Integrac

1 Segun regla del trapecio simple

$$\int_a^b f(x)\,dx \simeq \int_a^b P_1(x)\,dx$$

$$P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$\int_a^b P_1(x)\,dx) = \int_a^b \frac{x-b}{a-b} f(a)\,dx + \int_a^b \frac{x-a}{b-a} f(b)\,dx$$

$$= \frac{f(a)}{a-b} \int_a^b x-b\,dx + \frac{f(b)}{b-a} \int_a^b x-a\,dx$$

$$\Rightarrow \int_a^b x-m\,dx = \frac{x^2}{2} - xm \Big|_a^b$$

$$= \frac{f(a)}{a-b}\left(\frac{b^2}{2} - b^2 - \left(\frac{a^2}{2} - ab\right)\right) + \dots$$

$$= \frac{f(a)}{a-b}\left(-\frac{b^2}{2} + ab - \frac{a^2}{2}\right) + \dots$$

$$= \frac{f(a)}{a-b} \cdot -\frac{(a-b)^2}{2} + \frac{f(b)}{b-a} \cdot -\frac{(a-b)^2}{2}$$

$$= \frac{f(a)}{2} \cdot b-a + \frac{f(b)}{2}\, b-a$$

$$\Rightarrow \int_a^b P_1(x)\,dx = \frac{b-a}{2}(f(a) + f(b))$$

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3 Segun metodo de Simpson $\frac{1}{3}$

$$\int_a^b f(x)\,dx \simeq \int_a^b P_2(x)$$

$\{(a,f(a)),\ (X_m, f(X_m)),\ (b, f(b))\}$

donde $X_m = \frac{a+b}{2}$

$$P_2(X) = \frac{(X-b)(X-X_m)}{(a-b)(a-X_m)} f(a) + \frac{(X-a)(X-b)}{(X_m-a)(X_m-b)} f(X_m) + \frac{(X-a)(X-X_m)}{(b-a)(b-X_m)} f(b)$$

Si expresamos cada punto como $X = a+\lambda h$, siendo h la separacion entre puntos en el intervalo $[a,b]$.

entonces:

$$\frac{(X-b)(X-X_m)}{(a-b)(a-X_m)} = \frac{(a+\lambda h-b)(a+\lambda h-X_m)}{2h^2} = \frac{(\lambda h-2h)(\lambda h-h)}{2h^2} = \frac{(\lambda-2)(\lambda-1)}{2}$$

$$\frac{(X-a)(X-b)}{(X_m-a)(X_m-b)} = \frac{\lambda h(\lambda h-2h)}{-h^2} = -\lambda(\lambda-2)$$

$$\frac{(X-a)(X-X_m)}{(b-a)(b-X_m)} = \frac{\lambda h(\lambda h-h)}{2h^2} = \frac{\lambda(\lambda-1)}{2}$$

$$\Rightarrow P_2(X) = \frac{(\lambda-2)(\lambda-1)}{2} f(a) - \lambda(\lambda-2) f(X_m) + \frac{\lambda(\lambda-1)}{2} f(b)$$

$$\Rightarrow \int_a^b P_2(x)\,dx = \int_0^2 \frac{h(\lambda-2)(\lambda-1)}{2} f(a)\,d\lambda - \int_0^2 h\lambda(\lambda-2) f(x_m)\,d\lambda + \int_0^2 \frac{h\lambda(\lambda-1)}{2} f(b)\,d\lambda$$

Se Sustituye $x = a+\lambda h$ y $dx = h\,d\lambda$

$$\int_0^2 (\lambda-2)(\lambda-1)\,d\lambda = \int_0^2 \lambda^2 - 3\lambda + 2\,d\lambda = \frac{\lambda^3}{3} - \frac{3\lambda^2}{2} + 2\lambda \Big|_0^2 = \frac{8}{3} - 6 + 4$$

$$= \frac{2}{3}$$

$$\int_0^2 \lambda^2 - \lambda 2\,d\lambda = \frac{\lambda^3}{3} - \frac{\cancel{2}\lambda^2}{\cancel{2}} \Big|_0^2 = \frac{8}{3} - 4 = -\frac{4}{3}$$

$$\int_0^2 \lambda^2 - \lambda\,d\lambda = \frac{\lambda^3}{3} - \frac{\lambda^2}{2} \Big|_0^2 = \frac{8}{3} - 2 = \frac{2}{3}$$

$$\int_a^b P_2(x)\,dx = \frac{h}{\cancel{2}} \cdot \frac{\cancel{2}}{3} f(a) + \frac{h4}{3} f(x_m) + \frac{h}{\cancel{2}} \cdot \frac{\cancel{2}}{3} f(b)$$

$$= \frac{h}{3}\left(f(a) + 4 f(x_m) + f(b) \right)$$

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