1.

$$u = 2^y(x^2+1)$$

$$\frac{\partial^2 u}{\partial x^2} x \ln 2 = \frac{\partial^2 u}{\partial x \partial y}$$

$$u_x = \frac{\partial}{\partial x}(2^y(x^2+1)) = \frac{\partial}{\partial x}(2^y x^2 + 2^y) = 2^y 2x + 0 = 2^{y+1}x$$

$$u_{xx} = \frac{\partial}{\partial x}(2^{y+1}x) = 2^{y+1}$$

$$u_{xy}\frac{\partial}{\partial y}(2^{y+1}x) = x\frac{\partial}{\partial y}(2^{y+1}) = x \ln 2 \times 2^{y+1}$$

$$u_{xx} x \ln 2 = (2^{y+1}) x \ln 2 = x \ln 2 \times 2^{y+1}$$

$$\text{Отже, } \frac{\partial^2 u}{\partial x^2} x \ln 2 = \frac{\partial^2 u}{\partial x \partial y}$$

2.

$$z = xy - 2x - y, x = 3, x = 0, y = 0, y = 4$$

$$\begin{cases} z_x = \frac{\partial z}{\partial x} = y - 2 = 0 \to y = 2 \\ z_y = \frac{\partial z}{\partial y} = x - 1 = 0 \to x = 1 \end{cases}$$

$$z(1,2) = 2 - 2 - 2 = -2$$

$$z(0,y) = 0y - 0 - y = -y \to \begin{cases} y = 0 \to z = 0 \\ y = 4 \to z = -4 \end{cases}$$

$$z(3,y) = 3y - 6 - y = 2y - 6 \to \begin{cases} y = 0 \to z = -6 \\ y = 4 \to z = 2 \end{cases}$$

$$z(x,0) = 0x - 2x - 0 = -2x \to \begin{cases} x = 0 \to z = 0 \\ x = 3 \to z = -6 \end{cases}$$

$$z(x,4) = 4x - 2x - 4 = 2x - 4 \to \begin{cases} x = 0 \to z = -4 \\ x = 3 \to z = 2 \end{cases}$$

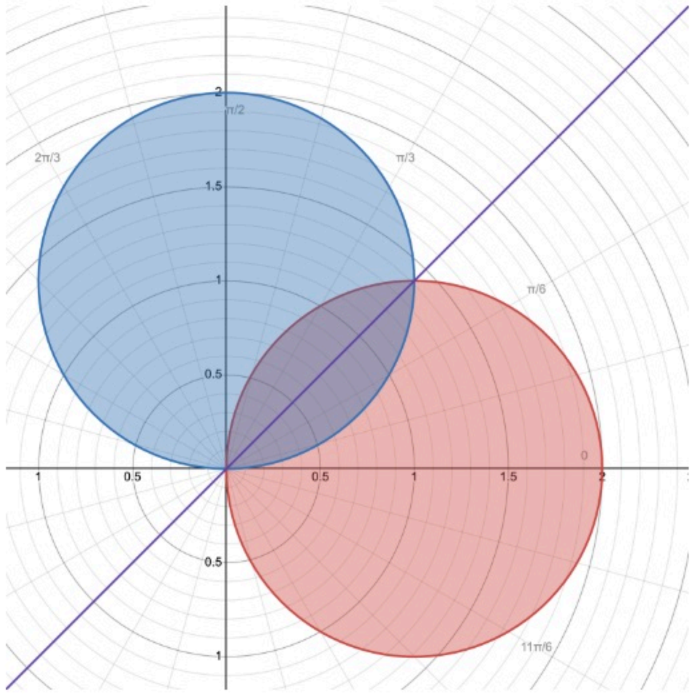
x	y	z
1	2	-2
0	0	0
0	4	-4
3	0	-6
3	4	2

$$z_{max} = 2$$
$$z_{min} = -6$$

3.

$$\iint \sqrt{x^2+y^2}dxdy$$

$$D : (x-1)^2 + y^2 \leq 1, x^2 + (y-1)^2 \leq 1$$



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$\sqrt{x^2+y^2} = r$$
$$dxdy = rdrd\theta$$

$$(r\cos\theta - 1)^2 + (r\sin\theta)^2 \leq 1 \Rightarrow r^2 - 2r\cos\theta + 1 \leq 1 \Rightarrow r \leq 2\cos\theta$$
$$(r\cos\theta)^2 + (r\sin\theta - 1)^2 \leq 1 \Rightarrow r^2 - 2r\sin\theta + 1 \leq 1 \Rightarrow r \leq 2\sin\theta$$

$$\int_0^{\pi/4}\int_0^{2\sin\theta} r^2 dr d\theta + \int_{\pi/4}^{\pi/2}\int_0^{2\cos\theta} r^2 dr d\theta$$

$$\int_0^{\pi/4} d\theta \int_0^{2\sin\theta} r^2 dr = \int_0^{\pi/4} \frac{r^3}{3}\Bigg|_0^{2\sin\theta} d\theta = \int_0^{\pi/4} \frac{(2\sin\theta)^3}{3} d\theta = \frac{8}{3}\int_0^{\pi/4} \sin^3\theta d\theta = \frac{8}{3}\left(-\cos\theta + \frac{\cos^3\theta}{3}\right)\Bigg|_0^{\pi/4} = \frac{8}{3}\left(\frac{-5\sqrt{2}}{12} + \frac{2}{3}\right)$$

$$\int_{\pi/4}^{\pi/2} d\theta \int_0^{2\cos\theta} r^2 dr = \int_{\pi/4}^{\pi/2} \frac{r^3}{3}\Bigg|_0^{2\cos\theta} d\theta = \int_{\pi/4}^{\pi/2} \frac{(2\cos\theta)^3}{3} d\theta = \frac{8}{3}\int_{\pi/4}^{\pi/2} \cos^3\theta d\theta = \frac{8}{3}\left(\sin\theta - \frac{\sin^3\theta}{3}\right)\Bigg|_{\pi/4}^{\pi/2} = \frac{8}{3}\left(\frac{-5\sqrt{2}}{12} + \frac{2}{3}\right)$$

$$\int_0^{\pi/4}\int_0^{2\sin\theta} r^2 dr d\theta + \int_{\pi/4}^{\pi/2}\int_0^{2\cos\theta} r^2 dr d\theta = \frac{2}{3}\left(\frac{16-10\sqrt{2}}{3}\right) = \frac{32-20\sqrt{2}}{9}$$

4.

$$\iiint (x^2+y) z dx dy dz$$

$$V : x^2 + y - z^2 = -1, x^2 + y^2 = 9, y \le x, y \ge \frac{x}{\sqrt{3}}$$

$$x^2 + y - z^2 = -1 \Rightarrow z \in \left[-\sqrt{x^2+y+1}, \sqrt{x^2+y+1}\right] \Rightarrow A = \sqrt{x^2+y+1}$$

$$\iiint (x^2+y) z dx dy dz = \int\int_{xy} dx dy \int_{z=-A}^{A} (x^2+y) z dz = \int\int_{xy} (x^2+y)\frac{z^2}{2}\Bigg|_{-A}^{A} dx dy = \int\int_{xy} (x^2+y)\times 0 \; dx dy = 0$$