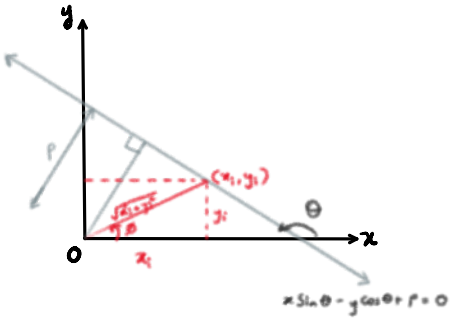
HW3 Problem 1

Tuesday, February 26, 2019 9:41 PM

Equation of line:

$$x \sin\theta - y\cos\theta + \rho = 0 \quad \text{——— (1)}$$



For (x_i, y_i) on the line,

from the figure,

$$\cos\phi = \frac{x_i}{\sqrt{x_i^2+y_i^2}} \quad \& \quad \sin\phi = \frac{y_i}{\sqrt{x_i^2+y_i^2}}$$

$\therefore$ (1) gives

$$\frac{x_i}{\sqrt{x_i^2+y_i^2}}\sin\theta - \frac{y_i}{\sqrt{x_i^2+y_i^2}}\cos\theta + \frac{\rho}{\sqrt{x_i^2+y_i^2}} = 0$$

$$\Rightarrow \cos\phi\sin\theta - \sin\phi\cos\theta + \frac{\rho}{\sqrt{x_i^2+y_i^2}} = 0$$

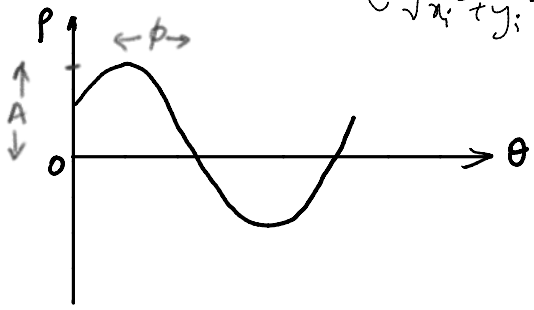
$$\Rightarrow \rho = \sqrt{x_i^2+y_i^2}\,(\sin\phi\cos\theta - \cos\phi\sin\theta)$$

$$\therefore \boxed{\rho = \sqrt{x_i^2+y_i^2}\,\sin(\phi-\theta)}$$

It is a sinusoid in $\rho-\theta$ Hough space w/h

Amplitude, $A = \sqrt{x_i^2+y_i^2}$

$$\text{phase} = \phi = \sin^{-1}\left(\frac{y_i}{\sqrt{x_i^2+y_i^2}}\right)$$



Period of the sinusoid $= \frac{2\pi}{\omega} = \frac{2\pi}{-1} = -2\pi$

$\therefore$ It doesn't vary with point (x_i, y_i)