \tag{16}$$

where $f_i(l_i) \geq f_i(l'_i)$ for $l_i < l'_i$, and $f_i(i) = 1$. Concretely according to Chen et al. [6], the form of f_i can be

$$f_i(l_i) = \frac{n + 1 - l_i}{n + 1 - i} \tag{17}$$

However up to now, a general form of prioritized measure for the weakly ordered prioritizations has not been developed. As an important basis of ELECTRE in prioritized MCDM environment, we here introduce a new form of prioritized measure for the weakly ordered prioritizations. We assume that there are q distinct prioritized hierarchies, $H_1 \succ \cdots \succ H_q$, for the set of criteria, $C = \{c_1, c_2, ..., c_n\}$. Without loss of generality, we assume that the criteria are sorted in descending prioritizations, then $H_1 = \{c_1, ..., c_{n_1}\}$, $H_2 = \{c_{n_1+1}, ..., c_{n_1+n_2}\}$, …, $H_q = \{c_{\sum_{k=1}^{q-1} n_k+1}, ..., c_n\}$. In such a case, for a subset $A = \{c_{l_1}, c_{l_2}, ..., c_{l_s}\}$ of C, the prioritized measure of A can be

$$\mu(A) = \sum_{i=1}^{s} w_{l_i} \cdot f_{\eta(i)}[\eta(l_i)] \tag{18}$$

where η: $\{1, ..., n\} \to \{1, ..., q\}$ and if $c_i \in H_k$ then $\eta(i) = k$. $f_{\eta(i)}$ is the same with f_i in (16): (1) $f_{\eta(i)}$ is a monotonic decreasing function, and (2) if $\eta(i) = \eta(l_i)$ then $f_{\eta(i)}[\eta(l_i)] = 1$. Specially, if there is only one criterion in each hierarchy, there will be n distinct prioritized hierarchies for C and we have $H_i = \{c_i\}$ for $i \in \{1, 2, .., n\}$. According to the definition, such weakly ordered prioritizations are called strictly ordered ones. In this case, we have $\eta(i) = i$, because $c_i \in H_k$ iff $k = i$. Therefore, (18) is reduced into (16).

4. Concordance for prioritized criteria

In this section, we will reformulate the expressions of concordance indices based on fuzzy measures, and then contribute to validate the concordance in the situation of prioritized criteria.

4.1. Reformulation of concordances based on fuzzy measures

Here, we reformulate the concordance indices for ELECTRE-I, ELECTRE-Iv and ELECTRE-IS with the form of fuzzy measures.

1) ELECTRE-I and ELECTRE-Iv

In the preceding, we define the index set of criteria, $N = \{1, 2, ..., n\}$, and its subsets: N^+_{jk}, $N^=_{jk}$ and N^-_{jk}. If we construct a probability measure ξ on N such that $\xi(B) = \sum_{i \in B} w_i$ for $B \subseteq N$, we can reformulate (5) as

$$I_{jk} = \xi\left(N^+_{jk} \bigcup N^=_{jk}\right) = \xi\left(N^+_{jk}\right) + \xi\left(N^=_{jk}\right) \tag{19}$$

If substituting a fuzzy measure μ for the probability measure ξ above, we will obtain more general concordance indices:

$$I_{jk} = \mu\left(N^+_{jk} \bigcup N^=_{jk}\right) \tag{20}$$

The new concordance indices meet the requirement of actual multicriteria decision making (MCDM) problems more easily than the traditional ones. For example, there usually exist interdependences among criteria in actual MCDM problems, and the interdependences cannot be described well by weights of criteria, thus it lacks a good mechanism to evaluate the interdependences for the traditional concordance validation in ELECTRE. A typical instance is the correlation between mathematics and physics. Generally speaking, if a student is good at mathematics, it is more possible for the student to get good scores in physics tests, and vice versa. Therefore, we shall take into account this kind of correlations in order to assess capabilities of students accurately in some situations. If a student has good records in both mathematics and physics, the correlation shall be involved and the weights of mathematics and physics shall be reduced reasonably for his/her assessment; otherwise, we keep the weights unchanged. In such a case, the weights of mathematics and physics, possibly including other courses, are varying for different students. Fuzzy measures together with fuzzy integrals are proved to be an appropriate tool to cope with such a kind of problems [19,30], which is the reason to introduce fuzzy measure to concordance index in this paper.

2) ELECTRE-IS

Before reformulate the concordance index for ELECTRE-IS, we define a generalized fuzzy measure:

Definition 4. Let Ω be the universal of discuss, and $\tilde{\mu}$ is a function mapping from fuzzy subsets on Ω to the unit interval [0, 1]. Then $\tilde{\mu}$ is called a *generalized fuzzy measure*, if satisfying: 1) $\tilde{\mu}(\emptyset) = 0$, $\tilde{\mu}(\Omega) = 1$; 2) $\tilde{\mu}(\tilde{A}) \le \tilde{\mu}(\tilde{B})$ for $\tilde{A} \subseteq \tilde{B}$, where $\tilde{A}$, $\tilde{B}$ are fuzzy subsets on Ω.

In such a case, we discuss a domain $N = \{1, 2, ..., n\}$, and then build a fuzzy index set for $x_j \mathcal{S} x_k$ (another form of (9) and (10)):

$$\tilde{N}_{jk} = \{(i, \rho(i))\} \tag{21}$$

where $i \in N$ and $\rho(i)$ is the membership degree of i with

$$\rho(i) = \begin{cases} 1 & \text{if } c_i(x_j) + q_i(c_i(x_j)) \ge c_i(x_k) \\ \dfrac{c_i(x_j) + p_i(c_i(x_j)) - c_i(x_k)}{p_i(c_i(x_j)) - q_i(c_i(x_j))} & \text{if } c_i(x_j) + q_i(c_i(x_j)) < c_i(x_k) \le c_i(x_j) + p_i(c_i(x_j)) \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

If there is a generalized fuzzy measure $\tilde{\mu}: N \to [0, 1]$ with

$$\tilde{\mu}(\tilde{A}) = \sum_{i \in N} \rho_{\tilde{A}}(i) \cdot w_i \tag{23}$$

where $\tilde{A} = \{(i, \rho_{\tilde{A}}(i))\}$ is a fuzzy set on N, then (11) can be reformulated as

$$I_{jk} = \tilde{\mu}(\tilde{N}_{jk}) = \sum_{i \in N} \rho(i) \cdot w_i \tag{24}$$

From the above, the concordance indices of ELECTRE-I and its extended versions can be derived on the basis of fuzzy measures or generalized fuzzy measures.

4.2. Generalized prioritized measure

Before exploring the concordance index of ELECTRE-IS in the environment of prioritized MCDM, we shall introduce a new concept, generalized prioritized measures, for fuzzy subsets on N.

Definition 5. Let Ω be the universal of discuss, and $\tilde{\mu}$ is a function mapping from fuzzy subsets on Ω to the unit interval [0, 1]. Then $\tilde{\mu}$ is called a *generalized prioritized measure*, if satisfying:

1) $\tilde{\mu}(\emptyset) = 0$, $\tilde{\mu}(\Omega) = 1$;
2) $\tilde{\mu}(\tilde{A}) \leq \tilde{\mu}(\tilde{B})$ for $\tilde{A} \subseteq \tilde{B}$, where $\tilde{A}$, $\tilde{B}$ are fuzzy subsets on Ω;
3) suppose $\tilde{A} = \{(c_i, \rho_{\tilde{A}}(c_i)\}$ and $\tilde{B} = \{(c_i, \rho_{\tilde{B}}(c_i)\}$ $(i \in \{1, 2, ..., n\})$ are two fuzzy subsets on Ω, and $\forall i \in \{1, ..., n\} \backslash \{j, k\}$ we have $\rho_{\tilde{A}}(c_i) = \rho_{\tilde{B}}(c_i)$ but $\rho_{\tilde{A}}(c_j) = \rho_{\tilde{B}}(c_k)$ and $\rho_{\tilde{A}}(c_k) = \rho_{\tilde{B}}(c_j)$, then

$$\left.\begin{array}{r} c_j \succ c_k \\ \rho_{\tilde{A}}(c_j) \geq \rho_{\tilde{A}}(c_k) \end{array}\right\} \Rightarrow \tilde{\mu}(\tilde{A}) \geq \tilde{\mu}(\tilde{B})$$

We assume that there are four fuzzy subsets:

$$\tilde{A} = \{(c_1, 0.8), (c_2, 1), (c_3, 0.9), (c_4, 0.7)\},\ \tilde{B}_1 = \{(c_1, 0.8), (c_2, 0.9), (c_3, 1), (c_4, 0.7)\}$$
$$\tilde{B}_2 = \{(c_1, 0.8), (c_2, 0.8), (c_3, 1), (c_4, 0.7)\},\ \tilde{B}_3 = \{(c_1, 0.8), (c_2, 0.7), (c_3, 1), (c_4, 0.8)\}$$

on the universal of discuss $\{c_1, c_2, c_3, c_4\}$ with $c_1 \succ c_2 \succ c_3 \succ c_4$ (see Fig. 2). According to the 3rd property in the above definition, we have the generalized prioritized measure of $\tilde{A}$ is not less than that of $\tilde{B}_1$, i.e., $\tilde{\mu}(\tilde{A}) \geq \tilde{\mu}(\tilde{B}_1)$. And similarly, $\tilde{\mu}(\tilde{B}_2) \geq \tilde{\mu}(\tilde{B}_3)$. Besides, according to the 2nd property, we have $\tilde{\mu}(\tilde{B}_1) \geq \tilde{\mu}(\tilde{B}_2)$, because $\tilde{B}_1 \supset \tilde{B}_2$. From the above, we have $\tilde{\mu}(\tilde{A}) \geq \tilde{\mu}(\tilde{B}_1) \geq \tilde{\mu}(\tilde{B}_2) \geq \tilde{\mu}(\tilde{B}_3)$. In such a case, we can compare the generalized prioritized measures of $\tilde{A}$ and $\tilde{B}_3$ exactly, which cannot be straightly drawn based on the three properties in Definition 5. Therefore, by using the 2nd and 3rd properties repeatedly, the generalized prioritized measure can be applied into more complicated environment.

Suppose there are n criteria, $c_1, c_2, ..., c_n$, with prioritizations among them, then a possible form of generalized prioritized measure can be

$$\tilde{\mu}(\tilde{A}) = \sum_{i=1}^{n} w_i \cdot \lfloor \rho \rfloor_{\tilde{A}}(c_i) \tag{25}$$

where $\tilde{A} = \{(c_i, \rho_{\tilde{A}}(c_i) | i = 1, 2, ..., n\}$ is a fuzzy subset, w_i $(i = 1, 2, ..., n)$ denote weights of criteria, and

$$\lfloor \rho \rfloor_{\tilde{A}}(c_i) = \min_{j \in \{j | c_j \succ c_i\} \bigcup \{i\}} \{\rho_{\tilde{A}}(c_j)\}$$

If (25) is used to calculate the generalized prioritized measures of $\tilde{A}$ in Fig. 2, we have

$$\lfloor \rho \rfloor_{\tilde{A}}(c_1) = 0.8,\ \lfloor \rho \rfloor_{\tilde{A}}(c_2) = 0.8,\ \lfloor \rho \rfloor_{\tilde{A}}(c_3) = 0.8,\ \lfloor \rho \rfloor_{\tilde{A}}(c_4) = 0.7$$

If $w_1 = w_2 = w_3 = w_4 = 0.25$, then $\tilde{\mu}(\tilde{A}) = 0.775$. Besides, we have $\tilde{\mu}(\tilde{B}_1) = 0.775$, $\tilde{\mu}(\tilde{B}_2) = 0.775$, $\tilde{\mu}(\tilde{B}_3) = 0.725$. In such a case, $\tilde{\mu}(\tilde{A}) = \tilde{\mu}(\tilde{B}_1) = \tilde{\mu}(\tilde{B}_2) > \tilde{\mu}(\tilde{B}_3)$, which is consistent with the above analysis.

4.3. Concordance validation in prioritized MCDM

If the fuzzy measure μ in (20) is replaced with a prioritized measure with the form of (16) or (18), we will derive a concordance index for ELECTRE-I or ELECTRE-Iv in the prioritized MCDM environment. Besides, we also can derive concordance indices in ELECTRE-IS for prioritized MCDM problems, if we replace the generalized fuzzy measure in (24) with the generalized prioritized measure in (25). In the following, we take an example to illustrate how to calculate a concordance index for two actions under a set of prioritized criteria.

Example 1. We assume there are a set of prioritized criteria, $C = \{c_1, c_2, c_3, c_4\}$, in a prioritized MCDM problem, where $c_1 \succ c_2 \succ c_3 \succ c_4$. For two actions x_1 and x_2, if $c_1(x_1) > c_1(x_2)$, $c_2(x_1) = c_2(x_2)$, $c_3(x_1) < c_3(x_2)$ and $c_4(x_1) > c_4(x_2)$, we calculate the sets of indices according to (2)–(4):

$$N^{+}_{1,2} = N^{-}_{2,1} = \{1, 4\},\ N^{=}_{1,2} = N^{=}_{2,1} = \{2\},\ N^{-}_{1,2} = N^{+}_{2,1} = \{3\}$$

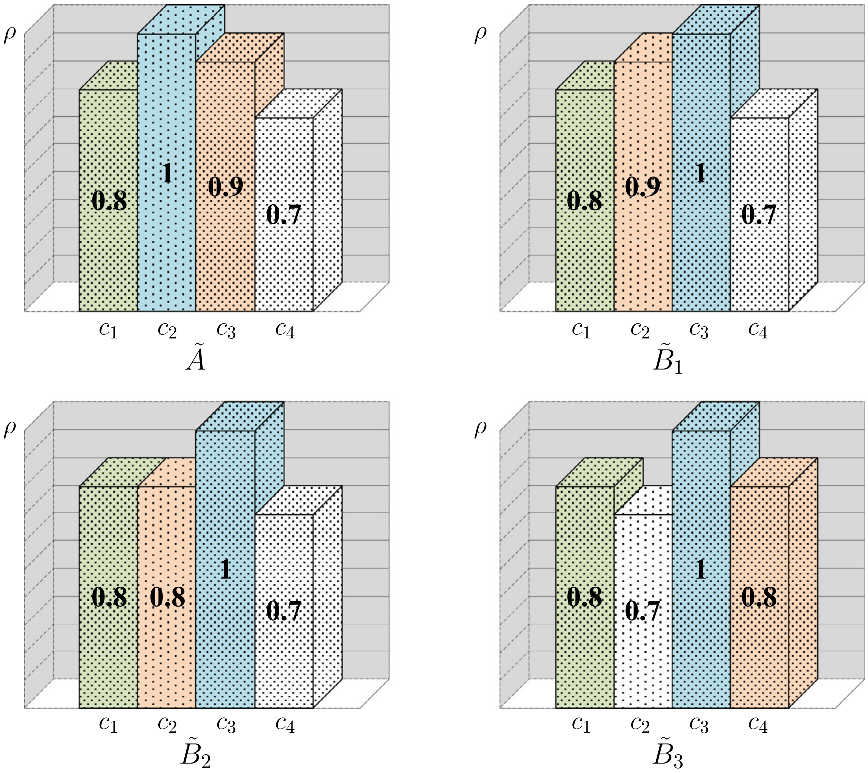

Fig. 2. Four fuzzy subsets.

Suppose the weights of criteria are given, $w_1 = 0.3$, $w_2 = 0.27$, $w_3 = 0.23$ and $w_4 = 0.2$. According to (16) and (20), we have

$$I_{1,2} = \mu(\{1, 2, 4\}) = w_1 \cdot f_1(1) + w_2 \cdot f_2(2) + w_4 \cdot f_3(4) = 0.67$$

$$I_{2,1} = \mu(\{2, 3\}) = w_2 \cdot f_1(2) + w_3 \cdot f_2(3) = 0.356$$

5. Discordance for prioritized criteria

In this section, we will reformulate the expressions of discordance indices based on digraphs, and then contribute to validate the discordance in the situation of prioritized criteria.

5.1. Reformulation of concordances based on digraphs

None of the criteria should oppose too strongly to the assertion $x_j\mathcal{S}x_k$ for ELECTRE-I, ELECTRE-Iv or ELECTRE-IS. In other words, if a criterion c_i is against the discordance condition of $x_j\mathcal{S}x_k$, such negation cannot be compensated/traded off by other criteria. There will be bilateral relations between any pair of criteria, and such relations can be represented by a complete digraph (see Fig. 3). Suppose there are n criteria, c_1, c_2, ..., c_n, and we can construct a complete digraph, $G = (C, E)$, based on these criteria, where $C = \{c_1, c_2, ..., c_n\}$, $E = \{(c_i, c_j)|c_i, c_j \in C\}$. In Fig. 3, an arc from c_1 to c_2, (c_1, c_2), means the negation of c_1 to the discordance condition of $x_j\mathcal{S}x_k$ is not allowed to be compensated by c_2. Also, we have a backward arc (c_2, c_1) in the digraph G.

In the following, we will design a new method to calculate the discordance indices based on the above complete digraph. First of all, we calculate a veto vector, $\boldsymbol{v} = (v_1, v_2, ..., v_n)$, for each assertion $x_j\mathcal{S}x_k$. In ELECTRE-I, we have

$$v_i = \frac{c_i(x_k) - c_i(x_j)}{v} \tag{26}$$

where v denotes a uniform veto threshold as in (6). Similarly, for ELETRE-Iv and ELECTRE-IS, we have

$$v_i = \frac{c_i(x_k) - c_i(x_j)}{v_i(c_i(x_j))} \tag{27}$$

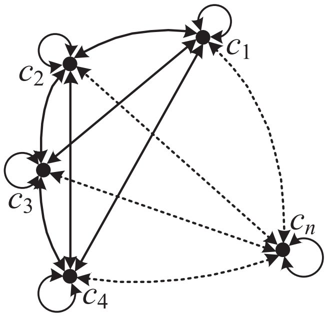

Fig. 3. A complete digraph of criteria.

and

$$v_i = \frac{c_i(x_k) - c_i(x_j) + q_i(c_i(x_k)) \cdot \eta_i}{v_i(c_i(x_j))} \tag{28}$$

The meanings of symbols in (27) and (28) refer to (8) and (15), respectively.

As stated in Section 2.2, if at least one element is not less than 1 in the veto vector $\boldsymbol{v}$, the assertion will be refused by the discordance. Here, we will design an equivalent expression to calculate discordance indices based on some existing concepts of the graph theory.

Definition 6. (adjacency matrix). The *adjacency matrix* of $G = (C, E)$ with n vertices is an $n \times n$ matrix $\boldsymbol{M} = (\varepsilon_{ij})_{n\times n}$ where

$$\varepsilon_{ij} = \begin{cases} 1, & \text{if } e_{ij} \in E \\ 0, & \text{otherwise} \end{cases}$$

Definition 7. (composition). Let $\boldsymbol{X} = (x_{ij})_{m\times n}$ and $\boldsymbol{Y} = (y_{ij})_{n\times s}$ be two matrixes, then the *composition* of $\boldsymbol{X}$ and $\boldsymbol{Y}$ can be denoted as $\boldsymbol{Z} = \boldsymbol{X} \circ \boldsymbol{Y}$, where $\boldsymbol{Z} = (z_{ij})_{m\times s}$ with $z_{ij} = \max_{k=1,2,\dots,n}\{\min(x_{ik}, y_{kj})\}$.

According to Definition 6, the adjacency matrix of the complete digraph $G = (C, E)$ in Fig. 3 is an all 1's matrix

$$\boldsymbol{M} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

By calculating the composition of $\boldsymbol{v}$ and $\boldsymbol{M}$, we derive a discordance vector, $\boldsymbol{d} = \boldsymbol{v} \circ \boldsymbol{M}$. Then the discordance index of the assertion $x_j \mathcal{S} x_k$ can be calculated by

$$D_{jk} = \frac{\sum_{i=1}^{n} d_i}{n} \tag{29}$$

where $\boldsymbol{d} = (d_1, d_2, \dots, d_n)$. If $D_{jk} \geq 1$, the assertion $x_j \mathcal{S} x_k$ will be refused; otherwise, the discordance exerts no power.

For example, if at least one element in $\boldsymbol{v}$ is not less than 1, $\boldsymbol{d}$ will be all 1's vector, and $D_{jk} = 1$ be the largest value; otherwise, $D_{jk} < 1$. Therefore, the discordance validation based on (29) is equivalent with those based on (6), (8) and (15).

5.2. Discordance validation in prioritized multicriteria decision making (MCDM)

In order to validate the discordance, we first construct a complete digraph for criteria in the last subsection. Why we shall construct a complete digraph but not a general one. That is because the above method focuses on a kind of simplified MCDM problems, in which criteria are of equivalent power or status and there exists identical influence between any pair of criteria. Actually, the assumption does not always stand. For example, the criteria with high priorities have more power to influence others than those with low priorities in the prioritized MCDM environment, thus a digraph for prioritized criteria is less likely to be balanced. Here, we try to construct a digraph based on the prioritizations among criteria, which will be used to validate discordance like the method in the last subsection.

Suppose there are n prioritized criteria, $C = \{c_1, c_2, \dots, c_n\}$, in a MCDM problem. C can be partitioned into q distinct prioritized hierarchies, H_1, H_2, ..., H_q, where $H_k \succ H_l$ if $k < l$. We then construct a digraph for these prioritized criteria, denoted as $G = (C, E)$, where E is a set of 2-tuples:

1) if $c_{i_1}, c_{i_2} \in H_k$, i.e., c_{i_1} is as equally prioritized as c_{i_2} in the set C, then we have $(c_{i_1}, c_{i_2}) \in E$ and $(c_{i_2}, c_{i_1}) \in E$;
2) if $c_{i_1} \in H_k$ and $c_{i_2} \in H_l$, where $H_k \succ H_l$, then $(c_{i_1}, c_{i_2}) \in E$ and $(c_{i_2}, c_{i_1}) \notin E$;
3) for all $c_i \in C$, we have $(c_i, c_i) \in E$.

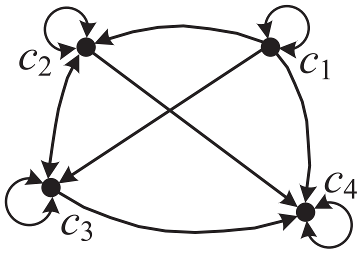

Fig. 4. A digraph of prioritized criteria.

Example 2. We assume there are four criteria, $C = \{c_1, c_2, c_3, c_4\}$, and the four criteria can be partitioned into three prioritized hierarchies, $H_1 = \{c_1\} \succ H_2 = \{c_2, c_3\} \succ H_3 = \{c_4\}$. According to the above method, we can construct a digraph $G = (C, E)$ (see Fig. 4), where

$$E = \{(c_1, c_1), (c_1, c_2), (c_1, c_3), (c_1, c_4), (c_2, c_2), (c_2, c_3), (c_2, c_4), (c_3, c_2), (c_3, c_3), (c_3, c_4), (c_4, c_4)\}$$

Its adjacency matrix is

$$\boldsymbol{M} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{30}$$

Here $\boldsymbol{M}$ is not an all 1' matrix differing from the adjacency matrix of the complete digraph in Fig. 3, resulting from the influence of prioritizations to discordance validations: the criteria with higher priorities have more power to refuse the assertion.

Similar to the method in the last subsection, we next calculate the veto vector $\boldsymbol{v}$, and then the discordance vector $\boldsymbol{d} = \boldsymbol{v} \circ \boldsymbol{M}$. As stated in the last subsection, if one element is not less than 1 in $\boldsymbol{v}$, $\boldsymbol{d}$ will be all 1's. However, in the environment of prioritized MCDM, we will have a different result.

For example, after calculated by (26), (27) and (28) in Example 2,

1) if $\boldsymbol{v} = (1, 0, 0, 0)$, then $\boldsymbol{d} = (1, 1, 1, 1)$;
2) if $\boldsymbol{v} = (0, 1, 0, 0)$, then $\boldsymbol{d} = (0, 1, 1, 1)$;
3) if $\boldsymbol{v} = (0, 0, 0, 1)$, then $\boldsymbol{d} = (0, 0, 0, 1)$.

The values in $\boldsymbol{d}$ tell the differences of criteria: 1) if the difference of $c_1(x_k)$ to $c_1(x_j)$ is large enough to reach the veto threshold, i.e., $v_1 = 1$ in $\boldsymbol{v}$, the assertion will be not valid; 2) the criterion c_2 has less power to refuse the assertion; and 3) c_4 is the worst.

Let us see other two special cases for Example 2:

1) if $\boldsymbol{v} = (0.5, -0.3, 0.7, 1.2)$, then $\boldsymbol{d} = (0.5, 0.7, 0.7, 1)$ and $D_{jk} = 0.725$ by (29);
2) if $\boldsymbol{v} = (0.8, -0.3, -0.3, -0.3)$, then $\boldsymbol{d} = (0.8, 0.8, 0.8, 0.8)$ and $D_{jk} = 0.8$.

The assertion is more easily refused in the 2nd case, which means the criteria with higher priorities expert more power in the process of the discordance validation.

However, if D_{jk} is also calculated by (29), the condition, that the discordance expert its efficiency against the assertion, will be quite strict. In such a case, if and only if the corresponding element in $\boldsymbol{v}$ is not less than 1 for at least one criterion with the highest priority, we have an all 1's $\boldsymbol{d}$ and $D_{jk} = 1$. Therefore, we shall weak the condition by introducing a parameter θ ($\in (0, 1]$):

$$D_{jk} = \frac{\sum_{i=1}^{n} d_i}{\theta n} \tag{31}$$

Similarly, if $D_{jk} \geq 1$, the assertion is no longer valid; otherwise, the discordance exerts no power against the assertion. For example, if θ is given as 0.75, the assertion will be refused by the discordance in the 2nd case ($0.8 > 0.75$) but not in the 1st case ($0.725 < 0.75$), for the above two special cases.

6. Constructions of outranking relations in the prioritized MCDM environment

According to the concordance validation (Section 4.3) and the discordance validation (Section 5.2), we design procedures to validate the assertion $x_j \mathcal{S} x_k$ in the environment of prioritized MCDM.

6.1. A procedure of ELECTRE-I

Suppose there are n criteria, $C = \{c_1, c_2, ..., c_n\}$, in a prioritized multicriteria decision making (MCDM) problem. We here validate an outranking $x_j \mathcal{S} x_k$, for two actions/alternatives x_j and x_k, referring to ELECTRE-I.

Procedure 1.

Step 1. Concordance validation

(1-1) Determine three sets of indices, N^+_{jk}, $N^=_{jk}$ and N^-_{jk}, based on $c_i(x_j)$ and $c_i(x_k)$ $(i = 1, 2, ..., n)$ according to (2)–(4), where

$$N^+_{jk} = \{i \in N | c_i(x_j) > c_i(x_k)\},\ N^=_{jk} = \{i \in N | c_i(x_j) = c_i(x_k)\}$$
$$N^-_{jk} = \{i \in N | c_i(x_j) < c_i(x_k)\}$$

(1-2) Determine a prioritized measure μ by (18), and calculate a concordance index $I_{jk} = \mu(N^+_{jk} \bigcup N^=_{jk})$;

(1-3) For a given concordance level, α, if $I_{jk} \geq \alpha$, the assertion $x_j \mathcal{S} x_k$ is validated by the concordance.

Step 2. Discordance validation

(2-1) For a given veto threshold, ν, calculate each element of a veto vector $\boldsymbol{v} = (v_1, ..., v_n)$ by (26), where

$$v_i = \frac{c_i(x_k) - c_i(x_j)}{\nu}$$

(2-2) Construct a digraph $G = (C, E)$ to describe the prioritizations among criteria according to the method in Section 5.2 (above Example 2);

(2-3) Derive the adjacency matrix $\boldsymbol{M}$ of the digraph G according to Definition 6, and then $\boldsymbol{d} = (d_1, d_2, ..., d_n) = \boldsymbol{v} \circ \boldsymbol{M}$;

(2-4) Calculate a discordance index by using (31) for a given θ:

$$D_{jk} = \frac{\sum_{i=1}^{n} d_i}{\theta n}$$

(2-5) If $D_{jk} < 1$, the assertion $x_j \mathcal{S} x_k$ passes the validation of the discordance.

If the assertion $x_j \mathcal{S} x_k$ passes both the concordance validation and the discordance validation, there exists an outranking relation $x_j \mathcal{S} x_k$; otherwise, the outranking relation $x_j \mathcal{S} x_k$ does not exist.

Besides, we have the following theorem:

Theorem 1. An assertion $x_j \mathcal{S} x_k$, supported by the concordance and discordance validations in Procedure 1, is an outranking relation.

Proof. According to the statements in Section 2.1, any relation, satisfying both the weak transitivity and the reflexivity, must be an outranking relation, thus we here prove the assertion satisfying the above two properties.

1) Proof of the weak transitivity.

We assume that there is another alternative x_l, and we have $x_k \trianglerighteq x_l$, which means $c_i(x_k) \geq c_i(x_l)$ for all $i \in \{1, 2, ..., n\}$ and $c_{i'}(x_k) > c_{i'}(x_l)$ for at least one $i' \in \{1, 2, ..., n\}$. In such a case, if $c_i(x_k) < c_i(x_l)$, then we have $c_i(x_j) < c_i(x_k)$. Thus $N^-_{jl} \subseteq N^-_{jk}$, and $(N^+_{jk} \bigcup N^=_{jk}) \subseteq (N^+_{jl} \bigcup N^=_{jl})$ (see (2)–(4)). Because any prioritized measure μ must be monotone increasing according to Definition 3, we have

$$\mu\left(N^+_{jl} \bigcup N^=_{jl}\right) \geq \mu\left(N^+_{jk} \bigcup N^=_{jk}\right)$$

i.e., $I_{jl} \geq I_{jk} \geq \alpha$. The concordance validates $x_j \mathcal{S} x_l$.

Because we just construct one digraph $G = (C, E)$ according to the prioritizations among criteria for a prioritized MCDM problem being solved, the adjacency matrix $\boldsymbol{M}$ is the same during the discordance validation of $x_j \mathcal{S} x_k$ with that of $x_j \mathcal{S} x_l$. We then calculate veto vectors $\boldsymbol{v}_{(jk)} = (v_{(jk),1}, v_{(jk),2}, ..., v_{(jk),n})$ and $\boldsymbol{v}_{(jl)} = (v_{(jl),1}, v_{(jl),2}, ..., v_{(jl),n})$ for $x_j \mathcal{S} x_k$ and $x_j \mathcal{S} x_l$ by (26) respectively. Because $c_i(x_k) \geq c_i(x_l)$, we have $v_{(jk),\,i} \geq v_{(jl),\,i}$ for all $i \in \{1, ..., n\}$, and then any element of the discordance vector $\boldsymbol{d}_{(jk)}$ is not less than the corresponding element of $\boldsymbol{d}_{(jl)}$, i.e., $d_{(jk),\,i} \geq d_{(jl),\,i}$ for $\forall i \in \{1, ..., n\}$. Therefore, we have $D_{jl} \leq D_{jk} < 1$, in other words, the discordance validates $x_j \mathcal{S} x_l$.

In this case, let x_j, x_k and x_l be three alternatives described by a set of prioritized criteria $C = \{c_1, c_2, ..., c_n\}$, where $x_j \mathcal{S} x_k$ is validated by Procedure 1, and $x_k \trianglerighteq x_l$, then $x_j \mathcal{S} x_l$ can also pass the validations during Procedure 1.

Similarly, we can prove another form of weak transitivity:

$$\left.\begin{array}{l} x_j \trianglerighteq x_k \\ x_k \mathcal{S} x_l \end{array}\right\} \Rightarrow x_j \mathcal{S} x_l$$

1) Proof of the reflexivity.

We here validate whether $x_j\mathcal{S}x_j$ is supported by the concordance and discordance validations in Procedure 1. In such a case, we have $N_{jj}^{=}=N$ and $N_{jj}^{+}=N_{jj}^{-}=\emptyset$. Because $\mu(N)=1$, then $I_{jj}=\mu(N_{jj}^{+}\bigcup N_{jj}^{=})=1>\alpha$. Therefore, $x_j\mathcal{S}x_j$ is validated by the concordance.

According to (26), we have $\upsilon_i=0$ for $\forall i\in\{1,\ldots,n\}$, i.e., the veto vector $\boldsymbol{\upsilon}$ is all 0's, thus the discordance vector $\boldsymbol{d}$ is also all 0's, and $D_{jj}=0$. Therefore, the discordance exerts no power against $x_j\mathcal{S}x_j$.

From the above, $x_j\mathcal{S}x_j$ can be validated as an outranking relation during Procedure 1.

Therefore, the theorem holds. □

6.2. A procedure of ELECTRE-Iv

Similar to Procedure 1, we here design another procedure so as to validate an assertion $x_j\mathcal{S}x_k$ for a prioritized MCDM problem based on the method of ELECTRE-Iv.

Procedure 2.

Step 1. Concordance validation

(1-1)–(1-3) are the same with Procedure 1.

Step 2. Discordance validation

(2-1) For given veto thresholds, $\nu_i(i=1,2,\ldots,n)$, calculate each element of a veto vector $\boldsymbol{\upsilon}=(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)$ by (27), where

$$\upsilon_i=\frac{c_i(x_k)-c_i(x_j)}{\nu_i(c_i(x_j))}$$

(2-2)–(2-5) are the same with Procedure 1.

If the assertion $x_j\mathcal{S}x_k$ passes both the concordance validation and the discordance validation, there exists an outranking relation $x_j\mathcal{S}x_k$; otherwise, the outranking relation $x_j\mathcal{S}x_k$ does not exist.

Similarly, we have the following theorem:

Theorem 2. An assertion $x_j\mathcal{S}x_k$, supported by the concordance and discordance validations in Procedure 2, is an outranking relation.

6.3. A procedure of ELECTRE-IS

We here propose Procedure 3 based on ELECTRE-IS.

Procedure 3.

Step 1. Concordance validation

(1-1) Determine a fuzzy index set, $\tilde{N}_{jk}=\{(i,\rho(i))\}$, according to (21);

(1-2) Determine a generalized prioritized measure $\tilde{\mu}$ by (25), and calculate a concordance index $I_{jk}=\tilde{\mu}(\tilde{N}_{jk})$;

(1-3) For a given concordance level, α, if $I_{jk}\geq\alpha$, the assertion $x_j\mathcal{S}x_k$ is validated by the concordance.

Step 2. Discordance validation

(2-1) For given veto thresholds, $\nu_i(i=1,2,\ldots,n)$, calculate each element of a veto vector $\boldsymbol{\upsilon}=(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)$ by (28), where

$$\upsilon_i=\frac{c_i(x_k)-c_i(x_j)+q_i(c_i(x_k))\cdot\eta_i}{\nu_i(c_i(x_j))}$$

and η_i refers to (14).

(2-2)–(2-3) are the same with Procedure 1;

(2-4) Calculate a discordance index, D_{jk}, by using (31) for a given θ, where

$$\max_{i,j}\left\{\frac{q_i(c_i(x_j))\eta'_i}{\nu_i(c_i(x_j))},0\right\}<\theta\leq 1\quad\text{and}\quad\eta'_i=\frac{-w_i}{1-\alpha-w_i}\tag{32}$$

(2-5) If $D_{jk}<1$, the assertion $x_j\mathcal{S}x_k$ passes the validation of the discordance.

If the assertion $x_j\mathcal{S}x_k$ passes both the concordance validation and the discordance validation, there exists an outranking relation $x_j\mathcal{S}x_k$; otherwise, the outranking relation $x_j\mathcal{S}x_k$ does not exist.

Note: The limitation of θ in the above Step 2-4 is to ensure the reflexivity of the outranking relation $x_j\mathcal{S}x_k$.

Similarly, we have the following theorem:

Theorem 3. An assertion $x_j\mathcal{S}x_k$, supported by the concordance and discordance validations in Procedure 3, is an outranking relation.

Table 1
Satisfaction degree.

$c_j(x_i)$	c_1	c_2	c_3	c_4	c_5	c_6
x_1	0.6	0.85	0.55	0.85	0.9	0.45
x_2	0.4	0.9	0.8	0.75	0.6	0.8
x_3	0.55	0.8	0.9	0.65	0.7	0.6
x_4	0.4	0.5	0.5	0.6	0.4	0.8
x_5	0.75	0.7	0.85	0.45	0.6	0.75

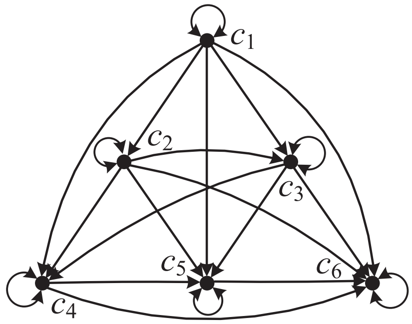

Fig. 5. The digraph for prioritizations among criteria.

7. Illustrative example

In order to illustrate how to deal with prioritized multicriteria decision making (MCDM) problems with the help of ELECTRE methods introduced in this paper, we take a simple example below.

Example 3. Suppose there are six criteria, $C = \{c_1, c_2, ..., c_6\}$, and five alternatives, $X = \{x_1, x_2, ..., x_5\}$, in a prioritized MCDM problem, where $c_1 \succ c_2 \succ ... \succ c_6$. All satisfaction degree $c_j(x_i)$ $(i = 1, ..., 5;\ j = 1, ..., 6)$ are given in Table 1.

Here, we validate an outranking $x_1 \mathcal{S} x_2$ according to Procedure 1 above, and the validations of other outrankings will be the same.

Firstly, we validate the concordance of $x_1 \mathcal{S} x_2$.

Step 1-1. According to (2)–(4), we have

$$N_{12}^{+} = \{1, 4, 5\}, N_{12}^{-} = \{2, 3, 6\}, N_{12}^{=} = \emptyset$$

Step 1-2. Because the prioritizations among the above criteria are strictly ordered prioritizations, we calculate the concordance index I_{12} by using (16) and (17):

$$I_{12} = \mu\left(N_{12}^{+} \bigcup N_{12}^{=}\right) = 0.35$$

where $w_1 = w_2 = ... = w_6 = 1/6$ were given as the weights of criteria.

Step 1-3. If the concordance level is given as $\alpha = 0.35$, the $x_1 \mathcal{S} x_2$ is then validated by the concordance since $I_{12} \geq \alpha$.

Secondly, we validate the discordance of $x_1 \mathcal{S} x_2$.

Step 2-1. Let the veto threshold $\nu = 0.2$, according to (26) we calculate the veto vector of $x_1 \mathcal{S} x_2$: $\boldsymbol{v} = (-1, 0.25, 1.25, -0.5, -1.5, 1.75)$.

Step 2-2. We then construct the digraph of prioritizations among criteria, $G = (C, E)$, (see Fig. 5) where

$$\begin{aligned} E = \{&(c_1, c_1), (c_1, c_2), (c_1, c_3), (c_1, c_4), (c_1, c_5), (c_1, c_6),\\ &(c_2, c_2), (c_2, c_3), (c_2, c_4), (c_2, c_5), (c_2, c_6), (c_3, c_3),\\ &(c_3, c_4), (c_3, c_5), (c_3, c_6), (c_4, c_4), (c_4, c_5), (c_4, c_6),\\ &(c_5, c_5), (c_5, c_6), (c_6, c_6)\} \end{aligned}$$

Step 2-3. The adjacency matrix of G can be

$$\boldsymbol{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

And then, we calculate the discordance vector of $x_1 \mathcal{S} x_2$, $\boldsymbol{d} = \boldsymbol{v} \circ \boldsymbol{M} = (0, 0.25, 1, 1, 1, 1)$.

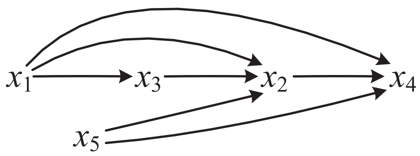

Fig. 6. Outranking relations.

Step 2-4. Suppose $\theta = 0.9$, then the discordance index of $x_1 \mathcal{S} x_2$ can be $D_{12} = 0.787$.

Step 2-5. Because $D_{12} < 1$, the discordance exerts no power to refuse the assertion $x_1 \mathcal{S} x_2$.

From the above, the assertion $x_1 \mathcal{S} x_2$ is true.

Similarly, we calculate the concordance and discordance indices between alternatives pairwise:

$$\boldsymbol{I} = \begin{pmatrix} - & 0.35 & 0.569 & 0.833 & 0.322 \\ 0.314 & - & 0.281 & 1 & 0.378 \\ 0.144 & 0.383 & - & 0.833 & 0.508 \\ 0.028 & 0.2 & 0.028 & - & 0.117 \\ 0.342 & 0.383 & 0.2 & 0.611 & - \end{pmatrix}$$

and

$$\boldsymbol{D} = \begin{pmatrix} - & 0.787 & 0.741 & 0.185 & 1.019 \\ 1.111 & - & 0.833 & 0 & 1.111 \\ 0.694 & 0.556 & - & 0.185 & 1.111 \\ 1.111 & 0.926 & 1.065 & - & 1.111 \\ 0.833 & 0.926 & 0.741 & 0.417 & - \end{pmatrix}$$

where I_{ij} and D_{ij} denote the concordance index and the discordance index of $x_i \mathcal{S} x_j$ $(i \neq j \in \{1, ..., 5\})$ respectively. For the given concordance level α above, if $I_{ij} \geq \alpha$ in $\boldsymbol{I}$ and meanwhile $D_{ij} < 1$ in $\boldsymbol{D}$, the corresponding $x_i \mathcal{S} x_j$ is validated as an outranking relation. In such a case, we have the following outranking relations (see Fig. 6):

$$x_1 \mathcal{S} x_2, x_1 \mathcal{S} x_3, x_1 \mathcal{S} x_4, x_2 \mathcal{S} x_4, x_3 \mathcal{S} x_2, x_3 \mathcal{S} x_4, x_5 \mathcal{S} x_2, x_5 \mathcal{S} x_4$$

Because x_1 and x_5 are not outranked by other alternatives, these two alternatives can be selected as the best ones.

For the same example, Yu et al. [50] ranked alternatives: $x_1 \to x_3 \to x_5 \to x_2 \to x_4$, which can be regarded as a special order resulting from the above outranking relations. Actually, our methods are more flexible, since there are more adjustable parameters: weights of criteria, the concordance level, the veto threshold, and the parameter θ.

However, if the method in [44] is used to solve the example after setting the same expectation levels to [50], the ranking order of alternatives is $x_3 \to x_5 \to x_1 \to \{x_2, x_4\}$. As stated by Yu et al. [50], the method in [44] (or similar method presented by [40–43,47]) fails when comparing x_2 and x_4. Intuitively, x_2 is more likely better than x_4 as our method concludes, because x_2 is not worse than x_4 for each criterion (x_2 dominates x_4). Therefore, our method inherits the advantages of the method in [50], which can effectively overcome the incomparable problems in the situations that the satisfaction degrees under the criterion with the highest priority fail to reach their expectations.

8. Concluding remarks

We have introduced ELECTRE methods into the problems of multicriteria decision making (MCDM) with prioritizations among criteria, called prioritized MCDM problems in this paper. Considering traditional ELECTRE methods mainly focused on MCDM problems with independent criteria, we have reformulate the expressions of concordance and discordance indices based on fuzzy measures and digraphs respectively, so as to obtain generalized forms of the two indices which were key points to extend traditional ELECTRE methods. And then we have devised approaches of concordance and discordance validations for the situation of prioritized MCDM. Finally, we have proved the assertion, validated by methods proposed in this paper, certainly is an outranking relation.

This paper has two characteristics: 1) it provides a new idea to solve the prioritized MCDM problems; and 2) the application scope of ELECTRE methods is extended because of the reformulations of concordance and discordance. However, we have just discussed ELECTRE-I and its improved versions in this paper, but not all ELECTRE methods. Actually as stated in Introduction, there are lots of ELECTRE methods with different functionalities. Therefore in our future work, it is worth studying how to apply the idea of reformulations of concordance and discordance to other ELECTRE methods, such as ELECTRE-II, ELECTRE-III, ELECTRE-IV, etc., to solve the prioritized MCDM problem. Meanwhile it is meaningful to study more fully ELECTRE methods for complex MCDM situations. Besides in my opinion, we may explore how to introduce granular computing techniques [7,29,36–38,45] into prioritized MCDM problems for fuzzy decision making environment, which must be valuable in the future.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (No. 71501186).

References

[1] R. Benayoun, B. Roy, B. Sussman, ELECTRE: Une method pour guider le choix en présence de points de vue multiple, Note de travail 49, SEMA-METRA International, Direction Scientifique, 1966.
[2] J.P. Brans, Ph. Vincke, B. Mareschal, How to select and how to rank projects: the PROMETHEE method, Eur. J. Oper. Res. 24 (1986) 228–238.
[3] P. Buffet, J. Grémy, M. Marc, B. Sussmann, Peut-on choisir en tenant compte de critères multiples ? Une method ELECTRE et trois applications, Rev. METRA VI (1967) 283–316.
[4] A. Calizaya, O. Meixner, L. Bengtsson, R. Berndtsson, Multi-criteria decision analysis (MCDA) for integrated water resources management (IWRM) in the Lake Poopo basin, Bolivia, Water Res. Manag. 24 (2010) 2267–2289.
[5] L.H. Chen, Z.S. Xu, A new prioritized multi-criteria outranking method: the prioritized PROMETHEE, J. Intell. Fuzzy Syst 29 (2015) 2099–2110.
[6] L.H. Chen, Z.S. Xu, X.H. Yu, Prioritized measure-guided aggregation operators, IEEE. Trans. Fuzzy. Syst. 22 (2014) 1127–1138.
[7] S. Das, S. Kar, T. Pal, Robust decision making using intuitionistic fuzzy numbers, Granul. Comput. 2 (2017) 41–54.
[8] P. Ekel, I. Kokshenev, R. Parreiras, W. Pedrycz, J. Pereira Jr., Multiobjective and multiattribute decision making in a fuzzy environment and their power engineering applications, Inf. Sci. 361–362 (2016) 100–119.
[9] X. Gou, Z.S. Xu, H.C. Liao, Hesitant fuzzy linguistic entropy and cross-entropy measures and alternative queuing method for multiple criteria decision making, Inf. Sci. 388–389 (2017) 225–246.
[10] J. Grolleau, J. Tergny, Manuel de reference du programme ELECTRE II, Document de travail 24, SEMA-METRA International, Direction Scientifique, 1971.
[11] A.N. Haq, G. Kannan, A hybrid normalized multi-criteria decision making for the vendor selection in a supply chain model, Int. J. Manag. Decis. Mak. 8 (2007) 601–622.
[12] J. Hugonnard, B. Roy, Le plan d'extension du metro en banlieue parisienne, un cas type d'application de l'analyse multicritère, Cah. Sci. Rev. Transp. 6 (1982) 77–108.
[13] I. Kaliszewski, J. Miroforidis, D. Podkopaev, Interactive multiple criteria decision making based on preference driven evolutionary multiobjective optimization with controllable accuracy, Eur. J. Oper. Res. 216 (2012) 188–199.
[14] P. Koele, Multiple Attribute Decision Making: An Introduction, in: K.P. Yoon, C.L. Hwang (Eds.), 1995 Sage University Paper Series On Quantitative Applications in the Social Sciences, Thousand Oaks, CA: Sage.
[15] H. Kuang, D.M. Kilgour, K.W. Hipel, Grey-based PROMETHEE II with application to evaluation of source water protection strategies, Inf. Sci. 294 (2015) 376–389.
[16] Q. Lei, Z.S. Xu, H. Bustince, J. Fernandez, Intuitionistic fuzzy integrals based on Archimedean t-conorms and t-norms, Inf. Sci. 327 (2016) 57–70.
[17] I. Linkov, F.K. Satterstrom, J. Steevens, E. Ferguson, R.C. Pleus, Multi-criteria decision analysis and environmental risk assessment for nanomaterials, J. Nanopart. Res. 9 (2007) 543–554.
[18] L. Maystre, J. Pictet, J. Simos, Les Méthodes Multicritères ELECTRE, Presses Polytechniques et Universitaires Romandes, Lausanne, 1994.
[19] T. Murofushi, M. Sugeno, Fuzzy measures and fuzzy integrals, in: M. Grabisch, T. Murofushi, M. Sugeno (Eds.), Fuzzy Measures and Integrals – Theory and Applications, Physica Verlag, 2000, pp. 3–41.
[20] Y. Ouyang, Improved minimax disparity model for obtaining OWA operator weights: Issue of multiple solutions, Inf. Sci. 320 (2015) 101–106.
[21] S.D. Pohekar, M. Ramachandran, Appication of multi-criteria decision making to sustainable energy planning – a review, Renew. Sustain. Energy Rev 8 (2014) 365–381.
[22] Z. Ren, Z.S. Xu, H. Wang, Dual hesitant fuzzy VIKOR method for multi-criteria group decision making based on fuzzy measure and new comparison method, Inf. Sci. 388–389 (2017) 1–16.
[23] A. Rodríguez, F. Ortega, R. Concepción, An intuitionistic method for the selection of a risk management approach to information technology projects, Inf. Sci. 375 (2017) 202–218.
[24] B. Roy, Classement et choix en présence de points de vue multiples (la méthode ELECTRE), Rev. Fr. Inf. Rech. Opér. 8 (1968) 57–75.
[25] B. Roy, How outranking relation help multiple criteria decision making, in: J.L. Cochrane, M. Zeleny (Eds.), Multiple Criteria Decision Making, University of South Carolina Press, Columbia, South Carolina, 1973.
[26] B. Roy, The outranking approach and the foundations of ELECTRE methods, Theor. Decis. 31 (1991) 49–73.
[27] B. Roy, M. Présent, D. Silhol, A programming method for determining with Paris metro stations should be renovated, Eur. J. Oper. Res. 24 (1986) 318–334.
[28] B. Roy, J. Skalka, ELECTRE IS: Aspects méthodologiques et Guide d'utilisation, Document du LAMSADE 30, Université Paris Dauphine, 1984.
[29] A. Skowron, A. Jankowski, S. Dutta, Interactive granular computing, Granul. Comput. 1 (2016) 95–113.
[30] M. Sugeno, (Ph.D. thesis), Tokyo Institute of Technology, 1974.
[31] W.H. Tsai, W.C. Chou, Building an integrated multi-criteria decision-making model based on DEMATEL and ANP for selecting the risk management system of banking, Int. J. Manag. Enterp. Dev. 8 (2010) 358–382.
[32] S.-P. Wan, G.-L. Xu, J.-Y. Dong, Supplier selection using ANP and ELECTRE II in interval 2-tuple linguistic environment, Inf. Sci. 385–386 (2017) 19–38.
[33] G. Wang, S.H. Huang, J.P. Dismukes, Product-driven supply chain selection using integrated multi-criteria decision-making methodology, Int. J. Prod. Econ. 91 (2004) 1–15.
[34] J.J. Wang, Y.Y. Jing, C.F. Zhang, J.H. Zhao, Review on multi-criteria decision analysis aid in sustainable energy decision-making, Renew. Sust. Energy Rev 13 (2009) 2263–2278.
[35] S.Q. Weng, G.H. Huang, Y.P. Li, An integrated scenario-based multi-criteria decision support system for water resources management and planning – a case study in the Haihe River Basin, Expert. Syst. Appl. 37 (2010) 8242–8254.
[36] G. Wilke, E. Portmann, Granular computing as a basis of human-data interaction: a cognitive cities use case, Granul. Comput. 1 (2016) 181–197.
[37] Z. Xu, X. Gou, An overview of interval-valued intuitionistic fuzzy information aggregations and applications, Granul. Comput 2 (2017) 13–39.
[38] Z. Xu, H. Wang, Managing multi-granularity linguistic information in qualitative group decision making: an overview, Granul. Comput. 1 (2016) 21–35.
[39] R.R. Yager, On ordered weighted averaging aggregation operators in multicriteria decision making, IEEE. Trans. Syst. Man Cybern. B 18 (1988) 183–190.
[40] R.R. Yager, Modeling prioritized multicriteria decision making, IEEE. Trans. Syst. Man Cybern. 34 (2004) 2396–2404.
[41] R.R. Yager, Prioritized aggregation operators, Int. J. Approx. Reason. 48 (2008) 263–274.
[42] R.R. Yager, Prioritized OWA aggregation, Fuzzy Optim. Decis. Mak. 8 (2009) 245–262.
[43] R.R. Yager, C.L. Walker, E.A. Walker, A prioritized measure for multi-criteria aggregation and its Shapley index, in: Proceedings of 2011 Annual Meeting of the North America on Fuzzy Information Processing Society, 2011, pp. 1–4.
[44] H.B. Yan, V.N. Huynh, Y. Nakamori, T. Murai, On prioritized weighted aggregation in multi-criteria decision making, Expert Syst. Appl. 38 (2011) 812–823.
[45] Y. Yao, A triarchic theory of granular computing, Granul. Comput 1 (2016) 145–157.
[46] W. Yu, (Ph.D. thesis), in, Université Paris-Dauphine, 1992.
[47] X.H. Yu, Z.S. Xu, Prioritized intuitionistic fuzzy aggregation operators, Inf. Fusion 14 (2013) 108–116.
[48] X.H. Yu, Z.S. Xu, S.S. Liu, Prioritized multi-criteria decision making based on preference relations, Comput. Ind. Eng. 66 (2013) 104–115.

[49] X.H. Yu, Z.S. Xu, S.S. Liu, Generalized prioritized multicriteria aggregation, Int. J. Intell. Syst. 28 (2013) 1141–1161.
[50] X.H. Yu, Z.S. Xu, Y. Ma, Prioritized multi-criteria decision making based on the idea of PROMETHEE, in: Y. Shi, Y. Xi, P. Wolcott, Y. Tian, J. Li, D. Berg, Z. Chen, E. HerreraViedma, G. Kou, H. Lee, Y. Peng, L. Yu (Eds.), Proceedings of the First International Conference on Information Technology and Quantitative Management, Elsevier Science Bv, Amsterdam, 2013, pp. 449–456.