



GRB Afterglow Modeling

Vsevolod Nedora

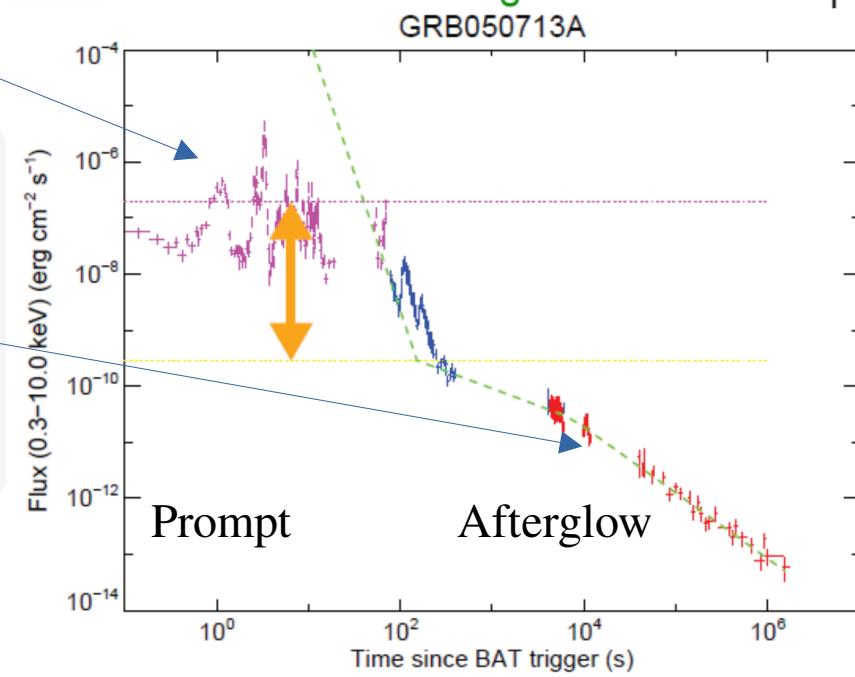
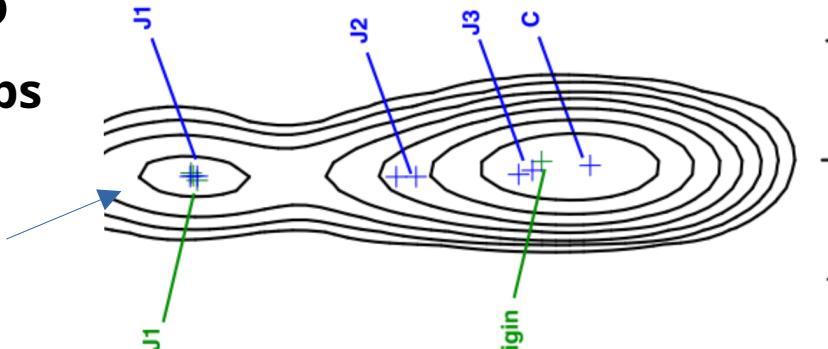
Bremen, Group Retreat
30 September 2024



Observables

Light curves, spectra, and sky maps

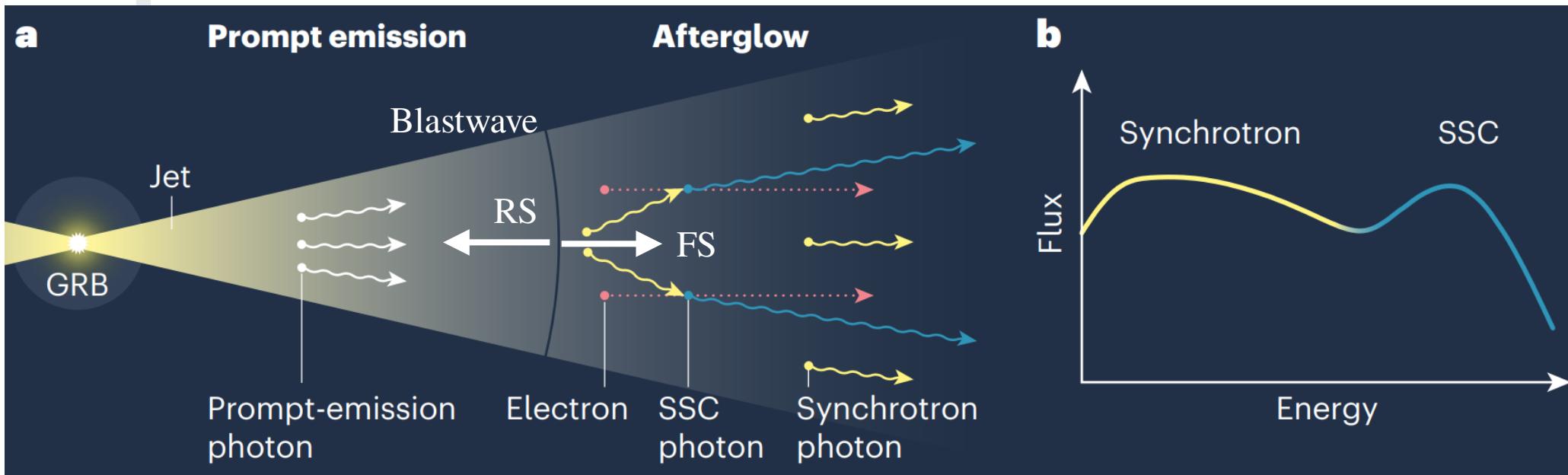
- **Sky map** – intensity distribution projected onto a plane orthogonal to the line of sight (observed usually in radio band)
- **Prompt emission** (oscillatory, early) produced by **internal shocks** or magnetic reconnection inside the jet; synchrotron, SSC, thermal emission
- **Afterglow** (smooth, late) produced by **external shocks** (forward/reverse); synchrotron emission (based on spectra)



Mechanism

Synchrotron radiation from shocks in jet head

- Jet head has **Forward** and **Reverse** shocks (**collisionless shocks**)
- Magnetic field amplification and particle acceleration at shocks
- Synchrotron and inverse Compton emission from shock downstream
- **Afterglow depends on shock microphysics and jet dynamics**

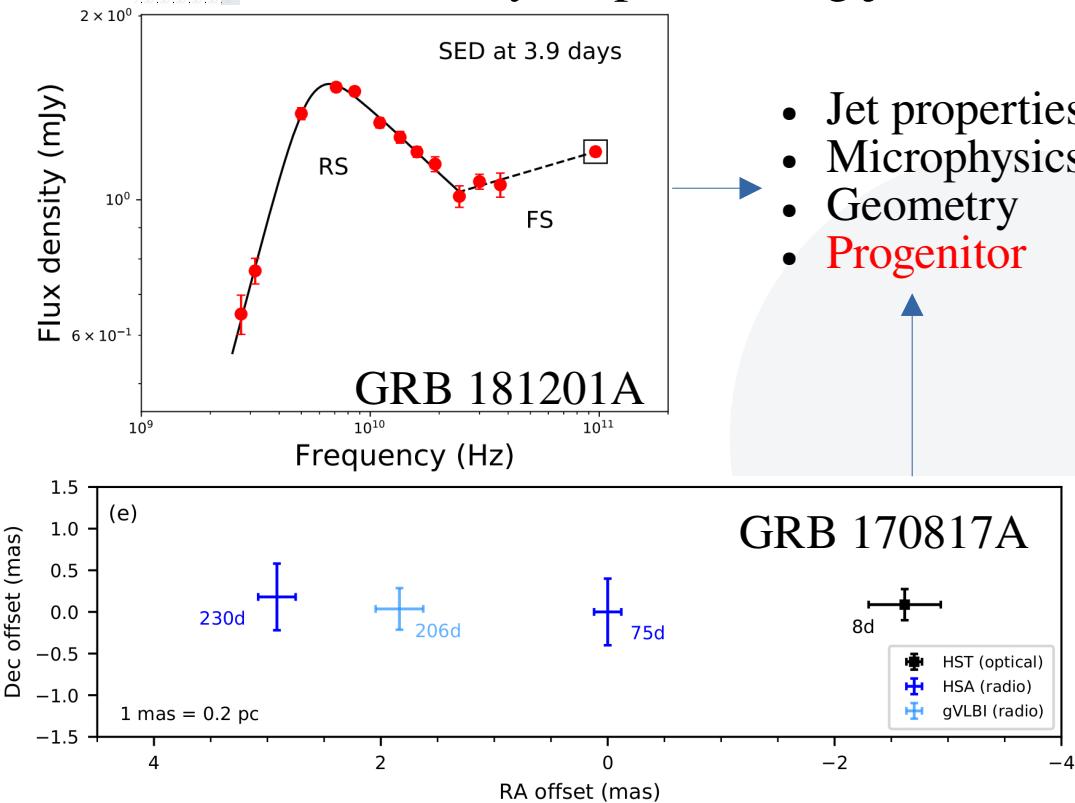




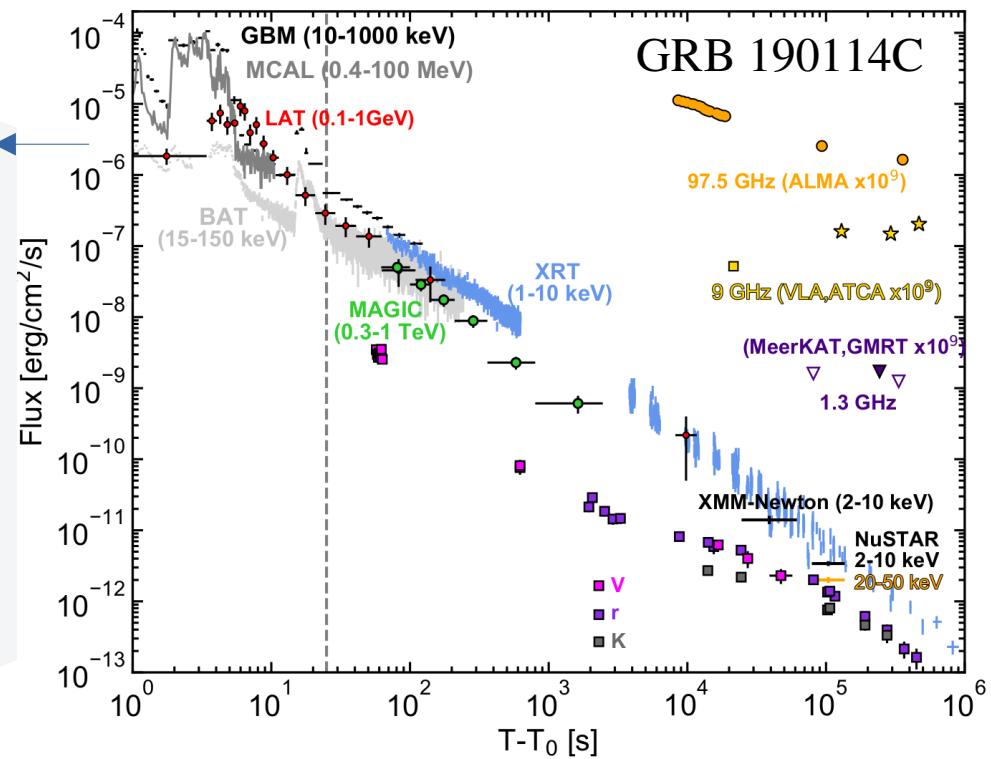
Afterglow examples

FS & RS emission; VHE emission; sky map properties

- Synchrotron, power-law spectrum from forward (reverse) shocks
- GeV emission from inverse Compton scattering
- Radio sky maps showing jetted outflow moving off-axis



- Jet properties
- Microphysics
- Geometry
- Progenitor



Model Dynamics

Energy conservation; thin-shell approximation; semi-analytic

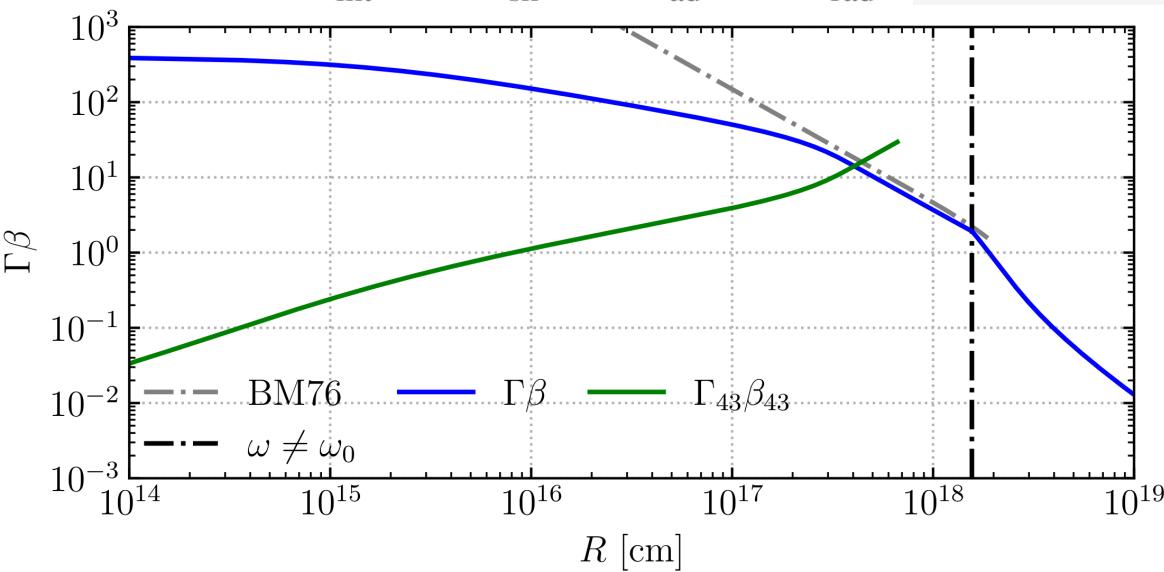
Total blast wave energy

$$E_{\text{tot}} = \Gamma_0 M_{0,4} c^2 + \Gamma M_{0,3} c^2 + \Gamma m c^2 + \Gamma_{\text{eff},3} E'_{\text{int},3} + \Gamma_{\text{eff},2} E'_{\text{int},2}.$$

Energy conservation equation

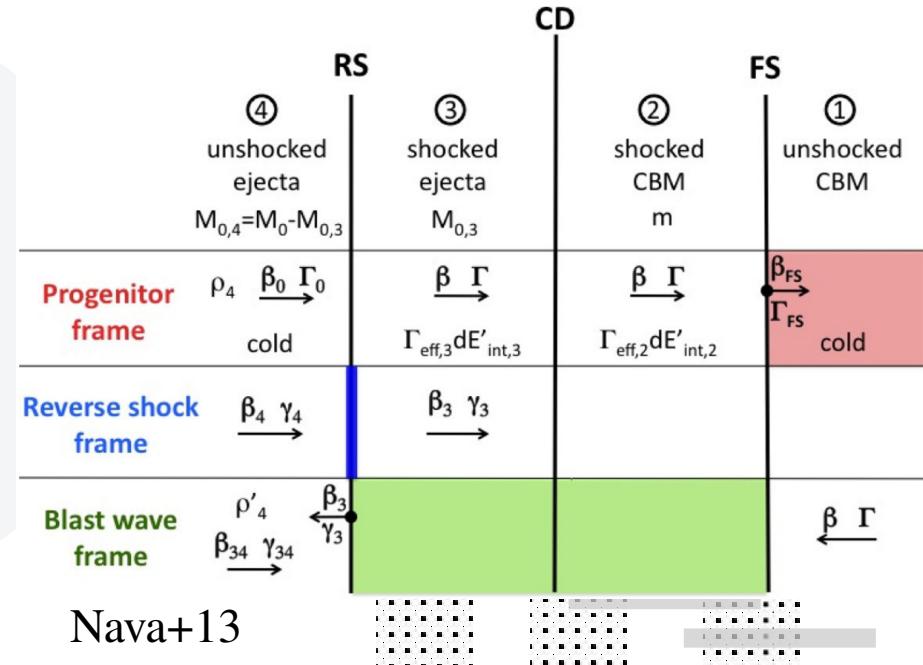
$$dE_{\text{tot}} = dm c^2 + \Gamma_{\text{eff},2} dE'_{\text{rad},2} + \Gamma_{\text{eff},3} dE'_{\text{rad},3}.$$

$$dE'_{\text{int}} = dE'_{\text{sh}} + dE'_{\text{ad}} + dE'_{\text{rad}}.$$



Numerics:

Adaptive step-size
Explicit ODE solver
DOP8(53)





Model Micropysics

Collisionless shocks; power-law electron distribution; cooling

Energy Density $e' = E'_{\text{int}}/V'$

Magnetic Field $B' = \sqrt{8\pi\epsilon_b e'}$

Injection spectrum

$$dN_e/d\gamma_e \propto \gamma_e^{-p} \text{ for } \gamma_e \in (\gamma_{e; m}, \gamma_{e; M})$$

Electron distribution evolution

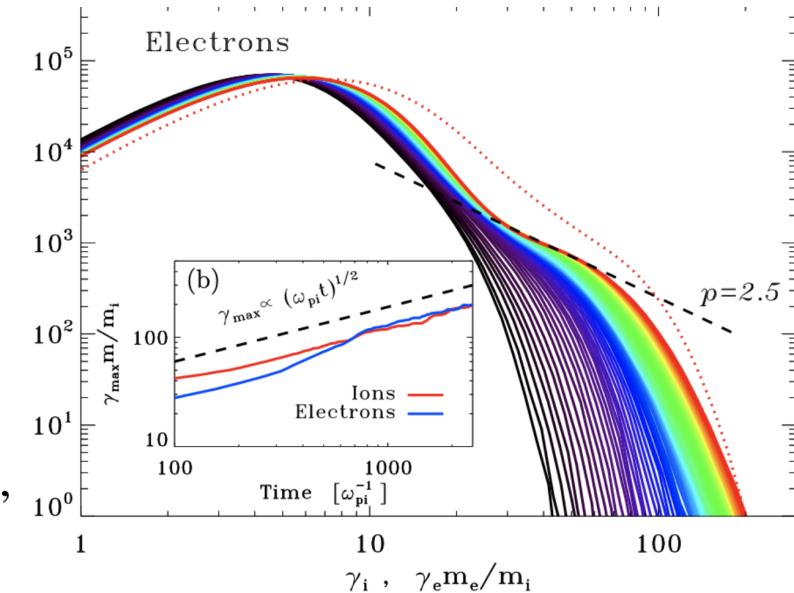
$$\frac{\partial}{\partial t'} \frac{dN_e'}{d\gamma_e'} + \frac{\partial}{\partial \gamma_e'} \left(\dot{\gamma}_{e,\text{tot}}' \frac{dN_e'}{d\gamma_e'} \right) = d\dot{N}_{e,0}' = \dot{N}_e' \frac{p-1}{\gamma_m'} \left(\frac{\gamma_e'}{\gamma_m'} \right)^{-p} d\gamma_e',$$

$$\dot{\gamma}_{e,\text{tot}}' = \dot{\gamma}_{e,\text{adi}}' + \dot{\gamma}_{e,\text{syn}}' + \dot{\gamma}_{e,\text{IC}}'.$$

Numerics:

Fully-implicit PDE solver; unconditionally stable, particle number preserving
Chang-Cooper scheme

Sub-stepping to respect CFL condition instead of directly following CFL
timesteps

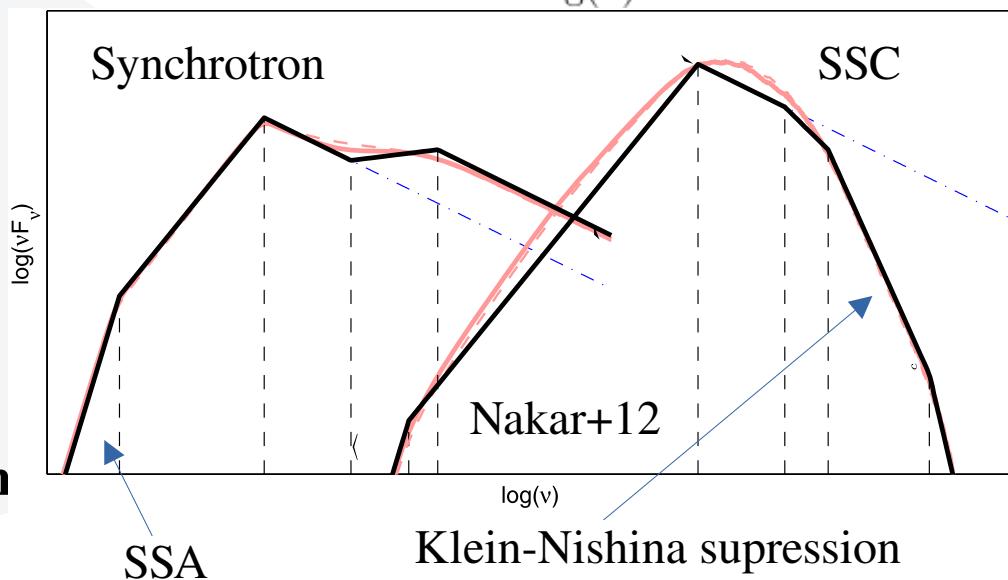
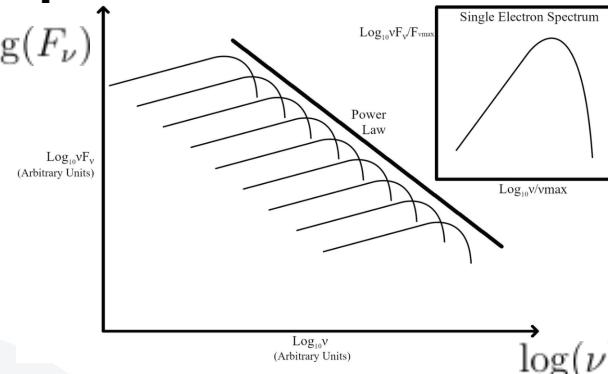




Modeling Radiation

Synchrotron emission & absorption; SSC emission; PP & EBL losses

- **Synchrotron emission:** convolving single electron emissivity with electron distribution
- **Synchrotron self-absorption (SSA):** convolving derivative of the distribution with emissivity
- **Synchrotron self-Compton (SSC):** convolving scattering kernel with photon density spectrum. Klein-Nishina corrections reduce VHE
- **Pair-production and external background light (EBL) absorption** further reduce VHE





GRB afterglow modeling

GRB ejecta lateral structure; relativistic effects; EATS integration

Radiation intensity (observer frame)

$$I_\nu = \frac{j_\nu}{\alpha_\nu} (1 - e^{-\tau_\nu}) \quad \tau_\nu \approx -\alpha_\nu \Delta R / \mu'$$

Thickness of the emitting region depends on the observer

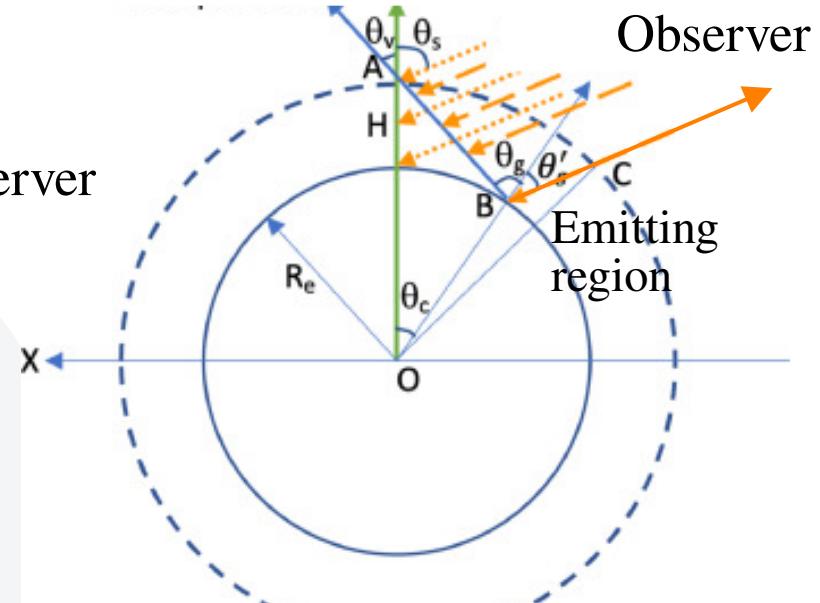
$$\Delta R = \Delta R' / (1 - \mu \beta_{\text{sh}})$$

Equal-arrival time surface integration

$$F_\nu = \frac{1+z}{2\pi d_L^2} \sum_i^{\text{layers}} \int_{\theta_{i;l}}^{\theta_{i;h}} \int_{\phi_0=0}^{\phi_1=\pi/2} I_{i,\nu}(\theta, \phi) d\theta d\phi$$

Numerics:

Adaptive quadrature for flux density evaluation and iterative algorithm for sky map resolving integration

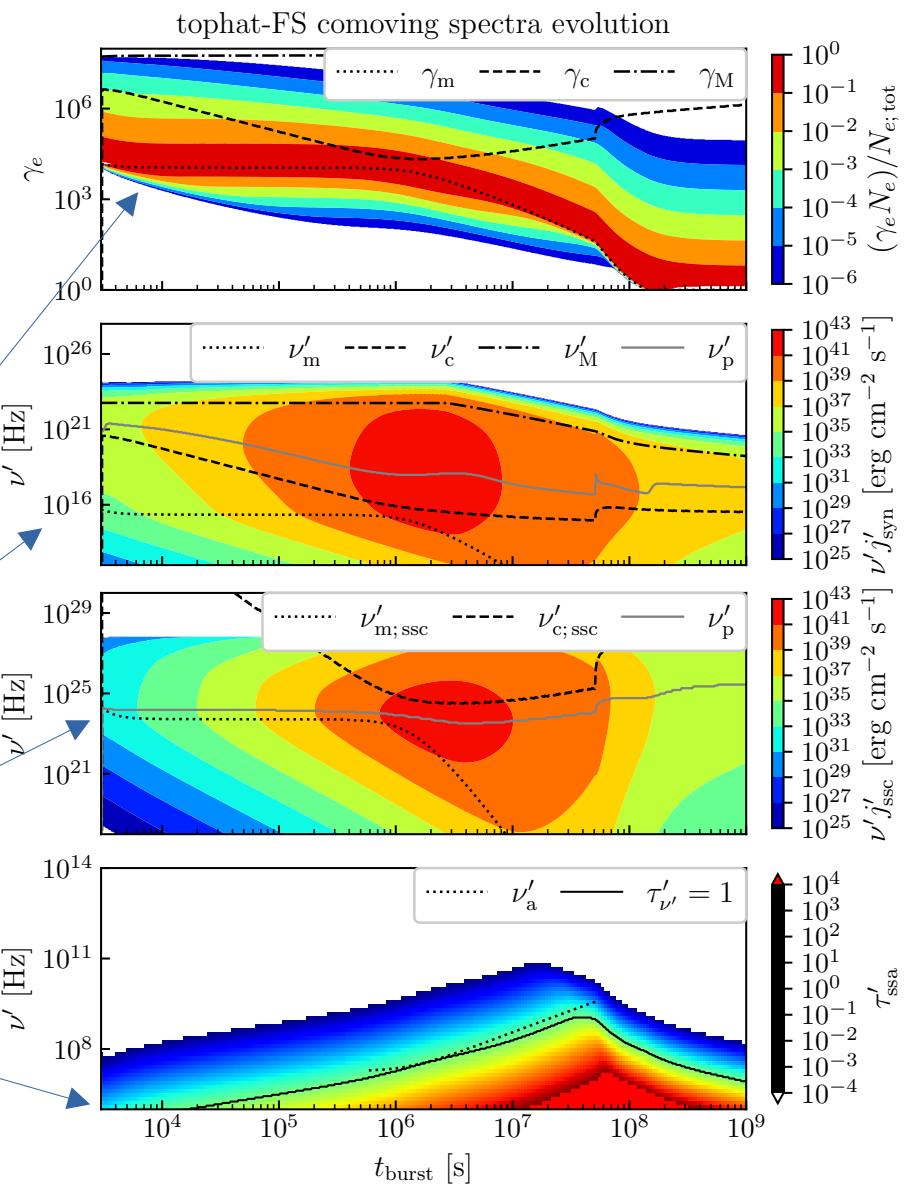




Afterglow Modeling results

Comoving spectra evolution

- **Electron spectrum:** Cooling of injected electrons; evolution of injection spectrum
- **Synchrotron emission:** peaks at the onset of deceleration
- **SSC spectrum** follows the synchrotron one – echo at higher frequencies
- **SSA** affects primarily lower frequencies; and most prominent at late deceleration stages.



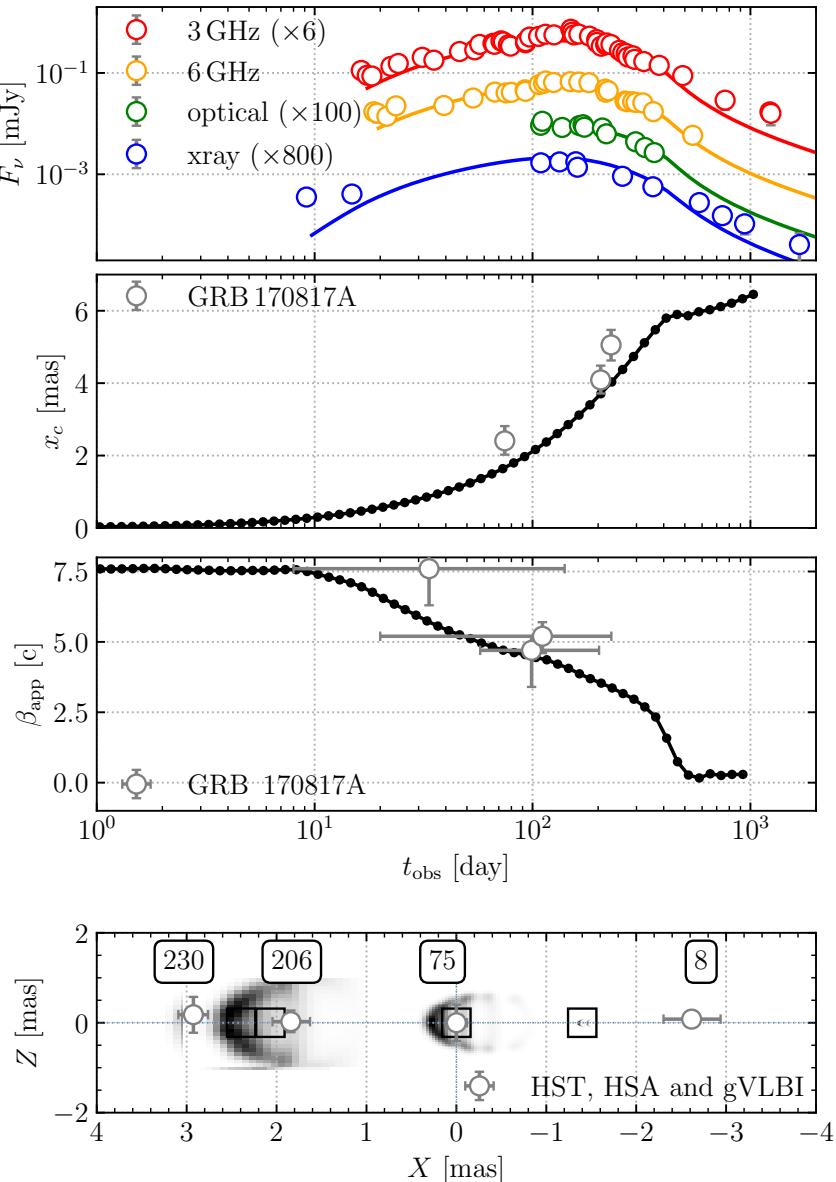


GRB 170817A

Modeling Example

Comparison with observables

- Jet structure: Gaussian jet with 20 angular layers (blastwaves)
- Synchrotron only; no VHE detected
- Observed off-axis (~ 20 deg.) shows slow rising early light curve
- Sky map related quantities: flux centroid motion constraint geometry and energetics
- Numerically expensive runs prohibit Bayesian parameter inference

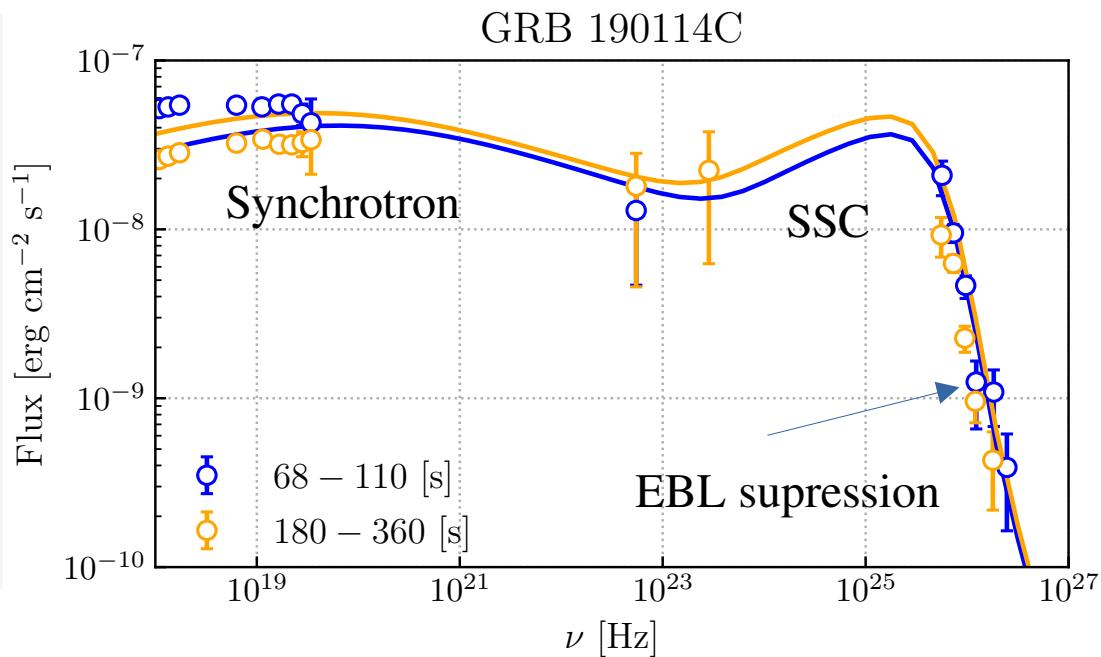


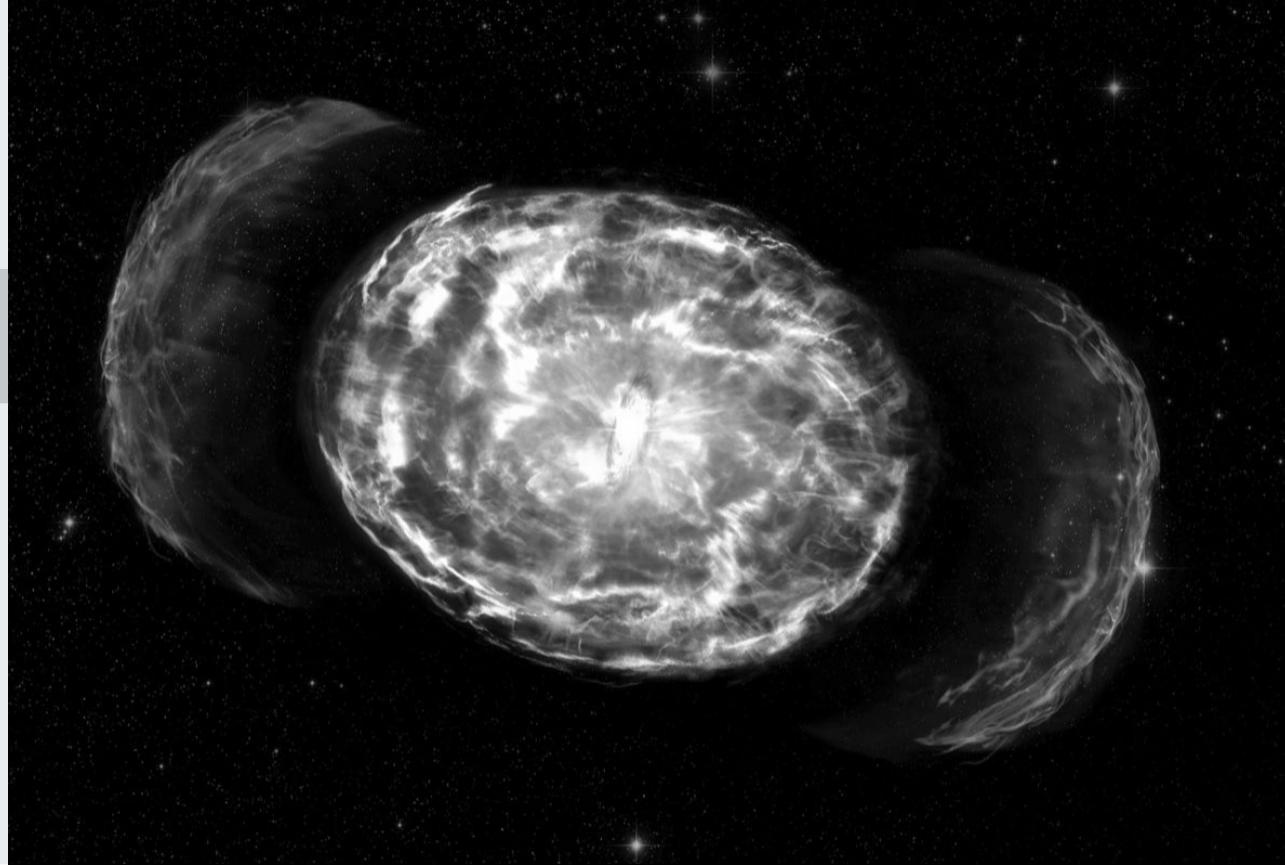


VHE afterglow modeling example

GRB 190114C VHE spectra from SSC mechanism

- Grid-search analysis (20k runs)
- Top-hat jet observed on-axis
- VHE is produced by SSC but is shaped by EBL absorption
- Model parameters of best-fitting model are close to the literature values





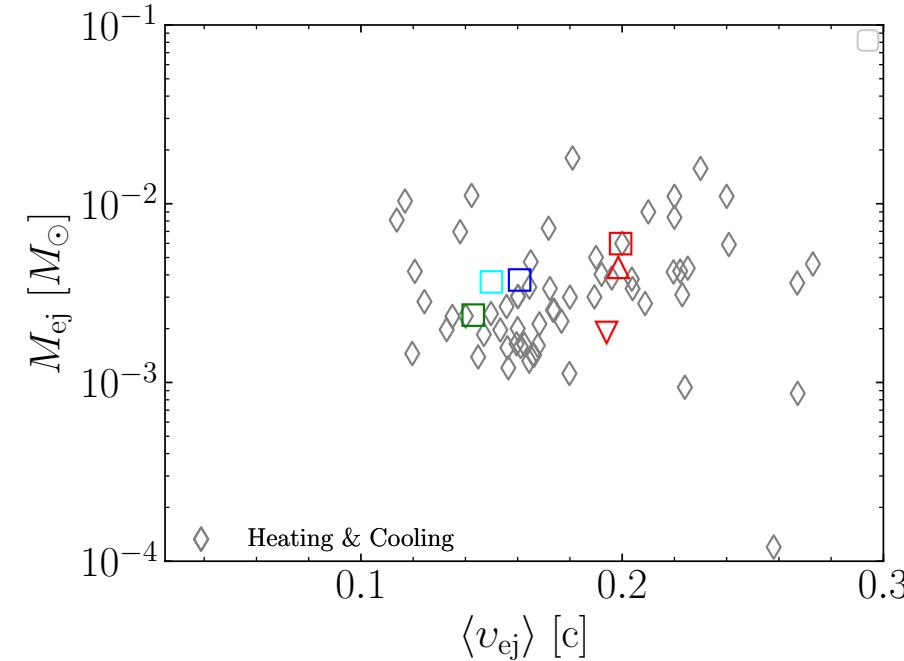
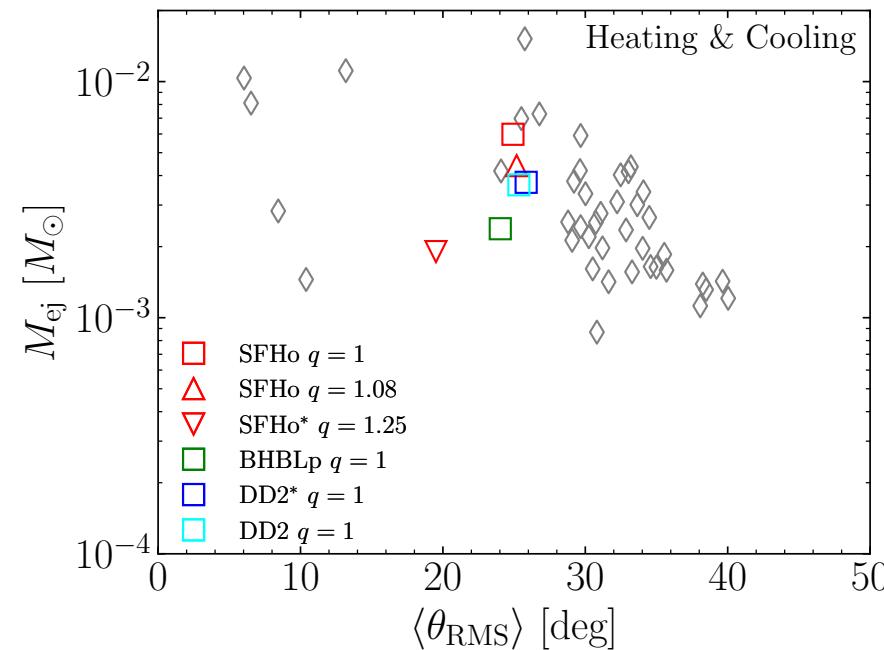
Kilonova Afterglow Modeling



Kilonova Ejecta

Dynamical ejecta from binary neutron star merger simulations

- 5 BNS merger simulations: high resolution; M1 neutrino transport; MHD effects
- Goal: analyze origin of the ejecta fast tail; statistics of the ejecta kinetic energy distribution and kilonova afterglow light curves

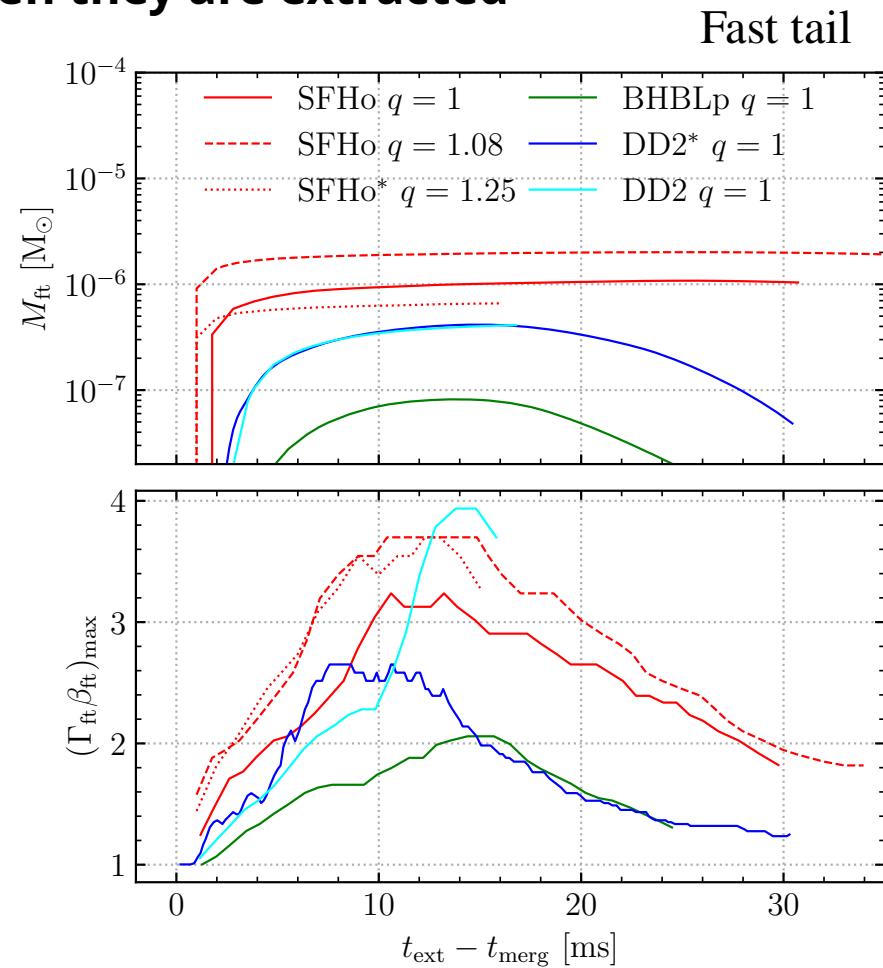
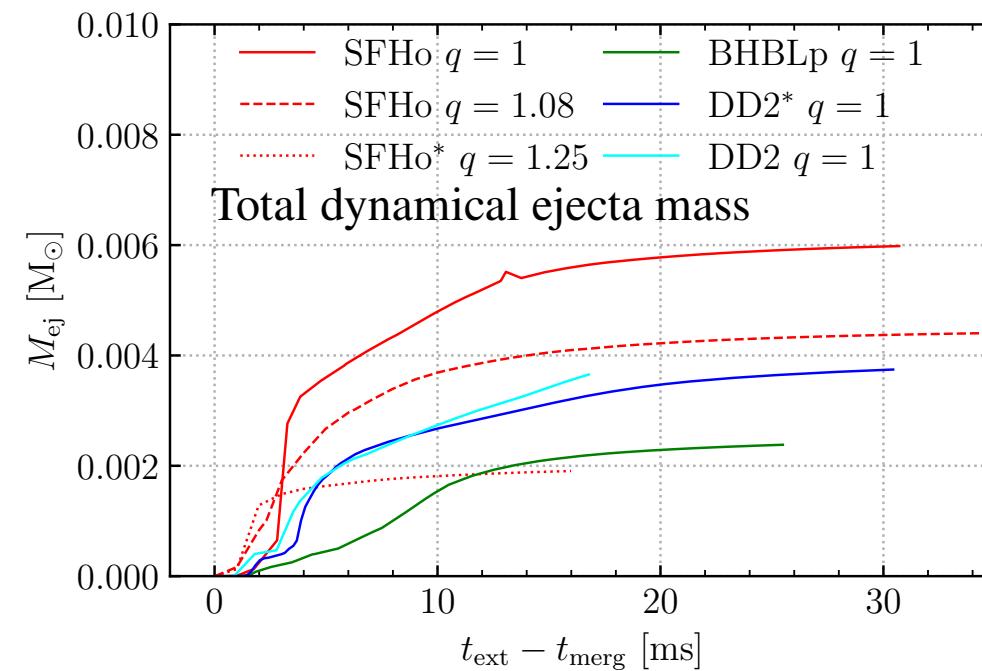




Ejecta Extraction Time

Ejecta properties depend on when they are extracted

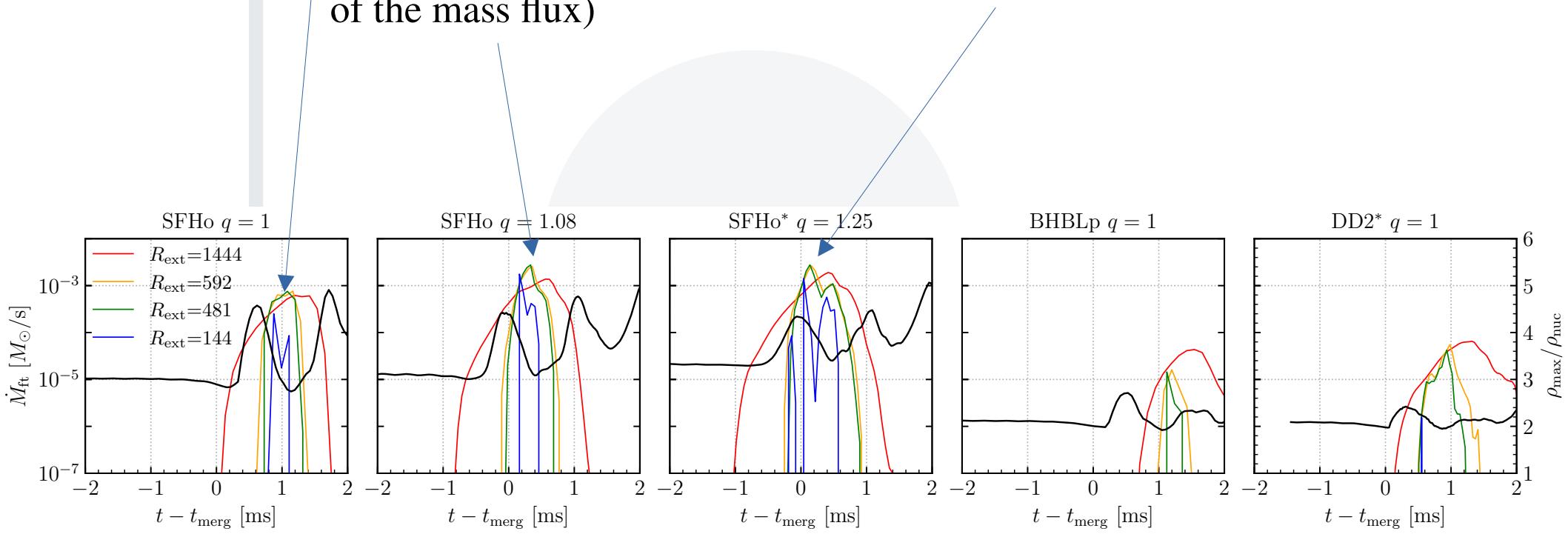
- Mass and maximum momentum saturate before the end of dynamical phase – possibly due to interactions with atmosphere

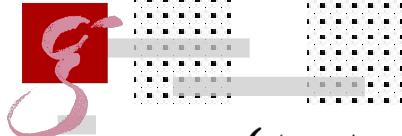


Fast Tail Origin

Comparing mass flux through with maximum density

- Mass flux of the fast tail seem to coincide with the **core bounce**, traced by the maximum density evolution
- **Other mechanisms**, however may contributed (see double peaked structure of the mass flux)

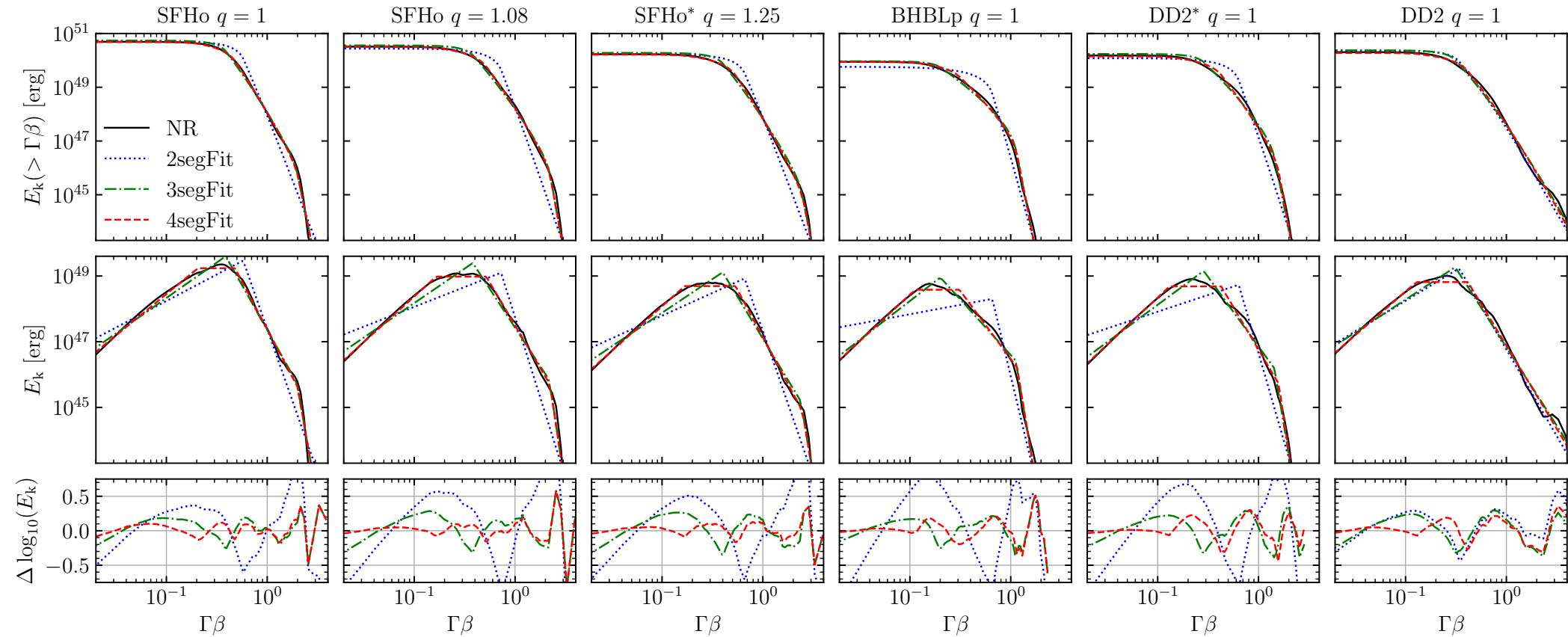




Fitting formula:

$$E_k = \mathcal{E}_0 \begin{cases} (\mathcal{M}/\mathcal{M}_0)^{k_1} & \text{if } \mathcal{M}_l \leq \mathcal{M} < \mathcal{M}_0 \\ (\mathcal{M}/\mathcal{M}_0)^{k_2} & \text{if } \mathcal{M}_0 \leq \mathcal{M} < \mathcal{M}_1 \\ (\mathcal{M}/\mathcal{M}_1)^{k_3} (\mathcal{M}_1/\mathcal{M}_0)^{k_2} & \text{if } \mathcal{M} > \mathcal{M}_1, \end{cases}$$

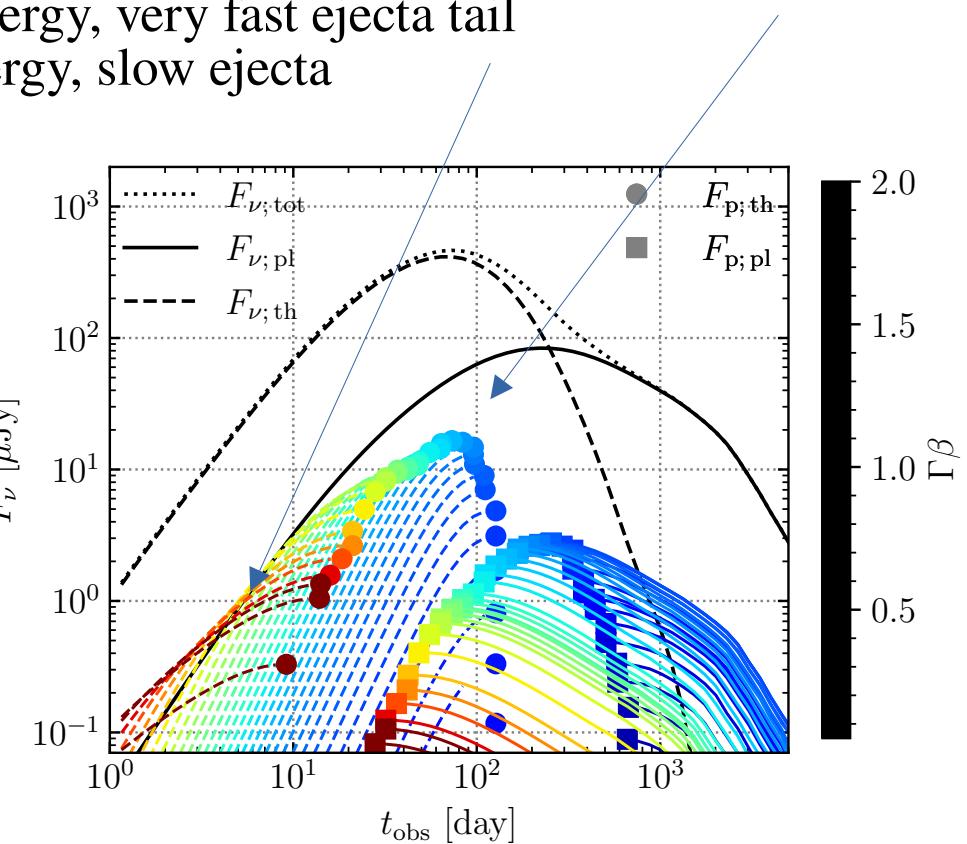
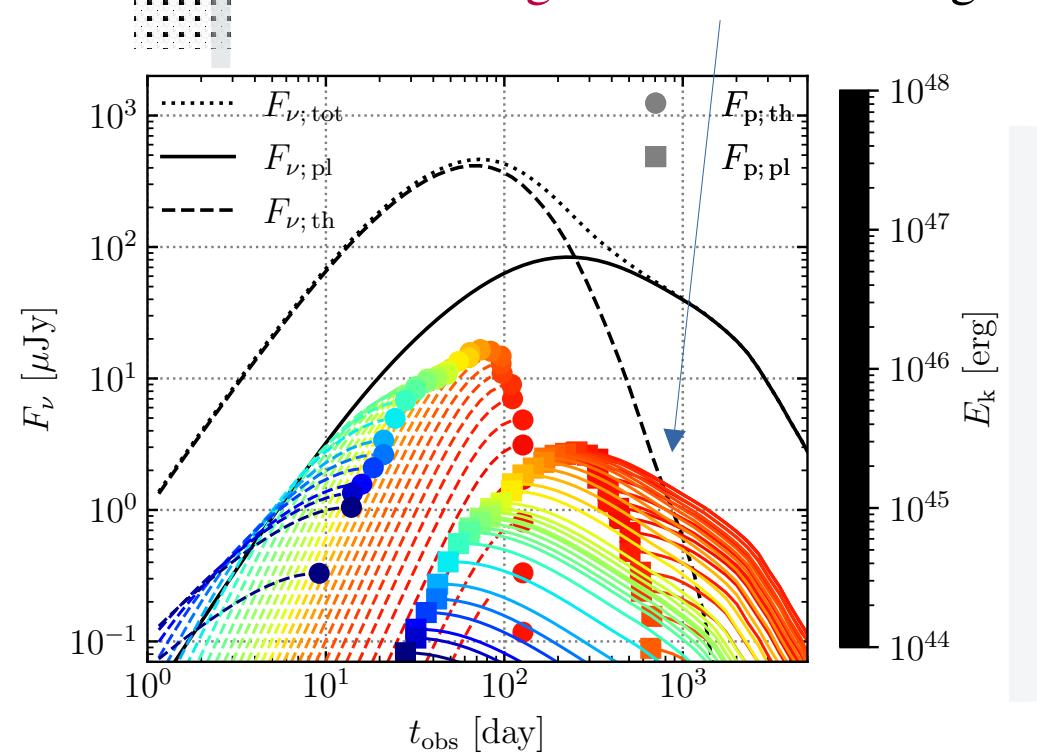
Ejecta kinetic energy



Afterglow Light Curves

Which part of the ejecta generates what part of the light curve

- Light curve peak is produced by moderately fast and moderately energetic ejecta
- Early rising part comes from low-energy, very fast ejecta tail
- Late afterglow comes from high-energy, slow ejecta

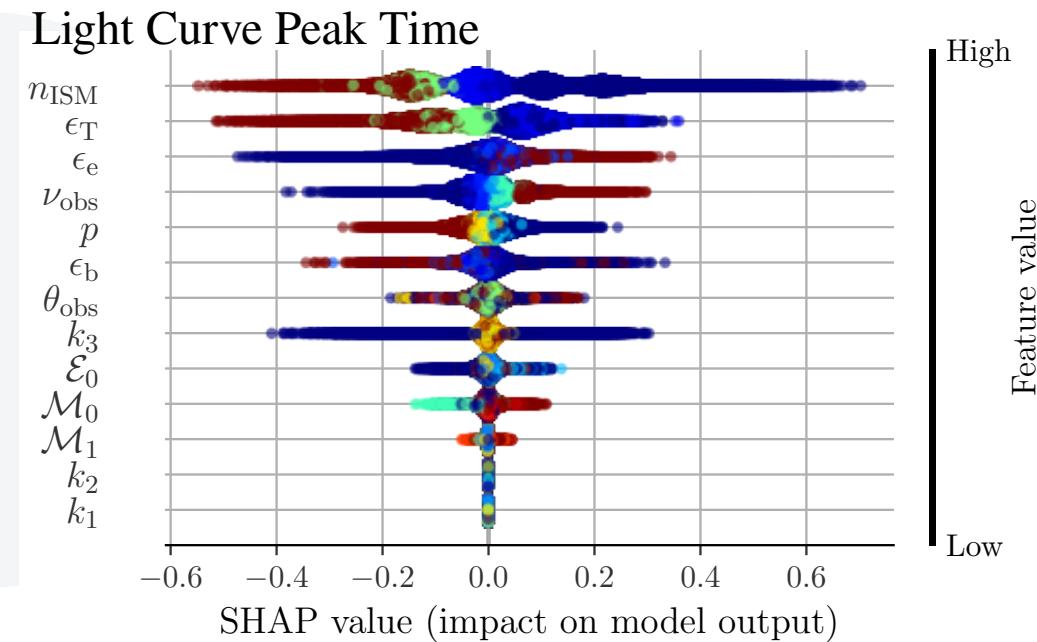
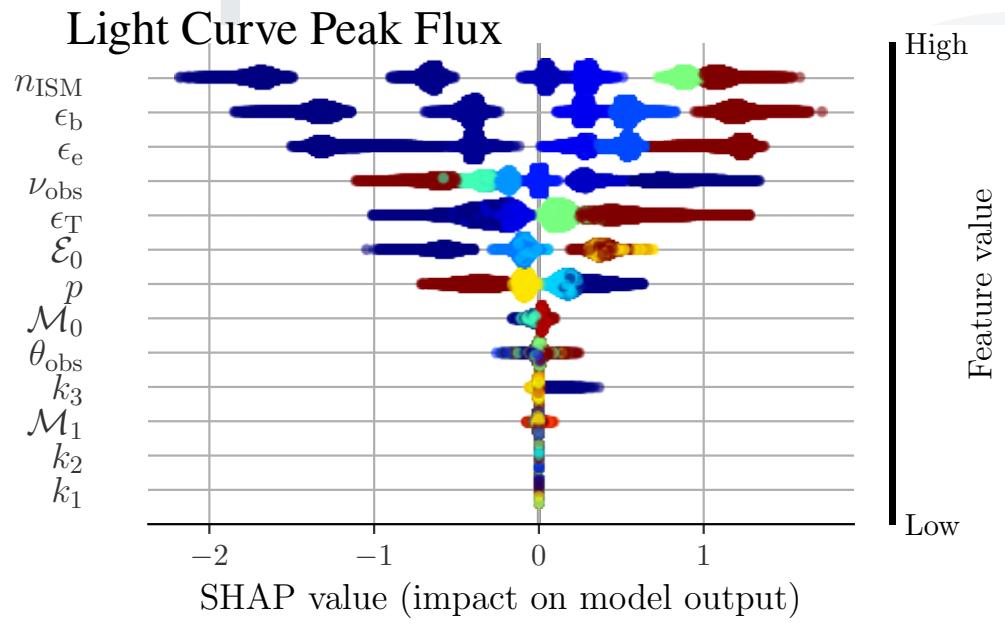




Afterglow Statistics

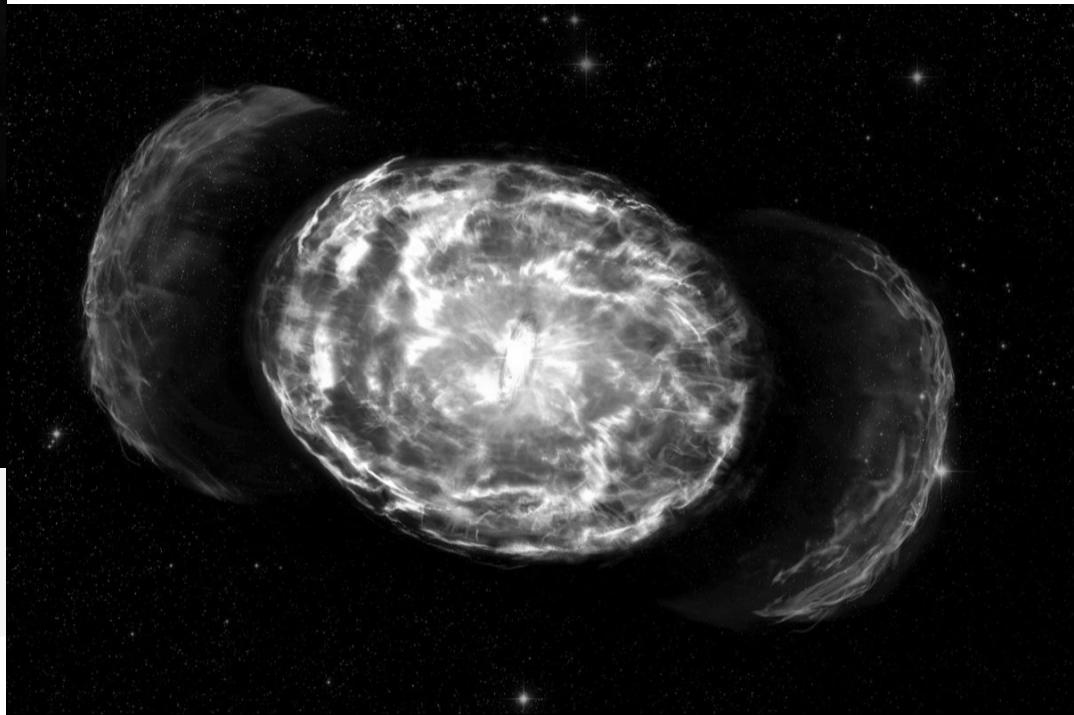
Which model parameter is the most important

- Surrogate machine learning model (XGBoost) trained on large grid of simulations
- Feature importance using Shapley values:
$$\phi_i = \sum_{S \in N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} [f(S \cup \{i\}) - f(S)]$$
- **Most important features:** ISM density and microphysics (not ejecta parameters)





Thank you for your attention



Building Afterglow Model: FS & RS dynamics

Total blast wave energy

$$E_{\text{tot}} = \Gamma_0 M_{0,4} c^2 + \Gamma M_{0,3} c^2 + \Gamma m c^2 + \Gamma_{\text{eff},3} E'_{\text{int},3} + \Gamma_{\text{eff},2} E'_{\text{int},2}.$$

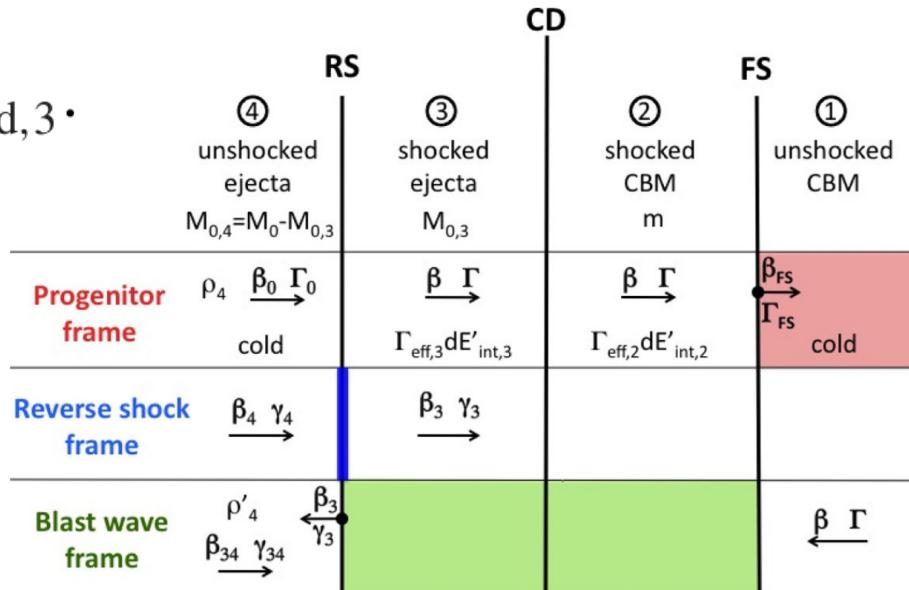
Energy conservation equation

$$dE_{\text{tot}} = dm c^2 + \Gamma_{\text{eff},2} dE'_{\text{rad},2} + \Gamma_{\text{eff},3} dE'_{\text{rad},3}.$$

$$dE'_{\text{int}} = dE'_{\text{sh}} + dE'_{\text{ad}} + dE'_{\text{rad}}.$$

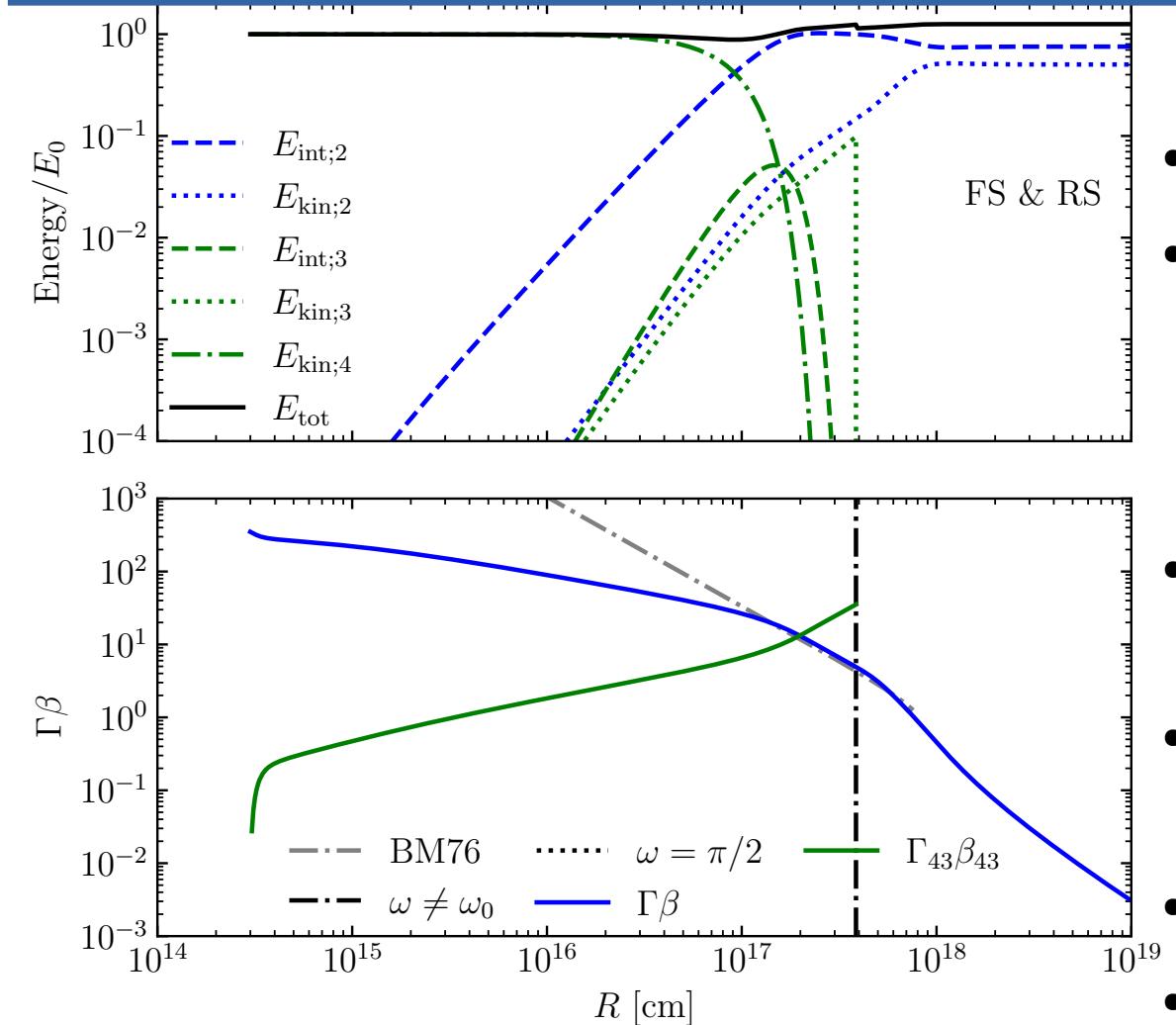
Evolution equation: after some algebra...

$$\begin{aligned} \frac{d\Gamma}{dR} = & - \frac{(\Gamma_{\text{eff},2} + 1)(\Gamma - 1) \frac{dm c^2}{dR} + \Gamma_{\text{eff},2} dE'_{\text{ad},2}}{(M_{0,3} + m)c^2 + E'_{\text{int},2} \frac{d\Gamma_{\text{eff},2}}{d\Gamma} + E'_{\text{int},3} \frac{d\Gamma_{\text{eff},3}}{d\Gamma}} \\ & - \frac{(\Gamma - \Gamma_0 - \Gamma_{\text{eff},3} + \Gamma_{\text{eff},3}\gamma_{34}) \frac{dM_{0,3}c^2}{dR} + \Gamma_{\text{eff},3} dE'_{\text{ad},3}}{(M_{0,3} + m)c^2 + E'_{\text{int},2} \frac{d\Gamma_{\text{eff},2}}{d\Gamma} + E'_{\text{int},3} \frac{d\Gamma_{\text{eff},3}}{d\Gamma}} \end{aligned}$$



Adaptive ODE solve used

Building Afterglow Model: FS & RSexample



Energy;

- Reverse shock shocks ejecta
- Total energy \sim conserved

Dynamics

- Initial deceleration when RS is crossing ejecta
- Short standard deceleration (BM solution)
- Lateral spreading
- Non-relativistic regime

Building Afterglow: Electron Distribution

Electron distribution evolution

$$\frac{\partial}{\partial t'} \frac{dN_e'}{d\gamma_e'} + \frac{\partial}{\partial \gamma_e'} \left(\dot{\gamma}_{e,\text{tot}'} \frac{dN_e'}{d\gamma_e'} \right) = d\dot{N}_{e,0}' = N_e' \frac{p-1}{\gamma_m'} \left(\frac{\gamma_e'}{\gamma_m'} \right)^{-p} d\gamma_e',$$

Electron energy loss

$$\dot{\gamma}_{e,\text{tot}'} = \dot{\gamma}_{e,\text{adi}'} + \dot{\gamma}_{e,\text{syn}'} + \dot{\gamma}_{e,\text{IC}'},$$

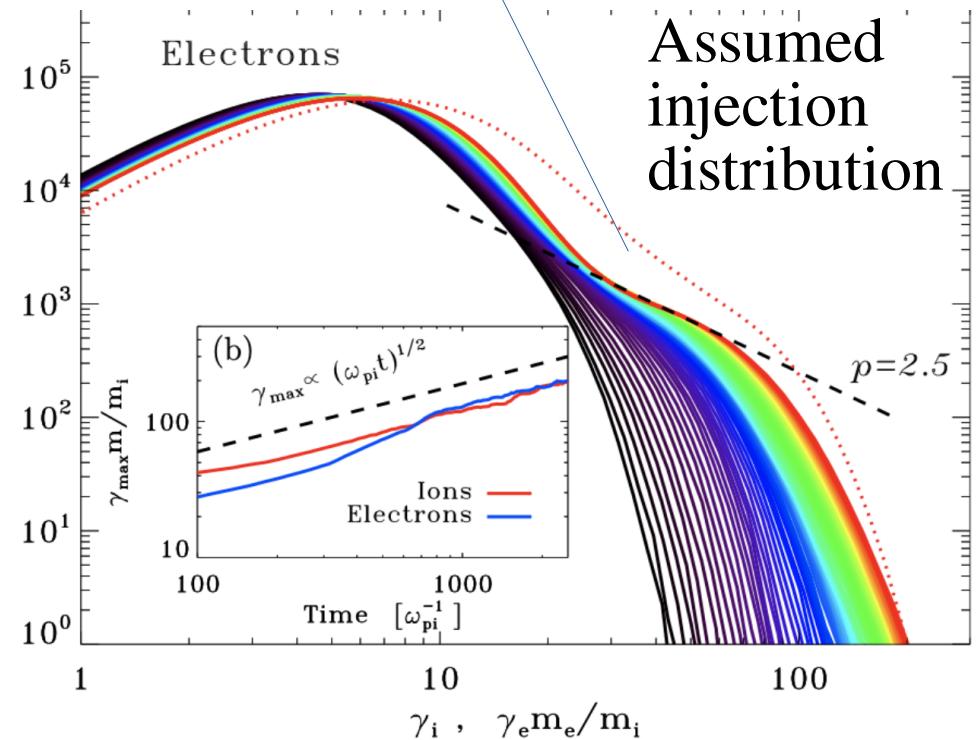
Synchrotron cooling

$$\dot{\gamma}'_{e,\text{syn}} = -\frac{\sigma_T B'^2 \gamma_e'^2}{6\pi m_e c}.$$

Adiabatic cooling

$$\dot{\gamma}'_{e,\text{adi}} = \frac{1}{3} \gamma'_e \frac{d \ln n_e'}{dt'} = -\frac{1}{2} \frac{\gamma'_e}{R} \frac{dR}{dt'}$$

Electron injection



Building Afterglow: SSC cooling

Electrons lose energy by up-scattering (syn) photons

Connects the electron population with existing radiation field

SSC cooling

$$\dot{\gamma}'_{e, \text{IC}} = -\frac{1}{m_e c^2} \frac{3\sigma_T c}{4\gamma_e'^2} \int_{\nu_{\min}'}^{\nu_{\max}'} \frac{n_{\nu'} d\nu'}{\nu'} \\ \int_{\nu_{\text{IC}, \min}'}^{\nu_{\text{IC}, \max}'} h\nu_{\text{IC}'} d\nu_{\text{IC}'} F(q, g),$$

SSC scattering cross-section

$$F(q, g) = 2q \ln q + (1 + 2q)(1 - q) + \frac{1}{2} \frac{(4qg)^2}{1 + 4qg}(1 - q),$$

$$g = \frac{\gamma_e' h\nu'}{m_e c^2}, \quad w = \frac{h\nu_{\text{IC}'}}{\gamma_e' m_e c^2}, \quad \text{and} \quad q = \frac{w}{4g(1-w)}.$$

Seed photon density

$$n_{\nu'} \approx \frac{T'}{h\nu'} \frac{\sqrt{3} e^3 B'}{m_e c^2} \int_{\gamma_{e, \min}'}^{\gamma_{e, \max}'} \\ \times n'(\gamma_e') \mathcal{R}(\nu'/\nu_c') d\gamma_e',$$

Escape time

$$T' \approx \Delta'/c$$

$$n'(\gamma_e') = \frac{dN_e'/d\gamma_e'}{4\pi\Delta'R^2}$$

Building Afterglow: Synchrotron Emission

Convolving the emission of a single electron gyrating around magnetic field lines with electron distribution function

Synchrotron emission power

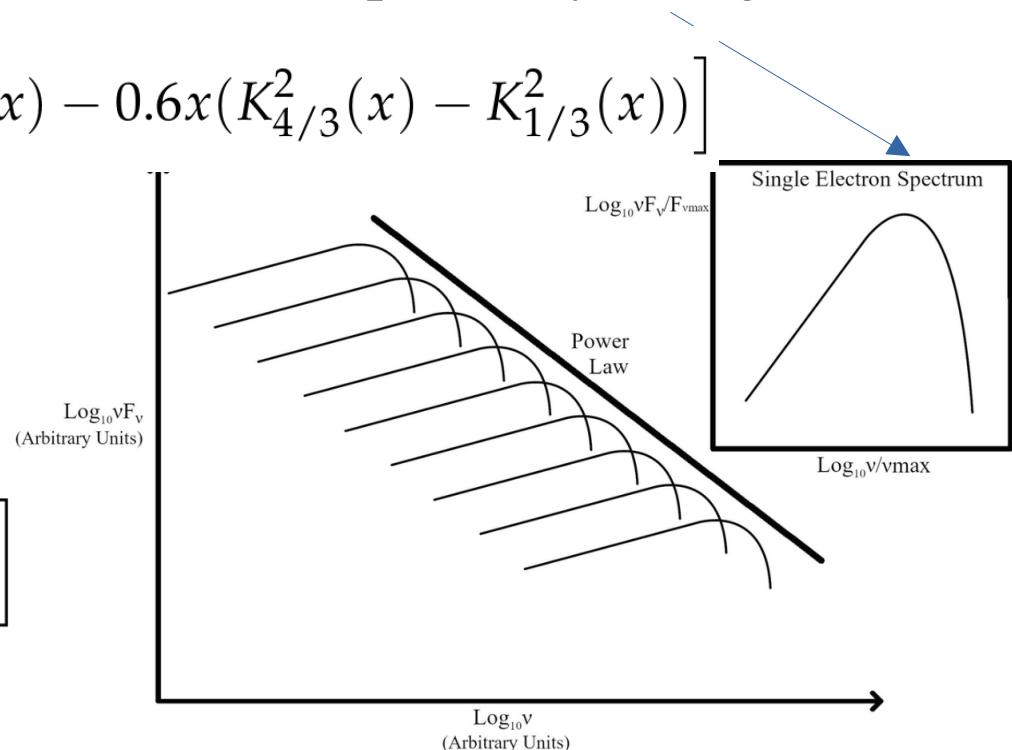
$$P_{\nu'}^{syn}(\nu', \gamma) = \frac{2\sqrt{3}e^3 B'}{m_e c^2} x^2 \left[K_{4/3}(x) K_{1/3}(x) - 0.6x(K_{4/3}^2(x) - K_{1/3}^2(x)) \right]$$

$$P_{\nu'}^{syn}(\nu') = \int P_{\nu'}^{syn}(\nu', \gamma) \frac{dN}{d\gamma} d\gamma$$

Synchrotron self-absorption (SSA)

$$\alpha_\nu = -\frac{1}{8\pi\nu'^2 m_e} \int d\gamma P'(\gamma, \nu') \gamma^2 \frac{\partial}{\partial \gamma} \left[\frac{N(\gamma)}{\gamma^2} \right]$$

Emission power by a single electron



Building Afterglow: SSC emission

SSC emission power

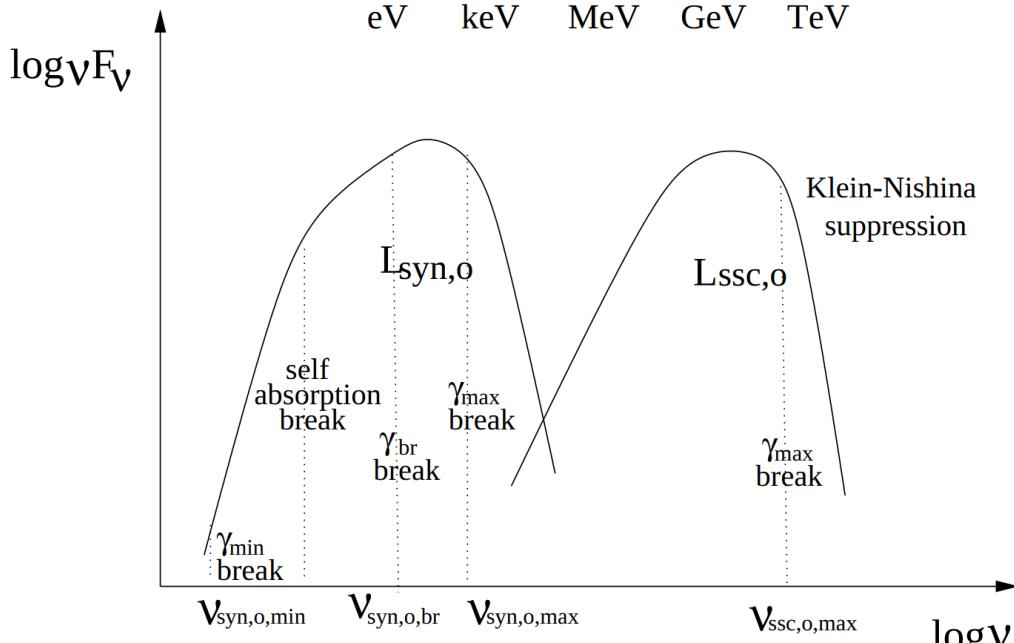
$$P'_{\text{ssc}}(\nu'_{\text{ssc}}) = \int \int \frac{dN'_e}{d\gamma'_e} h\nu'_{\text{ssc}} \frac{dN'_\gamma}{dt'd\nu'_{\text{ssc}}} d\gamma'_e.$$

$$\frac{dN'_\gamma}{dt'd\nu'_{\text{ssc}}} = \frac{3\sigma_T c}{4\gamma_e'^2} \frac{n_{\nu'} d\nu'}{\nu'} F(q, g).$$

$$n_{\nu'} \approx \frac{T'}{h\nu'} \frac{\sqrt{3} e^3 B'}{m_e c^2} \int_{\gamma_{e,\min}'}^{\gamma_{e,\max}'}$$

$$\times n'(\gamma_e') \mathcal{R}(\nu'/\nu_c') d\gamma_e',$$

$$F(q, g) = 2q \ln q + (1 + 2q)(1 - q) + \frac{1}{2} \frac{(4qg)^2}{1 + 4gq}(1 - q),$$



Example: sky map

Sky map – intensity distribution projected onto a plane orthogonal to the line of sight.

Properties:

- Centroid position
- Size

Shows:

- geometry of the ejecta
- jet inclination angle

