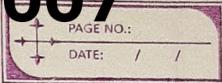


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Q1 Form the partial differential -----
--- from the following:

$$a) \left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) + \left(\frac{z^2}{c^2}\right) = 1$$

differentiating w.r.t x

$$\frac{2x}{a^2} + \frac{2z}{c^2} \frac{dz}{dx} = 0 \quad \text{--- (i)}$$

differentiating w.r.t x again

$$\frac{2}{a^2} + \frac{2z}{c^2} \frac{d^2z}{dx^2} + \frac{2}{c^2} \left(\frac{dz}{dx}\right)^2$$

$$= \frac{d^2z}{dx^2} + \left(\frac{dz}{dx}\right)^2 - \frac{c^2}{a^2} \quad \text{--- (ii)}$$

Solving (i)

$$\frac{z dz}{xdx} = \frac{-c^2}{a^2} \quad \text{--- (iii)}$$

using (ii) & (iii)

$$\left[\frac{z d^2z}{dx^2} + \left(\frac{dz}{dx}\right)^2 = \frac{z dz}{xdx} \right] //$$

$$b) (x-h)^2 + (y-k)^2 + z^2 = c^2$$

differentiating w.r.t x

$$2(x-h) + 2z \beta = 0$$

$$(x-h) = -z \beta \quad \text{--- (i)}$$

differentiating w.r.t y

$$2(y-k) + 2z \gamma = 0$$

$$(y-k) = -z \gamma \quad \text{--- (ii)}$$

putting value of $(x-a)$ & $y-b$ in original
 \log^n

$$(-2p)^2 + (-2q)^2 + z^2 = c^2$$

$$\boxed{z^2(1+p^2+q^2) = c^2}$$

c) $z = (x^2+a)(y^2+b)$

differentiating w.r.t x

$$p = 2x(y^2+b) \quad \text{--- i)}$$

differentiating w.r.t y

$$q = 2y(x^2+a) \quad \text{--- ii)}$$

i) \times ii)

$$pq = 4xy(x^2+a)(y^2+b)$$

$$\boxed{pq = 4xyz}$$

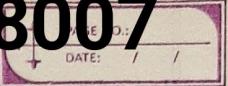
Q2 → Form partial ----- following

a) $z = e^{xy} f(x-y)$

differentiating w.r.t x

$$p = e^{xy} f'(x-y) \quad \text{--- i)}$$

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differentiating w.r.t y

$$g = ne^{ny} f(x-y) - e^{ny} f'(x-y) \quad \text{---(i)}$$

$$(i) + (ii)$$

$$p+g = ne^{ny} f(x-y)$$

$$\boxed{p+g = nz}$$

b) $z = f(x+ay) + g(x-ay)$

differentiating w.r.t x

$$p = f'(x+ay) + g'(x-ay)$$

differentiating again w.r.t x

$$\frac{d^2 z}{dx^2} = f''(x+ay) + g''(x-ay) \quad \text{---(i)}$$

differentiating original eqn w.r.t y twice

$$\frac{d^2 z}{dy^2} = a^2 f''(x+ay) + a^2 g''(x-ay)$$

$$\frac{d^2 z}{dy^2} = a^2 (f''(x+ay) + g''(x-ay)) = a^2 \frac{d^2 z}{dx^2} \quad (\text{using (i)})$$

$$\boxed{\frac{d^2 z}{dy^2} = a^2 \frac{d^2 z}{dx^2}}$$

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c) $z = f(x+iy) + F(x-iy)$

differentiating w.r.t x twice

$$\frac{d^2 z}{dx^2} = f''(x+iy) + F''(x-iy) \quad \text{---(i)}$$

differentiating w.r.t y twice

$$\frac{d^2 z}{dy^2} = i^2 f''(x+iy) + i^2 F''(x-iy)$$

$$\frac{d^2 z}{dy^2} = i^2 (f''(x+iy) + F''(x-iy)) = i^2 \frac{d^2 z}{dx^2}$$

$$\boxed{\frac{d^2 z}{dy^2} + \frac{d^2 z}{dx^2} = 0}$$

d) $f(x+y+z, x^2+y^2-z^2) = 0$

$$u = x+y+z$$

$$v = x^2+y^2-z^2$$

$$\frac{du}{dx} = 1+\beta$$

$$\frac{dv}{dy} = 1+2$$

$$\frac{dv}{dx} = 2x - 2zb$$

$$\frac{dv}{dy} = 2y - 2zg$$

$$f(u, v) = 0$$

differentiating w.r.t x

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} + \frac{df}{dv} \cdot \frac{dv}{dx} = 0$$

$$\frac{df}{dx} = (1+\beta) \frac{df}{du} + (2x - 2zb) \frac{df}{dv} \quad \text{---(i)}$$

diff f w.r.t y

$$\frac{df}{dy} = \left(\frac{df}{du} \cdot \frac{du}{dy} \right) + \left(\frac{df}{dv} \cdot \frac{dv}{dy} \right)$$

$$\frac{df}{dy} = (1+g) \frac{df}{du} + (2y - 2zg) \frac{df}{dv} \quad \text{(ii)}$$

from (i) & (ii)

$$\begin{vmatrix} 1+p & 2x-2zp \\ 1+g & 2y-2zg \end{vmatrix} = 0$$

$$\text{or } (1+p)(2y-2zg) = (1+g)(2x-2zp)$$

$$\boxed{p(y+z) - g(x+z) = x-y}$$

Q3 → Solve the following PDE by Lagrange's method

$$a) p \cos(x+y) + q \sin(x+y) = z$$

$$\frac{dx}{\cos(x+y)} - \frac{dy}{\sin(x+y)} = \frac{dz}{z} = \frac{dx+dy}{\cos(x+y)+\sin(x+y)}$$

L(i) L(ii) L(iii) L(iv)

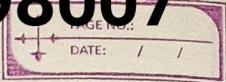
$$(iii) = (iv)$$

$$\frac{dz}{z} = \frac{dx+dy}{\sqrt{2}\sin(x+y+\pi/4)}$$

$$\int \frac{dz}{z} = \frac{1}{\sqrt{2}} \int \cosec(x+y+\pi/4) d(x+y)$$

$$\Rightarrow z^{\sqrt{2}} \cot\left(\frac{x+y}{2} + \frac{\pi}{8}\right) = C_1$$

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$$\frac{dx - dy}{\cos(x+y) - \sin(x+y)} = \frac{dx + dy}{\cos(x+y) + \sin(x+y)}$$

$$\frac{\cos(x+y) - \sin(x+y)}{\cos(x+y) + \sin(x+y)} d(x+y) = d(x-y)$$

$$\log(\sin(x+y) + \cos(x+y)) = x-y + C_2$$

~~$\cos(x+y) + \sin(x+y)$~~

$$\phi(C_1, C_2)$$

$$= \boxed{\phi\left(z^{\sqrt{2}} \cot\left(\frac{x+y+\frac{\pi}{8}}{2}\right), \log[\cos(x+y) + \sin(x+y)] - x+y\right)} = 0$$

$$b) yz\beta + xz\alpha = xy$$

$$\frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{xy}$$

L(i)

L(ii)

L(iii)

$$i) = ii)$$

$$\frac{dx}{yz} = \frac{dy}{xz}$$

$$x^2 - y^2 = C_1 \quad \text{- (1)}$$

$$ii) = iii)$$

$$\frac{dy}{xz} = \frac{dz}{xy}$$

$$y^2 - z^2 = C_2 \quad \text{- (2)}$$

$$\boxed{\phi(x^2 - y^2, y^2 - z^2)} = 0$$

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$$c) x(y^2 - z) p - y(x^2 + z) q = z(x^2 - y^2)$$

$$\frac{dx}{x(y^2 - z)} = \frac{dy}{-y(x^2 + z)} = \frac{dz}{z(x^2 - y^2)}$$

taking multipliers $x, y, -1$

$$\frac{dx}{x(y^2 - z)} = \frac{dy}{-y(x^2 + z)} = \frac{dz}{z(x^2 - y^2)} = \frac{x dx + y dy - dz}{0}$$

$$C_1 = \frac{x^2}{2} + \frac{y^2}{2} - z$$

$$C_1' = x^2 + y^2 - 2z \quad \text{-} \textcircled{a}$$

taking multipliers $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0 \Rightarrow \log x + \log y + \log z = 0 \text{ } C_2$$

$$xyz = C_2' \quad \text{-} \textcircled{b}$$

$$\boxed{\phi(x^2 + y^2 - 2z, xyz) = 0}$$

$$d) (z^2 - 2yz - y^2) p + (xy + xz) q = xy - xz$$

$$\frac{dx}{z^2 - 2yz - y^2} = \frac{dy}{xy + xz} = \frac{dz}{xy - xz}$$

using multiplier (x, y, z)

$$xdx + ydy + zdz = 0 \Rightarrow x^2 + y^2 + z^2 = C, \quad \text{-} \textcircled{c}$$

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$$\frac{dy}{xy+xz} = \frac{dz}{xy-xz}$$

$$\frac{y^2}{2} - (zdy + ydz) + \frac{z^2}{2} = C_2$$

$$C_2 = \frac{y^2}{2} - \frac{z^2}{2} - yz$$

$$\phi\left(x^2+y^2+z^2, \frac{y^2}{2} - \frac{z^2}{2} - yz\right) = 0$$

c) $y^2p - xyq = x(z-2y)$

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

$$\Rightarrow \frac{dx}{y^2} = \frac{dy}{-xy} \Rightarrow -x \, dx = y \, dy$$

$$-\frac{x^2}{2} - \frac{y^2}{2} = C_1 \Rightarrow x^2 + y^2 = C_1 \quad \text{---(a)}$$

$$\Rightarrow \frac{dz}{x(z-2y)} = \frac{dy}{-xy} \Rightarrow y \, dz - 2y \, dy + \cancel{y \, dz} - z \, dy = 0$$

$$\Rightarrow zy - y^2 = C_2 \quad \text{---(b)}$$

$$\phi(x^2+y^2, zy-y^2) = 0$$

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$$f) \quad \rho + 3y = 5z + \tan(y - 3x)$$

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y - 3x)}$$

$$\textcircled{3} \quad \frac{dx}{1} = \frac{dy}{3} \Rightarrow 3x - y = C_1 \quad -\textcircled{a}$$

$$\frac{dx}{1} = \frac{dz}{5z + \tan(y - 3x)}$$

$$x = \frac{1}{5} \ln(5z + \tan(y - 3x)) + C_2$$

$$\textcircled{5x} \quad e^{5x} = C_2 [5z + \tan(y - 3x)]$$

$$C_2 = e^{-5x} [5z + \tan(y - 3x)]$$

$$\boxed{\phi(y - 3x - y, e^{-5x} [5z + \tan(y - 3x)]) = 0}$$

$$g) \quad \frac{x du}{dx} + \frac{y du}{dy} + z \frac{du}{dz} = xyz$$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{du}{xyz}$$

\textcircled{i} \quad \textcircled{ii} \quad \textcircled{iii} \quad \textcircled{iv}

\textcircled{i} & \textcircled{ii}

$$\log x = \log y + C_1$$

$$C_1 = x/y \quad -\textcircled{a}$$

\textcircled{ii} & \textcircled{iii}

$$\log y = \log z + C_2$$

$$C_2 = y/z \quad -\textcircled{b}$$

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$$\frac{du}{xyz} = \frac{yzdx + xzdy + xydz}{3xyz}$$

$$3\int du = xyz + C_3$$

$$3u - xyz = C_3$$

$$\boxed{\phi\left(\frac{x}{y}, \frac{y}{z}, 3u - xyz\right) = 0}$$

Q4 - Obtain the complete solⁿ of the following PDEs using standard form

a) $p+q = pq$

$$z = ax + by + c \quad (\text{let})$$

$$d = p = a \Rightarrow q = b$$

$$a+b = ab$$

$$[p+q = pq]$$

$$a = b(a-1)$$

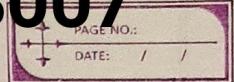
$$b = \frac{a}{(a-1)}$$

$$\boxed{z = ax + \left(\frac{a}{a-1}\right)y + c}$$

b) $x^2p^2 + y^2q^2 = z^2$

$$p = \frac{dz}{dx} \quad q = \frac{dz}{dy}$$

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$$\frac{x^2}{z^2} p^2 + \frac{y^2}{z^2} q^2 = 1$$

$$\frac{(dz/z)^2}{(dx/x)^2} + \frac{(dz/z)^2}{(dy/y)^2} = 1$$

$$\text{Let } z = \log z \quad x = \log x \quad y = \log y$$

$$\left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2 = 1$$

$p^2 + q^2 = 1$

$$\text{If } z = Ax + By + C \leftarrow B = \sqrt{1-A^2}$$

$$\log z = A \log x + \sqrt{1-A^2} \log y + C$$

$$z = C x^A y^{\sqrt{1-A^2}}$$

c) $p^2 + q^2 = z$

$$x = x + ay + b$$

$$z = f(x)$$

$$p = \frac{dz}{dx} = \frac{dz}{dx} \cdot \frac{dx}{dx} = \frac{dz}{dx}$$

$$q = \frac{dz}{dy} = \frac{dz}{dx} \cdot \frac{dx}{dy} = a \frac{dz}{dx}$$

$$\therefore z = p^2 + q^2 = \left(\frac{dz}{dx}\right)^2 + a^2 \left(\frac{dz}{dx}\right)^2$$

$$\frac{dz}{dx} = \sqrt{\frac{z}{1+a^2}}$$

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$$\frac{dx}{\sqrt{1+a^2}} = \frac{dz}{\sqrt{2}} \Rightarrow \frac{x}{\sqrt{1+a^2}} = 2\sqrt{z}$$

$$\boxed{(x+ay+b)^2 = 4(1+a^2)z} //$$

d) $p^3 + q^3 = 2xz$

$$X = x + ay$$

$$\left(\frac{dz}{dx}\right)^3 + a^3 \left(\frac{dz}{dx}\right)^2 = 3a \left(\frac{dz}{dx}\right) z$$

$$\frac{dz}{dx} = \frac{3az}{\cancel{4a^3}(1+a^3)}$$

$$\int \frac{dz}{z} = \frac{3ax}{(1+a^3)} \Rightarrow \ln z = \frac{3ax}{1+a^3} + C$$

$$\boxed{z = e^{\frac{3a(x+ay)}{1+a^3} + C}} //$$

e) $p^2 - x = -q^2 - y$

$$p^2 - x = -(q^2 - y)$$

$$p = \pm \sqrt{x+a} \quad q = \pm \sqrt{y-a}$$

$$dz = pdx + qdy \Rightarrow \pm \int \sqrt{x+a} dx + \int \sqrt{y-a} dy$$

$$\boxed{z = \frac{2}{3} \left(\pm (x+a)^{3/2} \pm (y-a)^{3/2} \right)} //$$

$$f) px + qy + \log pq = z$$

$$z = ax + by + c$$

$$p=a \quad q=b \quad c = \log pq$$

$$\boxed{z = ax + by + \log(ab)} //$$

Q5 Solve the following PDEs by charpit's method.

a) $z = pq$

~~let~~ let $f = z - pq = 0 \quad \text{--- (i)}$

$$f_p = -q \quad f_x = 0 \quad f_y = 0 \quad f_z = 1$$

$$\frac{dx}{-q} = \frac{dy}{-p} = \frac{dz}{-pq} = \frac{dp}{p} = \frac{dq}{q}$$

$$\frac{dp}{p} = \frac{dq}{q} \Rightarrow \log p = \log q + \log c$$

$$\boxed{p = cq} \quad \text{--- (ii)}$$

$$z - pq = 0 \Rightarrow \cancel{z - pq^2} \quad z - cq^2 = 0$$

$$q = \sqrt{z/c} \quad p = \cancel{c}\sqrt{cz}$$

$$dz = pdx + qdy \Rightarrow dz = \sqrt{cz} dx + \sqrt{\frac{z}{c}} dy$$

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$$2\sqrt{cz} = cx + y + c'$$

b) $px + qy = pq$

$$f = px - qy - pq \quad \textcircled{i}$$

$$\frac{dx}{x-z} = \frac{dy}{y-p} = \frac{dz}{p(x-z) + q(y-p)} = \frac{dp}{-p} = \frac{dq}{-q}$$

$$\frac{dp}{p} = \frac{dq}{q} \Rightarrow p = qc \quad \textcircled{ii}$$

$$f = qc x - qy - cq^2 = 0$$

$$q = \frac{cx+y}{c} \quad p = \alpha cx + y$$

$$dz = pdx + qdy \Rightarrow \boxed{2cz = (cx+y)^2 + 2c'}$$

c) $(p^2 + q^2)y = q^2z$

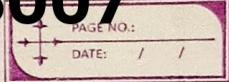
$$f = (p^2 + q^2)y - q^2z = 0$$

$$\frac{dx}{2py} = \frac{dy}{2qy - z} = \frac{dz}{p(2py) + q(2qy - z)} = \frac{dp}{-pq} = \frac{dq}{p^2}$$

$$-\frac{dp}{pq} = \frac{dz}{p^2} \Rightarrow p^2 + q^2 = c^2 \quad \textcircled{iii}$$

$$\textcircled{iv} \quad c^2y = qz -$$

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$$g = \frac{c^2 y}{z}$$

$$p = \frac{c}{z} \sqrt{z^2 - a^2 y^2}$$

$$dz = px + gy dy$$

$$z dz - c^2 y dy = ca \sqrt{z^2 - a^2 y^2} dx$$

$$\text{act } \cancel{\text{ad}} \quad cdx = \frac{1}{b} \left[\frac{z dz - a^2 y dy}{\sqrt{z^2 - a^2 y^2}} \right]$$

$$\boxed{z^2 = (cx + b)^2 + c^2 y^2}$$

d) $z^2 = pgxy$

$$f: z^2 - pgxy = 0$$

$$\frac{dx}{gxy} = \frac{dy}{-pgy} = \frac{dz}{-2pgxy} = \frac{dp}{-(-pgy + 2pz)} = \frac{dz}{-(pgx + 2paz)}$$

~~$$\int pdx + ndy$$~~

$$\int \frac{pdz + ndy}{paz} = \int \frac{2dy + ydz}{yz}$$

$$\int \frac{d(paz)}{paz} = \int \frac{d(2y)}{yz} \Rightarrow paz = a^2 g y \quad \text{(ii)}$$

$$z^2 = \frac{a^2 gy}{x} \Rightarrow z^2 = a^2 g^2 y^2$$

$$g = \frac{z}{ay} \quad p = \frac{a^2 y}{x}$$

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$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$\ln z = \ln(x^a y^b)$$

$$\boxed{z = x^a y^b}$$

e) $z = px + qy + p^2 + q^2$

$$f = px + qy + p^2 + q^2 - z = 0$$

$$\frac{dx}{x+2p} = \frac{dy}{y+2q} = \frac{dz}{p(x+2p) + q(y+2q)} = \frac{dp}{0} = \frac{dq}{0}$$

$$\begin{aligned} dp = 0 &\Rightarrow p = a \\ dq = 0 &\Rightarrow q = b \end{aligned}$$

$$\boxed{z = ax + by + a^2 + b^2}$$

f) $(p+q)(xp + yq) = 1$

$$\text{Let } z = px + qy$$

$$(p+q)z = 1$$

$$X = x + ay$$

$$p = \frac{dz}{dx} \cdot \frac{dx}{dx} = \frac{\partial z}{\partial x}$$

$$q = \frac{dz}{dx} \cdot \frac{dx}{dy} = \frac{a \partial z}{\partial x}$$

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$$\left(\frac{dz}{dx} + a \frac{dz}{dx} \right) z = 1$$

$$z dz = \frac{1}{1+a} dx$$

$$(1+a) \frac{z^2}{2} = \frac{x+c}{1+a}$$

$$z = \frac{\sqrt{2} \sqrt{x+ay+c}}{\sqrt{1+a}}$$

Q6 → Obtain the general solⁿ ----- $z=1, y=zx$

$$(2y^2+z)p + (y+2x)q = 4xy - z$$

$$\frac{dx}{2y^2+z} = \frac{dy}{y+2x} = \frac{dz}{2yz-z} = \frac{dx+dz}{2y^2+z+2yx-z}$$

(i) (ii) (iii) (iv)

(ii) & (iv)

$$\frac{dy}{y+2x} = \frac{d(x+z)}{2y(y+2x)} \Rightarrow d(x+z) = 2y dy$$

$$\Rightarrow x+z = y^2 + C_1$$

$$C_1 = x - y^2 + z \quad (i)$$

using multiplier $2x, -2, -y$

$$2x dx - 2 dy - y dz = 0$$

$$x^2 - yz = C_2$$

$$\cancel{\phi(x-yz, x^2-yz)} \left[\phi(x-y^2+z, x^2-yz) = 0 \right]$$

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for particular solution \rightarrow

$$\cancel{C_1 + \lambda C_2 = a}$$

$$(x - y^2 + z) + \lambda(x^2 - yz) = \text{const.}$$

for $(x=y)$ & $(z=1)$

$$(x - x^2 + 1) + \lambda(x^2 - x) = \text{const.}$$

$\therefore \lambda = 1$ for a to be const.

for ~~$\lambda = 1$~~ , $\lambda = 1$, $a = 1$

$$\therefore (x - y^2 + z) + (x^2 - yz) = 1$$

$$\cancel{\boxed{z(1-y) + x - y^2 + x^2 = 1}}$$

Q7 \rightarrow Find the equation of the surface satisfying
 $t = 6x^3y$ & containing the two lines
 $y=0, z=0$ & $y=1, z=1$

$$t = 6x^3y \Rightarrow \frac{dt}{dy} = 6x^3y$$

$$g = 3x^3y^2 + f(x) \leftarrow \left(g = \frac{dz}{dy}\right)$$

$$z = x^3y^3 + \int f(x) dy$$

$$z = x^3y^3 + y f(x) + g(x)$$

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for $y=z=0$

$$0 = 0+0+g(x)$$
$$g(x)=0 \quad \text{---(i)}$$

for $y=z=1$

$$1 = x^3 + f(x)$$

$$f(x) = 1-x^3 \quad \text{---(ii)}$$

$$\therefore \boxed{z = x^3 + y^3 + y(1-x^3)} //$$