# GRB Afterglow Modeling

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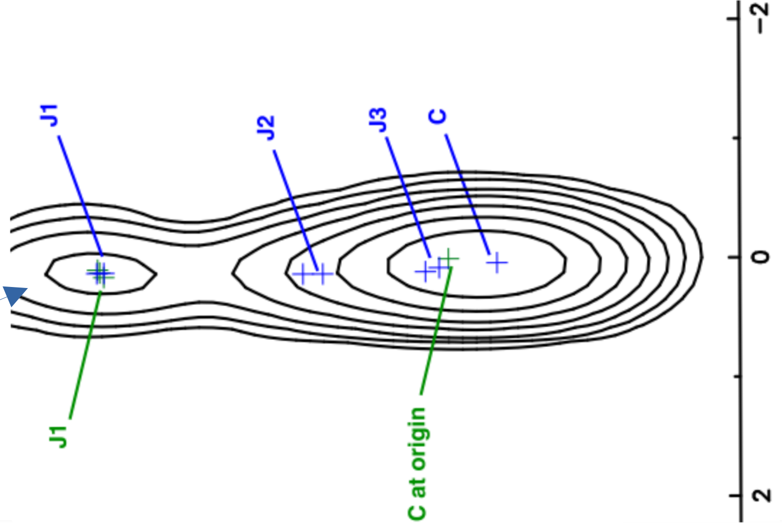
# Observables

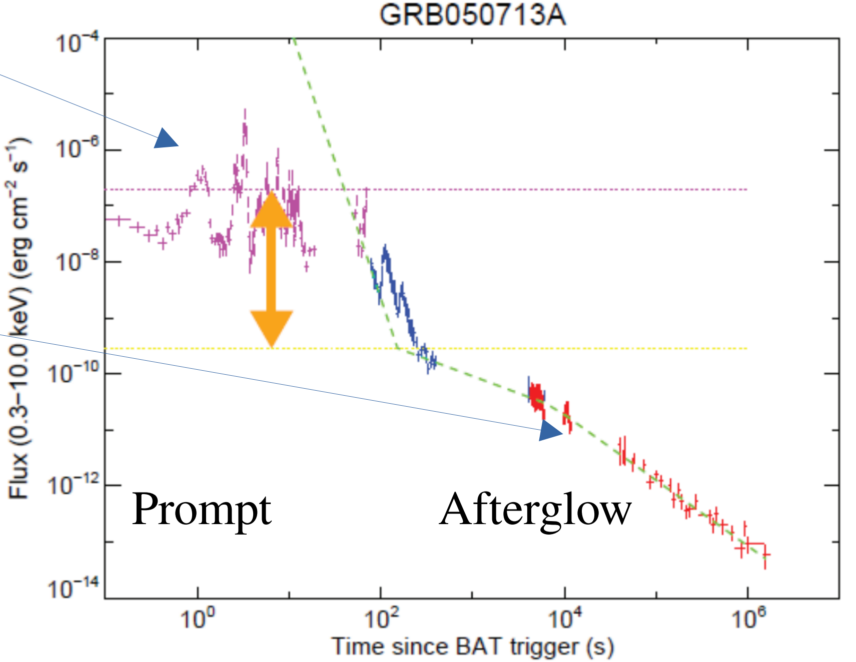
**Light curves, spectra, and sky maps**

- **Sky map** – intensity distribution projected onto a plane orthogonal to the line of sight (observed usually in radio band)

- **Prompt emission** (oscillatory, early) produced by internal shocks or magnetic reconnection inside the jet;
  synchrotron, SSC, thermal emission

- **Afterglow** (smooth, late) produced by external shocks (forward/reverse);
  synchrotron emission (based on spectra)



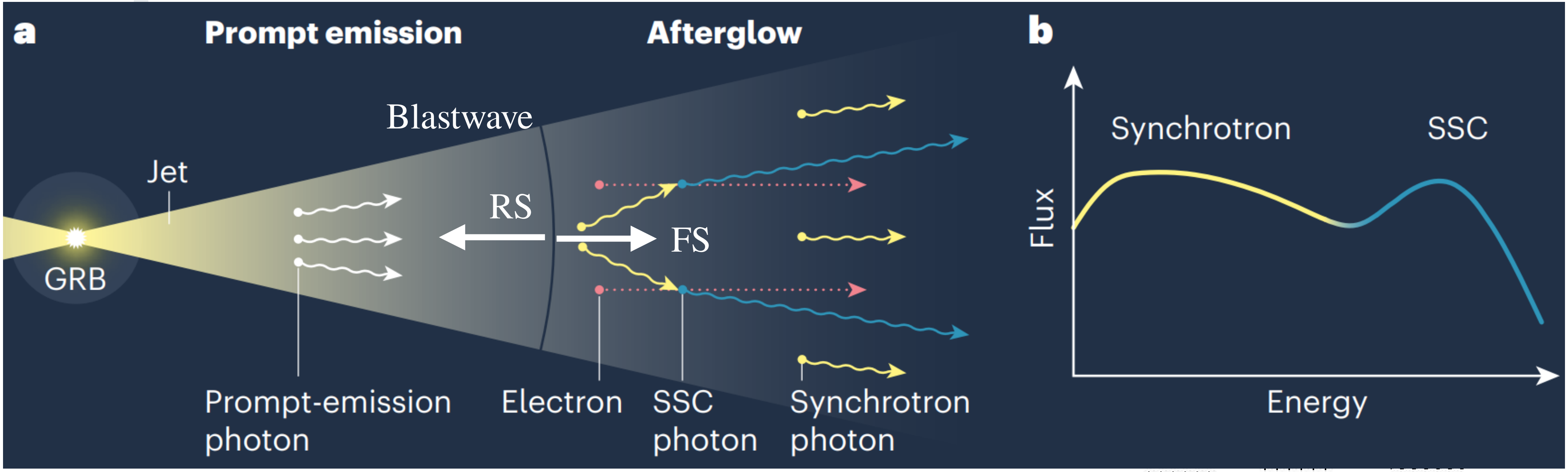



# Mechanism

## Synchrotron radiation from shocks in jet head

- Jet head has Forward and Reverse shocks (collisionless shocks)
- Magnetic field amplification and particle acceleration at shocks
- Synchrotron and inverse Compton emission from shock downstream

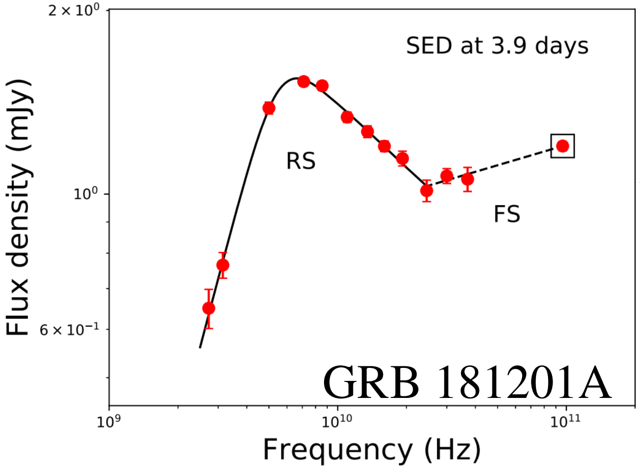
- **Afterglow depends on shock microphysics and jet dynamics**



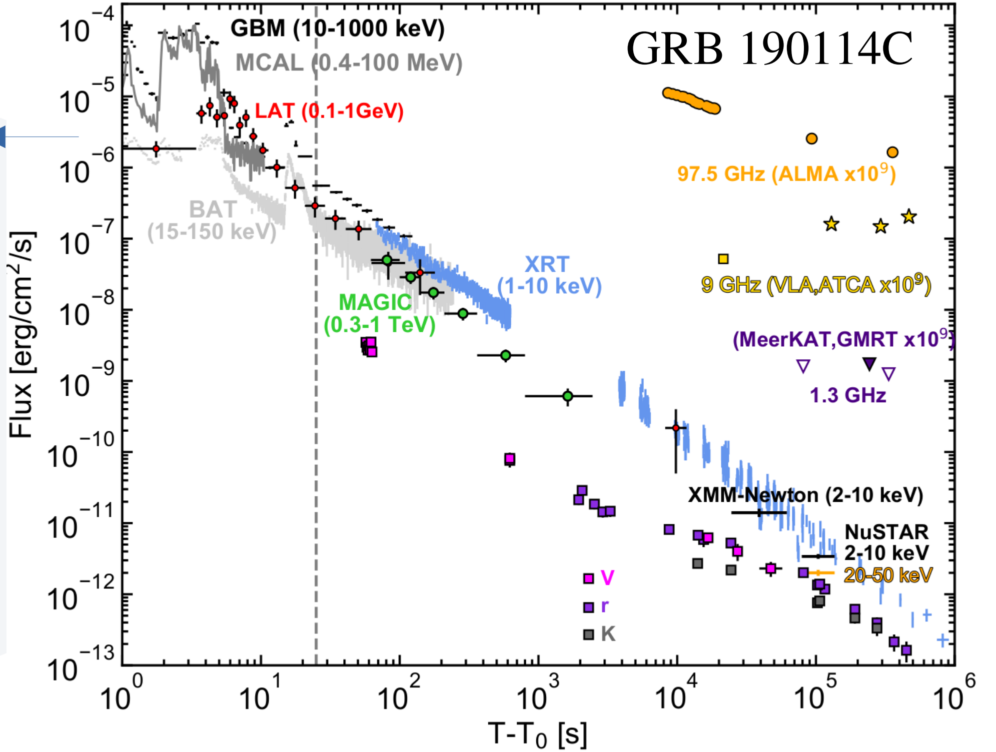
# Afterglow examples

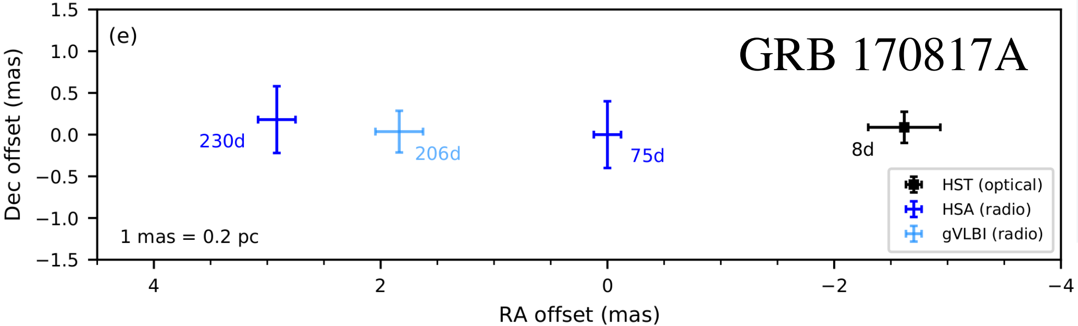
## FS & RS emission; VHE emission; sky map properties

- Synchrotron, power-law spectrum from forward (reverse) shocks
- GeV emission from inverse Compton scattering
- Radio sky maps showing jetted outflow moving off-axis




- Jet properties
- Microphysics
- Geometry
- Progenitor



# Model Dynamics

**Energy conservation; thin-shell approximation; sem-analytic**

Total blast wave energy

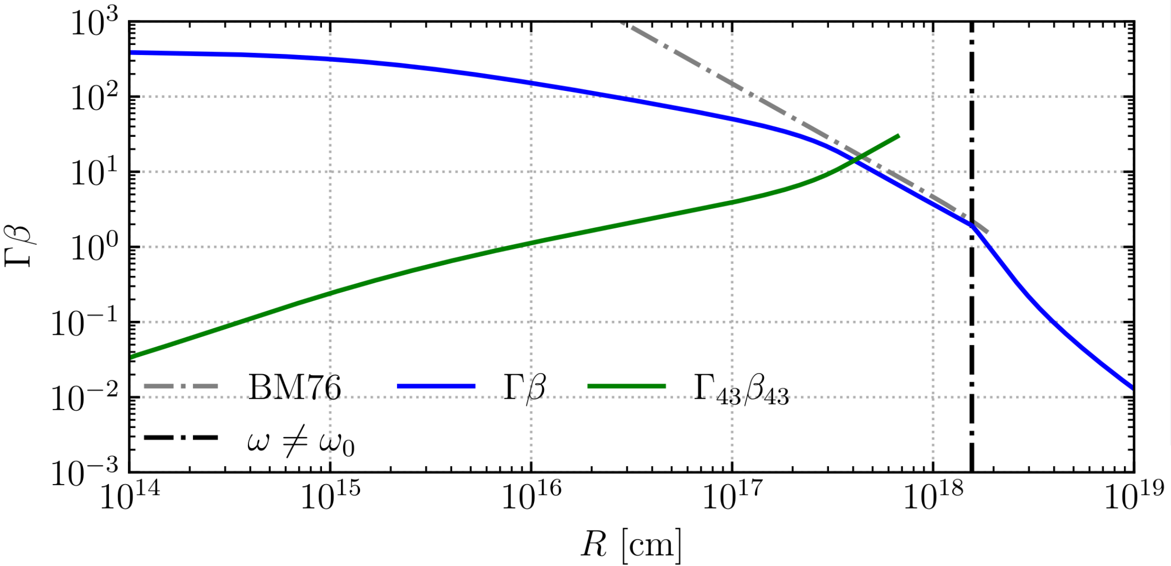
$$E_{\rm tot} = \Gamma_0 M_{0,4}\, c^2 + \Gamma M_{0,3}\, c^2 + \Gamma m\, c^2 + \Gamma_{\rm eff,3} E'_{\rm int,3} + \Gamma_{\rm eff,2} E'_{\rm int,2}.$$

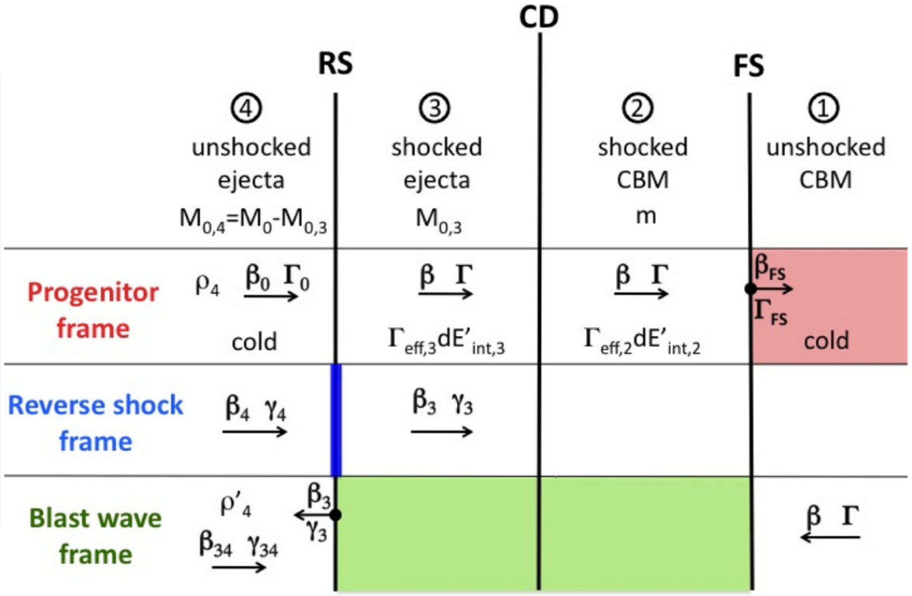
Energy conservation equation

$$dE_{\rm tot} = dm\, c^2 + \Gamma_{\rm eff,2} dE'_{\rm rad,2} + \Gamma_{\rm eff,3} dE'_{\rm rad,3}.$$

$$dE'_{\rm int} = dE'_{\rm sh} + dE'_{\rm ad} + dE'_{\rm rad}.$$

**Numerics**:
Adaptive step-size
Explicit ODE solver
DOP8(53)

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# Model Microphysics

## Collisionless shocks; power-law electron distribution; cooling

Energy Density $e' = E'_{\mathrm{int}}/V'$

Magnetic Field $B' = \sqrt{8\pi\epsilon_{\mathrm{b}} e'}$

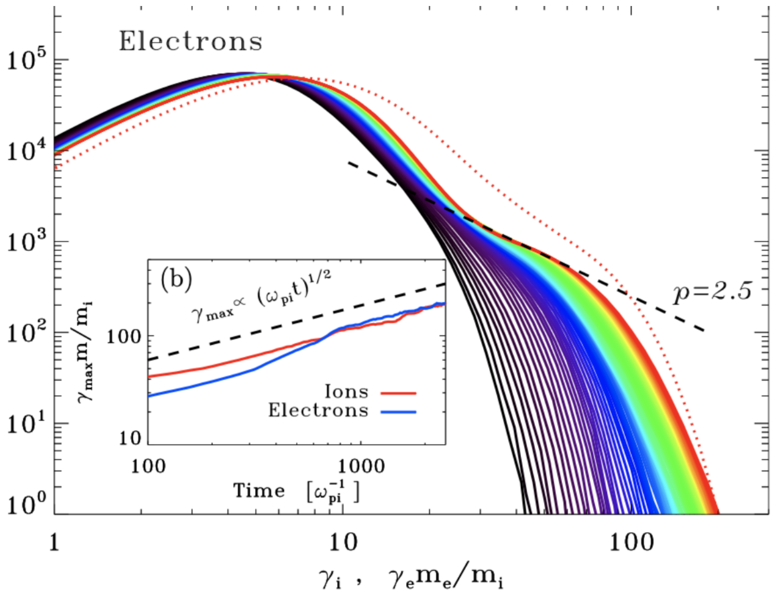
Injection spectrum

$$dN_e/d\gamma_e \propto \gamma_e^{-p} \text{ for } \gamma_e \in (\gamma_{e;\,\mathrm{m}}, \gamma_{e;\,\mathrm{M}})$$

Electron distribution evolution — Electron injection

$$\frac{\partial}{\partial t'}\frac{dN_e'}{d\gamma_e'} + \frac{\partial}{\partial \gamma_e'}\left(\dot{\gamma}_{\mathrm{e,tot}}'\frac{dN_e'}{d\gamma_e'}\right) = d\dot{N}'_{\mathrm{e},0} = \dot{N}'_{\mathrm{e}}\frac{p-1}{\gamma'_{\mathrm{m}}}\left(\frac{\gamma_e'}{\gamma'_{\mathrm{m}}}\right)^{-p} d\gamma_e',$$

$$\dot{\gamma}_{\mathrm{e,tot}}' = \dot{\gamma}_{\mathrm{e,adi}}' + \dot{\gamma}_{\mathrm{e,syn}}' + \dot{\gamma}_{\mathrm{e,IC}}'.$$




**Numerics**:

Fully-implicit PDE solver; unconditionally stable, particle number preserving Chang-Cooper scheme

Sub-stepping to respect CFL condition instead of directly following CFL timesteps

# Modeling Radiation

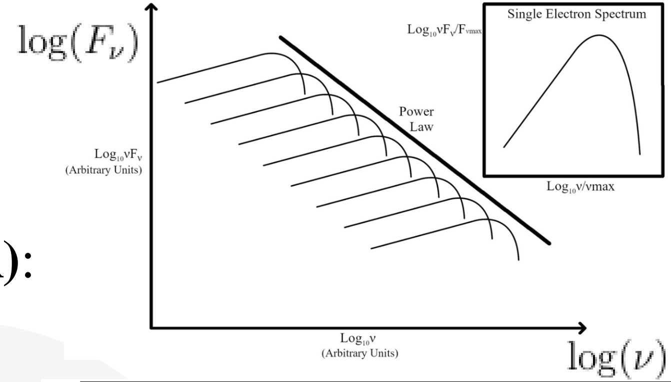
## Synchrotron emission & absorption; SSC emission; PP & EBL losses

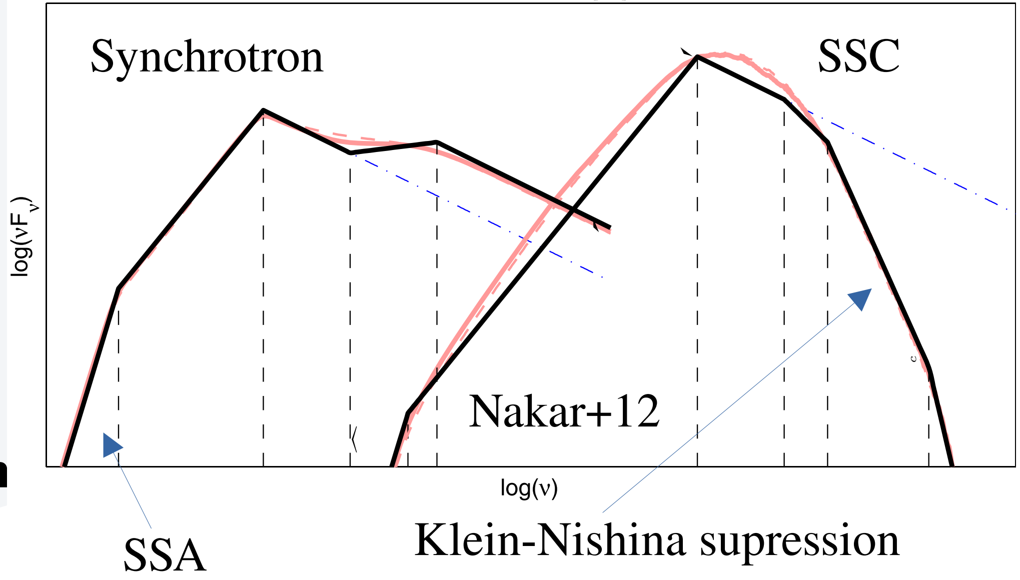
- **Synchrotron emission**: convolving single electron emissivity with electron distribution

- **Synchrotron self-absorption (SSA)**: convolving derivative of the distribution with emissivity

- **Synchrotron self-Compton (SSC)**: convolving scattering kernel with photon density spectrum. Klein-Nishina corrections reduce VHE

- **Pair-production and external background light (EBL) absorption** further reduce VHE



# GRB afterglow modeling

## GRB ejecta lateral structure; relativistic effects; EATS integration

Radiation intensity (observer frame)

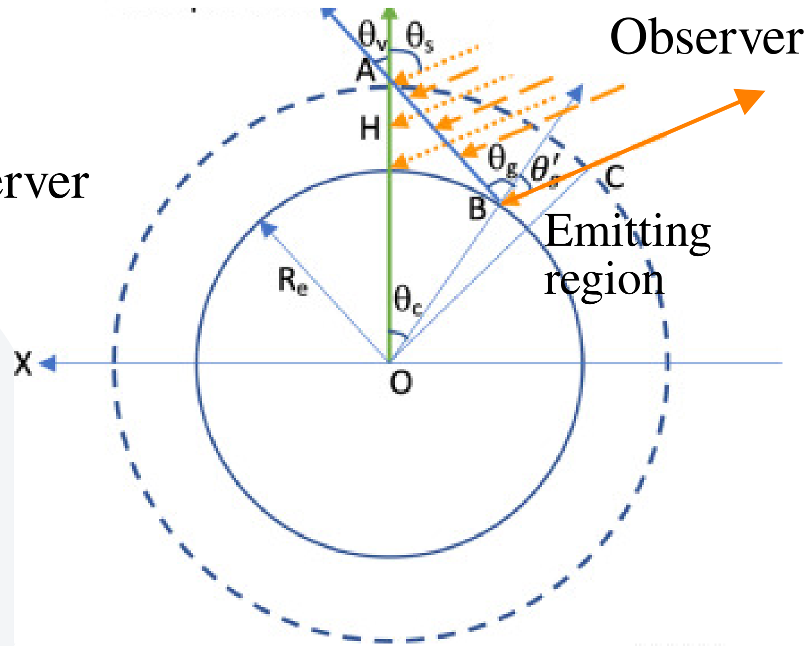
$$I_\nu = \frac{j_\nu}{\alpha_\nu}(1 - e^{-\tau_\nu}) \qquad \tau_\nu \approxeq -\alpha_\nu \Delta R/\mu'$$

Thickness of the emitting region depends on the observer

$$\Delta R = \Delta R'/(1-\mu\beta_{\rm sh})$$

Equal-arrival time surface integration

$$F_\nu = \frac{1+z}{2\pi d_L^2} \sum_i^{\text{layers}} \int_{\theta_{i;l}}^{\theta_{i;h}} \int_{\phi_0=0}^{\phi_1=\pi/2} I_{i,\nu}(\theta,\phi) d\theta d\phi$$

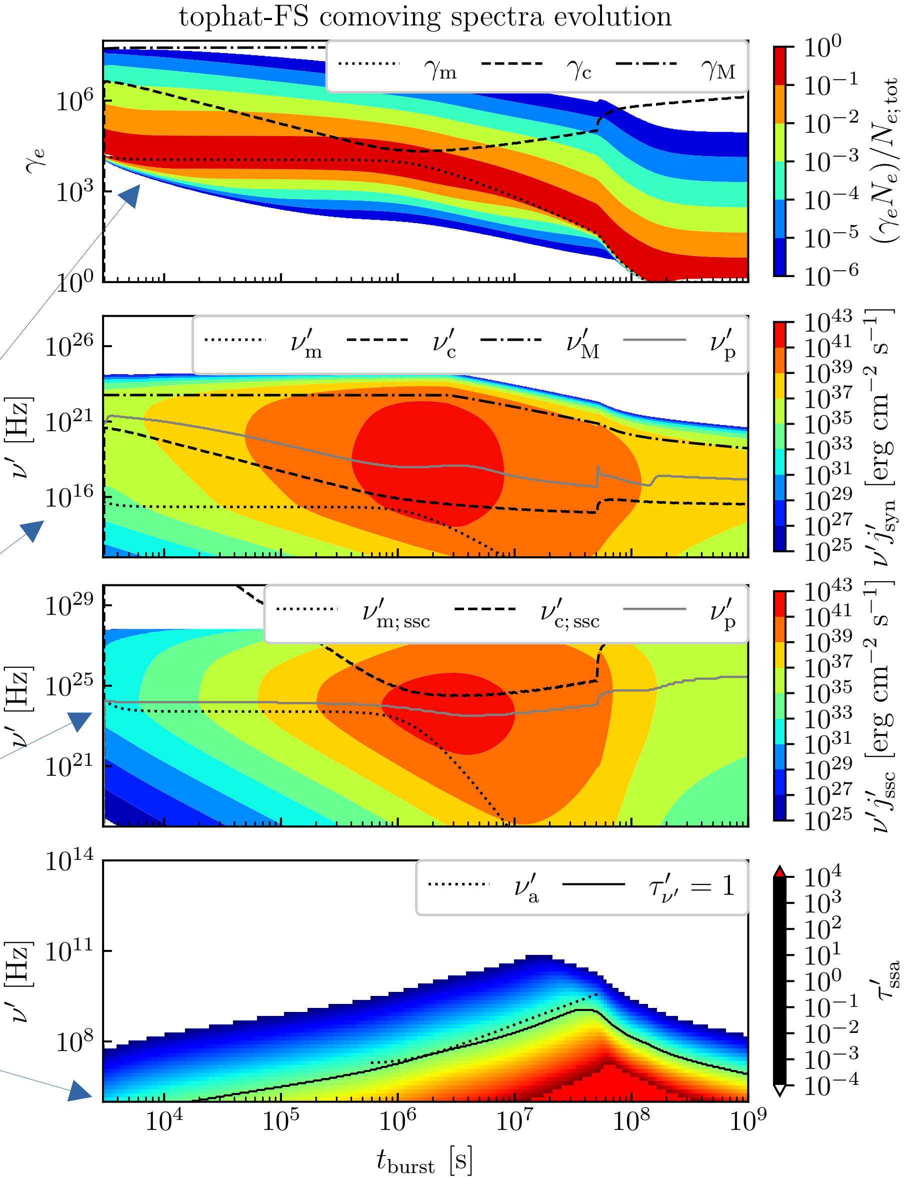



**Numerics**:
Adaptive quadrature for flux density evaluation and iterative algorithm for sky map resolving integration

# Afterglow Modeling results

### Comoving spectra evolution

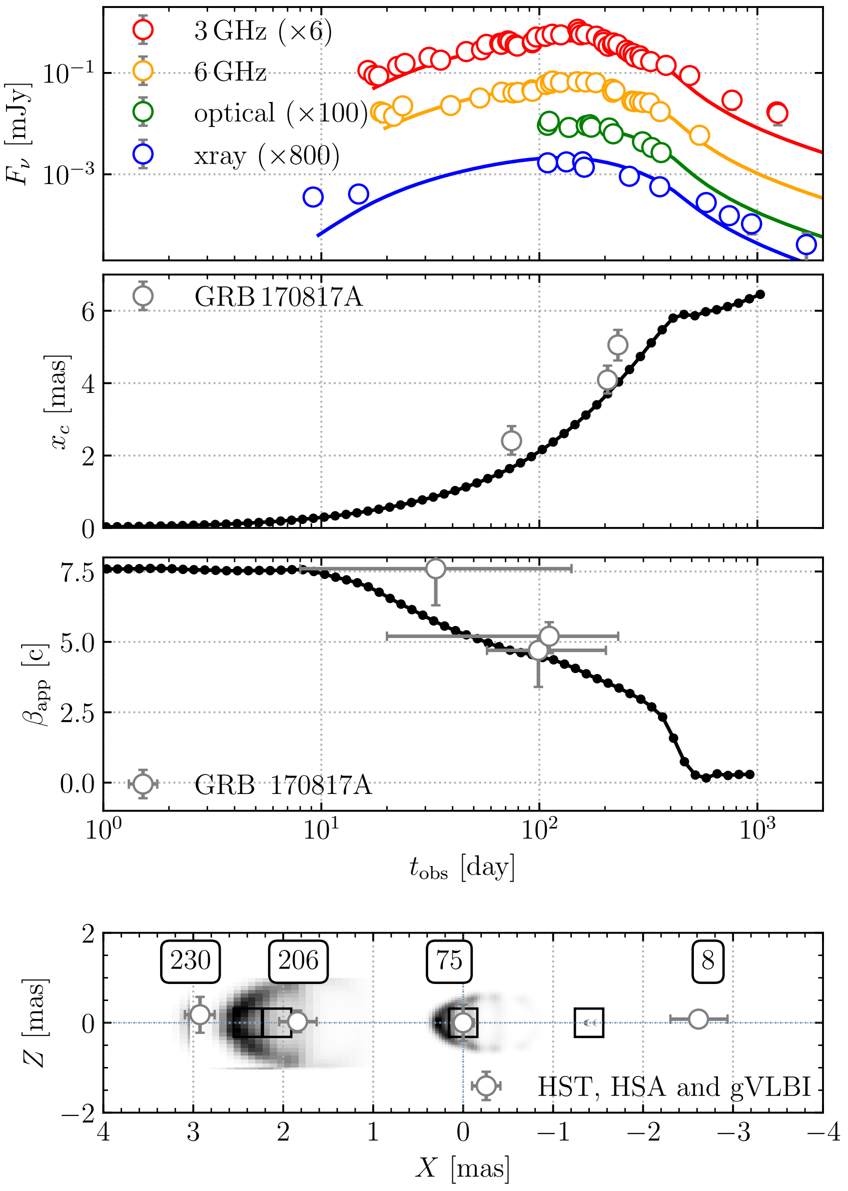
- Electron spectrum: Cooling of injected electrons; evolution of injection spectrum
- Synchrotron emission: peaks at the onset of deceleration
- SSC spectrum follows the synchrotron one – echo at higher frequencies
- SSA affects primarily lower frequencies; and most prominent at late deceleration stages.



# GRB 170817A Modeling Example

### Comparison with observables

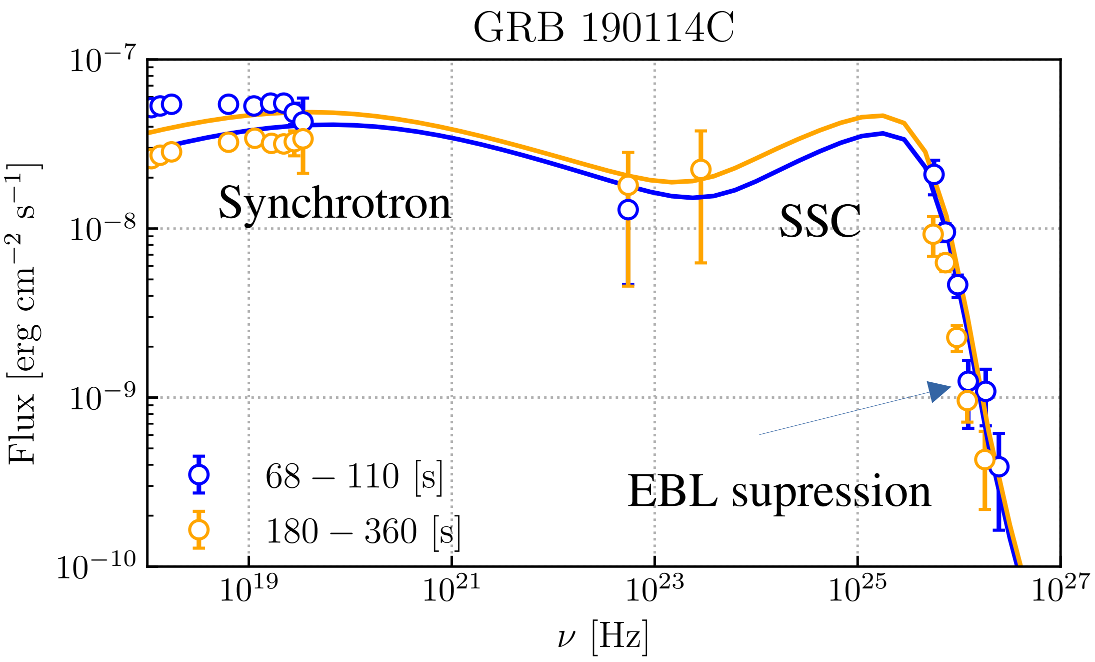
- Jet structure: Gaussian jet with 20 angular layers (blastwaves)
- Synchrotron only; no VHE detected
- Observed off-axis (~20 deg.) shows slow rising early light curve
- Sky map related quantities: flux centroid motion constraint geometry and energetics
- Numerically expensive runs prohibit Bayesian parameter inference

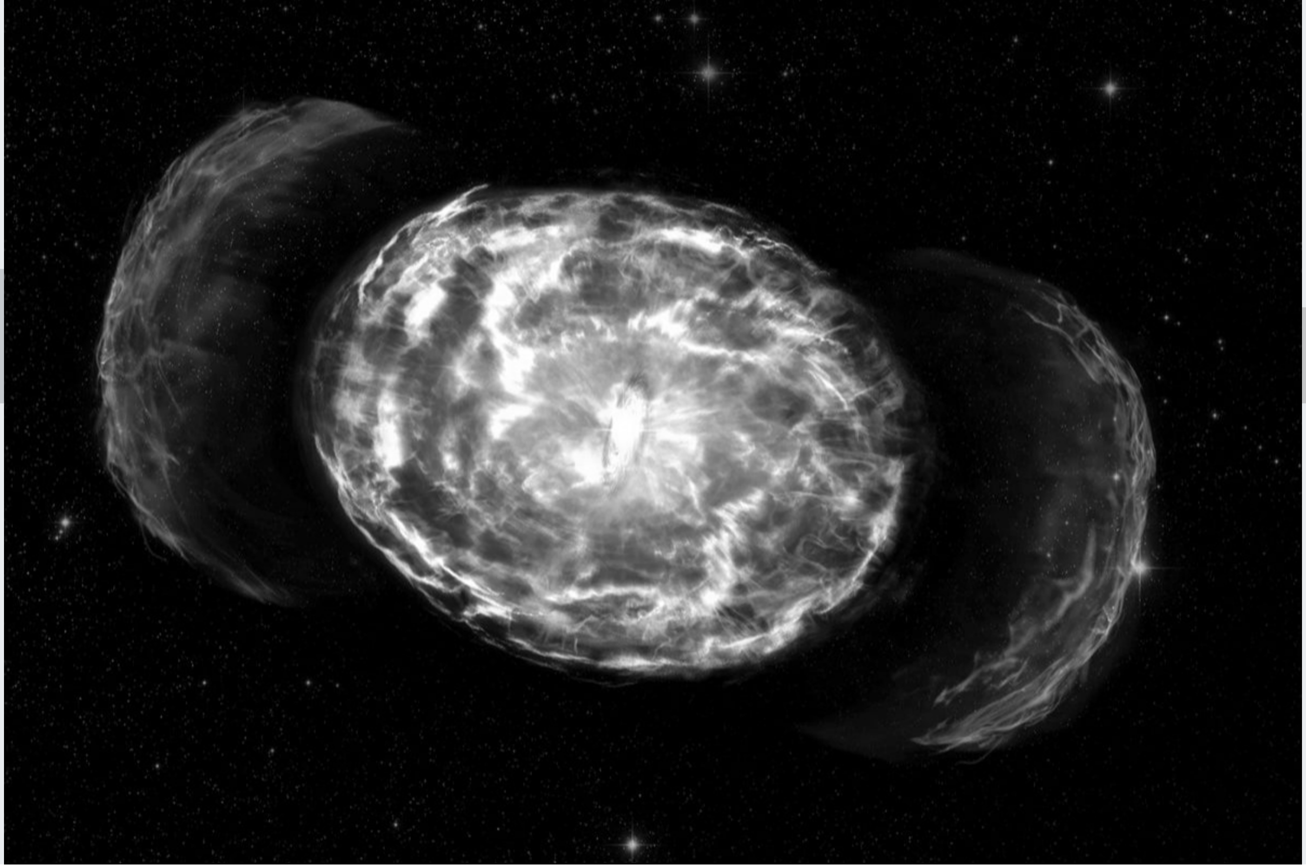


# VHE afterglow modeling example

## GRB 190114C VHE spectra from SSC mechanism

- Grid-search analysis (20k runs)
- Top-hat jet observed on-axis
- VHE is produced by SSC but is shaped by EBL absorption
- Model parameters of best-fitting model are close to the literature values



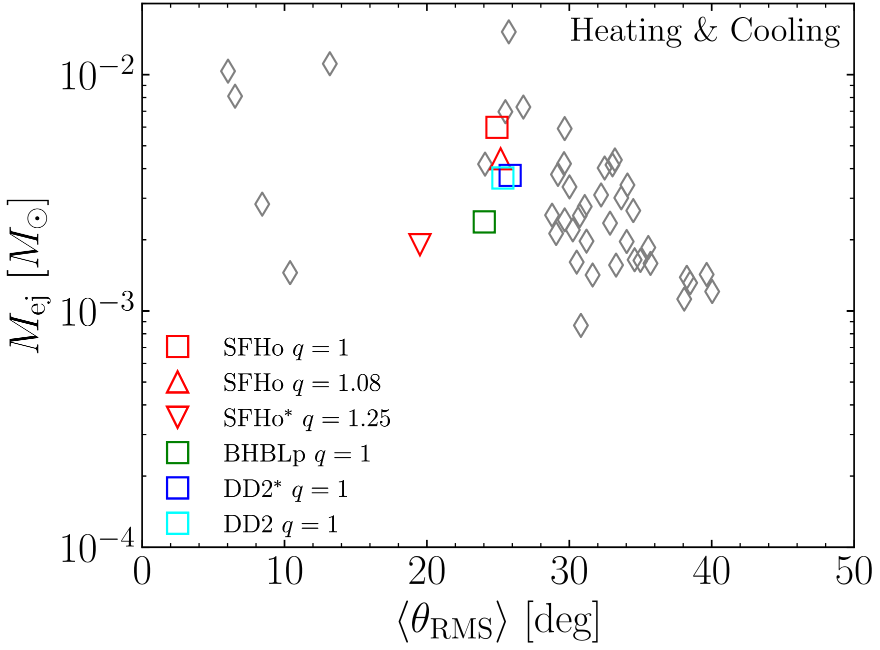


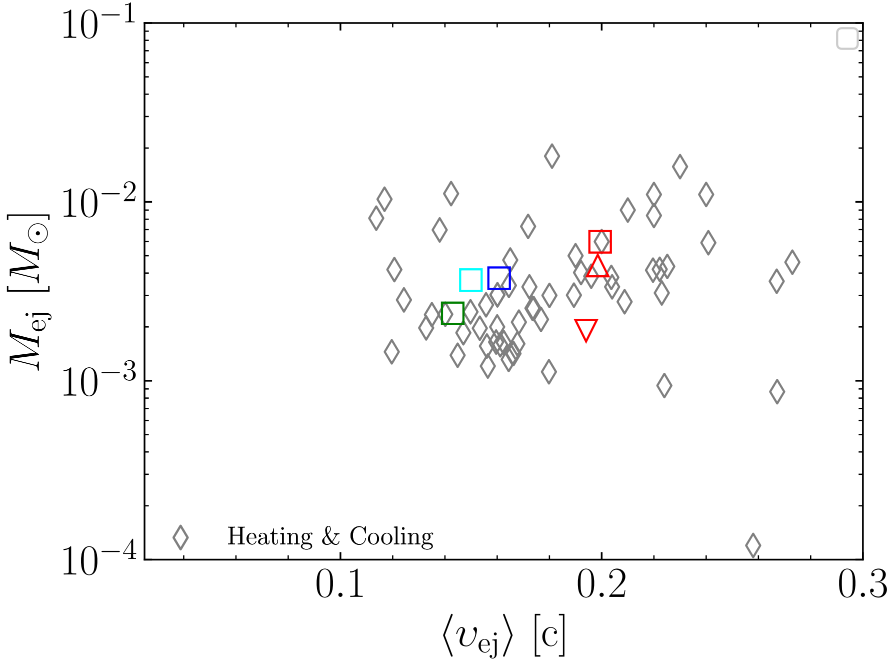
# Kilonova Afterglow Modeling

# Kilonova Ejecta

## Dynamical ejecta from binary neutron star merger simulations

- 5 BNS merger simulations: high resolution; M1 neutrino transport; MHD effects
- Goal: analyze origin of the ejecta fast tail; statistics of the ejecta kinetic energy distribution and kilonova afterglow light curves

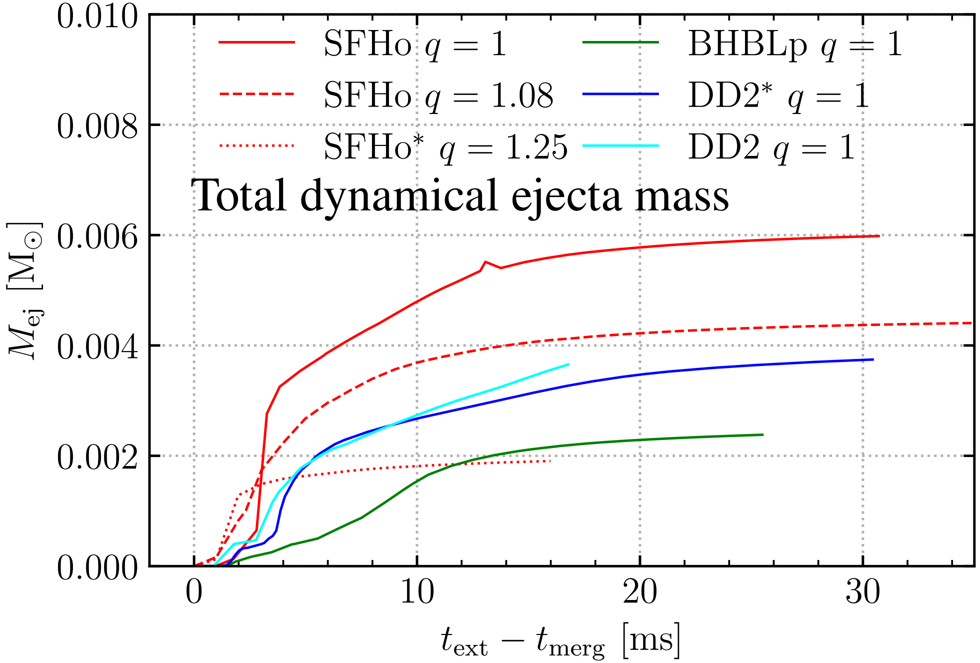


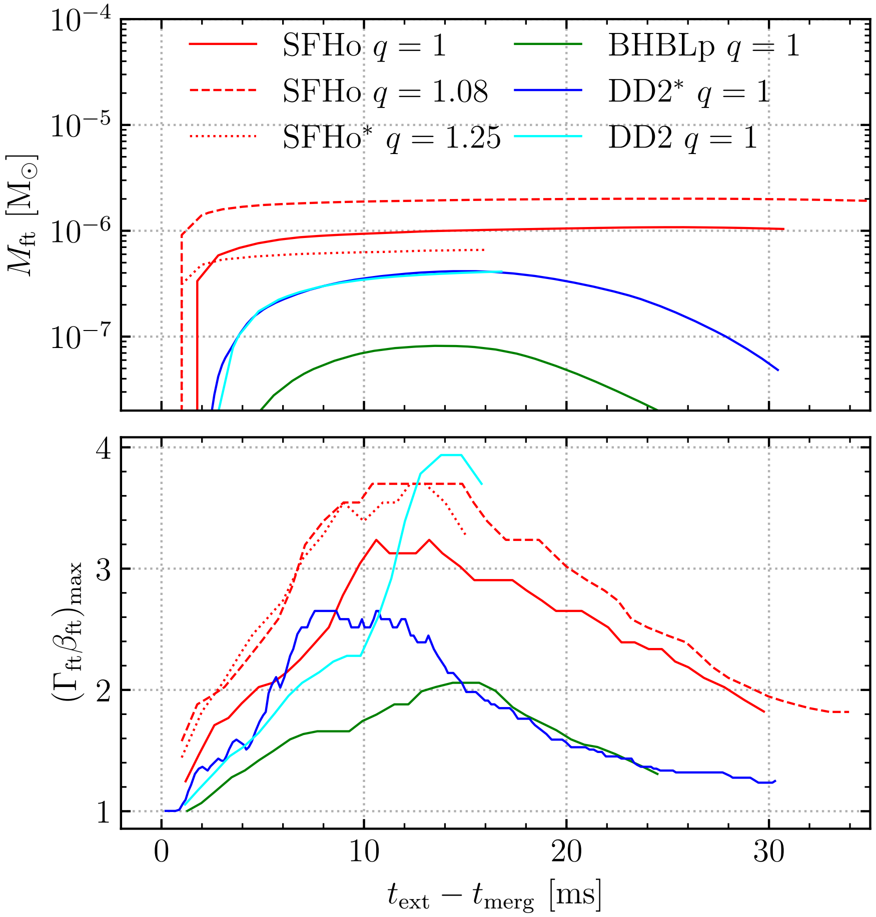
# Ejecta Extraction Time

## Ejecta properties depend on when they are extracted

- Mass and maximum momentum saturate before the end of dynamical phase – possibly due to interactions with atmosphere




Fast tail

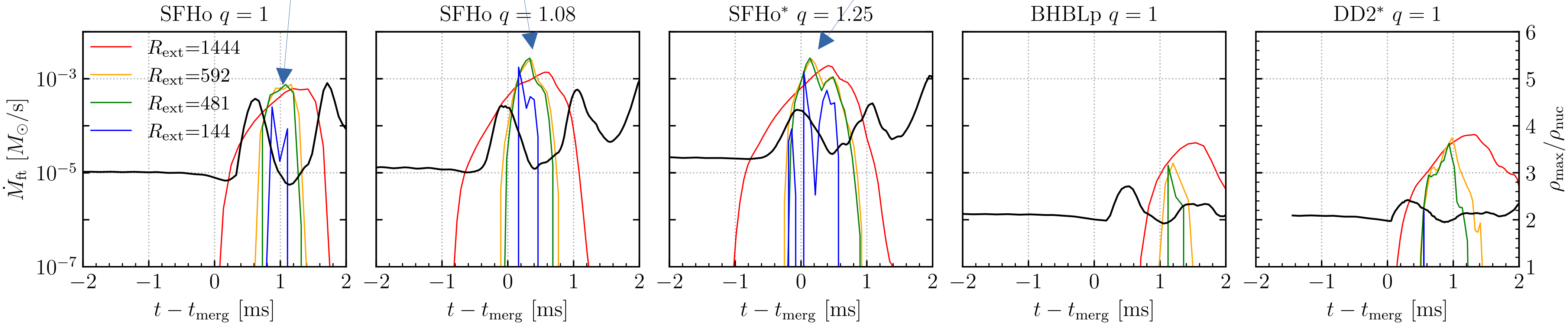


# Fast Tail Origin

## Comparing mass flux through with maximum density

- Mass flux of the fast tail seem to coincide with the core bounce, traced by the maximum density evolution

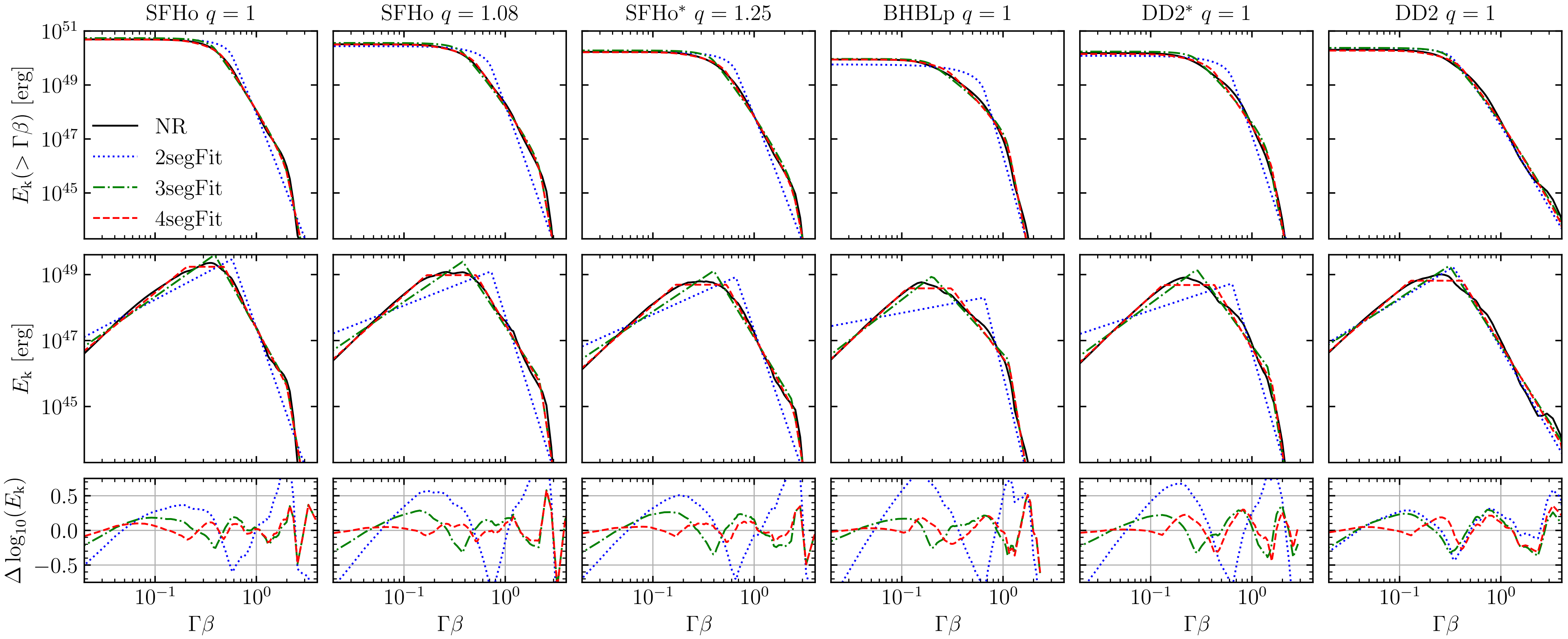
- Other mechanisms, however may contributed (see double peaked structure of the mass flux)



# Ejecta kinetic energy

Fitting formula:

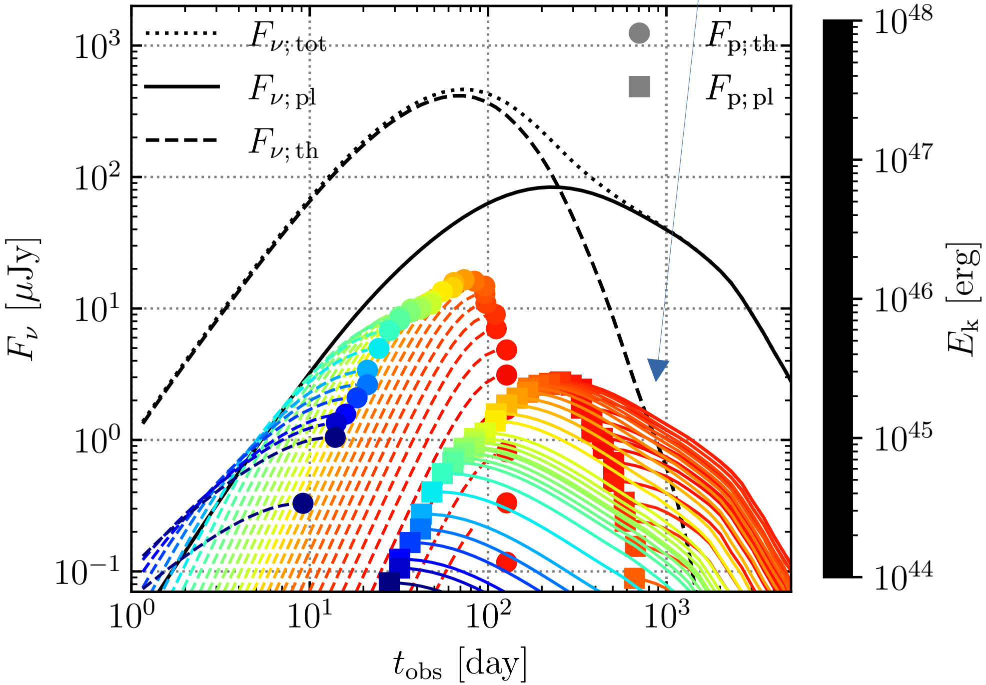
$$E_\mathrm{k}=\mathcal{E}_0 \begin{cases} (\mathcal{M}/\mathcal{M}_0)^{k_1} & \text{if } \mathcal{M}_l \le \mathcal{M} < \mathcal{M}_0 \\ (\mathcal{M}/\mathcal{M}_0)^{k_2} & \text{if } \mathcal{M}_0 \le \mathcal{M} < \mathcal{M}_1 \\ (\mathcal{M}/\mathcal{M}_1)^{k_3}(\mathcal{M}_1/\mathcal{M}_0)^{k_2} & \text{if } \mathcal{M} > \mathcal{M}_1 \,, \end{cases}$$

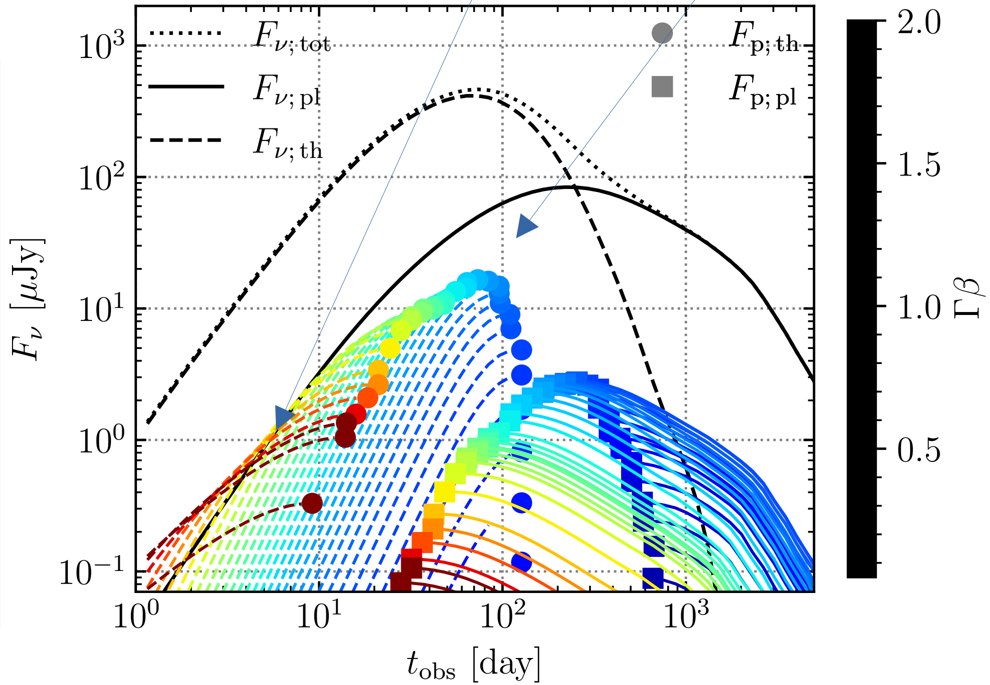


# Afterglow Light Curves

## Which part of the ejecta generates what part of the light curve

- Light curve peak is produced by moderately fast and moderately energetic ejecta
- Early rising part comes from low-energy, very fast ejecta tail
- Late afterglow comes from high-energy, slow ejecta



# Afterglow Statistics

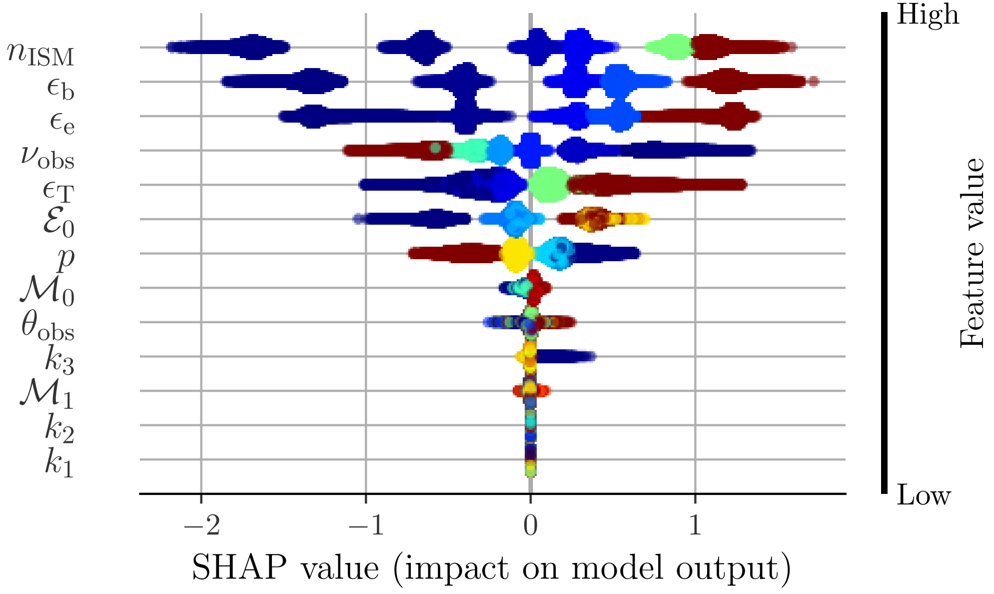
## Which model parameter is the most important

- Surrogate machine learning model (XGBoost) trained on large grid of simulations
- Feature importance using Shapley values:

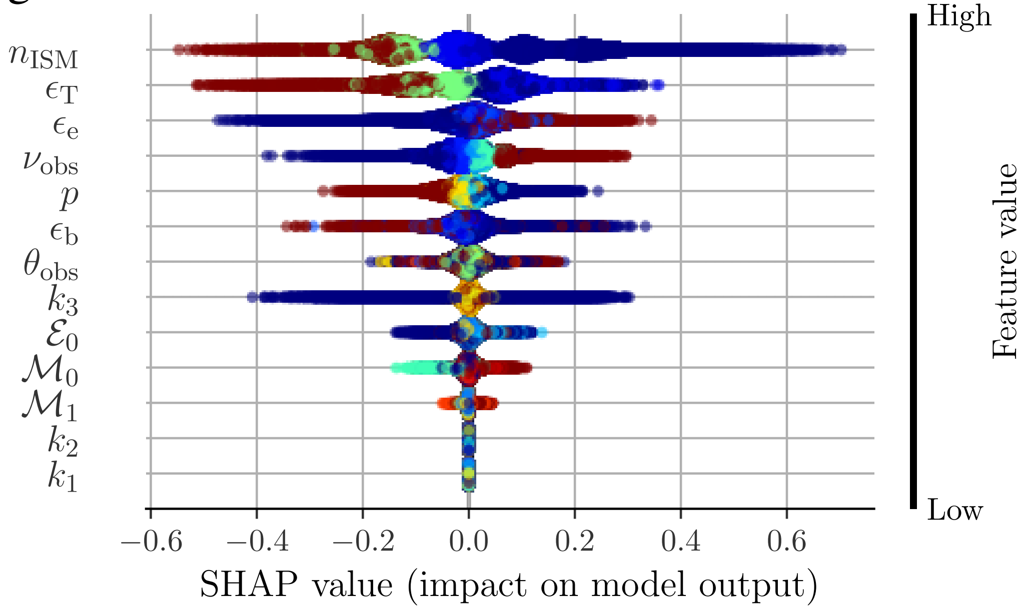
$$\phi_i = \sum_{S \in N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} [f(S \cup \{i\} - f(S))]$$

- Most important features: ISM density and microphysics (not ejecta parameters)

Light Curve Peak Flux




Light Curve Peak Time



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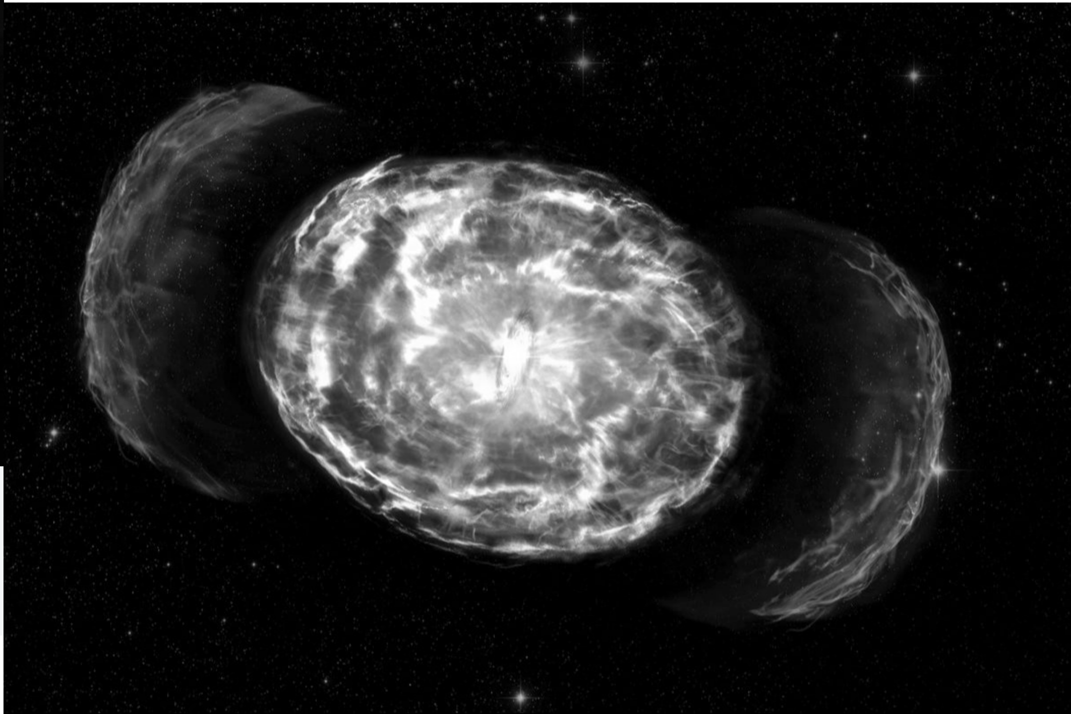


# Building Afterglow Model: FS & RS dynamics

**Total blast wave energy**

$$E_{\rm tot} = \Gamma_0 M_{0,4}\, c^2 + \Gamma M_{0,3}\, c^2 + \Gamma m\, c^2 + \Gamma_{\rm eff,3} E'_{\rm int,3} + \Gamma_{\rm eff,2} E'_{\rm int,2}.$$

**Energy conservation equation**

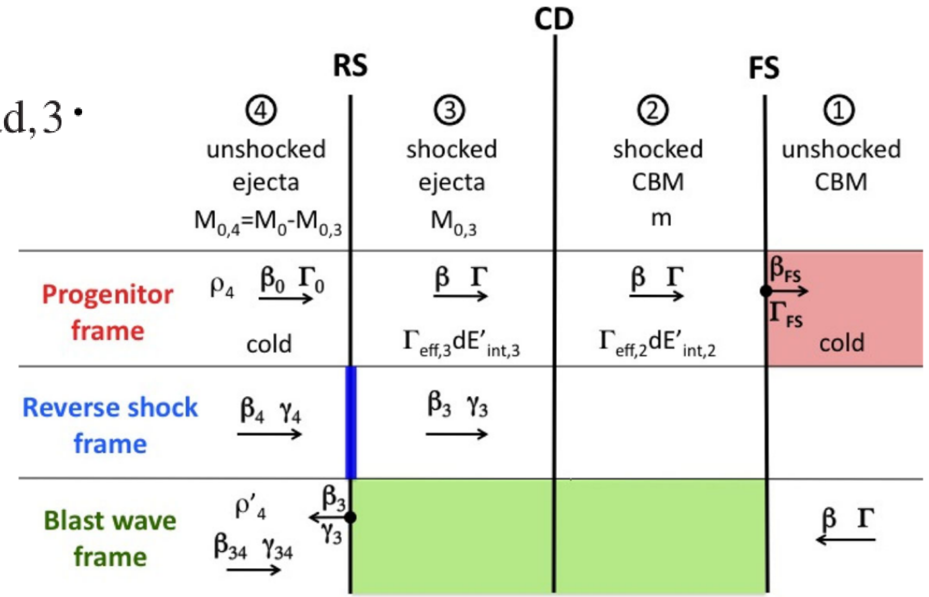
$$\mathrm{d}E_{\rm tot} = \mathrm{d}m\, c^2 + \Gamma_{\rm eff,2} \mathrm{d}E'_{\rm rad,2} + \Gamma_{\rm eff,3} \mathrm{d}E'_{\rm rad,3}.$$

$$\mathrm{d}E'_{\rm int} = \mathrm{d}E'_{\rm sh} + \mathrm{d}E'_{\rm ad} + \mathrm{d}E'_{\rm rad}.$$

**Evolution equation:** after some algebra...

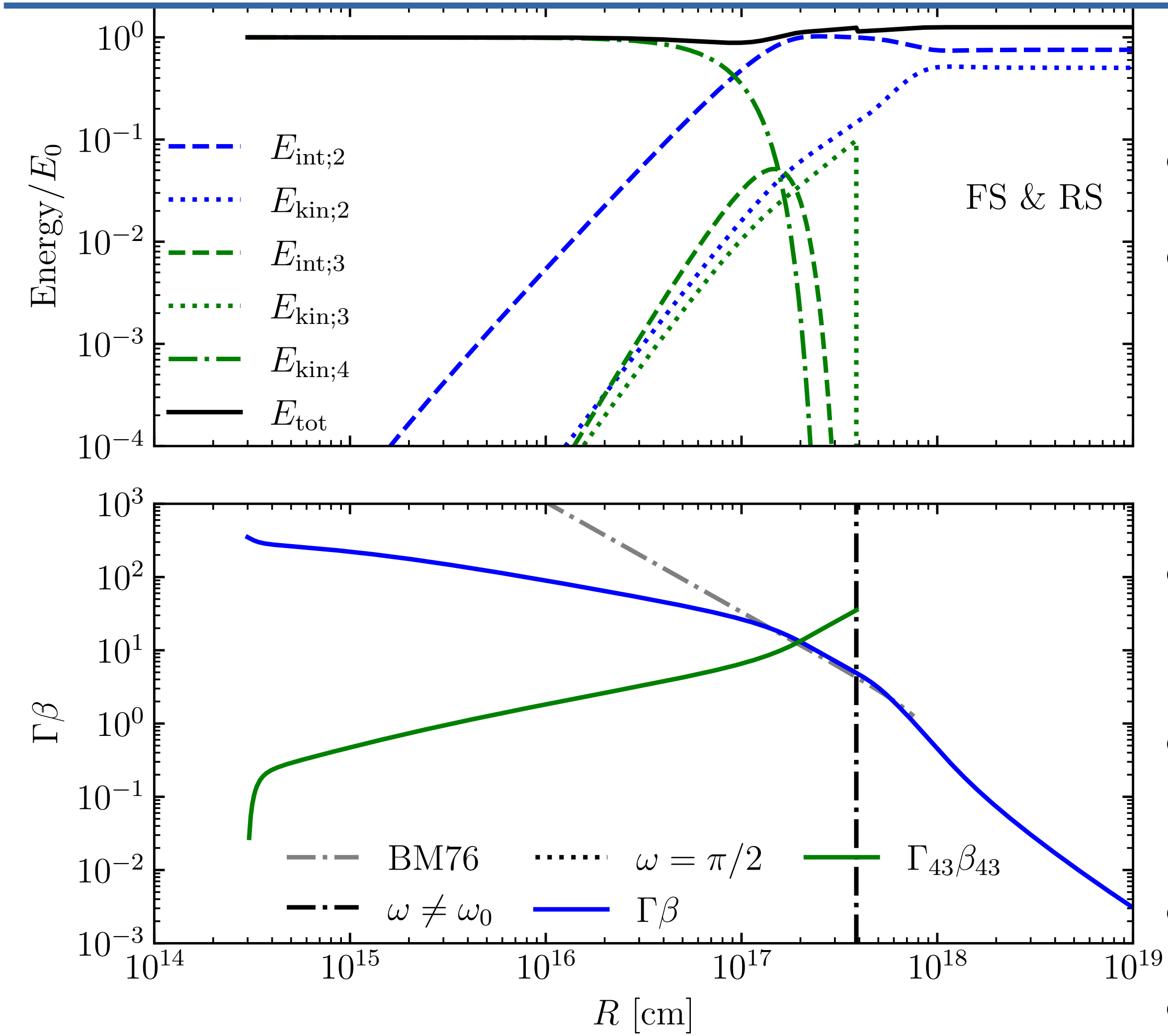
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}R} = -\frac{(\Gamma_{\rm eff,2}+1)(\Gamma-1)\frac{\mathrm{d}m\, c^2}{\mathrm{d}R} + \Gamma_{\rm eff,2}\mathrm{d}E'_{\rm ad,2}}{(M_{0,3}+m)c^2 + E'_{\rm int,2}\frac{\mathrm{d}\Gamma_{\rm eff,2}}{\mathrm{d}\Gamma} + E'_{\rm int,3}\frac{\mathrm{d}\Gamma_{\rm eff,3}}{\mathrm{d}\Gamma}}$$

$$-\frac{(\Gamma - \Gamma_0 - \Gamma_{\rm eff,3} + \Gamma_{\rm eff,3}\gamma_{34})\frac{\mathrm{d}M_{0,3}c^2}{\mathrm{d}R} + \Gamma_{\rm eff,3}\mathrm{d}E'_{\rm ad,3}}{(M_{0,3}+m)c^2 + E'_{\rm int,2}\frac{\mathrm{d}\Gamma_{\rm eff,2}}{\mathrm{d}\Gamma} + E'_{\rm int,3}\frac{\mathrm{d}\Gamma_{\rm eff,3}}{\mathrm{d}\Gamma}}$$




Adaptive ODE solve used

# Building Afterglow Model: FS & RSexample




**Energy**;

- Reverse shock shocks ejecta
- Total energy ~ conserved

**Dynamics**

- Initial deceleration when RS is crossing ejecta
- Short standard deceleration (BM solution)
- Lateral spreading
- Non-relativistic regime

# Building Afterglow: Electron Distribution

## Electron distribution evolution

$$\frac{\partial}{\partial t'}\frac{dN_e{}'}{d\gamma_e{}'} + \frac{\partial}{\partial \gamma_e{}'}\left(\dot{\gamma}_{\mathrm{e,tot}}{}'\frac{dN_e{}'}{d\gamma_e{}'}\right) = d\dot{N}'_{\mathrm{e},0}$$

## Electron injection

$$d\dot{N}'_{\mathrm{e},0} = \dot{N}'_{\mathrm{e}}\frac{p-1}{\gamma'_{\mathrm{m}}}\left(\frac{\gamma_e{}'}{\gamma'_{\mathrm{m}}}\right)^{-p} d\gamma_e{}',$$

## Electron energy loss

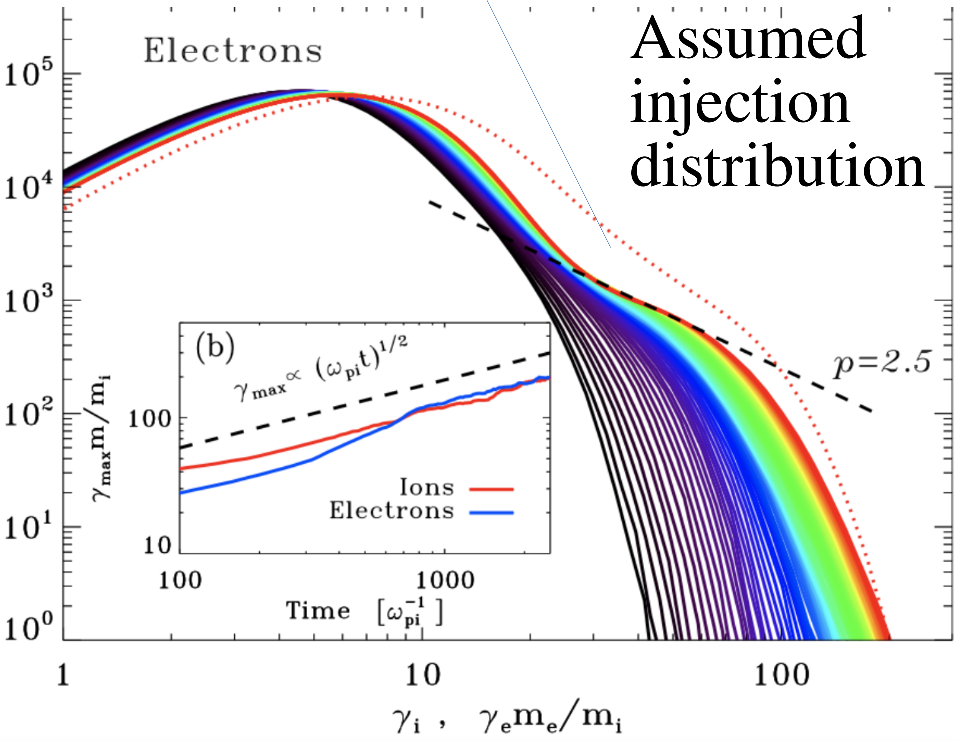
$$\dot{\gamma}_{\mathrm{e,tot}}{}' = \dot{\gamma}_{\mathrm{e,adi}}{}' + \dot{\gamma}_{\mathrm{e,syn}}{}' + \dot{\gamma}_{\mathrm{e,IC}}{}'.$$

## Synchrotron cooling

$$\dot{\gamma}'_{\mathrm{e,syn}} = -\frac{\sigma_T B'^2 \gamma_e{}'^2}{6\pi m_e c}.$$

## Adiabatic cooling

$$\dot{\gamma}'_{\mathrm{e,adi}} = \frac{1}{3}\gamma'_e \frac{d\ln n'_e}{dt'} = -\frac{1}{2}\frac{\gamma'_e}{R}\frac{dR}{dt'}$$




Assumed injection distribution

# Building Afterglow: SSC cooling

Electrons lose energy by up-scattering (syn) photons

Connects the electron population with existing radiation field

SSC cooling

$$\dot{\gamma}'_{\mathrm{e,IC}} = -\frac{1}{m_e c^2}\frac{3\sigma_T c}{4{\gamma_e}'^2}\int_{\nu_{\min}'}^{\nu_{\max}'}\frac{n_{\nu'}d\nu'}{\nu'} \int_{\nu_{\mathrm{IC,min}}'}^{\nu_{\mathrm{IC,max}}'} h\nu_{\mathrm{IC}}' d\nu_{\mathrm{IC}}' F(q,\, g),$$

Seed photon density

$$n_{\nu'} \approx \frac{T'}{h\nu'}\frac{\sqrt{3}\, e^3 B'}{m_e c^2}\int_{\gamma_{\mathrm{e,min}}'}^{\gamma_{\mathrm{e,max}}'} \times n'({\gamma_e}')\mathcal{R}(\nu'/{\nu_c}')d{\gamma_e}',$$

Escape time

$$T' \approx \Delta'/c$$

SSC scattering cross-section

$$F(q,\, g) = 2q\ln q + (1+2q)(1-q) + \frac{1}{2}\frac{(4qg)^2}{1+4gq}(1-q),$$

$$g = \frac{{\gamma_e}' h\nu'}{m_e c^2}, \quad w = \frac{h{\nu_{\mathrm{IC}}}'}{{\gamma_e}' m_e c^2}, \quad \text{and} \quad q = \frac{w}{4g(1-w)}.$$

$$n'({\gamma_e}') = \frac{d{N_e}'/d{\gamma_e}'}{4\pi\Delta' R^2}$$

# Building Afterglow: Synchrotron Emission

Convolving the emission of a single electron gyrating around magnetic field lines with electron distribution function

Synchrotron emission power

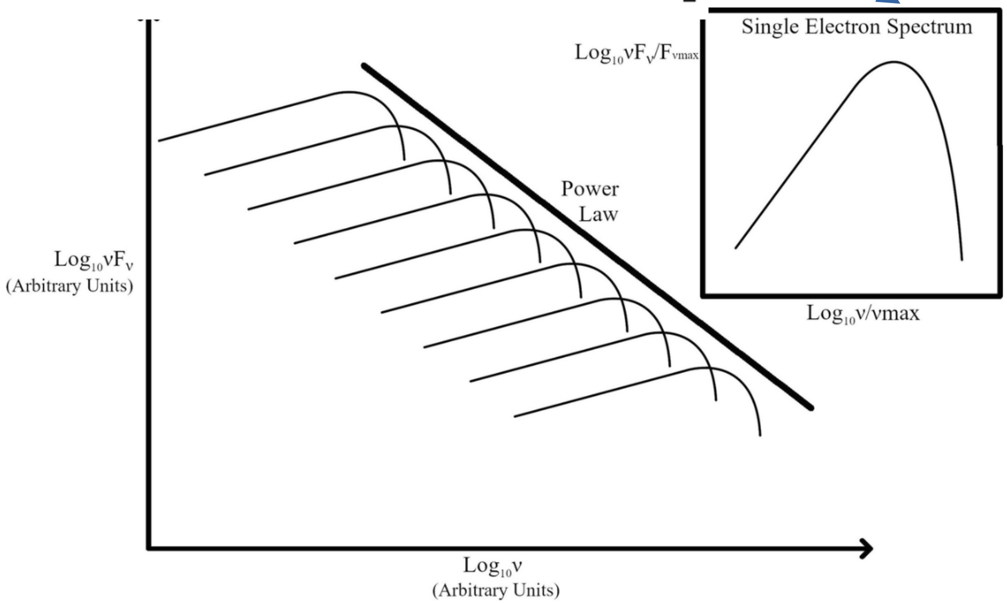
Emission power by a single electron

$$P'^{syn}_{\nu'}(\nu',\gamma) = \frac{2\sqrt{3}\,e^3\,B'}{m_e\,c^2}\,x^2\left[K_{4/3}(x)\,K_{1/3}(x) - 0.6x(K^2_{4/3}(x) - K^2_{1/3}(x))\right]$$

$$P'^{syn}_{\nu'}(\nu') = \int P'^{syn}_{\nu'}(\nu',\gamma)\frac{dN}{d\gamma}d\gamma$$

Synchrotron self-absorption (SSA)

$$\alpha_\nu = -\frac{1}{8\pi\nu'^2 m_e}\int d\gamma P'(\gamma,\nu')\gamma^2\frac{\partial}{\partial\gamma}\left[\frac{N(\gamma)}{\gamma^2}\right]$$



# Building Afterglow: SSC emission

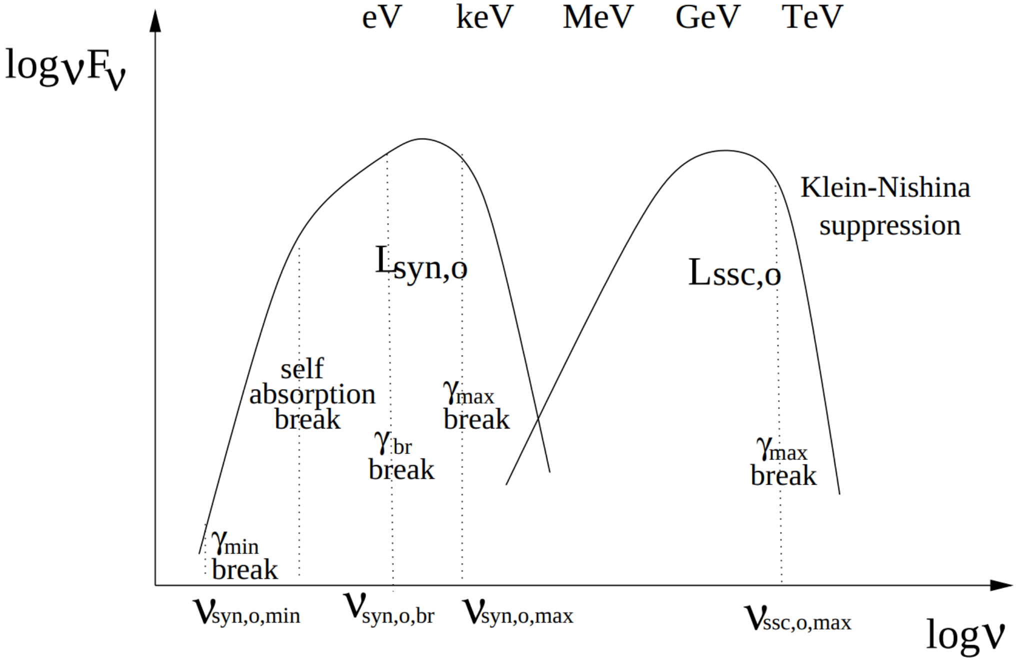
## SSC emission power

$$P'_{\rm ssc}(\nu_{\rm ssc}') = \int\int \frac{dN_{\rm e}'}{d\gamma_e'} h\nu_{\rm ssc}' \frac{dN_\gamma'}{dt' d\nu_{\rm ssc}'} d\gamma_e'.$$

$$\frac{dN_\gamma'}{dt' d\nu_{\rm ssc}'} = \frac{3\sigma_T c}{4\gamma_e'^2} \frac{n_{\nu'} d\nu'}{\nu'} F(q, g).$$

$$n_{\nu'} \approx \frac{T'}{h\nu'} \frac{\sqrt{3}\, e^3 B'}{m_e c^2} \int_{\gamma_{\rm e,min}'}^{\gamma_{\rm e,max}'} \times n'(\gamma_e') \mathcal{R}(\nu'/\nu_c') d\gamma_e',$$

$$F(q, g) = 2q \ln q + (1 + 2q)(1 - q) + \frac{1}{2}\frac{(4qg)^2}{1 + 4qg}(1 - q),$$



# Example: sky map

**Sky map** – intensity distribution projected onto a plane orthogonal to the line of sight.

**Properties**:
- Centroid position
- Size

**Shows:**
- geometry of the ejecta
- jet inclination angle

Flux centroid

Origin

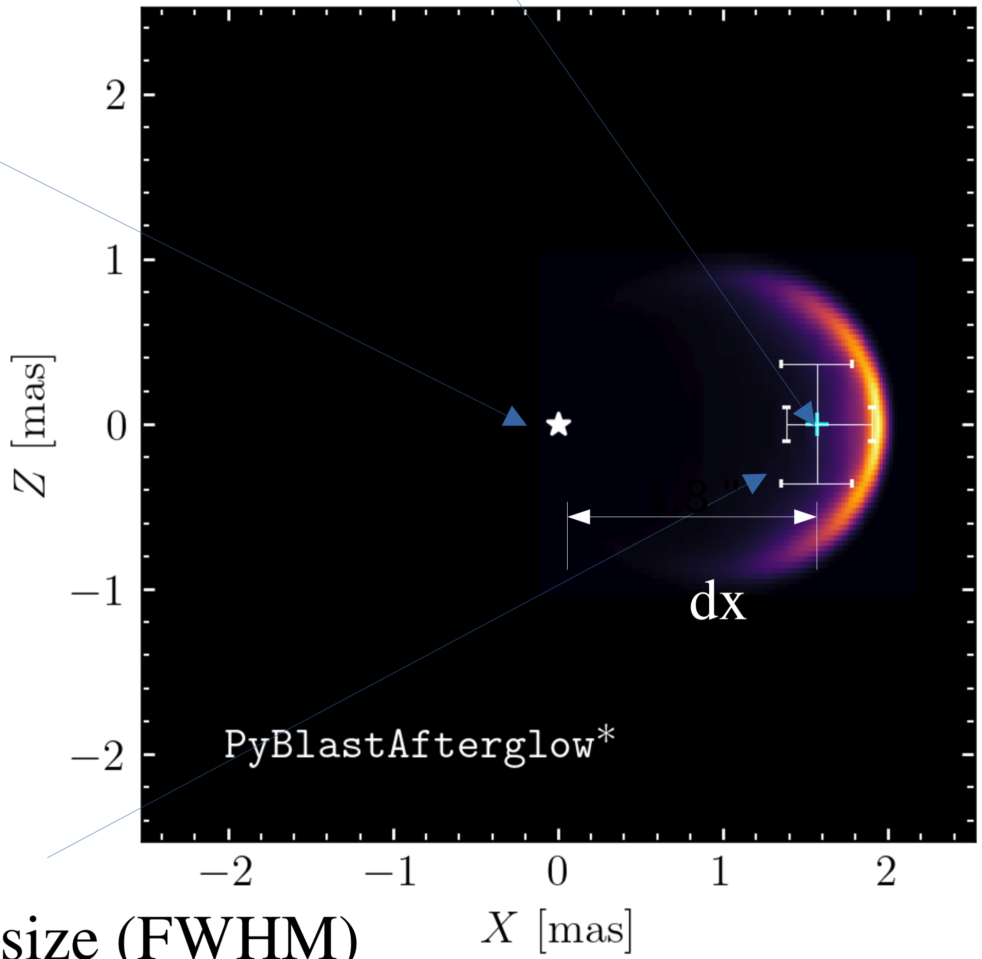



Image size (FWHM)