Computer Vision, Spring 2019 - Homework 5

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Written Assignments

Problem 1

a) Given a Lambertian surface with albedo ρ_d, for a surface point with normal vector $\bar{n}$ receiving a light of intensity I in direction $\bar{s}$, the scene radiance L is given by

$$L = \frac{\rho_d}{\pi} I(\bar{n}.\bar{s})$$

Therefore, for a surface point receiving light of equal intensity I from two directions $\bar{s_1}$ and $\bar{s_2}$, the radiance is

$$\begin{aligned} L &= \frac{\rho_d}{\pi} I(\bar{n}.\bar{s_1}) + \frac{\rho_d}{\pi} I(\bar{n}.\bar{s_2}) \\ &= \frac{\rho_d}{\pi} I(\bar{n}.(\bar{s_1} + \bar{s_2})) \\ &= \frac{\rho_d}{\pi} I(\bar{n}.\frac{(\bar{s_1} + \bar{s_2})||\bar{s_1} + \bar{s_2}||}{||\bar{s_1} + \bar{s_2}||}) \\ &= \frac{\rho_d}{\pi}\ (||\bar{s_1} + \bar{s_2}||I)\ (\bar{n}.\frac{(\bar{s_1} + \bar{s_2})}{||\bar{s_1} + \bar{s_2}||}) \end{aligned}$$

Therefore, the “effective” intensity of the light source is $||\bar{s_1} + \bar{s_2}||I$ and the unit normal vector in the “effective” direction is $s_3 = \frac{\bar{s_1} + \bar{s_2}}{||\bar{s_1} + \bar{s_2}||}$, where $||\bar{v}||$ is the L_2 norm of the vector $\bar{v}$.

b) If the two light sources have unequal intensities I_1 and I_2, the radiance is given by

$$\begin{aligned} L &= \frac{\rho_d}{\pi} I_1(\bar{n}.\bar{s_1}) + \frac{\rho_d}{\pi} I_2(\bar{n}.\bar{s_2}) \\ &= \frac{\rho_d}{\pi} (\bar{n}.(I_1\bar{s_1} + I_2\bar{s_2})) \\ &= \frac{\rho_d}{\pi} (\bar{n}.\frac{(I_1\bar{s_1} + I_2\bar{s_2})||I_1\bar{s_1} + I_2\bar{s_2}||}{||I_1\bar{s_1} + I_2\bar{s_2}||}) \\ &= \frac{\rho_d}{\pi}\ (||I_1\bar{s_1} + I_2\bar{s_2}||)\ (\bar{n}.\frac{(I_1\bar{s_1} + I_2\bar{s_2})}{||I_1\bar{s_1} + I_2\bar{s_2}||}) \end{aligned}$$

Therefore, the “effective” intensity of the light source is $||I_1\bar{s_1} + I_2\bar{s_2}||$ and the unit normal vector in the “effective” direction is $s_3 = \frac{I_1\bar{s_1} + I_2\bar{s_2}}{||I_1\bar{s_1} + I_2\bar{s_2}||}$.