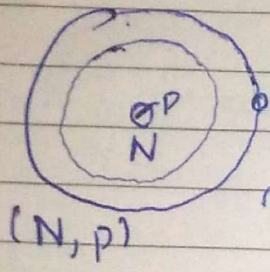
Electrostatics

Study of → force b/w charged particle system
→ Enery stored in the system
→ Work done on the charge particle system while distrubuting the electric field



$n_p = n_e$

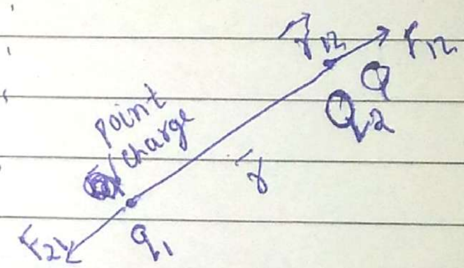
Elementary charge

$e = 1.6 \times 10^{-19}$ C

Coulomb's law

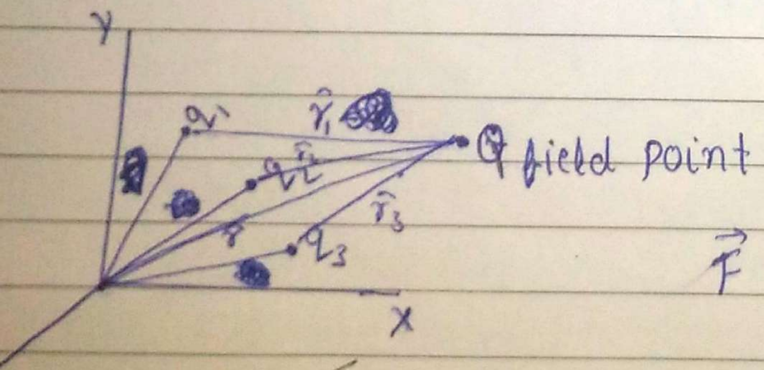
$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$4\pi\varepsilon_0$ → permittivity of free space
8.85×10^{-12} C^2/Nm^2



$$F_{21} = K \frac{q_1 q_2}{|r_{21}|^2} \hat{r}_{21}$$

Concept of Electric Field



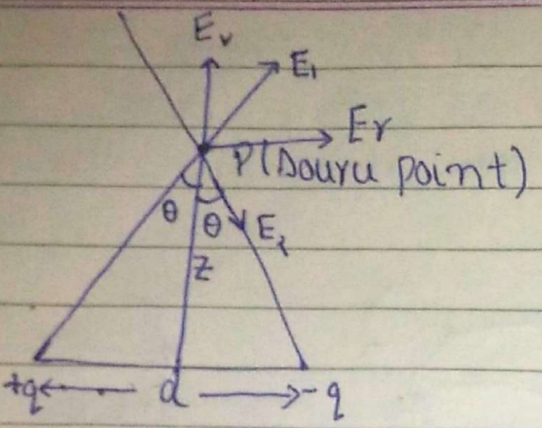
If we have several charge points $q_1, q_2, \ldots, q_n$ at a distance $r_1, r_2, \ldots, r_n$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{q_n}{r_n^2}\hat{r}_n\right)$$

$$\therefore \boxed{E(r) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i}$$

↑ from field point



$$|E_1| = |E_2|$$

$$\therefore \; E_v = 0$$

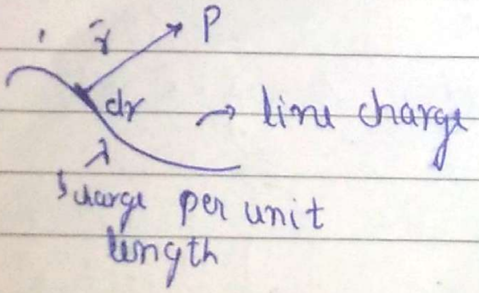
$$E_r = \frac{2}{4\pi\epsilon_0}\left(\frac{q \sin\theta}{r^2}\right)$$

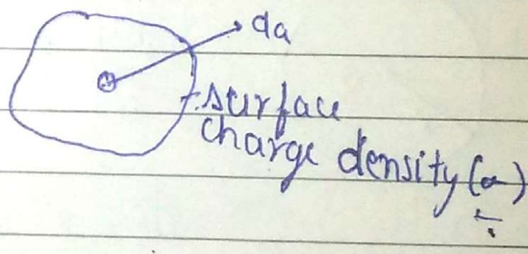
where, $r^2 = \left(\frac{d}{2}\right)^2 + z^2$

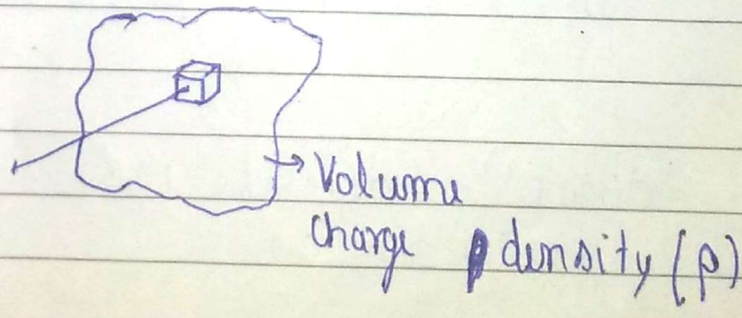
$$\therefore \; \boxed{E_r = \frac{1}{4\pi\epsilon_0} \frac{qd}{\left(z^2 + \frac{d^2}{4}\right)^{3/2}}}$$

If source point is very far away, i.e., $z >> d$

$$\therefore \; E_r = \frac{1}{4\pi\epsilon_0} \frac{qd}{z^3} \hat{r}$$

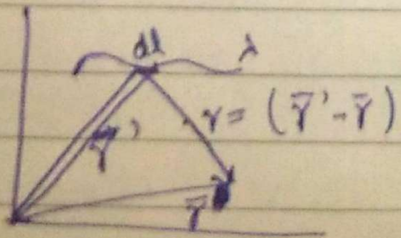






$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

Eg.



$$E = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \, d\vec{r}' \, (r - r')}{|r - r'|^3}$$

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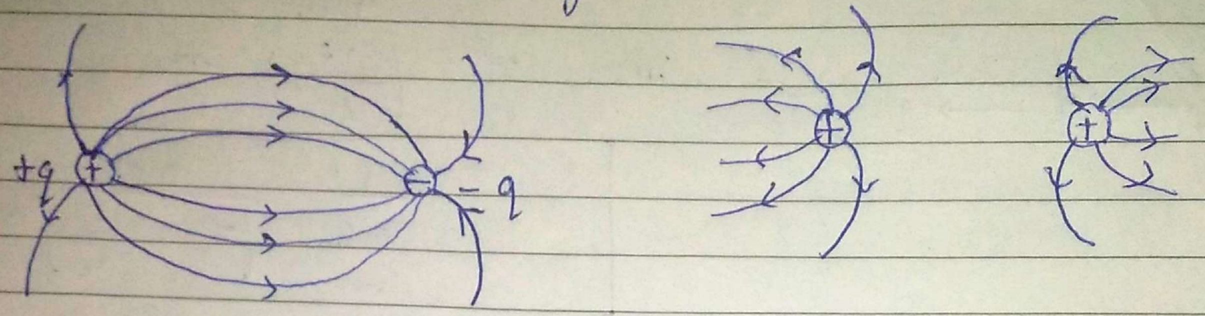
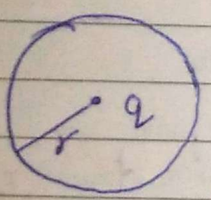
Electric field lines v/s Electric vector field

The direct^n is tangent at any point.
No of field lines per unit area gives the idea about the field strength.



Gauss's Law



For point charge and a spherical surface, the total flux going through the spherical surface $\phi = \oint \bar{E} \cdot d\bar{s}$

$d\bar{s} = r^2 \sin\theta \, d\theta \, d\phi$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{q\hat{r}}{4\pi\epsilon_0 r^2} r^2 \sin\theta \, d\theta \, \hat{r} \, d\phi$$

$$= \frac{q}{\epsilon_0}$$

Therefore, ϕ doesn't depents upon the shape of the surface.

Suppose instead of a single charge at the origin we have a bunch of charge scattered in the surface, therefore flux through that encloses them all $\oint \bar{E} \cdot d\bar{s} = \sum_{i=1}^{n} \oint \bar{E}_i \cdot d\bar{s} = \frac{Q_{enclo}}{\epsilon_0}$

total charge enclosed inside the particular surface.

We know that

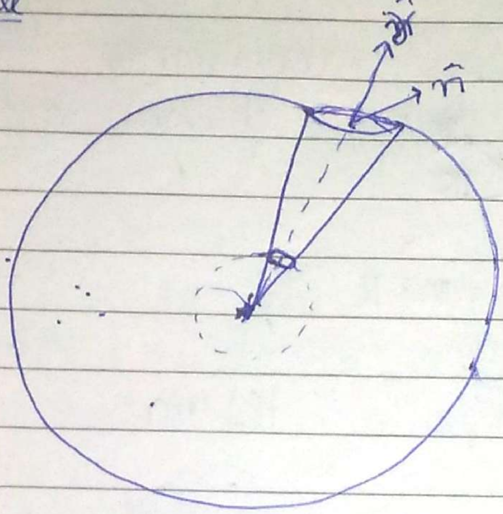
$$\oint \bar{E} \cdot d\bar{s} = \underbrace{\int (\bar{\nabla} \cdot \bar{E})\, dv}_{\text{divergence}} = \int_V \left(\frac{\rho}{\epsilon_0}\right) dv$$

charge density

$$\bar{\nabla} \cdot \bar{E} = \rho/\epsilon_0$$

$$Q_{enclosed} = \int_V \rho \, dv$$

Solid Angle



solid Angle

$$\boxed{d\Omega = \frac{\hat{n} \cdot \hat{r}\, da}{r^2}}$$

The total solid Angle

$$\oint \frac{\hat{n} \cdot \hat{r}\, da}{r^2} = 4\pi$$

The flux of electric field originated from the charge can be calculated

$$\phi = \int \bar{E} \cdot \hat{n}\, da = \int \frac{q\, \hat{r} \cdot \hat{n}\, da}{4\pi\epsilon_0 r^2} = \frac{q}{\epsilon_0} = \frac{Q_{enclosed}}{\epsilon_0}$$

Application of Gauss's law

$$\oint \bar{E}\, d\bar{a} = \frac{Q_{enclosed}}{\epsilon_0}$$

To find out Electric field.

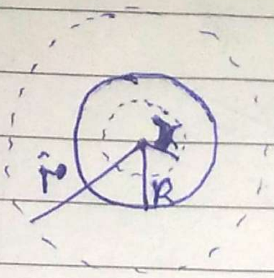
i) E should be uniform on the Gaussian surface

ii) $\bar{E}$ & $d\bar{a}$ must have some definite notation

$\bar{E} \parallel d\bar{a}$ or $\bar{E} \perp d\bar{a}$

$\cancel{\dfrac{q}{4\pi\epsilon_0 r^2}}$

① Uniform charged hollow surface



uniformly charged distribution

$$\oint \mathcal{E} \cdot da = \frac{q}{\epsilon_0} \qquad r > R$$

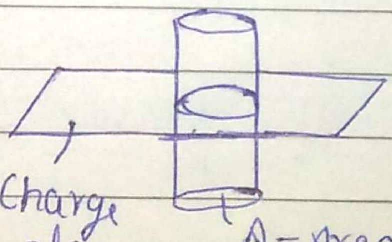
$$\oint |\mathcal{E}| \cdot da = |\mathcal{E}| \oint da \qquad |\mathcal{E}|\, 4\pi r^2$$

Electric field inside | E.F. outside

$$\mathcal{E}_{inc} \int da = 0 \qquad \mathcal{E}_{out} = \frac{q}{4\pi\epsilon_0} \cdot \frac{\hat{r}}{r^2}$$

Infinite plane σ = charge density

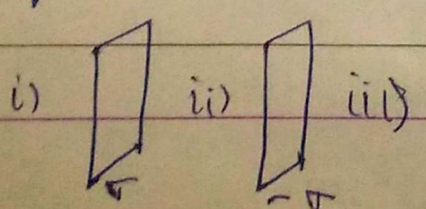
$$\oint |\bar{\mathcal{E}}|\, d\bar{a} = \frac{\sigma A}{\epsilon_0}$$



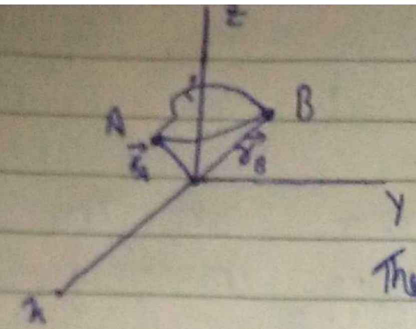
$$|\mathcal{E}| \cdot \int d\bar{a} = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{|\mathcal{E}| = \frac{\sigma}{\epsilon_0}\hat{n}}$$

Two infinite planes carry equal & opp. uniform chare of $\pm\sigma$



Find the EF in each of three regions



$$\mathcal{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

The line integral of $\vec{\mathcal{E}}$ $\int_A^B \vec{\mathcal{E}} \cdot d\vec{l}$

Therefore, $\vec{\mathcal{E}} \cdot d\vec{l} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} dr$

$$\int_A^B \vec{\mathcal{E}} \cdot d\vec{l} = \int_A^B \frac{q\, dr}{4\pi\epsilon_0 r^2} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

According of stoke theorem

$$\oint (\nabla \times v)\, da = \int \vec{\nabla} \cdot dl$$

$$\nabla \times v\, da = \nabla \cdot dl$$

The electric field is conserved

<u>Definition of Electrostatic potential</u>

$\vec{\nabla} \times \vec{\mathcal{E}} = 0$, the line of an $\vec{\mathcal{E}}$ around the closed loop is zero.

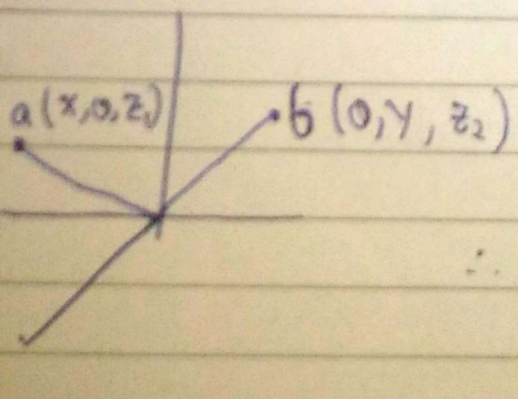
The line integral is independent of path, we can find a funct^n V

$$V(r) = -\int_a^r \vec{\mathcal{E}} \cdot d\vec{l}$$

$a \rightarrow$ any reference point

Scaler Quantity.

The function V is dependent on the point r. This is called potential

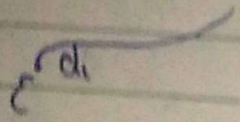


$$V_{(b)} = -\int_0^b \bar{\mathcal{E}}\, d\bar{l} \qquad V_{(a)} = -\int_0^a \bar{\mathcal{E}} \cdot d\bar{l}$$

$$\therefore V_{(b)} - V_{(a)} = -\int_a^b \vec{\mathcal{E}} \cdot d\bar{l}$$

(Potential diff.)

Suppose, we wish to move a charge q a distance di in an electric field $\vec{E}$.



The force acting on the charge particle having charge q

$$\vec{F}_E = q\vec{E}\,\hat{m}$$

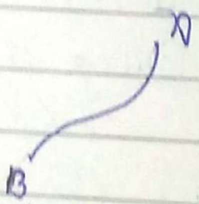
Therefore, we must apply eq. & opposite force to the force associated with E.F.

The differential work done by the external force moving q $[-q\,E\cdot\hat{m}\,dl = -q\,E\cdot dl]$

$$\therefore\ -dW = -q\,E\cdot dl$$

The work required to move the charge from infinite to finite distance

$$W = -q\int_{\text{infinite}}^{\text{finite}} E\cdot dl$$



Suppose we have a non-uniform electric field

$$E = y\hat{i} + x\hat{j} + z\hat{k}$$

determine the work done on a charge 2q from B(1,0,1) to A(0.8,0.6,1) along the shortest root.

Ans
$$W = -2q\int_B^A E\cdot d\vec{l}$$

$$= -2q\int_B^A (y\,dx + x\,dy + z\,dz)$$

$$= -q\int_B^A xy + \frac{z^2}{}$$

$$= -q\,[\,4\times0.8\times0.6\,]$$

Electric potential of a system ~~[illegible]~~
i) Uniformly charged hollow sphere

$V(\infty) = 0$, $V(0) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l} = \frac{q}{4\pi\epsilon_0 r}$

On the surface $V(R) = \frac{q}{4\pi\epsilon_0 R}$

Inside $= -\int_{\infty}^{R} \vec{E} \cdot d\vec{r}$