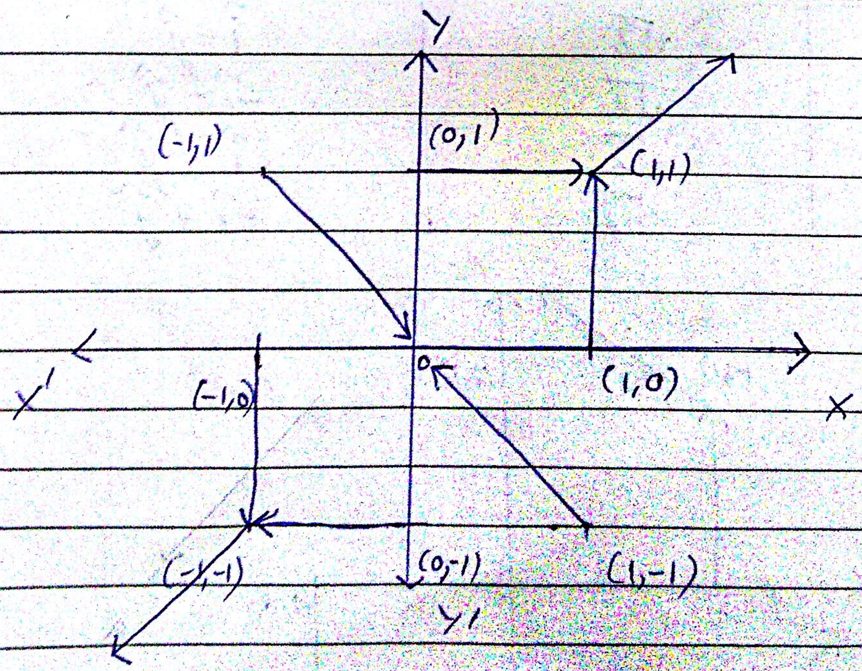
P-II (Electrodynamics)

Assignment - 1

Q1) a) $V = y\hat{i} + x\hat{j}$

	Magnitude	direction
(1,0)	1	$\hat{j}$
(0,1)	1	$\hat{i}$
(-1,0)	1	$-\hat{j}$
(0,-1)	1	$-\hat{i}$
(1,1)	$\sqrt{2}$	$\hat{i}+\hat{j}$
(1,-1)	$\sqrt{2}$	$-\hat{i}+\hat{j}$
(-1,1)	$\sqrt{2}$	$\hat{i}-\hat{j}$
(-1,-1)	$\sqrt{2}$	$-\hat{i}-\hat{j}$



Divergence $\vec{\nabla}\cdot V = \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(x)$

$= 0 + 0 = 0$

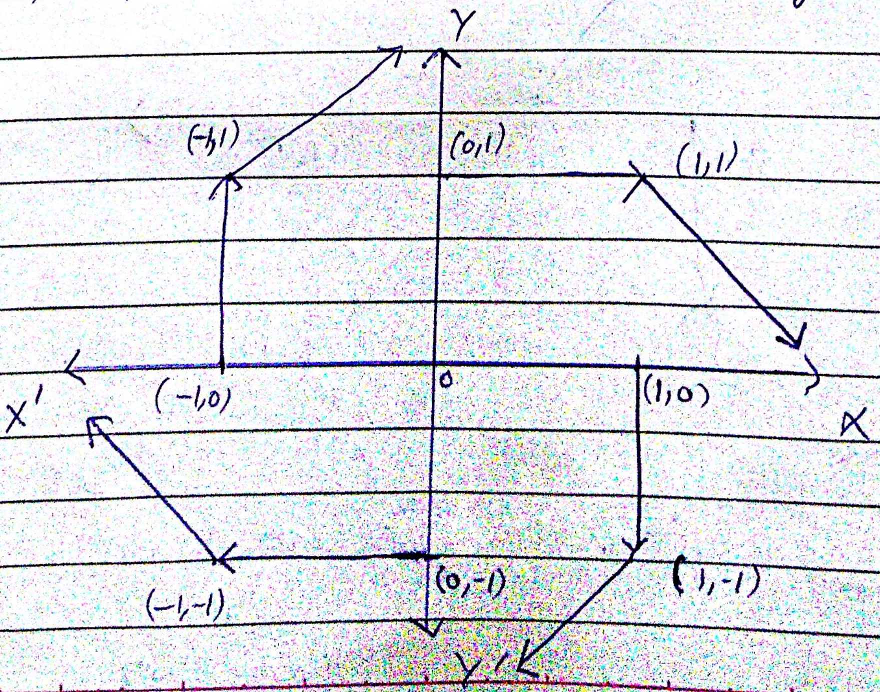
Curl $\vec{\nabla}\times V = \hat{x}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \hat{y}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) + \hat{z}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$

$= \hat{x}\left(0 - \frac{\partial (x)}{\partial z}\right) + \hat{y}\left(\frac{\partial}{\partial z}(y) - 0\right) + \hat{z}\left(\frac{\partial (x)}{\partial x} - \frac{\partial (y)}{\partial y}\right)$

$= 0 + 0 + \hat{z}(1-1) = 0$

b) $V = y\hat{i} - x\hat{j}$

	Magnitude	Direction
(1,0)	1	$-\hat{j}$
(0,1)	1	$\hat{i}$
(-1,0)	1	$\hat{j}$
(0,-1)	1	$-\hat{i}$
(1,1)	$\sqrt{2}$	$\hat{i}-\hat{j}$
(1,-1)	$\sqrt{2}$	$-\hat{i}-\hat{j}$
(-1,1)	$\sqrt{2}$	$\hat{i}+\hat{j}$
(-1,-1)	$\sqrt{2}$	$-\hat{i}+\hat{j}$



$$\bar{\nabla} \cdot V = \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(-x)$$

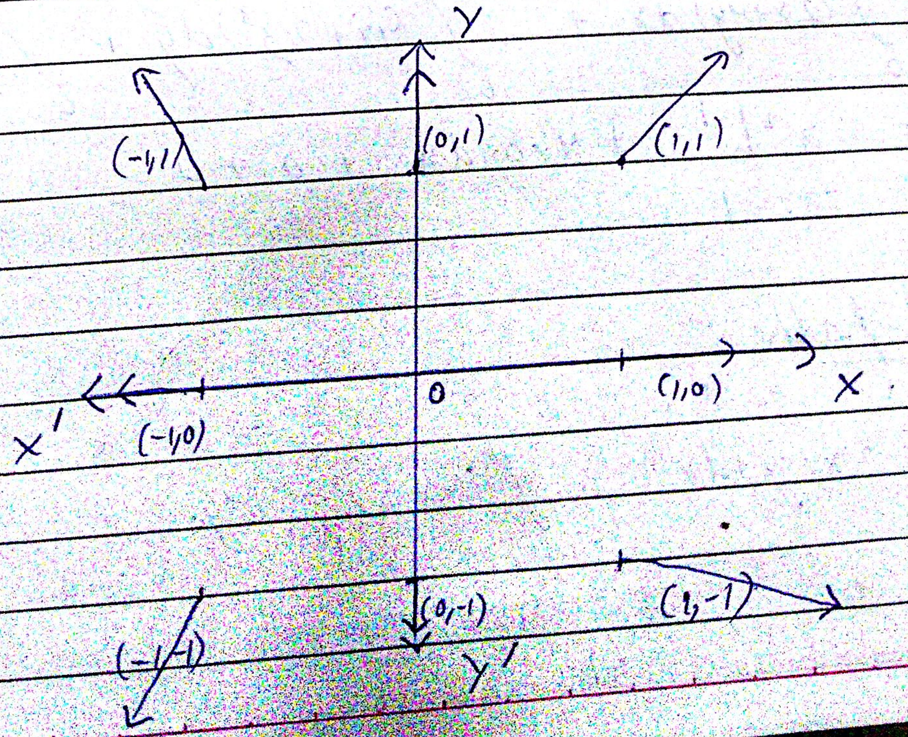
$$0 + 0 = 0$$

$$\bar{\nabla} \times V = 0 + 0 + \hat{z}\left(\frac{\partial}{\partial x}(-x) - \frac{\partial}{\partial y}(y)\right)$$

$$= -2\hat{z}$$

c) $\vec{V} = \frac{x}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\hat{j}$

	Magnitude	direction
(1,0)	1	$\hat{i}$
(0,1)	1	$\hat{j}$
(-1,0)	1	$-\hat{i}$
(0,-1)	1	$-\hat{j}$
(1,1)	1	$\hat{i} + \hat{j}$
(1,-1)	1	$\hat{i} - \hat{j}$
(-1,1)	1	$-\hat{i} + \hat{j}$
(-1,-1)	1	$-\hat{i} - \hat{j}$



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$$\bar{\nabla}\cdot V = \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

$$= \frac{x^2}{(x^2+y^2)^{3/2}} + \frac{y^2}{(x^2+y^2)^{3/2}} = \frac{1}{\sqrt{x^2+y^2}}$$

$$\bar{\nabla}\times V = 0 + 0 + \hat{z}\left(\frac{2xy}{2\sqrt{x^2+y^2}} - \frac{2xy}{2\sqrt{x^2+y^2}}\right)$$

$$= 0$$

Q2) $\int_a^b \vec{\nabla}T\cdot d\vec{l} = T(b) - T(a)$.

$b = (1\,1\,1)$. $a = (0\,0\,0)$.

$$\nabla T = (2x+4y)\hat{i} + (4x+2z^3)\hat{j} + (6yz^2)\hat{k}$$

$$= \int \nabla\vec{T}\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

for I^st path

$$\int_0^1 (2x+4y)\,dx + \int_0^0 (4x+2z^3)\,dy + \int_0^0 6yz^2\,dz$$

$$= x^2\Big|_0^1 + 4\times 0\times dx + 0 + 0$$

$$= 1$$

for II^nd path

$$\int_1^1 (2x+4y)\,dx + \int_0^1 (4x+2z^3)\,dy + \int_0^0 6yz^2\,dz$$

$$0 + 4\times 1 + 0 + 0$$

$$= 4.$$

for III path:

$$\int_1^1 (2x+4y)\,dx + \int_1^1 (4x+2z^3)\,dy + \int_0^1 6yz^2\,dz$$

$$= 6^2 \times 1 \times \frac{z^3}{3} = 2.$$

$$I^{st} + II^{nd} + III^{rd} = 1+4+2 = 7.$$

$$T(1,1,1) - T(0,0,0) = 1+4+2-0-0-0 = 7.$$

Hence verified.

b) Exactly same as (a)

c) $z = x^2$; $y = x$

$dz = 2x\,dx$ $\quad dy = dx.$

$$\int \vec{\nabla} T \cdot d\vec{l} = \int_0^1 (2x+4y)\,dx + (4x+2z^3)\,dx + (6yz^2)\,2x\,dx.$$

$$= \int_0^1 (2x+4x)\,dx + (4x+2x^6)\,dx + 12x^6\,dx$$

$$= \frac{\overset{5}{\cancel{10}}x^2}{2} + \frac{\overset{2}{\cancel{14}}x^7}{\cancel{7}}\Big|_0^1$$

$$= 5+2 = 7.$$

$T(1,1,1) - T(0,0,0) = 7.$ Hence verified.

Q3) $a = (x+y)\hat{i} + (y+x)\hat{j}$

1) $\int \{(x+y)\hat{i} + (y+x)\hat{j}\}\,(dx\,\hat{i} + dy\,\hat{j})$ $\quad$ y^{2}

$y^2 = x$

$2y\,dy = dx$

$$\int (x+y)\,dx + (x+y)\,dy$$

$$\int_1^2 (y^2+y)\,2y\,dy + (y^2+y)\,dy$$

$$\int_1^2 (2y^3+3y^2+y)\,dy$$

$$= \frac{\cancel{2}y^4}{\cancel{4}\,2} + \frac{\cancel{3}y^3}{\cancel{3}} + \frac{y^2}{2}\Big|_1^2 = \frac{16}{2} + 8 + \frac{4}{2} - \frac{1}{2} - 1 - \frac{1}{2}$$

$$= \underline{16}$$

2) $x = 2u^2+u+1$, $y = 1+u^2$

$dx = (4u+1)\,du$ $dy = 2u\,du$

$$\int (3u^2+u+2)(4u+1)\,du + (3u^2+u+2)\,2u\,du$$

for y fr 1 to 2

$u = 0$ to 1

fr x fro 1 to 4.

$u = 0, -\frac{1}{2}, \frac{3}{2}, 1$

so $u = 0$ to 1

$$\int_0^1 (12u^3+4u^2+8u+3u^2+u+2+6u^3+2u^2+4u)\,du$$

$$= \int_0^1 (18u^3+9u^2+13u+2)\,du$$

$$= \frac{\overset{9}{\cancel{18}}u^4}{\cancel{4}\,2} + \frac{\cancel{9}\,3\,y^3}{\cancel{3}} + \frac{13u^2}{2} + 2u\Big|_0^1$$

$$= \frac{9}{2} + 3 + \frac{13}{2} + 2 - 0$$

$$= 11+3+2 = \underline{16}$$

Path 1 + Path 2

3) $\left\{ \int_1^4 (x+1)\,dx + \int_1^1 (x+1)\,dy \right\} + \left\{ \int_4^4 (4+y)\,dx + \int_1^2 (4+y)\,dy \right\}$

$= \frac{x^2}{2}+x \Big|_1^4 + 0 + 0 + 4y+\frac{y^2}{2}\Big|_1^2$

$= \frac{16}{2}+4-\frac{1}{2}-1+0+0+8+\frac{4}{2}-4-\frac{1}{2}$

$= 22-4-2$

$= \underline{16}$