

Assignment - 1

① - Asymptotic notations are languages that allow us to analyze an algorithm running time by identifying its behaviour as the input size of algorithm.

Types-

(i). Big O - commonly used for worst case, and gives upper bound for growth rate of runtime of algorithm.

Eg - Big O notation for linear search is  $O(n)$

(ii) - Big Omega - It is notation used for least complexity, it provides with an asymptotic lower bound.

Eg - Big Omega for linear search is  $\Omega(n)$

(iii) - Theta - used for tight bound on growth rate of runtime of algo.

Eg - Theta of linear search is  $\Theta(n)$

(iv) - Small Omega - to denote lower bound

② - for  $i=1$  to  $n$   
     $\{$

$i = i + 2,$

$\}$

$\Rightarrow O(\log n)$

③-  $T(n) = 3T(n-1)$   
 $T(1) = 1$

$$T(2) = 3T(1) = 3$$

$$T(3) = 3T(2) = 9$$

$$T(4) = 3T(3) = 27$$

⋮

$$T(n) = (n-1)^3$$

$$\text{Time complexity} = (n-1)^3$$

④-  $T(n) = 2(T(n-1) - 1)$

$$T(n-1) = 2T(n-2) - 1$$

$$T(n) = 4T(n-2) - 2 - 1$$

$$T(n-2) = 2T(n-3) - 1$$

$$T(n) = 8T(n-3) - 4 - 2 - 1$$

$$T(n-3) = 2T(n-4) - 1$$

$$T(n) = 16T(n-4) - 8 - 4 - 2 - 1$$

$$T(n) = 2^k - 2^3 - 2^2 - 2^1 - 2^0$$

$$= O(1)$$

⑤-  $S \quad i$

$$1 \quad 1$$

$$3 \quad 2$$

$$6 \quad 3$$

$$10 \quad 4$$

$$O(\sqrt{n})$$

⑥-  $i * i = n$

$$i^2 = n$$

$$i = \sqrt{n}$$

$$O(\sqrt{n})$$



⑦.  $O(n \log n)$

⑧ - Total  $T = O(n \log n)$

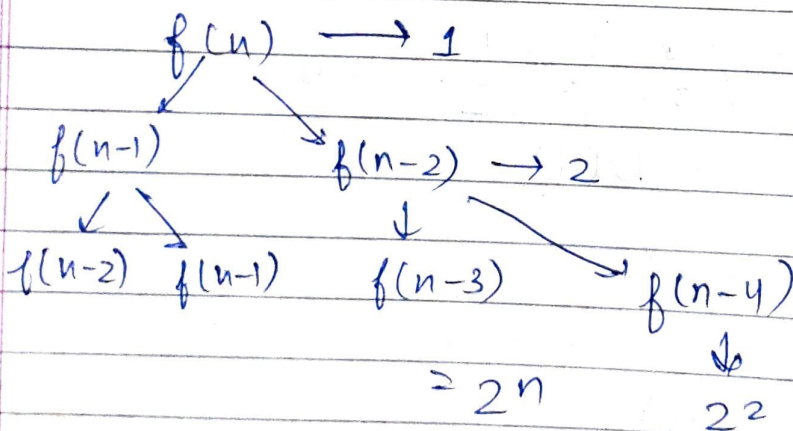
⑩.  $n^t$  is  $O(c^k)$  as per example of -  
 when we take  $n=2, k=2, c=2$   
 then,  $2^2 \leq 2^2$  so  $c^k$  is upper limit of  $n^t$ .

⑪.

$i=1$	$i=0$
1	1
2	3
3	6
4	10

Series is nearly dependant on  $i$  as  $2^i$  so  $O(2^n)$

⑫. Space complexity  $= O(n)$  as clear call of  $(n-1)$



Time complexity  $= O(2^n)$

⑬.  $n \log n$   
 for  $i=0; i < n; i++$   
 for  $j=0; j < n; j++$   
 $c++;$

$n^3$   
 for  $i=0; i < n; i++$   
 for  $j=0; j < n; j++$   
 for  $k=0; k < n; k++$   
 $c++;$

$\log(\log n)$

```
int funct (int n)
{
```

```
    if (n == 1)
```

```
        return n;
```

```
    else
```

```
        return funct(5n) + funct(5n);
```

⑭.  $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2$

$a = 2,$

$b = 2, c = 1$

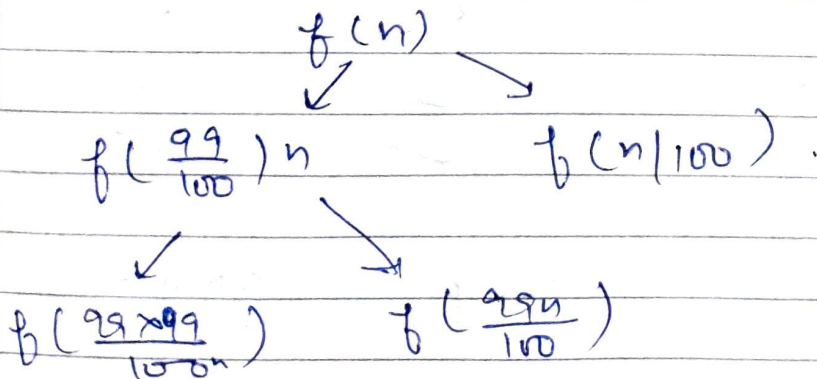
$f(n) > n^k \quad \& \quad n^2 > 1$

$O(n^2)$

⑮.  $O(n \sqrt{n})$

⑯.  ~~$O(\log(\log n))$~~   $O(\log \log n)$

⑰.  $T(n) = T\left(\frac{99}{100}n\right) + T\left(\frac{n}{100}\right)$



$= O(\log n)$

⑱. a)  $100 < \log \log n < \sqrt{n} < n \log(1) < n \log n < n^2 < 2^n < 25n < n$

b)  $1 < \log \log n < \sqrt{\log n} < \log^2 n < \log n < 2 \log n < n < n < 4n < n^2$   
 $\leq n! < 2(2n)^n$

c)  $96 < \log_2 n < \log_2 n < \log n < \log n < n \log n < n \log_2 n < 8n^2 < 7n^3 < 8^n < n!$

19 - linear(arr, key)

{

for (int i=0; i<n; i++)

if (arr[i] == key)

return i;

return -1;

20 - int(arr, n)

{

if (n <= 1) return;

Recursively for (n-1) elements

Insert sort(arr, n)

Iterations -

Insert(arr, n)

{

for (i=1; i<n; i++)

{

Pick arr[i] & insert into arr[0, ..., i-1]

}

	Stable	Inplace	Online
Bubble sort	✓	✓	✗
Selection sort	✗	✓	✗
Insertion sort	✓	✓	✓

22 -

	Best	Avg	Worst
Bubble	$O(n^2)$	$O(n^2)$	$O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion	$O(n)$	$O(n)$	$O(n^2)$



(23) = Recursive -

Binary (arr, l, n, key)

{

if ( $l < n$ )

{

mid =  $l + (n - l) / 2$ ;

if (arr[mid] == key) return 1;

if (key < arr[mid])

Binary (l, mid - 1, key);

else

Binary (mid + 1, n, key)

}

Iterative

while ( $l < n$ )

{

mid =  $l + (n - 1) / 2$

if (arr[mid] == key) return 1;

if (key < arr[mid])

n = mid - 1;

else

l = mid + 1;

}