1.1

$$\int \frac{x-5}{\sqrt{x^2+2x+5}}dx = \left| \begin{array}{c} u=x+1 \\ x=u-1 \\ dx=du \end{array} \right| = \int \frac{u-6}{\sqrt{u^2-2u+1+2u-3}}du = \int \frac{u-6}{\sqrt{u^2+4}}du = \int \frac{u}{\sqrt{u^2+4}}du - 6\int \frac{du}{\sqrt{u^2+4}} =$$

$$= \left| \begin{array}{c} v=u^2+4 \\ dv/2=udu \end{array} \right| = \left| \int \frac{1}{2\sqrt{v}}dv = \frac{2\sqrt{v}}{2} = \sqrt{u^2+4} \right| = \sqrt{u^2+4} - 6\ln|u+\sqrt{u^2+4}| + C$$

$$= \sqrt{(x+1)^2+4} - 6\ln|x+1+\sqrt{(x+1)^2+4}| + C$$

1.2

$$\int \sqrt[5]{2+\cos^2 x}\sin 2x dx = 2\int \sqrt[5]{2+\cos^2 x}\sin x\cos x dx = \left| \begin{array}{c} u=2+\cos^2 x \\ du=-2\sin x\cos x dx \\ -du/2=\sin x\cos x \end{array} \right|$$

$$= 2\int \sqrt[5]{u}\left(\frac{-du}{2}\right) = -\int \sqrt[5]{u}du = -\left(\frac{u^{6/5}}{6/5}\right) + C = -\frac{5}{6}u^{6/5} + C = -\frac{5}{6}(2+\cos^2 x)^{6/5} + C$$

2.1

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \sqrt{1-x^2}dx = \left| \begin{array}{c} x=\sin u \to dx=\cos u du \\ x=1/2 \to u=\pi/6 \\ x=\sqrt{3}/2 \to u=\pi/3 \end{array} \right| = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin^2 u}\cos u du = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2 u du$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1+\cos 2u}{2}du = \frac{1}{2}\left(u+\frac{\sin 2u}{2}\right)\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{1}{2}\left(\pi/3+\frac{\sin 2\pi/3}{2}\right) - \frac{1}{2}\left(\pi/6+\frac{\sin \pi/3}{2}\right)$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{8} - \frac{\pi}{12} - \frac{\sqrt{3}}{8} = \frac{\pi}{12}$$

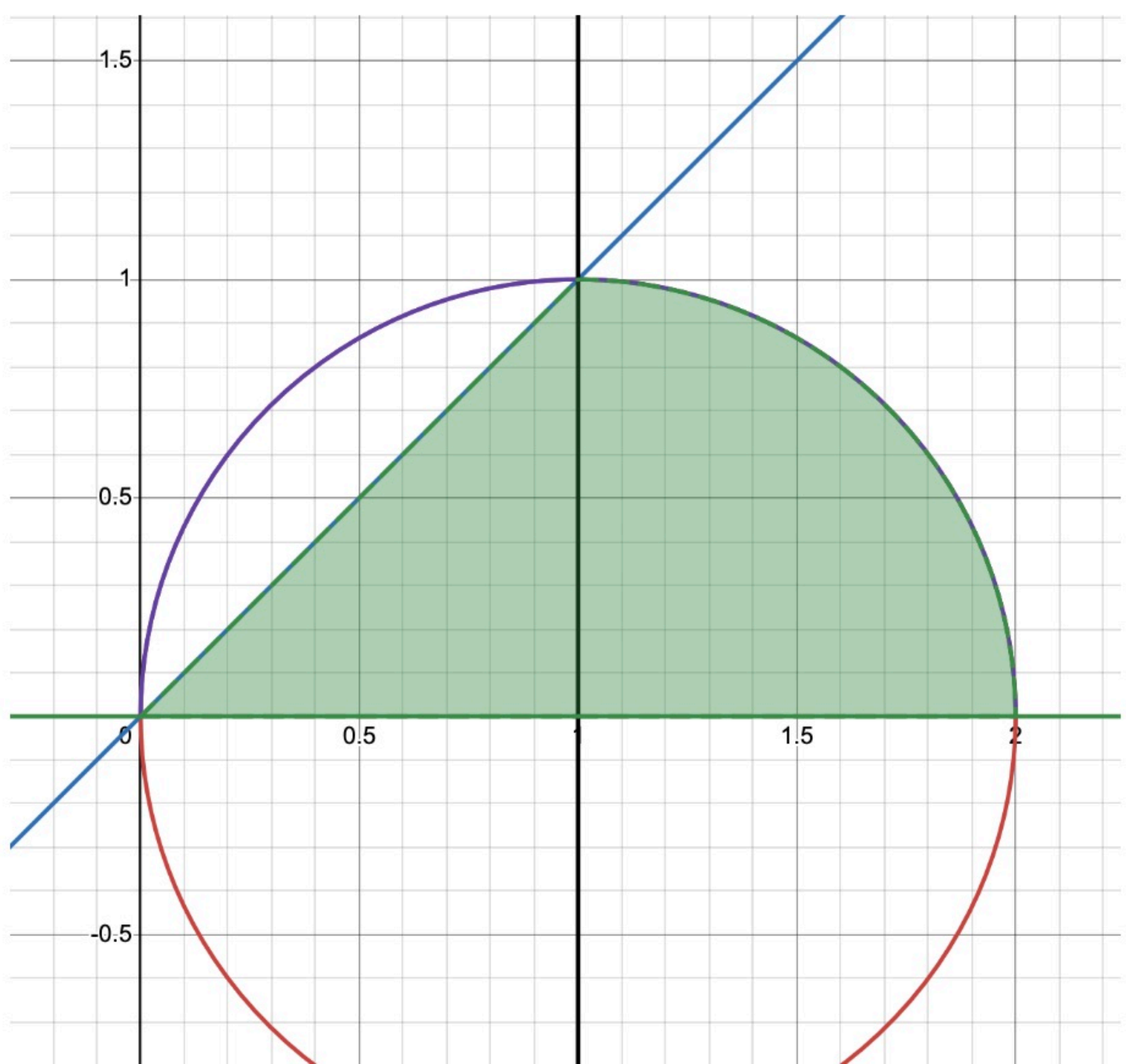
2.2

$$\int_2^{\infty} \frac{dx}{x\sqrt{x^2-1}} = \left| \begin{array}{c} u = \sqrt{x^2-1} \\ du = \frac{x}{\sqrt{x^2-1}} dx \\ dx = \frac{\sqrt{x^2-1}}{x} du \end{array} \right| = \int_2^{\infty} \frac{du}{x^2} = \int_2^{\infty} \frac{du}{u^2+1} = \int_2^{\infty} \operatorname{arctg} u + C = \int_2^{\infty} \operatorname{arctg} \sqrt{x^2-1} + C$$

$$\lim_{b\to\infty} \int_2^{b} \frac{dx}{x\sqrt{x^2-1}} = \lim_{b\to\infty} \left(\operatorname{arctg} \sqrt{b^2-1} - \operatorname{arctg} \sqrt{2^2-1} \right) = \pi/2 - \pi/3 = \pi/6$$

3

$$(x+1)^2 + y^2 = 1$$
$$y = x$$
$$y = 0$$



$$S = \int_0^1 x dx + \int_1^2 \sqrt{1-(x-1)^2} dx$$

$$\int_0^1 x dx = \frac{1}{2}$$

$$\int \sqrt{1-(x-1)^2} dx = \left| \begin{array}{c} u = x-1 \\ du = dx \end{array} \right| = \int \sqrt{1-u^2} du = \left| \begin{array}{c} v = \arcsin u \\ u = \sin v \\ \cos^2 v = 1-u^2 \\ du = \cos v dv \end{array} \right|$$

$$= \int \cos^2 v dv = \int \frac{1+\cos 2v}{2} dv = \frac{1}{2}\int dv + \frac{1}{2}\int \cos 2v dv$$

$$= \frac{v}{2} + \frac{\sin 2v}{4} + C = \frac{\arcsin u}{2} + \frac{u\sqrt{1-u^2}}{4} + C = \frac{\arcsin(x-1)}{2} + \frac{(x-1)\sqrt{2x-x^2}}{4} + C$$

$$\int_1^2 \sqrt{1-(x-1)^2} dx = \left(\frac{\arcsin(1)}{2} + 0\right) - (0+0) = \frac{\pi}{4}$$

$$S = \int_0^1 x dx + \int_1^2 \sqrt{1-(x-1)^2} dx = \frac{1}{2} + \frac{\pi}{4}$$

Виконаємо перевірку. Оскільки площа 1 фігури дорівнює половині площі квадрата зі стороною 1, а 2 - четвертині площі круга з радіусом 1:

$$S = \frac{1*1}{2} + \frac{\pi \times 1^2}{4} = \frac{1}{2} + \frac{\pi}{4}$$