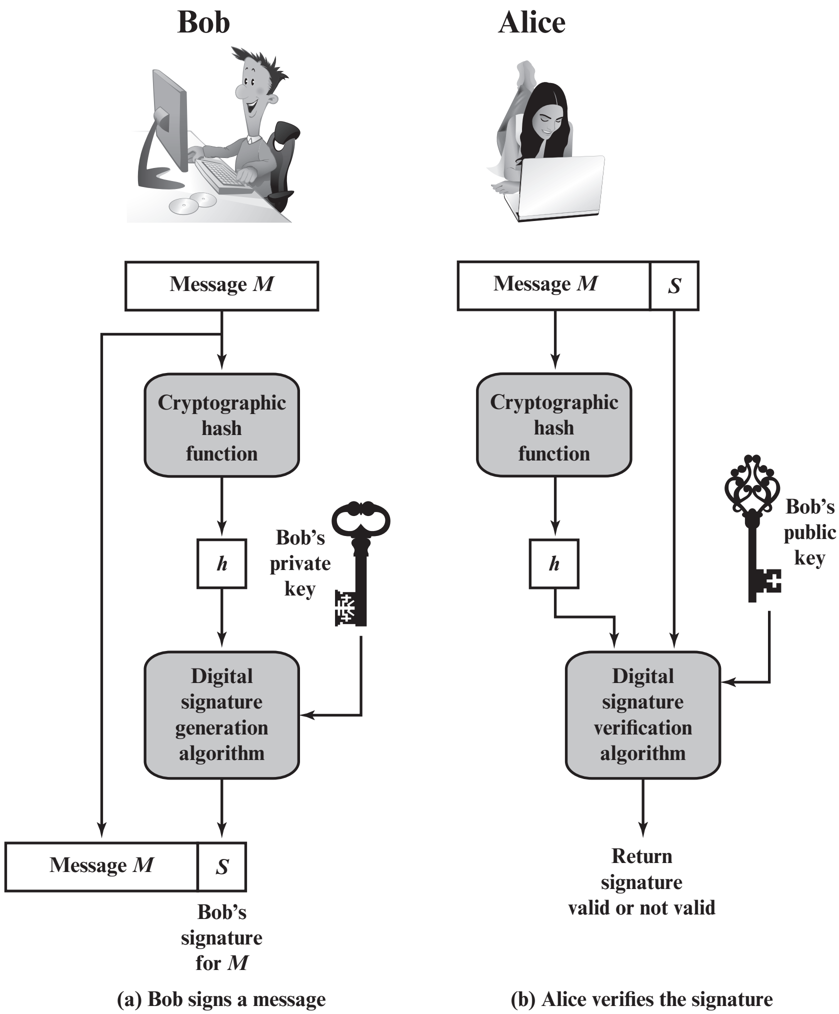


Figure 13.1 Simplified Depiction of Essential Elements of Digital Signature Process

13.1 DIGITAL SIGNATURES

Properties

Message authentication protects two parties who exchange messages from any third party. However, it does not protect the two parties against each other. Several forms of dispute between the two parties are possible.

For example, suppose that John sends an authenticated message to Mary, using one of the schemes of Figure 12.1. Consider the following disputes that could arise.

1. Mary may forge a different message and claim that it came from John. Mary would simply have to create a message and append an authentication code using the key that John and Mary share.
2. John can deny sending the message. Because it is possible for Mary to forge a message, there is no way to prove that John did in fact send the message.

Both scenarios are of legitimate concern. Here is an example of the first scenario: An electronic funds transfer takes place, and the receiver increases the amount of funds transferred and claims that the larger amount had arrived from the sender. An example of the second scenario is that an electronic mail message contains instructions to a stockbroker for a transaction that subsequently turns out badly. The sender pretends that the message was never sent.

In situations where there is not complete trust between sender and receiver, something more than authentication is needed. The most attractive solution to this problem is the digital signature. The digital signature must have the following properties:

- It must verify the author and the date and time of the signature.
- It must authenticate the contents at the time of the signature.
- It must be verifiable by third parties, to resolve disputes.

Thus, the digital signature function includes the authentication function.

Attacks and Forgeries

[GOLD88] lists the following types of attacks, in order of increasing severity. Here A denotes the user whose signature method is being attacked, and C denotes the attacker.

- **Key-only attack:** C only knows A's public key.
- **Known message attack:** C is given access to a set of messages and their signatures.
- **Generic chosen message attack:** C chooses a list of messages before attempting to breaks A's signature scheme, independent of A's public key. C then obtains from A valid signatures for the chosen messages. The attack is generic, because it does not depend on A's public key; the same attack is used against everyone.
- **Directed chosen message attack:** Similar to the generic attack, except that the list of messages to be signed is chosen after C knows A's public key but before any signatures are seen.
- **Adaptive chosen message attack:** C is allowed to use A as an "oracle." This means that C may request from A signatures of messages that depend on previously obtained message-signature pairs.

[GOLD88] then defines success at breaking a signature scheme as an outcome in which C can do any of the following with a non-negligible probability:

- **Total break:** C determines A's private key.
- **Universal forgery:** C finds an efficient signing algorithm that provides an equivalent way of constructing signatures on arbitrary messages.
- **Selective forgery:** C forges a signature for a particular message chosen by C.
- **Existential forgery:** C forges a signature for at least one message. C has no control over the message. Consequently, this forgery may only be a minor nuisance to A.

Digital Signature Requirements

On the basis of the properties and attacks just discussed, we can formulate the following requirements for a digital signature.

- The signature must be a bit pattern that depends on the message being signed.
- The signature must use some information only known to the sender to prevent both forgery and denial.
- It must be relatively easy to produce the digital signature.
- It must be relatively easy to recognize and verify the digital signature.
- It must be computationally infeasible to forge a digital signature, either by constructing a new message for an existing digital signature or by constructing a fraudulent digital signature for a given message.
- It must be practical to retain a copy of the digital signature in storage.

A secure hash function, embedded in a scheme such as that of Figure 13.1, provides a basis for satisfying these requirements. However, care must be taken in the design of the details of the scheme.

Direct Digital Signature

The term **direct digital signature** refers to a digital signature scheme that involves only the communicating parties (source, destination). It is assumed that the destination knows the public key of the source.

Confidentiality can be provided by encrypting the entire message plus signature with a shared secret key (symmetric encryption). Note that it is important to perform the signature function first and then an outer confidentiality function. In case of dispute, some third party must view the message and its signature. If the signature is calculated on an encrypted message, then the third party also needs access to the decryption key to read the original message. However, if the signature is the inner operation, then the recipient can store the plaintext message and its signature for later use in dispute resolution.

The validity of the scheme just described depends on the security of the sender's private key. If a sender later wishes to deny sending a particular message, the sender can claim that the private key was lost or stolen and that someone else forged his or her signature. Administrative controls relating to the security of private keys can be employed to thwart or at least weaken this ploy, but the threat is still there, at least to some degree. One example is to require every signed message to include a **timestamp** (date and time) and to require prompt reporting of compromised keys to a central authority.

Another threat is that a private key might actually be stolen from X at time T. The opponent can then send a message signed with X's signature and stamped with a time before or equal to T.

The universally accepted technique for dealing with these threats is the use of a digital certificate and certificate authorities. We defer a discussion of this topic until Chapter 14, and focus in this chapter on digital signature algorithms.

13.2 ELGAMAL DIGITAL SIGNATURE SCHEME

Before examining the NIST Digital Signature Algorithm, it will be helpful to understand the Elgamal and Schnorr signature schemes. Recall from Chapter 10, that the Elgamal encryption scheme is designed to enable encryption by a user's public key with decryption by the user's private key. The Elgamal signature scheme involves the use of the private key for digital signature generation and the public key for digital signature verification [ELGA84, ELGA85].

Before proceeding, we need a result from number theory. Recall from Chapter 2 that for a prime number q, if α is a primitive root of q, then

$$\alpha, \alpha^2, \ldots, \alpha^{q-1}$$

are distinct (mod q). It can be shown that, if α is a primitive root of q, then

1. For any integer m, $\alpha^m \equiv 1 \pmod{q}$ if and only if $m \equiv 0 \pmod{q-1}$.
2. For any integers, i, j, $\alpha^i \equiv \alpha^j \pmod{q}$ if and only if $i \equiv j \pmod{q-1}$.

As with Elgamal encryption, the global elements of **Elgamal digital signature** are a prime number q and α, which is a primitive root of q. User A generates a private/public key pair as follows.

1. Generate a random integer X_A, such that $1 < X_A < q - 1$.
2. Compute $Y_A = \alpha^{X_A} \bmod q$.
3. A's private key is X_A; A's pubic key is $\{q, \alpha, Y_A\}$.

To sign a message M, user A first computes the hash $m = \mathrm{H}(M)$, such that m is an integer in the range $0 \le m \le q - 1$. A then forms a digital signature as follows.

1. Choose a random integer K such that $1 \le K \le q - 1$ and $\gcd(K, q - 1) = 1$. That is, K is relatively prime to $q - 1$.
2. Compute $S_1 = \alpha^K \bmod q$. Note that this is the same as the computation of C_1 for Elgamal encryption.
3. Compute $K^{-1} \bmod (q - 1)$. That is, compute the inverse of K modulo $q - 1$.
4. Compute $S_2 = K^{-1}(m - X_A S_1) \bmod (q - 1)$.
5. The signature consists of the pair (S_1, S_2).

The first part of this scheme is the generation of a private/public key pair, which consists of the following steps.

1. Choose primes p and q, such that q is a prime factor of $p - 1$.
2. Choose an integer a, such that $a^q = 1 \bmod p$. The values a, p, and q comprise a global public key that can be common to a group of users.
3. Choose a random integer s with $0 < s < q$. This is the user's private key.
4. Calculate $v = a^{-s} \bmod p$. This is the user's public key.

A user with private key s and public key v generates a signature as follows.

1. Choose a random integer r with $0 < r < q$ and compute $x = a^r \bmod p$. This computation is a preprocessing stage independent of the message M to be signed.
2. Concatenate the message with x and hash the result to compute the value e:

$$e = \mathrm{H}(M \| x)$$

3. Compute $y = (r + se) \bmod q$. The signature consists of the pair (e, y).

Any other user can verify the signature as follows.

1. Compute $x' = a^y v^e \bmod p$.
2. Verify that $e = \mathrm{H}\,(M \| x')$.

To see that the verification works, observe that

$$x' \equiv a^y v^e \equiv a^y a^{-se} \equiv a^{y-se} \equiv a^r \equiv x \pmod p$$

Hence, $\mathrm{H}\,(M \| x') = \mathrm{H}\,(M \| x)$.

13.4 NIST DIGITAL SIGNATURE ALGORITHM

The National Institute of Standards and Technology (NIST) has published Federal Information Processing Standard FIPS 186, known as the Digital Signature Algorithm (DSA). The DSA makes use of the Secure Hash Algorithm (SHA) described in Chapter 12. The DSA was originally proposed in 1991 and revised in 1993 in response to public feedback concerning the security of the scheme. There was a further minor revision in 1996. In 2000, an expanded version of the standard was issued as FIPS 186-2, subsequently updated to FIPS 186-3 in 2009, and FIPS 186-4 in 2013. This latest version also incorporates digital signature algorithms based on RSA and on elliptic curve cryptography. In this section, we discuss DSA.

The DSA Approach

The DSA uses an algorithm that is designed to provide only the digital signature function. Unlike RSA, it cannot be used for encryption or key exchange. Nevertheless, it is a public-key technique.

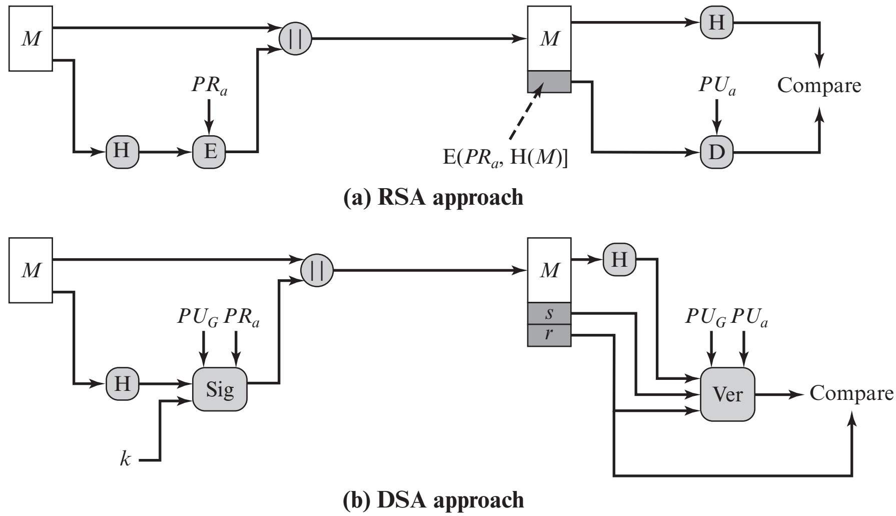


Figure 13.2 Two Approaches to Digital Signatures

Figure 13.2 contrasts the DSA approach for generating digital signatures to that used with RSA. In the RSA approach, the message to be signed is input to a hash function that produces a secure hash code of fixed length. This hash code is then encrypted using the sender's private key to form the signature. Both the message and the signature are then transmitted. The recipient takes the message and produces a hash code. The recipient also decrypts the signature using the sender's public key. If the calculated hash code matches the decrypted signature, the signature is accepted as valid. Because only the sender knows the private key, only the sender could have produced a valid signature.

The DSA approach also makes use of a hash function. The hash code is provided as input to a signature function along with a random number k generated for this particular signature. The signature function also depends on the sender's private key (PR_a) and a set of parameters known to a group of communicating principals. We can consider this set to constitute a global public key (PU_G).[1] The result is a signature consisting of two components, labeled s and r.

At the receiving end, the hash code of the incoming message is generated. The hash code and the signature are inputs to a verification function. The verification function also depends on the global public key as well as the sender's public key (PU_a), which is paired with the sender's private key. The output of the verification function is a value that is equal to the signature component r if the signature is valid. The signature function is such that only the sender, with knowledge of the private key, could have produced the valid signature.

We turn now to the details of the algorithm.

[1]It is also possible to allow these additional parameters to vary with each user so that they are a part of a user's public key. In practice, it is more likely that a global public key will be used that is separate from each user's public key.

Global Public-Key Components

p prime number where $2^{L-1} < p < 2^{L}$ for $512 \leq L \leq 1024$ and L a multiple of 64; i.e., bit length L between 512 and 1024 bits in increments of 64 bits

q prime divisor of $(p - 1)$, where $2^{N-1} < q < 2^{N}$ i.e., bit length of N bits

g $= h(\mathrm{p} - 1)/\mathrm{q}$ is an exponent mod p, where h is any integer with $1 < h < (p - 1)$ such that $h^{(p-1)/q} \bmod p > 1$

User's Private Key

x random or pseudorandom integer with $0 < x < q$

User's Public Key

$y = g^{x} \bmod p$

User's Per-Message Secret Number

k random or pseudorandom integer with $0 < k < q$

Signing

$r = (g^{k} \bmod p) \bmod q$

$s = [k^{-1}(H(M) + xr)] \bmod q$

Signature $= (r, s)$

Verifying

$w = (s')^{-1} \bmod q$

$u_1 = [H(M')w] \bmod q$

$u_2 = (r')w \bmod q$

$v = [(g^{u1} y^{u2}) \bmod p] \bmod q$

TEST: $v = r'$

M = message to be signed

$\mathrm{H}(M)$ = hash of M using SHA-1

M', r', s' = received versions of M, r, s

Figure 13.3 The Digital Signature Algorithm (DSA)

The Digital Signature Algorithm

DSA is based on the difficulty of computing discrete logarithms (see Chapter 2) and is based on schemes originally presented by Elgamal [ELGA85] and Schnorr [SCHN91].

Figure 13.3 summarizes the algorithm. There are three parameters that are public and can be common to a group of users. An N-bit prime number q is chosen. Next, a prime number p is selected with a length between 512 and 1024 bits such that q divides $(p - 1)$. Finally, g is chosen to be of the form $h^{(p-1)/q} \bmod p$, where h is an integer between 1 and $(p - 1)$ with the restriction that g must be greater than 1.[2] Thus, the global public-key components of DSA are the same as in the Schnorr signature scheme.

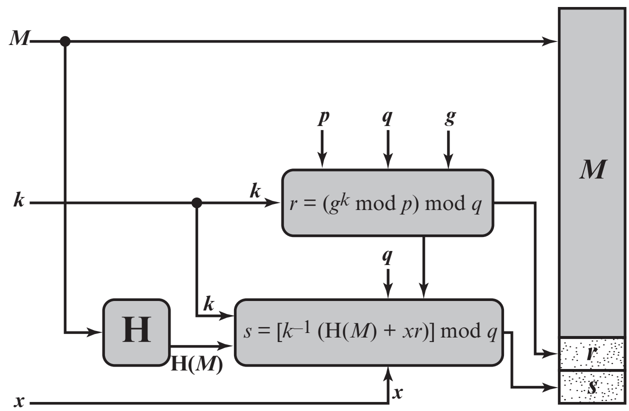
With these parameters in hand, each user selects a private key and generates a public key. The private key x must be a number from 1 to $(q - 1)$ and should be chosen randomly or pseudorandomly. The public key is calculated from the private key as $y = g^{x} \bmod p$. The calculation of y given x is relatively straightforward. However, given the public key y, it is believed to be computationally infeasible to determine x, which is the discrete logarithm of y to the base g, mod p (see Chapter 2).

[2]In number-theoretic terms, g is of order q mod p; see Chapter 2.

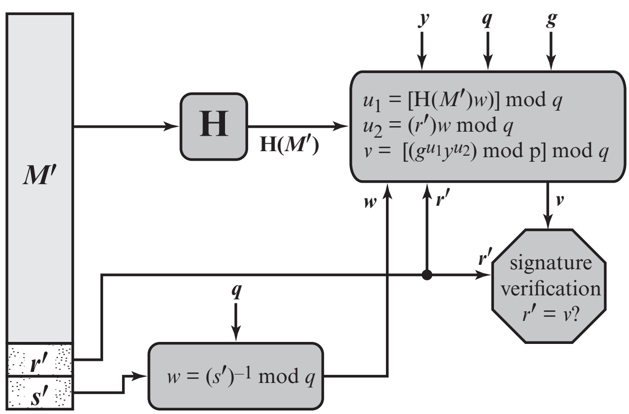
The signature of a message M consists of the pair of numbers r and s, which are functions of the public key components (p, q, g), the user's private key (x), the hash code of the message H(M), and an additional integer k that should be generated randomly or pseudorandomly and be unique for each signing.

Let M, r', and s' be the received versions of M, r, and s, respectively. Verification is performed using the formulas shown in Figure 13.3. The receiver generates a quantity v that is a function of the public key components, the sender's public key, the hash code of the incoming message, and the received versions of r and s. If this quantity matches the r component of the signature, then the signature is validated.

Figure 13.4 depicts the functions of signing and verifying.



(a) Signing



(b) Verifying

Figure 13.4 DSA Signing and Verifying

The structure of the algorithm, as revealed in Figure 13.4, is quite interesting. Note that the test at the end is on the value r, which does not depend on the message at all. Instead, r is a function of k and the three global public-key components. The multiplicative inverse of k (mod q) is passed to a function that also has as inputs the message hash code and the user's private key. The structure of this function is such that the receiver can recover r using the incoming message and signature, the public key of the user, and the global public key. It is certainly not obvious from Figure 13.3 or Figure 13.4 that such a scheme would work. A proof is provided in Appendix K.

Given the difficulty of taking discrete logarithms, it is infeasible for an opponent to recover k from r or to recover x from s.

Another point worth noting is that the only computationally demanding task in signature generation is the exponential calculation $g^k \bmod p$. Because this value does not depend on the message to be signed, it can be computed ahead of time. Indeed, a user could precalculate a number of values of r to be used to sign documents as needed. The only other somewhat demanding task is the determination of a multiplicative inverse, k^{-1}. Again, a number of these values can be precalculated.

13.5 ELLIPTIC CURVE DIGITAL SIGNATURE ALGORITHM

As was mentioned, the 2009 version of FIPS 186 includes a new digital signature technique based on elliptic curve cryptography, known as the **Elliptic Curve Digital Signature Algorithm (ECDSA)**. ECDSA is enjoying increasing acceptance due to the efficiency advantage of elliptic curve cryptography, which yields security comparable to that of other schemes with a smaller key bit length.

First we give a brief overview of the process involved in ECDSA. In essence, four elements are involved.

1. All those participating in the digital signature scheme use the same global domain parameters, which define an elliptic curve and a point of origin on the curve.
2. A signer must first generate a public, private key pair. For the private key, the signer selects a random or pseudorandom number. Using that random number and the point of origin, the signer computes another point on the elliptic curve. This is the signer's public key.
3. A hash value is generated for the message to be signed. Using the private key, the domain parameters, and the hash value, a signature is generated. The signature consists of two integers, r and s.
4. To verify the signature, the verifier uses as input the signer's public key, the domain parameters, and the integer s. The output is a value v that is compared to r. The signature is verified if $v = r$.

Let us examine each of these four elements in turn.