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find eigen values and eigen vectors of

$$A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$$

$$\begin{array}{r} 12 \\ 12 \\ \hline 132 \end{array}$$

$(12-\lambda)(-11-\lambda) - (-51)\times 2 = 0$

$-(12-\lambda)(11+\lambda) + 102 = 0$

$[A - \lambda I_2] = 0 \Rightarrow \begin{bmatrix} 12-\lambda & -51 \\ 2 & -11-\lambda \end{bmatrix}$

$-132 + 11\lambda - 12\lambda - \lambda^2 + 102 = 0$

$\lambda^2 - \lambda - 30 = 0$

$\lambda = 6, -5$

eigen vector corresponding to $\lambda = 6$

$(A - \lambda I_2)X = 0$

$$\begin{pmatrix} 6 & -51 \\ 2 & -17 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Apply $R_1 \to \frac{1}{6} R_1$

$$\begin{pmatrix} 1 & -51/6 \\ 2 & -17 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -51/6 \\ 0 & -17 + \frac{51}{3} \end{pmatrix} = \cancel{\begin{pmatrix} 1 & -51/6 \\ 0 & 0 \end{pmatrix}}$$

$$\begin{bmatrix} 1 & -51/6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$x_1 - \frac{51}{6} x_2 = 0$

let $x_2 = a \Rightarrow x_1 = \frac{17a}{2}$

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$$[x_1, x_2] = \left[\left(\frac{17}{2}\right)a, a\right]$$

$$= \frac{1}{2}a\,[17, 2]$$

eigen vectors corresponding to $\lambda = 6$ are non zero multiples of vector $[17, 2]$

eigen space : set of all non zero eigen vector

$$E_6 = \{a(17, 2) : a \in R\}$$

eigen vector corresponding to $\lambda = -5$

$$(A - 5I)X = 0$$

$$\begin{bmatrix} 17 & -51 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Apply $R_1 \to \frac{1}{17}R_1$ & $R_2 = \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & -3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$x_1 - 3x_2 = 0$$

let $x_2 = a$, $x_1 = 3a$

$$[x_1, x_2] = [3a, a] = a[3, 1]$$

eigen vector corresponding to $\lambda = -5$ is non zero multiple of $[3, 1]$

$$E_{-5} = \{a(3, 1) : a \in R\}$$

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$$A = \begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 2 & -3 \\ 1 & -1 & 2 \\ 3 & -4 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 2 & -3 \\ 0 & 1 & -1 \\ 3 & -4 & 7 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} -1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} -1-\lambda & 2 & -3 \\ 0 & 1-\lambda & -1 \\ 0 & 0 & 4-\lambda \end{bmatrix}$$

$$\begin{bmatrix} -4-\lambda & 8 & -12 \\ 6 & -6-\lambda & 12 \\ 6 & -8 & 14-\lambda \end{bmatrix}$$

$-(\lambda+1)(1-\lambda)(4-\lambda)=0$

$\lambda = 1, -1, 4$

$\lambda = 0, 2, 2$

$(A - \lambda I_3) X = 0$

$\lambda^3 - 4\lambda^2 + 4\lambda = 0$

eigen vector corresponding to $\lambda = 0$

$$\begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

using suitable row operatn we get

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$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + x_3 = 0$ $\qquad$ $x_3 = a$

$x_2 - x_3 = 0$ $\qquad$ $x_1 = -a$

$x_2 = a$

$[x_1, x_2, x_3] = [-a, a, a] = a[-1, 1, 1]$

eigenvectors corresponding to $\lambda = 0$ are non zero multiples of vector.

$$X_1 = [-1, 1, 1]$$

$$E_0 = \{ aX_1 : a \in R \}$$

Similarly for $\lambda = 2$

$(A - 2I_2)X = 0$

$$\begin{bmatrix} -6 & 8 & -12 \\ 6 & -8 & 12 \\ 6 & -8 & 12 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Apply suitable row operation we get

$$\begin{bmatrix} 1 & -4/3 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - \frac{4}{3}x_2 + 2x_3 = 0$

let $x_2 = a$

$x_3 = b$

$x_1 = \left(\frac{4}{3}\right)a - 2b$

$[x_1, x_2, x_3] = \left[\left(\frac{4}{3}\right)a - 2b, a, b\right]$

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$= \left[\left(\frac{4}{3}a, a, 0\right) + (-2, 0, b)\right]$

$= \frac{1}{3}a\,[4, 3, 0] + b(-2, 0, 1)$

$x_2 = [4, 3, 0]$

$x_3 = [-2, 0, 1]$

$E_2 = \{ a x_2 + b x_3 : a, b \in R \}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \overset{\text{eigen value}}{\lambda = 0, 0, 0}$$

For $\lambda = 0$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 = 0$

$x_3 = 0$

let $x_1 = a$

$X_1 = [a, 0, 0] = a(1, 0, 0)$

Properties of eigen values

# Sum of eigen value of matrix A = trace of matrix A = sum of diagonal entries

# Product of eigen values of a matrix = determinant of A

It further implies that determinant of A always vanishes iff atleast one eigen value is 0

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If $\lambda$ is eigenvalue of A then $\lambda^m$ is eigenvalue of $A^m$ where m is +ve integer & $\frac{1}{\lambda}$ is eigent value of $A^{-1}$ & $\lambda-\lambda$ is eigen valu of $A-KI$ where k is any real no.

Ex:- If $A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$

then $P = \begin{bmatrix} 17 & 3 \\ 2 & 1 \end{bmatrix}$

& $P^{-1}AP = \begin{bmatrix} 6 & 0 \\ 0 & -5 \end{bmatrix}$ verify?

Note:- If A is diagonal matrix i.e $P^{-1}AP = D$ then for any +ve integer n

$$A^n = PD^nP^{-1} \quad \text{for}$$
$$A^2 = [PD^{-1}P]^2 = PD^2P^{-1}$$
$$[PDP^{-1}][PDP^{-1}]$$

Determinat $A^2$ if

$$A = \begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$$

So $P = \begin{bmatrix} 4 & -2 & -1 \\ 3 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$P^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 3 & -4 & 7 \\ -3 & 4 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

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$$A^2 = PD^2P^{-1}$$

$$= \begin{bmatrix} 4 & -2 & -1 \\ 3 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & 7 \\ -3 & 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 16 & -24 \\ 12 & -12 & 24 \\ 12 & -16 & 28 \end{bmatrix}$$

Group:- A group is a set in which we can perform one operation (usually addition or multiplication)

Ring:- A ring is a set in which we can perform two operation (usually addition & multiplica

A ring is a group under addition & satisfies some of the properties of a group of multiplica

Field:- A field is a group under both addition & multiplication

Group:- A group is a set G which is closed under an operation + (i.e. for $x, y \in G$, ~~x+y~~ $x*y \in G$) and satisfies following properties

1. Identity: There is an element $\varepsilon$ in G such that for every element of $x \in G$
$$\varepsilon * x = x * \varepsilon = x$$
2. Inverse: for every $x$ in G there is an element $y \in G$ such that
$$x * y = y * x = \varepsilon$$

where ε is

3. Associativity
$x, y, z \in G$
$x * ($

Abilian gr

A Ring is a
operations
properties
1. Ring R
2. Associativ
3. Distributive
following

Field - A fi
Two operat
1) F is abili
2) F - {0} i
0} is a

where e is an identity

5. Associativity : The following identity holds for every $x, y, z \in G$

$$x * (y * z) = (x * y) * z$$

Abilian group $x * y = y * x \quad \forall\, x, y \in G$

A Ring is a set R which is closed under two operations + & $\times$ and satisfies the following properties

1. Ring R is an abilian group under addition
2. Associativity of $\times$ for every $a, b, c \in R$

$$a \times (b \times c) = (a \times b) \times c$$

3. Distributive property for every $a, b, c \in R$ the following identity hold

$$a \times (b + c) = a \times b + a \times c$$

$$(b + c) \times a = b \times a + c \times a$$

Field - A field is a set F which is closed under two operations + & $\times$ such that

1) F is abilian group under addition
2) $F - \{0\}$ i.e. {the set F without additive identity 0} is an abilian group under multiplication