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Assignment-2

Q1. Let x be any element in Σ^*. Let $P(y)$ denote the predicate that $||xy|| = ||x|| + ||y||$, where $y \in \Sigma^*$. Since $y \in \Sigma^*$, y can be the null word λ or a non-empty word.

Basis step: To show that $P(\lambda)$ is true; i.e;
$||x\lambda|| = ||x|| + ||\lambda||$:

Since $x\lambda = x$, $||x\lambda|| = ||x|| + 0$
$= ||x|| + ||\lambda||$;

So, $P(\lambda)$ is true.

Induction step:
Assume that $P(y)$ is true.
We must show that $P(ys)$ is also true:
We know : $xys = (xy)s$
Then $||xys|| = ||xy|| + 1$
$= (||x|| + ||y||) + 1$
$= ||x|| + (||y|| + 1)$
$= ||x|| + ||ys||$
Therefore, $P(ys)$ is true.
Hence by PMI, $||xy|| = ||x|| + ||y||$

Q3. If m pigeons are assigned to n pigeon holes then there must be a pigeon hole containing atleast

$$\left[\frac{m-1}{n}\right] + 1 \text{ pigeons}$$

here $m = 37$ $n = 36 = 26 + 10$ thus atleast 2 pigeons.

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Q4. $m = 12,305$

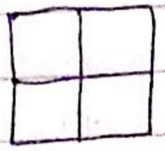
$n = 13$

$\Rightarrow \left\lfloor \frac{12304}{13} \right\rfloor + 1$

$\Rightarrow 947$

cost at least 947 refrigerator.

Q5. Let us divide the square into 4 $\frac{1}{2} \times \frac{1}{2}$ squar

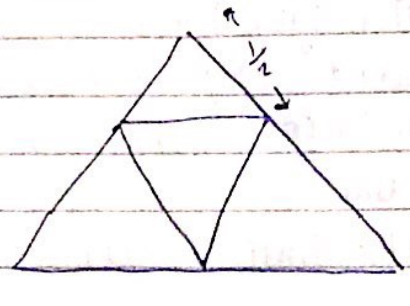


If five points in these four squares

$1 + \left\lfloor \frac{5-1}{4} \right\rfloor = 2$

Thus at least one square contain 2 points Max distance b/w the two points that is diagonal length is $\frac{1}{\sqrt{2}}$ thus at least two of them is no more than $\frac{\sqrt{2}}{2}$.

Q6.



Let us divide the ~~triang~~ triangle into 4 equilateral triangle

If five points in these four triangles

$\left\lfloor \frac{5-1}{4} \right\rfloor + 1 = 2$

Thus at least one triangle contain 2 points Max distance b/w the two points that is side length is $\frac{1}{2}$ thus at least two of them

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is no more than $\frac{1}{2}$.

Q8 LHS

$$\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2}$$

RHS $$\sum_{i=m}^{n} (n+m-i) = n(n-m) + m(n-m) - \sum_{i=m}^{n} i$$

$$= \frac{n^2 - m^2}{2} - \left(\frac{n-m}{2}\right)$$

$$= \frac{n(n+1)}{2} - \frac{m(m+1)}{2}$$

LHS = RHS;

Q9.

a) $$S = \sum_{i=m+1}^{n} (a_i - a_{i-1})$$

$$= \cancel{a_{m+1}} - a_m$$
$$\cancel{a_{m+2}} - \cancel{a_{m+1}}$$
$$\vdots \qquad \vdots$$
$$a_n - \cancel{a_{n-1}}$$

$$= a_n - a_m;$$

b) $$\sum_{i=1}^{n} \frac{1}{i(i+1)} = -a_1 + a_n$$

$$= 1 - \frac{1}{n+1};$$

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Q10. $\sum_{1 \le i \le j < 3} (a_i + a_j)$

$i=1$ $j=1$: $a_1 + a_1 = 2a_1$

$j=2$: $a_1 + a_2$

$i=2$ $j=2$: $2a_2$

Total : $\underline{3(a_1 + a_2)}$

Q11.

a) $m = p$ $\quad m \times n$

$n = q$

b) $p = r$ $\quad p \times q$

$q = s$

c) $q = r$ $\quad p \times q$

d) $m = n$ $\quad m \times m$

Q12. a) $\begin{bmatrix} 25 & 40 & 35 & 0 \\ 20 & 0 & 15 & 15 \\ 20 & 0 & 30 & 40 \end{bmatrix} \begin{bmatrix} 7 \\ 14 \\ 21 \\ 28 \end{bmatrix} = \begin{bmatrix} SL \\ L \\ U \end{bmatrix}$

SL gram intake = 1470

L gram intake 875

U gram intake = 1890

b) Total cost of the insulin:

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$$\begin{bmatrix} 1470 \\ 875 \\ 1890 \end{bmatrix} \begin{bmatrix} 10 & 11 & 12 \end{bmatrix} = 147005$$

+

c) $$\begin{bmatrix} 25 & 40 & 35 & 0 \\ 20 & 0 & 15 & 15 \\ 20 & 0 & 30 & 40 \end{bmatrix} \begin{bmatrix} 10 \\ 9 \\ 29 \\ 51 \end{bmatrix} = \begin{bmatrix} 2025 \\ 1250 \\ 2710 \end{bmatrix} \begin{matrix} \to \text{semi } L \\ \to L \\ \to U \end{matrix}$$

d) $$3 \times \begin{bmatrix} 1470 \\ 815 \\ 1890 \end{bmatrix} = \begin{bmatrix} 4410 \\ 2625 \\ 5670 \end{bmatrix}$$

Q13.

1024) 2076 (2
2048
28) 1024 (36
-1008
16) 28 (1
-16
12) 16 (1
-12
4) 12 (3
-12
×

Ans 4.

Q14. a) $1+2+ \_\_\_ 12 = \frac{12 \times 13}{2} = 78.$

b) $$\sum_{i=1}^{12} \frac{i(i+1)}{2}$$

$$= \sum_{1}^{12} \frac{i^2}{2} + \sum_{1}^{12} \frac{i}{2}$$

$$= \frac{12(12+1)(25)}{12} + \frac{12 \times 13}{4} = 325 + 39 = 364.$$

Premium

Q15

a) $$\sum_{i=1}^{n}(2i-1) = n^2$$

By PMI

$n = 2$

$$\sum_{i=1}^{2}(2i-1) = 1 + 3 = 4$$

$n = k$

$$\sum_{i=1}^{k}(2i-1) = k^2;$$

Thus it should be true for $k+1$

$$\sum_{i=1}^{k+1}(2i-1) = \sum_{i=1}^{k}(2i-1) + 2(k+1) - 1$$

$$= k^2 + 2k + 1 = (k+1)^2$$

Thus for $k+1$

$$\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$$

b) $n^4 + 2n^3 + n^2$ is divisible by 4

By PMI

$n = 2$

$16 + 16 + 4 = 36$

$n = k$

$k^4 + 2k^3 + k^2$ is divisible by 4

~~$k^4 \cdot 2k$~~ $k^4 + 2k^3 + k^2 = 4p_i = k^2(k+1)^2$

Thus this ~~is~~ should be true for $k+1$

$(k+1)^4 + 2(k+1)^3 + (k+1)^2$

$(k+1)^2 \; (k+1+1)^2$

$(k+1)^2 (k+2)^2$

$= (k^4 + 2k^3 + k^2) \; 4 \; (k^3 + 3k^2 + 3k + 1)$

$= 4q$

$P(k+1)$: True

By PMI $P(n)$ is true $\forall \; n > 1$

Hence proved.

Q16. a) $n \leftarrow 0$

$$\sum_{1}^{n} (2i-1)$$

$= n(n+1) - n$

$= n^2$

b) ~~$n(n+$~~

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Q2. Let $a_1, a_2 \ldots a_{n+1}$ denote the integers selected we can write each of them as a product of a power of 2 and an odd integer i.e.

$$a_i = 2^{e_i} b_i \text{ where } 1 \leq i \leq n+1 \text{ and } e_i \geq 0$$

The integers $b_1, b_2 \ldots b_{n+1}$ are odd positive integers $\leq 2n$ since there are exactly n odd positive integer $\leq 2n$, by the pigeonhole principle, two of the elements $b_1, b_2, \ldots b_{n+1}$ must be equal

say, $b_i = b_j$

i.e. $a_i = 2^{e_i} b_j = 2^{e_j} b_i$

Thus, if $e_i < e_j$ then $a_i | a_j$

if $e_j < e_i$, then $a_j | a_i$

Hence proved.