23.1

$$\int_2^3 x\ln(x-1)dx = \left|\begin{array}{c} t=x-1 \\ x=t+1 \\ dx=dt \\ x=2\to t=1 \\ x=3\to t=2 \end{array}\right| = \int_1^2 (t+1)\ln t dt = \int_1^2 t\ln t dt + \int_1^2 \ln t dt =$$

$$= \left| \begin{array}{cc} u=\ln t & v=\frac{t^2}{2} \\ du=\frac{dt}{t} & dv=tdt \end{array} \quad \int t\ln t dt = \frac{t^2}{2}\ln t - \int \frac{t^2}{2t}dt = \frac{t^2}{2}\ln t - \frac{1}{2}\int t dt = \frac{t^2}{2}\ln t - \frac{t^2}{4} \right| =$$

$$= \left| \int_1^2 t\ln t dt = \left(\frac{t^2}{2}\ln t - \frac{t^2}{4}\right)\Bigg|_1^2 = 2\ln 2 - 1 - \left(\frac{1}{2}\ln 1 - \frac{1}{4}\right) = 2\ln 2 - \frac{3}{4} \right| =$$

$$= \left| \int_1^2 \ln t dt = t\ln t - t\Big|_1^2 = 2\ln 2 - 2 - (\ln 1 - 1) = 2\ln 2 - 1 \right| =$$

$$= 2\ln 2 - \frac{3}{4} + 2\ln 2 - 1 = \frac{16\ln 2 - 7}{4}$$

23.2

$$\int_0^1 \frac{3x^4+3x^2+1}{x^2+1}dx = \int_0^1 \frac{1}{x^2+1} + 3x^2 dx = \int_0^1 \frac{1}{x^2+1}dx + 3\int_0^1 x^2 dx =$$

$$= \operatorname{arctg} x\Big|_0^1 + x^3\Big|_0^1 = \frac{\pi}{4} + 1$$

23.3

$$\int_3^{29} \frac{\sqrt[3]{(x-2)^2}}{3+\sqrt[3]{(x-2)^2}}dx = \left| \begin{matrix} t=\sqrt[3]{x-2} \to x = t^3+2 \\ dt = \frac{dx}{3t^2} \to dx = 3t^2dt \\ x=3 \to t=1 \\ x=29 \to t=3 \end{matrix} \right| = \int_1^3 \frac{t^2}{3+t^2}3t^2dt = 3\int_1^3 \frac{t^4}{3+t^2}dt =$$

$$= \left| \frac{t^4}{t^2+3} = t^2-3+\frac{9}{t^2+3} \right| = 3\int_1^3 \left(t^2-3+\frac{9}{t^2+3}\right)dt = 3\int_1^3 t^2dt - 9\int_1^3 dt + 27\int_1^3 \frac{dt}{t^2+3} =$$

$$\frac{3t^3}{3}\bigg|_1^3 - 9t\bigg|_1^3 + \frac{27}{\sqrt{3}}\arctan\frac{t}{\sqrt{3}}\bigg|_1^3 = 27-1-18+\frac{27\pi}{3\sqrt{3}} - \frac{27\pi}{6\sqrt{3}} = 8+\frac{3\sqrt{3}\pi}{2}$$

23.4

$$\int_{\frac{\pi}{2}}^{\pi} 2^8\sin^8 x dx = 2^8\int_{\frac{\pi}{2}}^{\pi}\sin^8 x dx = 2^8 \times \frac{7!!}{8!!}\times\frac{\pi}{2} = \frac{256\times 35\pi}{256} = 35\pi$$

23.5

$$\int_0^2 x^2\sqrt{4-x^2}dx = \int_0^2 x^2\sqrt{2^2-x^2}dx = \left| \begin{matrix} x = 2\sin t \\ t = \arcsin x/2 \\ 4-x^2 = 4\cos^2 t \\ dx = 2\cos t dt \\ x=0 \to t=0 \\ x=2 \to t=\pi/2 \end{matrix} \right| =$$

$$= \int_0^{\frac{\pi}{2}} 4\sin^2 t \times 2\cos t \times 2\cos t dt = \int_0^{\frac{\pi}{2}} 16\sin^2 t\cos^2 t dt = 4\int_0^{\frac{\pi}{2}}\sin^2 2t dt =$$

$$= |u=2x| = 4\int_0^{\frac{\pi}{2}}\frac{\sin^2 u}{2}du = 2\int_0^{\frac{\pi}{2}}\frac{1-\cos 2u}{2}du = \int_0^{\frac{\pi}{2}} 1du - \int_0^{\frac{\pi}{2}}\cos 2u du =$$

$$= u - \frac{\sin 2u}{2} = 2t - \frac{\sin 4t}{2} = 2\pi/2 - \frac{\sin 4\pi/2}{2} + \frac{\sin 0}{2} = \pi$$

23.6

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos^3 x}{\sqrt{\sin x}}dx = \left| \begin{array}{c} u = \sin x \\ du = \cos x dx \end{array} \right| =$$

$$= \left| \int (1-u^2)u^{-1/2}du = \int u^{-1/2}du - \int u^{3/2}du = 2u^{1/2} - \frac{2}{5}u^{5/2} + C \right| =$$

$$= (2\sqrt{\sin x} - \frac{2}{5}\sin^{5/2} x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = (2 - \frac{2}{5}) - \left(\frac{2}{\sqrt[4]{2}} - \frac{2}{5 \times 2^{5/4}}\right) =$$

$$= \frac{8}{5} - \left(2 \times 2^{-1/4} - \frac{2^{-1/4}}{5}\right) = \frac{8}{5} - \frac{9 \times 2^{-1/4}}{5} = \frac{8 - 9 \times 2^{-1/4}}{5}$$

24.1

$$\int_1^{\infty} \frac{xdx}{\sqrt{16x^4 - 1}} = \lim_{b\to\infty} \int_1^b \frac{xdx}{\sqrt{16x^4-1}} = \left| \begin{array}{c} u = 4x^2 \\ du = 8xdx \\ x = 1 \to u = 4 \end{array} \right| = \lim_{b\to\infty}\left(\frac{1}{8}\int_4^b \frac{du}{\sqrt{u^2-1}}\right) =$$

$$= \lim_{b\to\infty}\left(\frac{1}{8}\ln\left|u + \sqrt{u^2-1}\right| \Big|_4^b\right) = \lim_{b\to\infty}\left(\frac{1}{8}\left(\ln|b + \sqrt{b^2-1}| - \ln|4+\sqrt{15}|\right)\right) =$$

$$= \frac{1}{8}\left(\ln|\infty + \infty| - \ln|4 + \sqrt{15}|\right) = \infty - \text{Інтеграл розбіжний}$$

24.2

$$\int_0^1 \frac{dx}{\sqrt[3]{2-4x}} = \left| 2-4x=0 \to x=\frac{1}{2} \right| \int_0^{1/2} \frac{dx}{\sqrt[3]{2-4x}} + \int_{1/2}^1 \frac{dx}{\sqrt[3]{2-4x}} =$$

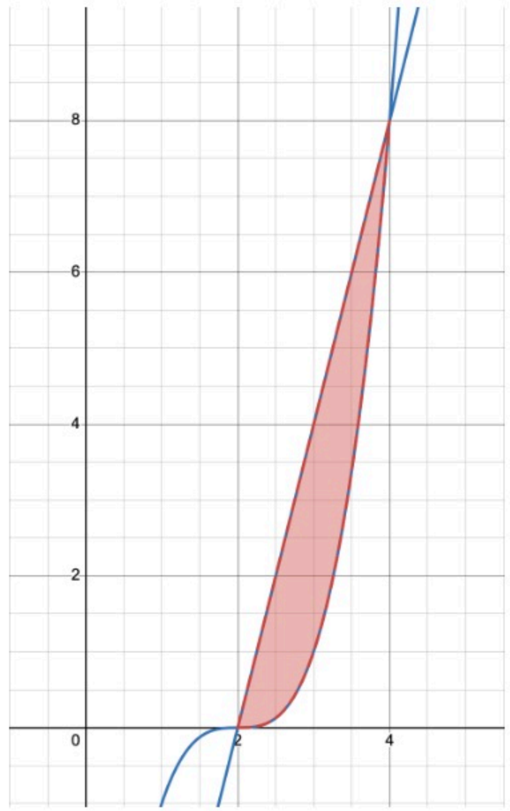
$$= \left| \begin{array}{c} u=2-4x \\ du=-4dx \to dx=-\frac{du}{4} \\ x=0 \to u=2 \\ x=1/2 \to u=0 \\ x=1 \to u=-2 \end{array} \right| = \frac{1}{4}\int_0^2 u^{-1/3}du + \frac{1}{4}\int_{-2}^0 u^{-1/3}du =$$

$$= \frac{3}{8}\left(u^{2/3}\Big|_0^2 - u^{2/3}\Big|_{-2}^0 \right) = \frac{3}{8}\left(\sqrt[3]{4}-\sqrt[3]{4}\right) = 0$$

25.1

$$y = (x-2)^3, y = 4x - 8$$

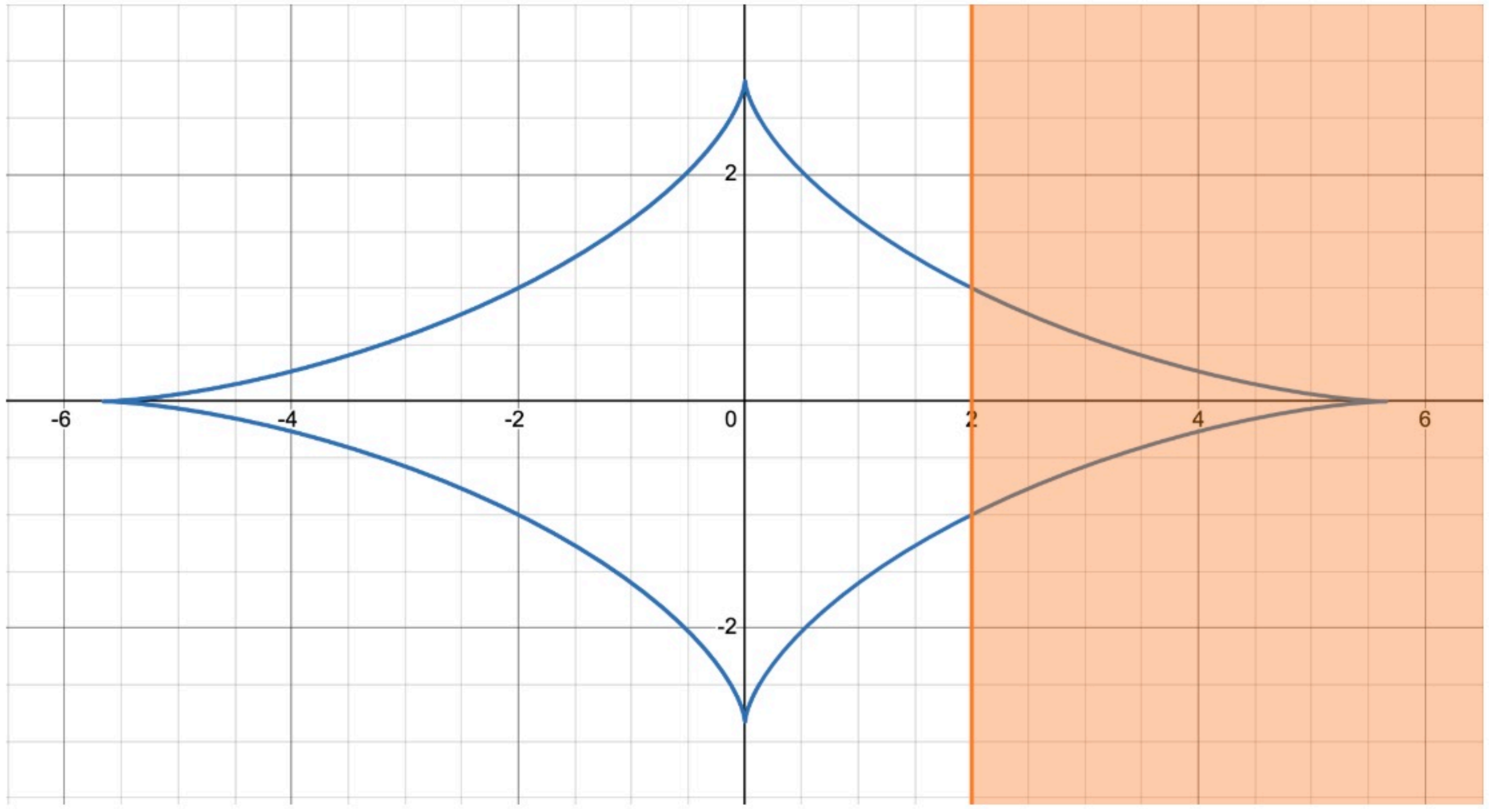
$$(x-2)^3 = 4(x-2) \to u = x - 2, u^3 = 4u \to u^2 = 4$$
$$u = 2, u = -2, u = 0$$
$$x = 0, x = 2, x = 4$$
$$x = 0 : y = 4 \times 0 - 8 = -8$$
$$x = 2 : y = 4 \times 2 - 8 = 0$$
$$x = 4 : y = 4 \times 4 - 8 = 8$$



$$S = 2\int_0^2 (4u - u^3)du = 2\int_0^2 4udu - 2\int_0^2 u^3du = 2\left(2u^2\Big|_0^2 - \frac{u^4}{4}\Big|_0^2\right) = 2(8-4) = 8$$

25.2

$$\begin{cases} x = 4\sqrt{2}\cos^3 t \\ y = 2\sqrt{2}\sin^3 t \end{cases} \quad x >= 2$$



$$4\sqrt{2}\cos^3 t = 2 \to \cos^3 t = \frac{1}{2\sqrt{2}} = 2^{-3/2} \to \cos t = \frac{1}{\sqrt{2}} \to t = \frac{\pi}{4}$$

$$S = -2\int_0^{\pi/4} 2\sqrt{2}\sin^3 t(4\sqrt{2}\cos^3 t)'dt = 96\int_0^{\pi/4}\sin^4 t\cos^2 tdt = \frac{96}{8}\int_0^{\pi/4}\sin^2 2t(1-\cos 2t)dt =$$

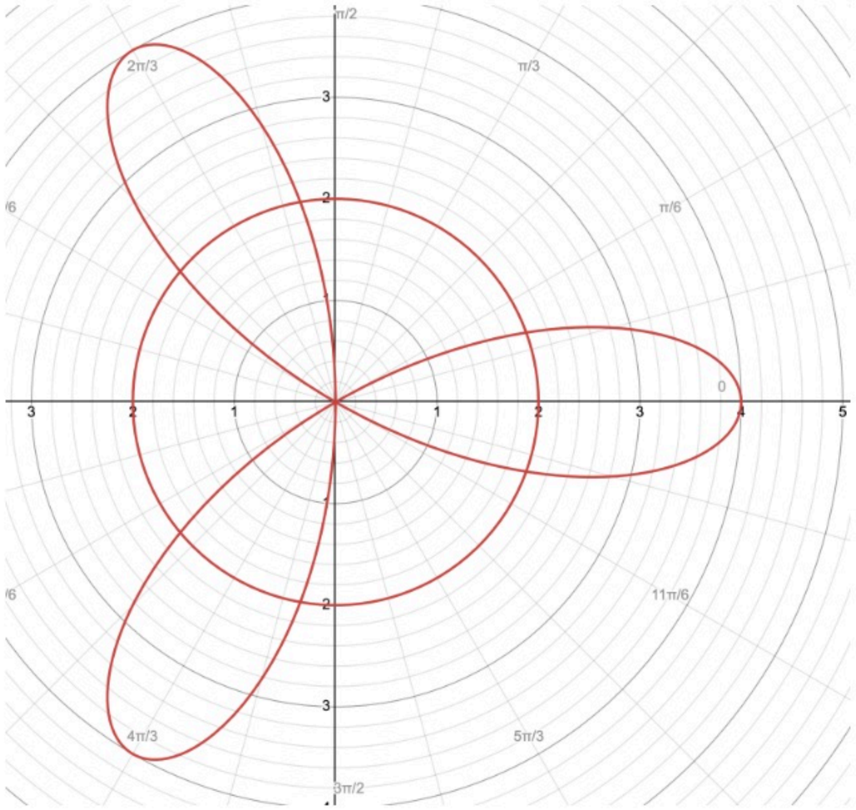
$$= 12\int_0^{\pi/4}\sin^2 2tdt - 12\int_0^{\pi/4}\sin^2 2t\cos 2tdt = 6\int_0^{\pi/4}(1-\cos 4t)dt - 6\frac{\sin^3 2t}{3}\Big|_0^{\pi/4} =$$

$$= 6\frac{4t-\sin 4t}{4}\Big|_0^{\pi/4} - 2\sin^3 2t\Big|_0^{\pi/4} = (\frac{6\pi}{4} - 0) - (-2-0) = \frac{3\pi}{2} - 2$$

25.3

$$p = 4\cos 3\phi, p >= 2$$

$$4\cos 3\phi = 2 \to \cos 3\phi = 1/2 \to \phi = \frac{\pi}{9}$$



$$S_1 = \frac{1}{2}\int_{-\pi/9}^{\pi/9}\left((4\cos 3\phi)^2 - 2^2\right)d\phi = \int_0^{\pi/9}\left(16\cos^2 3\phi - 4\right)d\phi =$$

$$= 16\int_0^{\pi/9}\cos^2(3\phi)d\phi - 4\int_0^{\pi/9}d\phi = \left|\int\cos^2(3\phi)d\phi = \frac{1}{2}\int(1+\cos 6\phi)d\phi\right| =$$

$$= 8\phi\Big|_0^{\pi/9} + \frac{8\sin 6\phi}{6}\Big|_0^{\pi/9} - 4\phi\Big|_0^{\pi/9} = \frac{8\pi}{9} + \frac{4\sin\frac{2\pi}{3}}{3} - \frac{4\pi}{9} = \frac{4\pi}{9} + \frac{2\sqrt{3}}{3}$$

$$S = 3S_1 = 3\left(\frac{4\pi}{9} + \frac{2\sqrt{3}}{3}\right) = \frac{12\pi}{9} + 2\sqrt{3} = \frac{4\pi}{3} + 2\sqrt{3}$$