Sample of Curriculum

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Open Access Course

Quantum Field Theory in Path Integral Approach I

May 2023 - Present

Graduate Level

- Weekly structured lectures on YouTube
- Chapter-wise presentation on Enabla
- Weekly Q&A sessions over Zoom (registration through Telegram)

Prerequisites

Helpful for conceptual purposes. Course is otherwise self-contained.

- Perturbation theory in non-relativistic quantum mechanics (NRQM).
- Lorentz representation of fields, especially spin 0, 1/2 and 1. Transformation laws.
- Canonical quantization of fields.
- Concept of gauge symmetries.

Content

1. Path integrals in non-relativistic quantum mechanics

- Motivation and arrival at Feynman's path integral statement.
- Add. Note 01 - Intuitive argument for Dirac's proposal.
- More concrete understanding of path integral.
- General properties of path integral.
- Demonstration using harmonic oscillator.
- Add. Note 02 - Distribution laws and functional derivatives.
- Calculation of path integral - Euclideanization and i-epsilon prescription.
- Calculation of path integral - Overall normalization.

2. Relativistic generalization of quantum mechanics

- Path integral for relativistic free particle (world-line theory).
- Quadratic form of world-line action.
- Gauge volume, gauge slice, gauge orbit.
- Working formula for the propagator.
- Summary of different approaches.
- Mathematical analogy between Klien-Gordon (KG) theory and non-relativistic harmonic oscillator.

- KG propagator and its equality to world-line propagator.
- Equivalence between world-line theory and spacetime QFT.
- Interpretation of insertions in world-line path integral.
- N-point Green's function in free and interacting world-line theory.
- Add. Note 03 – Schrödinger representation of QFT.

3. Path integral quantization of Klein-Gordon theory

- Derivation of QFT path integral statement from NRQM.
- Scalar QFT around a Poincaré invariant vacuum.
- Definition of Green's function.
- Introduction to Generating functional.
- Calculation of generating functional for free theory - diagrammatics.
- Generating functional in interacting theory - connected and disconnected diagrams.
- Organization of connected and disconnected diagrams.

4. Demonstration of perturbative QFT algorithm using phi-cubed theory

- Formulation of perturbative QFT – demonstration using phi-cubed theory.
- Perturbative evaluation of generating functional; Feynman rules in position space; origin of symmetry factors.
- Methods of computing symmetry factors.
- Symmetry factor for disconnected diagrams.
- Position space Green's function; Feynman rules; examples.
- Momentum space Green's function; Feynman rules; examples.
- Relation between h-bar and loop expansions.
- One point function or zero momentum tadpole.
- Tadpole removal.

5. 1PI quantum effective action

- One particle irreducible (1PI) and connected diagrams
- Effective action and effective potential
- Quadratic term in effective action
- Effective action as the generating functional for 1PI vertices
- Review of construction around Poincaré invariant vacuum and generalization
- Calculation of effective action in semi-classical expansion (including discussion of functional Gaussian integral)
- First quantum correction to effective action

6. General study of loop divergences

- Loop integrals associated to Feynman diagrams
- Feynman parametrization (identities)
- Proof of Feynman identities (Schwinger parametrization)
- Analytic continuation of loop integrals
- Euclidean momentum integrals

- Superficial degree of divergence - omega
- Behaviour of omega for power law potentials
- Weinberg's theorem
- Classification of divergences

7. Renormalization of phi-fourth theory in four dimensions

- The program of counter term renormalization
- Dimensional regularization
- Euclidean momentum integration
- Explicit example
- Partial-p method
- Explicit example I2
- Explicit example I3
- Use of Feynman parametrization to calculate I3
- Renormalization of phi-fourth in D=4: warm-up
- Calculation of Tadpole
- Calculation of Fish
- Calculation of Double Scoop
- Calculation of Setting Sun
- Summary of results
- Counter terms at first order in coupling
- Counter terms at second order in coupling
- Comments on third order renormalization
- Polynomial Residue Theorem
- Bare and renormalized quantities and their relations
- Renormalization prescription
- Scale dependence, beta function and Landau pole
- First glimpse of asymptotic freedom
- Scenarios of asymptotic behavior – UV and IR fixed points, asymptotic freedom
- Renormalization Group Equation and anomalous dimension