CS766 - HW1

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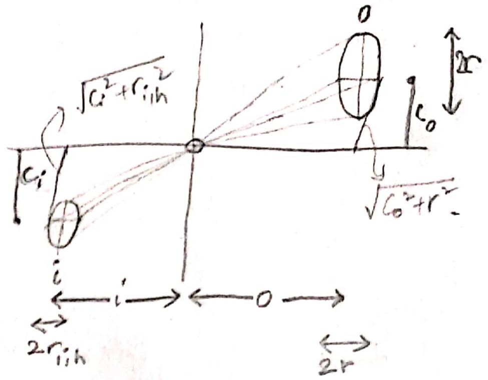
Problem 1:

(a) The image of a circular disk through a pinhole camera on a plane parallel to the image plane is a circle with a different radius. To see why, assume horizontal width of image is $2r_{i,h}$ & vertical width is $2r_{i,v}$



From similar triangles,

$$\frac{c_i}{c_o} = \frac{i}{o} = \frac{r_{i,v}}{r},$$

Also, $$\frac{\sqrt{c_o^2+r^2}}{o} = \frac{\sqrt{c_i^2+r_{i,h}^2}}{i} \Rightarrow \frac{c_i}{c_o} = \frac{r_{i,h}}{r}$$

$\therefore r_{i,v} = r_{i,h} \Rightarrow$ The image is a circle.

(b) Let r_i : radius of image, r_o : radius of object.

$\Rightarrow \pi r_i^2 = 1mm^2$

Magnification, $m = \frac{r_i}{r_o} = \frac{f}{z_o} = \frac{f}{1m}$

Distance of disk is doubled.

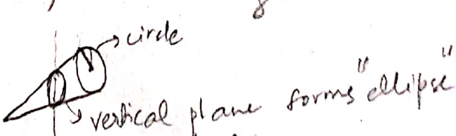
Let new radius $= r_i'$

$$\Rightarrow m' = \frac{r_i'}{r_o} = \frac{f}{2m} \Rightarrow r_i' = \frac{f r_o}{2} = \frac{r_i}{2}$$

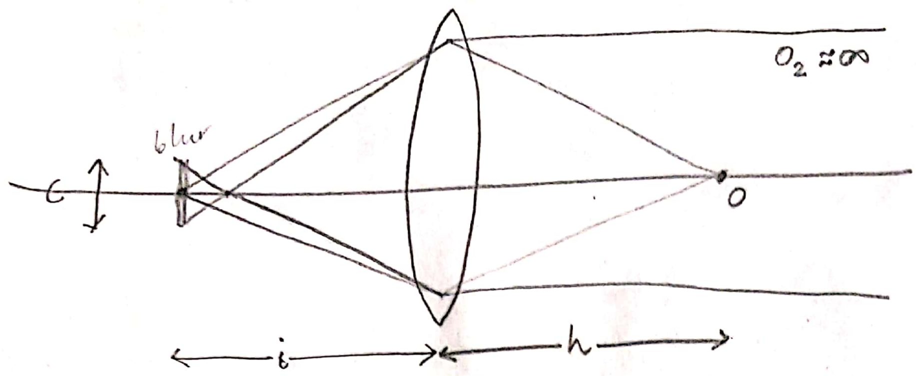
$$\therefore \text{Area of image} = \pi r_i'^2 = \pi\left(\frac{r_i}{2}\right)^2 = \frac{\pi r_i^2}{4} = \frac{1}{4}mm^2$$

(c) To understand the image formation of a sphere, consider all the lines through the pin hole that are tangential to the sphere. They form a cone, (with a tilted base) Now, lets extend this cone on the other side of the pin hole to see how it intersects the image plane. It is easy to see that the intersection is an "Ellipse" if fully fits on image plane.

Special case: When the image plane & the line through the center of the sphere & the pinhole is perpendicular, the elliptical image becomes a circle. Other possibilities include parabola & hyperbola if image plane isn't big enough.



Problem - 2:



For a maximum blur circle width 'c', if 'o_1' & 'o_2' are nearest & farthest distances, then from Gaussian Lens Law, the depth of field,

$$o_2 - o_1 = \frac{2of^2cN(o-f)}{f^4 - c^2N^2(o-f)^2}$$

The hyperfocal distance, H = the distance of object from lens to keep objects at '∞' focused ($o_2 \to \infty$)

$\Rightarrow$ the denominator of depth of field expression $\to 0$

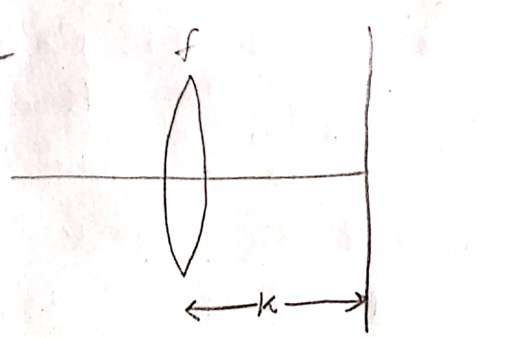
$\Rightarrow \quad f^4 - c^2N^2(o-f)^2 = 0 \quad$ (where $o = H$)

$\Rightarrow \quad f^2 = cN(H-f)$

$\Rightarrow \quad H - f = \frac{f^2}{cN}$

$\Rightarrow \quad \boxed{H = f + \frac{f^2}{cN}}$

Problem - 3 (a):



Let 'i' be the distance from lens where the image of scene line will be formed.

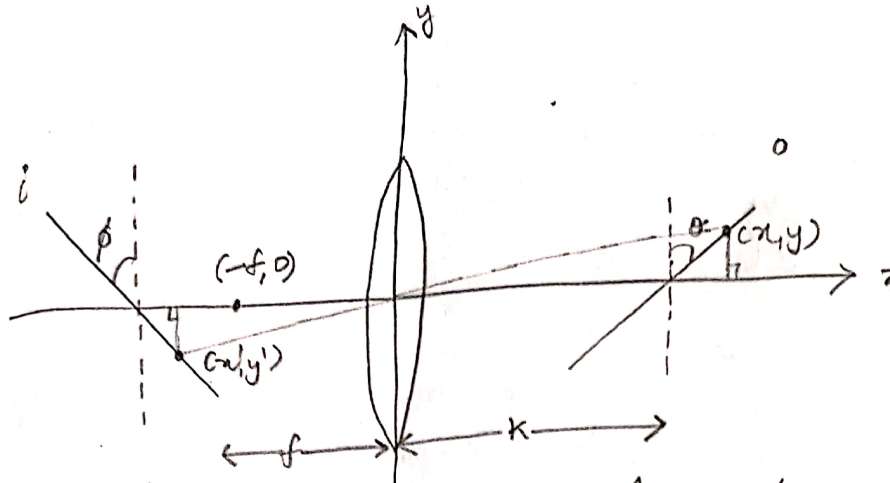
From Gaussian Lens Law:

$$\frac{1}{i} + \frac{1}{k} = \frac{1}{f} \Rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{k} = \frac{k-f}{kf}$$

$$\Rightarrow \boxed{i = \frac{kf}{k-f}}$$

Since Gauss law depends only on object distance, & since all points are at same distance, image is also a vertical line parallel to the scene line.

3(b):



Consider a coordinate system centered at lens as shown above.
Since the object is a straight line, consider its equation as

$$\boxed{y = m_o x + c_o}$$

Consider a point (x,y) on the line (object).
Let (x',y') be the position of its image.
From Gaussian Lens Law, $\frac{1}{-x'} + \frac{1}{x} = \frac{1}{f}$

$$\Rightarrow \frac{1}{x} = \frac{1}{x'} + \frac{1}{f} = \frac{f+x'}{x'f}$$

$$\Rightarrow x = \frac{x'f}{f+x'}$$

Also, magnification, $\frac{h_i}{h_o} = \frac{i}{o}$ $\Rightarrow$ $\frac{-y'}{y} = \frac{-x'}{x}$
(from similar Δles)

$$\Rightarrow y = \frac{xy'}{x'} = \left(\frac{\cancel{x'}f}{f+x'}\right)\frac{y'}{\cancel{x'}}$$

$$\Rightarrow y = \frac{y'f}{f+x'}$$

Image:
Locus of (x',y'): We know (x,y) satisfies $y = m_o x + c_o$.

$$\Rightarrow \frac{y'f}{f+x'} = m_o\left(\frac{x'f}{f+x'}\right) + c_o$$

$$\Rightarrow \frac{y'f}{\cancel{f+x'}} = \frac{m_o x'f + c_o f + c_o x'}{\cancel{f+x'}} = x'(m_o f + c_o) + c_o f$$

$$\Rightarrow \boxed{y' = x'\left(m_o + \frac{c_o}{f}\right) + c_o}$$

$\therefore$ Image is also a straight line that's tilted!

(c) Object line : $y = m_0 x + c_0$

Given $y = 0 \Rightarrow x = k \Rightarrow \boxed{-m_0 k = c_0}$

$$\text{slope} = \tan\left(\frac{\pi}{2} - \theta\right) = m_0$$

$$\Rightarrow \cot\theta = m_0$$

Image line : $y = x\left(m_0 + \frac{c_0}{f}\right) + c_0$

$$\text{slope} = \tan\left(\frac{\pi}{2} + \phi\right) = m_0 + \frac{c_0}{f} = m_0 - \frac{m_0 k}{f}$$

$$\Rightarrow -\cot\phi = m_0\left(1 - \frac{k}{f}\right) = \cot\theta\left(\frac{f-k}{f}\right)$$

$$\Rightarrow \tan\theta = -\tan\phi\left(\frac{f-k}{f}\right)$$

$$\Rightarrow \boxed{\tan\phi = \frac{f}{k-f}\tan\theta}$$