The LNMIIT Jaipur

Discrete Mathematics Structures (DMS)

Mid Term Exam 2017

M. Marks 30

Roll No.

Note: Partial marks can be given only in Question 2 & 5, & there is no other partial marking in other remaining questions. Solve questions carefully. All the best.

Question 1

Marks [1+2+2=5]

a) Represent the relation R on the given set A in a digraph. R={(a, b) | b=a+2}, A={2,4,5,6}

b) Find the connectivity relation of the relation R={(a, b), (a, c), (b, a), (c, a)} on {a, b, c}

OR

Find the number of times the statement x <-- x + 1 is executed by following loop.

```
for i = 1 to n do
    for j = 1 to i do
        for k = 1 to j do
            x←-x+1
```

c) Determine if the given elements {a, b} and {b} are comparable in the poset (A, ⊆), where A denotes the power set of {a, b, c},

Question 2

Marks [3.5+1.5+1=6]

a) Construct a Hasse diagram for each poset. (A, R), where A = {a, b, c} and R= {(a, a), (a, b), (b, b), (b, c), (c, c)}.

b) Prove that C(2n, n) is an even integer for every n >= 1.

c) In how many ways can 10 quarters in a piggy bank be distributed among 7 people?

Question 3

Marks [1.5 +2+1.5=5]

a) Let A be a 10-element subset of the set {1, 2,... 20}. Determine if A has two five-element subsets that yield the same sum of the elements.

b) Let A and B be two finite sets with |A| = m and |B| = n. How many bijections can be defined from A to B (assume m = n)?

c) Find the number of two-digit numerals that can be formed using the digits 0, 3, 5, 6, and, 9 and that contain no repeated digits

OR

Using the pigeonhole principle, prove that the cardinality of a finite set is unique.

Question 4

Marks [1.5+2+1.5=5]

a) Using the sets A= {a, b, e, h}, B = {b, c, e, f, h}, C= {c, d, f, g}, and U = { a , . . . , h}, find the binary representation of each set and then compute (A⊕B)-C

b) If 10 points are selected inside an equilateral triangle of unit side, then at least two of them are no more than 1/3 of a unit apart.

OR

An important problem in computer science is to determine whether or not a given expression is legally parenthesized. For example, (()), ( ) (), and (() ()) are validly paired sequences of parentheses, but) (), ( ) (, and) ( ) (are not. Define the set S of sequences of legally paired parentheses recursively.

c) Let a_n denote the number of times the assignment statement x <- x + 1 is executed by each nested for loop. Define a_n recursively.

```
for i = 1 to n do
   for j = 1 to i  do
      x <-x + 1
```

Question 5

Marks [2.5+3.5+3=9]

a) Prove that the predicate P(n) in the following algorithm is a loop invariant. Algorithm sum (x, y) (* This algorithm prints the sum of two nonnegative integers x and y. *)

0. Start (* algorithm *)
I. Sum = x
2. Count = 0 (* counter *)
3. while count < y do
4. Sum =Sum+1
5. Count=Count + 1
7. end while
8. End (* algorithm *)

$P(n): x = q_n y + r_n$

P (n) : $q_n y + r_n$ where q_n and r_n denote the quotient and the remainder after n iterations.

b) Solve $a_n = 7a_{n-1} - 10a_{n-2} + n^2$, , *where* $a_0 = 0$ $a_1 = 1$

c) Using generating functions, solve following LHRRWCC $a_n = a_{n-1} + 2$, *where* $a_1 = 1$.