Modeling Gamma-Ray Burst Afterglows

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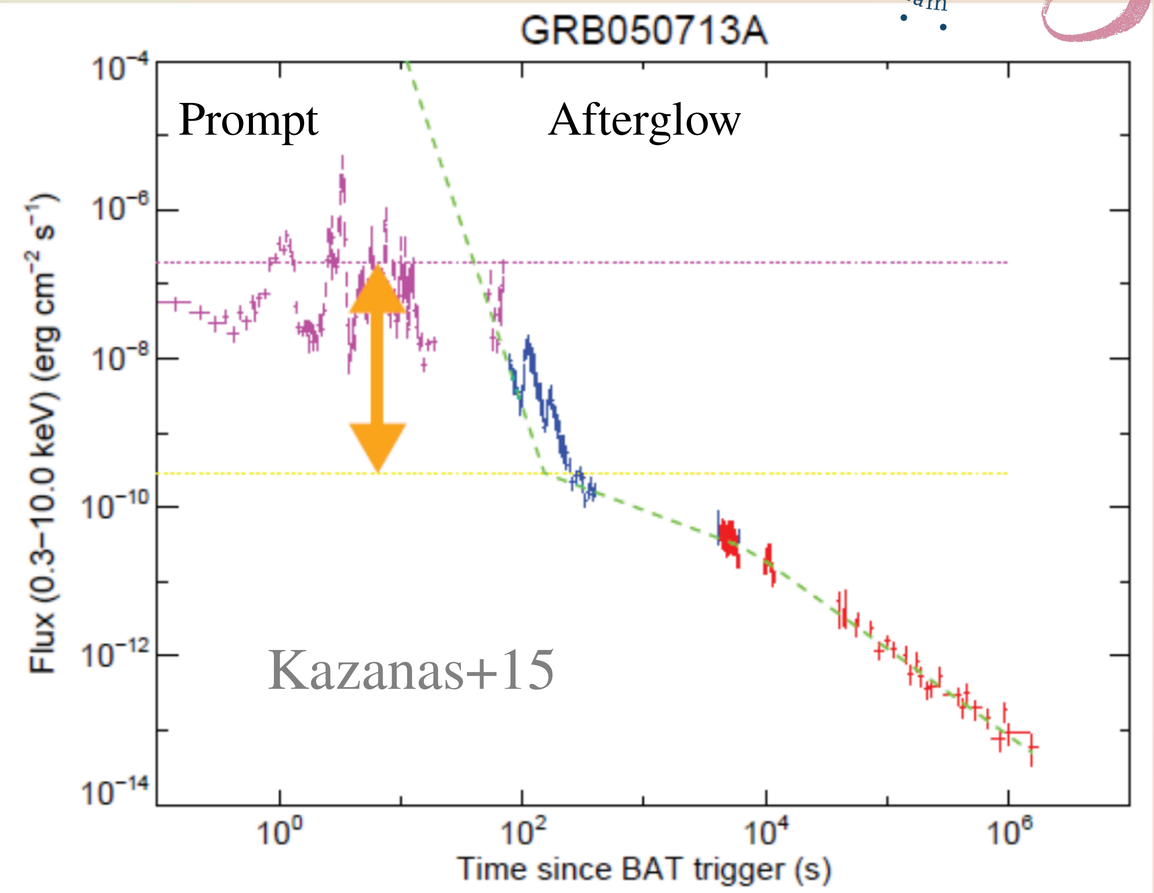
Structure

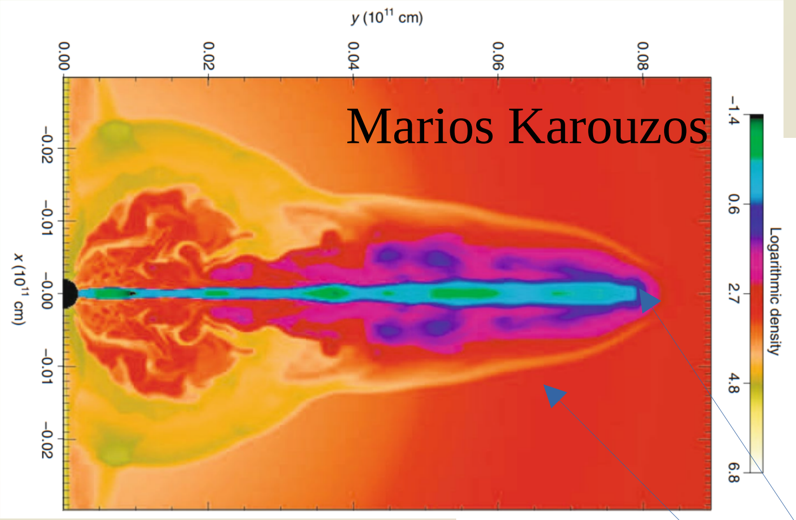
- General picture and basic properties
- Examples of observations
- Building an afterglow model
  - Dynamics
  - Microphysics and radiation
  - Observables
- Comparison with other models
- Building a surrogate model

Introduction

- **Gamma-Ray Bursts**
- **Prompt emission** (oscillatory, early) *Internal shocks* or magnetic reconnection
  Convert fraction of jet energy to radiation
- **Afterglow** (smooth, late)
  External shocks (forward/reverse)
  Synchrotron emission (based on spectra)

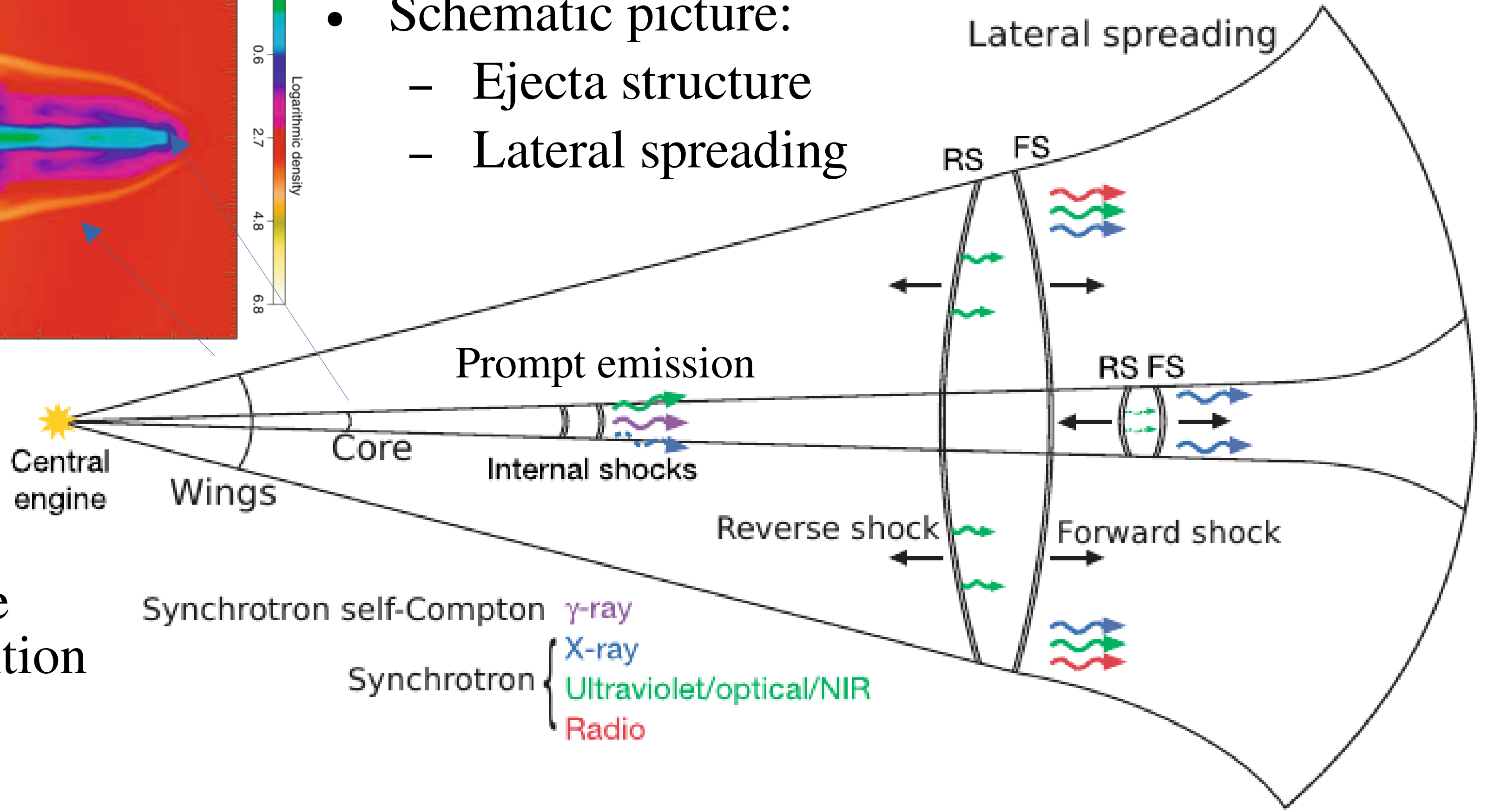


General Picture: Jet



- Schematic picture:
  - Ejecta structure
  - Lateral spreading

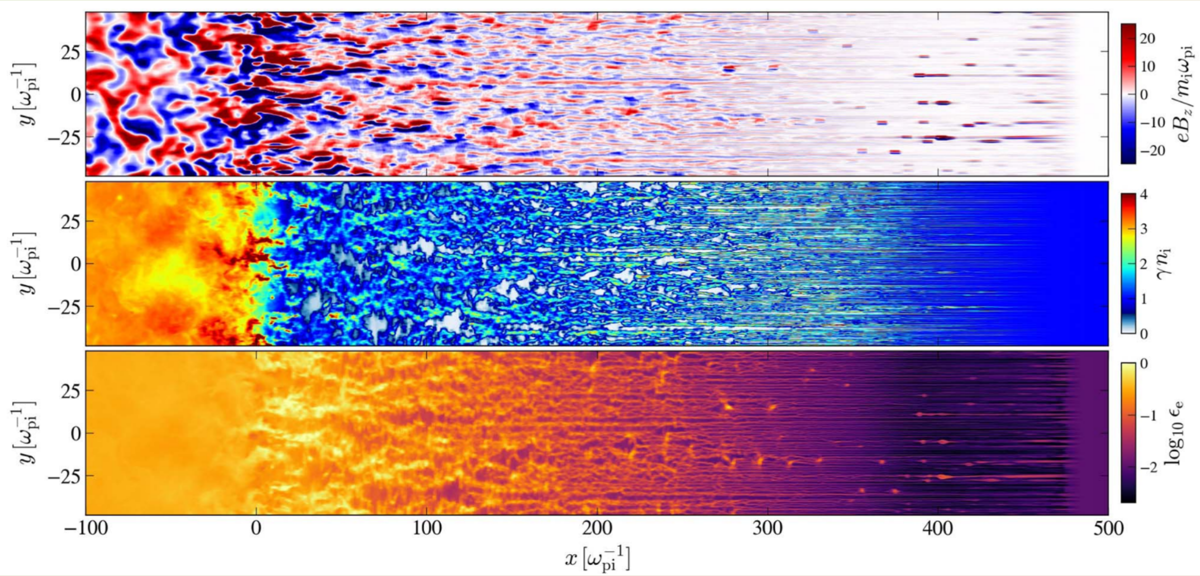
- **Engine**:
  disk accretion
  - burst energy
  - ejecta structure
  - ejecta composition

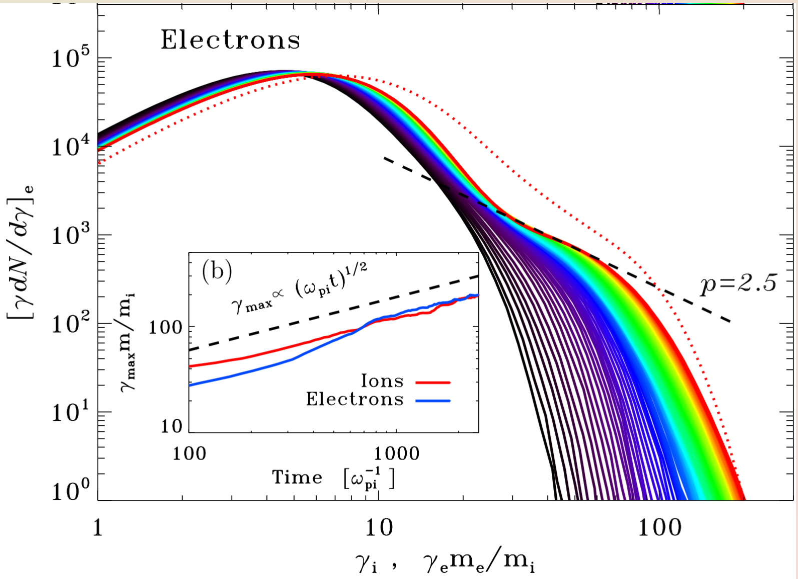


General Picture: Shocks

Vanthieghem+22

Sironi+13



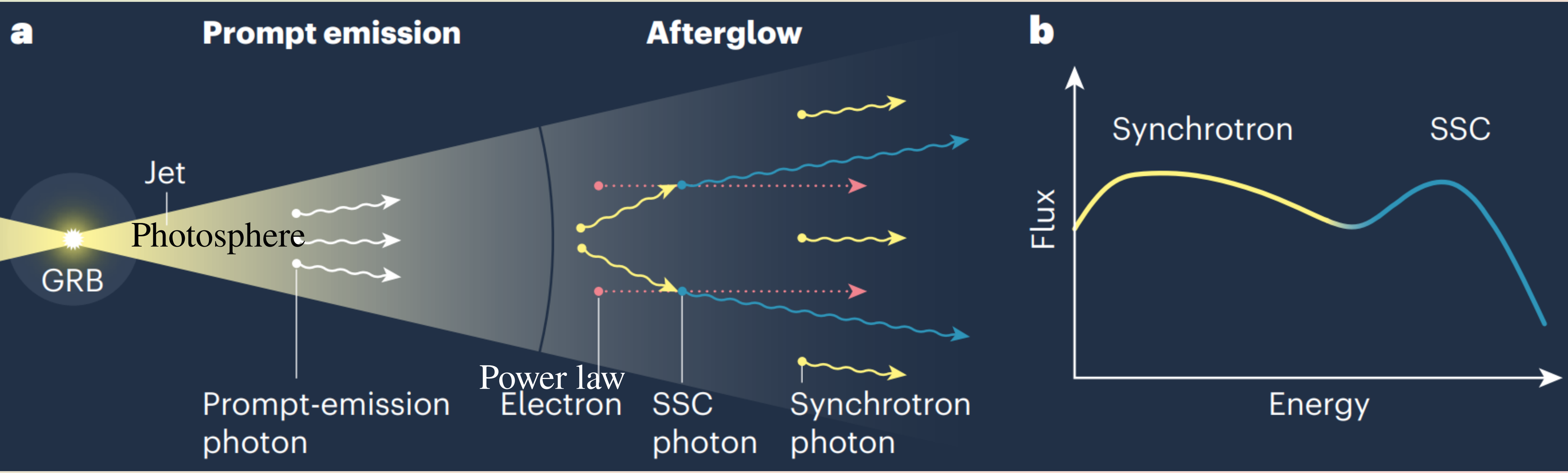


- **Collisionless shocks**
  (few gyro-radii scale)
- MHD instabilities
  magnetic field amplification
- **Particle acceleration**
  fermi first order acceleration →
  thermal core + power-law tail)

} *Synchrotron Radiation*

General Picture: Summary

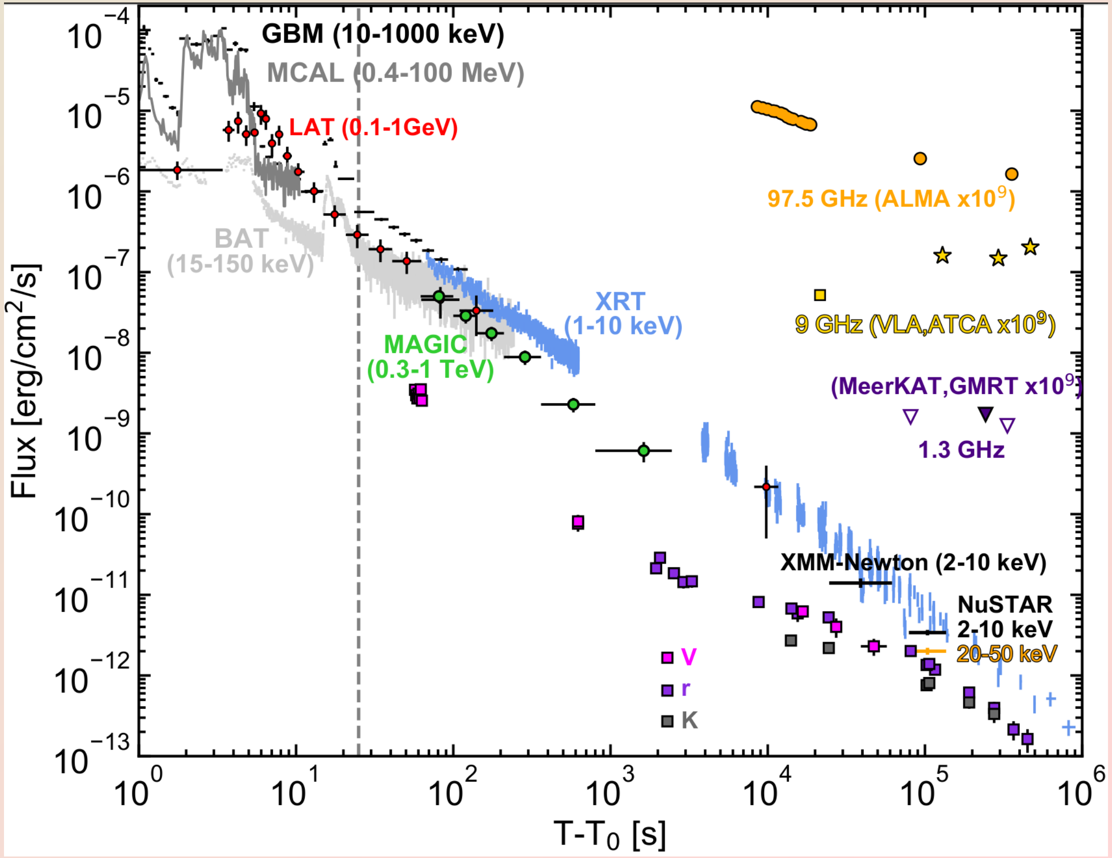
From Bing Theodore Zhang (Zhang+19)



- **Jet structure:** (energy/velocity angular dependency)
- **Ejecta dynamics:** coasting; deceleration; spreading
- **Collisionless shocks:** magnetic field amplification & article acceleration: Fermi first order acceleration; Power-law tail + thermal core

Examples: Very High Energy Afterglow

- GRB 190114C (Acciari+20, MAGIC Collaboration)
  TeV photons have distinct spectrum
  Explanation: SSC
  (KN regime & g-g absorption)
- GRB 190114C,
  GRB 180720B,
  GRB 221009A



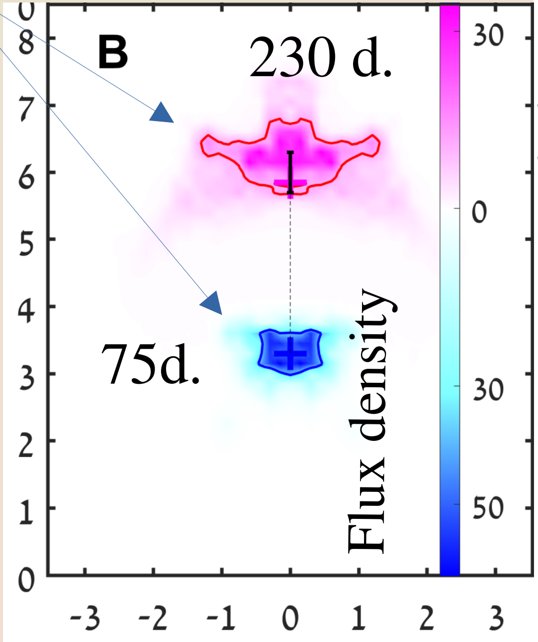
Possible sources of VHE

- Proton synchrotron
- Synchrotron self-Compton
- External inverse Compton

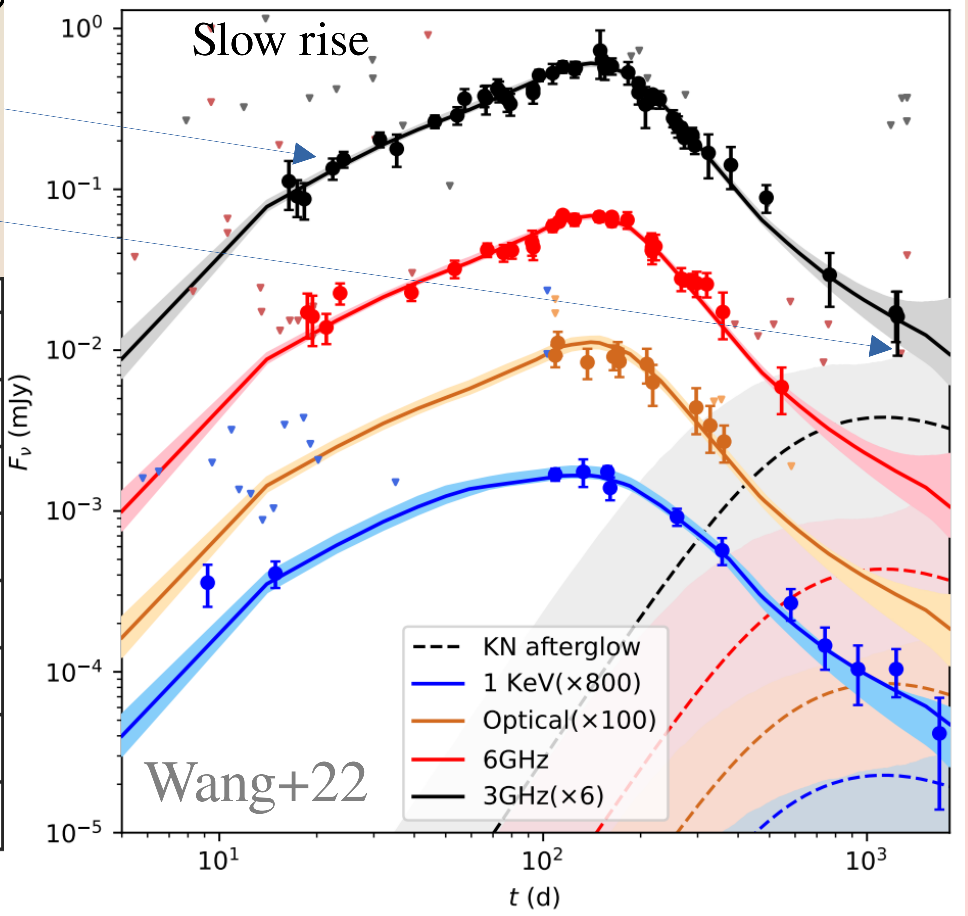
Examples: Structure & Radio Images

- GRB 170817A
  Slow rise (not an off-axis tophat GRB)
  Explanation:
  - jet lateral structure; off-axis observer
  Shallow decay: lateral spreading

- VLA observed radio image (centroid motion gives additional constrains)

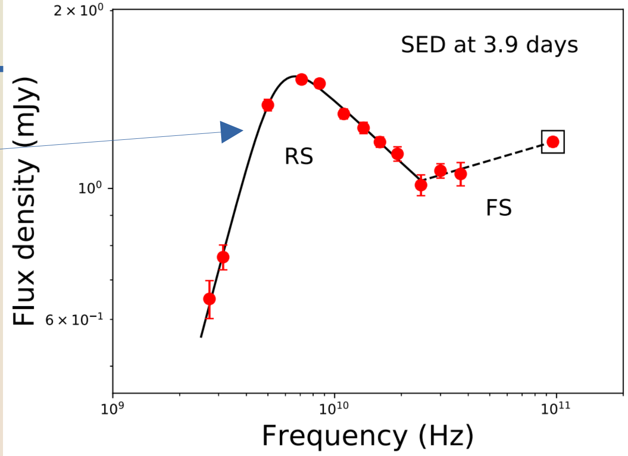


Mooley+18

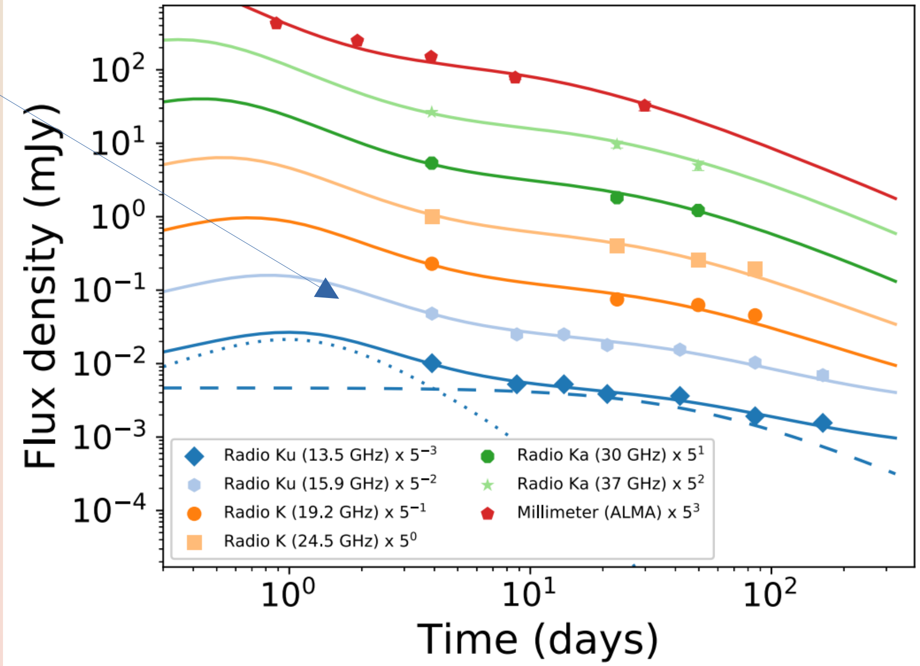


Examples: Reverse Shock

- GRB 181201A
  Distinct signature at 3.9 days, in radio (emission from mildly relativistic reverse shock)



(Laskar+19)



- GRB 160821B reverse shock & kilonova & energy injection (Lamb+20)

- GRB 190114C also showed early decaying IR component attributed to the reverse shock

State of the afterglow modeling & Goal

- Numeric codes (BoxFit…) - Computationally expensive
- Semi-analytic codes (afterglowpy…) - FS & synchrotron only

- Code requirements for multimessenger studies

  - **Speed** (suitable for inference or NN training) – semi-analytic dynamics;

  - **Jet structure** (layers with different properties);

  - **FS & RS**;

  - **Synchrotron & SSC** – numerical evolution of electron spectra;

  - **Multiple observables**: spectra & sky maps

  - **Extendable** (possibility to implement new physics & approximants)

Building an Afterglow Model

Building Afterglow Model: FS only example

Total blast wave energy (assume ejecta is <u>already</u> shocked)

$$E_{\rm tot} = \Gamma(M_0 + m)\, c^2 + \Gamma_{\rm eff}\, E'_{\rm int},$$

Energy conservation equation

$$\mathrm{d}\left[\Gamma(M_0 + m)c^2 + \Gamma_{\rm eff}\, E'_{\rm int}\right] = \mathrm{d}m\, c^2 + \Gamma_{\rm eff}\mathrm{d}E'_{\rm rad}.$$

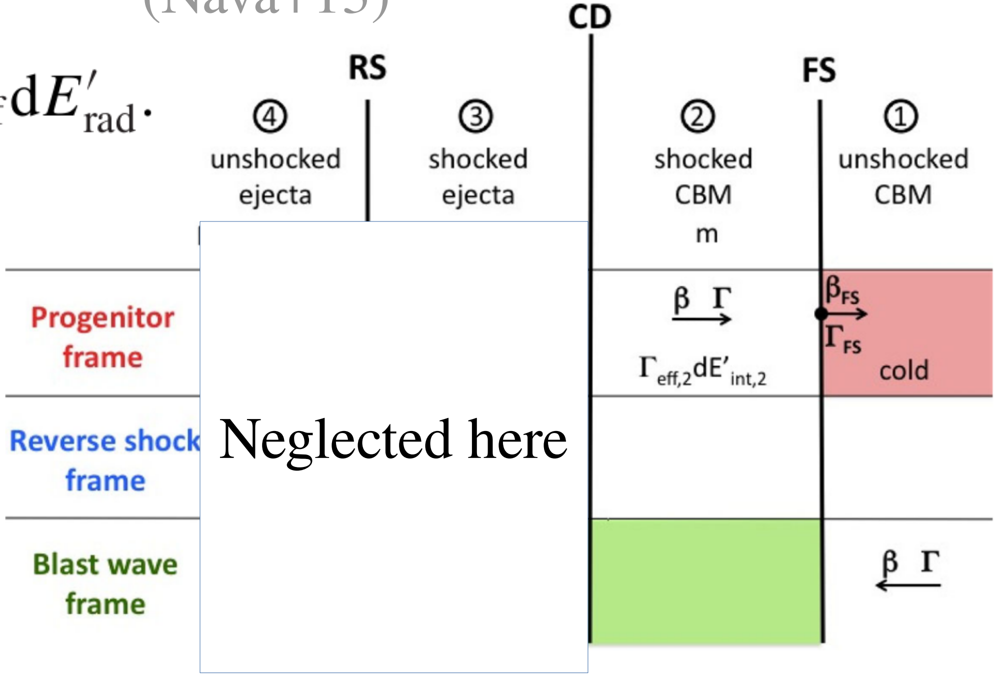
$$\mathrm{d}E'_{\rm int} = \mathrm{d}E'_{\rm sh} + \mathrm{d}E'_{\rm ad} + \mathrm{d}E'_{\rm rad}.$$

Evolution equation: after some algebra…

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}R} = -\frac{(\Gamma_{\rm eff}+1)(\Gamma-1)\, c^2 \frac{\mathrm{d}m}{\mathrm{d}R} + \Gamma_{\rm eff}\frac{\mathrm{d}E'_{\rm ad}}{\mathrm{d}R}}{(M_0 + m)\, c^2 + E'_{\rm int}\frac{\mathrm{d}\Gamma_{\rm eff}}{\mathrm{d}\Gamma}},$$

Final set of ODEs we solve numerically

Schematic picture of a Blast wave (Nava+13)

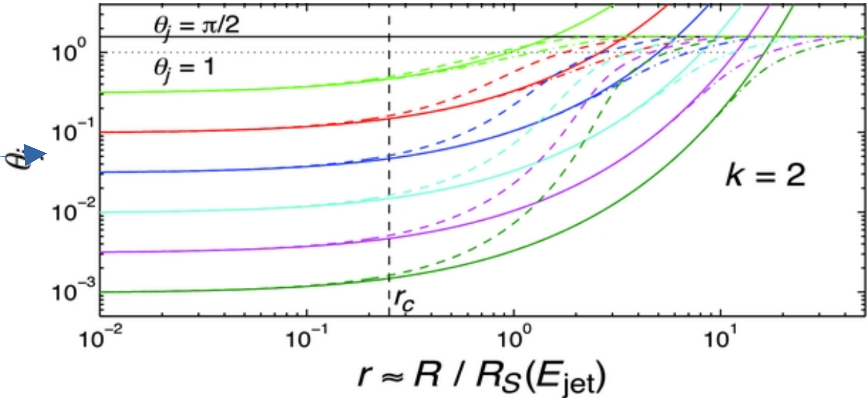


Building Afterglow Model: lateral spreading

Required 2D HD simulations. Using approximants. Many options...

Calibrated with early HD simulation (Granot+12)

$$\frac{d\omega}{dR} = R^{-1}\Gamma^{-1-a}$$



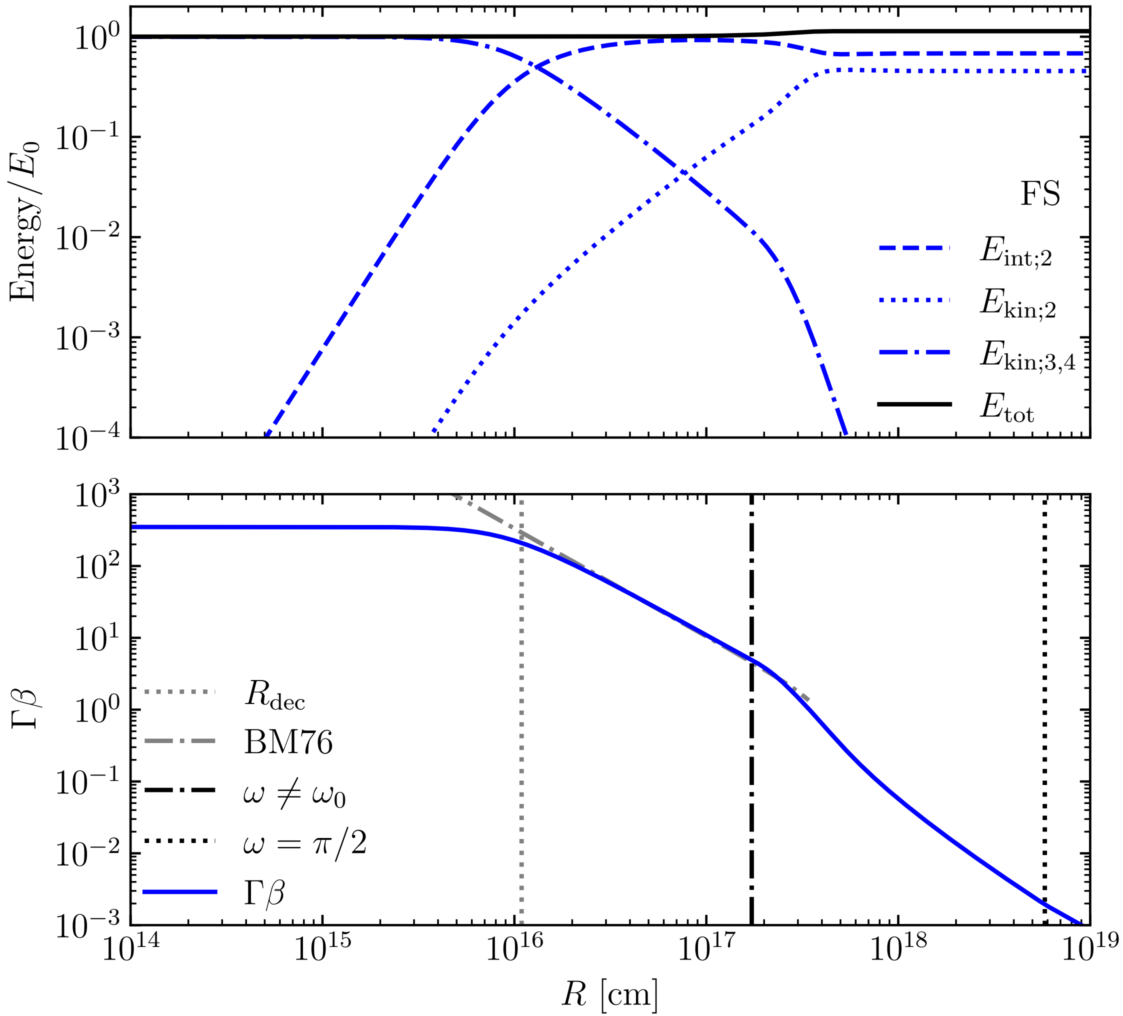
Sound-speed based formulation (Huang+00)

$$\frac{d\omega}{dR} = \frac{c_s}{R\Gamma\beta c} \qquad c_s^2 = \frac{\hat{\gamma}p'}{\rho'}\left[\frac{(\hat{\gamma}-1)\rho'}{(\hat{\gamma}-1)\rho' + \hat{\gamma}\rho'}\right]c^2 = \frac{\hat{\gamma}(\hat{\gamma}-1)(\Gamma-1)}{1+\hat{\gamma}(\Gamma-1)}c^2$$

Structured-jet aware formulation (Ryan+20)

$$\frac{d\omega}{dt} = v_\perp \frac{c}{R} \times \begin{cases} 1 & \text{core} \\ \tan(\omega_0/2)/\tan(\omega_c/2) & \text{wings} \end{cases} \qquad v_\perp = \frac{1}{2}\frac{\beta_{\rm sh}}{\Gamma}\sqrt{\frac{2(\Gamma\beta)^2+3}{4(\Gamma\beta)^2+3}}$$

Building Afterglow Model: FS only example



Energy:

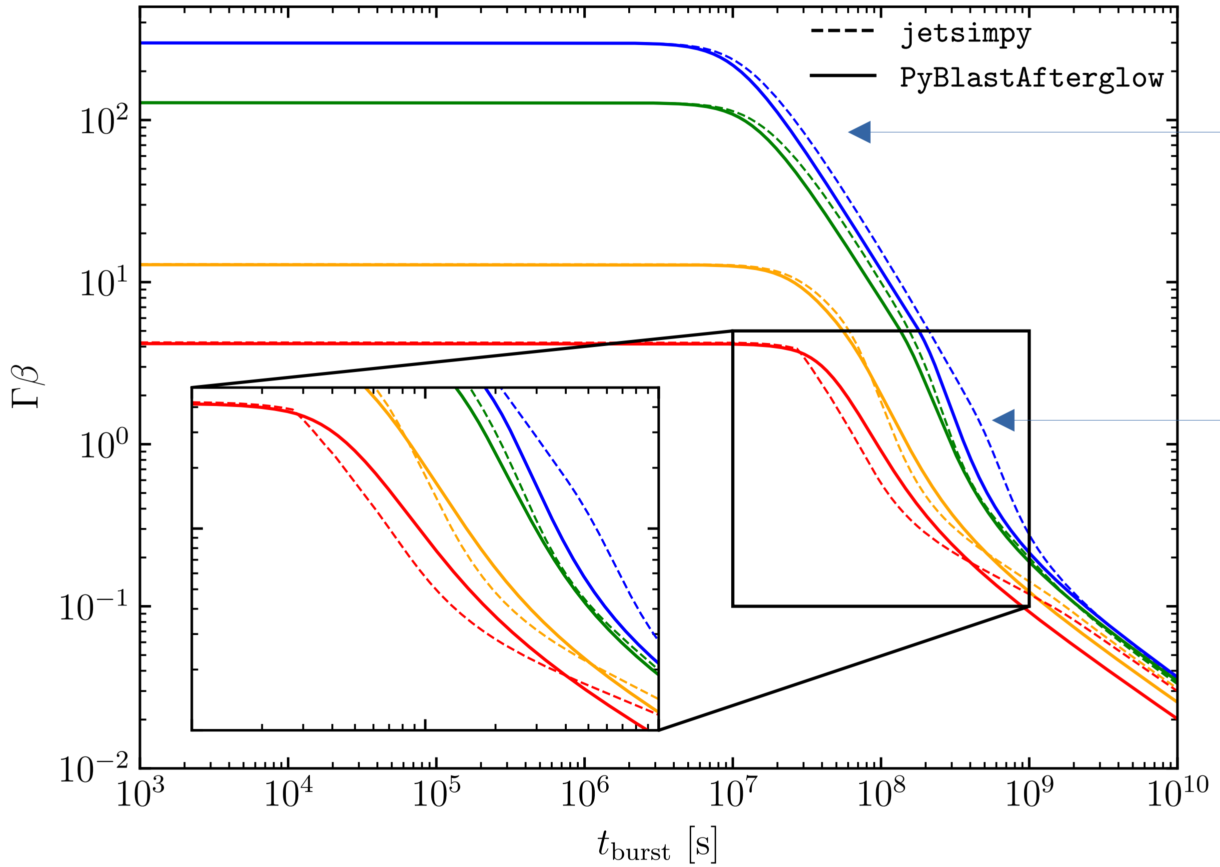
- kinetic → internal energy
- total energy is conserved c

Dynamics:

- free coasting (constant momentum)
- deceleration (BM solution)
- lateral spreading
- non-relativistic stage (ST solution)

Building Afterglow Model: Comparison

- Dynamics with lateral spreading against 2D thin-shell hydrodynamics model Structure and Spreading



calibration to BM can be added

spreading prescription
can be improved

Building Afterglow Model: FS & RS dynamics

Total blast wave energy

$$E_{\rm tot} = \Gamma_0 M_{0,4}\, c^2 + \Gamma M_{0,3}\, c^2 + \Gamma m\, c^2 + \Gamma_{\rm eff,3} E'_{\rm int,3} + \Gamma_{\rm eff,2} E'_{\rm int,2}.$$

Energy conservation equation

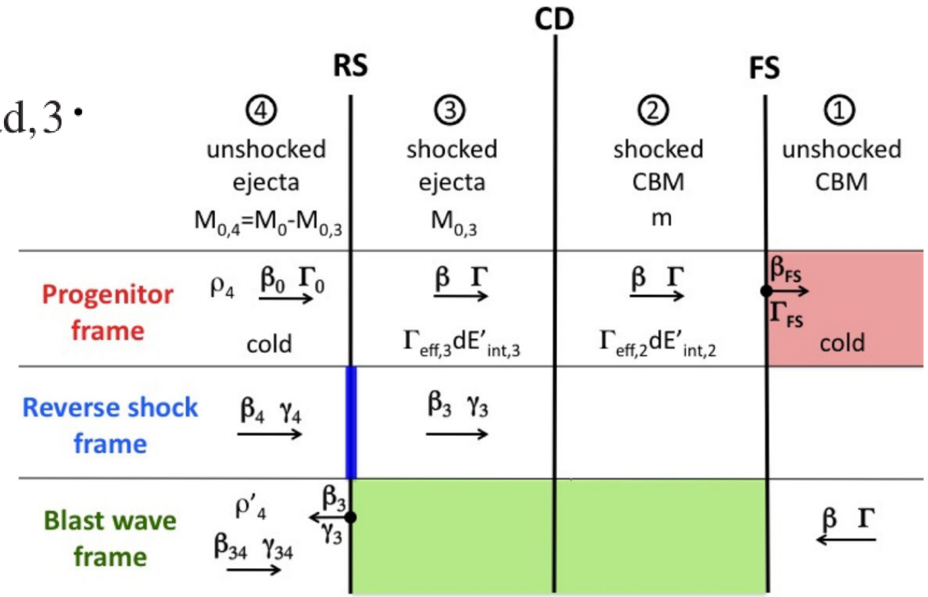
$$dE_{\rm tot} = dm\, c^2 + \Gamma_{\rm eff,2} dE'_{\rm rad,2} + \Gamma_{\rm eff,3} dE'_{\rm rad,3}.$$

$$dE'_{\rm int} = dE'_{\rm sh} + dE'_{\rm ad} + dE'_{\rm rad}.$$

Evolution equation: after some algebra...

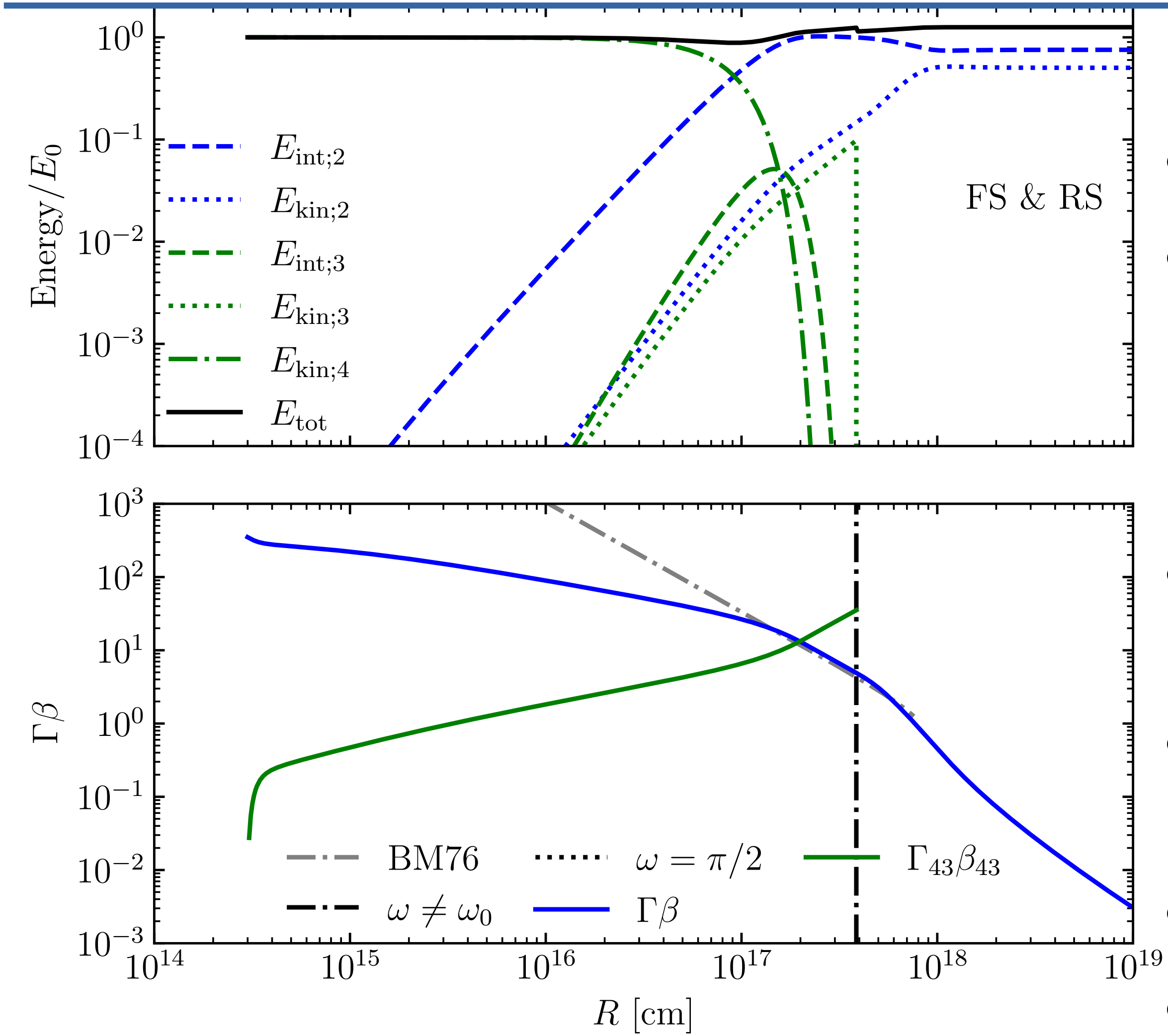
$$\frac{d\Gamma}{dR} = -\frac{(\Gamma_{\rm eff,2}+1)(\Gamma-1)\frac{dm\,c^2}{dR} + \Gamma_{\rm eff,2} dE'_{\rm ad,2}}{(M_{0,3}+m)c^2 + E'_{\rm int,2}\frac{d\Gamma_{\rm eff,2}}{d\Gamma} + E'_{\rm int,3}\frac{d\Gamma_{\rm eff,3}}{d\Gamma}}$$

$$-\frac{(\Gamma-\Gamma_0-\Gamma_{\rm eff,3}+\Gamma_{\rm eff,3}\gamma_{34})\frac{dM_{0,3}c^2}{dR} + \Gamma_{\rm eff,3} dE'_{\rm ad,3}}{(M_{0,3}+m)c^2 + E'_{\rm int,2}\frac{d\Gamma_{\rm eff,2}}{d\Gamma} + E'_{\rm int,3}\frac{d\Gamma_{\rm eff,3}}{d\Gamma}}$$



Adaptive ODE solve used

Building Afterglow Model: FS & RSexample



Energy;

- Reverse shock shocks ejecta
- Total energy ~ conserved

Dynamics

- Initial deceleration when RS is crossing ejecta
- Short standard deceleration (BM solution)
- Lateral spreading
- Non-relativistic regime

Building Afterglow: Shock Microphysics

Energy Density $e' = E'_{\rm int}/V'$

Magnetic Field $B' = \sqrt{8\pi\epsilon_{\rm b}e'}$

Free parameter

Injection spectrum

$$dN_e/d\gamma_e \propto \gamma_e^{-p} \text{ for } \gamma_e \in (\gamma_{e;\,\rm m},\, \gamma_{e;\,\rm M})$$

Spectral limits

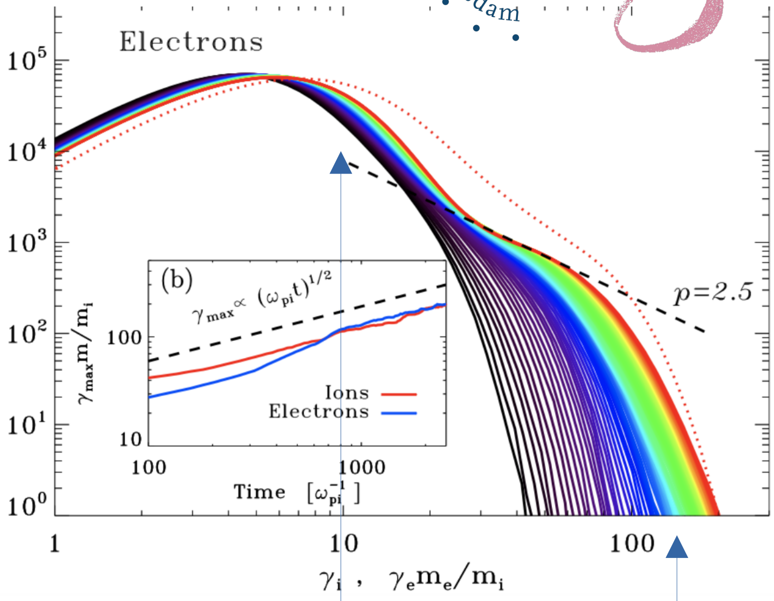
$$\gamma_{e;\,\rm M} = \sqrt{\frac{6\pi q_e}{\sigma_T B' \zeta(1+\tilde{Y})}}$$

Compton parameter

Microphysics parameter

$$\epsilon_{\rm e}\frac{e'}{\rho'}\frac{m_p}{m_e c^2} = \langle\gamma_e\rangle = \left(\frac{p-1}{p-2}\right)\left(\frac{\gamma_{e;\,\rm M}^{-p+2} - \gamma_{e;\,\rm m}^{-p+2}}{\gamma_{e;\,\rm M}^{-p+1} - \gamma_{e;\,\rm m}^{-p+1}}\right)$$

Free parameter



M

m

Solving for minimum

Building Afterglow: Electron Evolution

Electron distribution evolution

$$\frac{\partial}{\partial t'}\frac{dN_e{}'}{d\gamma_e{}'} + \frac{\partial}{\partial \gamma_e{}'}\left(\dot{\gamma}_{\rm e,tot}{}'\frac{dN_e{}'}{d\gamma_e{}'}\right) = d\dot{N}'_{\rm e,0}$$

Electron injection

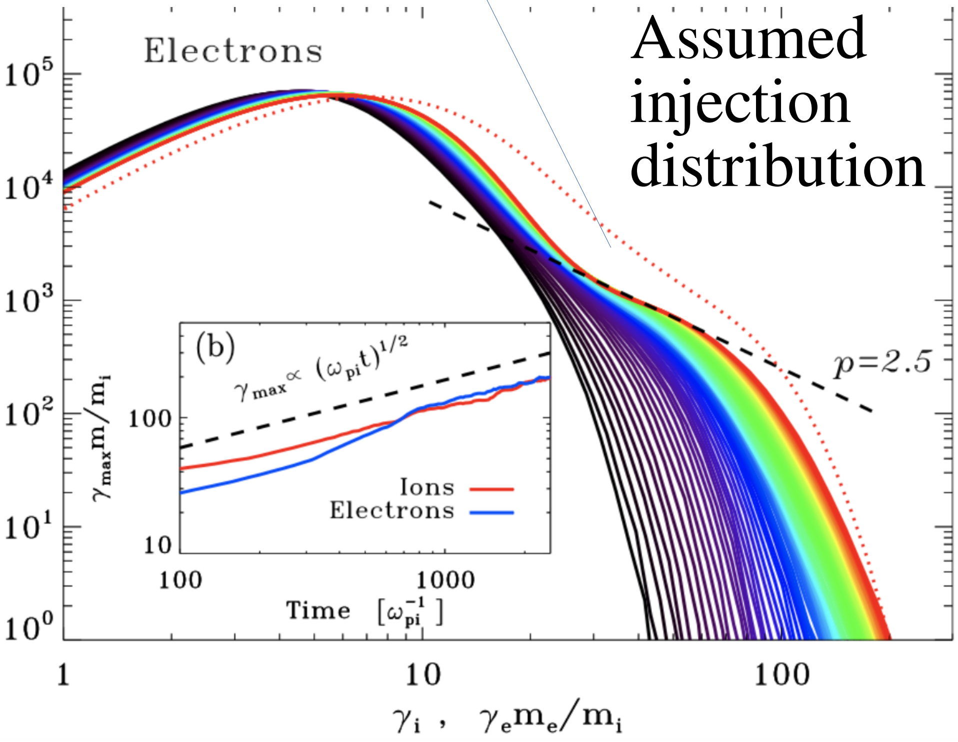
$$d\dot{N}'_{\rm e,0} = \dot{N}'_{\rm e}\frac{p-1}{\gamma'_{\rm m}}\left(\frac{\gamma_e{}'}{\gamma'_{\rm m}}\right)^{-p} d\gamma_e{}',$$

Electron energy loss

$$\dot{\gamma}_{\rm e,tot}{}' = \dot{\gamma}_{\rm e,adi}{}' + \dot{\gamma}_{\rm e,syn}{}' + \dot{\gamma}_{\rm e,IC}{}'.$$

Solved using Chang-Cooper scheme with Substeps for cooling

Assumed injection distribution



Building Afterglow: Cooling terms

Cooling terms

$$\dot{\gamma}_{\rm syn} = -\frac{4}{3}\left(\frac{\sigma_T B'^2}{8\pi m_e c}\right)\gamma_e^2 \,.$$

Synchrotron cooling

$$\dot{\gamma}_{\rm adi} = \frac{d\ln(V')}{dR}\left(\frac{\gamma_e^2-1}{3\gamma_e}\right),$$

Adiabatic cooling

SSC cooling

$$\dot{\gamma}_{\rm ssc} = \frac{3}{4}\frac{h\sigma_T}{m_e c\gamma_e^2}\int_{\tilde{\nu}'_0}^{\tilde{\nu}'_1}\frac{n_{\tilde{\nu}'}}{\tilde{\nu}'}\left[\int_{\nu'_0}^{\nu'_1} h\nu' K(\tilde{\nu}',\gamma_e,\nu')d\nu'\right]d\tilde{\nu}'$$

SSC kernel

$$K(\tilde{\varepsilon}',\gamma_e,\varepsilon) = 2q\ln(q) + (1+2q)(1-q) + 0.5(1-q)\frac{(4\gamma_e\tilde{\varepsilon}q)^2}{(1+4\gamma_e\tilde{\varepsilon})}$$

Building Afterglow: Synchrotron Emission

Convolving the emission of a single electron gyrating around magnetic field lines with electron distribution function

Emission power by a single electron

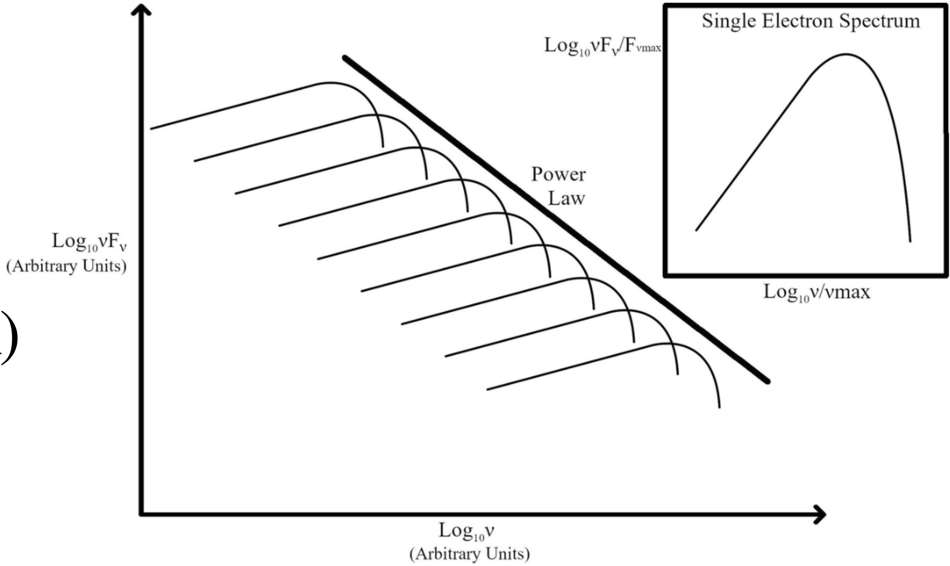
$$j'_{\rm syn}(\nu', \gamma_e) = \frac{3q_e^3 B'}{m_e c^2} 2y^2 \Big(K_{4/3}(y) K_{1/3}(y) - \frac{3}{5} y \big(K_{4/3}^2(y) - K_{1/3}^2(y) \big) \Big) ,$$

$$y = \nu' / \nu'_{\rm crit}$$

$$j'_{\rm syn}(\nu') = \int_{\gamma_{e;0}}^{\gamma_{e;1}} N_e(\gamma_e) j'_{\rm syn}(\nu', \gamma_e) d\gamma_e ,$$

Synchrotron self-absorption (SSA)

$$\alpha'_{\rm syn}(\nu') = -\frac{1}{8\pi m_e \nu'^2} \times \int_{\gamma_{e;0}}^{\gamma_{e;1}} j'_{\rm syn}(\nu', \gamma_e) \gamma_e^2 \frac{\partial}{\partial \gamma_e} \Big[\frac{N_e(\gamma_e)}{\gamma_e^2} \Big] d\gamma_e$$



Building Afterglow: SSC emission

Synchrotron self-Compton spectrum

$$j'_{\mathrm{ssc}}(\nu', \gamma_e) = \frac{3}{4} h \sigma_T c \frac{\nu'}{\gamma_e^2} \int_{\nu'_0}^{\nu'_1} \frac{n_{\tilde{\nu}'}}{\tilde{\nu}'} F(\tilde{\varepsilon}', \gamma_e, \varepsilon') d\tilde{\nu}' \,,$$

Seed photon spectrum

$$n_{\nu'} = n_{\nu';\,\mathrm{syn}} + n_{\nu';\,\mathrm{ssc}} = \left(\frac{j'_{\mathrm{syn}}(\nu')}{h\nu'} + \frac{j'_{\mathrm{ssc}}(\nu')}{h\nu'} \right) \frac{\Delta t'}{V'} \,,$$

Comoving escape time

$$\Delta t' = \Delta R'/c = \Delta R \Gamma_{\mathrm{sh}}/c$$

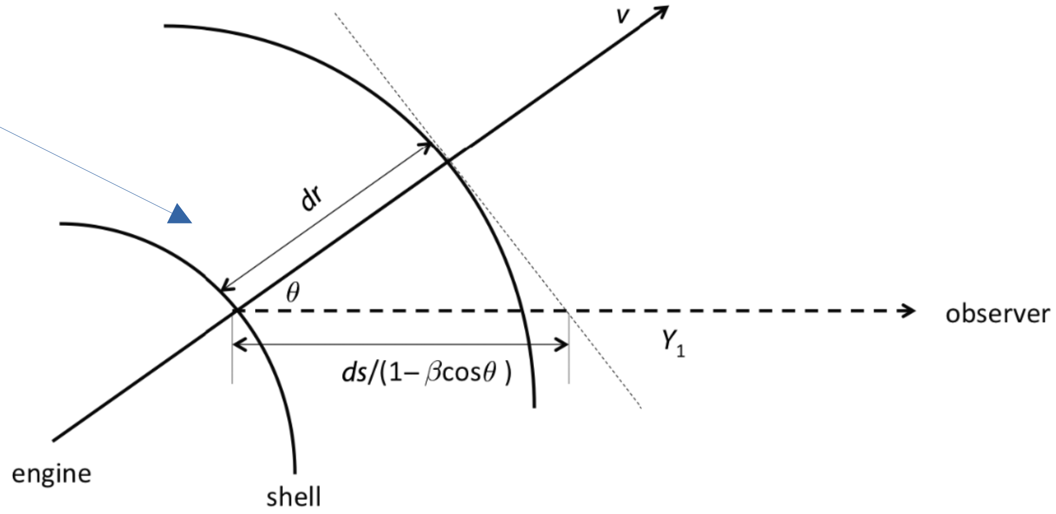
Building Afterglow: Intensity and Flux Density

From comoving emissivities to observed radiation flux density

Radiation intensity (observer frame)

$$I_\nu = \frac{j_\nu}{\alpha_\nu}(1 - e^{-\tau_\nu}) \quad \tau_\nu \approx -\alpha_\nu \Delta R/\mu'$$

Thickness of the emitting region depends on the observer



Equal-arrival time surface integration

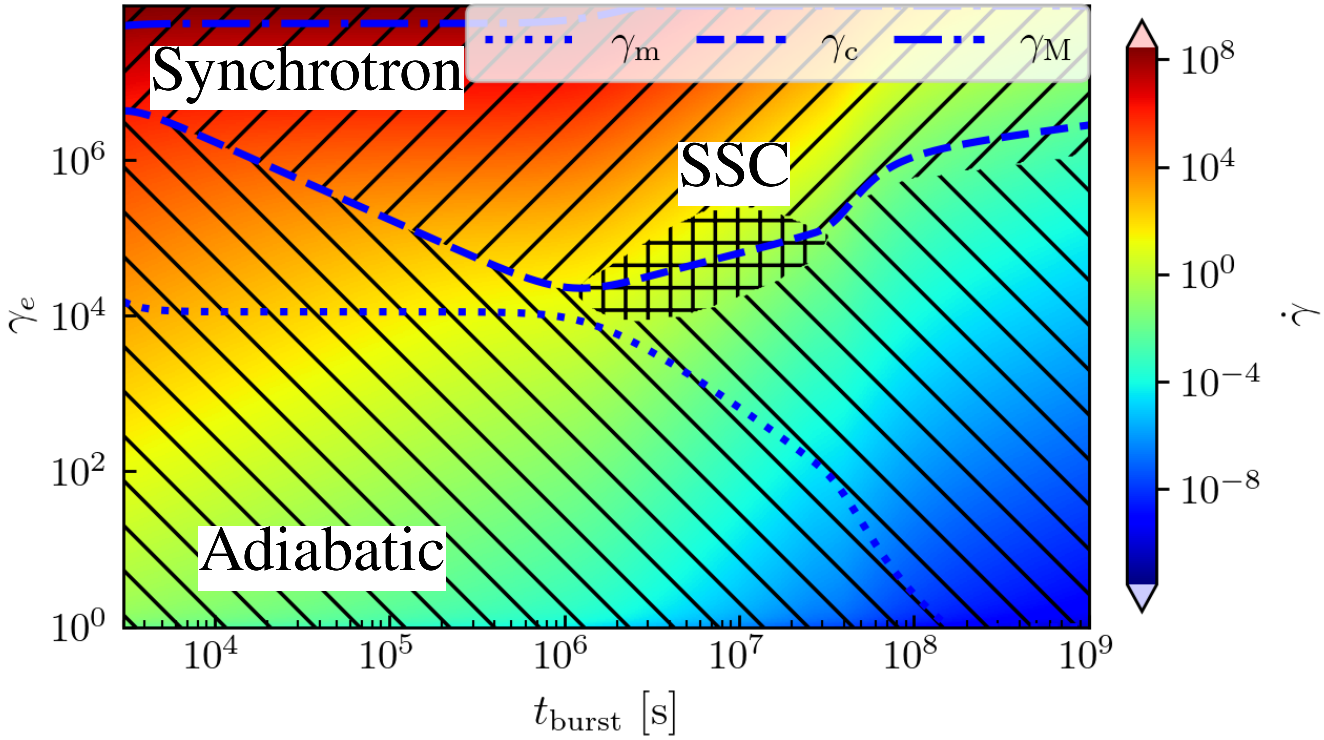
$$F_\nu = \frac{1+z}{2\pi d_L^2} \sum_i^{\text{layers}} \int_{\theta_{i;l}}^{\theta_{i;h}} \int_{\phi_0=0}^{\phi_1=\pi/2} I_{i,\nu}(\theta,\phi)\, d\theta d\phi$$

Preliminary results: Electron Cooling

Cooling of an electron population in shock downstream

All three cooling mechanisms are important

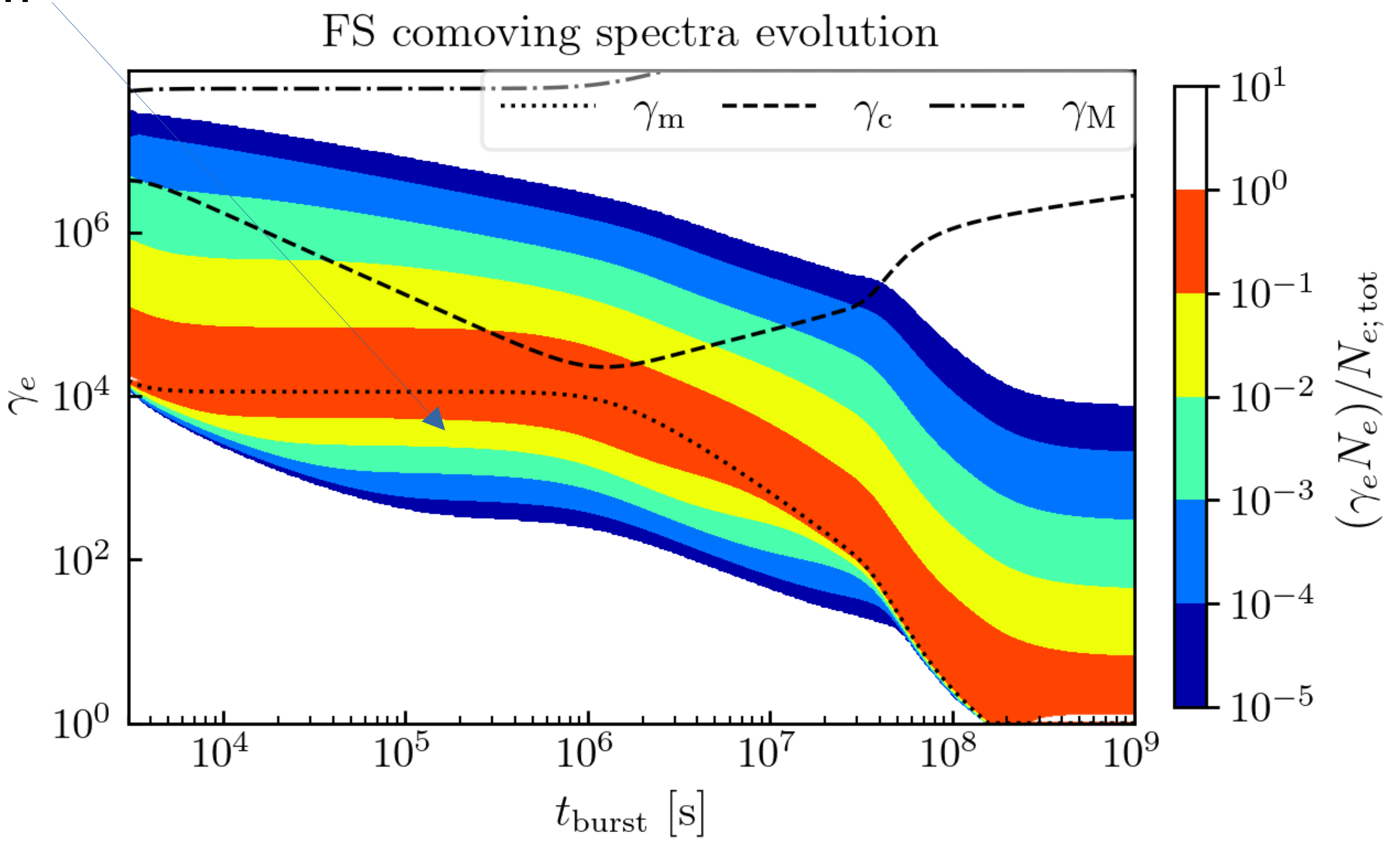
$$\gamma_{e;\,\mathrm{c}} = \frac{6\pi m_e c}{\sigma_T t' B'^2 (1+\tilde{Y})} \,.$$



Preliminary results: Electron Spectrum

Comoving electron spectrum evolution

Cooled part of the spectrum

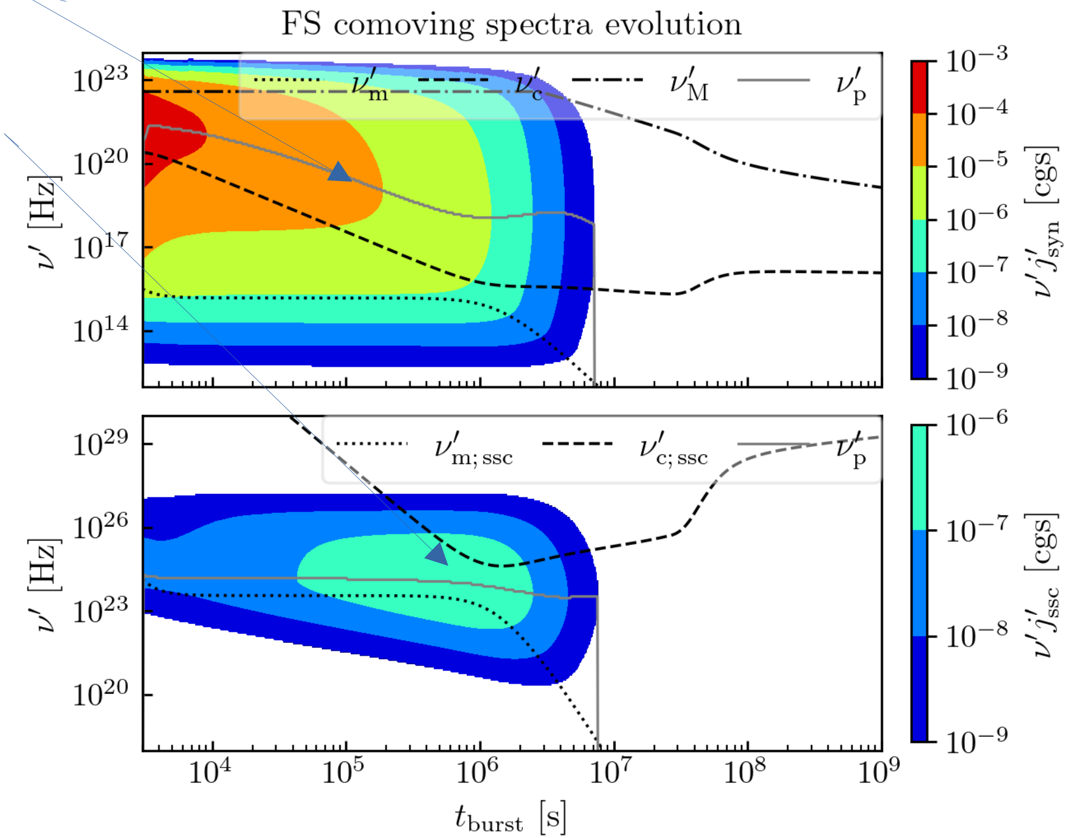


Preliminary results: Radiation Spectra

Comoving synchrotron and SSC spectra

Numerical spectrum peak > analytical

SSC reaches peak at BW deceleration

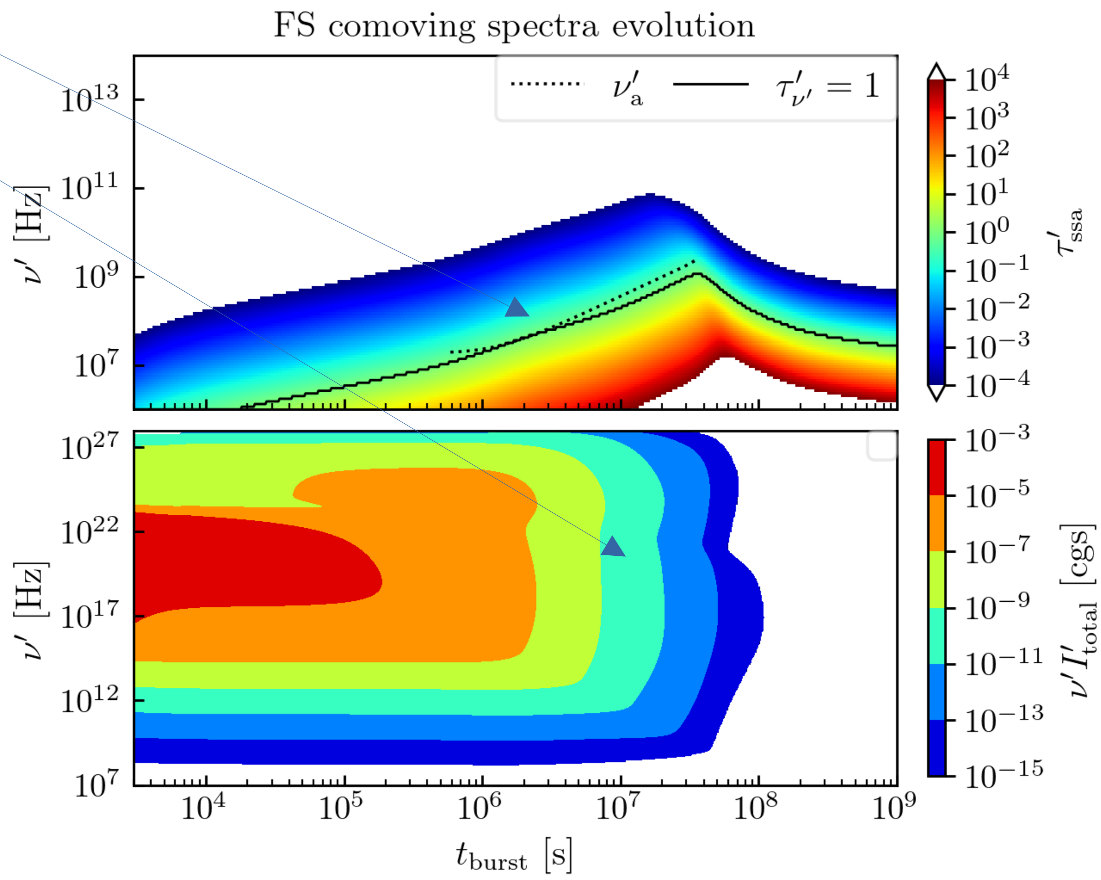


Preliminary results: Radiation Spectra

Comoving synchrotron and SSC spectra

Spectrum is primarily optically thin

Intensity spectrum is double-peaked

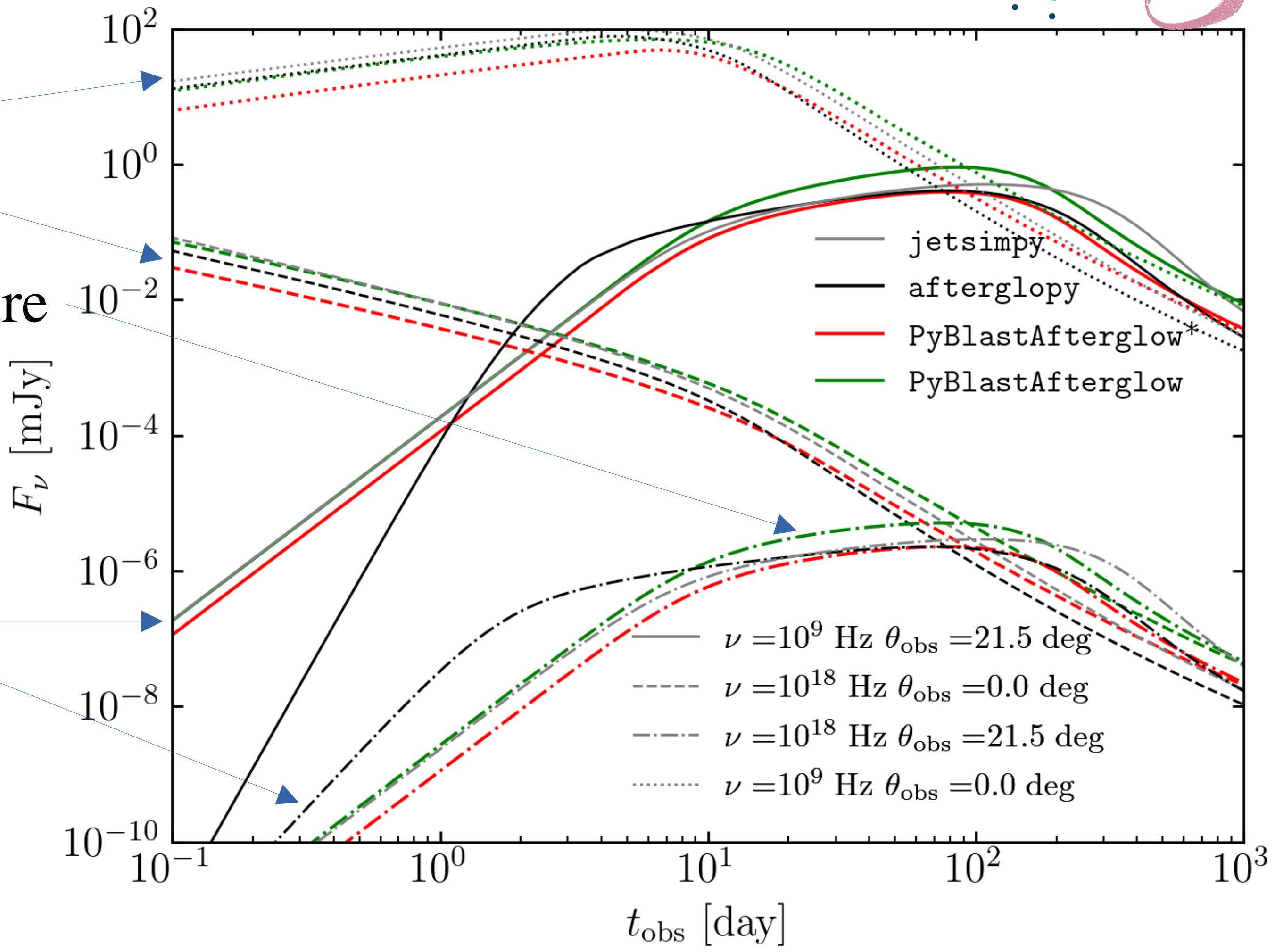


Comparison with afterglowpyl; jetsimpy

On-axis afterglow

Shallow slope due to structure

Off-axis afterglow



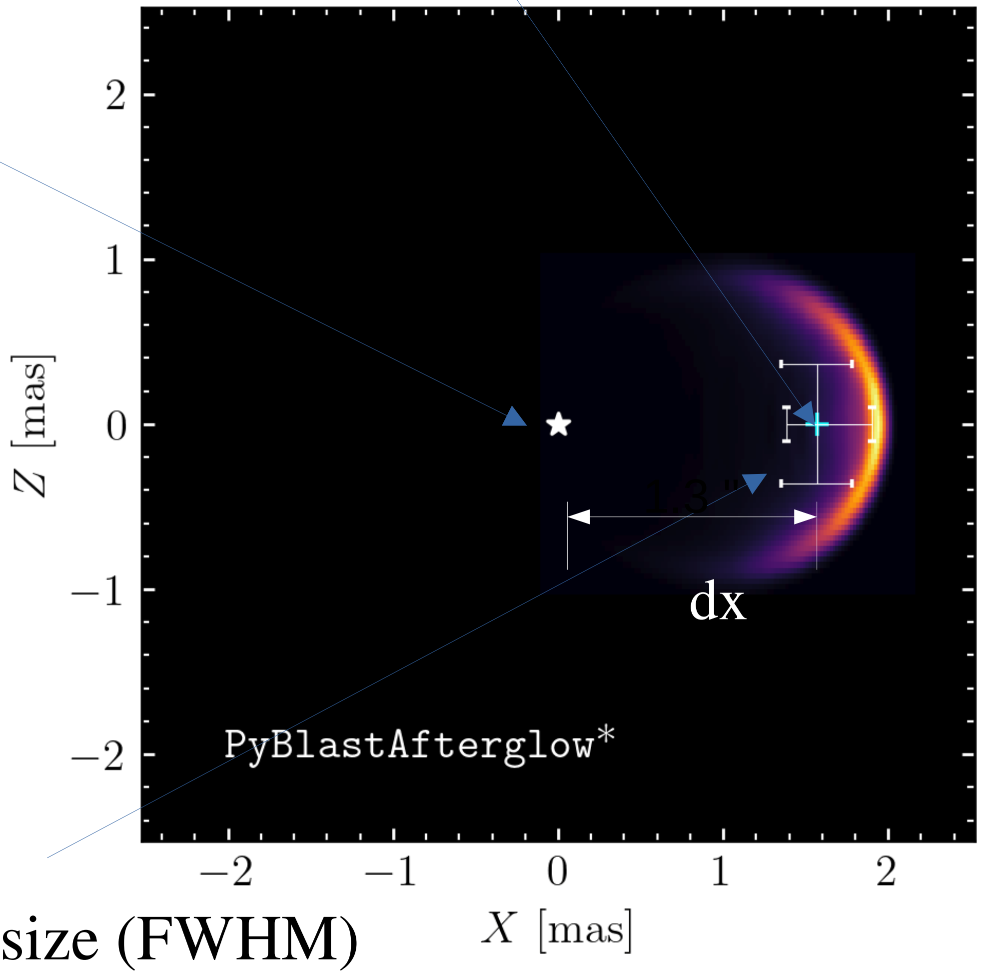
Comparison:
Gaussian Jet
Forward Shock only
Synchrotron only

Example: sky map

Sky map – intensity distribution projected onto a plane ortogonal to the line of sight.

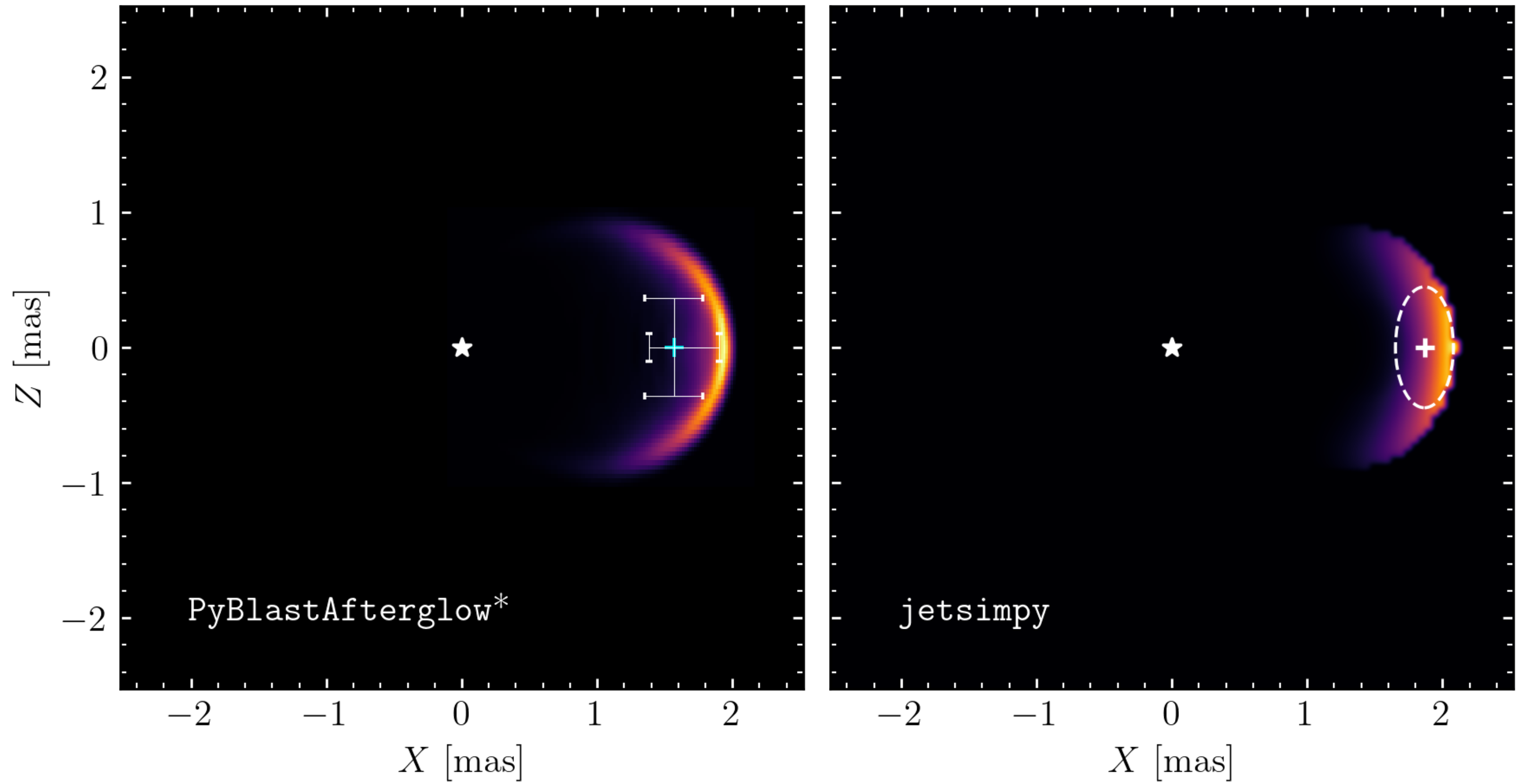
Properties:
- Centroid position
- Size

Shows:
- geometry of the ejecta
- jet inclination angle



Comparison with jetsimpy

Comparison:
Gaussian Jet
Forward Shock on
Synchrotron only



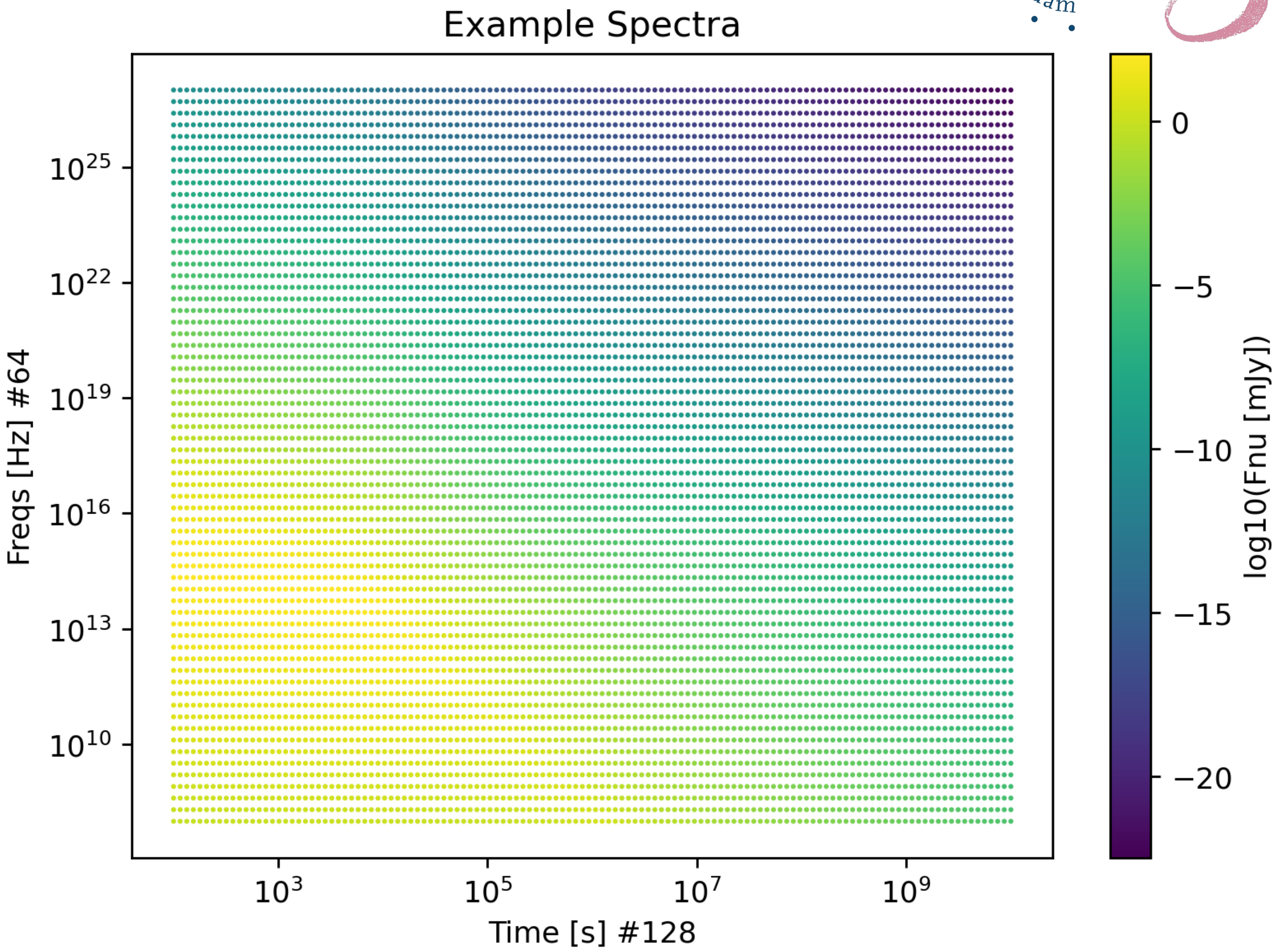
Building a Surrogate Model: Dataset

Building train dataset (with afterglowpy)

- Set of time-dependent Spectra (images)

~20 Gb for ~6400 spectra

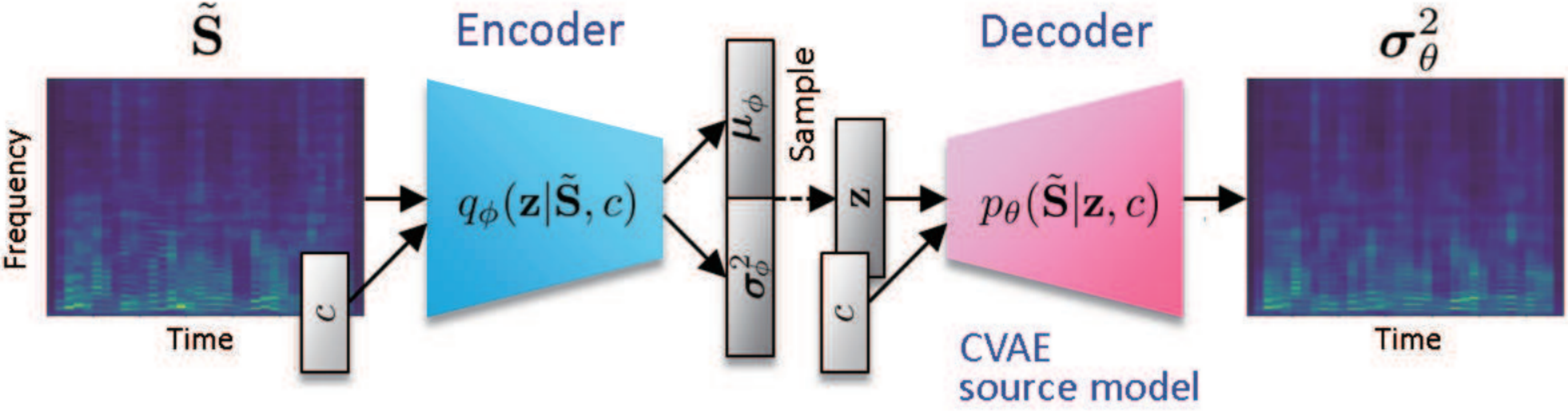
Parameters:
"n_ism", "theta_obs", "Eiso_c", "Gamma0c", "theta_w", "p_fs", "eps_e_fs", "eps_b_fs"



Building a Surrogate Model: Architecture

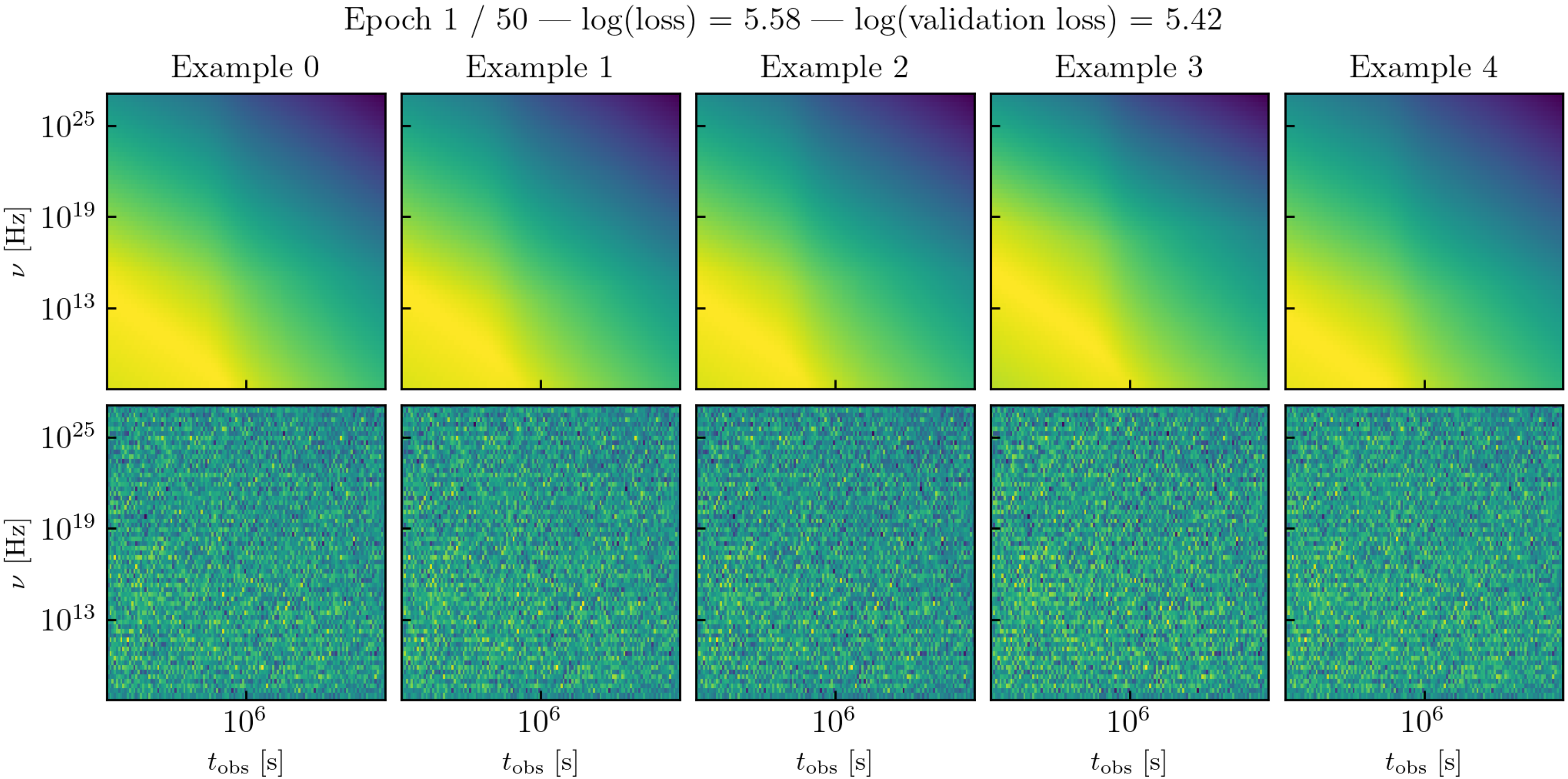
Neural networds; generative model;
conditional variational autoencoder (cVAE)

Kameoka+18



Building a Surrogate Model: Training

Training model on a 1000 spectra : Starting Epoch



Building a Surrogate Model: Training

Training model on a 1000 spectra : Last Epoch