1. Given a positive integers n and a given base b, prove that some power of b lies in the range [n, bn].

2. Given a list L of integers, a_1, a_2, . . . , a_n, and an integer M, describe an algorithm that finds the largest subset of L whose sum is at most M. Your algorithm should run in linear time.

3. Given a sorted array of distinct integers A[1,..,n], you want to find out whether there is an index I for which A[i]=i. Give a effective algorithm that runs in time complexity of log(n).

4. Given an array A, write an efficient algorithm that will decide the elements of the array into 2 halves such that the sum of integers in each part is equal.

5. In a typical weighing balance, there is a 'n' set of individual weights (a1, a2, a3.. an). Given a comodity of weight 't', design a O(nt) algorithm to determine the subset of weights which when added together turns up to 't'.

6. With the rise in the costs of the bus tickets, students were asked to design an efficient algorithm to reach from a particular source to a destination with minimum cost. The cost of travel accross each source destination pair(between any two places) is being provided apriorily. You can imagine this to be a matrix of set of start stations to be along the rows and the set of destinations accross the coloumn of the matrix with value k(i,j) determining the cost of travel from node i to node j. Given this matrix, design an algorithm that can give the minimum cost of travel from any given source to any destination.
   Note: Due to increasing traffic, there are restrictions in the directions in which you can move. Considering the matrix being provided, you can either move diagonlly left upwards, diagonally right upwards or only upwards.

7. Given two strings A and B of lengths 'm' and 'n', design an algorithm to compute the minimum cost of transforming string A to string B, given the cost of adding a character to A is C_a, deleting a character is C_d and replacing a character is C_r.

8. A small variation from the regular coin exchange problem is that, along with finding the minimum number of denominations (d_1, $d_2..d_n$) each of number x_1, $x_2..x_m$(x_1 number of denomination d_1 are present) to pay a particular change 't', there is a constraint on the number of coins that can be used out of the total set of coins x should be less than or equal to a given constant 'c'. Design an algorithm which can tell such whether the given change 't' can be obtained by atmost 'c' denominations.

9. Given an n by n array of positive integers (a_{ij}) such that i¿1 and j¡n, rolled into a cylinder, so that the top and bottom rows are glued together. A path is to be threaded from the entry side of the cylinder to the exit side, subject to the restriction that from the given square (i,j) it is possible to move to (i+1, j), (i+1, j-1) or (i+1, j+1). The path may begin at any position on the entry side and end at any position on the exit side. The cost of such a path is the sum of the integers in the squares through which it passes. Provide a dyanamic programming solution for the same and prove that its complexity is of the order of n^2.

10. A fresher from IIITB joins an hardware manufacturering unit of INTEL. He enters into the integration team of the company. He has the following instructions from his instructor. A logical '1' indicates the component is working and a logical '0' means the component is malfunctioning. He has been asked to combine these components with connectives 'and', 'or' and 'xor'. Given, there are 'n' components, Design an efficient algorithm to find out number of ways of grouping all the components so that the resultant state of the system is '1'.

    Note: Given the states 0,1,1 one such combination that yields true is given by the grouping of the paranthesis ((0 or 1) and 1) = (1). Find the total number of all such combinations.

11. Suppose you have k sorted arrays, each with n elements, and you want to combine them into a single sorted array of kn elements. Design an effective algorithm to do the same.

12. There exists a tile board of size 2^k * 2^k which needs to be filled with one single tile and 2^{2k} - 1 L-shaped groups (of 3 tiles). Design an effective approach that can bipartition the board into four segments recursively,

and place a L-grouped set of 3 tiles in the center at the parts that have no extra tile.

13. A Prof. of IIITb says that its asymptotically faster to square an n-bit integer than to multiply two n-bit integers. If you agree with the same, justify it. If you do not agree, provide reasons for the same.

14. An ornithologist observes a one-one mapping between birds on one side of a bridge and their respective nests on the other side. There are 'n' birds b(1), b(2), b(3)..b(n) and n nests N(1), N(2), N(3)..N(n). Find an efficient algorithm to maximize the mapping between the birds and the nests such that no two birds cross over in the path to the nest. A bird is assumed to go to its own nest.