Modeling Gamma-Ray Burst Afterglows

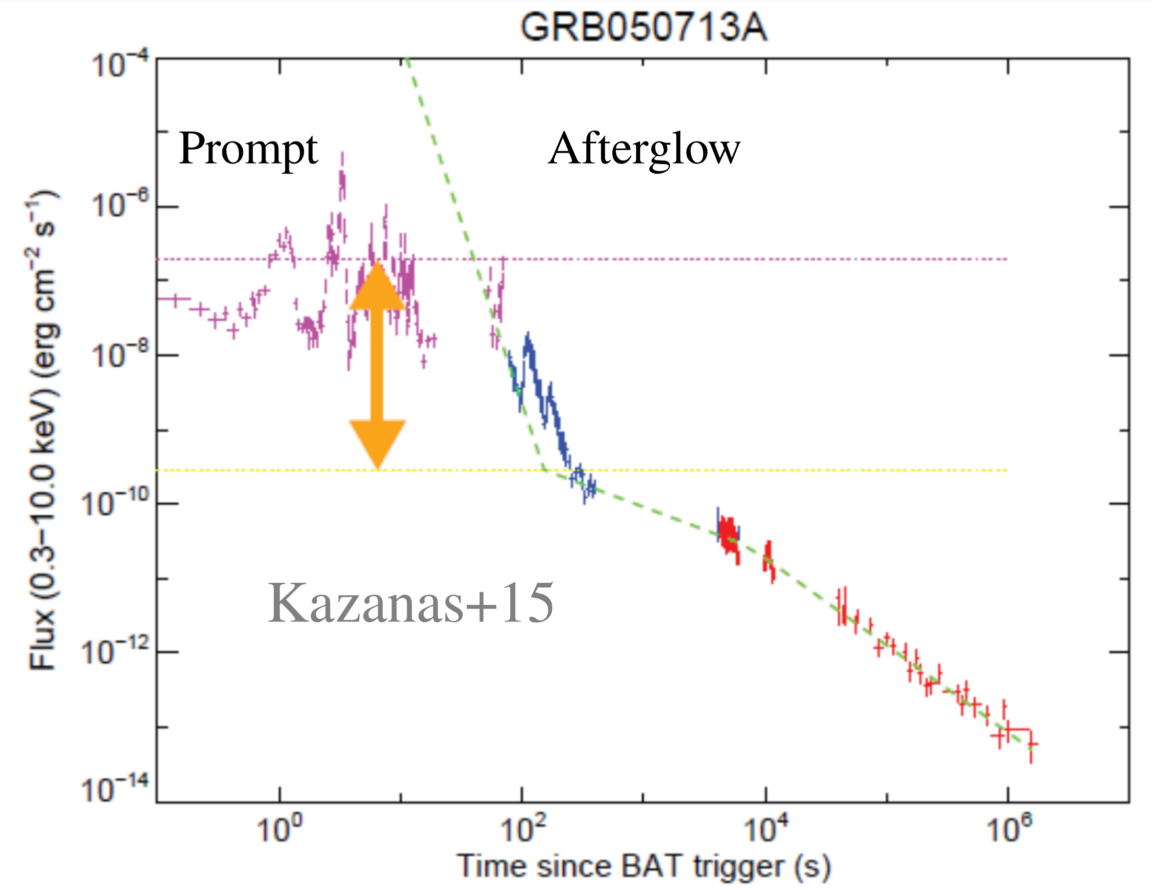
Vsevolod Nedora

Introduction

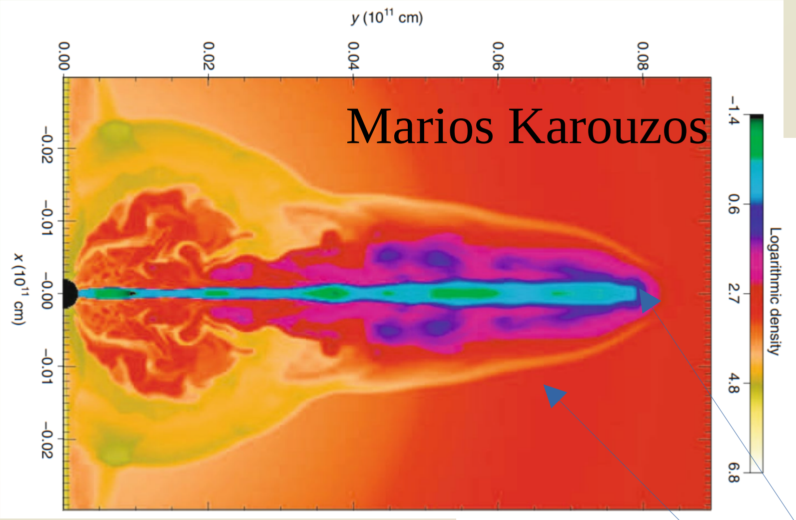
- **Gamma-Ray Bursts**

  Prompt emission
  (oscillatory, early)
  Internal shocks or magnetic reconnection
  Convert fraction of jet energy to radiation

  Afterglow
  (smooth, late)
  External shocks
  (forward/reverse)
  Synchrotron emission (based on spectra)

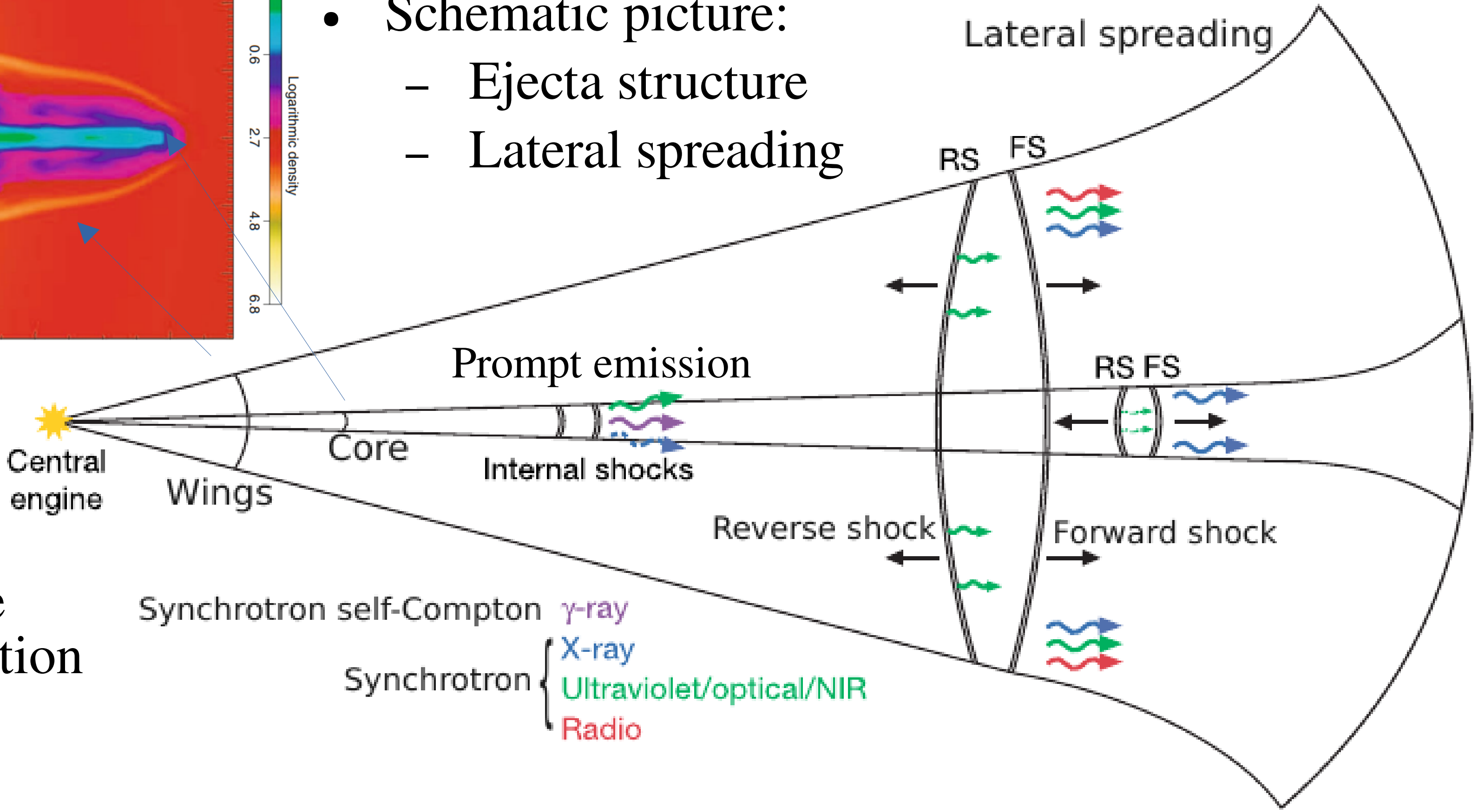


General Picture: Origin of GRBs



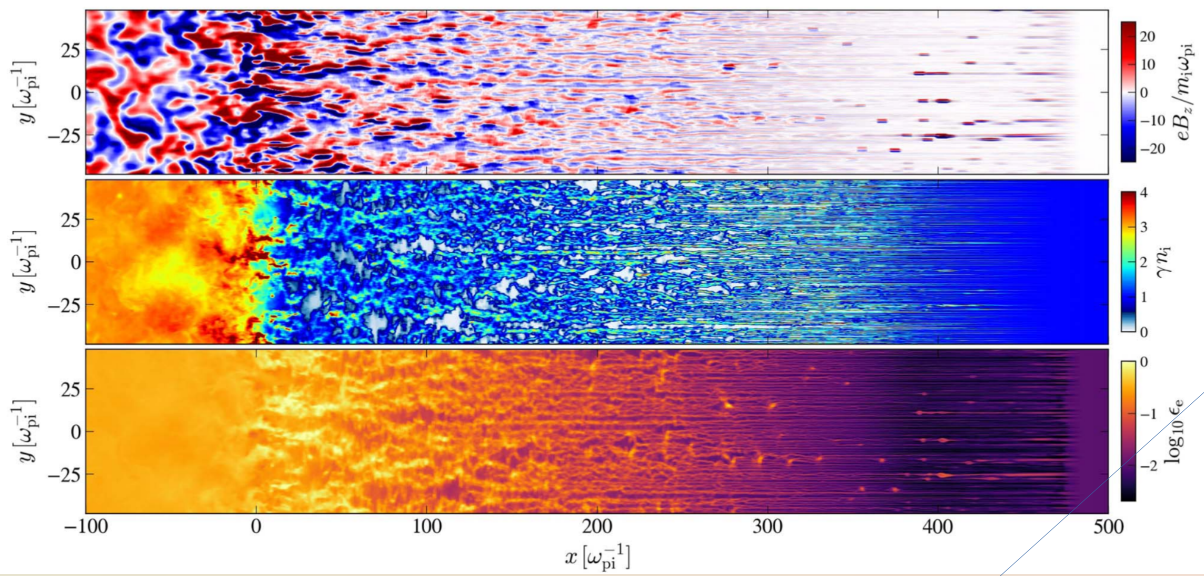
Marios Karouzos

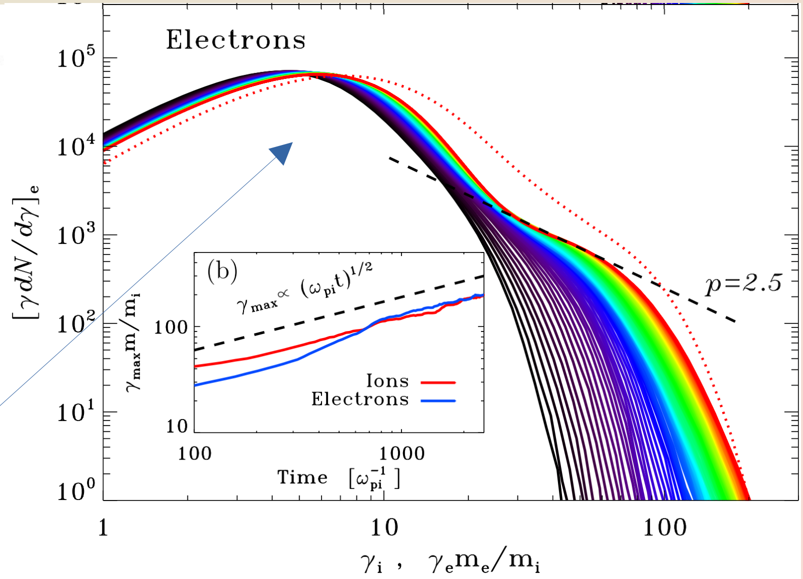
- Schematic picture:
  - Ejecta structure
  - Lateral spreading



- **<u>Engine</u>**:
  disk accretion
  - burst energy
  - ejecta structure
  - ejecta composition

General Picture: Colisionless Shocks





Vanthieghem+22

Sironi+13

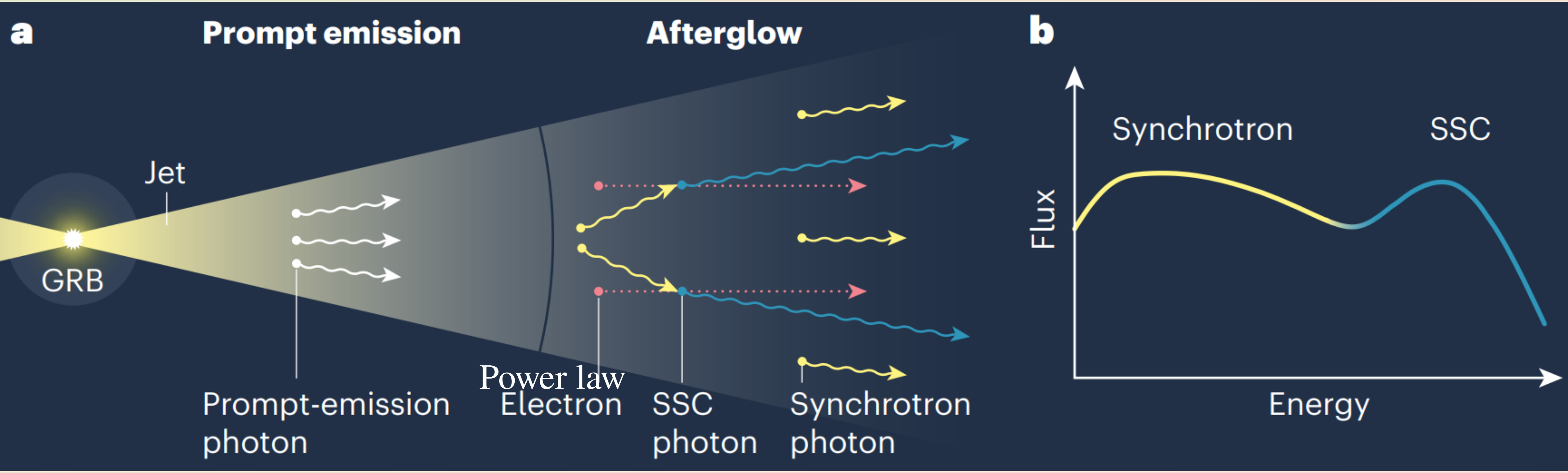
Collisionless shocks
- mediated by electromagnetic processes
- few gyro-radii scale

- At shocks:
  - **magnetic field amplification**
  - **particle acceleration**
    fermi first order acceleration →
    thermal core + power-law tail)

} *Synchrotron Radiation*

General Picture: Summary

From Bing Theodore Zhang (Zhang+19)



- **Jet structure:** (energy/velocity angular dependency)
- **Ejecta dynamics:** coasting; deceleration; spreading
- **Collisionless shocks:** magnetic field amplification & article acceleration:
- Synchrotron emission (self-absorption, inverse-Compton...)

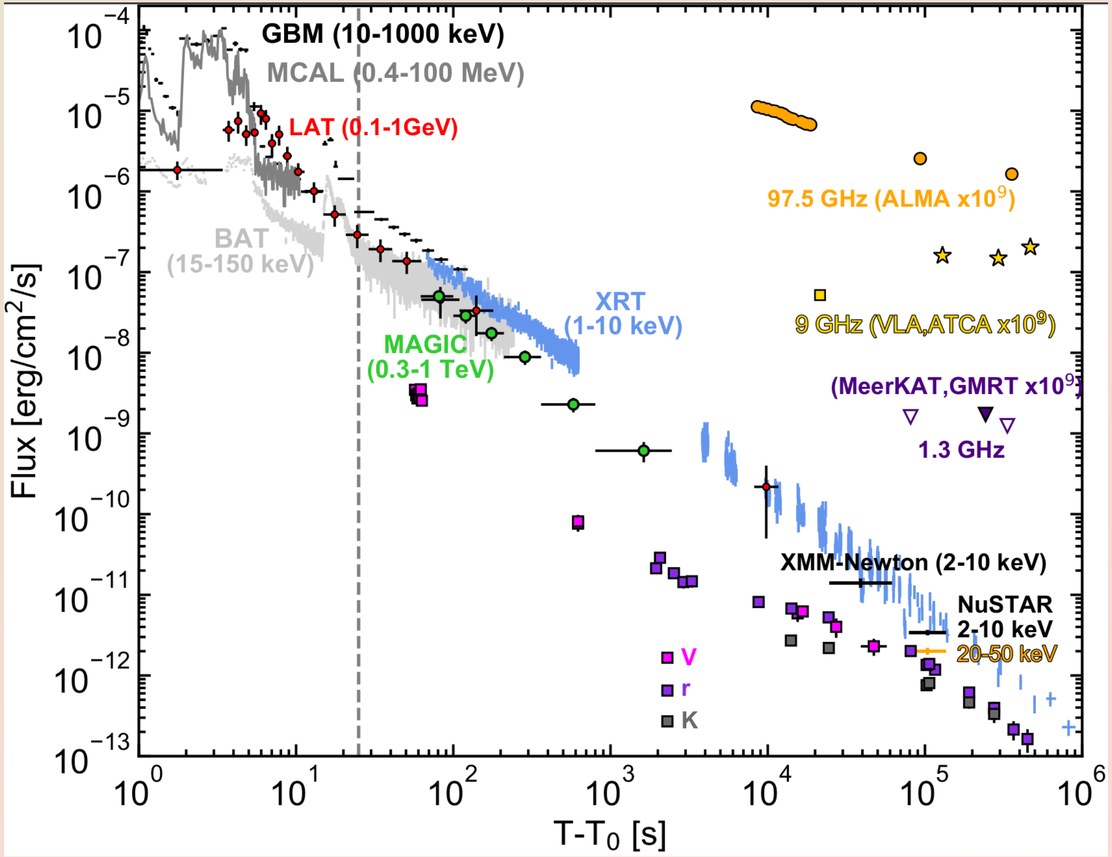
Examples: Very High Energy Afterglow

- GRB 190114C (Acciari+20, MAGIC Collaboration)
  TeV photons had distinct spectrum
  Leading explanation: **SSC**
  (KN regime & g-g absorption)

- Other examples of VHE:
  _ GRB 180720B
  - GRB 221009A

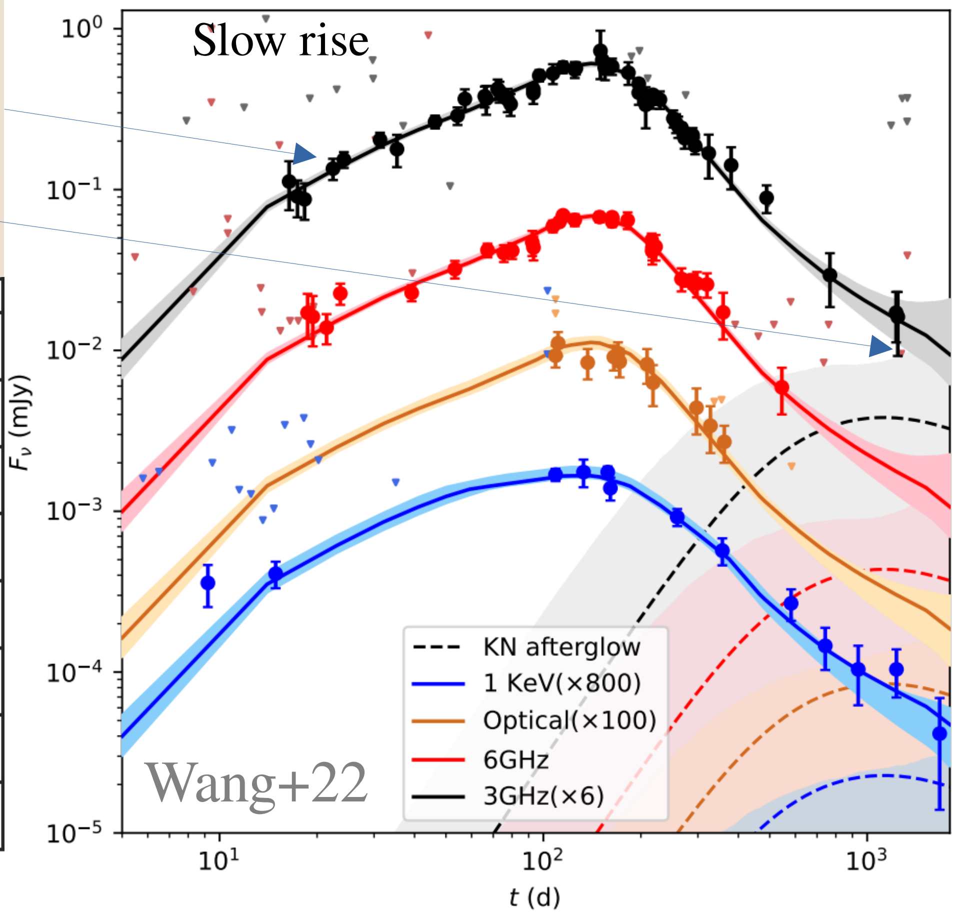
  Other possible sources of VHE

- Proton synchrotron
- Synchrotron self-Compton
- External inverse Compton

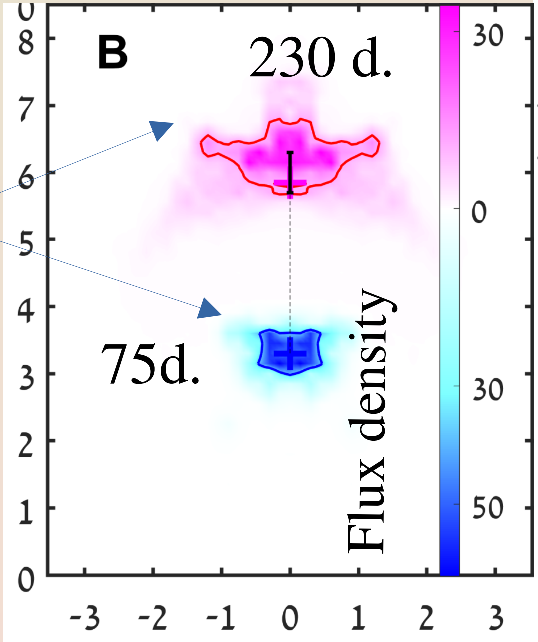


Examples: Structure & Radio Images

- GRB 170817A showed
  slow rise (not an off-axis tophat GRB)
  Leading explanation:
  - structured jet observed off-axis
  Shallow decay: lateral spreading



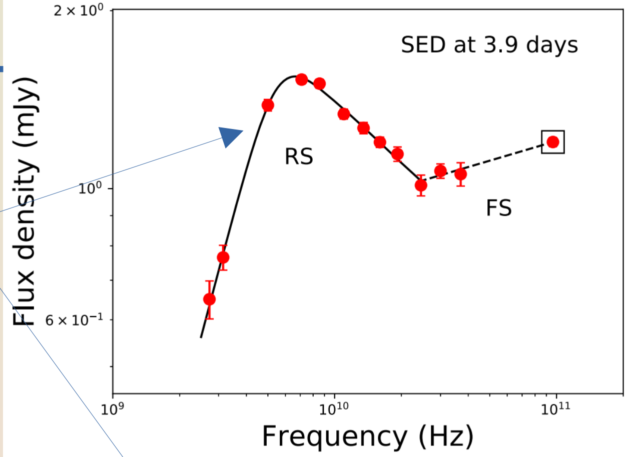
- VLA observed radio image (*centroid motion gives additional constrains*)



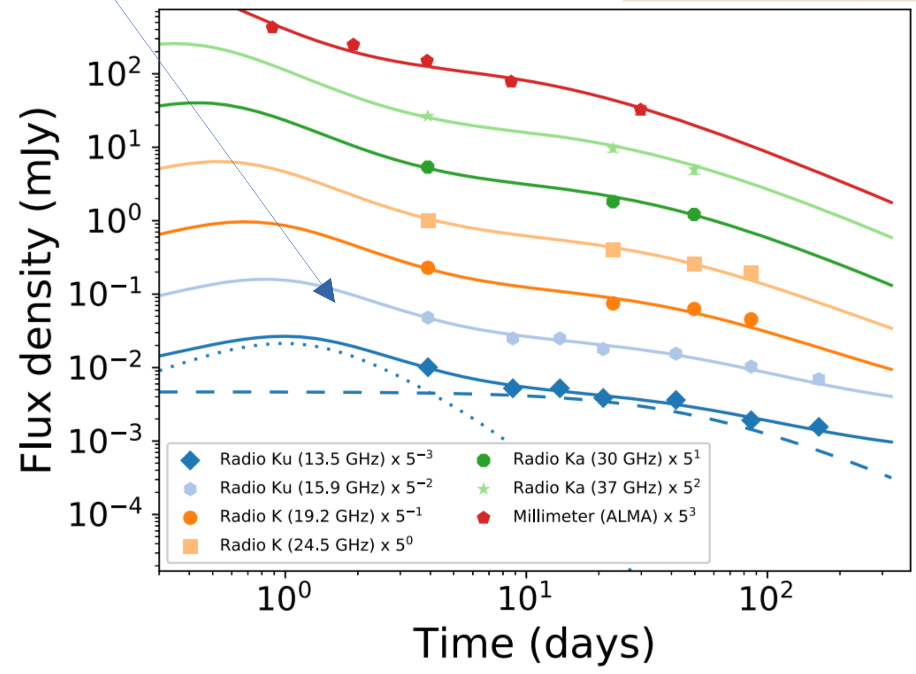
Mooley+18

Examples: Reverse Shock

- GRB 181201A showed **distinct spectra** at 3.9 days, in radio *Leading explanation*: emission from mildly relativistic reverse shock)
- Other examples:
  - GRB 160821B showed reverse shock & kilonova & energy injection (Lamb+20)
  - GRB 190114C also showed early decaying IR component attributed to the reverse shock



(Laskar+19)



State of the afterglow modeling & Goal

- Numeric codes (BoxFit…) - Computationally expensive.
- Semi-analytic codes (afterglowpy…) - FS & synchrotron only.

- Code requirements for multimessenger studies (*semi-numeric code*):

  - **speed** (suitable for inference or NN training) – semi-analytic dynamics;

  - **jet structure** (layers with different properties);

  - **FS & RS**;

  - **synchrotron & SSC** – numerical evolution of electron spectra;

  - **multiple observables**: spectra & sky maps;

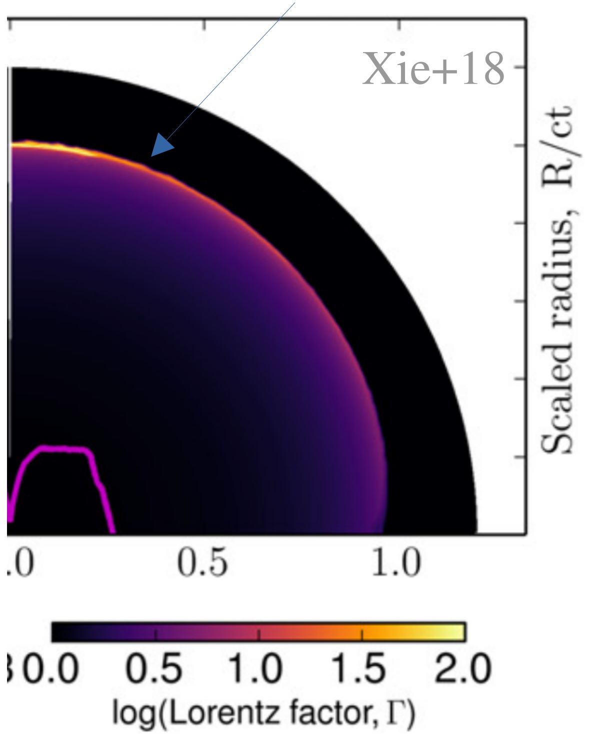
  - **extendable** (possibility to implement new physics & approximants).

Building an Afterglow Model

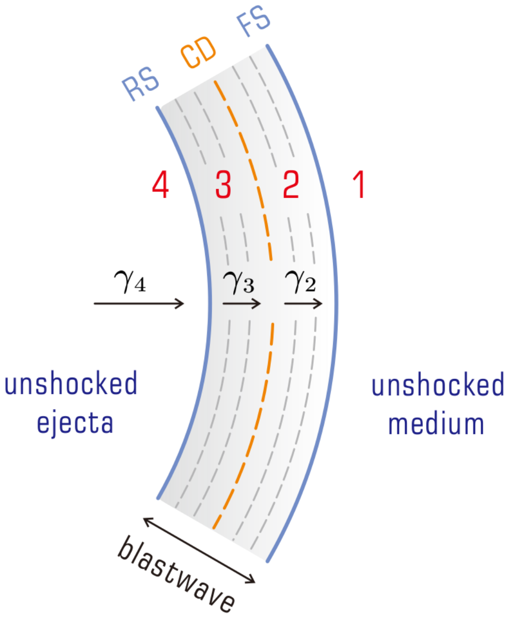
Building Afterglow Model: FS & RS dynamics

Numerical HD simulation of a jet

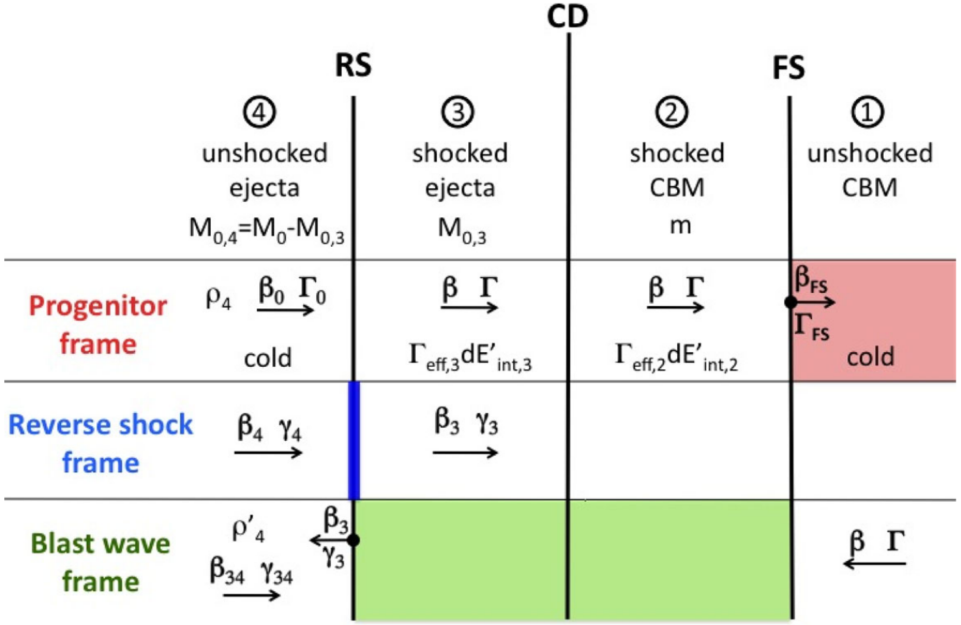
Thin shell forms



Thin shell approximation



Z. L. Zhang+22



Nava+13

Write evolution equations for the System (blastwave) Lorentz factor

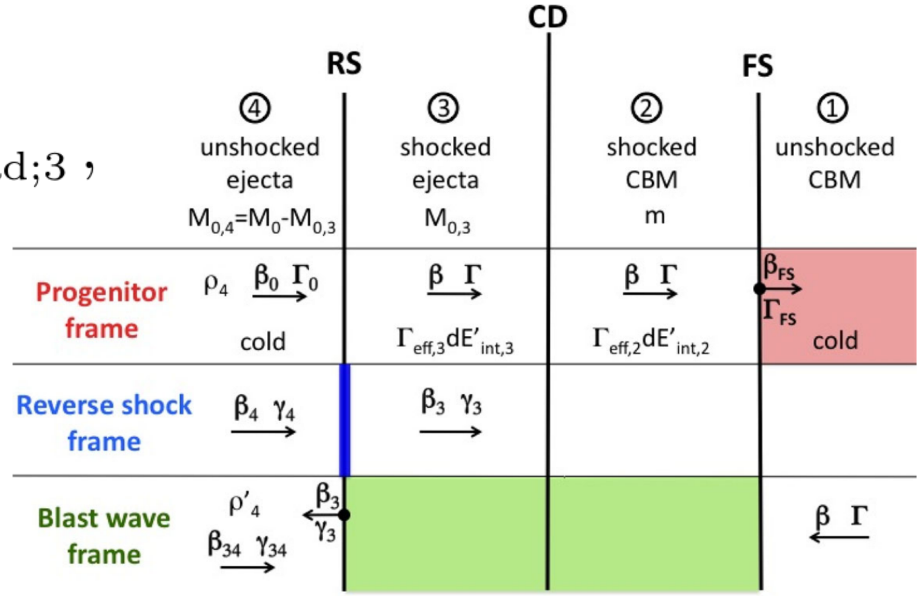
Building Afterglow Model: FS & RS dynamics

Total blast wave energy

$$E_{\rm tot} = \Gamma_0 m_{0;4} c^2 + \Gamma m_{0;3} c^2 + \Gamma_{\rm eff;3} E'_{\rm int;3}$$
$$+ \Gamma_0 m_{0;2} c^2 + \Gamma_{\rm eff;2} E'_{\rm int;2} \,.$$

Energy conservation equation

$$dE_{\rm tot} = \Gamma dm c^2 + \Gamma_{\rm eff;2} dE'_{\rm rad;2} + \Gamma_{\rm eff;3} dE'_{\rm rad;3} \,,$$
$$dE'_{\rm int;2} = dE'_{\rm sh;2} + dE'_{\rm ad;2} + dE'_{\rm rad;2} \,,$$
$$dE'_{\rm int;3} = dE'_{\rm sh;3} + dE'_{\rm ad;3} + dE'_{\rm rad;3} \,,$$



Accreted mass and lateral spreading

$$dm = 2\pi\rho\left[(1-\cos(\omega)) + \frac{1}{3}\sin(\omega) r d\omega\right] r^2 \,.$$

$$\frac{d\omega}{dt} = v_{\perp}\frac{c}{R} \times \begin{cases} 1 & \text{core} \\ \tan(\omega_0/2)/\tan(\omega_c/2) & \text{wings} \end{cases}$$

sound-speed-based;
calibrated with numerical simulations

Building Afterglow Model: FS & RS dynamics

Evolution equation:
after some algebra...

$$d\Gamma = -\frac{N}{D}\,,\text{ where}$$

$$\begin{aligned}
N =& (\Gamma - 1)(\Gamma_{\text{eff};2} + 1)dm \\
&+ (\Gamma - \Gamma_0 + \Gamma_{\text{eff};3}(\Gamma_{43} - 1))dm_{0;3} \\
&+ \Gamma_{\text{eff};2}(\hat{\gamma}_2 - 1)E'_{\text{int};2}(d\ln m - d\ln\rho) \\
&- \Gamma_{\text{eff};3}(\hat{\gamma}_3 - 1)E'_{\text{int};3}(d\ln m_{0;3} - d\ln\rho_4)
\end{aligned}$$

$$\begin{aligned}
D =& (m + m_{0;3}) + E'_{\text{int};2}\frac{d\Gamma_{\text{eff};2}}{d\Gamma} + E'_{\text{int};3}\frac{d\Gamma_{\text{eff};3}}{d\Gamma} \\
&+ \frac{\Gamma_{\text{eff};2}}{\Gamma}(\hat{\gamma}_2 - 1)E'_{\text{int};2} + \frac{\Gamma_{\text{eff};3}}{\Gamma_{43}}(\hat{\gamma}_3 - 1)E'_{\text{int};3}\frac{d\Gamma_{43}}{d\Gamma}\,.
\end{aligned}$$

Assumed ejecta shell density profile

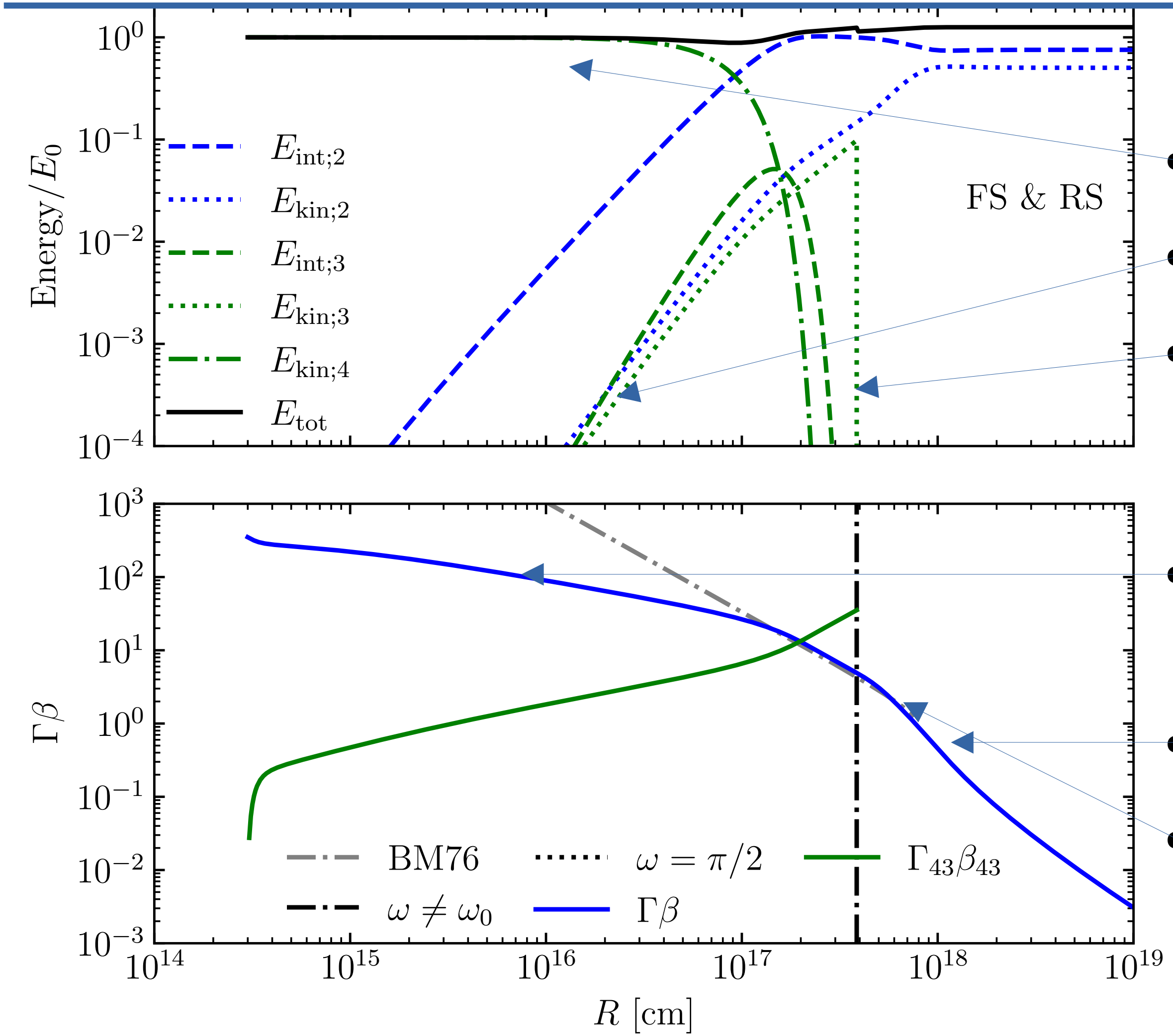
$$\rho_4 = \frac{M_0}{2\pi(1 - \cos(\omega))R_{\text{rsh}}\Delta_0}\exp\left(-\frac{\Delta_4}{\Delta_0}\right),$$

Mass accreted by RS

$$\frac{dm_{0;3}}{dR} = 2\pi R_{\text{rsh}}^2(1 - \cos(\omega))\rho_4\frac{d\Delta'_4}{dR},$$

System of ODEs is solved numerical with DOP8(53) method

Building Afterglow Model: FS & RSexample



During the evolution

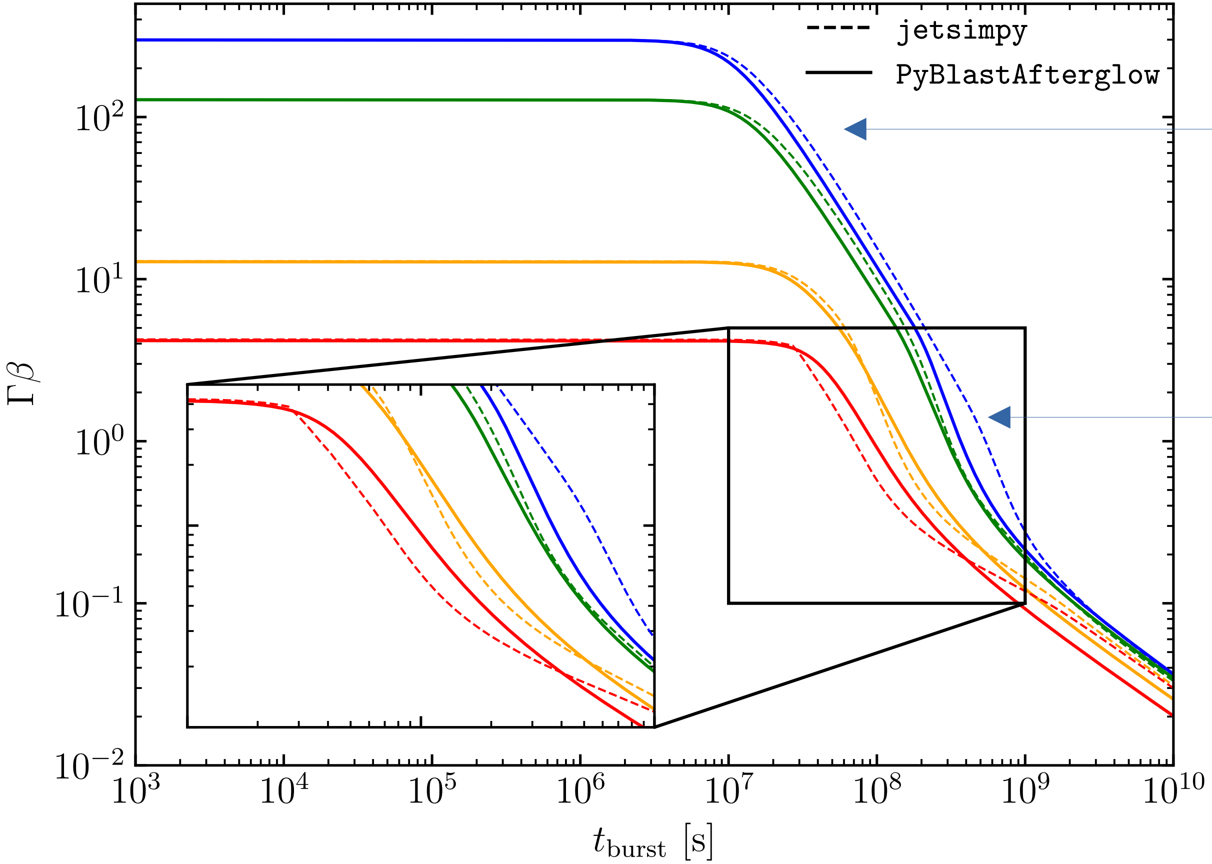
- Total energy ~ conserved
- Reverse shock *shocks* ejecta
- RS crosses the ejecta

Dynamics

- deceleration when RS is crossing ejecta
- Lateral spreading
- BM solution is recovered close to RS crossing

Building Afterglow Model: Comparison

- Dynamics with lateral spreading against 2D thin-shell hydrodynamics model (using structure and Spreading)



calibration to BM can be added

spreading prescription can be improved

Building Afterglow: Shock Microphysics

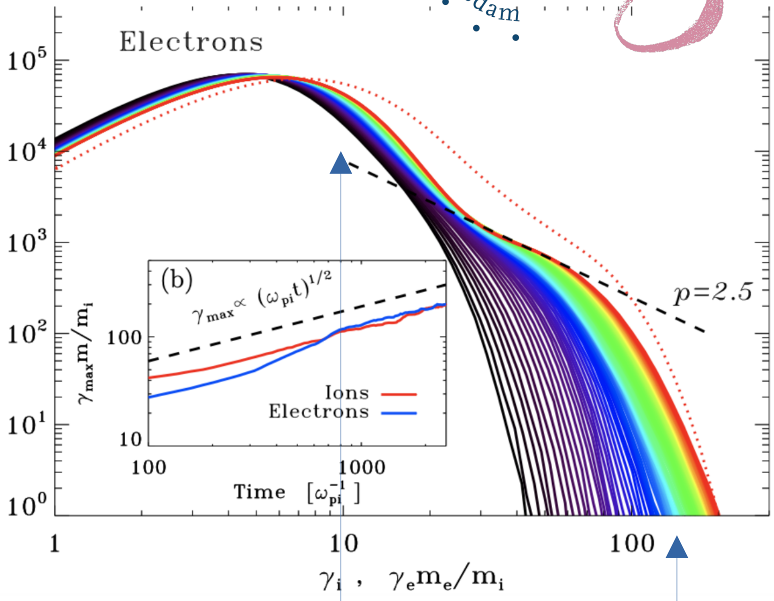
Energy Density $e' = E'_{\rm int}/V'$

Magnetic Field $B' = \sqrt{8\pi\epsilon_{\rm b}e'}$

Free parameter

Injection spectrum

$$dN_e/d\gamma_e \propto \gamma_e^{-p} \text{ for } \gamma_e \in (\gamma_{e;\,\rm m}, \gamma_{e;\,\rm M})$$



M

m

Limits of the injection spectrum

$$\gamma_{e;\,\rm M} = \sqrt{\frac{6\pi q_e}{\sigma_T B' \zeta (1+\tilde{Y})}}$$

$$\epsilon_{\rm e}\frac{e'}{\rho'}\frac{m_p}{m_e c^2} = \langle\gamma_e\rangle = \left(\frac{p-1}{p-2}\right)\left(\frac{\gamma_{e;\,\rm M}^{-p+2} - \gamma_{e;\,\rm m}^{-p+2}}{\gamma_{e;\,\rm M}^{-p+1} - \gamma_{e;\,\rm m}^{-p+1}}\right)$$

Free parameter

Solving for minimum

Building Afterglow: Electron Evolution

Electron distribution evolution

Source term

$$\frac{\partial}{\partial t'}\frac{dN_e{}'}{d\gamma_e{}'} + \frac{\partial}{\partial \gamma_e{}'}\left(\dot{\gamma}_{\rm e,tot}{}'\frac{dN_e{}'}{d\gamma_e{}'}\right) = d\dot{N}'_{\rm e,0} = \dot{N}'_{\rm e}\frac{p-1}{\gamma'_{\rm m}}\left(\frac{\gamma_e{}'}{\gamma'_{\rm m}}\right)^{-p} d\gamma_e{}',$$

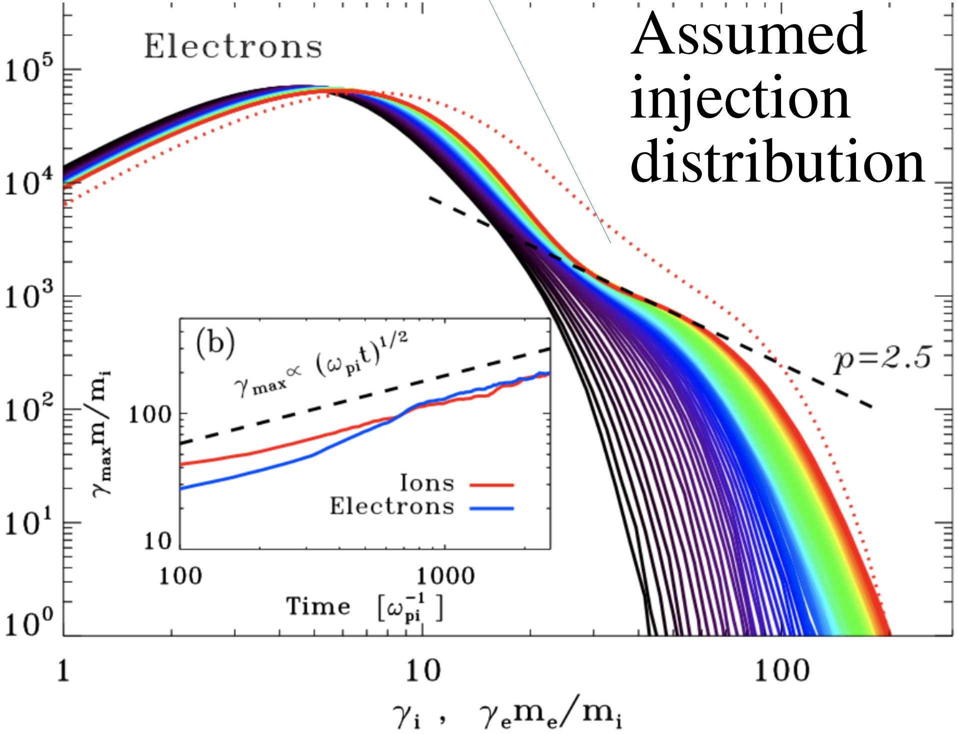
Electron energy loss

$$\dot{\gamma}_{\rm e,tot}{}' = \dot{\gamma}_{\rm e,adi}{}' + \dot{\gamma}_{\rm e,syn}{}' + \dot{\gamma}_{\rm e,IC}{}'.$$

Solved using Chang-Cooper scheme with substeps for cooling.

Numerically compute

- synchrotron emissivity $j'_{\rm syn}(\nu')$,
- SSA coefficient $\alpha'_{\rm syn}(\nu')$,
- SSC emissivity $j'_{\rm ssc}(\nu')$.



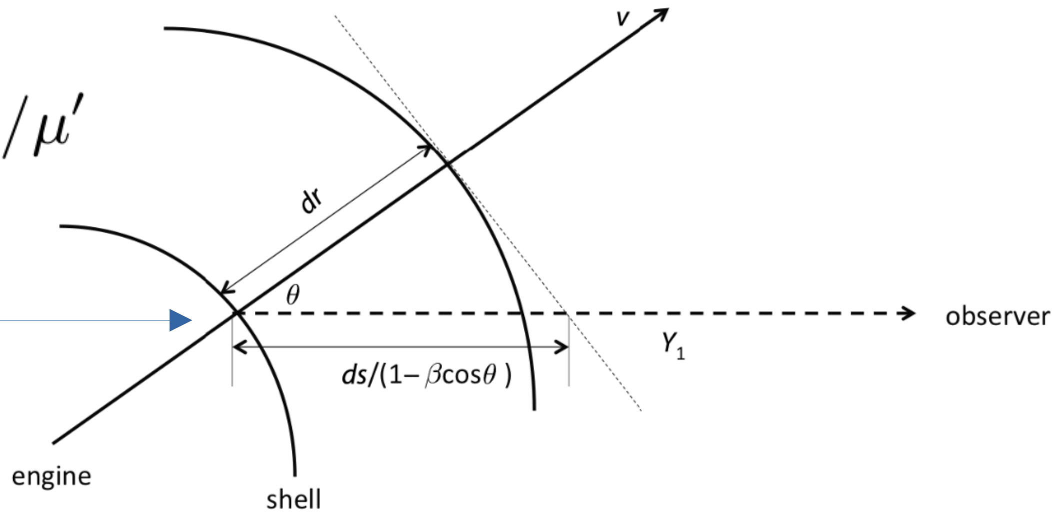
Assumed injection distribution

Building Afterglow: Intensity and Flux Density

Radiation transport in a thin, homogeneous shell.

Radiation intensity (*observer* frame)

$$I_\nu = \frac{j_\nu}{\alpha_\nu}(1 - e^{-\tau_\nu}) \quad \tau_\nu \approx -\alpha_\nu \Delta R/\mu'$$



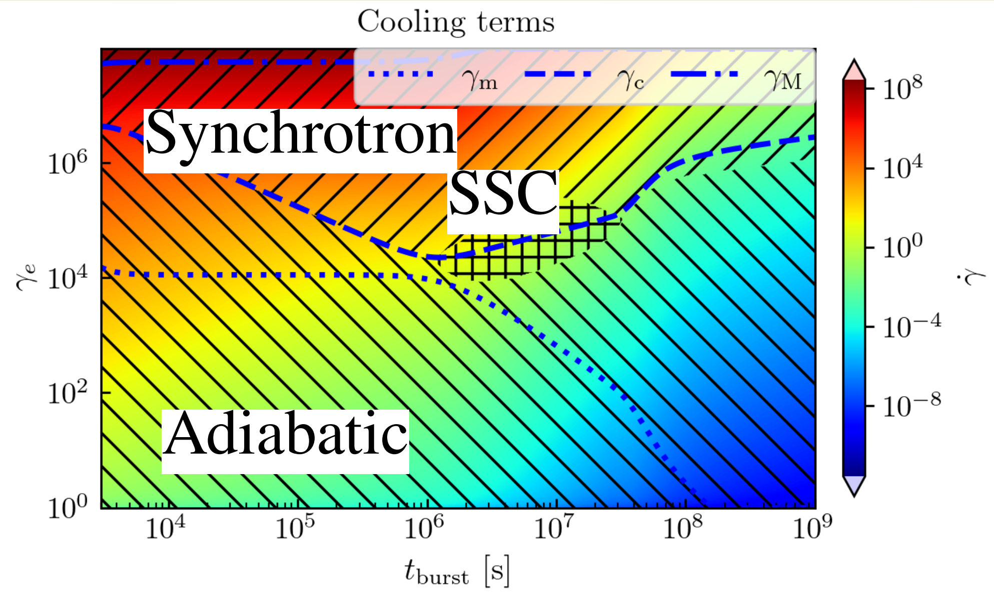
Thickness of the emitting region depends on the observer.

Equal-arrival time surface integration

$$F_\nu = \frac{1+z}{2\pi d_L^2} \sum_i^{\text{layers}} \int_{\theta_{i;l}}^{\theta_{i;h}} \int_{\phi_0=0}^{\phi_1=\pi/2} I_{i,\nu}(\theta,\phi) d\theta d\phi$$

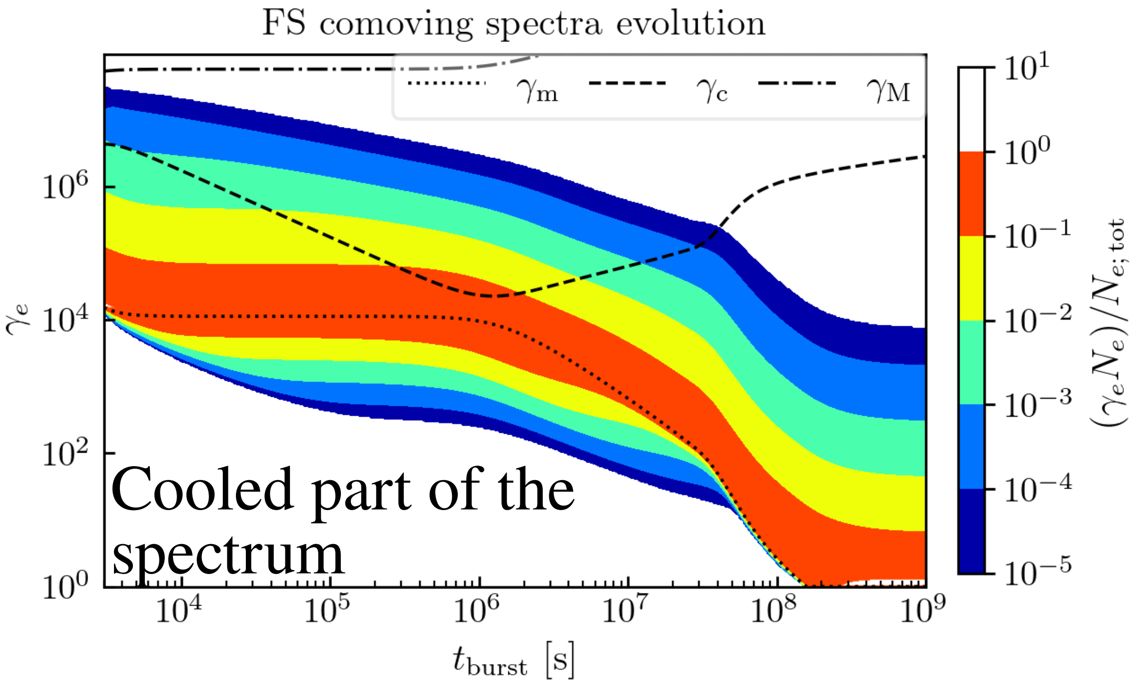
Preliminary Results

Preliminary results: Electron Spectrum



Dominant cooling process

Electron spectrum evolution

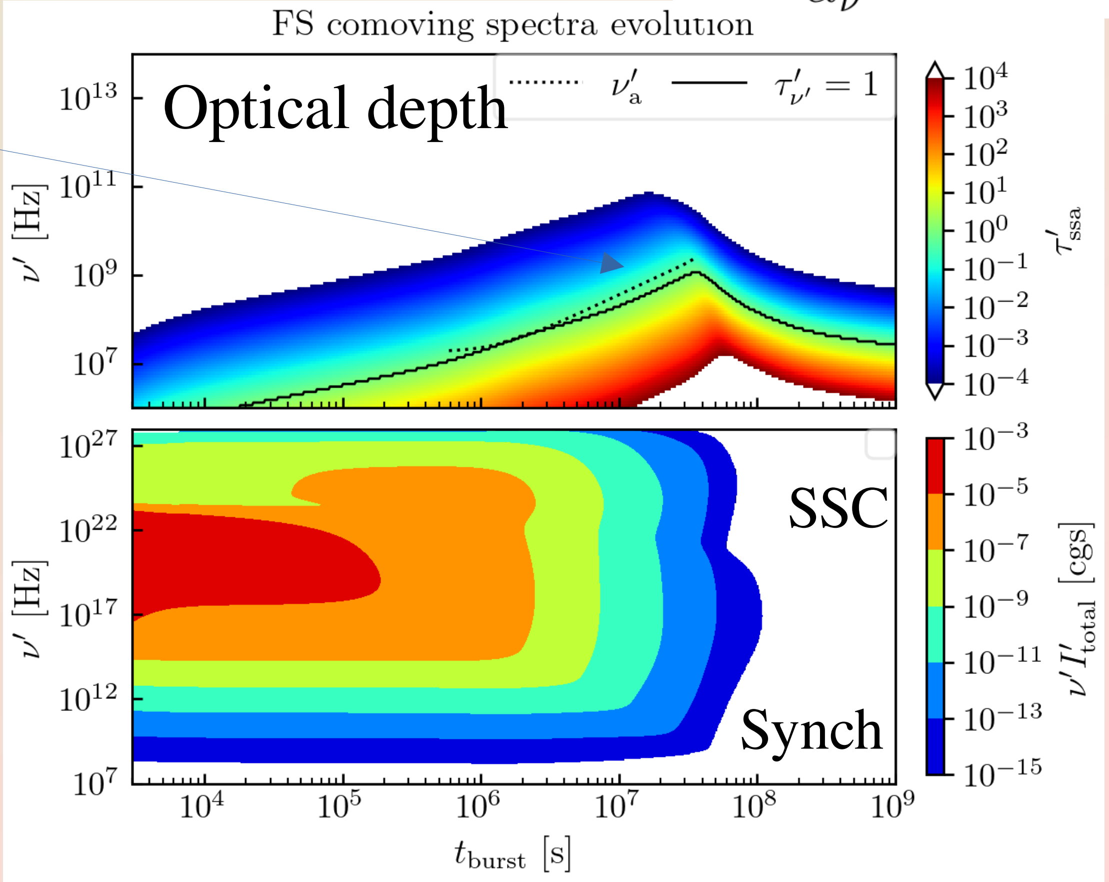


Preliminary results: Radiation Spectra

$$I'_\nu = \frac{j'_\nu}{\alpha'_\nu}(1 - e^{-\tau}),$$

Spectrum is primarily optically thin

Comoving synchrotron and SSC spectra



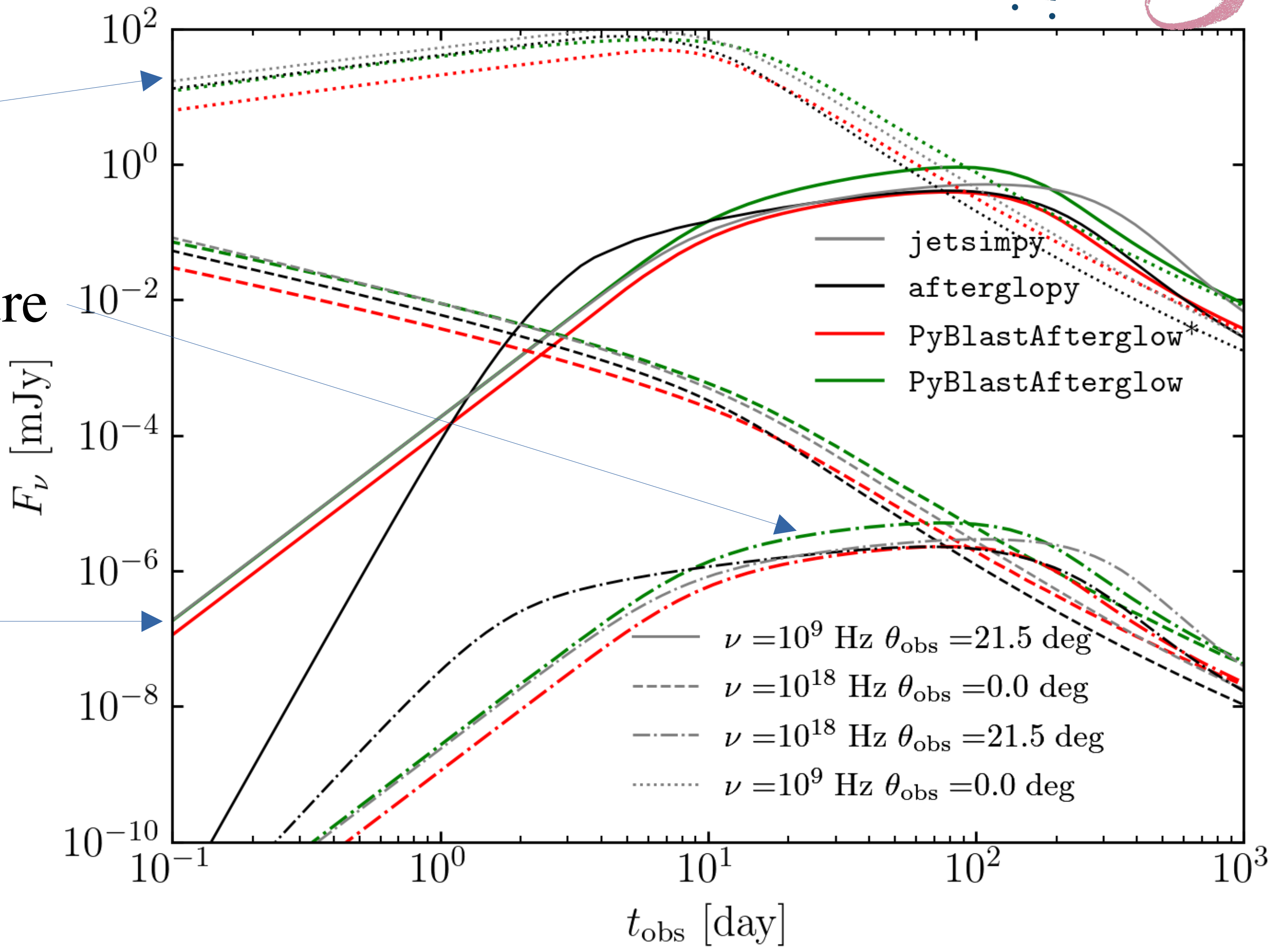
Comparison with afterglowpy; jetsimpy

On-axis afterglow

Shallow slope due to structure

Off-axis afterglow

Comparison:
Gaussian jet;
forward shock only;
synchrotron only.



Example: sky map

Sky map – intensity distribution projected onto a plane orthogonal to the line of sight.

Properties:
- centroid position,
- image size.

Sky map shows:
- geometry of the ejecta,
- jet Lorentz factor,
- jet inclination angle.

Origin

Flux centroid

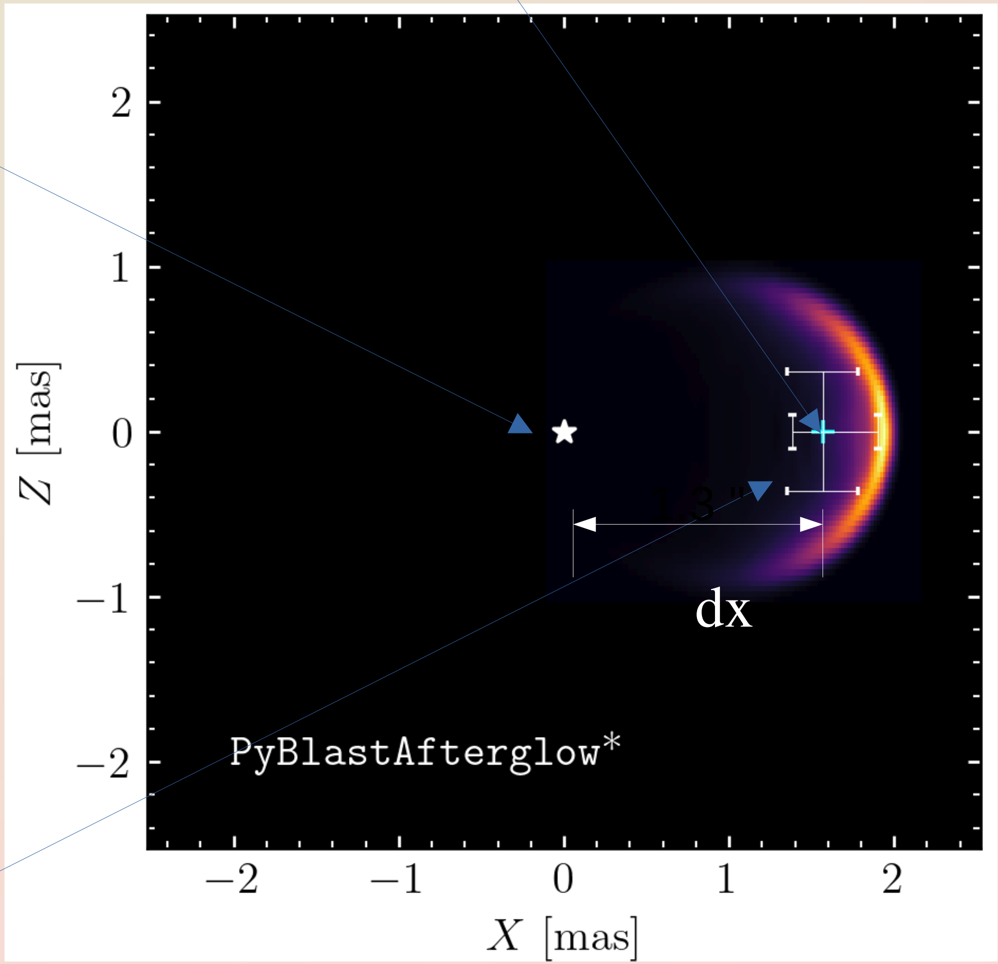
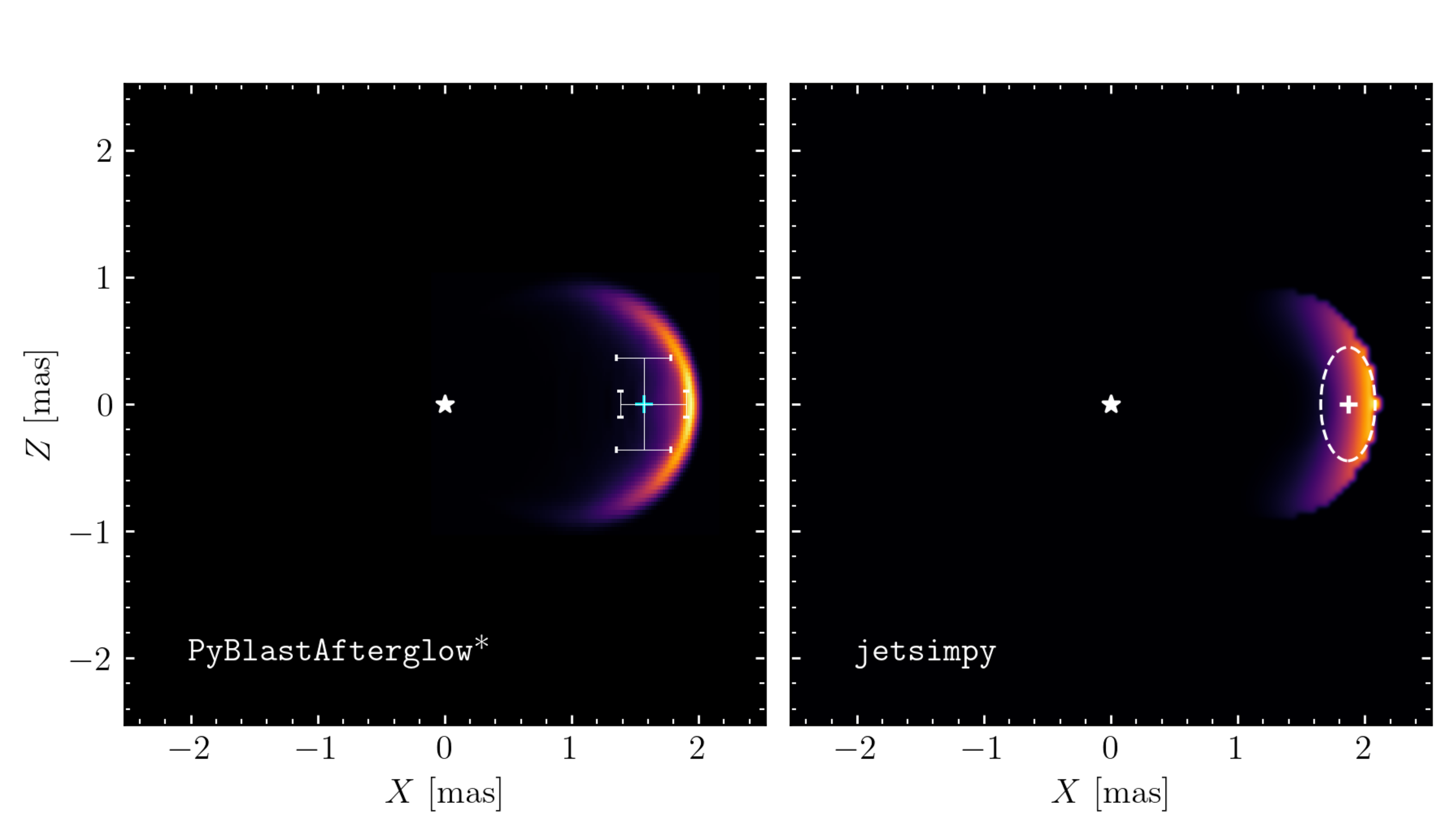


Image size (FWHM)

Comparison with jetsimpy

Comparison: Gaussian structure; forward shock only; synchrotron only



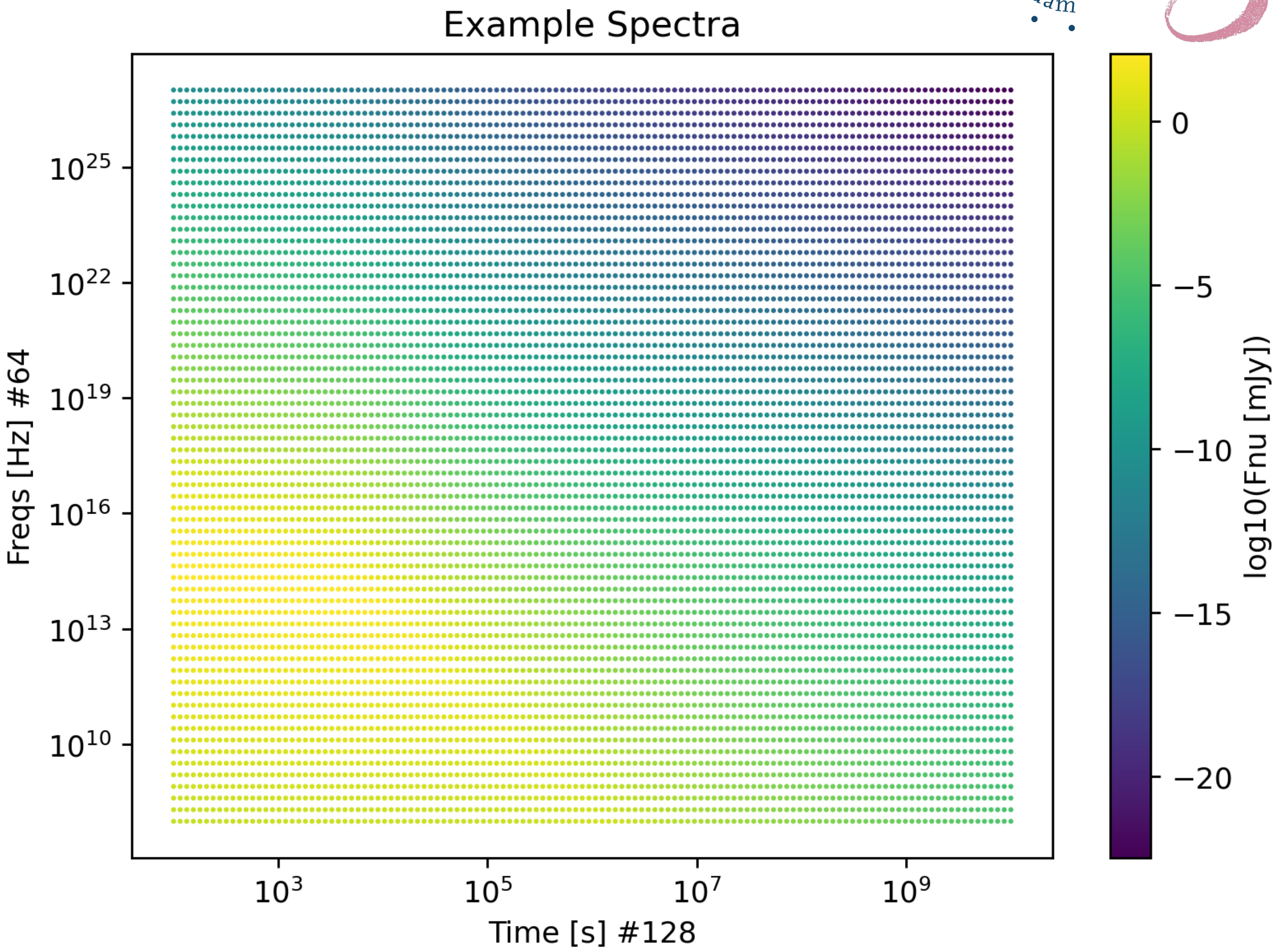
Building a Surrogate Model: Dataset

Building train dataset (with afterglowpy)

Data:
set of time-dependent spectra for tophat jet.

Features:
- Microphysics (3)
- Jet structure (3)
- Observer (3)

Metadata:
- 10000 spectra,
- 26 GB training set.



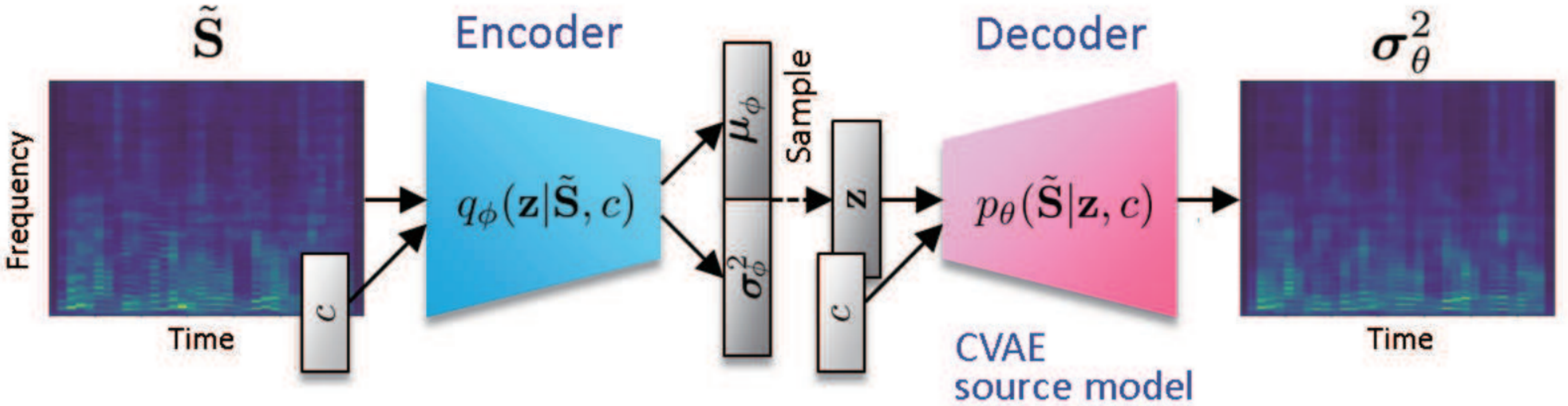
Building a Surrogate Model: Architecture

Neural network:

- generative model
    - conditional variational autoencoder (cVAE)

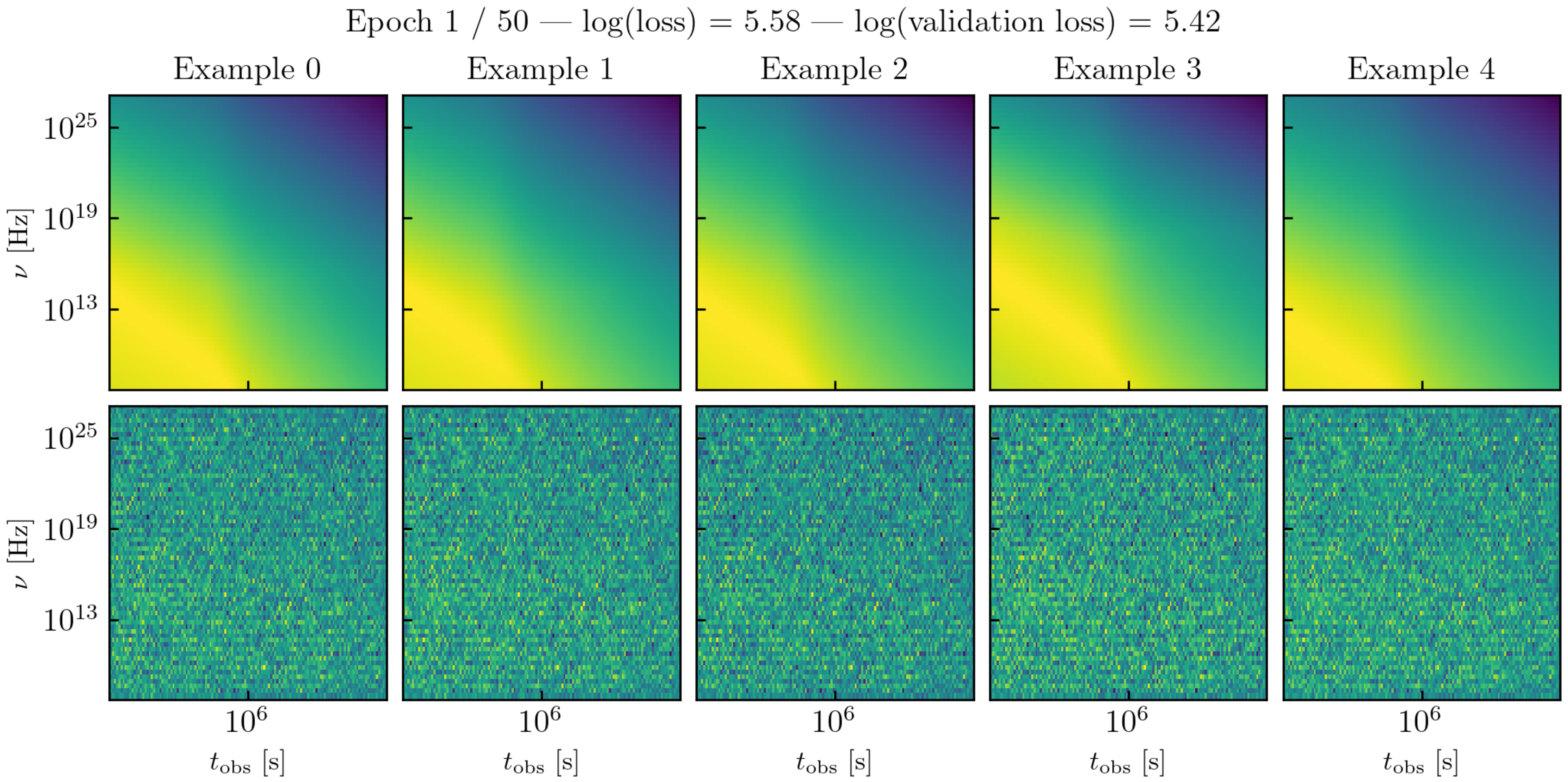
Model allows to create a latent space that represents the space of real spectra from which we can sample new spectra

Kameoka+18



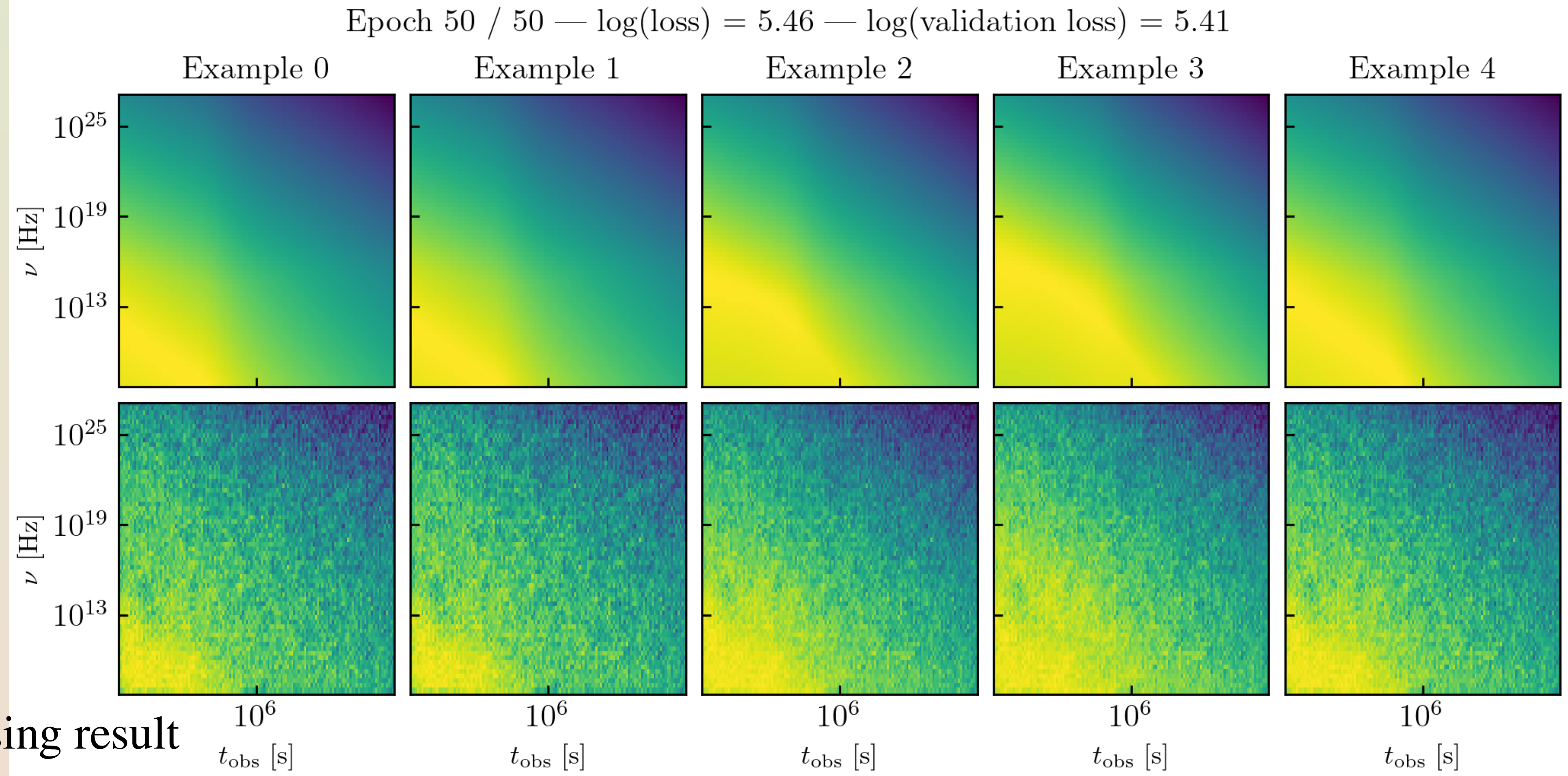
Building a Surrogate Model: Training

Training model on a 1000 spectra : starting epoch



Building a Surrogate Model: Training

Training model on a 1000 spectra : last epoch



Promising result

Summary

- **NMMA** – using observational data and models for trainsients to constrain the fundamental physics
- **PyBlastAfterglow** – semi-numeric GRB afterglow model with advanced physics input that generates spectra and sky maps, fast enough to train a surrogate model. Key features:
  - GRB ejecta structure (non-interacting layers)
  - Forward & reverse shock dynamics and emission
  - Numerical evolution of electron distribution
  - Synchrotron, synchrotron self-absorption, synchrotron self-Compton
  - Spectra and sky maps

Building Afterglow: Cooling terms

Cooling terms

$$\dot{\gamma}_{\rm syn} = -\frac{4}{3}\left(\frac{\sigma_T {B'}^2}{8\pi m_e c}\right)\gamma_e^2 \,.$$

Synchrotron cooling

$$\dot{\gamma}_{\rm adi} = \frac{d\ln(V')}{dR}\left(\frac{\gamma_e^2 - 1}{3\gamma_e}\right),$$

Adiabatic cooling

SSC cooling

$$\dot{\gamma}_{\rm ssc} = \frac{3}{4}\frac{h\sigma_T}{m_e c \gamma_e^2}\int_{\tilde{\nu}'_0}^{\tilde{\nu}'_1}\frac{n_{\tilde{\nu}'}}{\tilde{\nu}'}\left[\int_{\nu'_0}^{\nu'_1} h\nu' K(\tilde{\nu}', \gamma_e, \nu')d\nu'\right]d\tilde{\nu}'$$

SSC kernel

$$K(\tilde{\varepsilon}', \gamma_e, \varepsilon) = 2q\ln(q) + (1+2q)(1-q) + 0.5(1-q)\frac{(4\gamma_e\tilde{\varepsilon}q)^2}{(1+4\gamma_e\tilde{\varepsilon})}$$

Building Afterglow: Synchrotron Emission

Convolving the emission of a single electron gyrating around magnetic field lines with electron distribution function

$$j'_{\rm syn}(\nu',\gamma_e) = \frac{3q_e^3 B'}{m_e c^2} 2y^2 \Big(K_{4/3}(y) K_{1/3}(y) - \frac{3}{5} y \big(K^2_{4/3}(y) - K^2_{1/3}(y) \big) \Big) \,,$$

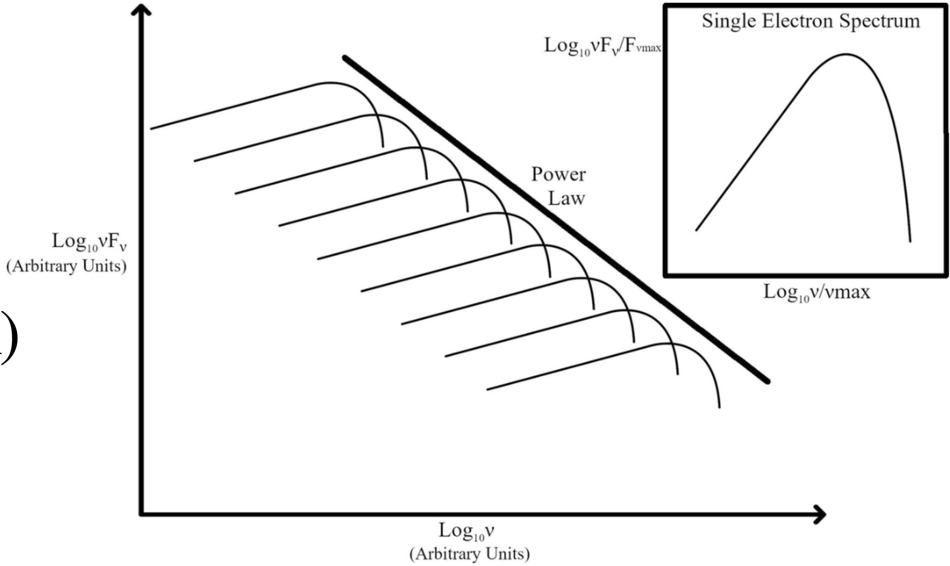
$$y = \nu'/\nu'_{\rm crit}$$

Emission power by a single electron

$$j'_{\rm syn}(\nu') = \int_{\gamma_{e;0}}^{\gamma_{e;1}} N_e(\gamma_e) j'_{\rm syn}(\nu',\gamma_e) d\gamma_e \,,$$

$$\alpha'_{\rm syn}(\nu') = -\frac{1}{8\pi m_e \nu'^2} \times \int_{\gamma_{e;0}}^{\gamma_{e;1}} j'_{\rm syn}(\nu',\gamma_e) \gamma_e^2 \frac{\partial}{\partial \gamma_e} \Big[\frac{N_e(\gamma_e)}{\gamma_e^2} \Big] d\gamma_e$$

Synchrotron self-absorption (SSA)



Building Afterglow: SSC emission

Synchrotron self-Compton spectrum

$$j'_{\rm ssc}(\nu',\gamma_e) = \frac{3}{4} h\sigma_T c \frac{\nu'}{\gamma_e^2} \int_{\nu'_0}^{\nu'_1} \frac{n_{\tilde{\nu}'}}{\tilde{\nu}'} F(\tilde{\varepsilon}',\gamma_e,\varepsilon') d\tilde{\nu}' \,,$$

Seed photon spectrum

$$n_{\nu'} = n_{\nu';\,\rm syn} + n_{\nu';\,\rm ssc} = \left(\frac{j'_{\rm syn}(\nu')}{h\nu'} + \frac{j'_{\rm ssc}(\nu')}{h\nu'} \right) \frac{\Delta t'}{V'} \,,$$

Comoving escape time

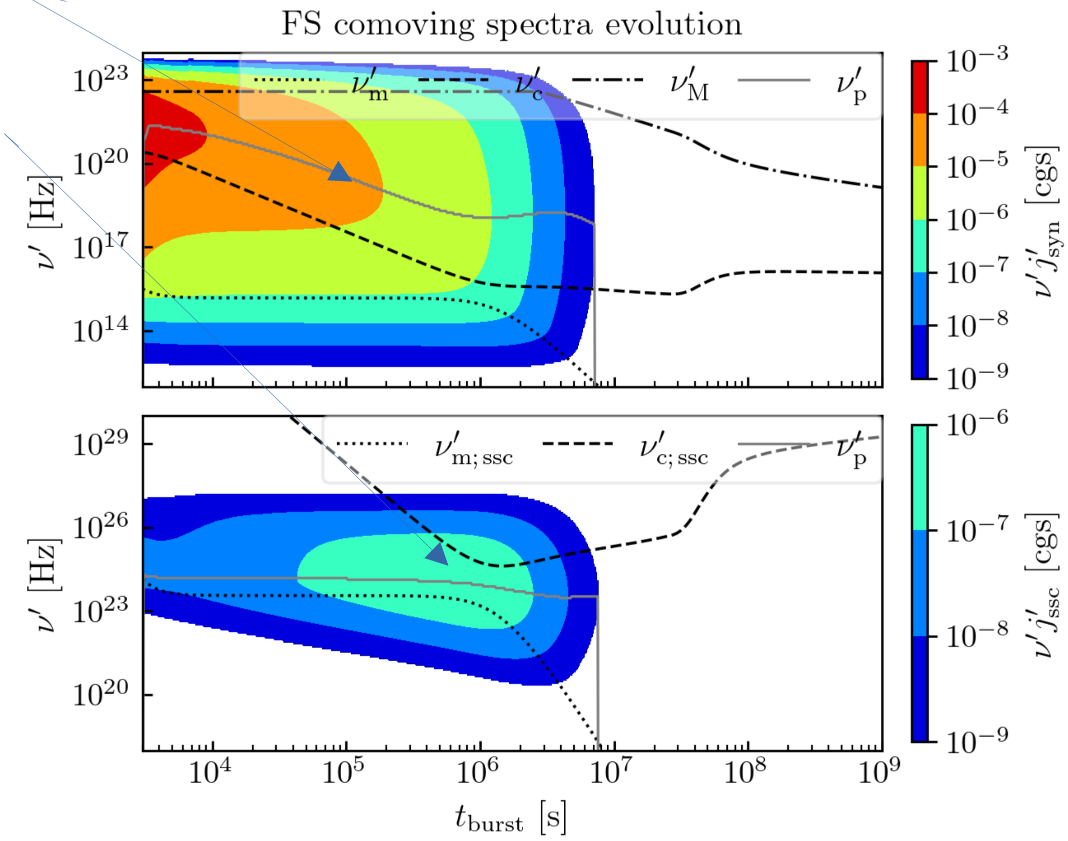
$$\Delta t' = \Delta R'/c = \Delta R \Gamma_{\rm sh}/c$$

Preliminary results: Radiation Spectra

Comoving synchrotron and SSC spectra

Numerical spectrum peak > analytical

SSC reaches peak at BW deceleration



Building Afterglow Model: FS & RS dynamics

Total blast wave energy

$$E_{\rm tot} = \Gamma_0 M_{0,4}\, c^2 + \Gamma M_{0,3}\, c^2 + \Gamma m\, c^2 + \Gamma_{\rm eff,3} E'_{\rm int,3} + \Gamma_{\rm eff,2} E'_{\rm int,2}.$$

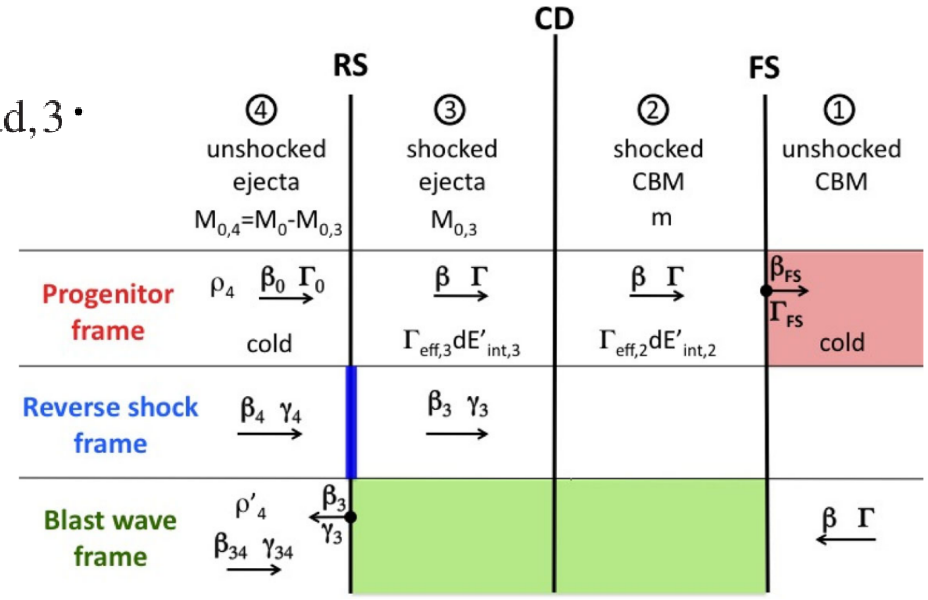
Energy conservation equation

$$\mathrm{d}E_{\rm tot} = \mathrm{d}m\, c^2 + \Gamma_{\rm eff,2}\mathrm{d}E'_{\rm rad,2} + \Gamma_{\rm eff,3}\mathrm{d}E'_{\rm rad,3}.$$

$$\mathrm{d}E'_{\rm int} = \mathrm{d}E'_{\rm sh} + \mathrm{d}E'_{\rm ad} + \mathrm{d}E'_{\rm rad}.$$

Evolution equation: after some algebra...

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}R} = -\frac{(\Gamma_{\rm eff,2}+1)(\Gamma-1)\frac{\mathrm{d}m\,c^2}{\mathrm{d}R} + \Gamma_{\rm eff,2}\mathrm{d}E'_{\rm ad,2}}{(M_{0,3}+m)c^2 + E'_{\rm int,2}\frac{\mathrm{d}\Gamma_{\rm eff,2}}{\mathrm{d}\Gamma} + E'_{\rm int,3}\frac{\mathrm{d}\Gamma_{\rm eff,3}}{\mathrm{d}\Gamma}} - \frac{(\Gamma-\Gamma_0-\Gamma_{\rm eff,3}+\Gamma_{\rm eff,3}\gamma_{34})\frac{\mathrm{d}M_{0,3}c^2}{\mathrm{d}R} + \Gamma_{\rm eff,3}\mathrm{d}E'_{\rm ad,3}}{(M_{0,3}+m)c^2 + E'_{\rm int,2}\frac{\mathrm{d}\Gamma_{\rm eff,2}}{\mathrm{d}\Gamma} + E'_{\rm int,3}\frac{\mathrm{d}\Gamma_{\rm eff,3}}{\mathrm{d}\Gamma}}$$



Adaptive ODE solve used

Building Afterglow Model: FS only example

Total blast wave energy (assume ejecta is <u>already</u> shocked)

$$E_{\rm tot} = \Gamma (M_0 + m)\, c^2 + \Gamma_{\rm eff}\, E'_{\rm int},$$

Energy conservation equation

$$\mathrm{d}\left[\Gamma (M_0 + m) c^2 + \Gamma_{\rm eff} E'_{\rm int}\right] = \mathrm{d}m\, c^2 + \Gamma_{\rm eff} \mathrm{d}E'_{\rm rad}.$$

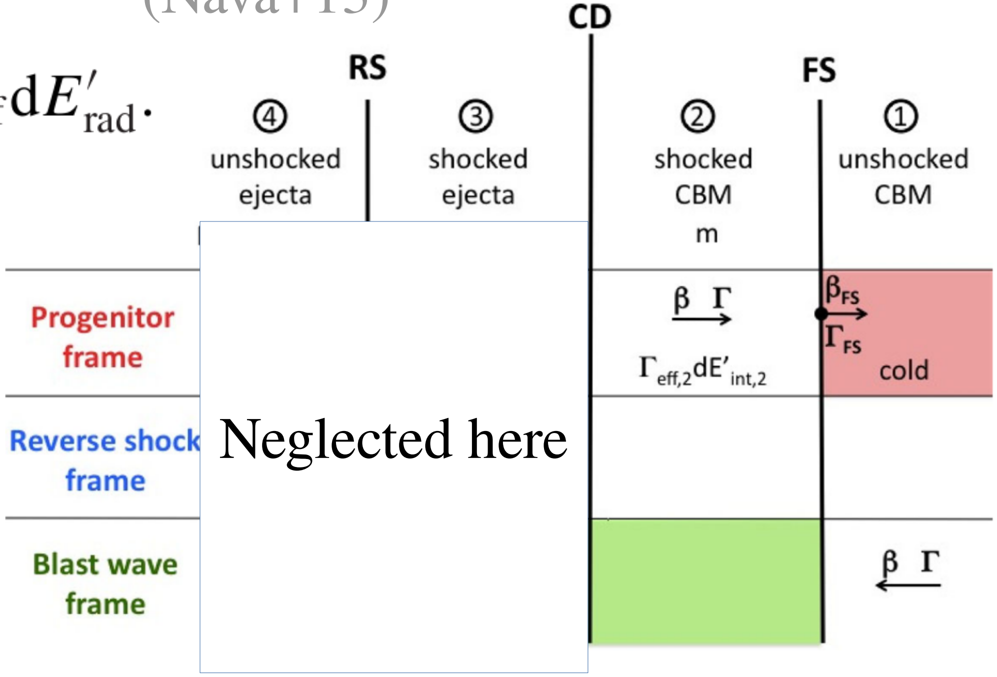
$$\mathrm{d}E'_{\rm int} = \mathrm{d}E'_{\rm sh} + \mathrm{d}E'_{\rm ad} + \mathrm{d}E'_{\rm rad}.$$

Evolution equation: after some algebra…

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}R} = -\frac{(\Gamma_{\rm eff} + 1)(\Gamma - 1)\, c^2 \frac{\mathrm{d}m}{\mathrm{d}R} + \Gamma_{\rm eff} \frac{\mathrm{d}E'_{\rm ad}}{\mathrm{d}R}}{(M_0 + m)\, c^2 + E'_{\rm int} \frac{\mathrm{d}\Gamma_{\rm eff}}{\mathrm{d}\Gamma}},$$

Final set of ODEs we solve numerically

Schematic picture of a Blast wave (Nava+13)

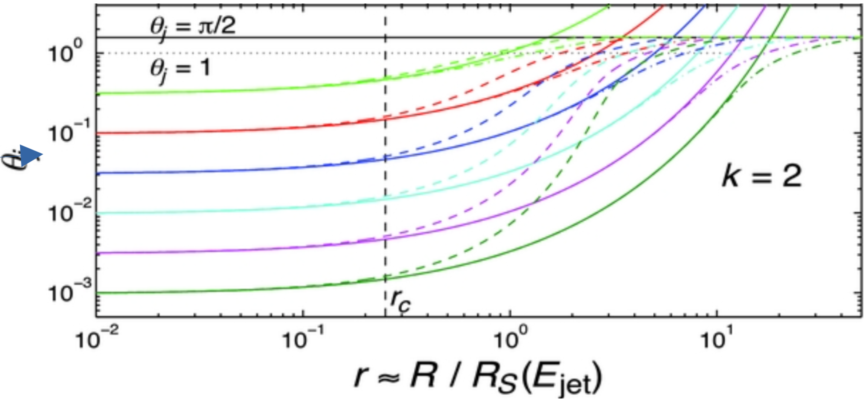


Building Afterglow Model: lateral spreading

Required 2D HD simulations. Using approximants. Many options...

Calibrated with early HD simulation (Granot+12)

$$\frac{d\omega}{dR} = R^{-1}\Gamma^{-1-a}$$



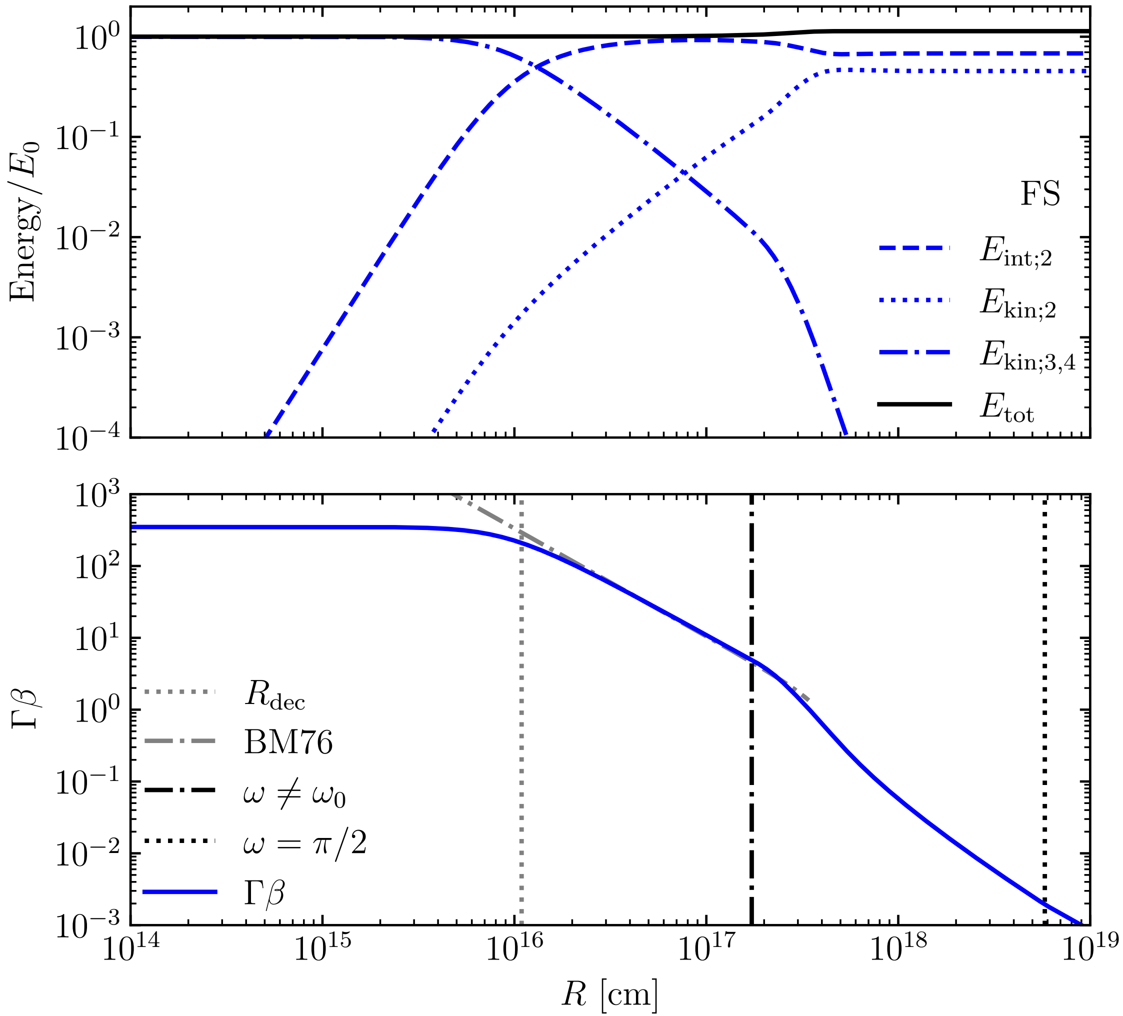
Sound-speed based formulation (Huang+00)

$$\frac{d\omega}{dR} = \frac{c_s}{R\Gamma\beta c} \qquad c_s^2 = \frac{\hat{\gamma}p'}{\rho'}\left[\frac{(\hat{\gamma}-1)\rho'}{(\hat{\gamma}-1)\rho' + \hat{\gamma}\rho'}\right]c^2 = \frac{\hat{\gamma}(\hat{\gamma}-1)(\Gamma-1)}{1+\hat{\gamma}(\Gamma-1)}c^2$$

Structured-jet aware formulation (Ryan+20)

$$\frac{d\omega}{dt} = v_\perp \frac{c}{R} \times \begin{cases} 1 & \text{core} \\ \tan(\omega_0/2)/\tan(\omega_c/2) & \text{wings} \end{cases} \qquad v_\perp = \frac{1}{2}\frac{\beta_{\rm sh}}{\Gamma}\sqrt{\frac{2(\Gamma\beta)^2+3}{4(\Gamma\beta)^2+3}}$$

Building Afterglow Model: FS only example



<u>Energy:</u>

- kinetic → internal energy
- total energy is conserved c

<u>Dynamics:</u>

- free coasting (constant momentum)
- deceleration (BM solution)
- lateral spreading
- non-relativistic stage (ST solution)