Modeling Gamma-Ray Burst Afterglows

Methods of PyBlastAfterglow

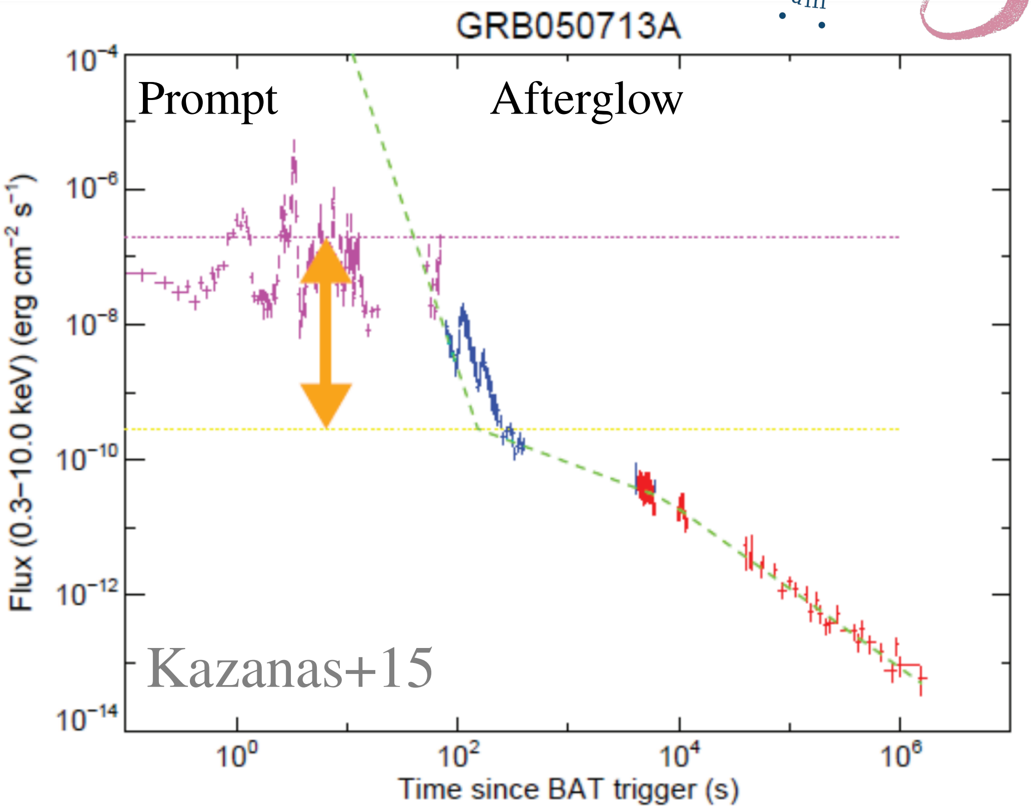
Universität Potsdam

Structure

- General picture and basic properties
- Examples of observations
- Building an afterglow model
  - Dynamics
  - Microphysics and radiation
  - Observables
- Comparison with other models
- Building a surrogate model

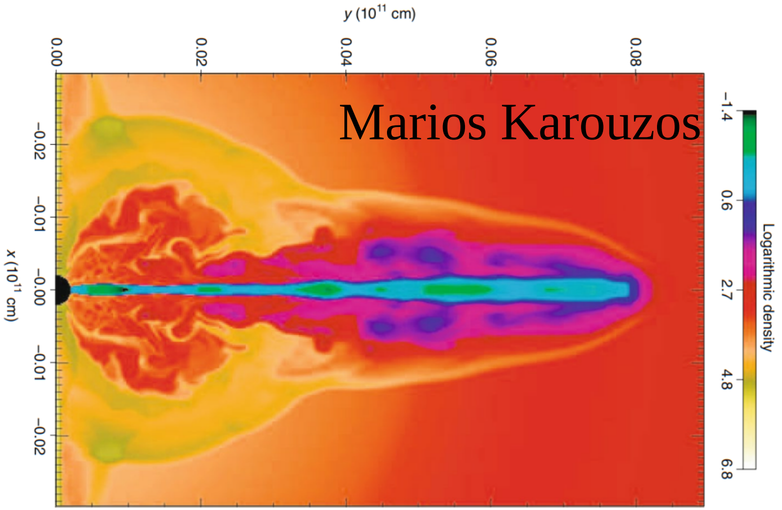
Introduction

- **Gamma-Ray Bursts**
- **Prompt emission** (oscillatory, early) *Internal shocks* or magnetic reconnection
  Convert fraction of jet energy to radiation
- **Afterglow** (smooth, late)
  External shocks (forward/reverse)
  Synchrotron emission (based on spectra)

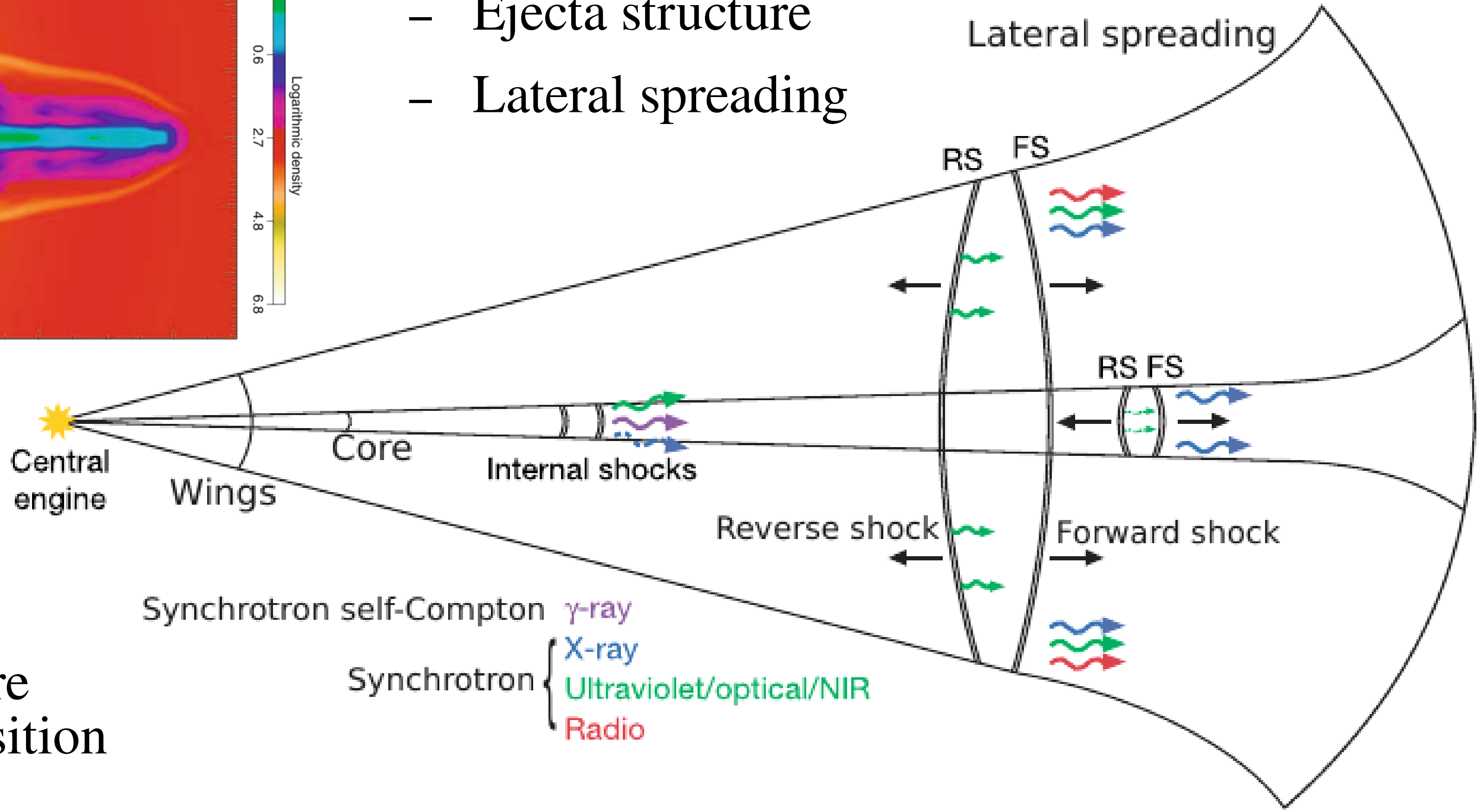


Jet → Internal dissipation (prompt emission) → Dissipation at external shocks (afterglow)

General Picture: Jet



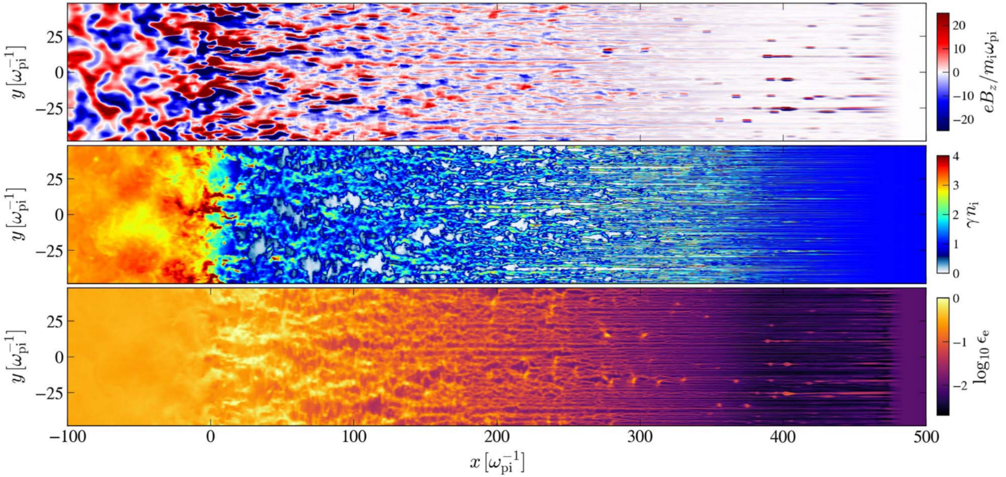
- Schematic picture:
  - Ejecta structure
  - Lateral spreading



- **<u>Engine</u>**:
  disk accretion
  - burst energy
  - ejecta structure
  - ejecta composition

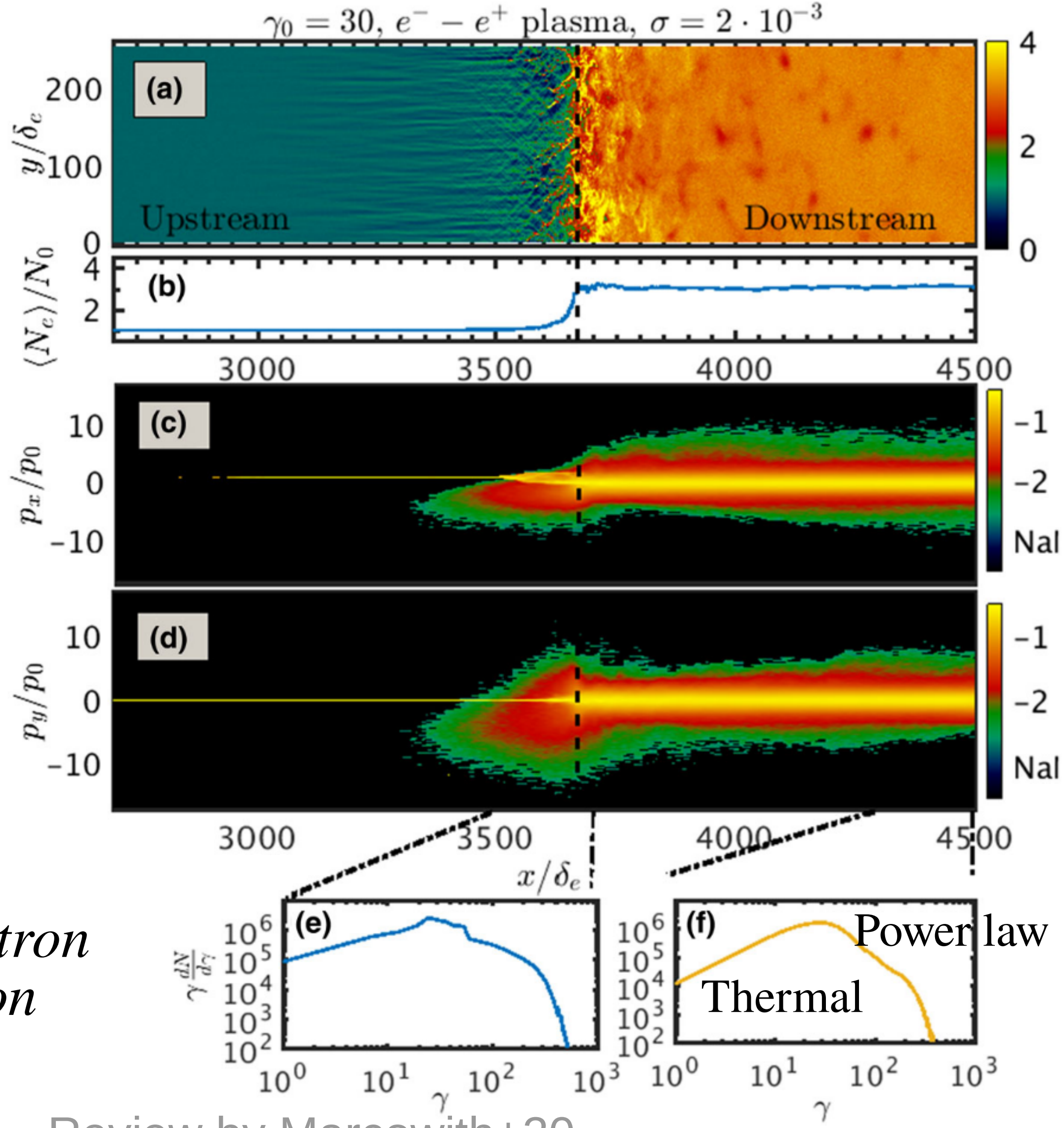
General Picture: Shocks

Simulation Vanthieghem+22



- **Collisionless shocks**
  (few gyro-radii scale)
- MHD instabilities
  magnetic field amplification
- **Particle acceleration**
  fermi first order acceleration → thermal core + power-law tail)

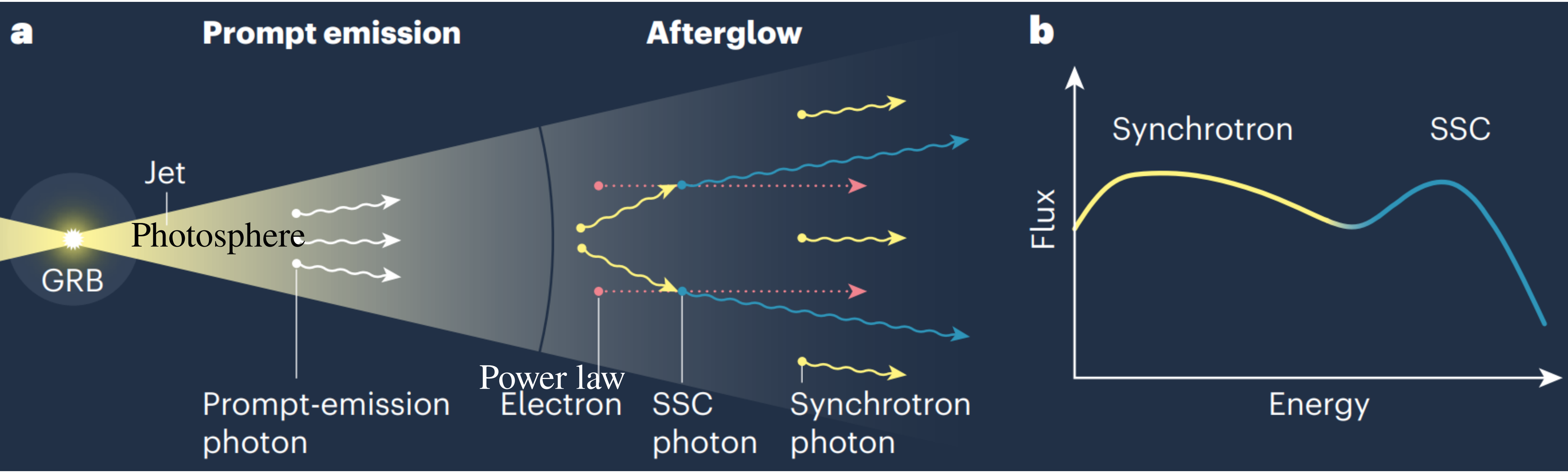
} *Synchrotron Radiation*



Review by Marcowith+20

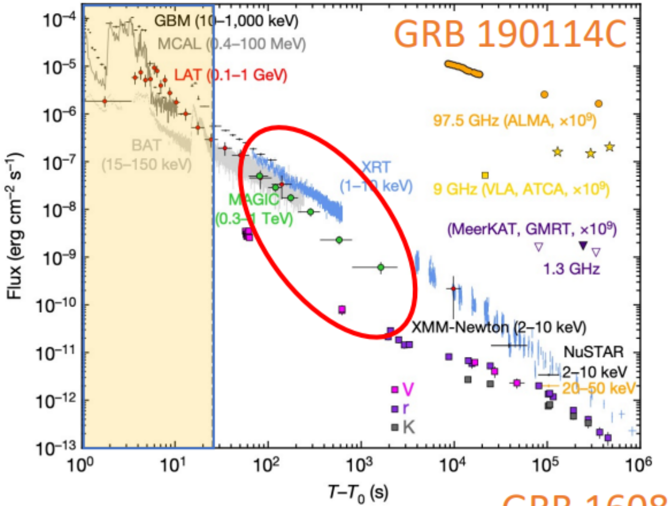
General Picture: Summary

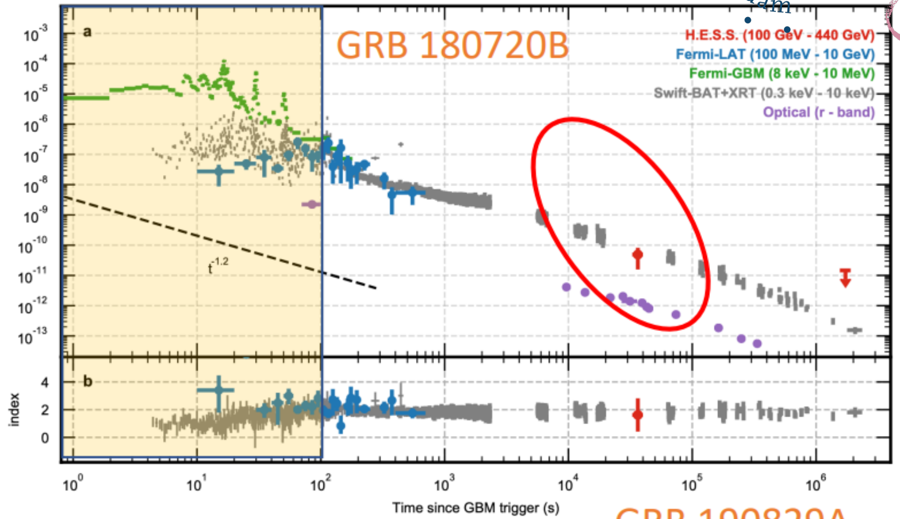
From Bing Theodore Zhang (Zhang+19)



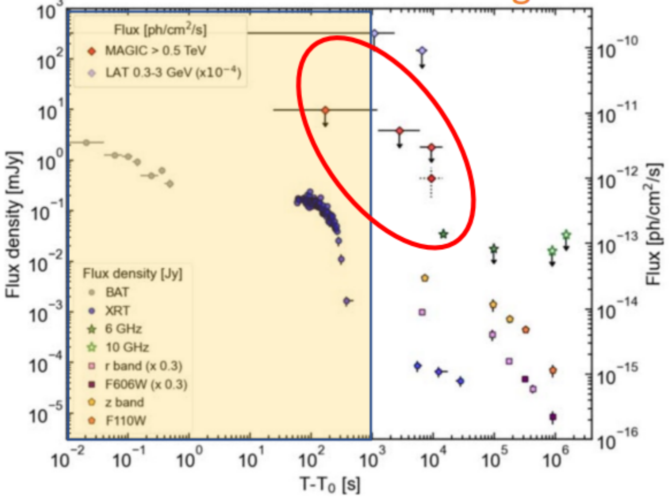
- **Jet structure:** (energy/velocity angular dependency)
- **Ejecta dynamics:** coasting; deceleration; spreading
- **Collisionless shocks:** magnetic field amplification & article acceleration: Fermi first order acceleration; Power-law tail + thermal core

Examples: Very High Energy Afterglow

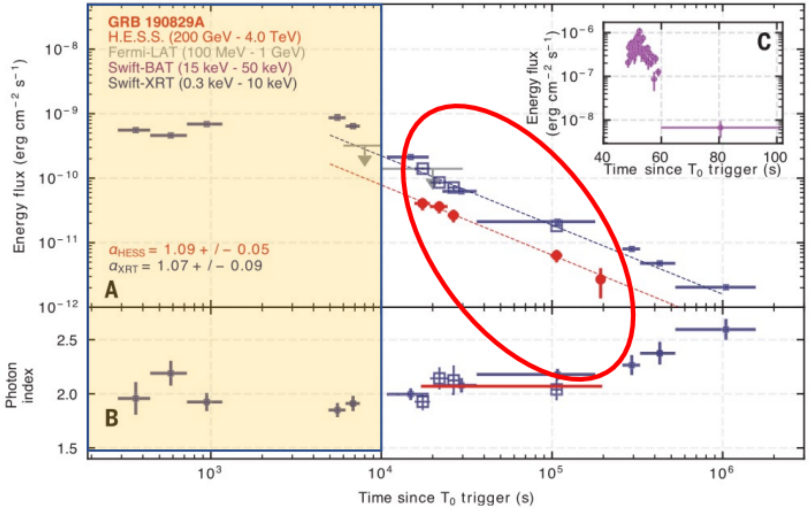




GRB 160821B ~ 3 sigma



GRB 190829A

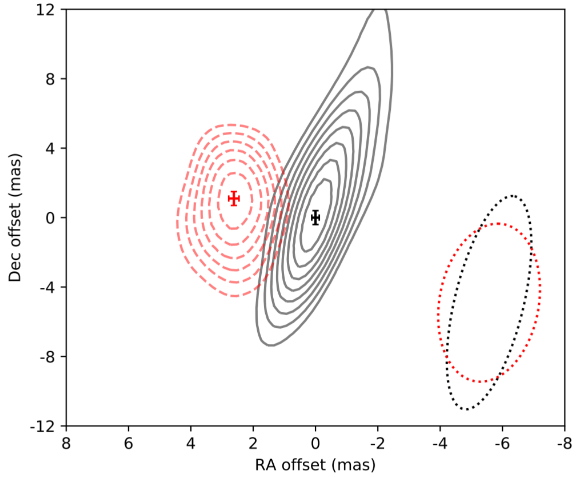


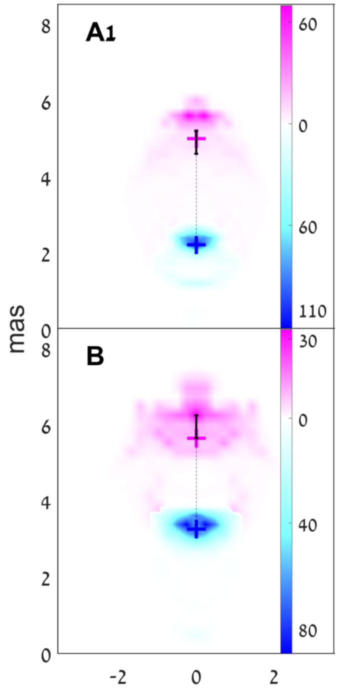
TeV photons from afterglow phase

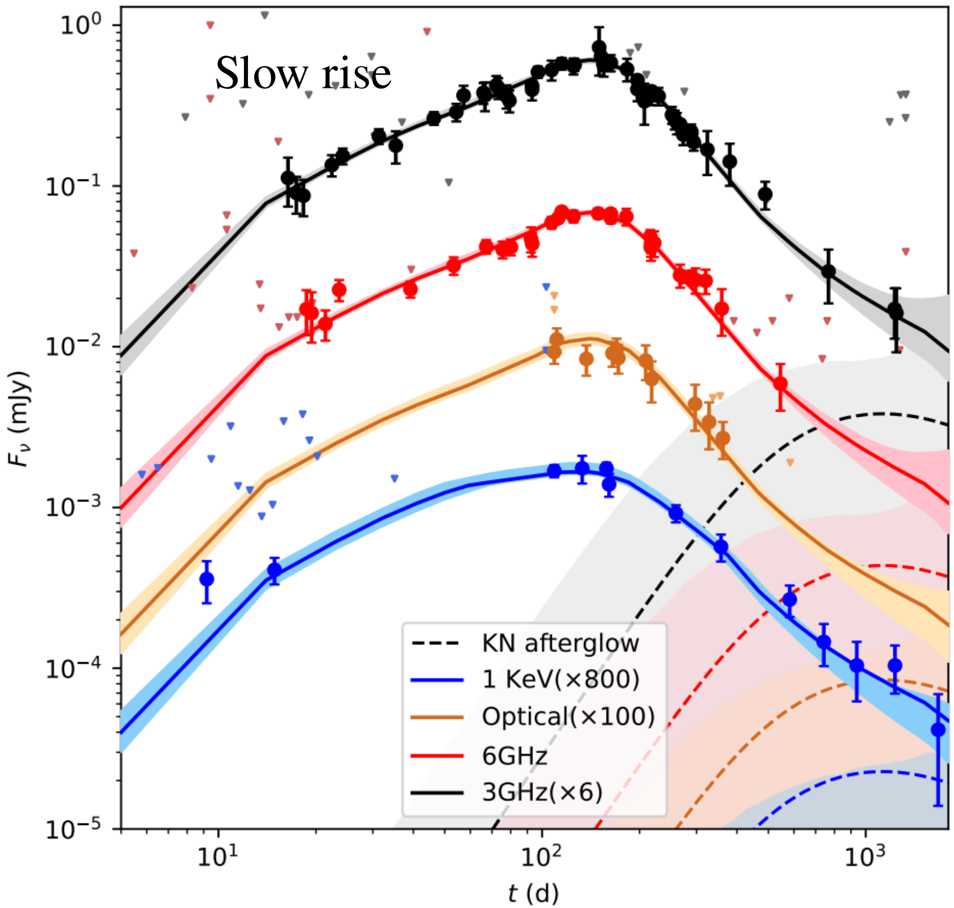
Opening the possibility to renovate and boost afterglow studies

Examples: Structure & Radio Images

- GRB 170817A off-axis & structured (2208.09121)
  - Refreshed shock (2005.12426)
- Synchrotron (semi-analytical)
- Flux centroid motion (1806.09693)

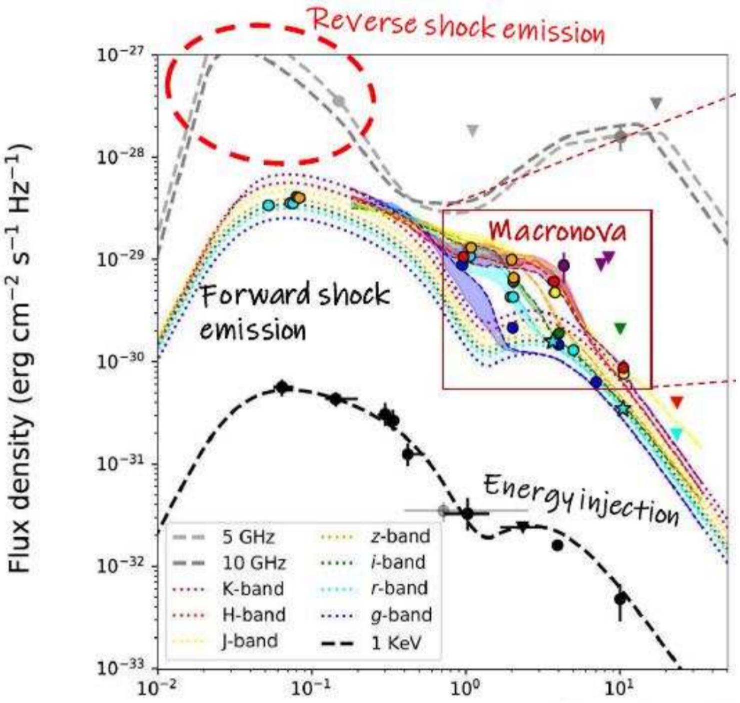


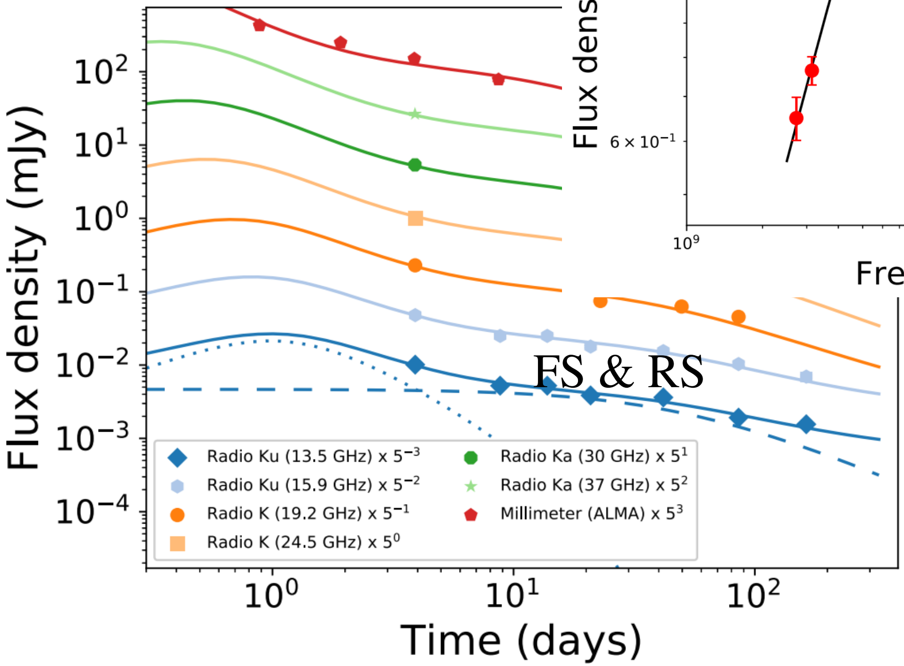


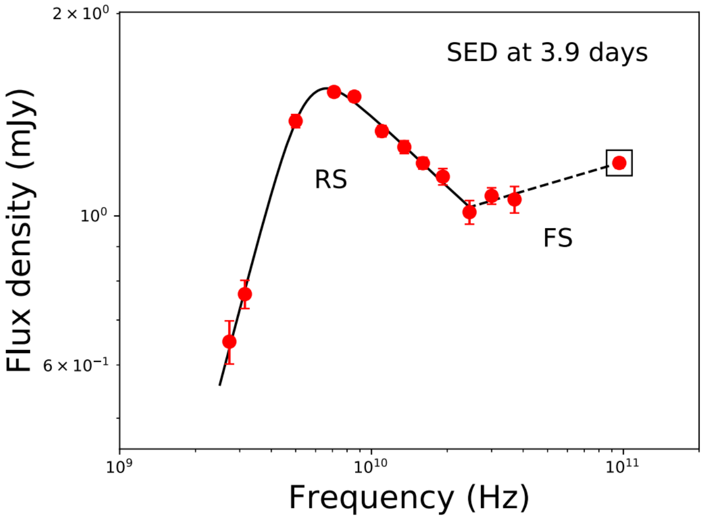


Examples: Reverse Shock

- GRB 181201A reverse shock (Laskar+19)
- GRB 160821B reverse shock
  & kilonova & energy injection (Lamb+20)

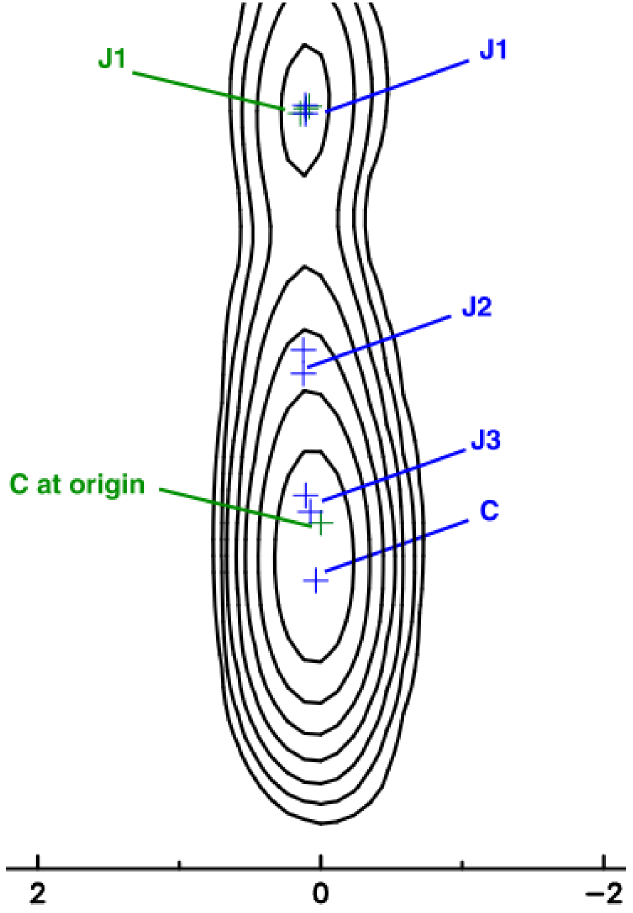


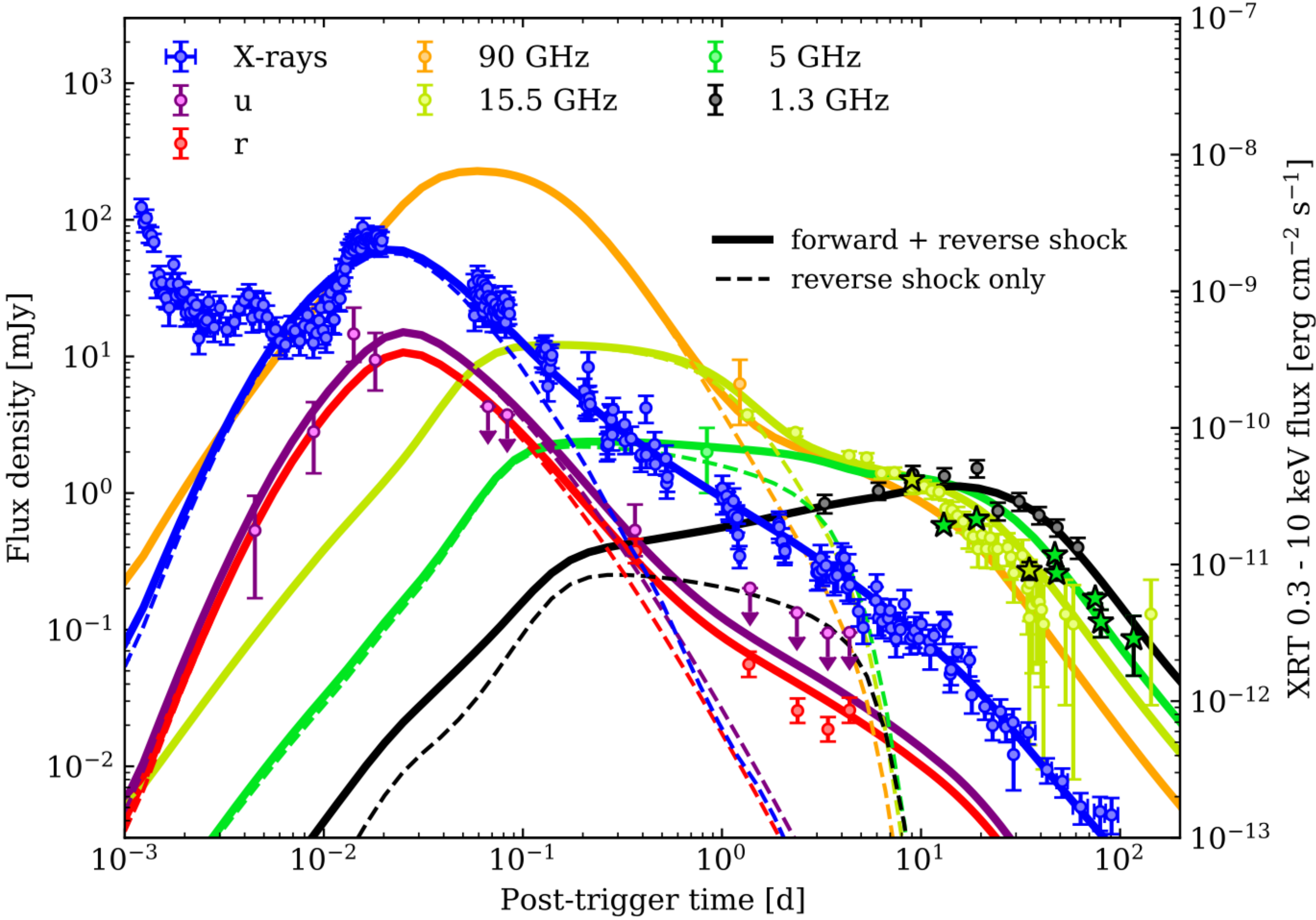




Examples: RS & SSC & Radio Images

- GRB 190829A (Salafia+22)

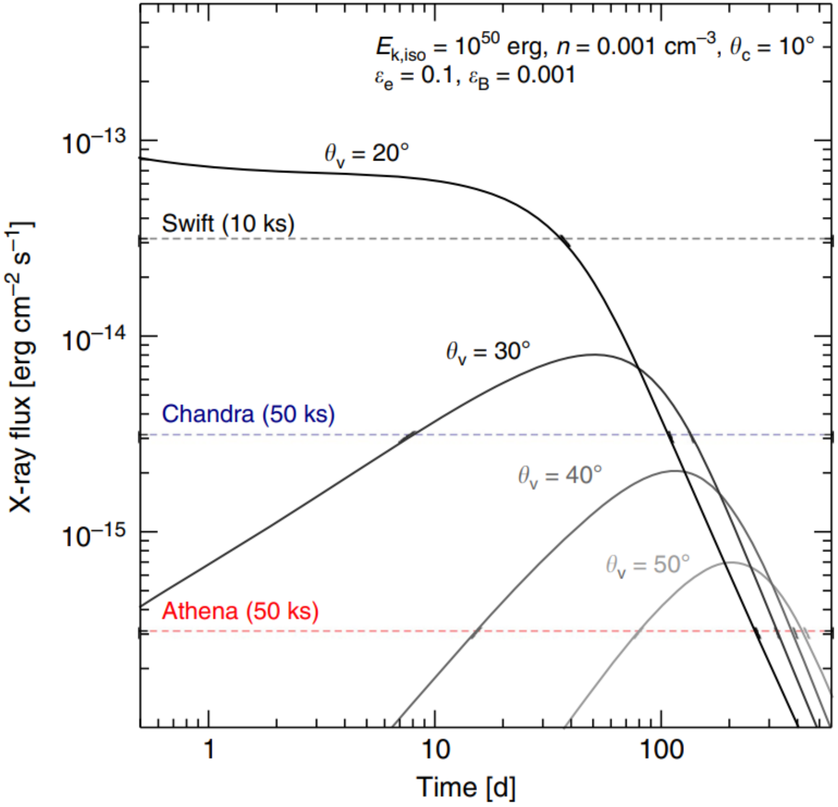




Observational Examples: Summary

- Prompt → Afterglow
- **Ejecta dynamics & geometry:** coasting; deceleration; spreading (jet break)
- **Spectrum evolution**: electron distribution evolution; synchrotron + synchrotron self-Compton;
- **Relativistic effects & EATS**: Beaming; Doppler shift

Observer angle dependency



Review by Marcowith+20

Ingredients for an Afterglow Model

- **Dynamics:** semi-analytic for computational reasons;
  - semi-analytic, energy conserving blast wave with FS and RS
  - independent layers representing different angular layers of the jet
- **Electron distribution** needs to be evolved (heating / cooling)
- **Radiation**: Synchrotron and SSC radiation (comoving spectra)
- **Observables:** Equal arrival time surface integration for spectra and sky maps (because of relativistic effects)

Building Afterglow Model: FS only example

Total blast wave energy (assume ejecta is <u>already</u> shocked)

$$E_{\rm tot} = \Gamma(M_0 + m)\,c^2 + \Gamma_{\rm eff}\,E'_{\rm int},$$

Energy conservation equation

$$\mathrm{d}\left[\Gamma(M_0 + m)c^2 + \Gamma_{\rm eff}\,E'_{\rm int}\right] = \mathrm{d}m\,c^2 + \Gamma_{\rm eff}\mathrm{d}E'_{\rm rad}.$$

$$\mathrm{d}E'_{\rm int} = \mathrm{d}E'_{\rm sh} + \mathrm{d}E'_{\rm ad} + \mathrm{d}E'_{\rm rad}.$$

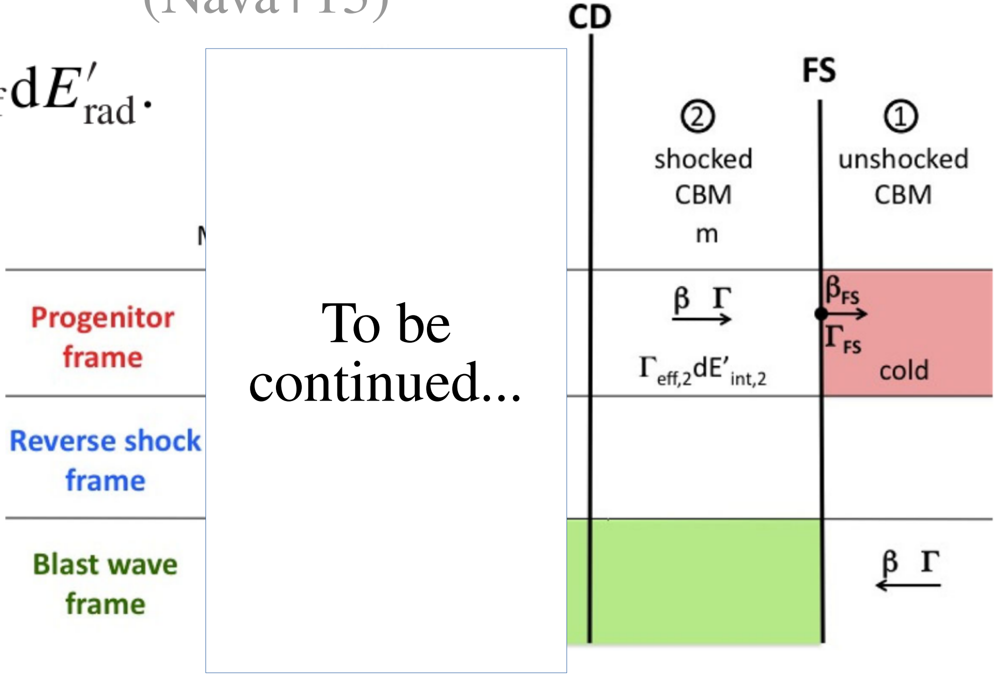
Evolution equation: after some algebra...

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}R} = -\frac{(\Gamma_{\rm eff}+1)(\Gamma-1)\,c^2\,\frac{\mathrm{d}m}{\mathrm{d}R} + \Gamma_{\rm eff}\,\frac{\mathrm{d}E'_{\rm ad}}{\mathrm{d}R}}{(M_0+m)\,c^2 + E'_{\rm int}\,\frac{\mathrm{d}\Gamma_{\rm eff}}{\mathrm{d}\Gamma}},$$

Where

$$\Gamma_{\rm eff} \equiv \frac{\hat{\gamma}\Gamma^2 - \hat{\gamma} + 1}{\Gamma},$$

Schematic picture of a Blast wave (Nava+13)



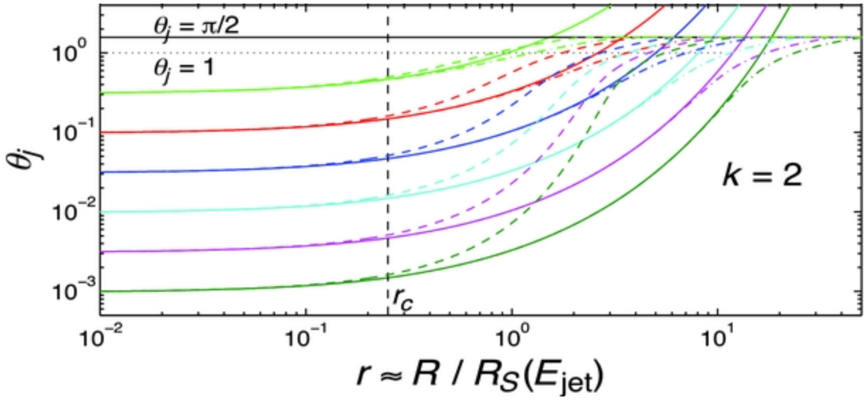
Final set of ODEs we solve numerically

Building Afterglow Model: lateral spreading

Required 2D HD simulations. Using approximants. Many options...

Calibrated with early HD simulation (Granot+12)

$$\frac{d\omega}{dR} = R^{-1}\Gamma^{-1-a}$$



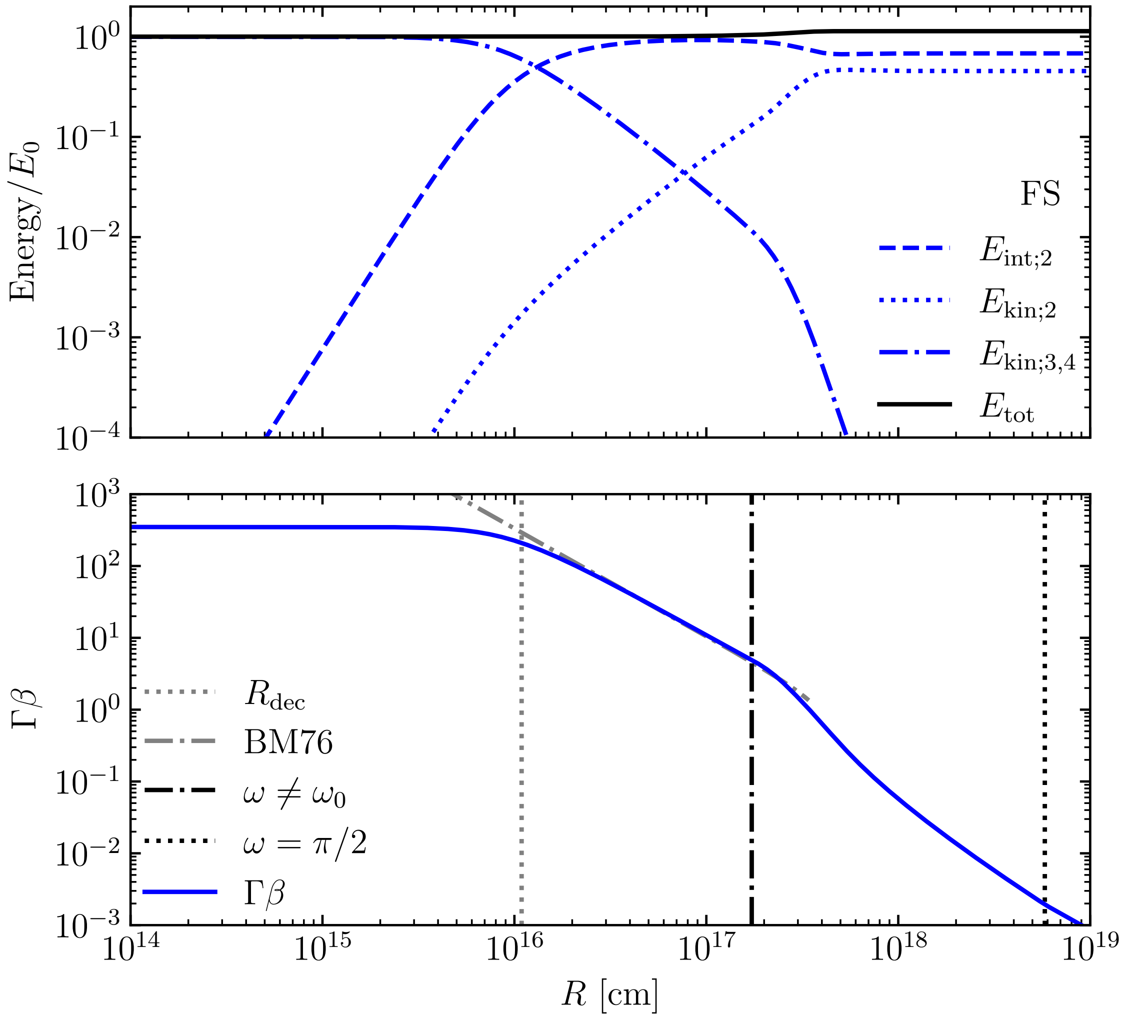
Sound-speed based formulation (Huang+00)

$$\frac{d\omega}{dR} = \frac{c_s}{R\Gamma\beta c} \qquad c_s^2 = \frac{\hat{\gamma}p'}{\rho'}\left[\frac{(\hat{\gamma}-1)\rho'}{(\hat{\gamma}-1)\rho' + \hat{\gamma}\rho'}\right]c^2 = \frac{\hat{\gamma}(\hat{\gamma}-1)(\Gamma-1)}{1+\hat{\gamma}(\Gamma-1)}c^2$$

Structured-jet aware formulation (Ryan+20)

$$\frac{d\omega}{dt} = v_{\perp}\frac{c}{R} \times \begin{cases} 1 & \text{core} \\ \tan(\omega_0/2)/\tan(\omega_c/2) & \text{wings} \end{cases} \qquad v_{\perp} = \frac{1}{2}\frac{\beta_{\rm sh}}{\Gamma}\sqrt{\frac{2(\Gamma\beta)^2+3}{4(\Gamma\beta)^2+3}}$$

Building Afterglow Model: FS only example



<u>Energy:</u>

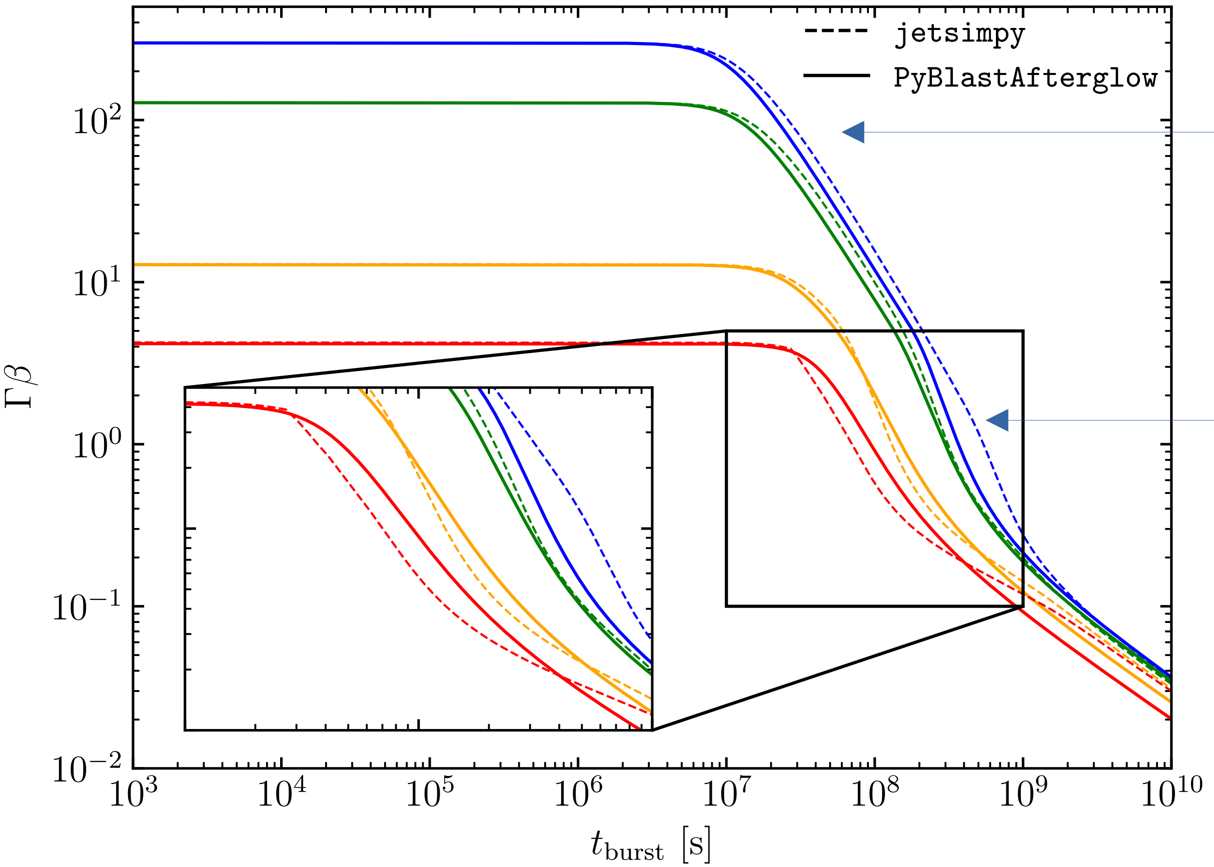
- kinetic → internal energy
- total energy is conserved c

<u>Dynamics:</u>

- free coasting (constant momentum)
- deceleration (BM solution)
- lateral spreading
- non-relativistic stage (ST solution)

Building Afterglow Model: Comparison

- Dynamics with lateral spreading against 2D thin-shell hydrodynamics model Structure and Spreading



calibration to BM can be added

spreading prescription can be improved

Building Afterglow Model: FS & RS dynamics

Total blast wave energy

$$E_{\rm tot} = \Gamma_0 M_{0,4}\, c^2 + \Gamma M_{0,3}\, c^2 + \Gamma m\, c^2 + \Gamma_{\rm eff,3} E'_{\rm int,3} + \Gamma_{\rm eff,2} E'_{\rm int,2}.$$

Energy conservation equation

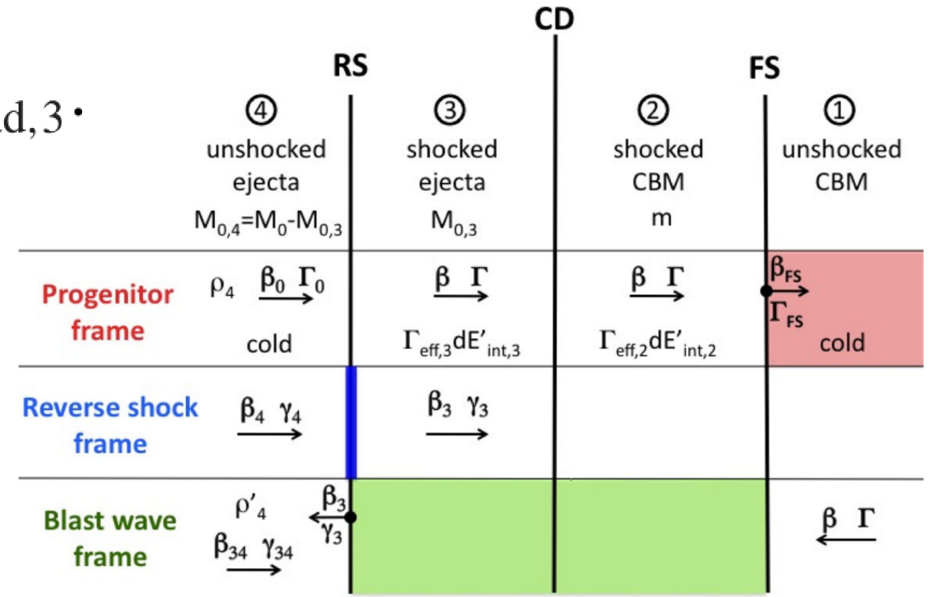
$$\mathrm{d}E_{\rm tot} = \mathrm{d}m\, c^2 + \Gamma_{\rm eff,2}\mathrm{d}E'_{\rm rad,2} + \Gamma_{\rm eff,3}\mathrm{d}E'_{\rm rad,3}.$$

$$\mathrm{d}E'_{\rm int} = \mathrm{d}E'_{\rm sh} + \mathrm{d}E'_{\rm ad} + \mathrm{d}E'_{\rm rad}.$$

Evolution equation: after some algebra...

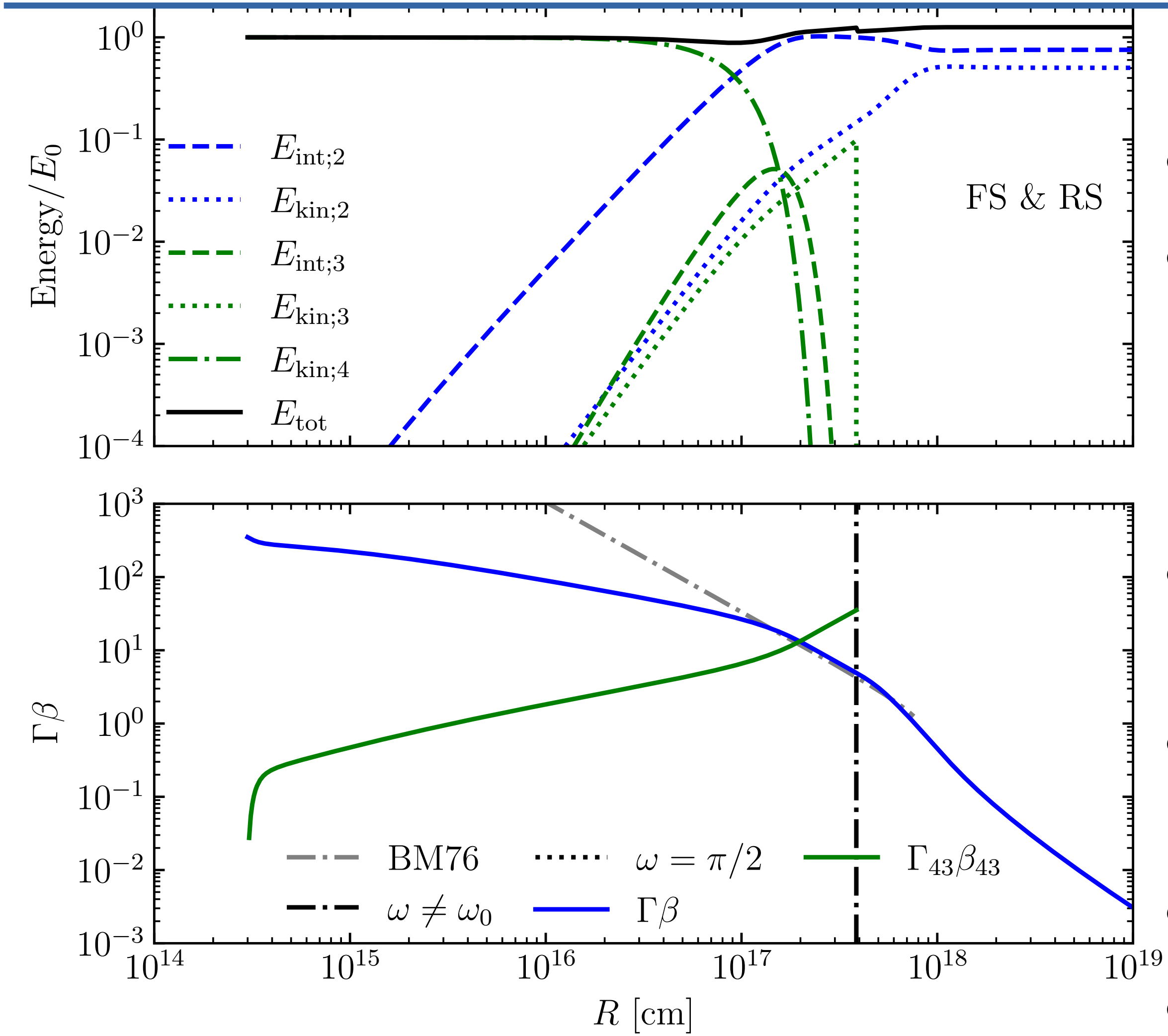
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}R} = -\frac{(\Gamma_{\rm eff,2}+1)(\Gamma-1)\frac{\mathrm{d}m\,c^2}{\mathrm{d}R} + \Gamma_{\rm eff,2}\mathrm{d}E'_{\rm ad,2}}{(M_{0,3}+m)c^2 + E'_{\rm int,2}\frac{\mathrm{d}\Gamma_{\rm eff,2}}{\mathrm{d}\Gamma} + E'_{\rm int,3}\frac{\mathrm{d}\Gamma_{\rm eff,3}}{\mathrm{d}\Gamma}}$$

$$-\frac{(\Gamma-\Gamma_0-\Gamma_{\rm eff,3}+\Gamma_{\rm eff,3}\gamma_{34})\frac{\mathrm{d}M_{0,3}c^2}{\mathrm{d}R} + \Gamma_{\rm eff,3}\mathrm{d}E'_{\rm ad,3}}{(M_{0,3}+m)c^2 + E'_{\rm int,2}\frac{\mathrm{d}\Gamma_{\rm eff,2}}{\mathrm{d}\Gamma} + E'_{\rm int,3}\frac{\mathrm{d}\Gamma_{\rm eff,3}}{\mathrm{d}\Gamma}}$$



Adaptive ODE solve used

Building Afterglow Model: FS & RSexample



Energy;

- Reverse shock shocks ejecta
- Total energy ~ conserved

Dynamics

- Initial deceleration when RS is crossing ejecta
- Short standard deceleration (BM solution)
- Lateral spreading
- Non-relativistic regime

Building Afterglow: Shock Microphysics

Energy Density $e' = E'_{\rm int}/V'$

Magnetic Field $B' = \sqrt{8\pi\epsilon_{\rm b} e'}$

Free parameter

Injection spectrum

$$dN_{\rm e}/d\gamma_{\rm e} \propto \gamma_{\rm e}^{-p} \text{ for } \gamma_{\rm e} \in (\gamma_{\rm e;\,m},\, \gamma_{\rm e;\,M})$$

Spectral limits

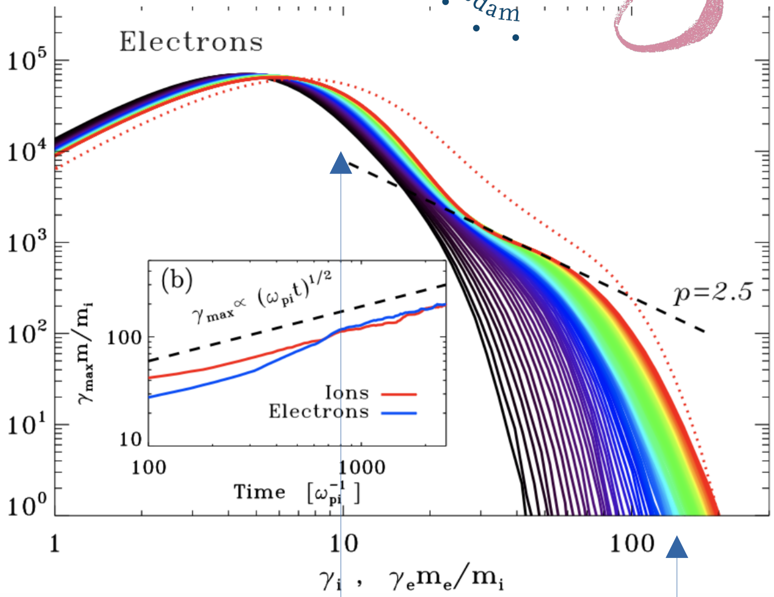
$$\gamma_{\rm e;\,M} = \sqrt{\frac{6\pi q_e}{\sigma_T B' \zeta (1+\tilde{Y})}}$$

Compton parameter

Microphysics parameter

$$\epsilon_{\rm e}\frac{e'}{\rho'}\frac{m_p}{m_e c^2} = \langle\gamma_e\rangle = \left(\frac{p-1}{p-2}\right)\left(\frac{\gamma_{\rm e;\,M}^{-p+2} - \gamma_{\rm e;\,m}^{-p+2}}{\gamma_{\rm e;\,M}^{-p+1} - \gamma_{\rm e;\,m}^{-p+1}}\right)$$

Free parameter



M

m

Solving for minimum

Building Afterglow: Electron Distribution

Electron distribution evolution

$$\frac{\partial}{\partial t'}\frac{dN_e{}'}{d\gamma_e{}'} + \frac{\partial}{\partial \gamma_e{}'}\left(\dot{\gamma}_{\mathrm{e,tot}}{}'\frac{dN_e{}'}{d\gamma_e{}'}\right) = d\dot{N}'_{\mathrm{e},0}$$

Electron injection

$$d\dot{N}'_{\mathrm{e},0} = \dot{N}'_{\mathrm{e}}\frac{p-1}{\gamma'_{\mathrm{m}}}\left(\frac{\gamma_e{}'}{\gamma'_{\mathrm{m}}}\right)^{-p} d\gamma_e{}',$$

Electron energy loss

$$\dot{\gamma}_{\mathrm{e,tot}}{}' = \dot{\gamma}_{\mathrm{e,adi}}{}' + \dot{\gamma}_{\mathrm{e,syn}}{}' + \dot{\gamma}_{\mathrm{e,IC}}{}'.$$

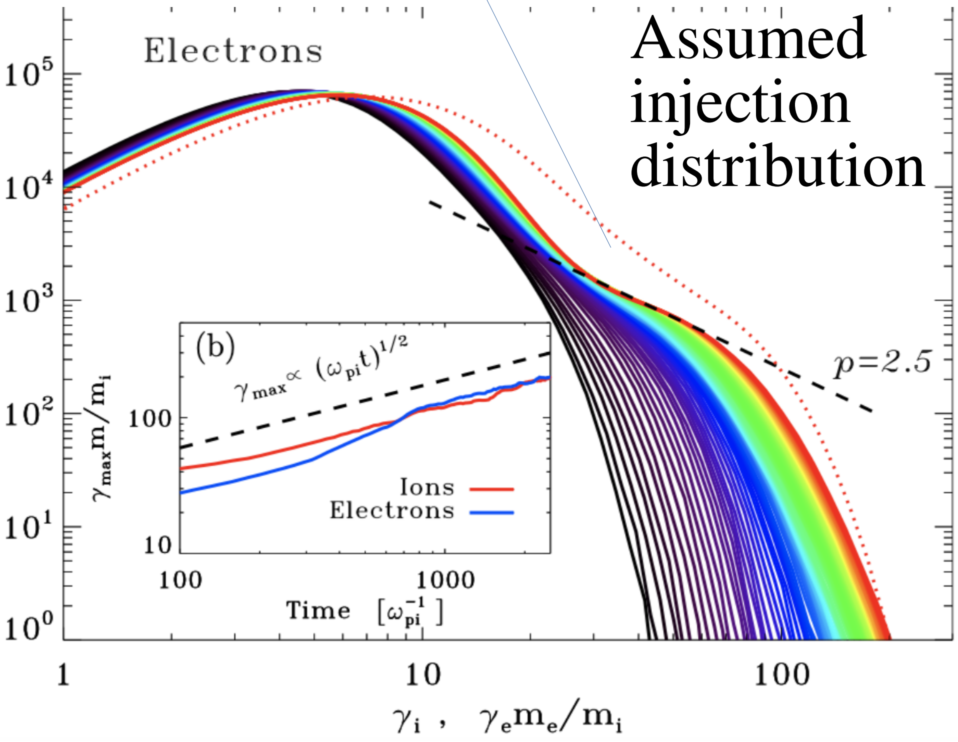
Synchrotron cooling

$$\dot{\gamma}'_{\mathrm{e,syn}} = -\frac{\sigma_T B'^2 \gamma_e{}'^2}{6\pi m_e c}.$$

Adiabatic cooling

$$\dot{\gamma}'_{\mathrm{e,adi}} = \frac{1}{3}\gamma'_e \frac{d\ln n'_e}{dt'} = -\frac{1}{2}\frac{\gamma'_e}{R}\frac{dR}{dt'}$$

Assumed injection distribution



Building Afterglow: SSC cooling

Electrons lose energy by up-scattering (syn) photons

Connects the electron population with existing radiation field

SSC cooling

$$\dot{\gamma}'_{\rm e,IC} = -\frac{1}{m_e c^2}\frac{3\sigma_T c}{4\gamma_e'^2}\int_{\nu_{\rm min}'}^{\nu_{\rm max}'}\frac{n_{\nu'}d\nu'}{\nu'}\int_{\nu_{\rm IC,min}'}^{\nu_{\rm IC,max}'} h\nu_{\rm IC}'d\nu_{\rm IC}'F(q,\, g),$$

Seed photon density

$$n_{\nu'} \approx \frac{T'}{h\nu'}\frac{\sqrt{3}\,e^3 B'}{m_e c^2}\int_{\gamma_{\rm e,min}'}^{\gamma_{\rm e,max}'} \times n'(\gamma_e')\mathcal{R}(\nu'/\nu_c')d\gamma_e',$$

Escape time

$$T' \approx \Delta'/c$$

SSC scattering cross-section

$$F(q,\, g) = 2q\ln q + (1+2q)(1-q) + \frac{1}{2}\frac{(4qg)^2}{1+4gq}(1-q),$$

$$g = \frac{\gamma_e' h\nu'}{m_e c^2},\quad w = \frac{h\nu_{\rm IC}'}{\gamma_e' m_e c^2},\quad \text{and}\quad q = \frac{w}{4g(1-w)}.$$

$$n'(\gamma_e') = \frac{dN_e'/d\gamma_e'}{4\pi\Delta' R^2}$$

Building Afterglow: Synchrotron Emission

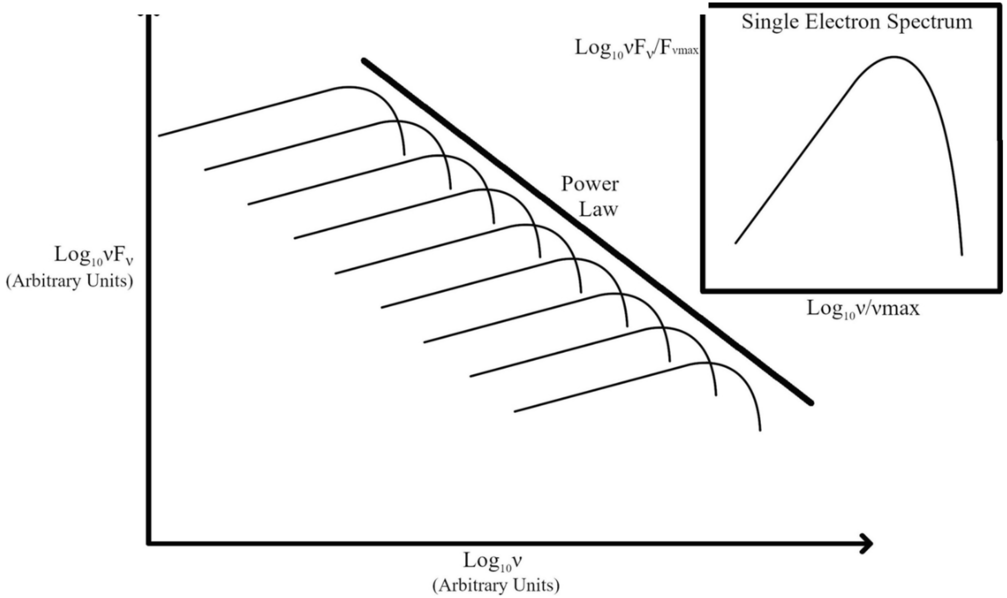
Convolving the emission of a single electron gyrating around magnetic field lines with electron distribution function

Synchrotron emission power

$$P'^{syn}_{\nu'}(\nu',\gamma) = \frac{2\sqrt{3}\,e^3\,B'}{m_e\,c^2}\,x^2\left[K_{4/3}(x)\,K_{1/3}(x) - 0.6x(K^2_{4/3}(x) - K^2_{1/3}(x))\right]$$

$$P'^{syn}_{\nu'}(\nu') = \int P'^{syn}_{\nu'}(\nu',\gamma)\frac{dN}{d\gamma}d\gamma$$

Emission power by a single electron



Synchrotron self-absorption (SSA)

$$\alpha_\nu = -\frac{1}{8\pi\nu'^2 m_e}\int d\gamma P'(\gamma,\nu')\gamma^2\frac{\partial}{\partial\gamma}\left[\frac{N(\gamma)}{\gamma^2}\right]$$

Building Afterglow: SSC emission

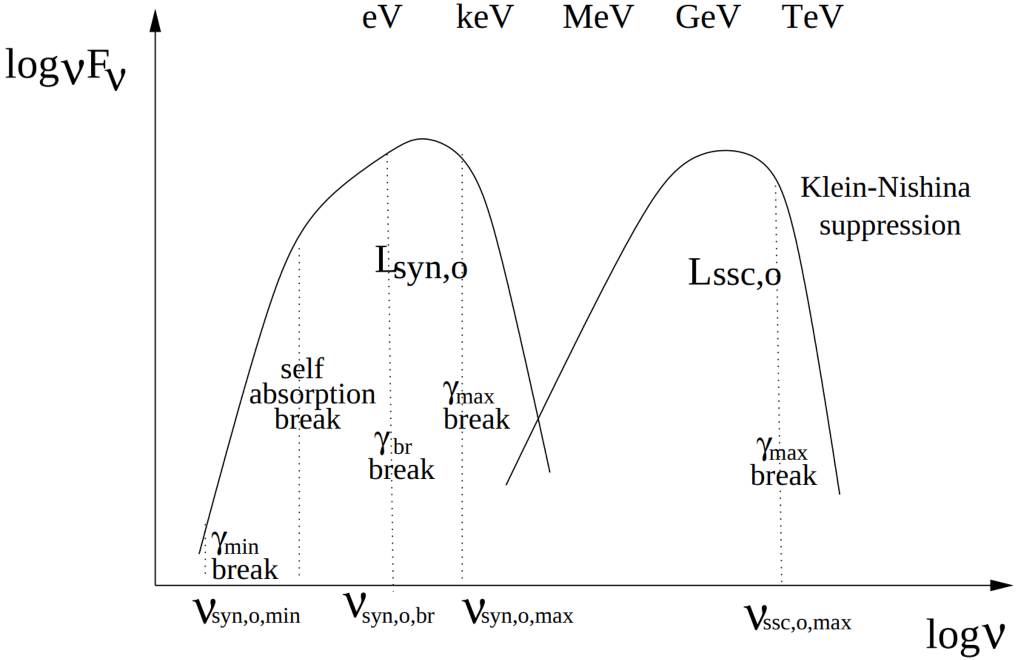
SSC emission power

$$P'_{\mathrm{ssc}}(\nu_{\mathrm{ssc}}') = \int\int \frac{dN_{\mathrm{e}}'}{d\gamma_e'} h\nu_{\mathrm{ssc}}' \frac{dN_\gamma'}{dt'd\nu_{\mathrm{ssc}}'} d\gamma_e'.$$

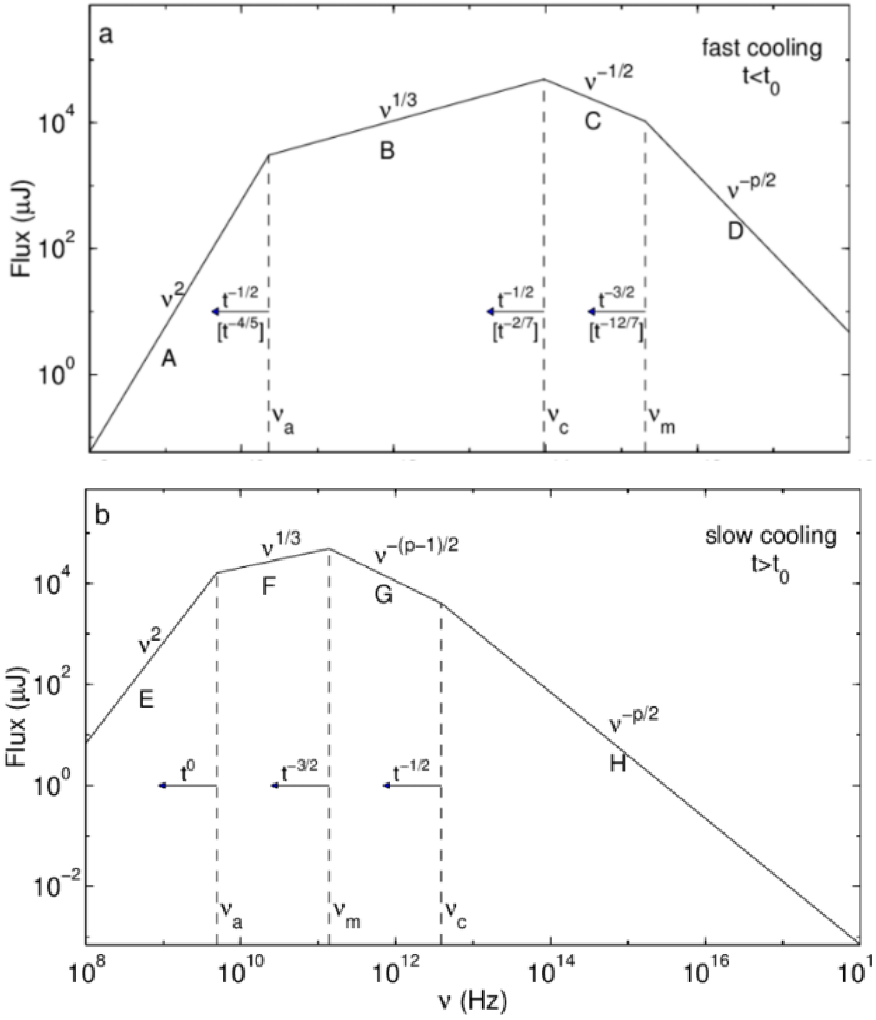
$$\frac{dN_\gamma'}{dt'd\nu_{\mathrm{ssc}}'} = \frac{3\sigma_T c}{4\gamma_e'^2}\frac{n_{\nu'}d\nu'}{\nu'}F(q,g).$$

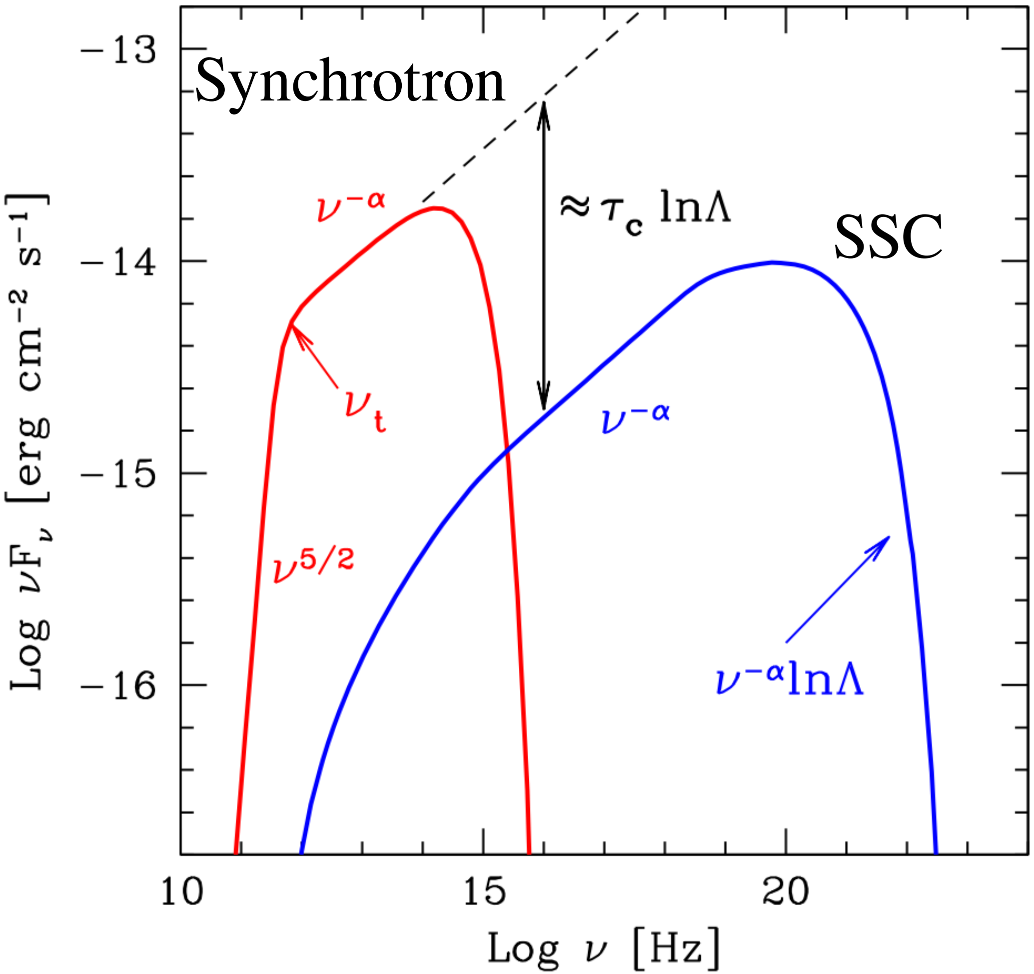
$$n_{\nu'} \approx \frac{T'}{h\nu'}\frac{\sqrt{3}e^3B'}{m_e c^2}\int_{\gamma_{\mathrm{e,min}}'}^{\gamma_{\mathrm{e,max}}'} \times n'(\gamma_e')\mathcal{R}(\nu'/\nu_c')d\gamma_e',$$

$$F(q,g) = 2q\ln q + (1+2q)(1-q) + \frac{1}{2}\frac{(4qg)^2}{1+4qg}(1-q),$$



Building Afterglow: Comoving Spectrum





Building Afterglow: Intensity and Flux Density

From comoving emissivities to observed radiation flux density

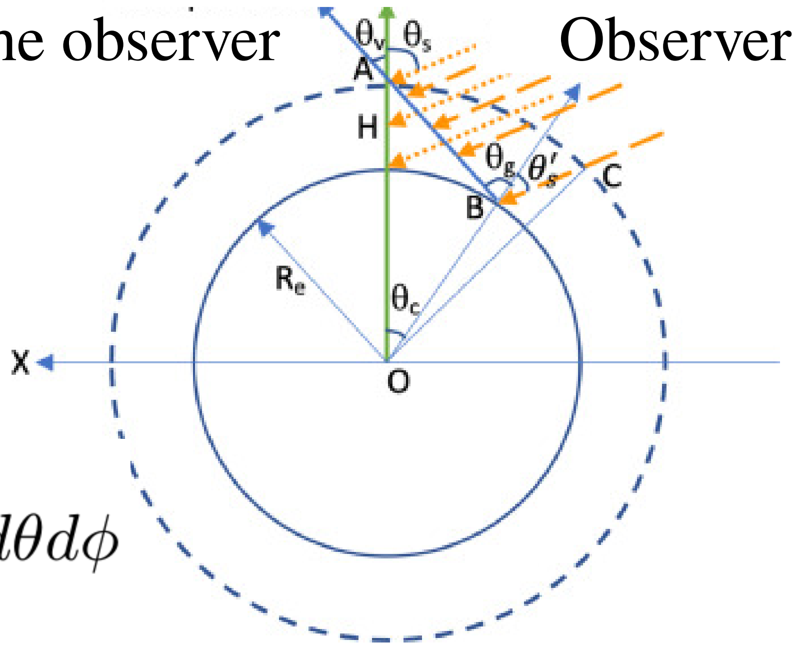
Radiation intensity (observer frame)

$$I_\nu = \frac{j_\nu}{\alpha_\nu}\left(1 - e^{-\tau_\nu}\right) \quad \tau_\nu \approxeq -\alpha_\nu \Delta R/\mu'$$

Thickness of the emitting region depends on the observer

$$\Delta R = \Delta R'/(1-\mu\beta_{\rm sh})$$

$$\mu' = \frac{\mu - \beta}{1-\beta\mu},$$ Light aberration



Equal-arrival time surface integration

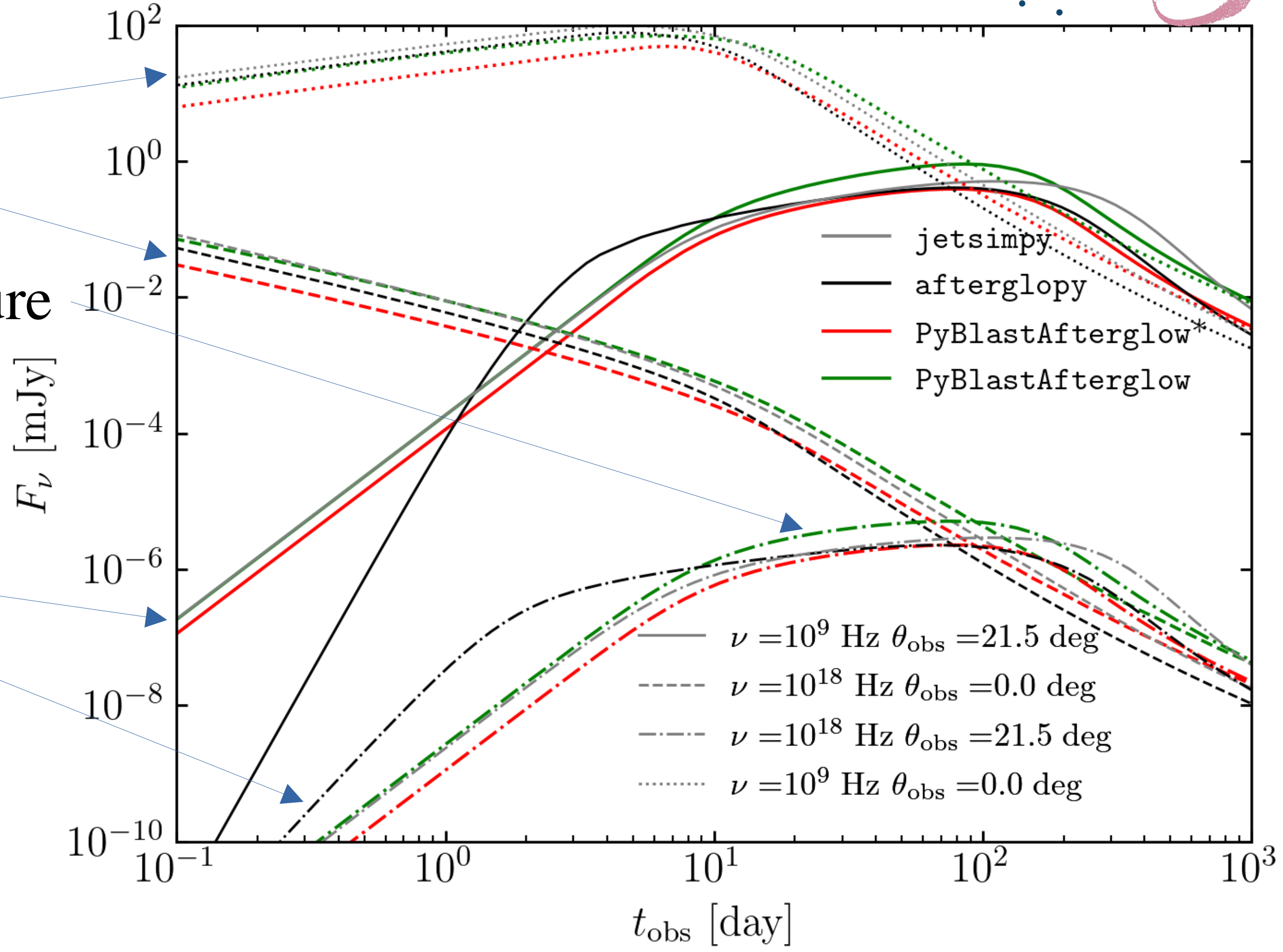
$$F_\nu = \frac{1+z}{2\pi d_L^2}\sum_i^{\text{layers}}\int_{\theta_{i;l}}^{\theta_{i;h}}\int_{\phi_0=0}^{\phi_1=\pi/2} I_{i,\nu}(\theta,\phi)d\theta d\phi$$

Comparison with afterglowpyl; jetsimpy

On-axis afterglow

Shallow slope due to structure

Off-axis afterglow



Comparison:
Gaussian Jet
Forward Shock only
Synchrotron only

Example: sky map

Sky map – intensity distribution projected onto a plane ortogonal to the line of sight.

Properties:
- Centroid position
- Size

Shows:
- geometry of the ejecta
- jet inclination angle

Flux centroid

Origin

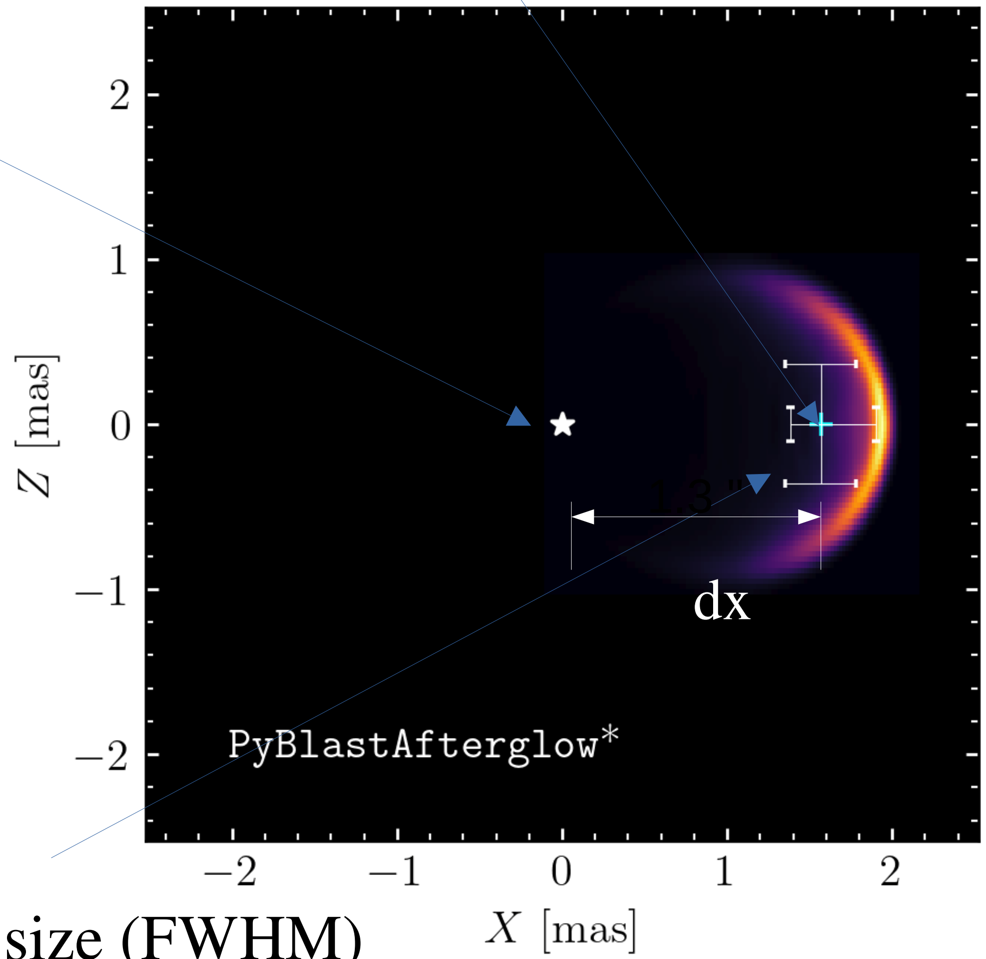
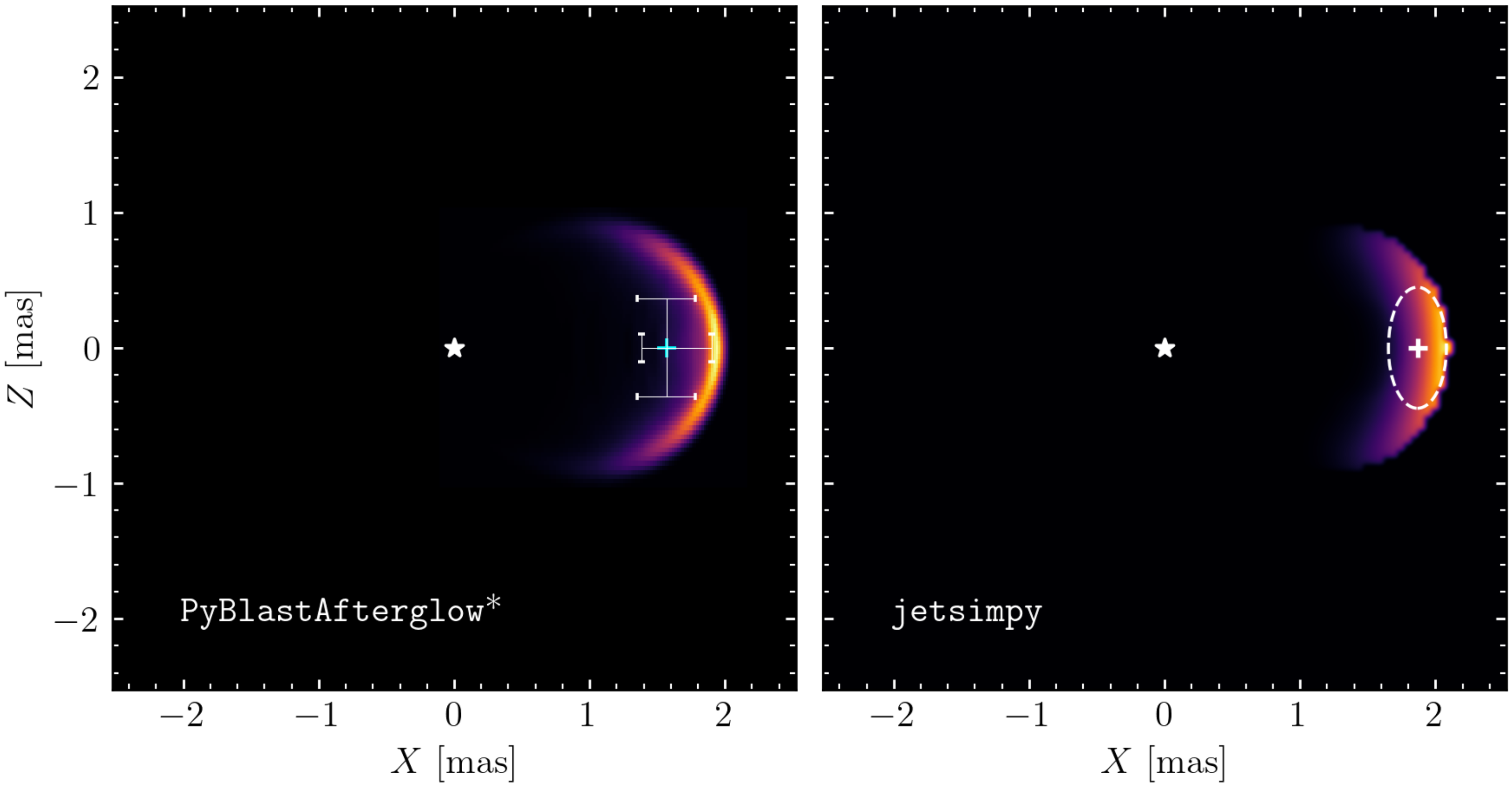


Image size (FWHM)

Comparison with jetsimpy

Comparison:
Gaussian Jet
Forward Shock only
Synchrotron only



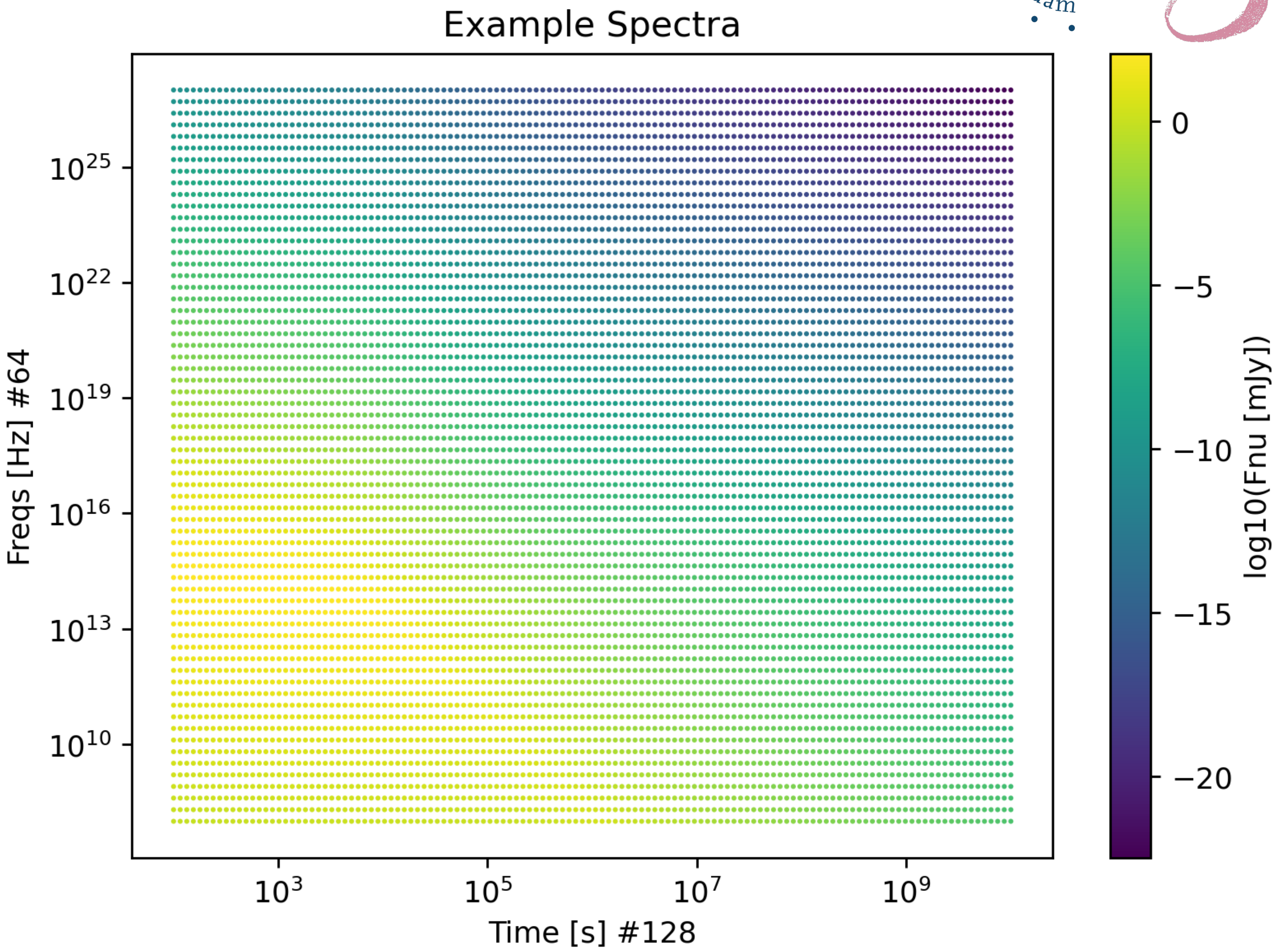
Building a Surrogate Model: Dataset

Building train dataset (with afterglowpy)

- Set of time-dependent Spectra (images)

~20 Gb for ~6400 spectra

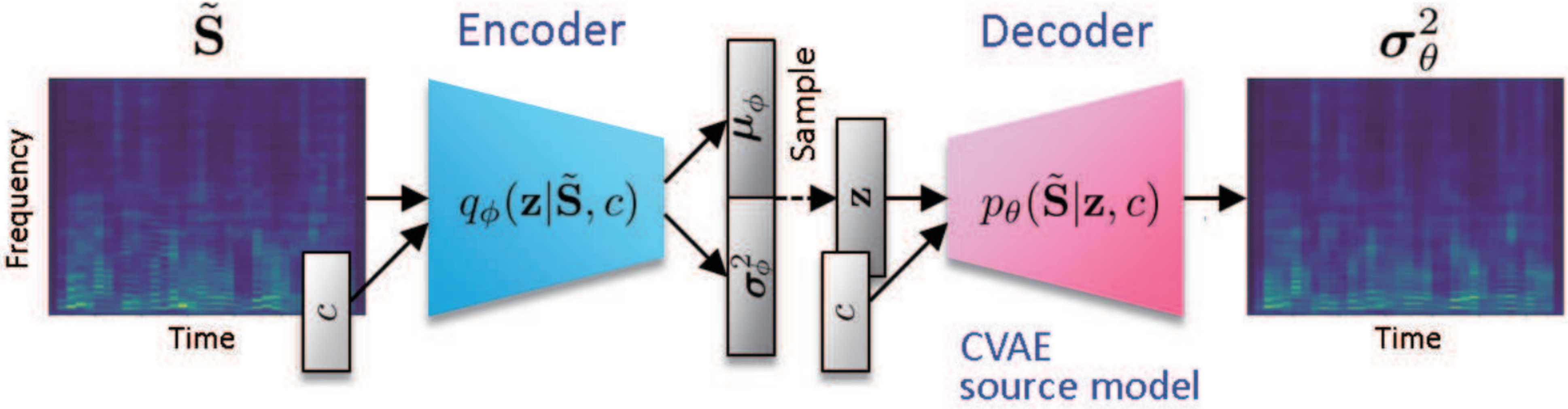
Parameters:
"n_ism", "theta_obs", "Eiso_c", "Gamma0c", "theta_c", "theta_w", "p_fs", "eps_e_fs", "eps_b_fs"



Building a Surrogate Model: Architecture

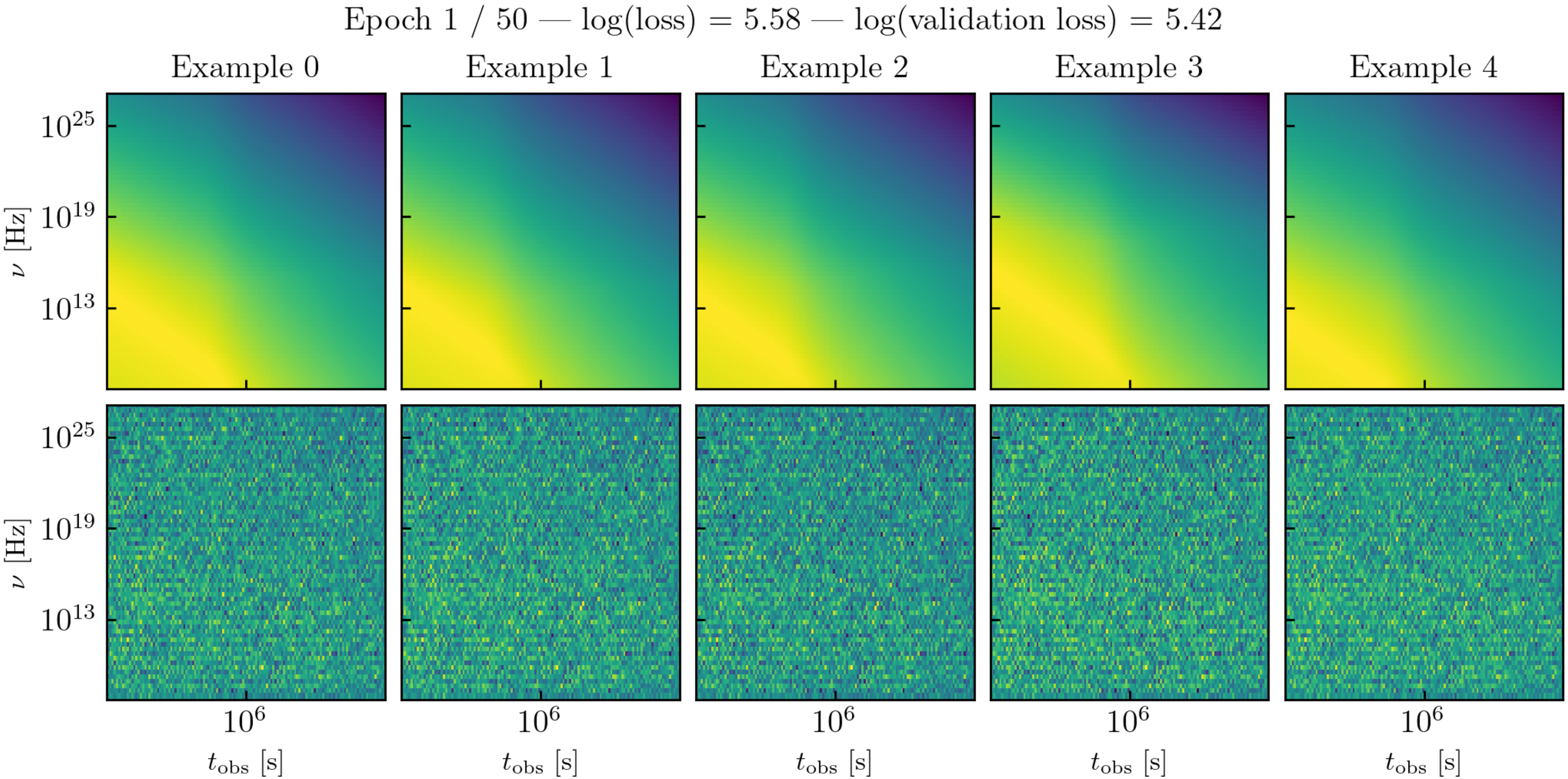
Neural networds; generative model; conditional variational autoencoder (cVAE)

Kameoka+18



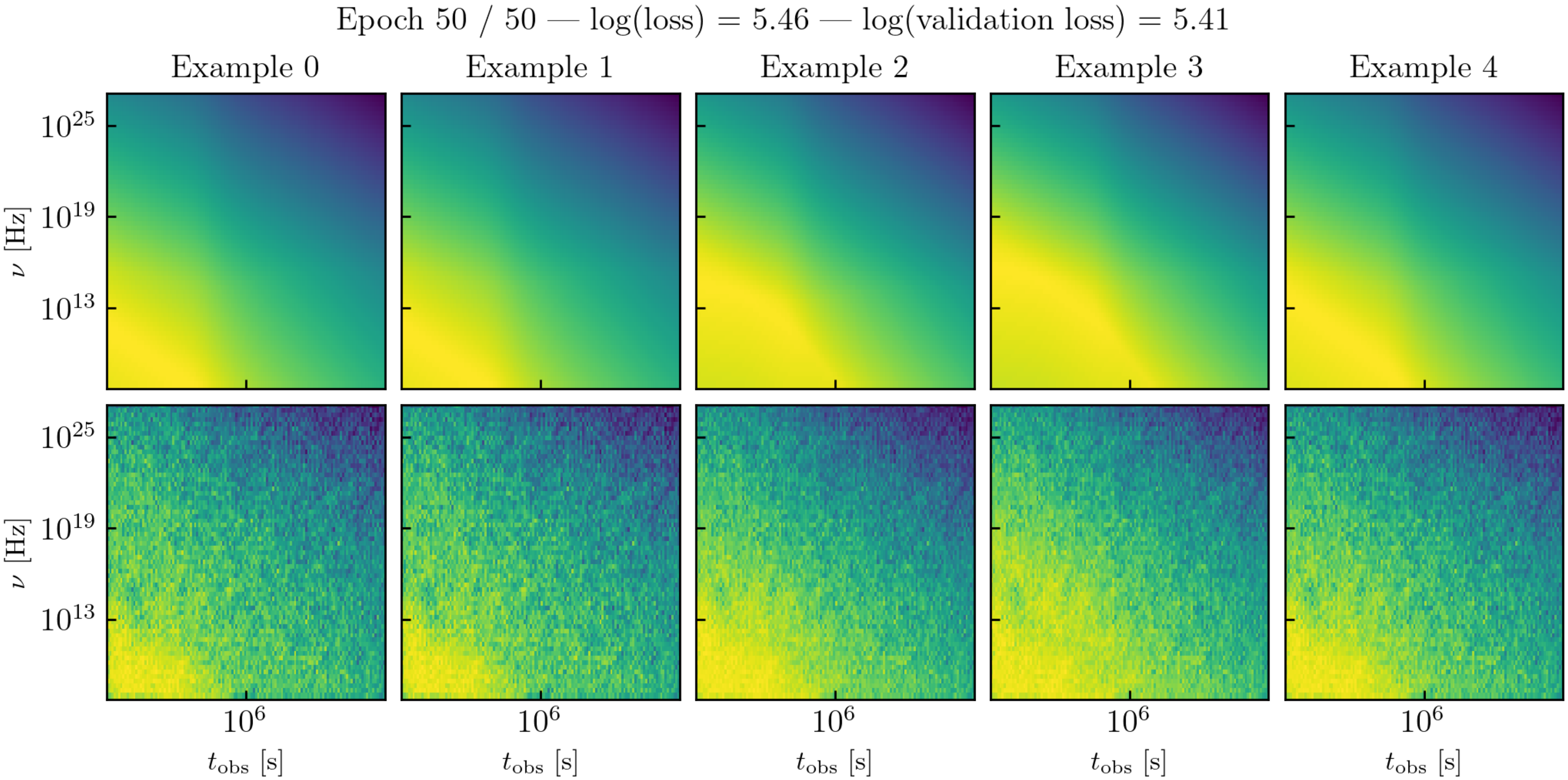
Building a Surrogate Model: Training

Training model on a 1000 spectra : Starting Epoch



Building a Surrogate Model: Training

Training model on a 1000 spectra : Last Epoch



Summary

- GRB afterglow model:
- Ejecta structure (jet structure)
- Blastwave dynamics with collisionless shocks; coasting; spreading
- Comoving electron distribution evolution
- Comoving radiation (synchrotron & SSC) calculation
- Integrating over EATS to obtain observables (spectra, sky maps)

- **Main work areas:**
  - Finishing the code (SSC; test cases; stability)
  - Finilizing surrate model
  - Integrating with NMMMA