Note: Today is going to consist of a lot of live examples instead of pre-written ones like some other notebooks

DSolve

Solving Simple, Ordinary Differential Equations

Lets solve $y'[x] + y[x] == a\,\mathrm{Sin}[x]$

In[ ]:=
```
DSolve[y'[x] + y[x] == a Sin[x], {y}, x]
```

Out[ ]=
$$\left\{\left\{y \to \mathrm{Function}\left[\{x\}, e^{-x} c_1 + \frac{1}{2} a\,(-\mathrm{Cos}[x] + \mathrm{Sin}[x])\right]\right\}\right\}$$

Now lets give it initial conditions as well

Lets solve $y'[x] + y[x] == a\,\mathrm{Sin}[x]$

y[0] = 1

In[ ]:=
```
sol = DSolve[{
    y'[x] + y[x] == 2 Sin[x],
    y[0] == 1
   }, {y}, x][[1]]
```

Out[ ]=
$$\{y \to \mathrm{Function}[\{x\}, -e^{-x} (-2 + e^{x} \mathrm{Cos}[x] - e^{x} \mathrm{Sin}[x])]\}$$

In[ ]:=
```
y[x] /. sol
```

Out[ ]=
$$-e^{-x} (-2 + e^{x} \mathrm{Cos}[x] - e^{x} \mathrm{Sin}[x])$$

In[ ]:=
```
y[3] /. sol
```

Out[ ]=
$$-\frac{-2 + e^3 \text{Cos}[3] - e^3 \text{Sin}[3]}{e^3}$$

In[ ]:=
```
y[3] /. sol // N
```

Out[ ]= 1.23069

Solving Systems of Ordinary Differential Equations

Let’s now look at an example where we want to solve a system of differential equations like:

y’[t] == a y[t] + b x[t]
x’[t] == c y[t] + d x[t]

In[ ]:=
```
sol = DSolve[{
    y'[t] == a y[t] + b x[t],
    x'[t] == c y[t] + d x[t],
    y[0] == 1,
    x[0] == 1
   }, {x, y}, t][[1]]
```

Out[ ]=

$$\Big\{x \to \text{Function}\Big[\{t\}, \frac{1}{2\sqrt{a^2+4bc-2ad+d^2}}\Big(a\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - 2c\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - d\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\;e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - a\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + 2c\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + d\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\;e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\Big)\Big],$$

$$y \to \text{Function}\Big[\{t\}, \frac{1}{2\sqrt{a^2+4bc-2ad+d^2}}\Big(-a\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - 2b\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + d\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\;e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + a\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + 2b\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} - d\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\;e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\Big)\Big]\Big\}$$

In[ ]:= `x[t] /. sol`

Out[ ]=
$$\frac{1}{2\sqrt{a^2+4bc-2ad+d^2}}\left(a\, e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - d\, e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\, e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - a\, e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + d\, e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\, e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\right) \mathbb{C}_1 - \frac{c\left(e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\right)\mathbb{C}_2}{\sqrt{a^2+4bc-2ad+d^2}}$$

In[ ]:= `y[t] /. sol`

Out[ ]=
$$-\frac{b\left(e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\right)\mathbb{C}_1}{\sqrt{a^2+4bc-2ad+d^2}} + \frac{1}{2\sqrt{a^2+4bc-2ad+d^2}}\left(-a\, e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + d\, e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\, e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + a\, e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} - d\, e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\, e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\right)\mathbb{C}_2$$

More complicated ODEs?

Can we solve this one?

y'[t] == a y[t] + b x[t] + Cos[t]

x’[t] == c y[t] + d x[t] + t

```
In[ ]:= sol = DSolve[{
      y'[t] == a y[t] + b x[t] + Cos[t],
      x'[t] == c y[t] + d x[t] + t
     }, {x, y}, t][[1]]
```

Out[ ]=

$$\Bigg\{x \to \text{Function}\Bigg[\{t\}, \frac{1}{2\sqrt{a^2+4bc-2ad+d^2}}\Bigg(a\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - d\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - a\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + d\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\Bigg)\,\mathbb{c}_1 - \frac{c\left(e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\right)\mathbb{c}_2}{\sqrt{a^2+4bc-2ad+d^2}} +$$

$$\frac{1}{4\left(a^2+4bc-2ad+d^2\right)}\Bigg(a\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - d\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - a\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + d\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} + \sqrt{a^2+4bc-2ad+d^2}\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\Bigg)$$

$$\Bigg(-\Bigg(\Bigg(16\,e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\Big(-a^3 - 3abc + a^2 d - bcd + a^2\sqrt{a^2+4bc-2ad+d^2} + bc\sqrt{a^2+4bc-2ad+d^2} + a^3\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t} + 3abc\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t} - a^2 d\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$

$$b\,c\,d\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t} + a^2 \sqrt{a^2+4bc-2ad+d^2}$$
$$e^{\sqrt{a^2+4bc-2ad+d^2}\,t} + b\,c\,\sqrt{a^2+4bc-2ad+d^2}$$
$$e^{\sqrt{a^2+4bc-2ad+d^2}\,t} + a^2 b\,c\,t + 2 b^2 c^2 t - a^3 d\,t - 3$$
$$a\,b\,c\,d\,t + a^2 d^2 t - a\,b\,c\,\sqrt{a^2+4bc-2ad+d^2}$$
$$t + a^2 d\,\sqrt{a^2+4bc-2ad+d^2}\;t - a^2 b\,c$$
$$e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\,t - 2 b^2 c^2 e^{\sqrt{a^2+4bc-2ad+d^2}\,t}$$
$$t + a^3 d\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\,t + 3\,a\,b\,c\,d$$
$$e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\,t - a^2 d^2 e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\,t - a$$
$$b\,c\,\sqrt{a^2+4bc-2ad+d^2}\;e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\,t +$$
$$a^2 d\,\sqrt{a^2+4bc-2ad+d^2}\;e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\,t\Big)\Big)\Big/$$
$$\Big(\Big(-a-d+\sqrt{a^2+4bc-2ad+d^2}\Big)^2$$
$$\Big(a+d+\sqrt{a^2+4bc-2ad+d^2}\Big)^2\Big)\Big) -$$
$$\Big(4\,c\,e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\,\Big(4\,a + a^3 + 4\,d +$$
$$3\,a^2 d + 3\,a\,d^2 + d^3 + 4\,\sqrt{a^2+4bc-2ad+d^2} -$$
$$a^2 \sqrt{a^2+4bc-2ad+d^2} - 2\,a\,d$$
$$\sqrt{a^2+4bc-2ad+d^2} - d^2 \sqrt{a^2+4bc-2ad+d^2} -$$
$$a\left(a^2+4bc-2ad+d^2\right) - d\left(a^2+4bc-2ad+d^2\right) +$$
$$\left(a^2+4bc-2ad+d^2\right)^{3/2} -$$
$$4\,a\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t} - a^3 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$4\,d\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t} - 3\,a^2 d\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$3\,a\,d^2 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} - d^3 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$
$$4\,\sqrt{a^2+4bc-2ad+d^2}\;e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$a^2 \sqrt{a^2+4bc-2ad+d^2}\;e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$2\,a\,d\,\sqrt{a^2+4bc-2ad+d^2}\;e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$d^2 \sqrt{a^2+4bc-2ad+d^2}\;e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$
$$a\left(a^2+4bc-2ad+d^2\right) e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$

$$d\left(a^2+4bc-2ad+d^2\right)e^{\sqrt{a^2+4bc-2ad+d^2}\,t}+$$
$$\left.\left.\left(a^2+4bc-2ad+d^2\right)^{3/2}e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\right)\mathrm{Cos}[t]\right)\Big/\left(\left(-2\,i-a-d+\sqrt{a^2+4bc-2ad+d^2}\right)\right.$$
$$\left(2\,i-a-d+\sqrt{a^2+4bc-2ad+d^2}\right)$$
$$\left(-2\,i+a+d+\sqrt{a^2+4bc-2ad+d^2}\right)$$
$$\left.\left(2\,i+a+d+\sqrt{a^2+4bc-2ad+d^2}\right)\right)-$$
$$\left(8\,c\,e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\left(-4-2a^2-4bc-2d^2+2a\sqrt{a^2+4bc-2ad+d^2}+2d\sqrt{a^2+4bc-2ad+d^2}+\right.\right.$$
$$4\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t}+a^2e^{\sqrt{a^2+4bc-2ad+d^2}\,t}+$$
$$2ad\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t}+d^2e^{\sqrt{a^2+4bc-2ad+d^2}\,t}+$$
$$2a\sqrt{a^2+4bc-2ad+d^2}\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t}+$$
$$2d\sqrt{a^2+4bc-2ad+d^2}\,e^{\sqrt{a^2+4bc-2ad+d^2}\,t}+$$
$$\left.\left.\left(a^2+4bc-2ad+d^2\right)e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\right)\mathrm{Sin}[t]\right)\Big/$$
$$\left(\left(-2\,i-a-d+\sqrt{a^2+4bc-2ad+d^2}\right)\right.$$
$$\left(2\,i-a-d+\sqrt{a^2+4bc-2ad+d^2}\right)$$
$$\left(-2\,i+a+d+\sqrt{a^2+4bc-2ad+d^2}\right)$$
$$\left.\left(2\,i+a+d+\sqrt{a^2+4bc-2ad+d^2}\right)\right)\Big)-$$
$$\frac{1}{2\left(a^2+4bc-2ad+d^2\right)}$$
$$c\left(e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t}-e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\right)$$

$$
\begin{aligned}
&\Bigg(\Bigg(16\, b\, e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4 b c-2 a d+d^2}\right) t} \\
&\quad\Big(-a^2-2 b c-d^2+a \sqrt{a^2+4 b c-2 a d+d^2}+ \\
&\quad d \sqrt{a^2+4 b c-2 a d+d^2}+a^2 e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}+ \\
&\quad 2 b c\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}+d^2 e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}+ \\
&\quad a \sqrt{a^2+4 b c-2 a d+d^2}\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}+ \\
&\quad d \sqrt{a^2+4 b c-2 a d+d^2}\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}+a b c t- \\
&\quad a^2 d t+b c d t-a d^2 t-b c \sqrt{a^2+4 b c-2 a d+d^2}\, t+ \\
&\quad a d \sqrt{a^2+4 b c-2 a d+d^2}\, t- \\
&\quad a b c\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t+a^2 d\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t- \\
&\quad b c d\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t+a d^2 e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t- \\
&\quad b c \sqrt{a^2+4 b c-2 a d+d^2}\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t+ \\
&\quad a d \sqrt{a^2+4 b c-2 a d+d^2}\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t\Big)\Bigg)\Big/ \\
&\quad\left(\left(-a-d+\sqrt{a^2+4 b c-2 a d+d^2}\right)^2\right. \\
&\quad\left.\left(a+d+\sqrt{a^2+4 b c-2 a d+d^2}\right)^2\right)+ \\
&\quad\Big(4\, e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4 b c-2 a d+d^2}\right) t}\,\Big(-4 a^2-8 b c+2 a^2 b c+ \\
&\quad 4 a d-a^3 d+4 a b c d-a^2 d^2+2 b c d^2+ \\
&\quad a d^3+d^4-4 a \sqrt{a^2+4 b c-2 a d+d^2}- \\
&\quad a^2 d \sqrt{a^2+4 b c-2 a d+d^2}-2 a d^2 \\
&\quad \sqrt{a^2+4 b c-2 a d+d^2}-d^3 \sqrt{a^2+4 b c-2 a d+d^2}- \\
&\quad 2 b c\left(a^2+4 b c-2 a d+d^2\right)+a d \\
&\quad \left(a^2+4 b c-2 a d+d^2\right)-d^2\left(a^2+4 b c-2 a d+d^2\right)+ \\
&\quad d\left(a^2+4 b c-2 a d+d^2\right)^{3/2}+4 a^2 e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}+ \\
&\quad 8 b c\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}-2 a^2 b c\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}- \\
&\quad 4 a d\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}+a^3 d\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}- \\
&\quad 4 a b c d\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}+
\end{aligned}
$$

$$a^2 d^2 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} - 2bcd^2 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$a d^3 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} - d^4 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$4a\sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$a^2 d\sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$2ad^2\sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$d^3\sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$
$$2bc\left(a^2+4bc-2ad+d^2\right) e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$ad\left(a^2+4bc-2ad+d^2\right) e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$
$$d^2\left(a^2+4bc-2ad+d^2\right) e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$
$$d\left(a^2+4bc-2ad+d^2\right)^{3/2} e^{\sqrt{a^2+4bc-2ad+d^2}\,t}\Big)$$
$$\mathrm{Cos}[t]\Big) \Big/ \Big(\Big(-2i - a - d + \sqrt{a^2+4bc-2ad+d^2}\Big)$$
$$\Big(2i - a - d + \sqrt{a^2+4bc-2ad+d^2}\Big)$$
$$\Big(-2i + a + d + \sqrt{a^2+4bc-2ad+d^2}\Big)$$
$$\Big(2i + a + d + \sqrt{a^2+4bc-2ad+d^2}\Big)\Big) +$$
$$\Big(4 e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\Big(4a - a^3 - 8abc -$$
$$4d + 3a^2 d - 8bcd + ad^2 - 3d^3 +$$
$$4\sqrt{a^2+4bc-2ad+d^2} - a^2\sqrt{a^2+4bc-2ad+d^2} +$$
$$2ad\sqrt{a^2+4bc-2ad+d^2} +$$
$$3d^2\sqrt{a^2+4bc-2ad+d^2} +$$
$$a\left(a^2+4bc-2ad+d^2\right) - d\left(a^2+4bc-2ad+d^2\right) +$$
$$\left(a^2+4bc-2ad+d^2\right)^{3/2} -$$
$$4a\, e^{\sqrt{a^2+4bc-2ad+d^2}\,t} + a^3 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$
$$8abc\, e^{\sqrt{a^2+4bc-2ad+d^2}\,t} + 4d\, e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$3a^2 d\, e^{\sqrt{a^2+4bc-2ad+d^2}\,t} + 8bcd\, e^{\sqrt{a^2+4bc-2ad+d^2}\,t} -$$
$$ad^2 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} + 3d^3 e^{\sqrt{a^2+4bc-2ad+d^2}\,t} +$$

$$4\sqrt{a^2+4bc-2ad+d^2}\ e^{\sqrt{a^2+4bc-2ad+d^2}\ t} -$$
$$a^2\sqrt{a^2+4bc-2ad+d^2}\ e^{\sqrt{a^2+4bc-2ad+d^2}\ t} +$$
$$2ad\sqrt{a^2+4bc-2ad+d^2}\ e^{\sqrt{a^2+4bc-2ad+d^2}\ t} +$$
$$3d^2\sqrt{a^2+4bc-2ad+d^2}\ e^{\sqrt{a^2+4bc-2ad+d^2}\ t} -$$
$$a\left(a^2+4bc-2ad+d^2\right)e^{\sqrt{a^2+4bc-2ad+d^2}\ t} +$$
$$d\left(a^2+4bc-2ad+d^2\right)e^{\sqrt{a^2+4bc-2ad+d^2}\ t} +$$
$$\left(a^2+4bc-2ad+d^2\right)^{3/2} e^{\sqrt{a^2+4bc-2ad+d^2}\ t}\Big)$$

$$\text{Sin}[t]\Big) \Big/ \Big(\Big(-2i-a-d+\sqrt{a^2+4bc-2ad+d^2}\Big)$$
$$\Big(2i-a-d+\sqrt{a^2+4bc-2ad+d^2}\Big)$$
$$\Big(-2i+a+d+\sqrt{a^2+4bc-2ad+d^2}\Big)\ \Big(2i+a+d+$$
$$\sqrt{a^2+4bc-2ad+d^2}\Big)\Big)\Big)\Big],\ y \to \text{Function}\Big[\{t\},$$

$$-\frac{b\left(e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} - e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t}\right)\mathbb{C}_1}{\sqrt{a^2+4bc-2ad+d^2}} +$$

$$\frac{1}{2\sqrt{a^2+4bc-2ad+d^2}}$$
$$\Big(-a$$
$$e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} +$$
$$d\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} +$$
$$\sqrt{a^2+4bc-2ad+d^2}$$
$$e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4bc-2ad+d^2}\right)t} +$$
$$a\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} -$$
$$d$$
$$e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right)t} +$$
$$\sqrt{a^2+4bc-2ad+d^2}$$

$$e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4 b c-2 a d+d^2}\right) t}\Bigg)$$

$$\mathbb{c}_2 - \frac{1}{2\left(a^2+4 b c-2 a d+d^2\right)}$$

$$b$$

$$\Bigg(e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4 b c-2 a d+d^2}\right) t} -$$

$$e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4 b c-2 a d+d^2}\right) t}\Bigg)$$

$$\Bigg(-\Bigg(\Bigg(16\, e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4 b c-2 a d+d^2}\right) t}\Bigg(-a^3-3 a b c+a^2$$

$$d - b c d + a^2\sqrt{a^2+4 b c-2 a d+d^2} + b c$$

$$\sqrt{a^2+4 b c-2 a d+d^2} + a^3 e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t} + 3$$

$$a b c\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t} - a^2 d\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t} +$$

$$b c d\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t} + a^2\sqrt{a^2+4 b c-2 a d+d^2}$$

$$e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t} + b c\sqrt{a^2+4 b c-2 a d+d^2}$$

$$e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t} + a^2 b c t + 2 b^2 c^2 t - a^3 d t - 3$$

$$a b c d t + a^2 d^2 t - a b c\sqrt{a^2+4 b c-2 a d+d^2}$$

$$t + a^2 d\sqrt{a^2+4 b c-2 a d+d^2}\, t - a^2 b c$$

$$e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t - 2 b^2 c^2 e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}$$

$$t + a^3 d\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t + 3 a b c d$$

$$e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t - a^2 d^2 e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t - a$$

$$b c\sqrt{a^2+4 b c-2 a d+d^2}\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t +$$

$$a^2 d\sqrt{a^2+4 b c-2 a d+d^2}\, e^{\sqrt{a^2+4 b c-2 a d+d^2}\, t}\, t\Bigg)\Bigg) \Big/$$

$$\Bigg(\left(-a-d+\sqrt{a^2+4 b c-2 a d+d^2}\right)^2$$

$$\left(a+d+\sqrt{a^2+4 b c-2 a d+d^2}\right)^2\Bigg)\Bigg) -$$

$$\Bigg(4 c\, e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4 b c-2 a d+d^2}\right) t}\Bigg(4 a + a^3 + 4 d +$$

$$3 a^2 d + 3 a d^2 + d^3 + 4\sqrt{a^2+4 b c-2 a d+d^2} -$$

$$
\begin{aligned}
&a^2 \sqrt{a^2+4bc-2ad+d^2} - 2ad \\
&\quad \sqrt{a^2+4bc-2ad+d^2} - d^2 \sqrt{a^2+4bc-2ad+d^2} - \\
&a \left(a^2+4bc-2ad+d^2\right) - d \left(a^2+4bc-2ad+d^2\right) + \\
&\left(a^2+4bc-2ad+d^2\right)^{3/2} - \\
&4\,a\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} - a^3\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} - \\
&4\,d\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} - 3\,a^2\,d\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} - \\
&3\,a\,d^2\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} - d^3\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + \\
&4 \sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\, t} - \\
&a^2 \sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\, t} - \\
&2\,a\,d \sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\, t} - \\
&d^2 \sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + \\
&a \left(a^2+4bc-2ad+d^2\right) e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + \\
&d \left(a^2+4bc-2ad+d^2\right) e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + \\
&\left(a^2+4bc-2ad+d^2\right)^{3/2} e^{\sqrt{a^2+4bc-2ad+d^2}\, t} \Big) \\
&\mathrm{Cos}[t]\Big) \Big/ \left(\left(-2\,i - a - d + \sqrt{a^2+4bc-2ad+d^2}\right)\right. \\
&\left(2\,i - a - d + \sqrt{a^2+4bc-2ad+d^2}\right) \\
&\left(-2\,i + a + d + \sqrt{a^2+4bc-2ad+d^2}\right) \\
&\left.\left(2\,i + a + d + \sqrt{a^2+4bc-2ad+d^2}\right)\right) - \\
&\Big(8\,c\, e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4bc-2ad+d^2}\right) t} \Big(-4 - 2a^2 - 4bc - 2d^2 + 2a \\
&\quad \sqrt{a^2+4bc-2ad+d^2} + 2d \sqrt{a^2+4bc-2ad+d^2} + \\
&4\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + a^2\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + \\
&2\,a\,d\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + d^2\, e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + \\
&2\,a \sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + \\
&2\,d \sqrt{a^2+4bc-2ad+d^2}\; e^{\sqrt{a^2+4bc-2ad+d^2}\, t} + \\
&\left(a^2+4bc-2ad+d^2\right) e^{\sqrt{a^2+4bc-2ad+d^2}\, t}\Big)\, \mathrm{Sin}[t]\Big) \Big/
\end{aligned}
$$

$$\left(\left(-2\,\mathrm{i}-a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\left(2\,\mathrm{i}-a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\left(-2\,\mathrm{i}+a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\left(2\,\mathrm{i}+a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\right)\right)+$$

$$\frac{1}{4\left(a^2+4\,b\,c-2\,a\,d+d^2\right)}\left(-a\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)t}+d\,e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)t}+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\;e^{\frac{1}{2}\left(a+d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)t}+a\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)t}-d\,e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)t}+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\;e^{\frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)t}\right)$$

$$\left(\left(16\,b\,e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)t}\left(-a^2-2\,b\,c-d^2+a\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}+d\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}+a^2\,e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+2\,b\,c\,e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+d^2\,e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+a\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\;e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+d\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\;e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+a\,b\,c\,t-a^2\,d\,t+b\,c\,d\,t-a\,d^2\,t-b\,c\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\;t+a\,d\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\;t-a\,b\,c\,e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}\,t+a^2\,d\,e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}\,t-b\,c\,d\,e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}\,t+a\,d^2\,e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}\,t-b\,c\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\;e^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}\,t+\right.\right.\right.$$

$$a\, d\, \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\; e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t}\; t\Big)\Big) \Big/$$

$$\Big(\Big(-a - d + \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\Big)^2$$

$$\Big(a + d + \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\Big)^2\Big) +$$

$$\Big(4\, e^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\right)\, t}\, \Big(-4\, a^2 - 8\, b\, c + 2\, a^2\, b\, c +$$

$$4\, a\, d - a^3\, d + 4\, a\, b\, c\, d - a^2\, d^2 + 2\, b\, c\, d^2 +$$

$$a\, d^3 + d^4 - 4\, a\, \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2} -$$

$$a^2\, d\, \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2} - 2\, a\, d^2$$

$$\sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2} - d^3\, \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2} -$$

$$2\, b\, c\, \left(a^2 + 4\, b\, c - 2\, a\, d + d^2\right) + a\, d$$

$$\left(a^2 + 4\, b\, c - 2\, a\, d + d^2\right) - d^2\, \left(a^2 + 4\, b\, c - 2\, a\, d + d^2\right) +$$

$$d\, \left(a^2 + 4\, b\, c - 2\, a\, d + d^2\right)^{3/2} + 4\, a^2\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} +$$

$$8\, b\, c\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} - 2\, a^2\, b\, c\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} -$$

$$4\, a\, d\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} + a^3\, d\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} -$$

$$4\, a\, b\, c\, d\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} +$$

$$a^2\, d^2\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} - 2\, b\, c\, d^2\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} -$$

$$a\, d^3\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} - d^4\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} -$$

$$4\, a\, \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\; e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} -$$

$$a^2\, d\, \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\; e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} -$$

$$2\, a\, d^2\, \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\; e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} -$$

$$d^3\, \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\; e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} +$$

$$2\, b\, c\, \left(a^2 + 4\, b\, c - 2\, a\, d + d^2\right)\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} -$$

$$a\, d\, \left(a^2 + 4\, b\, c - 2\, a\, d + d^2\right)\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} +$$

$$d^2\, \left(a^2 + 4\, b\, c - 2\, a\, d + d^2\right)\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t} +$$

$$d\, \left(a^2 + 4\, b\, c - 2\, a\, d + d^2\right)^{3/2}\, e^{\sqrt{a^2+4\, b\, c-2\, a\, d+d^2}\; t}\Big)$$

$$\mathrm{Cos}[t]\Big) \Big/ \Big(\Big(-2\, i - a - d + \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\Big)$$

$$\left(2\,\mathbb{i}-a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)$$
$$\left(-2\,\mathbb{i}+a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)$$
$$\left(2\,\mathbb{i}+a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\Big)+$$
$$\Bigg(4\,\mathbb{e}^{-\frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)t}\Big(4\,a-a^3-8\,a\,b\,c-$$
$$4\,d+3\,a^2\,d-8\,b\,c\,d+a\,d^2-3\,d^3+$$
$$4\,\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}-a^2\,\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}+$$
$$2\,a\,d\,\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}+$$
$$3\,d^2\,\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}+$$
$$a\left(a^2+4\,b\,c-2\,a\,d+d^2\right)-d\left(a^2+4\,b\,c-2\,a\,d+d^2\right)+$$
$$\left(a^2+4\,b\,c-2\,a\,d+d^2\right)^{3/2}-$$
$$4\,a\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+a^3\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+$$
$$8\,a\,b\,c\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+4\,d\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}-$$
$$3\,a^2\,d\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+8\,b\,c\,d\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}-$$
$$a\,d^2\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+3\,d^3\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+$$
$$4\,\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}-$$
$$a^2\,\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+$$
$$2\,a\,d\,\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+$$
$$3\,d^2\,\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}-$$
$$a\left(a^2+4\,b\,c-2\,a\,d+d^2\right)\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+$$
$$d\left(a^2+4\,b\,c-2\,a\,d+d^2\right)\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}+$$
$$\left(a^2+4\,b\,c-2\,a\,d+d^2\right)^{3/2}\mathbb{e}^{\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\,t}\Big)$$
$$\mathrm{Sin}[t]\Bigg)\Big/\left(\left(-2\,\mathbb{i}-a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\right.$$
$$\left(2\,\mathbb{i}-a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)$$
$$\left(-2\,\mathbb{i}+a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)$$

$$\left(2\, i + a + d + \sqrt{a^2 + 4\, b\, c - 2\, a\, d + d^2}\right)\Big)\Big)\Big]\Big\}$$

How about this one?

y'[t] == a y[t] + b x[t] + Cos[x[t]]
x'[t] == c y[t] + d x[t] + t

In[ ]:=
```
sol = DSolve[{
    y'[t] == a y[t] + b x[t] + Cos[x[t]],
    x'[t] == c y[t] + d x[t] + t
   }, {x, y}, t][[1]]
```

Out[ ]=
```
{y′[t] == Cos[x[t]] + b x[t] + a y[t], x′[t] == t + d x[t] + c y[t]}
```

Or what about this one?

```
y'[t] == Sin[e^y[t] t]
```

In[ ]:=
```
DSolve[{
   y'[t] == Sin[e^y[t] t]
  }, {y}, t]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[ ]=
```
DSolve[{y′[t] == Sin[e^y[t] t]}, {y}, t]
```

NDSolve

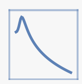
Solving Single ODEs Numerically

Let’s see how we can solve `y'[t] == Sin[ⅇ^y[t] t]` numerically if we know the initial conditions. For instance, let’s say that at time t=0 we know $y = \frac{1}{2}$ In other words:

$y[0] == \frac{1}{2}$

In[ ]:=
```
sol = NDSolve[{
    y'[t] == Sin[ⅇ^y[t] t],
    y[0] == 1/2
   }, {y}, {t, 0, 10}][[1]]
```

Out[ ]=
```
{y → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ]}
```

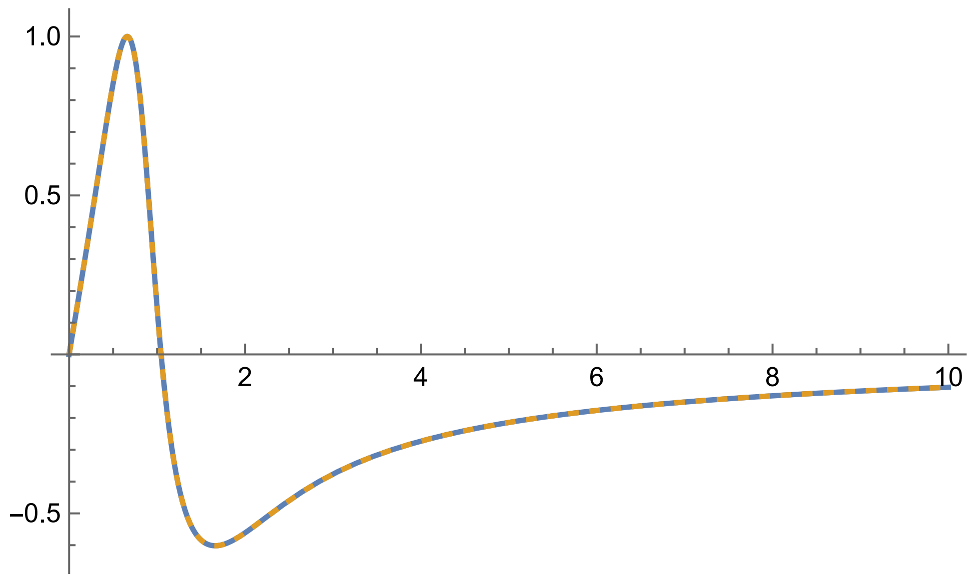


In[ ]:=
```
y'[5] /. sol
```

Out[ ]=
```
-0.214039
```

```
In[•]:= Plot[Evaluate[{y'[t], Sin[ⅇ^y[t] t]} /. sol], {t, 0, 10},
         PlotRange → All, PlotStyle → {Automatic, Dashed}]
```

Out[•]=



Initial Conditions in terms of derivatives

What if we had a similar differential equation of the form

$$y''[t] == \mathrm{Sin}\left[e^{y'[t]}\, t\right],$$
$$y[0] == \frac{1}{2},$$
$$y'[0] == \frac{1}{4}$$

In[ ]:=
```
NDSolve[{
    y''[t] == Sin[ⅇ^(y'[t]) t],
    y[0] == 1/2,
    y'[0] == 1/4
  }, {y}, {t, 0, 10}]
```

Out[ ]=
```
{{y → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ]}}
```

Solving Systems of ODEs Numerically

Let’s take a look at this example again

y’[t] == a y[t] + b x[t] + Cos[x[t]]
x’[t] == c y[t] + d x[t] + t

Where we will assume we know the values of $\begin{pmatrix} a & b \\ c & d \end{pmatrix} == \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

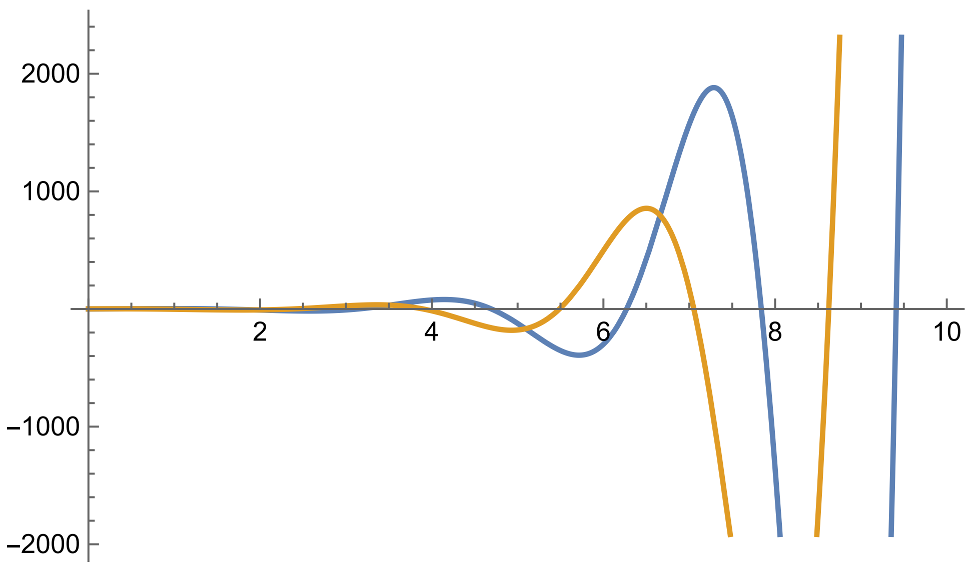
and that $\begin{pmatrix} y[0] \\ x[0] \end{pmatrix} == \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

In[ ]:=
```
sol = NDSolve[{
    y'[t] == 1 y[t] - 2 x[t] + Cos[x[t]],
    x'[t] == 2 y[t] + 1 x[t] + t,
    y[0] == 1,
    x[0] == 0
   }, {x, y}, {t, 0, 10}][[1]]
```

Out[ ]=
```
{x → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ],
 y → InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ]}
```

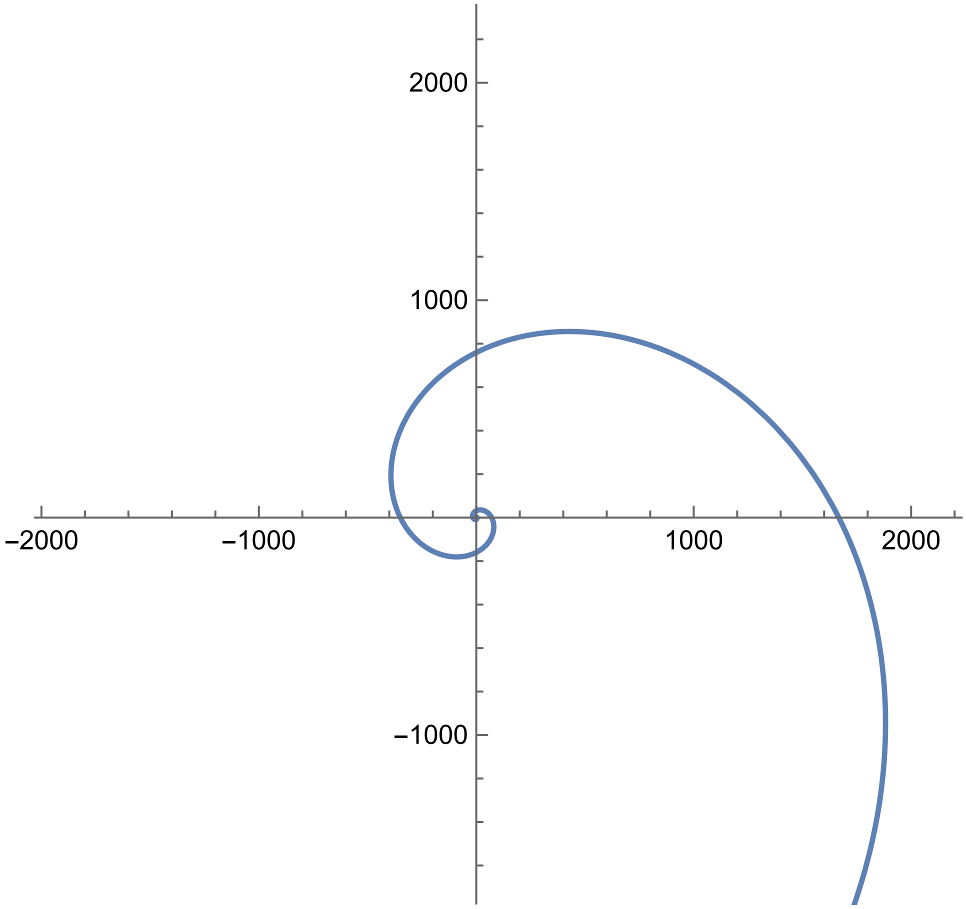
In[ ]:=
```
Plot[Evaluate[{x[t], y[t]} /. sol], {t, 0, 10}]
```

Out[ ]=



In[ ]:=
```
ParametricPlot[Evaluate[{x[t], y[t]} /. sol], {t, 0, 10}]
```

Out[ ]=



WhenEvents

In[ ]:=
```
sol = NDSolve[{
    y''[t] == -9.81,
    y[0] == 5,
    y'[0] == 0,
    WhenEvent[y[t] == 0, y'[t] → -0.95 y'[t]]
   }, y, {t, 0, 20}][[1]]
```

Out[ ]= {y → InterpolatingFunction[Domain: {{0., 20.}} Output: scalar]}

In[ ]:=
```
Plot[Evaluate[y[t] /. sol], {t, 0, 20}]
```

Out[ ]=