$$\int_{2}^{3} x \ln(x-1) dx = \begin{vmatrix} t = x - 1 \\ x = t + 1 \\ dx = dt \\ x = 2 \to t = 1 \\ x = 3 \to t = 2 \end{vmatrix} = \int_{1}^{2} (t+1) \ln t dt = \int_{1}^{2} t \ln t dt + \int_{1}^{2} \ln t dt =$$

$$= \begin{vmatrix} u = \ln t & v = \frac{t^{2}}{2} \\ du = \frac{dt}{t} & dv = t dt \end{vmatrix} \int t \ln t dt = \frac{t^{2}}{2} \ln t - \int \frac{t^{2}}{2t} dt = \frac{t^{2}}{2} \ln t - \frac{1}{2} \int t dt = \frac{t^{2}}{2} \ln t - \frac{t^{2}}{4} =$$

$$= \begin{vmatrix} \int_{1}^{2} t \ln t dt = \left(\frac{t^{2}}{2} \ln t - \frac{t^{2}}{4}\right) \Big|_{1}^{2} = 2 \ln 2 - 1 - \left(\frac{1}{2} \ln 1 - \frac{1}{4}\right) = 2 \ln 2 - \frac{3}{4} =$$

$$= \begin{vmatrix} \int_{1}^{2} \ln t dt = t \ln t - t \Big|_{1}^{2} = 2 \ln 2 - 2 - (\ln 1 - 1) = 2 \ln 2 - 1 =$$

$$= 2 \ln 2 - \frac{3}{4} + 2 \ln 2 - 1 = \frac{16 \ln 2 - 7}{4}$$

23.2

$$\int_0^1 rac{3x^4+3x^2+1}{x^2+1} dx = \int_0^1 rac{1}{x^2+1} + 3x^2 dx = \int_0^1 rac{1}{x^2+1} dx + 3 \int_0^1 x^2 dx = = rctg x igg|_0^1 + x^3 igg|_0^1 = rac{\pi}{4} + 1$$

$$\int_{3}^{29} rac{\sqrt[3]{(x-2)^2}}{3+\sqrt[3]{(x-2)^2}} dx = \left|egin{array}{c} t = \sqrt[3]{x-2} 
ightarrow x = t^3+2 \ dt = rac{dx}{3t^2} 
ightarrow dx = 3t^2 dt \ x = 3 
ightarrow t = 1 \ x = 29 
ightarrow t = 3 \end{array}
ight| = \int_{1}^{3} rac{t^2}{3+t^2} 3t^2 dt = 3 \int_{1}^{3} rac{t^4}{3+t^2} dt = 1$$

$$=\left|\frac{t^4}{t^2+3}=t^2-3+\frac{9}{t^2+3}\right|=3\int_1^3\left(t^2-3+\frac{9}{t^2+3}\right)dt=3\int_1^3t^2dt-9\int_1^3dt+27\int_1^3\frac{dt}{t^2+3}=$$

$$\left. rac{3t^3}{3} 
ight|_1^3 - 9t 
ight|_1^3 + rac{27}{\sqrt{3}} \arctan rac{t}{\sqrt{3}} 
ight|_1^3 = 27 - 1 - 18 + rac{27\pi}{3\sqrt{3}} - rac{27\pi}{6\sqrt{3}} = 8 + rac{3\sqrt{3}\pi}{2}$$

23.4

$$\int_{rac{\pi}{2}}^{\pi} 2^8 \sin^8 x dx = 2^8 \int_{rac{\pi}{2}}^{\pi} \sin^8 x dx = 2^8 imes rac{7!!}{8!!} imes rac{\pi}{2} = rac{256 imes 35\pi}{256} = 35\pi$$

23.5

$$\int_0^2 x^2 \sqrt{4-x^2} dx = \int_0^2 x^2 \sqrt{2^2-x^2} dx = egin{array}{c} x=2\sin t \ t=rcsin x/2 \ 4-x^2=4\cos^2 t \ dx=2\cos t dt \ x=0 o t=0 \ x=2 o t=\pi/2 \ \end{pmatrix} =$$

$$=\int_0^{rac{\pi}{2}} 4 \sin^2 t imes 2 \cos t imes 2 \cos t dt = \int_0^{rac{\pi}{2}} 16 \sin^2 t \cos^2 t dt = 4 \int_0^{rac{\pi}{2}} \sin^2 2t dt = = |u = 2x| = 4 \int_0^{rac{\pi}{2}} rac{\sin^2 u}{2} du = 2 \int_0^{rac{\pi}{2}} rac{1 - \cos 2u}{2} du = \int_0^{rac{\pi}{2}} 1 du - \int_0^{rac{\pi}{2}} \cos 2u du = = u - rac{\sin 2u}{2} = 2t - rac{\sin 4t}{2} = 2\pi/2 - rac{\sin 4\pi/2}{2} + rac{\sin 0}{2} = \pi$$

$$egin{aligned} \int_{rac{\pi}{4}}^{rac{\pi}{2}} rac{\cos^3 x}{\sqrt{\sin x}} dx &= \left|egin{array}{c} u = \sin x \ du = \cos x dx \end{array}
ight| = \ &= \left|\int (1-u^2) u^{-1/2} du = \int u^{-1/2} du - \int u^{3/2} du = 2 u^{1/2} - rac{2}{5} u^{5/2} + C 
ight| = \ &= \left(2 \sqrt{\sin x} - rac{2}{5} \sin^{5/2} x
ight) 
ight|_{rac{\pi}{4}}^{rac{\pi}{2}} &= (2-rac{2}{5}) - \left(rac{2}{\sqrt[4]{2}} - rac{2}{5 imes 2^{5/4}}
ight) = \ &= rac{8}{5} - \left(2 imes 2^{-1/4} - rac{2^{-1/4}}{5}
ight) = rac{8}{5} - rac{9 imes 2^{-1/4}}{5} = rac{8 - 9 imes 2^{-1/4}}{5} \end{aligned}$$

24.1

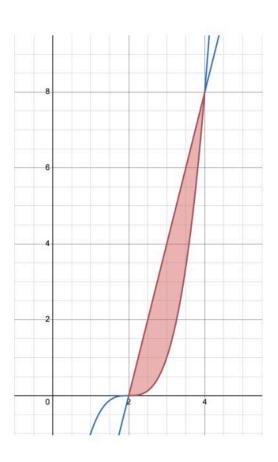
$$\int_{1}^{\infty} \frac{x dx}{\sqrt{16x^4 - 1}} = \lim_{b \to \infty} \int_{1}^{b} \frac{x dx}{\sqrt{16x^4 - 1}} = \begin{vmatrix} u = 4x^2 \\ du = 8x dx \\ x = 1 \to u = 4 \end{vmatrix} = \lim_{b \to \infty} \left( \frac{1}{8} \int_{4}^{b} \frac{du}{\sqrt{u^2 - 1}} \right) = \lim_{b \to \infty} \left( \frac{1}{8} \ln \left| u + \sqrt{u^2 - 1} \right| \right|_{4}^{b} \right) = \lim_{b \to \infty} \left( \frac{1}{8} \left( \ln |b + \sqrt{b^2 - 1}| - \ln |4 + \sqrt{15}| \right) \right) = \lim_{b \to \infty} \left( \ln |\infty + \infty| - \ln |4 + \sqrt{15}| \right) = \infty$$
 — Інтеграл розбіжний

$$\int_0^1 rac{dx}{\sqrt[3]{2-4x}} = \left|2-4x=0
ightarrow x = rac{1}{2}
ight|\int_0^{1/2} rac{dx}{\sqrt[3]{2-4x}} + \int_{1/2}^1 rac{dx}{\sqrt[3]{2-4x}} =$$

 $=rac{3}{8}\left(\left.u^{2/3}
ight|_{0}^{^{2}}-u^{2/3}
ight|_{-2}^{^{0}}
ight)=rac{3}{8}\left(\sqrt[3]{4}-\sqrt[3]{4}
ight)=0$ 

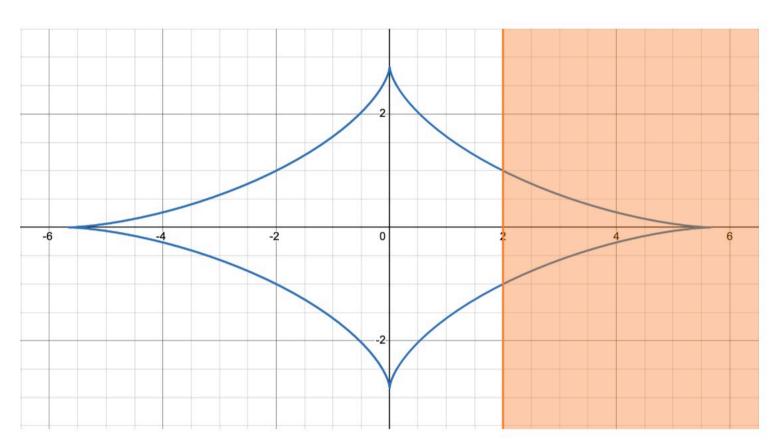
$$egin{aligned} \int_0^1 rac{dx}{\sqrt[3]{2-4x}} &= \left| 2-4x=0 
ightarrow x = rac{1}{2} 
ight| \int_0^{2/2} rac{dx}{\sqrt[3]{2-4x}} + \int_{1/2}^1 rac{dx}{\sqrt[3]{2-4x}} &= \ & u=2-4x \ du=-4dx 
ightarrow dx = -rac{du}{4} \ x=0 
ightarrow u=2 \ x=1/2 
ightarrow u=0 \ x=1 
ightarrow u=-2 \end{aligned} egin{aligned} &= rac{1}{4} \int_0^2 u^{-1/3} du + rac{1}{4} \int_{-2}^0 u^{-1/3} du = \ &= rac{1}{4} \int_0^2 u^{-1/3} du + rac{1}{4} \int_{-2}^0 u^{-1/3} du = \ &= 1 
ightarrow u=-2 \end{aligned}$$

$$y=(x-2)^3, y=4x-8$$
 $(x-2)^3=4(x-2) o u=x-2, u^3=4u o u^2=4$ 
 $u=2, u=-2, u=0$ 
 $x=0, x=2, x=4$ 
 $x=0: y=4 imes 0-8=-8$ 
 $x=2: y=4 imes 2-8=0$ 
 $x=4: y=4 imes 4-8=8$ 



$$S=2\int_{0}^{2}(4u-u^{3})du=2\int_{0}^{2}4udu-2\int_{0}^{2}u^{3}du=2\left(2u^{2}\Big|_{0}^{2}-rac{u^{4}}{4}\Big|_{0}^{2}
ight)=2(8-4)=8$$

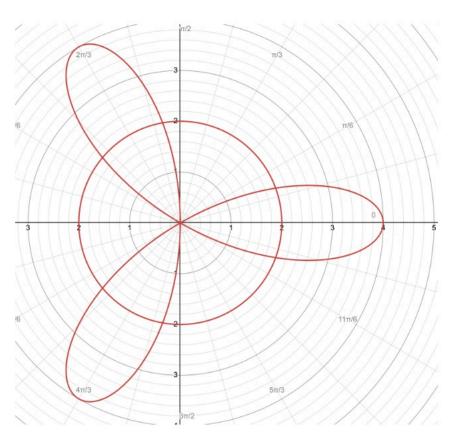
$$egin{cases} x = 4\sqrt{2}\cos^3 t \ y = 2\sqrt{2}\sin^3 t \end{cases} \quad x >= 2$$



$$egin{align*} 4\sqrt{2}\cos^3t = 2 
ightarrow \cos^3t = rac{1}{2\sqrt{2}} = 2^{-3/2} 
ightarrow \cos t = rac{1}{\sqrt{2}} 
ightarrow t = rac{\pi}{4} \ S = -2\int_0^{\pi/4} 2\sqrt{2}\sin^3t (4\sqrt{2}\cos^3t)'dt = 96\int_0^{\pi/4} \sin^4t \cos^2t dt = rac{96}{8}\int_0^{\pi/4} \sin^22t (1-\cos2t) dt = \ &= 12\int_0^{\pi/4} \sin^22t dt - 12\int_0^{\pi/4} \sin^22t \cos2t dt = 6\int_0^{\pi/4} (1-\cos4t) dt - 6rac{\sin^32t}{3}igg|_0^{\pi/4} = \ &= 6rac{4t-\sin4t}{4}igg|_0^{\pi/4} - 2\sin^32tigg|_0^{\pi/4} = (rac{6\pi}{4}-0) - (-2-0) = rac{3\pi}{2} - 2 \ &= 6\frac{4t-\sin4t}{4}igg|_0^{\pi/4} - 2\sin^32tigg|_0^{\pi/4} = (rac{6\pi}{4}-0) - (-2-0) = rac{3\pi}{2} - 2 \ &= 6\frac{4t-\sin4t}{4}igg|_0^{\pi/4} - 2\sin^32tigg|_0^{\pi/4} = (rac{6\pi}{4}-0) - (-2-0) = rac{3\pi}{2} - 2 \ &= 6\frac{4t-\sin4t}{4}igg|_0^{\pi/4} - 2\sin^32tigg|_0^{\pi/4} = (rac{6\pi}{4}-0) - (-2-0) = rac{3\pi}{2} - 2 \ &= 6\frac{4t-\sin4t}{4}igg|_0^{\pi/4} - 2\sin^32t \Big|_0^{\pi/4} = (rac{6\pi}{4}-0) - (-2-0) = rac{3\pi}{2} - 2 \ &= 6\frac{4t-\sin4t}{4}igg|_0^{\pi/4} - 2\sin^32t \Big|_0^{\pi/4} = (rac{6\pi}{4}-0) - (-2-0) = rac{3\pi}{2} - 2 \ &= 6\frac{4t-\sin4t}{4} - 2\frac{2}{2} + 2$$

$$p=4\cos 3\phi, p>=2$$

$$4\cos3\phi=2 o\cos3\phi=1/2 o\phi=rac{\pi}{9}$$



$$S_1 = rac{1}{2} \int_{-\pi/9}^{\pi/9} \left( (4\cos 3\phi)^2 - 2^2 
ight) d\phi = \int_0^{\pi/9} \left( 16\cos^2 3\phi - 4 
ight) d\phi =$$

$$=16\int_0^{\pi/9}\cos^2(3\phi)d\phi-4\int_0^{\pi/9}d\phi=\left|\int\cos^2(3\phi)d\phi=rac{1}{2}\int(1+\cos 6\phi)d\phi
ight|=$$

$$= 8\phiigg|_0^{\pi/9} + rac{8\sin 6\phi}{6}igg|_0^{\pi/9} - 4\phiigg|_0^{\pi/9} = rac{8\pi}{9} + rac{4\sinrac{2\pi}{3}}{3} - rac{4\pi}{9} = rac{4\pi}{9} + rac{2\sqrt{3}}{3}$$

$$S=3S_1=3\left(rac{4\pi}{9}+rac{2\sqrt{3}}{3}
ight)=rac{12\pi}{9}+2\sqrt{3}=rac{4\pi}{3}+2\sqrt{3}$$