$(a \vee b \vee c) \rightarrow (a \vee y) \wedge (\bar{y} \vee b \vee c)$ can't convert to 2-SAT using what we did earlier

⇓

2-SAT isn't polynomial time

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Ex HAMPATH = { <G, s, t> | There is a hamiltonian path from s to t }

3-SAT $\leq_p$ HAMPATH

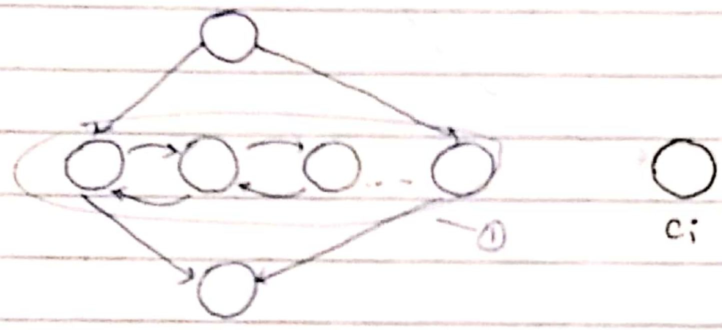
literal $(x_1, \bar{x}_1)$

$\phi = (a_1 \vee b_1 \vee c_1) \wedge (a_2 \vee b_2 \vee c_2) \ldots \wedge (a_k \vee b_k \vee c_k)$

↓ clause C_1 ↓ C_2 ↓ C_k

For each literal, we'll create a graph here.

x_1 :



For each clause, we'll add a node

Now, we've to connect the above components. (x_i and c_i)

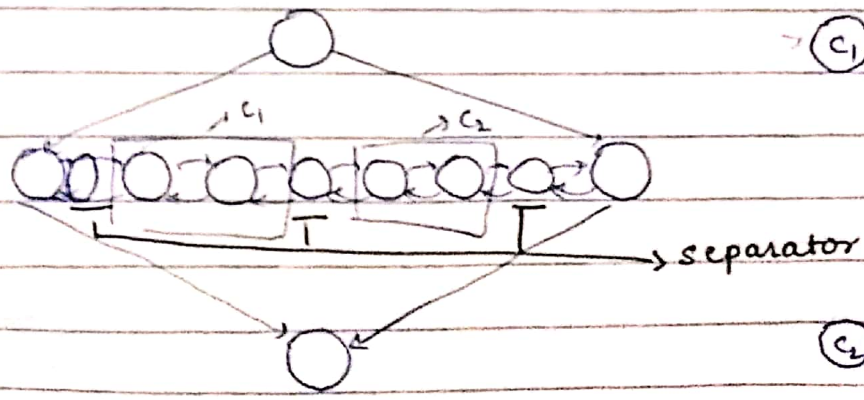
If $x_i \in C_i$, then we'll connect literal x_i to node c_i

① For each clause, we add 2 nodes separated with other classes by a node → separator node → at boundary also

$(a_1 \vee b_1 \vee c_1) \wedge (a_2 \vee b_2 \vee c_2)$

↓ C_1 ↓ C_2

a_1

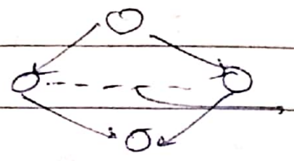


→ Will have similar structure of graph for every literal

↓

$x_1, x_2, \ldots$

→ If we've k-classes within the 3-SAT formula, no. of vertices in b/w (nodes) : $3k+1$



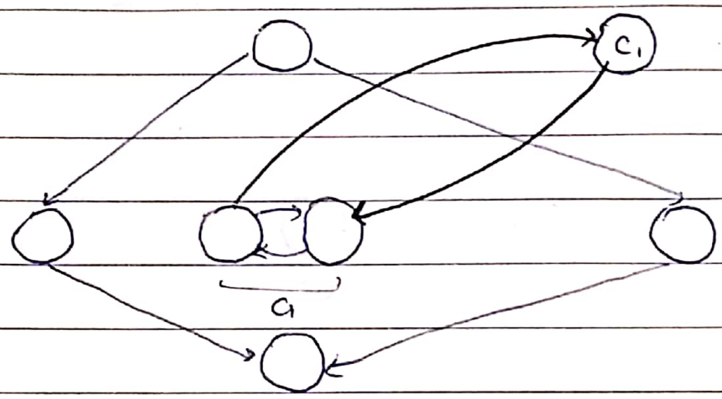
$3k+1$ no. of nodes (excluding boundary nodes)

• How to connect C_i with literal ?

For a particular class C_i, we've nodes inside literal graph

$a_1 \vee b \vee c$

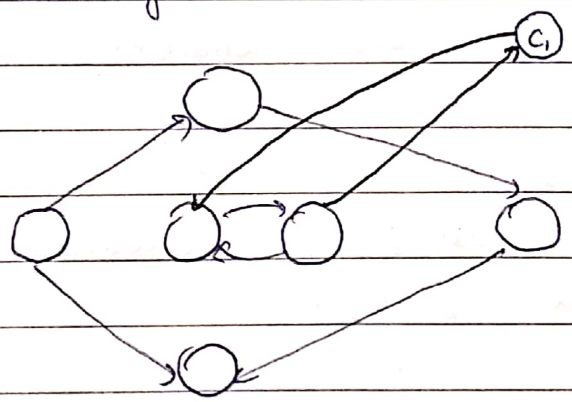
a



If $\bar{a} \in C_1$, we'll join it like : (reverse the arrow)

$\bar{a} \vee b \vee c$

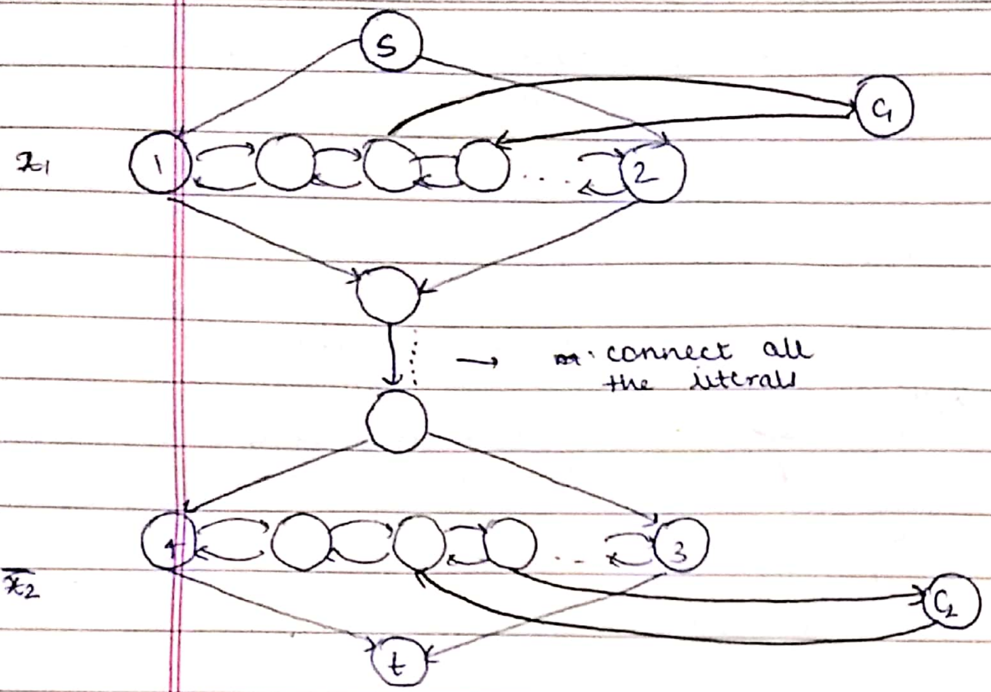
a



• If $x_i \notin C_j$: don't make any edge.

2

let $x_1 \in C_1$
$x_2 \in C_2$



- we're to get HAMPATH from this graph.

↓

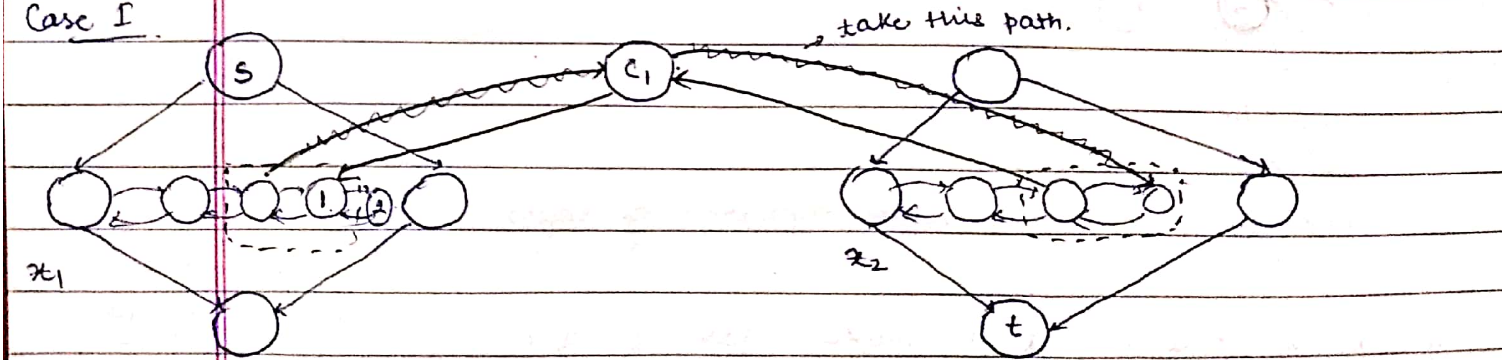
if $x_1 \in C_1$: go from s to 1 move inside, visit C_1, move (left → right) to 2 & move down further

$\bar{x}_2 \in C_2$ = go from s to 3 " " " C_2, move (right → left) to 4 & move down further.

problem: C_1 might be connected to other diamond structure also. we have to rule this possibility out in order to get Hamiltonian path.
↓ contradiction

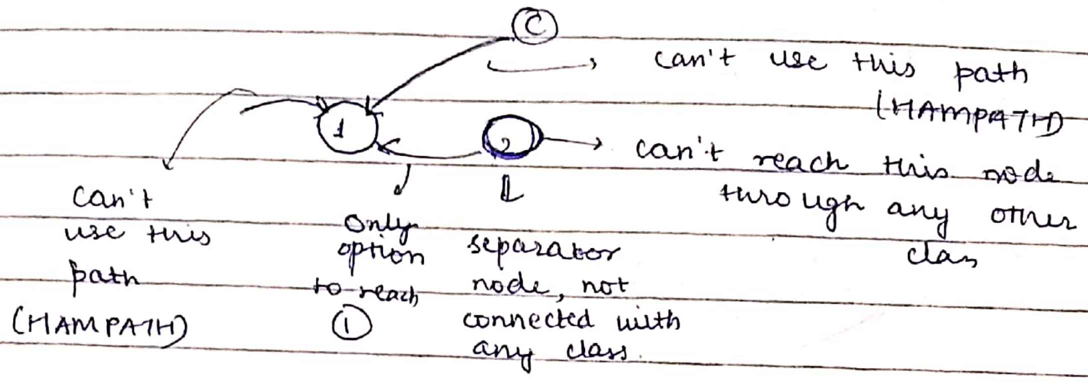
let it move to other diamond structure.

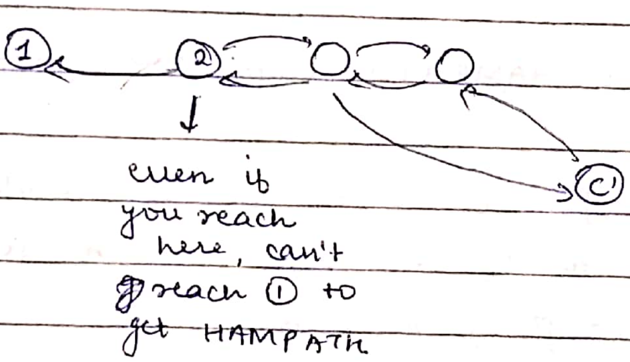
Case I



Problem here : we won't be able to cover certain vertices in both the diamond structures (either one of ① or ②)

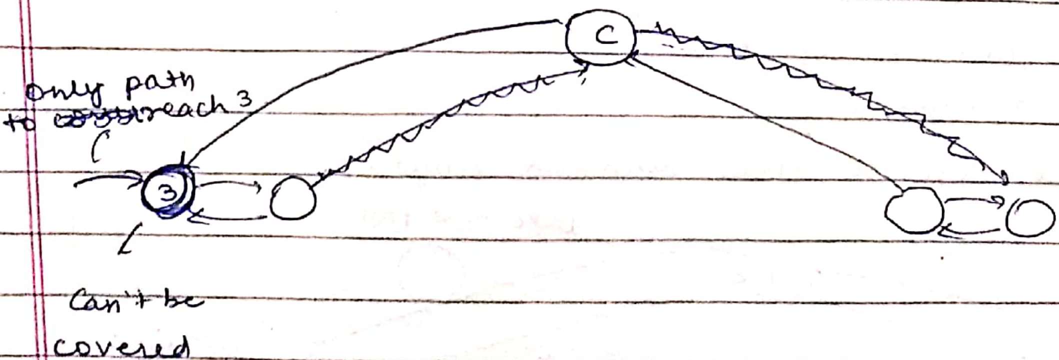
How can we reach 1 ?





Case II : Arrows to C are opposite. (compliment case)

Apply same reasoning



(HAMPATH : can't have repeated edges)

if x_1 is true : it'll make class C_1 true. If more than 1 literal : make any one true.

- Each class is atleast connected with one of diamond structures

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