ДЗ 1. Кінематика

① Дано:

$\vec{u}_0 = 10 \frac{м}{с}$

$\alpha_1 = 15°$

$\alpha_2 = 75°$

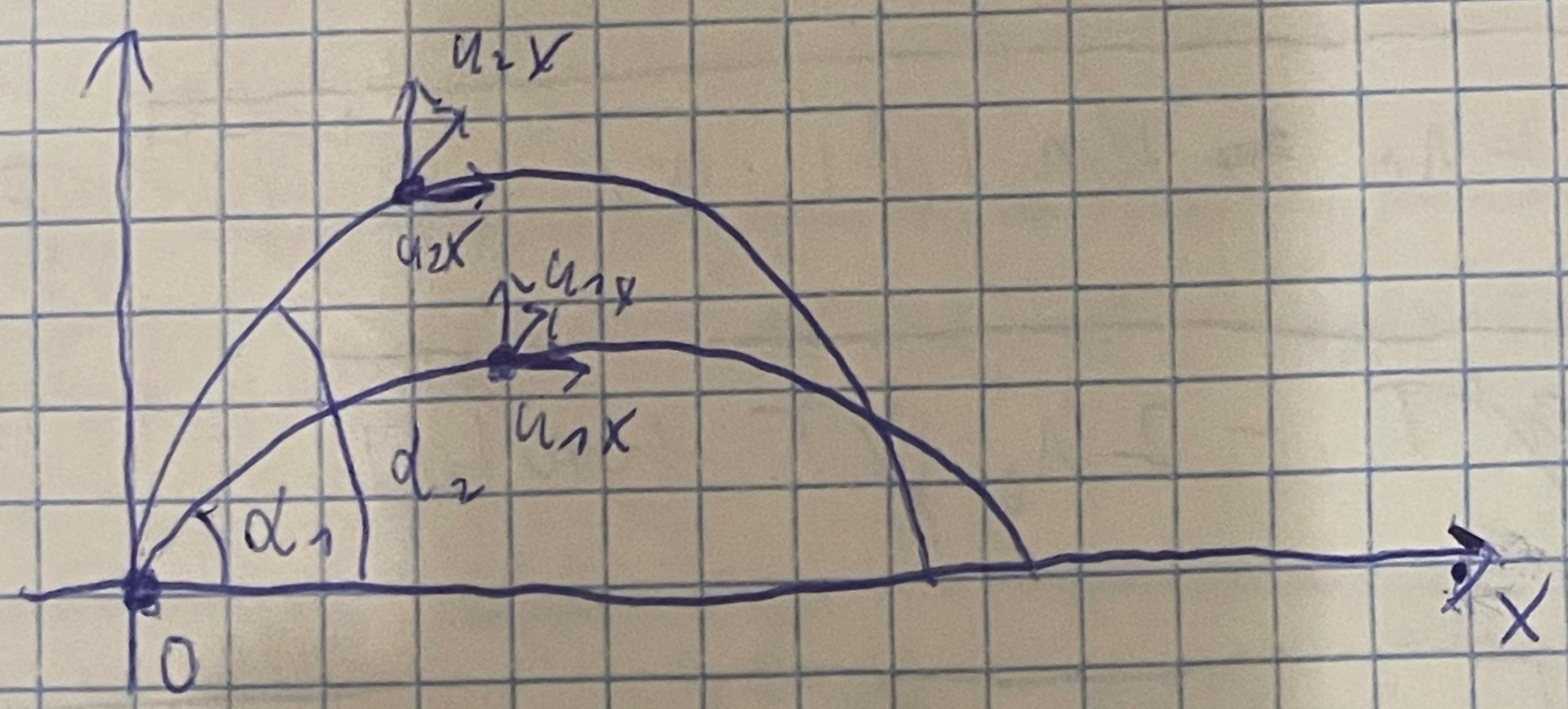
$T = 2,5с$

$g = 9,8 \frac{м}{с^2}$

$u_1(T) - ?$

$u_2(T) - ?$

$S - ?$



$$l = \frac{u_0^2 \sin(2\alpha)}{g} \Rightarrow l = max$$

$\alpha = 45° \Rightarrow$ тіло 1 буде далі

1) Швидкість через T:

1: $u_{1x}(T) = u_0 \cos(\alpha_1)$

$u_{1y}(T) = u_0 \cos(\alpha_1) - gT$

2: $u_{2x}(T) = u_0 \cos(\alpha_2) T$

$u_{2y}(T) = u_0 \sin(\alpha_2) - gT$

$$u = \sqrt{u_x^2 + u_y^2}$$

$$u(T) = \sqrt{(u_0 \cos(\alpha))^2 + (u_0 \sin(\alpha) - g)^2} =$$

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$$= \sqrt{u_0^2 \cos^2(\alpha) + u_0^2 \sin^2(\alpha) - 2u_0 \sin(\alpha)\, gT + g^2T^2} =$$

$$= \sqrt{u_0^2 - 2u_0 \sin(\alpha) \cdot gT + g^2T^2} =$$

$$= \sqrt{u_0^2 + g^2T^2 - 2u_0\, gT \sin(\alpha)}$$

$$u_1(T) = \sqrt{10^2 + 9{,}8^2 \cdot 2{,}5^2 - 2 \cdot 10 \cdot 9{,}8 \cdot 2{,}5 \cdot \sin(75)} =$$

$$= 23{,}95 \ \frac{м}{с}$$

$$u_2(T) = \sqrt{10^2 + 9{,}8^2 + 2 \cdot 10 \cdot 9{,}8 \cdot 2{,}5 - \sin(75)} =$$

$$= 15{,}07 \ \frac{м}{с}$$

Відстань S: $S = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$x(T) = u_0 \cos(\alpha) T$$

$$y(T) = u_0 \sin(\alpha) T - \frac{gT^2}{2}$$

$$S = \sqrt{(u_0 \cos(\alpha_1) T - u_0 \cos(\alpha_2) T)^2 + \left(u_0 \sin(\alpha_1) T - \frac{gT^2}{2} - \left(u_0 \sin(\alpha_2) T - \frac{gT^2}{2}\right)\right)^2} =$$

$$= \sqrt{(u_0 T (\cos(\alpha_1) - \cos(\alpha_2)))^2 + (T u_0 (\sin(\alpha_1) - \sin(\alpha_2)))^2} =$$

$$= \sqrt{T^2 u_0^2 ((\cos(\alpha_1) - \cos(\alpha_2))^2 + (\sin(\alpha_1) - \sin(\alpha_2))^2)} =$$

$$= T u_0 \sqrt{(\cos(\alpha_1) - \cos(\alpha_2))^2 + (\sin(\alpha_1) - \sin(\alpha_2))^2} =$$

$$= T u_0 \sqrt{2 - 2(\cos(\alpha_1)\cos(\alpha_2) + \sin(\alpha_1)\sin(\alpha_2))} =$$

$$= T u_0 \sqrt{2(1 - \cos(\alpha_1 - \alpha_2))} = T u_0 \sqrt{2 - 2\sin^2\left(\frac{\alpha_1 - \alpha_2}{2}\right)} =$$

$$= T u_0 2 \cdot \left| \sin\left(\frac{\alpha_1 - \alpha_2}{2}\right) \right| = 2 \cdot 10 \cdot 2,5 \cdot 0,5 =$$

$$= 25 \text{ м}.$$

Відповідь: $u_1(T) = 23,95 \frac{м}{с}$; $u_2(T) = 15,07 \frac{м}{с}$;

$S = 25$ м.

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④

① Дано:

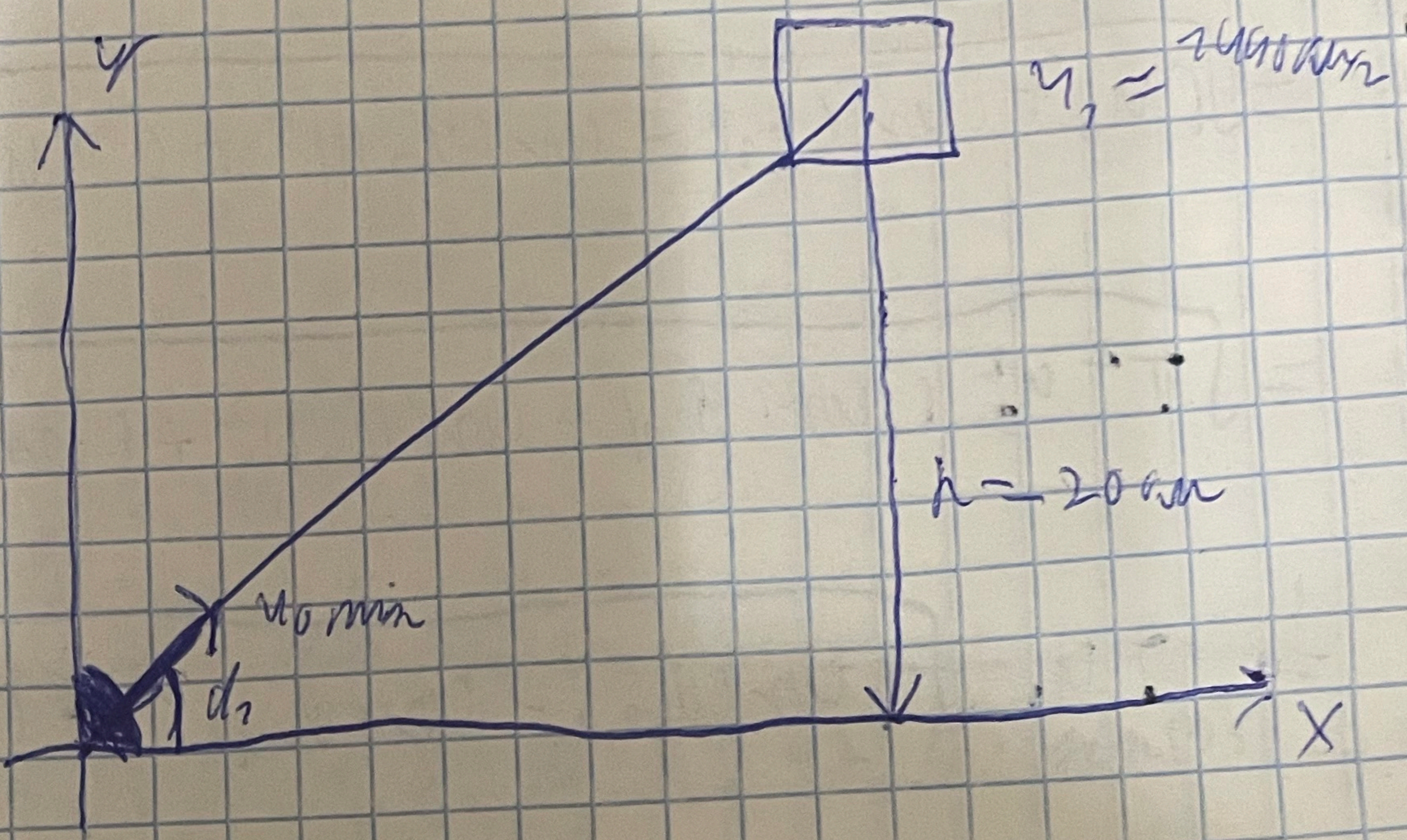
$v_1 = 1440 \frac{км}{год}$

$h = 20$ км

$\vec{g} = 9,8$ м/с

$\vec{v}_{0min}$ – ?

α_1 – ?



Швидкість т. в момент влучання на висоті h:

$v_{0x} = v_1$ ($v_x = v_{0x}$). Тоді:

$$h = \frac{v_0^2 \sin^2(\alpha)}{2g} \Rightarrow v_0^2 \sin^2(\alpha) = h\,2g \Rightarrow$$

$$\Rightarrow v_0 \sin(\alpha) = \sqrt{h\,2g}$$

$$v_x(t) = v_0 \cos(\alpha) \Rightarrow (v_x = v_{0x} = v_1) \Rightarrow$$

$$v_1 = v_0 \cos(\alpha)$$

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Мін. початкові швидкість в момент $t=0$:

$$\operatorname{tg}(\alpha) = \frac{v_0 \sin(\alpha)}{v_0 \cos(\alpha)} = \frac{\sqrt{h\,2g}}{v_1}$$

$$\operatorname{tg}(\alpha) = \frac{\sqrt{2 \cdot 9{,}8 \cdot 2000}}{400} = \frac{7\sqrt{5}}{10}$$

$$\alpha = \operatorname{arctg}\left(\frac{7\sqrt{5}}{10}\right) = 57{,}43^\circ$$

$$v_0 = \sqrt{v_x^2 + v_y^2} = \sqrt{(\sqrt{2gh})^2 + (v_1)^2} =$$

$$= \sqrt{2gh + v_1^2} = \sqrt{2 \cdot 9{,}8 \cdot 2000 + 160000} = 742{,}42\ \frac{м}{с}$$

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(6)

(3) Дано

$\vec{r} = \alpha t \vec{i} + \beta t^2 \vec{j}$

$y(x) - ?$ $|\vec{v}(t)| - ?$

$\vec{u}(t) - ?$ $\vec{a}(t) - ?$

$|\vec{a}(t)| - ?$

$\vec{u}(t) = \frac{d\vec{r}}{dt} =$

$= \frac{d}{dt}(\alpha t)\vec{i} + \frac{d}{dt}(\beta t^2)\vec{j} =$

$= \alpha \vec{i} + 2\beta t \vec{j}$

$\vec{a}(t) = \frac{d\vec{u}}{dt} = \frac{d}{dt}(\alpha)\vec{i} +$

$+ \frac{d}{dt}(2\beta t)\vec{j} = 2\beta \vec{j}$

$|\vec{u}(t)| = \sqrt{(\alpha)^2 + (2\beta t)^2} = \sqrt{\alpha^2 + 4\beta^2 t^2}$

$|\vec{a}(t)| = \sqrt{(2\beta)^2} = |2\beta| = 2|\beta|$

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \Rightarrow x = x(t),\ y = y(t),\ z = z(t)$

$x(t) = \alpha t \Rightarrow t = \frac{x}{\alpha}$ $\quad y(t) = \beta t^2 = \beta\left(\frac{x}{\alpha}\right)^2 =$

$= \frac{\beta x^2}{\alpha^2} \Rightarrow \underline{y(x) = \frac{\beta x^2}{\alpha^2}}$

В: $y(x) = \frac{\beta x^2}{\alpha^2}$; $\vec{u}(t) = \alpha\vec{i} + 2\beta t$; $\vec{a}(t) = 2\beta\vec{j}$

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② Дано

$\vec{r} = \alpha t \vec{i} + \beta t^2 \vec{j}$

$\alpha = 3$ м/с

$\beta = 2,25$ м/с

$t = 2$ с

$\vec{r}$ - ? v - ?

$y(x)$ - ?

$$v_x(t) = \int_0^t a_x\,dt = \int_0^t \alpha t\,dt = \frac{\alpha t^2}{2} = \frac{3 \cdot 2^2}{2} = 6 \text{ м/с}$$

$$v_y(t) = \int_0^t a_y\,dt = \int_0^t \beta t^2\,dt = \frac{\beta t^3}{3} = \frac{2,25 \cdot 2^{3=8}}{3} = \frac{18}{3} = 6 \text{ м/с}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 6^2} = 6\sqrt{2} \ \frac{м}{с}$$

$$x(t) = \int_0^t v_x\,dt = \int_0^t \frac{\alpha t^2}{2}\,dt = \frac{\alpha t^3}{6} = \frac{3 \cdot 2^3}{6} = 4 \text{ м}$$

$$y(t) = \int_0^t v_y\,dt = \int_0^t \frac{\beta t^3}{3}\,dt = \frac{\beta t^4}{12} = \frac{2,25 \cdot 2^4}{12} = 3 \text{ м}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2} = 5 \text{ м}$$

$$x = \frac{\alpha t^3}{6} \Rightarrow t^3 = \frac{6x}{\alpha} \Rightarrow t = \left(\frac{6x}{\alpha}\right)^{\frac{1}{3}}$$

$$y = \frac{\beta t^4}{12} = \frac{\beta}{12}\left(\frac{6x}{\alpha}\right)^{\frac{4}{3}} = \frac{2,25}{12}\left(\frac{6x}{3}\right)^{\frac{4}{3}} =$$

$$= \frac{9/4}{12} \cdot (2x)^{\frac{4}{3}} = \frac{3}{16}(2x)^{\frac{4}{3}}$$

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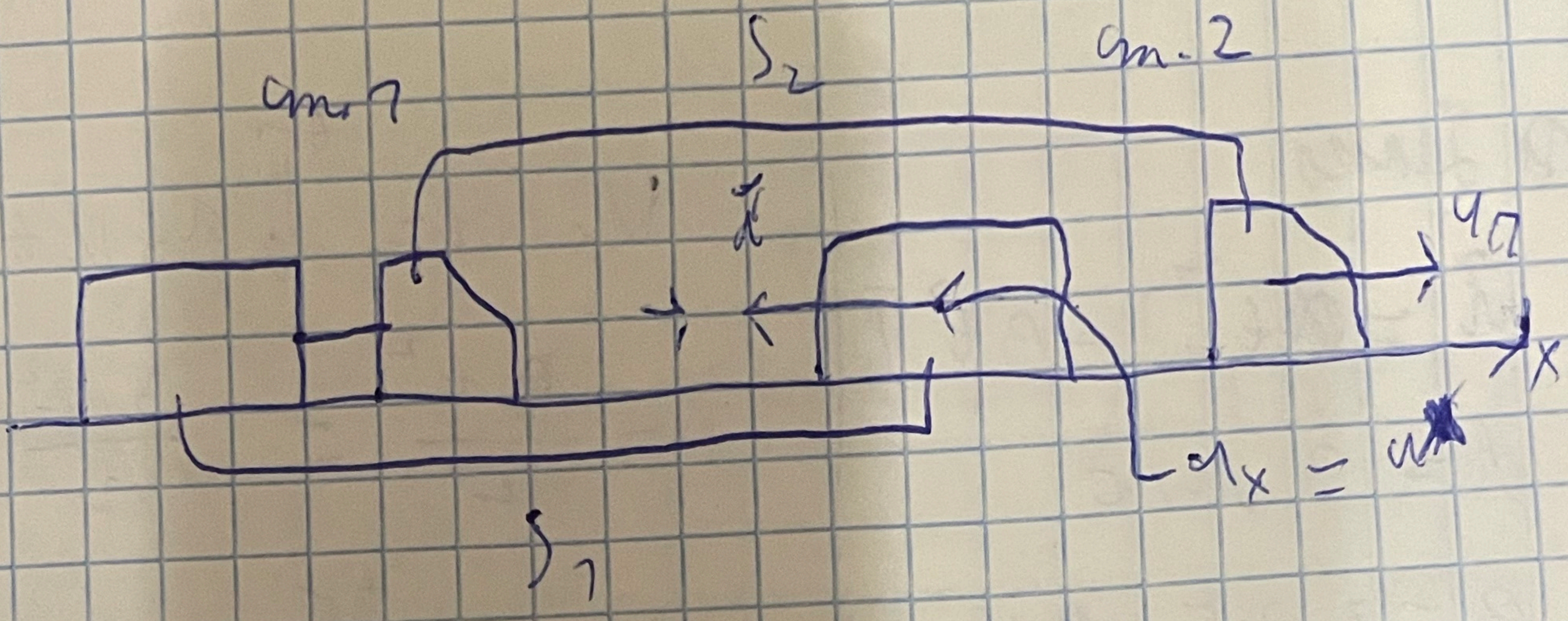
(5) Дано

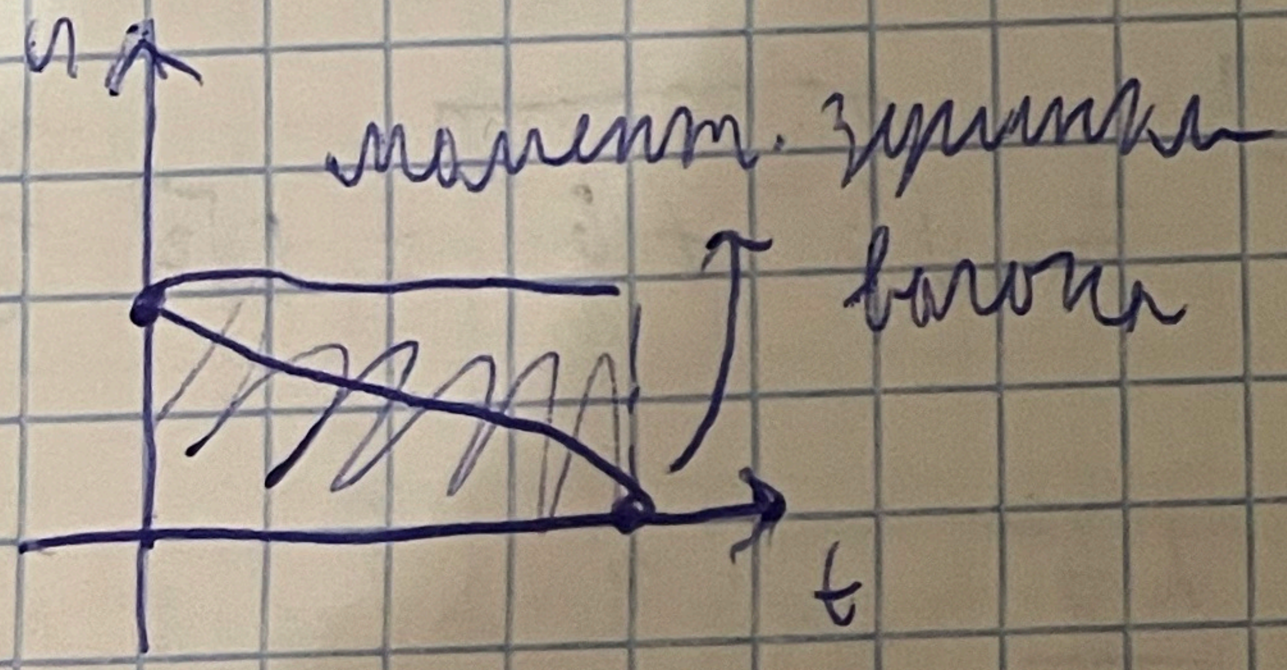
$v_п = const$

$S_1 - ?$



Для графіків рівномірн. та рівносповіл.:

$$a_x = \frac{v_x - v_{0x}}{t} \Rightarrow v_x = v_{0x} - a_x t \text{ (рівн. спов.)}$$



$$S_п = v_{0x} \cdot t \text{ (мом. зуп.)}$$

$$S_{x_1} = \frac{v_x^2 - v_{0x}^2}{2a_x} \Rightarrow$$

$$\Rightarrow S_{x1} = \frac{v_{0x}^2}{2\frac{v_{0x}}{t}} \Rightarrow S_{x1} = \frac{v_{0x}^2}{\frac{2v_x}{t}} \Rightarrow S_{x1} = \frac{1}{2} \cdot v_{0x} t$$

$$v_{0x} t = 2 S_{x1} \Rightarrow S_п = 2 S_{x1}$$

В: $S_п = 2 S_1$