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Ass8

Q1 $y = x^2 \sin x$ and $y = 0$ both are sol^n. of initial value problem

$$x^2 y'' - 4xy' + (x^2+6)y = 0,$$
$$y(0) = y'(0) = 0 \qquad x \in R.$$

$$y'' - \frac{4x}{x^2} y' + \frac{(x^2+6)y}{x^2} = 0$$

Q2 find curve $y = y(x)$ passing through origin for which $y'' = y'$ and line $y = x$ is tangent

$$y'' - y' = 0 \qquad , \qquad y(0) = 0$$
$$y'(x) = 1$$
$$p = y' \qquad\qquad y'(0) = 1$$
$$p' = y''$$
$$y(x) = e^x - 1$$

Q3 $y = \cos(ax+b)$

$$y' = \sin(ax+b) \times a$$
$$y' = -\sin(ax+b) \times a$$
$$y'' = -\cos(ax+b) \times a^2$$
$$y'' = -y \times a^2$$

$$\cos^{-1} y = ax+b$$
$$\frac{-1}{\sqrt{1-y^2}} \times y' = a$$
$$y'' = -y \frac{(y')^2}{(1-y^2)}$$

$$y''(1-y^2) = y(y')^2$$

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find the value of m such that $y = e^{mx}$.

(1) $y'' + 3y' + 2y = 0$

$m^2 e^{mx} + 3m e^{mx} + 2e^{mx} = 0$

$(m^2 + 3m + 2) = 0$

$m = \frac{-3 \pm \sqrt{9 - 4\times 2}}{2} = \frac{-3 \pm 1}{2} = -2, -1$

; $y'' + P(x)\, y' + q(x)\, y = r(x)$

$x \in I$

y_1 & y_2.

$y = ay_1 + by_2$

$(ay_1 + by_2)'' + p(x)\,(ay_1 + by_2)$
$+ q(x)\,(ay_1 + by_2) = r(x)$

$a(y_1'' + P(x)\,y_1' + q(x)\,y_1) + b(y_2'' + P(x)\,y_2'$
$+ q(x)\,y_2) =$
$r(x)$

$a(r(x)) + b(r(x)) = 1 \times r(x)$

$(a+b) = 1$ always

(6) $y'' + p(x)\,y' + q(x) = 0$ —— (1)

$p(x)$, $q(x)$ is cont^n on given interval I

$\underbrace{y = x,\ y = \sin x}_{LI}$ are sol^n of (1) or not

$W(y_1, y_2) = \begin{vmatrix} x & \sin x \\ 1 & \cos x \end{vmatrix} = x\cos x - \sin x$

$W(y_1(0), y_2(0)) = 0 \times \cos 0 - \sin 0 = 0$

at $x = 0$

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$W(y_1, y_2)(x) = 0$ when $x = 0$
y_1 & y_2 are not solⁿ of DE (1).

Q7 $y_1(x)$ & $y_2(x)$ are two solⁿ of a diffⁿ

$y'' + p(x)y' + q(x)y = 0$
$W(y_1, y_2) \neq 0$ y_1 & y_2 are LI.

$y_1'' + p(x)y + q(x)y = 0$ — (1)
$y_2'' + p(x)y_2'' + q(x)y_2 = 0$ — (2)

(2) $\times y_1$ — (1) $\times y_2$

$y_2 y_1'' - y_1 y_2'' + p(x)(y_1' y_2 - y_2 y_1') = 0$

$$p(x) = \frac{y_1 y_2'' - y_2 y_1''}{W(y_1, y_2)}$$

Calculate $q(x)$

Q8 $y = x \rightarrow$ tangent
$y'(x) = x$ $y(0) = 0$
$y'(x) = 0$ $y'' - y' = 0$

Q9 y_1 & y_2 are LI solⁿ
$y'' + p(x)y + q(x)y = 0$
between two consecutive zero of y_1
∃ one zero of y_2

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$y_1(x_1) = 0$

$y_2(x_2) = 0$

$\exists\, x^* \in (x_1, x_2)$ st $y_2(x^*) = 0$

$W(y_1 y_2)(x) = y_1 y_2' - y_2 y_1'$

Now let us assume that y_1 & y_2 have common zero's x.

$W(y_1 y_2) = 0$ But given that y_1 & y_2 are LI.

$W(y_1, y_2)(x') \neq 0 \Rightarrow$ contradiction

(9) Now, let x_1 & x_2 are two consecutive zeroes of y_1

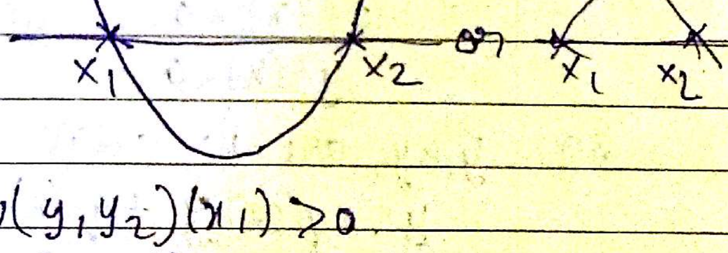
$y_1(x_1) = 0$

$y_1(x_2) = 0$

But $y_1' \neq 0$ & $y_2'(x) = 0$.

$W(y_1, y_2) = y_1 y_2' - y_2 y_1'$

$W(y_1, y_2)(x_1) = -y_2(x)\, y_1'(x_1)$



WLG, $W(y_1, y_2)(x_1) > 0$

$y_1(x_1) > 0$ then $y_1(x_2) < 0$ or vice versa

$y_1'(x_1) > 0$ and $y_1'(x_2) < 0$

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