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Parameter Estimation

1) Let $(x_1, x_2, \ldots)$ be random sample of size n taken from normal population with parameter mean $= \theta_1$ and Variance $= \theta_2$. Find the max. likelihood estimate of these two population parameter.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

$x_1, x_2, \ldots x_n \rightarrow$ Sample of size n

$$L(x_1, x_2, \ldots x_n) = f(x_1) \cdot f(x_2) \ldots \cdot f(x_n)$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) \cdots$$

Taking ln on both sides we get,

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

Taking derivative w.r.t. μ

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} - \frac{(2(x_i-\mu))}{2\sigma^2} = 0$$

$$= \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$= n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

Hence $\theta_1 = \bar{x}$ i.e. sample mean

$$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n}\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

Taking derivative w.r.t σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -n + \sum_{i=1}^{n} - \frac{(x_i-\mu)^2}{(\sigma^2)} = 0$$

$$-n + \sum_{i=1}^{n} - \frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

Hence $\theta_2 = \frac{1}{n}\sum^{n}(x_i-\mu)^2$

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2) Let $x_1, x_2, \ldots x_n$ be random sample from $B(m, \theta)$ distribution where $\theta \in \Theta = (0,1)$ is unknown and 'm' is +ve integer. Compute value of θ using M.L.E.

Binomial Distribution = ${}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1} \left(\log({}^nC_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i}\right)$$

$$\log L = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \log\theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n-x_i)$$

Diff. w.r.t θ

$$\frac{\partial}{\partial\theta} \log(L) = 0$$

$$\Rightarrow \frac{1}{\theta} \Sigma x_i - \frac{1}{1-\theta} \Sigma(n-x_i) = 0$$

$$\Rightarrow \frac{1}{\theta} \Sigma x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \Sigma x_i = 0$$

$$\Rightarrow \frac{1}{\theta(1-\theta)} \Sigma x_i = \frac{n^2}{1-\theta}$$

$$\Rightarrow \frac{\Sigma x_i}{\theta} = n^2$$

$$\boxed{\theta = \frac{\Sigma x_i}{n^2}}$$