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Decentralized adaptive neural control for high-order stochastic nonlinear strongly interconnected systems with unknown system dynamics

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ABSTRACT

This paper studies the problem of decentralized adaptive neural backstepping control for a class of high-order stochastic nonlinear systems with unknown strongly interconnected nonlinearity. During the control of the high-order nonlinear interconnected systems, only one adaptive parameter is used to overcome the over-parameterization problem, and radial basis function (RBF) neural networks are employed to tackle the difficulties brought about by completely unknown system dynamics and stochastic disturbances. In addition, to address the problem arising from high-order strongly interconnected nonlinearities with full states of the overall system, the variable separation technique is introduced based on the monotonically increasing property of the bounding functions. Next, a decentralized adaptive neural control method is proposed based on Lyapunov stability theory, in which the controller is designed to decrease the number of learning parameters. It is shown that the designed controller can ensure that all the signals in the closed-loop system are 4-Moment (or 2 Moment) semi-globally uniformly ultimately bounded (SGUUB) and the tracking error converges to a small neighborhood of the origin. Finally, two simulation examples are offered to illustrate the effectiveness of the proposed control scheme.

1. Introduction

Many real-life systems can be regarded as large-scale systems, such as transportation networks, power systems and multiagent systems. Out of these large-scale systems, of particular interest is a class of high-order stochastic nonlinear systems with completely unknown nonlinearities, which can describe a broad variety of practical systems, such as benchmark mechanical system [39], traffic networks and computer networks. At the same time, stochastic disturbances may cause performance degradation of practical systems or even lead to system instability [9]. In this work, we focus our attention on solving the problem of decentralized adaptive control for high-order large-scale stochastic nonlinear systems subject to unknown unmatched strong interconnections.

As is well-known, a power integrator method was applied to the control design for high-order nonlinear systems in [26,27]. The authors of [41] proposed the global adaptive control design for a class of high-order nonlinear systems with uncertainties by combining the homogeneous domination approach [23]. The problem of decentralized output-feedback control

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was studied in [12] for a class of large-scale high-order stochastic nonlinear systems, and the improved generalization of the homogeneous domination method about the system stabilization was developed in [25] for high-order nonlinear systems. However, these control schemes fail to achieve the tracking control task for high-order systems with completely unknown system dynamics, and therefore, there is a need to develop the approximation-based control methods.

Recently, fuzzy logic systems and neural networks have been extensively used in the modeling and controller design for various nonlinear systems [21,24]. The authors in [35,36] proposed a dynamic learning method from the adaptive control for strict-feedback systems based on the neural networks approximator and deterministic learning theory. In [40], a robust adaptive tracking control was presented using fuzzy logic systems, in which a method of estimating the norm of the weight vector rather than the weight vector itself was proposed. In [34], an adaptive neural control strategy was developed for non-lower triangular nonlinear systems. To overcome the difficulty caused by the nonstrict-feedback structure, a variable separation technique was developed in [2]. This technique was then applied to the constrained nonlinear nonstrict-feedback systems in [48] and switched stochastic nonlinear nonstrict-feedback systems in [43]. The pure-feedback structure was considered in [4], in which an adaptive tracking control for the switched stochastic nonlinear pure-feedback systems was presented with unknown backlash-like hysteresis. In [47], the approximation-based adaptive tracking control was investigated for multi-input multi-output (MIMO) nonlinear systems. The problem of adaptive fuzzy hierarchical sliding mode control was investigated in [45] for a class of MIMO nonlinear time-delay systems. In [15], a hybrid fuzzy control approach was developed for a class of MIMO strict-feedback nonlinear systems with unknown dead-zones.

In particular, the decentralized control for the interconnected and complex systems has long been a popular topic in the control community. In the decentralized control, given that the controller developed for each subsystem only contains local information, one of the main obstacles lies at dealing with the interconnections between all subsystems. In [32], an adaptive backstepping-based fuzzy control method was proposed for interconnected nonlinear systems. In [28], the decentralized adaptive fuzzy control for large-scale nonlinear systems was proposed, and the authors in [11,46] addressed the neural network-based decentralized stabilization problem for large-scale stochastic nonlinear systems. A class of the interconnected nonlinear pure-feedback systems was considered in [17] via dynamic surface control and fuzzy approximator techniques. The authors of [33] considered a class of stochastic nonlinear interconnected systems and proposed an adaptive neural tracking control scheme. In [3], the authors used a function approximator to construct the local controller for interconnected time-delay systems. In [13,14], the active fault-tolerant control was applied to the control of the large-scale nonlinear systems when actuator faults existed in the controlled system. In [8], the observer-based decentralized controller was designed for interconnected stochastic nonlinear time-delay systems. In [29], the switched uncertain nonlinear large-scale systems were considered subject to unmeasurable system states. A prescribed performance control technique was used in adaptive output-feedback control design for switched nonlinear systems in [18] and switched interconnected nonlinear systems in [16]. In [6], the decentralized adaptive neural control was investigated for high-order large-scale stochastic nonlinear systems. The authors in [44] dealt with adaptive neural control for switched high-order stochastic nonlinear systems. However, the above-mentioned control schemes for high-order systems have not taken into account the effects of strongly interconnected nonlinearity with full states of the overall system. Particularly, these studies have not accounted for completely unknown nonlinearity. Stochastic disturbances are prevalent in control engineering and thus worthy to be investigated. Note that, for the interconnected nonlinearity which depends on the full states of the entire system, most of the existing control methods become inapplicable. To the best of our knowledge, there are no results on the control of high-order stochastic nonlinear strongly interconnected systems with completely unknown nonlinearities.

With all these explanations, the objective of this work is to develop a decentralized adaptive control scheme for a class of high-order stochastic nonlinear large-scale systems subject to unknown strongly interconnected nonlinearities. In the controller design process, RBF neural networks will be used to approximate the unknown nonlinearities. Then, based on the backstepping design technique and adaptive control theory, we propose an approximation-based tracking control scheme to guarantee the boundedness of the closed-loop system in probability. Compared with previous works, the main advantages of this paper are summarized as follows.

(1) A new adaptive tracking control method is developed for high-order stochastic nonlinear strongly interconnected systems using stochastic Lyapunov approach. Compared with [42], this paper considers the stochastic disturbances, and contributes a method of practical value.
(2) This paper considers completely unknown strongly interconnected nonlinearities. Compared with existing control schemes for high-order systems [25,37], RBF neural networks approximation for decentralized control of high-order systems is achieved.
(3) In the control design of uncertain high-order systems, the number of the learning parameters is reduced by updating the estimation of the weights' norm instead of updating the weights, thus greatly reducing the online computation burden.

The remainder of this paper is organized as follows. Section 2 provides the problem formulation and preliminary results. The adaptive neural control design and the stability analysis are developed in Section 3. Section 4 offers two simulation examples. Finally, conclusions are drawn in Section 5.

2. Preliminaries and problem formulation

2.1. Preliminaries

Consider the following stochastic nonlinear system

$$dx = f(x,t)dt + h(x,t)d\omega, \forall x \in \mathbf{R}^n \tag{1}$$

where $x \in \mathbf{R}^n$ is the system state vector, $f: \mathbf{R}^n \times \mathbf{R}^+ \to \mathbf{R}^n$, $h: \mathbf{R}^n \times \mathbf{R}^+ \to \mathbf{R}^{n\times r}$ are locally Lipschitz functions and satisfy $f(0,t) = 0$, $h(0,t) = 0$. ω is an r-dimensional independent standard Brownian motion defined on the complete probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ with Ω being a sample space, F being $\sigma-$field, $\{F_t\}_{t\geq 0}$ being a filtration, and P being a probability measure.

Definition 1. [38]: For any given positive function $V(x, t) \in C^2$ $\mathbf{R}^n \to \mathbf{R}^+$, we define the differential operator L associated with the stochastic differential (1) as follows:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\left\{h^T \frac{\partial^2 V}{\partial x^2} h\right\} \tag{2}$$

where $Tr(A)$ denotes the trace of A.

Lemma 1. *[31]: For the stochastic nonlinear system (1), suppose there exists a $C^{2,1}$ function $V(x, t)$: $\mathbf{R}^n \times \mathbf{R}^+ \to \mathbf{R}^+$, two constants $c_1 > 0$ and $c_2 \geq 0$, class K_∞-functions μ_1 and μ_2 such that*

$$\mu_1(|x|) \leq V(x,t) \leq \mu_2(|x|) \tag{3}$$

$$LV \leq -c_1 V + c_2. \tag{4}$$

for all $x \in \mathbf{R}^n$ and $t > t_0$. Then, there is an unique strong solution of system (1) for each $x_0 \in \mathbf{R}^n$ and it satisfies

$$E(V(x,t)) \leq V(0)e^{-c_1 t} + \frac{c_2}{c_1}, \forall t > t_0 \tag{5}$$

Lemma 2. *(Young's inequality [5]): For $\forall(x, y) \in \mathbf{R}^2$, the following inequality holds:*

$$xy \leq \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q \tag{6}$$

where $\varepsilon > 0$, $p > 1$, $q > 1$, $(p-1)(q-1) = 1$.

2.2. Problem formulation

Consider the following uncertain strongly interconnected stochastic high-order nonlinear systems

$$\begin{cases} dx_{i,j_i} = (x_{i,j_i+1}^{p_{i,j_i}} + f_{i,j_i}(X))dt + g_{i,j_i}^T(\bar{x}_{i,j_i})d\omega_i, \\ dx_{i,n_i} = (u_i^{p_{i,n_i}} + f_{i,n_i}(X))dt + g_{i,n_i}^T(x_i)d\omega_i, \\ y_i = x_{i,1}, i = 1, \ldots, m, j_i = 1, \ldots, n_i - 1, n_i > 1. \end{cases} \tag{7}$$

where $X = [x_1^T, x_2^T, \ldots, x_m^T]^T$ represents the full states of the overall system with $x_i = [x_{i,1}, \ldots, x_{i,n_i}]^T \in \mathbf{R}^{n_i}$, $\bar{x}_{i,k} = [x_{i,1}, \ldots, x_{i,k}]^T \in \mathbf{R}^k$, $k = 1, \ldots, n_i$, is the system state vector. $y_i \in \mathbf{R}$ and $u_i \in \mathbf{R}$ are the system output and the control input of the ith subsystem, respectively. $p_{i,j_i} \in \mathbf{R}^* \triangleq \{\lambda \in R$: $\lambda \geq 1$ and λ is a ratio of odd integers}. $f_{i,j_i}(\cdot)$ and $g_{i,j_i}(\cdot)$ are unknown smooth nonlinear functions with $f_{i,j_i}(0) = 0$ and $g_{i,j_i}(0) = 0$. ω_i is an independent r_i-dimensional standard Brownian motion defined in (1).

Remark 1. The nonlinear functions $f_{i,j_i}(X)$ contain full states of the overall system. Thus, (7) is called the strongly interconnected nonlinear system. Note that (7) is different from the large-scale systems considered in [3,46] which only depend on the output of each subsystems. Compared with the existing methods for high-order systems [42], (7) incorporates the stochastic disturbances. In [44], single-input single-output high-order stochastic nonlinear systems were concerned. This paper investigates decentralized adaptive neural backstepping control for a class of high-order stochastic nonlinear systems with unknown strongly interconnected nonlinearity and (7) is of a more general structure than the existing results.

The objective of this paper is to design the decentralized adaptive controller such that all the signals in the closed-loop system are bounded and system output y_i can track the bounded reference signal y_{di} as closely as possible.

Assumption 1. [7] The reference signal y_{di} and its kth-order time derivatives $y_{di}^{(k)}$ are continuous and bounded for $k = 1, \ldots, n_i$. For simplicity, it is assumed that $|y_{di}| \leq d^*$, and $|y_{di}^{(k)}| \leq d^*$ with d^* being a constant.

Lemma 3. *[19] Let $p \in R^*$, $c \geq 1$ and x, y be real-valued functions, the following inequalities hold*

$$\left|x^p - y^p\right| \leq p|x-y|\left|x^{p-1} + y^{p-1}\right|$$
$$|x+y|^c \leq 2^{c-1}(|x|^c + |y|^c)$$

Assumption 2. [1,2,30] For the function $f_{i,j_i}(X)$, there exist strictly increasing smooth functions $\Phi_{i,j_i}(\cdot)$: $\mathbf{R}^+ \to \mathbf{R}^+$ with $\Phi_i(0) = 0$, such that for $i = 1, 2, \ldots, m$, $j_i = 1, 2, \ldots, n_i$, one has

$$|f_{i,j_i}(X)| \leq \Phi_{i,j_i}(\|X\|) \tag{8}$$

Remark 2. [2] Based on Assumption 2, there exists a strictly increasing smooth function $\Phi_{i,j_i}(\cdot)$ with $\Phi_{i,j_i}(0) = 0$. One has

$$\Phi_{i,j_i}\left(\sum_{k=1}^{n} a_k\right) \leq \Phi_{i,j_i}(na) \leq \sum_{k=1}^{n} \Phi_{i,j_i}(na_k) \tag{9}$$

where $a = \max_{1 \leq k \leq n}\{a_k\}$, $\sum_{k=1}^{n} a_k \leq na$.

There exists a continuous function $\psi_{i,j_i}(\cdot)$ satisfying $\Phi_{i,j_i}(s) = s\psi_{i,j_i}(s)$ such that

$$\Phi_{i,j_i}\left(\sum_{k=1}^{n} a_k\right) \leq \sum_{k=1}^{n} na_k \psi_{i,j_i}(na_k) \tag{10}$$

Such the property will be helpful for the backstepping design in this paper.

Lemma 4. *[22] For any variable $\eta \in \mathbf{R}$ and the constant $\varepsilon > 0$, the following inequality holds, $0 \leq |\eta| - \eta \tanh(\frac{\eta}{\varepsilon}) \leq \delta\varepsilon, \delta =$ 0.2785.*

2.3. RBF neural networks

In this paper, the continuous function $f(Z)$ can be approximated by RBF neural network (NN) over a compact set $\Omega_Z \subset \mathbf{R}^q$ for given arbitrary accuracy $\epsilon^* > 0$ as follows

$$f(Z) = W^{*\mathrm{T}} S(Z) + \epsilon(Z) \tag{11}$$

where $Z \in \Omega_Z \subset \mathbf{R}^q$ is the input vector with q being the NN input dimension. $W^* = [w_1^*, w_2^*, \cdots, w_l^*]^{\mathrm{T}} \in \mathbf{R}^l$ is the ideal NN weight vector with $l > 1$ being the NN node number. $\epsilon(Z)$ is the approximation error, $\|\epsilon(Z)\| \leq \epsilon^*$. $S(Z) = [s_1(Z), s_2(Z), \cdots, s_l(Z)]^{\mathrm{T}}$ means the basis function vector with $s_i(Z)$ being chosen as

$$s_i(Z) = \exp\left[\frac{-(Z-\xi_i)^{\mathrm{T}}(Z-\xi_i)}{\eta^2}\right], i = 1, \cdots, l \tag{12}$$

where $\xi_i = [\xi_{i1}, \xi_{i2}, \cdots, \xi_{iq}]^{\mathrm{T}}$ is the center of the receptive field and η is the width of Gaussian function.

The ideal constant weight vector is defined as

$$W^* := \arg\min_{\hat{W} \in R^l}\left\{\sup_{Z \in \Omega_Z} |f(Z) - \hat{W}^{\mathrm{T}} S(Z)|\right\} \tag{13}$$

where $\hat{W}$ denotes the estimate of W^*.

Lemma 5. *[10] Consider the Gaussian RBF NN (11). Let $\varrho: = \frac{1}{2}\min_{i \neq j}\|\xi_i - \xi_j\|$. q is the dimension of the NN input Z and η is the width of the Gaussian function (12), then an upper bound of $\|S(Z)\|$ is taken as*

$$\|S(Z)\| \leq \sum_{k=0}^{\infty} 3q(k+2)^{q-1} e^{-2\varrho^2 k^2/\eta^2} := s^* \tag{14}$$

where s^* is a limited value and is independent of neural input Z and the dimension l of the NN weights.

In fact, as the ideal constant weight vector, W^* needs to be estimated by $\hat{W}$. However, in this paper, we do not directly estimate the value of W^* but estimate the norm of W^*. Noting that $\|W^*\|^2$ is an unknown constant, a θ^* is used to express as $\|W^*\|^2 = b\theta^*$ with b being a positive constant. $\hat{\theta}$ denotes the estimate of θ^*, and the resulting estimation error $\tilde{\theta}$ is defined as $\tilde{\theta} = \hat{\theta} - \theta^*$.

Remark 3. The ideal weight $W^* \in \mathbf{R}^l$ has l unknown elements to be estimated. In the adaptive control design, the estimated values of these unknown constants are updated, which cause too many tuning parameters and directly increase the computational burden. By estimating the norms of the weight vectors instead of themselves, in this paper, only one positive unknown parameter θ^* need to be estimated. The replacement of the weight vector W^* by unknown constant θ^* has also been used in [40].

3. Adaptive control design and stability analysis

In this section, we give the design procedure, which is divided into n_i steps based on backstepping technique. Firstly, we define $p_i = \max\{p_{i,j_i}, i = 1, \dots, m, \quad j_i = 1, \dots, n_i\}$ with p_{i,j_i} being a positive odd integer. Then, the following coordinate transformation is introduced,

$$z_{i,1} = x_{i,1} - y_{di}, z_{i,j_i} = x_{i,j_i} - \alpha_{i,j_i-1}, i = 1, \dots, m, j_i = 2, \dots, n_i \tag{15}$$

where α_{i,j_i-1} is the virtual control signal which will be special later.

For $i = 1, \dots, m,\ j_i = 1, \dots, n_i - 1$, the virtual control signals α_{i,j_i} and the adaptive law $\dot{\hat{\theta}}_{i,j_i}$ are given as

$$\alpha_{i,j_i} = -z_{i,j_i}\left\{k_{i,j_i} + \eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}\hat{\theta}_{i,j_i}\left(S_{i,j_i}^{\mathrm{T}}(Z_{i,j_i})S_{i,j_i}(Z_{i,j_i})\right)^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}} + 1\right\}^{\frac{1}{p_{i,j_i}}} \tag{16}$$

where k_{i,j_i} is a positive design parameter; $\hat{\theta}_{i,j_i}$ is the estimator of θ^*_{i,j_i} with $\tilde{\theta}_{i,j_i} = \hat{\theta}_{i,j_i} - \theta^*_{i,j_i}$.

$$\dot{\hat{\theta}}_{i,j_i} = r_{i,j_i}\eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}\left(S_{i,j_i}^{\mathrm{T}}(Z_{i,j_i})S_{i,j_i}(Z_{i,j_i})\right)^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}} z_{i,j_i}^{p_i+3} - \sigma_{i,j_i}\hat{\theta}_{i,j_i}. \tag{17}$$

where σ_{i,j_i} is the design parameter. $Z_{i,j_i} = [x_{i,1}, \dots, x_{i,j_i}, y_{di}, \dots, y_{di}^{(j_i)}, \hat{\theta}_{i,1}, \dots, \hat{\theta}_{i,j_i-1}]^{\mathrm{T}}$.

For the given variable conversion $z_{i,j_i} = x_{i,j_i} - \alpha_{i,j_i-1}(i = 1, \dots, m,\ j_i = 1, \dots, n_i)$, $\alpha_{i,0} = y_{di}$, the following result holds

$$\begin{aligned}
\|X\| &\le \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}|x_{i,j_i}| \le \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}|z_{i,j_i}| + \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}|\alpha_{i,j_i-1}| \\
&= \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}|z_i| + \sum_{i=1}^{m}|y_{di}| \\
&\quad + \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}\left(\left\{k_{i,j_i} + \eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}\hat{\theta}_{i,j_i}\left(S_{i,j_i}^{\mathrm{T}}(Z_{i,j_i})S_{i,j_i}(Z_{i,j_i})\right)^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}} + 1\right\}^{\frac{1}{p_{i,j_i}}}\right)|z_{i,j_i}| \\
&\le \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}\Gamma_{i,j_i}(\hat{\theta}_{i,j_i})|z_{i,j_i}| + \bar{d}^*
\end{aligned} \tag{18}$$

where $\bar{d}^* = md^*$ with d^* being the upper bound of y_{di}. $\Gamma_{i,j_i}(\hat{\theta}_{i,j_i}) = 1 + \left\{k_{i,j_i} + \eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}\hat{\theta}_{i,j_i}\left(S_{i,j_i}^{\mathrm{T}}(Z_{i,j_i})S_{i,j_i}(Z_{i,j_i})\right)^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}} + 1\right\}^{\frac{1}{p_{i,j_i}}}$, $i = 1, \dots, m,\ j_i = 1, \dots, n_i$. $\Gamma_{i,n_i} = 1$.

Step i, 1: Since $z_{i,1} = x_{i,1} - y_{di}$, considering (7), it is obvious that

$$dz_{i,1} = (x_{i,2}^{p_{i,1}} + f_{i,1}(X) - \dot{y}_{di})dt + g_{i,1}^T(x_{i,1})d\omega \tag{19}$$

Choose the following Lyapunov function candidate

$$V_{i,1} = \frac{z_{i,1}^{p_i-p_{i,1}+4}}{p_i - p_{i,1} + 4} + \frac{\tilde{\theta}_{i,1}^2}{2r_{i,1}} \tag{20}$$

where $r_{i,\,1}$ is the positive design parameter.

By Itô differentiation of $V_{i,\,1}$ and considering (19), we can obtain

$$\begin{aligned}
LV_{i,1} &= z_{i,1}^{p_i-p_{i,1}+3}(x_{i,2}^{p_{i,1}} + f_{i,1}(X) - \dot{y}_{di}) \\
&\quad + \frac{p - p_{i,1} + 3}{2}g_{i,1}^T g_{i,1} z_{i,1}^{p_i-p_{i,1}+2} + \frac{\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}}{r_{i,1}}
\end{aligned} \tag{21}$$

By using the Young's inequality, one can get

$$\begin{aligned}
&\frac{p - p_{i,1} + 3}{2}g_{i,1}^T g_{i,1} z_{i,1}^{p_i-p_{i,1}+2} \\
&\le \frac{p_i - p_{i,1} + 2}{2}\xi_{i,1}^{\frac{p_i-p_{i,1}+3}{p_i-p_{i,1}+2}} z_{i,1}^{p_i-p_{i,1}+3}\|g_{i,1}\|^{\frac{2(p_i-p_{i,1}+3)}{p_i-p_{i,1}+2}} + \frac{1}{2}\xi_{i,1}^{-(p_i-p_{i,1}+3)}
\end{aligned} \tag{22}$$

where $\xi_{i,\,1}$ is the positive design parameter.

By applying Assumption 2, Lemma 2, (18), and Young's inequality, we have

$$\begin{aligned} z_{i,1}^{p_i-p_{i,1}+3} f_{i,1}(X) &\le |z_{i,1}^{p_i-p_{i,1}+3}||f_{i,1}(X)| \le |z_{i,1}^{p_i-p_{i,1}+3}|\Phi_{i,1}(\|X\|) \\ &\le |z_{i,1}^{p_i-p_{i,1}+3}|\Phi_{i,1}\Big(\sum_{l=1}^{m}\sum_{q=1}^{n_l}\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}| + \bar{d}^*\Big) \\ &\le |z_{i,1}^{p_i-p_{i,1}+3}|\sum_{l=1}^{m}\sum_{q=1}^{n_l}\Phi_{i,1}\big((M+1)|z_{l,q}|\Gamma_{l,q}(\hat{\theta}_{l,q})\big) + |z_{i,1}^{p_i-p_{i,1}+3}|\Phi_{i,1}\big((M+1)\bar{d}^*\big) \\ &\le |z_{i,1}^{p_i-p_{i,1}+3}|\sum_{l=1}^{m}\sum_{q=1}^{n_l}|z_{l,q}|\bar{\Phi}_{i,1}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) + |z_{i,1}^{p_i-p_{i,1}+3}|\Phi_{i,1}\big((M+1)\bar{d}^*\big) \\ &\le \frac{p_i+2}{p_i+3}Mz_{i,1}^{\frac{(p_i+3)(p_i-p_{i,1}+3)}{p_i+2}} + \frac{1}{p_i+3}\sum_{l=1}^{m}\sum_{q=1}^{n_l} z_{l,q}^{p_i+3}\bar{\Phi}_{i,1}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) \\ &\quad + |z_{i,1}^{p_i-p_{i,1}+3}|B_{i,1} \end{aligned} \tag{23}$$

where $M=\sum_{i=1}^{m} n_i$, $B_{i,1}=\Phi_{i,1}((M+1)\bar{d}^*)$, $\bar{\Phi}_{i,1}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) = (M+1)\Gamma_{l,q}\psi_{i,1}((M+1)\Gamma_{l,q}|z_{l,q}|)$.

Based on Lemma 4, one has

$$|z_{i,1}^{p_i-p_{i,1}+3}|B_{i,1} \le z_{i,1}^{p_i-p_{i,1}+3}B_{i,1}\tanh\left(\frac{z_{i,1}^{p_i-p_{i,1}+3}B_{i,1}}{\varepsilon_{i,1}}\right) + \delta\varepsilon_{i,1} \tag{24}$$

where $\varepsilon_{i,1}>0$.

Then, one can obtain

$$\begin{aligned} LV_{i,1} &\le z_{i,1}^{p_i-p_{i,1}+3}x_{i,2}^{p_{i,1}} + z_{i,1}^{p_i-p_{i,1}+3}\Bigg(B_{i,1}\tanh\left(\frac{z_{i,1}^{p_i-p_{i,1}+3}B_{i,1}}{\varepsilon_{i,1}}\right) \\ &\quad + \frac{p_i-p_{i,1}+2}{2}\xi_{i,1}^{\frac{p_i-p_{i,1}+3}{p_i-p_{i,1}+2}}\|g_{i,1}\|^{\frac{2(p_i-p_{i,1}+3)}{p_i-p_{i,1}+2}} + \frac{p_i+2}{p_i+3}Mz_{i,1}^{\frac{p_i-p_{i,1}+3}{p_i+2}} - \dot{y}_{di}\Bigg) + \frac{1}{2}\xi_{i,1}^{-(p_i-p_{i,1}+3)} \\ &\quad + \frac{\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}}{r_{i,1}} + \delta\varepsilon_{i,1} + \frac{1}{p_i+3}\sum_{l=1}^{m}\sum_{q=1}^{n_l} z_{l,q}^{p_i+3}\bar{\Phi}_{i,1}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) \end{aligned} \tag{25}$$

Define the unknown nonlinear $F_{i,1}(Z_{i,1})$ as

$$\begin{aligned} F_{i,1}(Z_{i,1}) &= B_{i,1}\tanh\left(\frac{z_{i,1}^{p_i-p_{i,1}+3}B_{i,1}}{\varepsilon_{i,1}}\right) - \dot{y}_{di} + \Psi_{i,1}(x_{i,1}) \\ &\quad + \frac{p_i-p_{i,1}+2}{2}\xi_{i,1}^{\frac{p_i-p_{i,1}+3}{p_i-p_{i,1}+2}}\|g_{i,1}\|^{\frac{2(p_i-p_{i,1}+3)}{p_i-p_{i,1}+2}} + \frac{p_i+2}{p_i+3}Mz_{i,1}^{\frac{p_i-p_{i,1}+3}{p_i+2}} \end{aligned} \tag{26}$$

where $\Psi_{i,1}(x_{i,1})$ will be defined later. $Z_{i,1}=[x_{i,1}, y_{di}, \dot{y}_{di}]^{\mathrm{T}} \in \Omega_{i,1}$. $\Psi_{i,1}(x_{i,1})$ is defined later.

A RBF NN $W_{i,1}^{*\mathrm{T}}S_{i,1}$ is employed to approximate the unknown function $F_{i,1}$ for any given $\epsilon_{i,1}^*>0$.

$$F_{i,1}(Z_{i,1}) = W_{i,1}^{*\mathrm{T}}S_{i,1}(Z_{i,1}) + \epsilon(Z_{i,1}),\ |\epsilon(Z_{i,1})| \le \epsilon_{i,1}^*. \tag{27}$$

where $\epsilon(Z_{i,1})$ is the approximation error.

Using the Young's inequality, one has:

$$\begin{aligned} z_{i,1}^{p_i-p_{i,1}+3}F_{i,1} &= z_{i,1}^{p_i-p_{i,1}+3}\big(W_{i,1}^{*\mathrm{T}}S_{i,1}(Z_{i,1}) + \epsilon(Z_{i,1})\big) \\ &\le \frac{p_i-p_{i,1}+3}{p_i+3}\eta_{i,1}^{\frac{p_i+3}{p_i-p_{i,1}+3}}z_{i,1}^{p_i+3}\|W_{i,1}^*\|^{\frac{p_i+3}{p_i-p_{i,1}+3}}(S_{i,1}^{\mathrm{T}}S_{i,1})^{\frac{p_i+3}{2(p_i-p_{i,1}+3)}} \\ &\quad + \frac{p_{i,1}}{p_i+3}\eta_{i,1}^{-\frac{p_i+3}{p_{i,1}}} + \frac{p_i-p_{i,1}+3}{p_i+3}z_{i,1}^{p_i+3} + \frac{p_{i,1}}{p_i+3}\epsilon_{i,1}^{*\frac{p_i+3}{p_{i,1}}} \\ &\le \eta_{i,1}^{\frac{p_i+3}{p_i-p_{i,1}+3}}z_{i,1}^{p_i+3}\theta_{i,1}^*(S_{i,1}^{\mathrm{T}}S_{i,1})^{\frac{p_i+3}{2(p_i-p_{i,1}+3)}} + \eta_{i,1}^{-\frac{p_i+3}{p_{i,1}}} + z_{i,1}^{p_i+3} + \epsilon_{i,1}^{*\frac{p_i+3}{p_{i,1}}} \end{aligned} \tag{28}$$

where $\theta_{i,1}^* = \|W_{i,1}^*\|^{\frac{p_i+3}{p_i-p_{i,1}+3}}$

Combining (25)–(28), one has

$$\begin{aligned} LV_{i,1} &\le z_{i,1}^{p_i-p_{i,1}+3}x_{i,2}^{p_{i,1}} + z_{i,1}^{p_i+3}\left(\eta_{i,1}^{\frac{p_i+3}{p_i-p_{i,1}+3}}\theta_{i,1}^*(S_{i,1}^{\mathrm{T}}S_{i,1})^{\frac{p_i+3}{2(p_i-p_{i,1}+3)}} + 1\right) + \eta_{i,1}^{-\frac{p_i+3}{p_{i,1}}} \\ &\quad + \epsilon_{i,1}^{*\frac{p_i+3}{p_{i,1}}} + \frac{1}{2}\xi_{i,1}^{-(p_i-p_{i,1}+3)} - z_{i,1}^{p_i-p_{i,1}+3}\Psi_{i,1}(x_{i,1}) \end{aligned}$$

$$+\frac{\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}}{r_{i,1}}+\delta\varepsilon_{i,1}+\frac{1}{p_i+3}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,1}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) \tag{29}$$

Then, the virtual control signal is designed as

$$\alpha_{i,1}=-z_{i,1}\left\{k_{i,1}+\eta_{i,1}^{\frac{p_i+3}{p_i-p_{i,1}+3}}\hat{\theta}_{i,1}(S_{i,1}^{\mathrm{T}}S_{i,1})^{\frac{p_i+3}{2(p_i-p_{i,1}+3)}}+1\right\}^{\frac{1}{p_{i,1}}}=-z_{i,1}\varrho_{i,1} \tag{30}$$

where $\hat{\theta}_{i,1}$ is the estimator of $\theta_{i,1}^{*}$ with $\tilde{\theta}_{i,1}=\hat{\theta}_{i,1}-\theta_{i,1}^{*}$. The adaptive law is designed as

$$\dot{\hat{\theta}}_{i,1}=r_{i,1}\eta_{i,1}^{\frac{p_i+3}{p_i-p_{i,1}+3}}(S_{i,1}^{\mathrm{T}}S_{i,1})^{\frac{p_i+3}{2(p_i-p_{i,1}+3)}}z_{i,1}^{p_i+3}-\sigma_{i,1}\hat{\theta}_{i,1}. \tag{31}$$

where $\sigma_{i,\,1}$ is the design parameter.

By substituting (30) and (31) into (29), and considering $-\frac{\sigma_{i,1}}{r_{i,1}}\tilde{\theta}_{i,1}\hat{\theta}_{i,1}\le-\frac{\sigma_{i,1}}{2r_{i,1}}\tilde{\theta}_{i,1}^{2}+\frac{\sigma_{i,1}}{2r_{i,1}}\theta_{i,1}^{*2}$, we have

$$\begin{aligned}LV_{i,1}\le&-k_{i,1}z_{i,1}^{p_i+3}+z_{i,1}^{p_i-p_{i,1}+3}(x_{i,2}^{p_{i,1}}-\alpha_{i,1}^{p_{i,1}})-\frac{\sigma_{i,1}}{2r_{i,1}}\tilde{\theta}_{i,1}^{2}+\frac{\sigma_{i,1}}{2r_{i,1}}\theta_{i,1}^{*2}\\&+\eta_{i,1}^{-\frac{p_i+3}{p_{i,1}}}+\epsilon_{i,1}^{*\frac{p_i+3}{p_{i,1}}}+\frac{1}{2}\xi_{i,1}^{-(p_i-p_{i,1}+3)}-z_{i,1}^{p_i-p_{i,1}+3}\Psi_{i,1}(x_{i,1})\\&+\delta\varepsilon_{i,1}+\frac{1}{p_i+3}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,1}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)\end{aligned} \tag{32}$$

Since $z_{i,2}=x_{i,2}-\alpha_{i,1}$, by using Lemma 3 and Young's inequality, we can obtain

$$\begin{aligned}z_{i,1}^{p_i-p_{i,1}+3}(x_{i,2}^{p_{i,1}}-\alpha_{i,1}^{p_{i,1}})\le&\left|z_{i,1}^{p_i-p_{i,1}+3}\right||x_{i,2}^{p_{i,1}}-\alpha_{i,1}^{p_{i,1}}|\\\le&\,p_{i,1}\left|z_{i,1}^{p_i-p_{i,1}+3}\right||z_{i,2}|\left|(z_{i,2}+\alpha_{i,1})^{p_{i,1}-1}+(z_{i,1}\varrho_{i,1})^{p_{i,1}-1}\right|\\\le&\,p_{i,1}\left|z_{i,1}^{p_i-p_{i,1}+3}\right||z_{i,2}|\left|2^{p_{i,1}-2}\left(z_{i,2}^{p_{i,1}-1}+(z_{i,1}\varrho_{i,1})^{p_{i,1}-1}\right)+(z_{i,1}\varrho_{i,1})^{p_{i,1}-1}\right|\\\le&\,z_{i,1}^{p_i+3}+z_{i,2}^{p_i-p_{i,2}+3}\bar{\varrho}_{i,1}\end{aligned} \tag{33}$$

where $\bar{\varrho}_{i,1}=z_{i,2}^{p_{i,2}}\left((2^{p_{i,1}-2}p_{i,1})^{(p_i+3/p_{i,1})}+(2^{p_{i,1}-2}p_{i,1}\varrho_{i,1}^{p_{i,1}-1})^{p_i+3}+(p_{i,1}\varrho_{i,1}^{p_{i,1}-1})^{p_i+3}\right)$.

Considering (33), (29) can be rearrange as

$$\begin{aligned}LV_{i,1}\le&-(k_{i,1}-1)z_{i,1}^{p_i+3}+z_{i,2}^{p_i-p_{i,2}+3}\bar{\varrho}_{i,1}-\frac{\sigma_{i,1}}{2r_{i,1}}\tilde{\theta}_{i,1}^{2}+\frac{\sigma_{i,1}}{2r_{i,1}}\theta_{i,1}^{*2}+\eta_{i,1}^{-\frac{p_i+3}{p_{i,1}}}\\&+\epsilon_{i,1}^{*\frac{p_i+3}{p_{i,1}}}+\frac{1}{2}\xi_{i,1}^{-(p_i-p_{i,1}+3)}-z_{i,1}^{p_i-p_{i,1}+3}\Psi_{i,1}(x_{i,1})+\delta\varepsilon_{i,1}\\&+\frac{1}{p_i+3}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,1}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)\end{aligned} \tag{34}$$

Step i, j_i $(i=1,\ldots,m;\ j_i=2\ldots n_i-1)$: A similar procedure is recursively used for each step j_i. Define $z_{i,j_i}=x_{i,j_i}-\alpha_{i,j_i-1}$ and its differential is

$$\begin{aligned}dz_{i,j_i}=&(x_{i,j_i+1}^{p_{i,j_i}}+f_{i,j_i}(X)-l\alpha_{i,j_i-1})dt\\&+\left(g_{i,j_i}-\sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}g_{i,k}\right)^{T}d\omega_i,\end{aligned} \tag{35}$$

where

$$\begin{aligned}l\alpha_{i,j_i-1}=&\sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}\left(x_{i,k+1}^{p_{i,k}}+f_{i,k}(X)\right)+\sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial\hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k}\\&+\sum_{k=0}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial y_{di}^{(k)}}y_{di}^{(k+1)}+\frac{1}{2}\sum_{p,q=1}^{j_i-1}\frac{\partial^2\alpha_{i,j_i-1}}{\partial x_{i,p}\partial x_{i,q}}g_{i,p}^{T}g_{i,q}\end{aligned} \tag{36}$$

Choose the Lyapunov function candidate V_{i,j_i} as

$$V_{i,j_i}=\frac{1}{p_i-p_{i,j_i}+4}z_{i,j_i}^{p_i-p_{i,j_i}+4}+\frac{1}{2r_{i,j_i}}\tilde{\theta}_{i,j_i}^{2} \tag{37}$$

where r_{i,j_i} is the positive design parameter.

The differential operator L of V_{i,j_i} is given by

$$\begin{aligned}LV_{i,j_i} &= z_{i,j_i}^{p_i-p_{i,j_i}+3}\Big(x_{i,j_i+1}^{p_{i,j_i}} + f_{i,j_i}(X) - l\alpha_{i,j_i-1}\Big) + \frac{1}{r_{i,j_i}}\tilde{\theta}_{i,j_i}\dot{\hat{\theta}}_{i,j_i} \\ &+ \frac{p_i - p_{i,j_i}+3}{2} z_{i,j_i}^{p_i-p_{i,j_i}+2}\Big(g_{i,j_i} - \sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}g_{i,k}\Big)^T\Big(g_{i,j_i} - \sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}g_{i,k}\Big)\end{aligned} \tag{38}$$

By Young's inequality, one can get

$$\begin{aligned}&\frac{p - p_{i,j_i}+3}{2}\Big(g_{i,j_i} - \sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}g_{i,k}\Big)^T\Big(g_{i,j_i} - \sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}g_{i,k}\Big)z_{i,j_i}^{p_i-p_{i,j_i}+2} \\ &\le \frac{p_i - p_{i,j_i}+2}{2}\xi_{i,j_i}^{\frac{p_i-p_{i,j_i}+3}{p_i-p_{i,j_i}+2}} z_{i,j_i}^{p_i-p_{i,j_i}+3}\Big\|g_{i,j_i} - \sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}g_{i,k}\Big\|^{\frac{2(p_i-p_{i,j_i}+3)}{p_i-p_{i,j_i}+2}} \\ &+ \frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,j_i}+3)}\end{aligned} \tag{39}$$

where ξ_{i,j_i} is the positive design parameter.

Let $\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,j_i}} = -1$

$$f_{i,j_i}(X) - \sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}f_{i,k}(X) = -\sum_{k=1}^{j_i}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}f_{i,k}(X) \tag{40}$$

Furthermore, similar to the derivations from (23), one has

$$\begin{aligned}&-z_{i,j_i}^{p_i-p_{i,j_i}+3}\sum_{k=1}^{j_i}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}f_{i,k}(X) \\ &\le \frac{p_i+2}{p_i+3}Mz_{i,j_i}^{\frac{(p_i+3)(p_i-p_{i,j_i}+3)}{p_i+2}}\sum_{k=1}^{j_i}\Big(\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}\Big)^{\frac{p_i+3}{p_i+2}} \\ &+ \frac{1}{p_i+3}\sum_{k=1}^{j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l} z_{l,q}^{p_i+3}\bar{\Phi}_{i,k}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) + |z_{i,j_i}^{p_i-p_{i,j_i}+3}|B_{i,j_i}\end{aligned} \tag{41}$$

where $M = \sum_{i=1}^m n_i$, $B_{i,j_i} = \sum_{k=1}^{j_i}\left|\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}\right|\Phi_{i,k}((M+1)\bar{d}^*)$, $\bar{\Phi}_{i,k}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) = (M+1)\Gamma_{l,q}\psi_{i,k}((M+1)\Gamma_{l,q}|z_{l,q}|)$, $k = 1, 2, \ldots, j_i$.

Based on Lemma 4, one has

$$|z_{i,j_i}^{p_i-p_{i,j_i}+3}|B_{i,j_i} \le z_{i,j_i}^{p_i-p_{i,j_i}+3}B_{i,j_i}\tanh\left(\frac{z_{i,j_i}^{p_i-p_{i,j_i}+3}B_{i,j_i}}{\varepsilon_{i,j_i}}\right) + \delta\varepsilon_{i,j_i} \tag{42}$$

where $\varepsilon_{i,j_i} > 0$.

Substituting (39) and (41) into (38), one has

$$\begin{aligned}LV_{i,j_i} &\le z_{i,j_i}^{p_i-p_{i,j_i}+3}x_{i,j_i+1}^{p_{i,j_i}} + z_{i,j_i}^{p_i-p_{i,j_i}+3}\Big(-l'\alpha_{i,j_i-1} \\ &+ \frac{p_i+2}{p_i+3}Mz_{i,j_i}^{\frac{p_i-p_{i,j_i}+3}{p_i+2}}\sum_{k=1}^{j_i}\big(\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}\big)^{\frac{p_i+3}{p_i+2}} \\ &+ \frac{p_i-p_{i,j_i}+2}{2}\xi_{i,j_i}^{\frac{p_i-p_{i,j_i}+3}{p_i-p_{i,j_i}+2}}\Big\|g_{i,j_i} - \sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}g_{i,k}\Big\|^{\frac{2(p_i-p_{i,j_i}+3)}{p_i-p_{i,j_i}+2}} \\ &+ B_{i,j_i}\tanh\Big(\frac{z_{i,j_i}^{p_i-p_{i,j_i}+3}B_{i,j_i}}{\varepsilon_{i,j_i}}\Big)\Big) + \delta\varepsilon_{i,j_i} + \frac{1}{r_{i,j_i}}\tilde{\theta}_{i,j_i}\dot{\hat{\theta}}_{i,j_i} \\ &+ \frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,j_i}+3)} + \frac{1}{p_i+3}\sum_{k=1}^{j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,k}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)\end{aligned} \tag{43}$$

where $l\alpha_{i,j_i-1} = l'\alpha_{i,j_i-1} + \sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}f_{i,k}(X)$.

The unknown nonlinear $F_{i,j_i}(Z_{i,j_i})$ is defined as

$$F_{i,j_i}(Z_{i,j_i}) = -l'\alpha_{i,j_i-1} + \frac{p_i+2}{p_i+3}Mz_{i,j_i}^{\frac{p_i-p_{i,j_i}+3}{p_i+2}}\sum_{k=1}^{j_i}\left(\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}\right)^{\frac{p_i+3}{p_i+2}} + \frac{p_i-p_{i,j_i}+2}{2}\xi_{i,j_i}^{\frac{p_i-p_{i,j_i}+3}{p_i-p_{i,j_i}+2}}\left\|g_{i,j_i}-\sum_{k=1}^{j_i-1}\frac{\partial\alpha_{i,j_i-1}}{\partial x_{i,k}}g_{i,k}\right\|^{\frac{2(p_i-p_{i,j_i}+3)}{p_i-p_{i,j_i}+2}} + B_{i,j_i}\tanh\left(\frac{z_{i,j_i}^{p_i-p_{i,j_i}+3}B_{i,j_i}}{\varepsilon_{i,j_i}}\right) + \bar{\varrho}_{i,j_i-1} + \Psi_{i,j_i}(x_{i,j_i}) \tag{44}$$

where $Z_{i,j_i} = [x_{i,1},\dots,x_{i,j_i},y_{di},\dots,y_{di}^{(j_i)},\hat{\theta}_{i,1},\dots,\hat{\theta}_{i,j_i-1}]^{\mathrm{T}} \in \Omega_{i,j_i}$. $\Psi_{i,j_i}(x_{i,j_i})$ is defined later.

A RBF NN $W_{i,j_i}^{*\mathrm{T}}S_{i,j_i}$ is employed to approximate the unknown function F_{i,j_i} for any given $\epsilon_{i,j_i}^* > 0$.

$$F_{i,j_i}(Z_{i,j_i}) = W_{i,j_i}^{*\mathrm{T}}S_{i,j_i}(Z_{i,j_i}) + \epsilon(Z_{i,j_i}),\ |\epsilon(Z_{i,j_i})| \le \epsilon_{i,j_i}^*. \tag{45}$$

where $\epsilon(Z_{i,j_i})$ is the approximation error.

Using the Young's inequality, one has:

$$\begin{aligned} z_{i,j_i}^{p_i-p_{i,j_i}+3}F_{i,j_i} &= z_{i,j_i}^{p_i-p_{i,j_i}+3}\left(W_{i,j_i}^{*\mathrm{T}}S_{i,j_i}(Z_{i,j_i})+\epsilon(Z_{i,j_i})\right)\\ &\le \frac{p_i-p_{i,j_i}+3}{p_i+3}\eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}z_{i,j_i}^{p_i+3}\|W_{i,j_i}^*\|^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}(S_{i,j_i}^{\mathrm{T}}S_{i,j_i})^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}}\\ &\quad+\frac{p_{i,j_i}}{p_i+3}\eta_{i,j_i}^{-\frac{p_i+3}{p_{i,j_i}}}+\frac{p_i-p_{i,j_i}+3}{p_i+3}z_{i,j_i}^{p_i+3}+\frac{p_{i,j_i}}{p_i+3}\epsilon_{i,j_i}^{*\frac{p_i+3}{p_{i,j_i}}}\\ &\le \eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}z_{i,j_i}^{p_i+3}\theta_{i,j_i}^*(S_{i,j_i}^{\mathrm{T}}S_{i,j_i})^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}}+\eta_{i,j_i}^{-\frac{p_i+3}{p_{i,j_i}}}+z_{i,j_i}^{p_i+3}+\epsilon_{i,j_i}^{*\frac{p_i+3}{p_{i,j_i}}} \end{aligned} \tag{46}$$

where $\theta_{i,j_i}^* = \|W_{i,j_i}^*\|^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}$

$$\begin{aligned} LV_{i,j_i} \le{}& z_{i,j_i}^{p_i-p_{i,j_i}+3}x_{i,j_i+1}^{p_{i,j_i}} + z_{i,j_i}^{p_i+3}\left(\eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}\theta_{i,j_i}^*(S_{i,j_i}^{\mathrm{T}}S_{i,j_i})^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}}+1\right)\\ &+\eta_{i,j_i}^{-\frac{p_i+3}{p_{i,j_i}}}+\epsilon_{i,j_i}^{*\frac{p_i+3}{p_{i,j_i}}}-z_{i,j_i}^{p_i-p_{i,j_i}+3}\bar{\varrho}_{i,j_i-1}+\frac{1}{r_{i,j_i}}\tilde{\theta}_{i,j_i}\dot{\hat{\theta}}_{i,j_i}+\delta\varepsilon_{i,j_i}-z_{i,j_i}^{p_i-p_{i,j_i}+3}\Psi_{i,j_i}(x_{i,j_i})\\ &+\frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,1}+3)}+\frac{1}{p_i+3}\sum_{k=1}^{j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,k}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) \end{aligned} \tag{47}$$

Then, the virtual control signal and the adaptive law are given as

$$\alpha_{i,j_i} = -z_{i,j_i}\left\{k_{i,j_i}+\eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}\hat{\theta}_{i,j_i}(S_{i,j_i}^{\mathrm{T}}S_{i,j_i})^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}}+1\right\}^{\frac{1}{p_{i,j_i}}} = -z_{i,j_i}\varrho_{i,j_i} \tag{48}$$

where k_{i,j_i} is a positive design parameter; $\hat{\theta}_{i,j_i}$ is the estimator of θ_{i,j_i}^* with $\tilde{\theta}_{i,j_i} = \hat{\theta}_{i,j_i} - \theta_{i,j_i}^*$.

$$\dot{\hat{\theta}}_{i,j_i} = r_{i,j_i}\eta_{i,j_i}^{\frac{p_i+3}{p_i-p_{i,j_i}+3}}(S_{i,j_i}^{\mathrm{T}}S_{i,j_i})^{\frac{p_i+3}{2(p_i-p_{i,j_i}+3)}}z_{i,j_i}^{p_i+3}-\sigma_{i,j_i}\hat{\theta}_{i,j_i}. \tag{49}$$

where σ_{i,j_i} is the design parameter.

Similar to (32), considering $-\frac{\sigma_{i,j_i}}{r_{i,j_i}}\tilde{\theta}_{i,j_i}\hat{\theta}_{i,j_i} \le -\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\tilde{\theta}_{i,j_i}^2+\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\theta_{i,j_i}^{*2}$, we have

$$\begin{aligned} LV_{i,j_i} \le{}& -k_{i,j_i}z_{i,j_i}^{p_i+3}+z_{i,j_i}^{p_i-p_{i,j_i}+3}\left(x_{i,j_i+1}^{p_{i,j_i}}-\alpha_{i,j_i}^{p_{i,j_i}}\right)+\eta_{i,j_i}^{-\frac{p_i+3}{p_{i,j_i}}}+\epsilon_{i,j_i}^{*\frac{p_i+3}{p_{i,j_i}}}\\ &-z_{i,j_i}^{p_i-p_{i,j_i}+3}\bar{\varrho}_{i,j_i-1}-\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\tilde{\theta}_{i,j_i}^2+\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\theta_{i,j_i}^{*2}+\delta\varepsilon_{i,j_i}-z_{i,j_i}^{p_i-p_{i,j_i}+3}\Psi_{i,j_i}(x_{i,j_i})\\ &+\frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,j_i}+3)}+\frac{1}{p_i+3}\sum_{k=1}^{j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,k}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) \end{aligned} \tag{50}$$

Since $z_{i,j_i} = x_{i,j_i} - \alpha_{i,j_i}$, similar to (33), we have

$$
\begin{aligned}
& z_{i,j_i}^{p_i-p_{i,j_i}+3}(x_{i,j_i+1}^{p_{i,j_i}}-\alpha_{i,j_i}^{p_{i,j_i}}) \le \left|z_{i,j_i}^{p_i-p_{i,j_i}+3}\right| |x_{i,j_i+1}^{p_{i,j_i}}-\alpha_{i,j_i}^{p_{i,j_i}}| \\
& \le p_{i,j_i}\left|z_{i,j_i}^{p_i-p_{i,j_i}+3}\right| |z_{i,j_i+1}| \left|(z_{i,j_i+1}+\alpha_{i,j_i})^{p_{i,j_i}-1}+(z_{i,j_i}\varrho_{i,j_i})^{p_{i,j_i}-1}\right| \\
& \le p_{i,j_i}\left|z_{i,j_i}^{p_i-p_{i,j_i}+3}\right| |z_{i,j_i+1}| \left|2^{p_{i,j_i}-2}\left(z_{i,j_i+1}^{p_{i,j_i}-1}+(z_{i,j_i}\varrho_{i,j_i})^{p_{i,j_i}-1}\right)+(z_{i,j_i}\varrho_{i,j_i})^{p_{i,j_i}-1}\right| \\
& \le z_{i,j_i}^{p_i+3}+z_{i,j_i+1}^{p_i-p_{i,j_i+1}+3}\bar{\varrho}_{i,j_i}
\end{aligned} \tag{51}
$$

where $\bar{\varrho}_{i,j_i}=z_{i,j_i+1}^{p_{i,j_i+1}}\left((2^{p_{i,j_i}-2}p_{i,j_i})^{(p_i+3/p_{i,j_i})}+(2^{p_{i,j_i}-2}p_{i,j_i}\varrho_{i,j_i}^{p_{i,j_i}-1})^{p_i+3}+(p_{i,j_i}\varrho_{i,j_i}^{p_{i,j_i}-1})^{p_i+3}\right)$.

Then, we have the following inequality.

$$
\begin{aligned}
LV_{i,j_i} \le & -(k_{i,j_i}-1)z_{i,j_i}^{p_i+3}+z_{i,j_i+1}^{p_i-p_{i,j_i+1}+3}\bar{\varrho}_{i,j_i}+\eta_{i,j_i}^{-\frac{p_i+3}{p_{i,j_i}}}+\epsilon_{i,j_i}^{*\frac{p_i+3}{p_{i,j_i}}} \\
& -z_{i,j_i}^{p_i-p_{i,j_i}+3}\bar{\varrho}_{i,j_i-1}-\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\tilde{\theta}_{i,j_i}^2+\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\theta_{i,j_i}^{*2}+\delta\varepsilon_{i,j_i}-z_{i,j_i}^{p_i-p_{i,j_i}+3}\Psi_{i,j_i}(x_{i,j_i}) \\
& +\frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,j_i}+3)}+\frac{1}{p_i+3}\sum_{k=1}^{j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,k}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)
\end{aligned} \tag{52}
$$

Step *i*, n_i: In this final step, $z_{i,n_i}=x_{i,n_i}-\alpha_{i,n_i-1}$ and its differential is

$$
dz_{i,n_i}=(u_i^{p_{i,n_i}}+f_{i,n_i}(X)-l\alpha_{i,n_i-1})dt+\Big(g_{i,n_i}-\sum_{k=1}^{j_n-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}g_{i,k}\Big)^T d\omega_i, \tag{53}
$$

where

$$
\begin{aligned}
l\alpha_{i,n_i-1} = & \sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}\left(x_{i,k+1}^{p_{i,k}}+f_{i,k}(X)\right)+\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k} \\
& +\sum_{k=0}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial y_{di}^{(k)}}y_{di}^{(k+1)}+\frac{1}{2}\sum_{p,q=1}^{n_i-1}\frac{\partial^2\alpha_{i,n_i-1}}{\partial x_{i,p}\partial x_{i,q}}g_{i,p}^T g_{i,q}
\end{aligned} \tag{54}
$$

Choose the Lyapunov function candidate V_{i,n_i} as

$$
V_{i,n_i}=\frac{1}{p_i-p_{i,n_i}+4}z_{i,n_i}^{p_i-p_{i,n_i}+4}+\frac{1}{2r_{i,n_i}}\tilde{\theta}_{i,n_i}^2 \tag{55}
$$

where r_{i,n_i} is the design parameter.

The differential operator L of V_{i,n_i} is given by

$$
\begin{aligned}
LV_{i,n_i} = & z_{i,n_i}^{p_i-p_{i,n_i}+3}\left(u_i^{p_{i,n_i}}+f_{i,n_i}(X)-l\alpha_{i,n_i-1}\right)+\frac{1}{r_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i} \\
& +\frac{p_i-p_{i,n_i}+3}{2}z_{i,n_i}^{p_i-p_{i,n_i}+2}\left(g_{i,n_i}-\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}g_{i,k}\right)^T\left(g_{i,n_i}-\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}g_{i,k}\right)
\end{aligned} \tag{56}
$$

By Young's inequality, one can get

$$
\begin{aligned}
& \frac{p-p_{i,n_i}+3}{2}\left(g_{i,n_i}-\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}g_{i,k}\right)^T\left(g_{i,n_i}-\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}g_{i,k}\right)z_{i,n_i}^{p_i-p_{i,n_i}+2} \\
& \le \frac{p_i-p_{i,n_i}+2}{2}\xi_{i,n_i}^{\frac{p_i-p_{i,n_i}+3}{p_i-p_{i,n_i}+2}}z_{i,n_i}^{p_i-p_{i,n_i}+3}\left\|g_{i,n_i}-\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}g_{i,k}\right\|^{\frac{2(p_i-p_{i,n_i}+3)}{p_i-p_{i,n_i}+2}} \\
& +\frac{1}{2}\xi_{i,n_i}^{-(p_i-p_{i,n_i}+3)}
\end{aligned} \tag{57}
$$

where ξ_{i,n_i} is the positive parameter.

Let $\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,n_i}}=-1$

$$
f_{i,n_i}(X)-\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}f_{i,k}(X)=-\sum_{k=1}^{n_i}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}f_{i,k}(X) \tag{58}
$$

$$
\begin{aligned}
&-z_{i,n_i}^{p_i-p_{i,n_i}+3}\sum_{k=1}^{n_i}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}f_{i,k}(X)\\
&\le\frac{p_i+2}{p_i+3}Mz_{i,n_i}^{\frac{(p_i+3)(p_i-p_{i,n_i}+3)}{p_i+2}}\sum_{k=1}^{n_i}\Big(\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}\Big)^{\frac{p_i+3}{p_i+2}}\\
&+\frac{1}{p_i+3}\sum_{k=1}^{n_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,k}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)+|z_{i,n_i}^{p_i-p_{i,n_i}+3}|B_{i,n_i}
\end{aligned}
\tag{59}
$$

where $M=\sum_{i=1}^{m}n_i$, $B_{i,n_i}=\sum_{k=1}^{n_i}\left|\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}\right|\Phi_{i,k}((M+1)\bar{d}^*)$, $\bar{\Phi}_{i,k}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)=(M+1)\Gamma_{l,q}\psi_{i,k}((M+1)\Gamma_{l,q}|z_{l,q}|)$, $k=1,2,\dots,n_i$.

Based on Lemma 4, one has

$$
|z_{i,n_i}^{p_i-p_{i,n_i}+3}|B_{i,n_i}\le z_{i,n_i}^{p_i-p_{i,n_i}+3}B_{i,n_i}\tanh\left(\frac{z_{i,n_i}^{p_i-p_{i,n_i}+3}B_{i,n_i}}{\varepsilon_{i,n_i}}\right)+\delta\varepsilon_{i,n_i}
\tag{60}
$$

where $\varepsilon_{i,n_i}>0$.

Substituting (57) and (59) into (56), one has

$$
\begin{aligned}
LV_{i,n_i}\le\ & z_{i,n_i}^{p_i-p_{i,n_i}+3}u_i^{p_{i,n_i}}+z_{i,n_i}^{p_i-p_{i,n_i}+3}\Big(-l'\alpha_{i,n_i-1}\\
&+\frac{p_i+2}{p_i+3}Mz_{i,n_i}^{\frac{p_i-p_{i,n_i}+3}{p_i+2}}\sum_{k=1}^{n_i}\left(\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}\right)^{\frac{p_i+3}{p_i+2}}+B_{i,n_i}\tanh\left(\frac{z_{i,n_i}^{p_i-p_{i,n_i}+3}B_{i,n_i}}{\varepsilon_{i,n_i}}\right)\\
&+\frac{p_i-p_{i,n_i}+2}{2}\xi_{i,n_i}^{\frac{p_i-p_{i,n_i}+3}{p_i-p_{i,n_i}+2}}\left\|g_{i,n_i}-\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}g_{i,k}\right\|^{\frac{2(p_i-p_{i,n_i}+3)}{p_i-p_{i,n_i}+2}}\Big)\\
&+\frac{1}{r_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i}+\frac{1}{2}\xi_{i,n_i}^{-(p_i-p_{i,n_i}+3)}+\delta\varepsilon_{i,n_i}\\
&+\frac{1}{p_i+3}\sum_{k=1}^{n_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}z_{l,q}^{p_i+3}\bar{\Phi}_{i,k}^{p_i+3}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)
\end{aligned}
\tag{61}
$$

where $l\alpha_{i,n_i-1}=l'\alpha_{i,n_i-1}+\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}f_{i,k}(X)$.

Define the unknown nonlinear $F_{i,n_i}(Z_{i,n_i})$ as

$$
\begin{aligned}
F_{i,n_i}(Z_{i,n_i})=\ & -l'\alpha_{i,n_i-1}+\frac{p_i+2}{p_i+3}Mz_{i,n_i}^{\frac{p_i-p_{i,n_i}+3}{p_i+2}}\sum_{k=1}^{n_i}\Big(\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}\Big)^{\frac{p_i+3}{p_i+2}}\\
&+B_{i,n_i}\tanh\left(\frac{z_{i,n_i}^{p_i-p_{i,n_i}+3}B_{i,n_i}}{\varepsilon_{i,n_i}}\right)+\bar{\varrho}_{i,n_i-1}+\Psi_{i,n_i}(x_{i,n_i})\\
&+\frac{p_i-p_{i,n_i}+2}{2}\xi_{i,n_i}^{\frac{p_i-p_{i,n_i}+3}{p_i-p_{i,n_i}+2}}\left\|g_{i,n_i}-\sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}g_{i,k}\right\|^{\frac{2(p_i-p_{i,n_i}+3)}{p_i-p_{i,n_i}+2}}
\end{aligned}
\tag{62}
$$

where $Z_{i,n_i}=[x_{i,1},\dots,x_{i,n_i},y_{di},\dots,y_{di}^{(n_i)},\hat{\theta}_{i,1},\dots,\hat{\theta}_{i,n_i-1}]^{\mathrm{T}}\in\Omega_{i,n_i}$. $\Psi_{i,n_i}(x_{i,n_i})$ is defined later.

A RBF NN $W_{i,n_i}^{*\mathrm{T}}S_{i,n_i}$ is employed to approximate the unknown function F_{i,n_i} for any given $\epsilon_{i,n_i}^*>0$.

$$
F_{i,n_i}(Z_{i,n_i})=W_{i,n_i}^{*\mathrm{T}}S_{i,n_i}(Z_{i,n_i})+\epsilon(Z_{i,n_i}),\ |\epsilon(Z_{i,n_i})|\le\epsilon_{i,n_i}^*.
\tag{63}
$$

where $\epsilon(Z_{i,n_i})$ is the approximation error.

Using the Young's inequality, one has:

$$
\begin{aligned}
z_{i,n_i}^{p_i-p_{i,n_i}+3}F_{i,n_i}&=z_{i,n_i}^{p_i-p_{i,n_i}+3}\big(W_{i,n_i}^{*\mathrm{T}}S_{i,n_i}(Z_{i,n_i})+\epsilon(Z_{i,n_i})\big)\\
&\le\eta_{i,n_i}^{\frac{p_i+3}{p_i-p_{i,n_i}+3}}z_{i,n_i}^{p_i+3}\theta_{i,n_i}^*(S_{i,n_i}^{\mathrm{T}}S_{i,n_i})^{\frac{p_i+3}{2(p_i-p_{i,n_i}+3)}}+\eta_{i,n_i}^{-\frac{p_i+3}{p_{i,n_i}}}\\
&\quad+z_{i,n_i}^{p_i+3}+\epsilon_{i,n_i}^{*\frac{p_i+3}{p_{i,n_i}}}
\end{aligned}
\tag{64}
$$

where $\theta^*_{i,n_i} = \|W^*_{i,n_i}\|^{\frac{p_i+3}{p_i-p_{i,n_i}+3}}$

Then, the control law and the adaptive parameter are given as

$$u_i = -z_{i,n_i}\left\{k_{i,n_i} + \eta_{i,n_i}^{\frac{p_i+3}{p_i-p_{i,n_i}+3}}\hat{\theta}_{i,n_i}(S^{\mathrm{T}}_{i,n_i}S_{i,n_i})^{\frac{p_i+3}{2(p_i-p_{i,n_i}+3)}} + 1\right\}^{\frac{1}{p_{i,n_i}}} \tag{65}$$

where k_{i,n_i} is a positive design parameter; $\hat{\theta}_{i,n_i}$ is the estimator of θ^*_{i,n_i} with $\tilde{\theta}_{i,n_i} = \hat{\theta}_{i,n_i} - \theta^*_{i,n_i}$.

$$\dot{\hat{\theta}}_{i,n_i} = r_{i,n_i}\eta_{i,n_i}^{\frac{p_i+3}{p_i-p_{i,n_i}+3}}(S^{\mathrm{T}}_{i,n_i}S_{i,n_i})^{\frac{p_i+3}{2(p_i-p_{i,n_i}+3)}}z^{p_i+3}_{i,n_i} - \sigma_{i,n_i}\hat{\theta}_{i,n_i}. \tag{66}$$

where σ_{i,n_i} is the design parameter.

Then, we have following inequality.

$$\begin{aligned}LV_{i,n_i} \le & -k_{i,n_i}z^{p_i+3}_{i,n_i} + \eta_{i,n_i}^{-\frac{p_i+3}{p_{i,n_i}}} + \epsilon^{*\frac{p_i+3}{p_{i,n_i}}}_{i,n_i} - z^{p_i-p_{i,n_i}+3}_{i,j_i}\bar{\varrho}_{i,n_i-1} \\ & -\frac{\sigma_{i,n_i}}{2r_{i,n_i}}\tilde{\theta}^2_{i,n_i} + \frac{\sigma_{i,n_i}}{2r_{i,n_i}}\theta^{*2}_{i,n_i} + \frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,n_i}+3)} - z^{p_i-p_{i,n_i}+3}_{i,n_i}\Psi_{i,n_i}(x_{i,n_i}) \\ & +\delta\varepsilon_{i,n_i} + \frac{1}{p_i+3}\sum_{k=1}^{n_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l} z^{p_i+3}_{l,q}\bar{\Phi}^{p_i+3}_{i,k}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)\end{aligned} \tag{67}$$

By rearranging sequence, one has

$$\begin{aligned}&\sum_{i=1}^{m}\left\{\frac{1}{p_i+3}\sum_{l=1}^{m}\sum_{q=1}^{n_l} z^{p_i+3}_{l,q}\bar{\Phi}^{p_i+3}_{i,1}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)\right. \\ &+\sum_{j_i=2}^{n_i-1}\frac{1}{p_i+3}\sum_{k=1}^{j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l} z^{p_i+3}_{l,q}\bar{\Phi}^{p_i+3}_{i,k}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) \\ &\left.\frac{1}{p_i+3}\sum_{k=1}^{n_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l} z^{p_i+3}_{l,q}\bar{\Phi}^{p_i+3}_{i,k}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|)\right\} \\ &= \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}\frac{1}{p_i+3}\sum_{k=1}^{j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l} z^{p_i+3}_{l,q}\bar{\Phi}^{p_i+3}_{i,k}(\Gamma_{l,q}(\hat{\theta}_{l,q})|z_{l,q}|) \\ &= \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}\frac{1}{p_i+3}z^{p_i+3}_{i,j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}\sum_{k=1}^{q}\bar{\Phi}^{p_i+3}_{l,k}(\Gamma_{i,j_i}(\hat{\theta}_{i,j_i})|z_{i,j_i}|)\end{aligned} \tag{68}$$

Define

$$\Psi_{i,1}(x_{i,1}) = \frac{1}{p_i+3}z^{p_{i,1}}_{i,1}\sum_{l=1}^{m}\sum_{q=1}^{n_l}\sum_{k=1}^{q}\bar{\Phi}^{p_i+3}_{l,k}(\Gamma_{i,1}(\hat{\theta}_{i,1})|z_{i,1}|) \tag{69}$$

$$\Psi_{i,j_i}(x_{i,j_i}) = \frac{1}{p_i+3}z^{p_{i,j_i}}_{i,j_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}\sum_{k=1}^{q}\bar{\Phi}^{p_i+3}_{l,k}(\Gamma_{i,j_i}(\hat{\theta}_{i,j_i})|z_{i,j_i}|) \tag{70}$$

$$\Psi_{i,n_i}(x_{i,n_i}) = \frac{1}{p_i+3}z^{p_{i,n_i}}_{i,n_i}\sum_{l=1}^{m}\sum_{q=1}^{n_l}\sum_{k=1}^{q}\bar{\Phi}^{p_i+3}_{l,k}(\Gamma_{i,n_i}(\hat{\theta}_{i,n_i})|z_{i,n_i}|) \tag{71}$$

Consider the whole Lyapunov function candidate as $V = \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}V_{i,j_i}$. From (34), (52) and (67), we have

$$\begin{aligned}LV \le & \sum_{i=1}^{m}\left\{-\sum_{j_i=1}^{n_i-1}(k_{i,j_i}-1)z^{p_i+3}_{i,j_i} - k_{i,n_i}z^{p_i+3}_{i,n_i} - \sum_{j_i=1}^{n_i}\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\tilde{\theta}^2_{i,j_i}\right\} \\ & + \sum_{i=1}^{m}\sum_{j_i=1}^{n_i}\left(\eta_{i,j_i}^{-\frac{p_i+3}{p_{i,j_i}}} + \epsilon^{*\frac{p_i+3}{p_{i,j_i}}}_{i,j_i} + \frac{\sigma_{i,j_i}}{2r_{i,j_i}}\theta^{*2}_{i,j_i} + \frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,j_i}+3)} + \delta\varepsilon_{i,j_i}\right)\end{aligned} \tag{72}$$

Choose $k_{i,j_i} - 1 > 0$, $k_{i,n_i} > 0$ and $\frac{\sigma_{i,j_i}}{2r_{i,j_i}} > 0$.

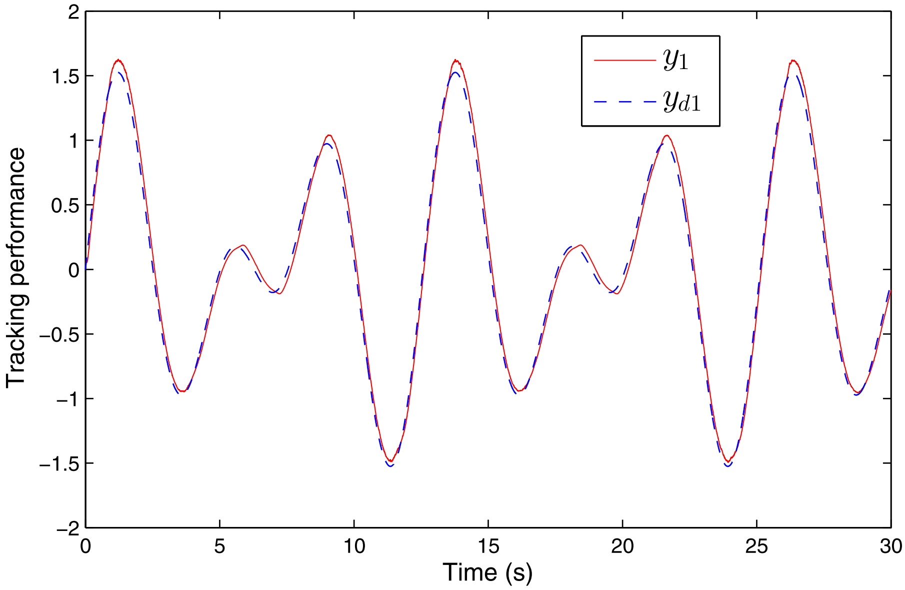

Fig. 1. System output y_1 and the reference signal y_{d1} for Example 1.

Then (72) can be rearranged as

$$\begin{aligned} LV &\le \sum_{i=1}^{m}\Big\{-\sum_{j_i=1}^{n_i-1}(k_{i,j_i}-1)z_{i,j_i}^{p_i+3}-k_{i,n_i}z_{i,n_i}^{p_i+3}-\sum_{j_i=1}^{n_i}\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\tilde{\theta}_{i,j_i}^{2}-\pi_i V_{Qi}\Big\} \\ &\quad+\sum_{i=1}^{m}\sum_{j_i=1}^{n_i}\Big(\eta_{i,j_i}^{-\frac{p_i+3}{p_{i,j_i}}}+\epsilon_{i,j_i}^{*\frac{p_i+3}{p_{i,j_i}}}+\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\theta_{i,j_i}^{*2}+\frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,j_i}+3)}+\delta\varepsilon_{i,j_i}\Big) \\ &\le -\alpha_0 V+\beta_0 \end{aligned} \tag{73}$$

where

$$\begin{aligned} &\alpha_0=\min_{\substack{1\le i\le m\\ 1\le j_i\le n_i}}\Big\{(p_i-p_{i,j_i}+4)(k_{i,j_i}-1)\beta_1^{*\frac{p_{i,j_i}-1}{p_i+3}},(p_i-p_{i,n_i}+4)k_{i,n_i}\beta_1^{*\frac{p_{i,n_i}-1}{p_i+3}},\sigma_{i,j_i}\Big\} \\ &\text{and, } \beta_0=\beta_1\Big(\sum_{i=1}^{m}\Big(\sum_{j_i=1}^{n_i-1}(k_{i,j_i}-1)+k_{i,n_i}\Big)+1\Big), \\ &\text{with}\quad \beta_1=\sum_{i=1}^{m}\sum_{j_i=1}^{n_i}\Big(\eta_{i,j_i}^{-\frac{p_i+3}{p_{i,j_i}}}+\epsilon_{i,j_i}^{*\frac{p_i+3}{p_{i,j_i}}}+\frac{\sigma_{i,j_i}}{2r_{i,j_i}}\theta_{i,j_i}^{*2}+\frac{1}{2}\xi_{i,j_i}^{-(p_i-p_{i,j_i}+3)}+\delta\varepsilon_{i,j_i}\Big). \end{aligned} \tag{74}$$

Based on [20], the following inequality can be easily obtained

$$\frac{dE[V]}{dt}\le -\alpha_0 E[V]+\beta_0 \tag{75}$$

Integrating the inequality (75), one has

$$0\le E[V]\le \Big(V(0)-\frac{\beta_0}{\alpha_0}\Big)e^{-\alpha_0 t}+\frac{\beta_0}{\alpha_0} \tag{76}$$

which implies that all the signals in the closed-loop system are SGUUB.

Set $D_0=\frac{\beta_0}{\alpha_0}$. It means that

$$E[V(t)]\le D_0,\ t\to\infty. \tag{77}$$

Then, we can obtain:

$$\lim_{t\to\infty}E[|z_{i,1}|]\le (D_0(p_i-p_{i,1}+4))^{\frac{1}{p_i-p_{i,1}+4}} \tag{78}$$

Now, we summarize the above discussion and present the results in the following theorem.

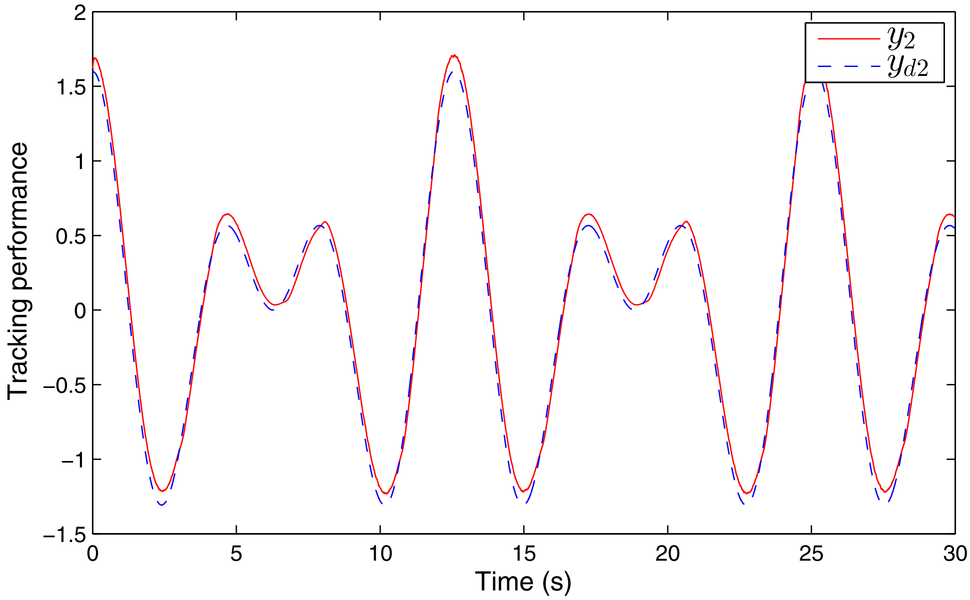

Fig. 2. System output y_2 and the reference signal y_{d2} for Example 1.

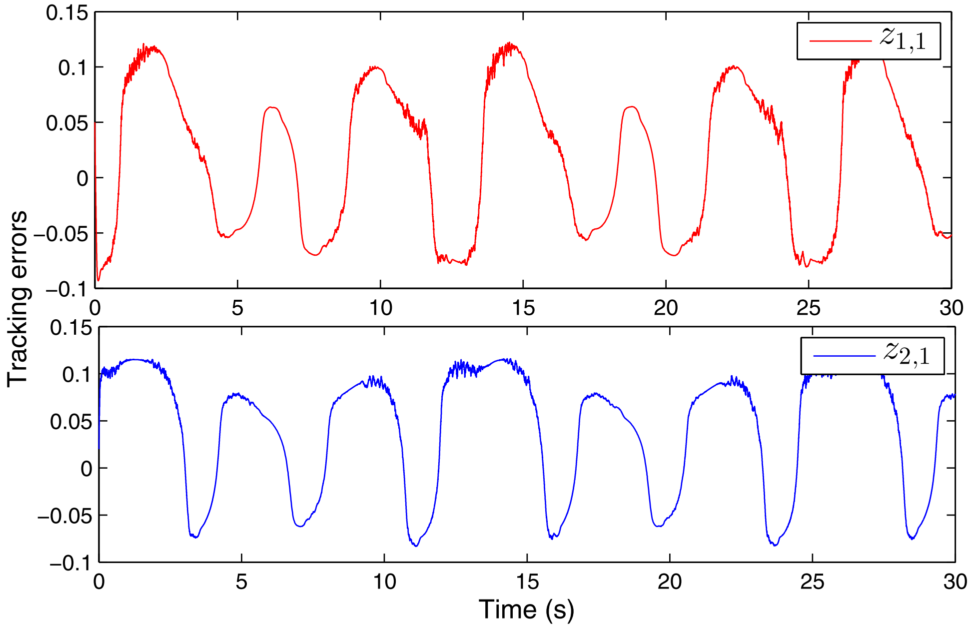

Fig. 3. Tracking errors $z_{1,\,1}$ and $z_{2,\,1}$ for Example 1.

Theorem 1. *Consider the plant (7) satisfying Assumption 1, the control law (65) and the adaptive laws (31),(49), and (66). Then, for bounded initial conditions, all closed-loop signals are bounded, and the output tracking error $z_{i,\,1}$ converges to a small neighborhood around the origin.*

4. Simulation study

In this section, to illustrate the effective of proposed control scheme, two examples including the strongly interconnected nonlinear system consisting of the third-order subsystems are provided.

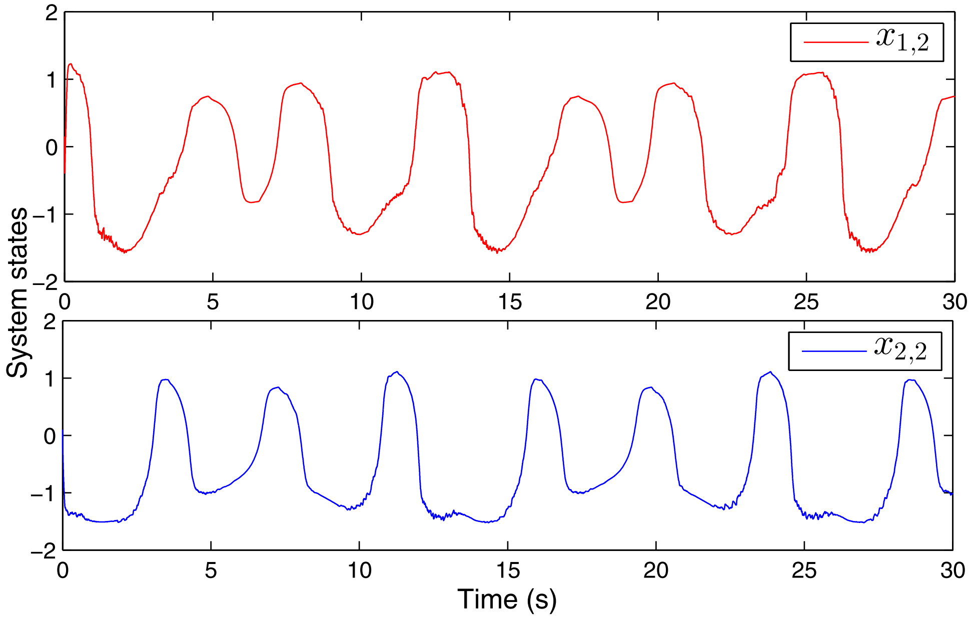

Fig. 4. System states $x_{1,2}$ and $x_{2,2}$ for Example 1.

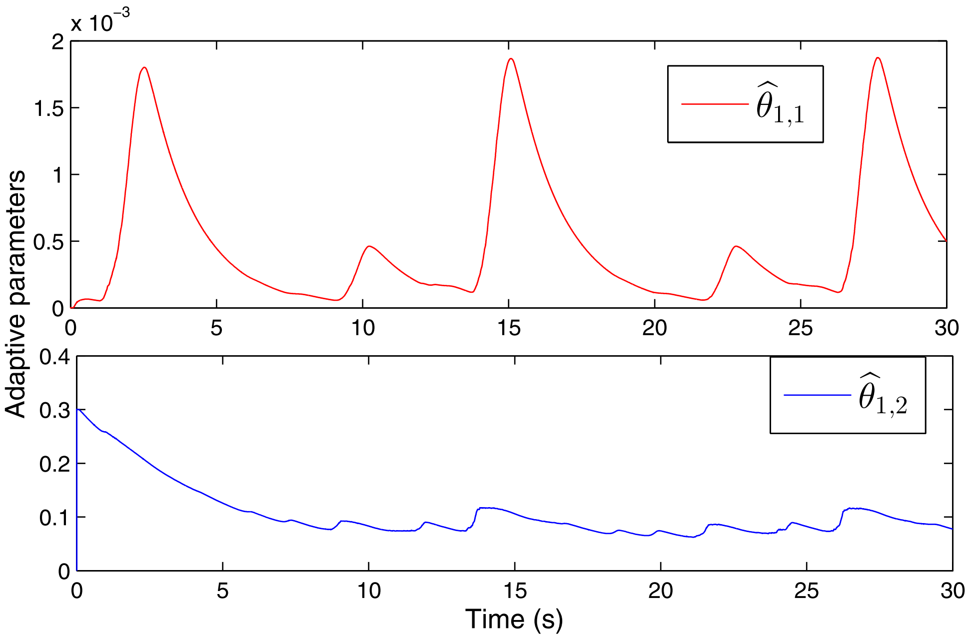

Fig. 5. Adaptive parameters $\hat{\theta}_{1,1}$ and $\hat{\theta}_{1,2}$ for Example 1.

Example 1. Consider the following high-order stochastic nonlinear interconnected system as

$$\begin{cases} dx_{i,1} = (x_{i,2}^{p_{i,1}} + f_{i,1}(x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}))dt + g_{i,1}(x_{i,1})d\omega, \\ dx_{i,2} = (u_i^{p_{i,2}} + f_{i,2}(x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}))dt + g_{i,2}(x_{i,1}, x_{i,2})d\omega, \\ y_i = x_{i,1}. \end{cases} \tag{79}$$

where $i = 1, 2$. $x_{i,1}$ and $x_{i,2}$ are the states of the system; y_i are the outputs of the system; u_i are the system inputs. $\dot{\omega}$ is chosen as the one-dimensional Gaussian white nose with zero mean and variance 1. $p_{1,1} = p_{2,1} = 3$, $p_{1,2} = p_{2,2} = 5$. $f_{1,1} = 0.6\sin(x_{1,1}x_{2,1}) + 0.4x_{1,2}x_{2,2}$, $g_{1,1} = 0.5x_{1,1}$. $f_{1,2} = x_{1,1}e^{-0.5x_{1,2}^2}/(1 + x_{2,1}^2 + x_{2,2}^2)$, $g_{1,2} = \cos(x_{1,2})/(1 + x_{1,1}^2)$. $f_{2,1} = 0.8x_{2,1}x_{2,2}$, $g_{2,1} = 1.1\sin(x_{2,1})$. $f_{2,2} = \sin(x_{1,1}x_{2,2})e^{-0.5x_{2,1}^2}$, $g_{2,2} = \cos(x_{2,2})x_{2,1}^2$.

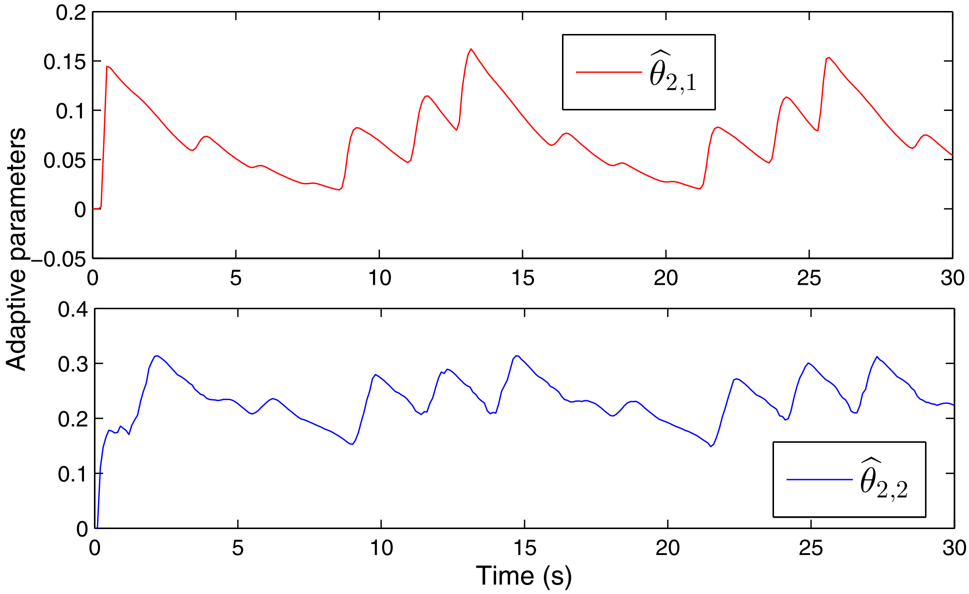

Fig. 6. Adaptive parameters $\hat{\theta}_{2,1}$ and $\hat{\theta}_{2,2}$ for Example 1.

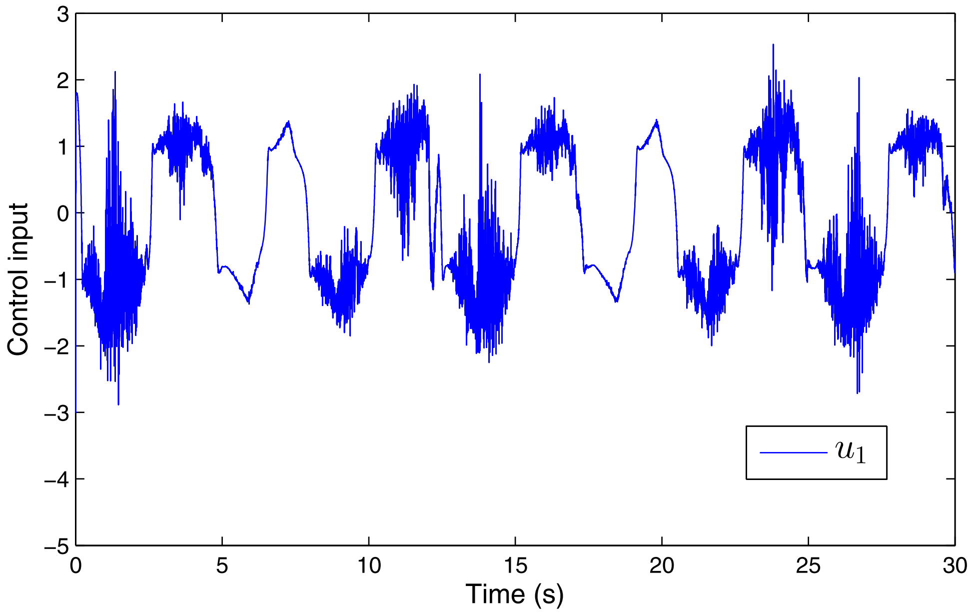

Fig. 7. Control input u_1 for Example 1.

The control objective is to design a decentralized adaptive neural controller such that all the signals remain bounded in probability and the system outputs y_i can track $y_{d1} = 0.8(\sin(t) + \sin(1.5t))$ and $y_{d2} = 0.8(\cos(t) + \cos(1.5t))$.

The virtual control signal $\alpha_{i,\,1}$ and the adaptive law $\dot{\hat{\theta}}_{i,1}$ are chosen as

$$\alpha_{i,1} = -z_{i,1}\left\{k_{i,1} + \eta_{i,1}^{\frac{p_i+3}{p_i-p_{i,1}+3}}\hat{\theta}_{i,1}(S_{i,1}^{\mathrm{T}}S_{i,1})^{\frac{p_i+3}{2(p_i-p_{i,1}+3)}} + 1\right\}^{\frac{1}{p_{i,1}}} \tag{80}$$

and

$$\dot{\hat{\theta}}_{i,1} = r_{i,1}\eta_{i,1}^{\frac{p_i+3}{p_i-p_{i,1}+3}}(S_{i,1}^{\mathrm{T}}S_{i,1})^{\frac{p_i+3}{2(p_i-p_{i,1}+3)}}z_{i,1}^{p_i+3} - \sigma_{i,1}\hat{\theta}_{i,1}. \tag{81}$$

where $z_{i,1} = x_{i,1} - y_{di}$.

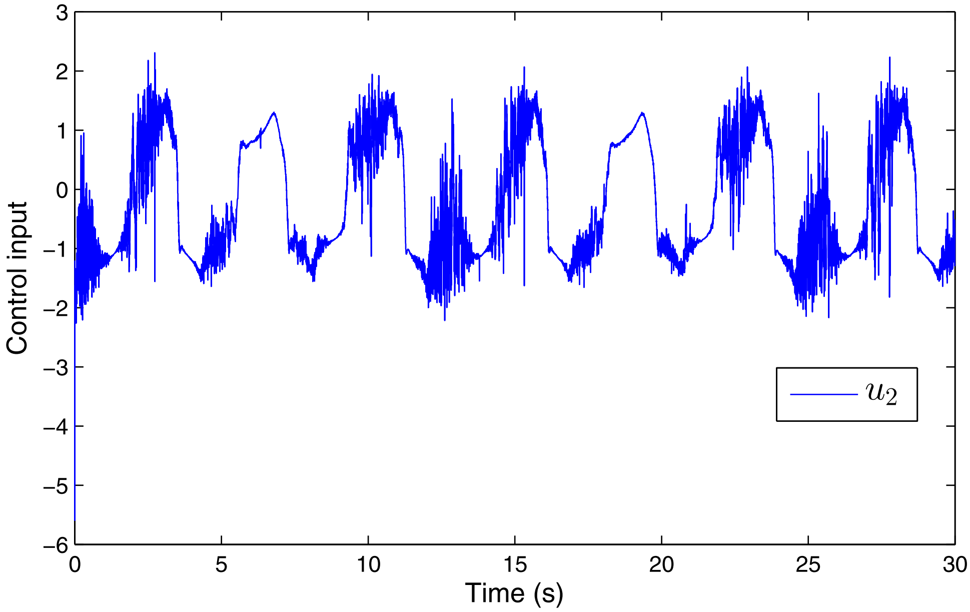

Fig. 8. Control input u_2 for Example 1.

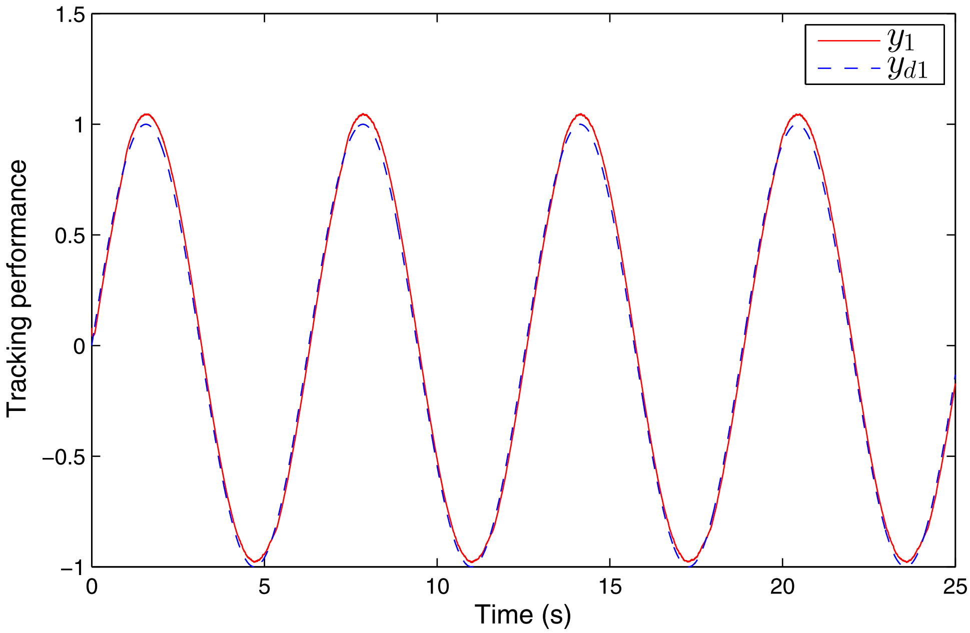

Fig. 9. System output y_1 and the reference signal y_{d1} for Example 2.

The control law u_i and the adaptive law $\dot{\hat{\theta}}_{i,2}$ are chosen as

$$u_i = -z_{i,2}\left\{k_{i,2} + \eta_{i,2}^{\frac{p_i+3}{p_i-p_{i,2}+3}}\hat{\theta}_{i,2}(S_{i,2}^{\mathrm{T}}S_{i,2})^{\frac{p_i+3}{2(p_i-p_{i,2}+3)}} + 1\right\}^{\frac{1}{p_{i,2}}} \tag{82}$$

and

$$\dot{\hat{\theta}}_{i,2} = r_{i,2}\eta_{i,1}^{\frac{p_i+3}{p_i-p_{i,2}+3}}(S_{i,2}^{\mathrm{T}}S_{i,2})^{\frac{p_i+3}{2(p_i-p_{i,2}+3)}}z_{i,2}^{p_i+3} - \sigma_{i,2}\hat{\theta}_{i,2}. \tag{83}$$

where $z_{i,2} = x_{i,2} - \alpha_{i,1}$.

The initial conditions $[x_{1,\,1}(0),\ x_{1,\,2}(0),\ x_{2,\,1}(0),\ x_{2,\,2}(0)]^{\mathrm{T}} = [0.05, 0.15, 1.62, 0.1]^{\mathrm{T}}$, $[\hat{\theta}_{1,1}(0), \hat{\theta}_{1,2}(0), \hat{\theta}_{2,1}(0), \hat{\theta}_{2,2}(0)]^{\mathrm{T}} = [0, 0, 0, 0]^{\mathrm{T}}$.

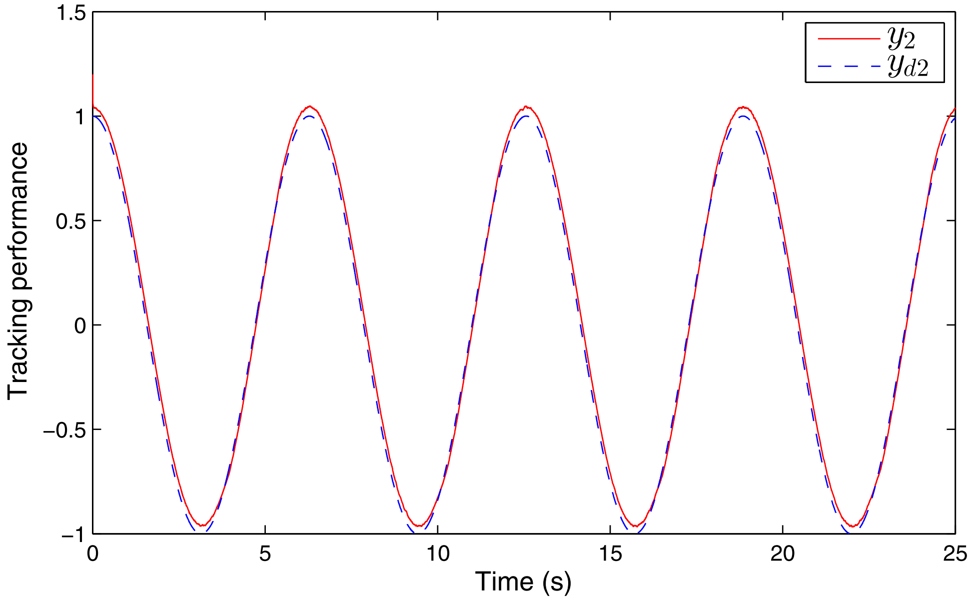

Fig. 10. System output y_2 and the reference signal y_{d2} for Example 2.

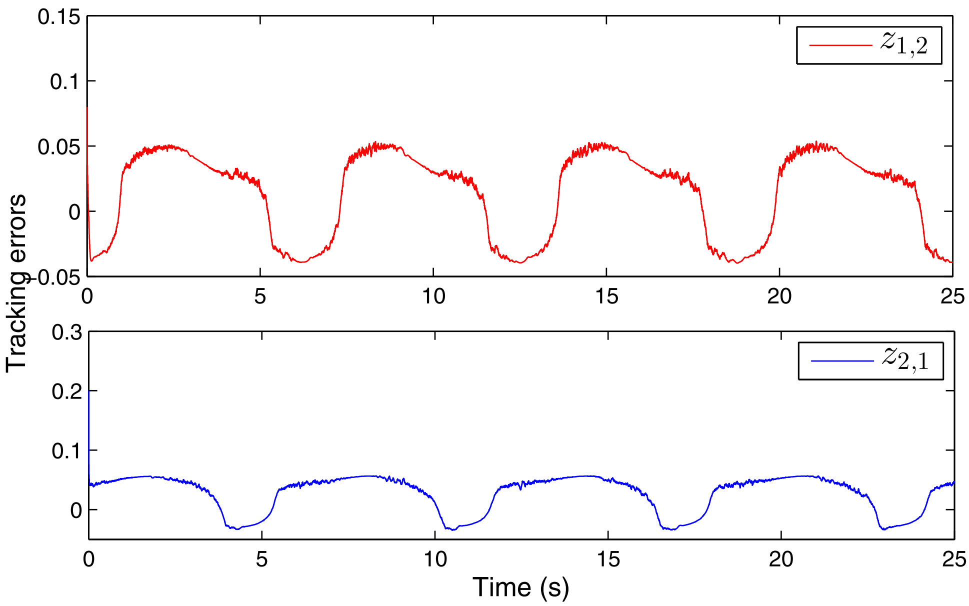

Fig. 11. Tracking errors $z_{1,\,1}$ and $z_{2,\,1}$ for Example 2.

In the simulation, design parameters are taken as follows: $k_{1,1}=12,\ k_{1,2}=16,\ k_{2,1}=18,\ k_{2,2}=19,\ r_{1,1}=r_{1,2}=r_{2,1}=r_{2,2}=1,\ \eta_{1,1}=\eta_{1,2}=\eta_{2,1}=\eta_{2,2}=6,\ \sigma_{1,1}=0.5,\ \sigma_{1,2}=0.6,\ \sigma_{2,1}=0.4,\ \sigma_{2,2}=0.2$. Based on the designed controller, RBF NNs are used to approximate the unknown nonlinear functions. Neural network $\hat{W}_{1,1}^{\mathrm{T}}S_{1,1}(Z_{1,1})$ with $Z_{1,1}=[x_{1,1},y_{d1},\dot{y}_{d1}]^T$ contains 6^3 nodes and centers are spaced evenly in the interval $[-1.2,1.2]\times[-1.8,1.8]\times[-2.0,2.0]$ with widths being equal to 0.45; and neural network $\hat{W}_{1,2}^{\mathrm{T}}S_{1,2}(Z_{1,2})$ with $Z_{1,2}=[x_{1,1},x_{1,2},y_{d1},\dot{y}_{d1},\ddot{y}_{d1},\hat{\theta}_{1,1}]^T$ contains 3^6 nodes and centers are spaced evenly in the interval $[-1.2,1.2]\times[-2.0,2.0]\times[-1.8,1.8]\times[-2.5,2.5]\times[-2.0,2.0]\times[-1.8,1.8]$ with widths being equal to 0.85. Similarly, neural network $\hat{W}_{2,1}^{\mathrm{T}}S_{2,1}(Z_{2,1})$ with $Z_{2,1}=[x_{2,1},y_{d2},\dot{y}_{d2}]^T$ contains 5^3 nodes with widths being equal to 0.48; neural network $\hat{W}_{2,2}^{\mathrm{T}}S_{2,2}(Z_{2,2})$ with $Z_{2,2}=[x_{2,1},x_{2,2},y_{d2},\dot{y}_{d2},\ddot{y}_{d2},\hat{\theta}_{2,1}]^T$ contains 3^6 nodes with widths being equal to 0.8.

The simulation results are shown in Figs. 1–8. Figs. 1 and 2 show the time trajectories of system outputs y_i and the reference signals y_{di}. Fig. 3 plots the tracking errors $z_{i,\,1}$. Fig. 4 shows the system states $x_{i,\,2}$. Figs. 5 and 6 give the time

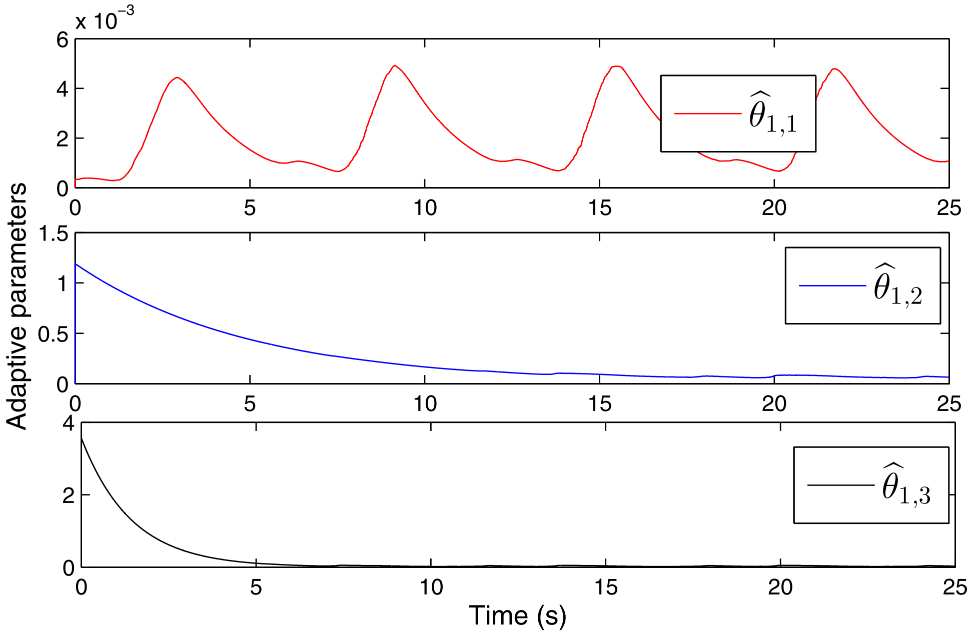

Fig. 12. Adaptive parameters $\hat{\theta}_{1,1}$, $\hat{\theta}_{1,2}$ and $\hat{\theta}_{1,3}$ for Example 2.

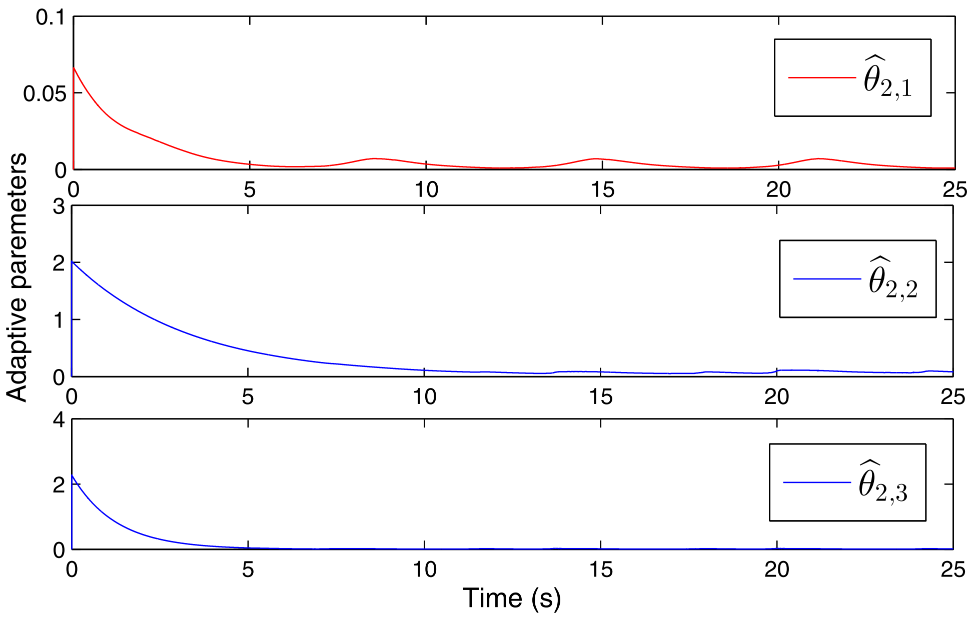

Fig. 13. Adaptive parameters $\hat{\theta}_{2,1}$, $\hat{\theta}_{2,2}$ and $\hat{\theta}_{2,3}$ for Example 2.

trajectories of the adaptive parameters $\hat{\theta}_{i,j_i}$, $i = 1, 2,\ j_i = 1, 2$. Figs. 7 and 8 show control signals u_1 and u_2. From Figs. 1–8, it can be seen that the proposed control schemes can ensure all the variables of the closed-loop system are bounded.

Example 2. To further illustrate the effectiveness of our design method, we employ the following high-order nonlinear system in which each subsystem is a third-order system.

$$\begin{cases} dx_{1,1} = (x_{1,2}^{p_{1,1}} + f_{1,1}(x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{2,3}))dt + g_{1,1}(x_{1,1})d\omega, \\ dx_{1,2} = (x_{1,3}^{p_{1,2}} + f_{1,2}(x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{2,3}))dt + g_{1,2}(x_{1,1}, x_{1,2})d\omega, \\ dx_{1,3} = (u_1^{p_{1,3}} + f_{1,2}(x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{2,3}))dt + g_{1,2}(x_{1,1}, x_{1,2}, x_{1,3})d\omega, \\ y_1 = x_{1,1}. \end{cases} \tag{84}$$

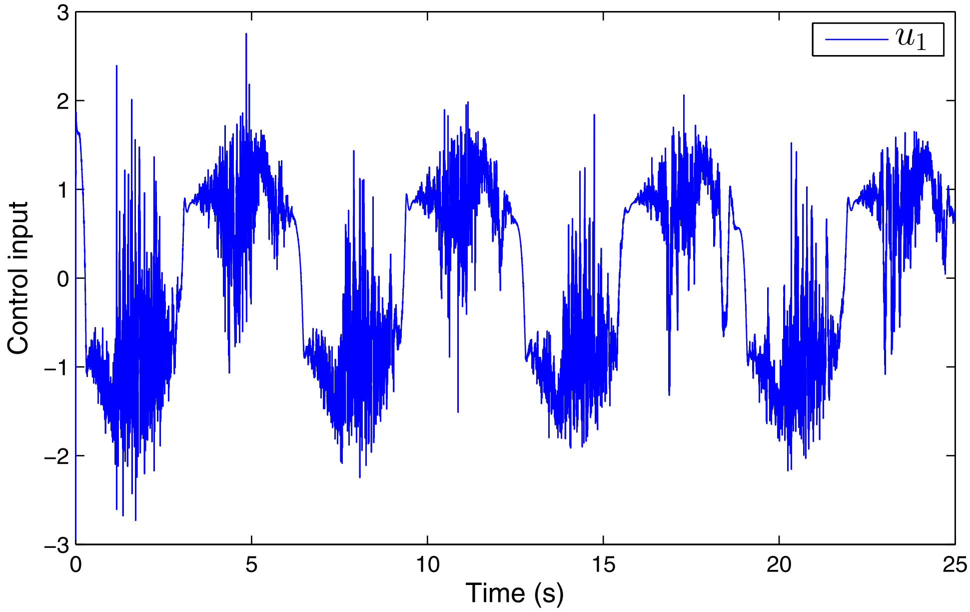

Fig. 14. Control input u_1 for Example 2.

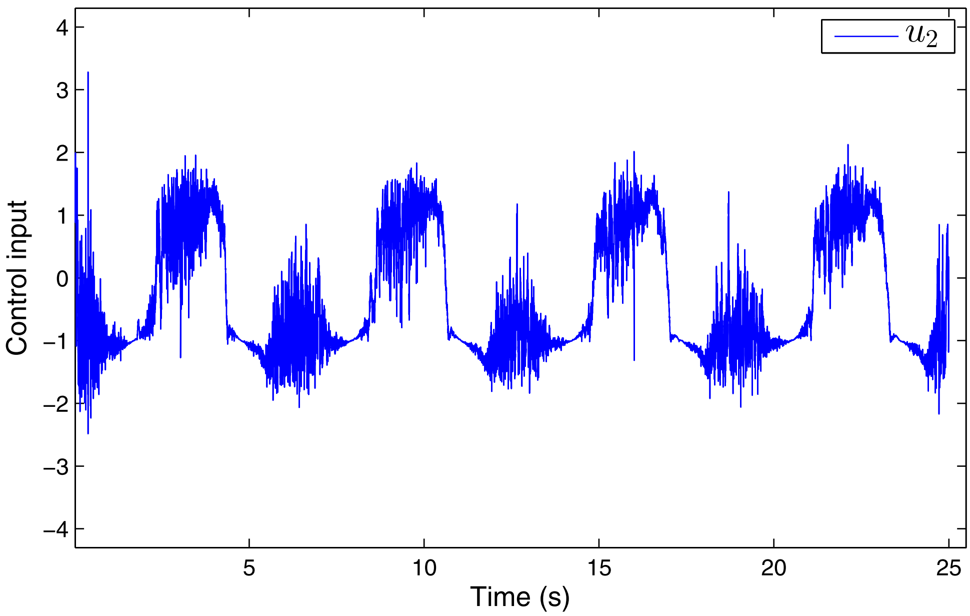

Fig. 15. Control input u_2 for Example 2.

$$\begin{cases} dx_{2,1} = (x_{2,2}^{p_{2,1}} + f_{2,1}(x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{2,3}))dt + g_{2,1}(x_{2,1})d\omega, \\ dx_{2,2} = (x_{2,3}^{p_{2,2}} + f_{2,2}(x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{2,3}))dt + g_{2,2}(x_{2,1}, x_{2,2})d\omega, \\ dx_{2,3} = (u_2^{p_{2,3}} + f_{2,3}(x_{1,1}, x_{1,2}, x_{1,3}, x_{2,1}, x_{2,2}, x_{2,3}))dt + g_{2,3}(x_{2,1}, x_{2,2}, x_{2,3})d\omega, \\ y_2 = x_{2,1}. \end{cases} \tag{85}$$

where the high-order terms $p_{1,1} = p_{2,1} = p_{1,2} = p_{2,2} = 3$, $p_{1,3} = p_{2,3} = 5$. $\dot{\omega}$ is chosen as in Example 1. $f_{1,1} = x_{1,1}x_{2,3} - 2x_{2,1}$, $f_{1,2} = 2\cos(x_{2,1}x_{2,2})$, $f_{1,3} = x_{1,3}\sin(x_{2,1})$. $f_{2,1} = 0.6x_{1,1}e^{-x_{2,2}^2}$, $f_{2,2} = 0.8x_{2,3} + 0.5x_{2,1}$, $f_{2,3} = x_{2,3}^2\cos(x_{1,3})$, $g_{1,1} = 0.3x_{1,1}^2$,

$g_{1,2} = x_{1,2}\sin(x_{1,1})$, $g_{1,3} = x_{1,3}/(1 + x_{1,1}^2)$, $g_{2,1} = x_{2,1}\sqrt{1 + x_{2,1}^2}$, $g_{2,2} = \cos(x_{2,1}x_{2,2})$, $g_{2,3} = 0.75x_{2,3}x_{2,1}$. The reference signals are given as $y_{d1} = \sin(t)$ and $y_{d2} = \cos(t)$.

The design parameters are given as, $k_{1,1} = 10$, $k_{1,2} = 12$, $k_{1,3} = 24$, $k_{2,1} = 15$, $k_{2,2} = 16$, $k_{2,3} = 30$, $r_{1,1} = r_{1,2} = r_{1,3} = r_{2,1} = r_{2,2} = r_{2,3} = 1$, $\eta_{1,1} = \eta_{1,2} = \eta_{1,3} = \eta_{2,1} = \eta_{2,2} = \eta_{2,3} = 5$, $\sigma_{1,1} = 0.7$, $\sigma_{1,2} = 0.2$, $\sigma_{1,3} = 0.15$, $\sigma_{2,1} = 0.1$, $\sigma_{2,2} = 0.08$, $\sigma_{2,3} = 0.15$. The initial conditions are given as $[x_{1,1}(0), x_{1,2}(0), x_{1,3}(0), x_{2,1}(0), x_{2,2}(0), x_{2,3}(0)]^{\mathrm{T}} = [0.08, 0.21, 0.3, 1.2, 0.2, 0.45]^{\mathrm{T}}$, $[\hat{\theta}_{1,1}(0), \hat{\theta}_{1,2}(0), \hat{\theta}_{1,3}(0), \hat{\theta}_{2,1}(0), \hat{\theta}_{2,2}(0), \hat{\theta}_{2,3}(0)]^{\mathrm{T}} = [0, 0, 0, 0, 0, 0]^{\mathrm{T}}$. Similar to Example 1, RBF NNs are used to approximate the unknown nonlinear functions.

The simulation results are shown in Figs. 9–15. Figs. 9 and 10 show the response of system output y_i and reference signal y_{di}, $i = 1, 2$. Figs. 14 and 15 give the control input of each subsystem. From these figures, it is clear that all close-loop signals are SGUUB and the proposed method is effective.

5. Conclusion

This paper investigates the problem of decentralized adaptive tracking control for a class of high-order stochastic nonlinear strongly interconnected systems with unknown system dynamics. In the controller design, only one adaptive parameter is designed to overcome over-parameterization problem, and RBF neural networks are utilized to approximate unknown nonlinearity. In addition, by using the backstepping technique, the new control law is constructed. It has been proved that the designed decentralized adaptive controller can ensure the stability of the closed-loop system and the tracking performance of the control system through the Lyapunov stability analysis.

This work can be extended toward several directions. One direction could be focused on the control problem of more complicated systems such as high-order stochastic nonlinear systems with time delays using efficient intelligent algorithms. Another direction that we are interested in exploring is how to develop observer-based control strategies for the high-order stochastic nonlinear system. These topics motivate our future research.

Acknowledgments

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