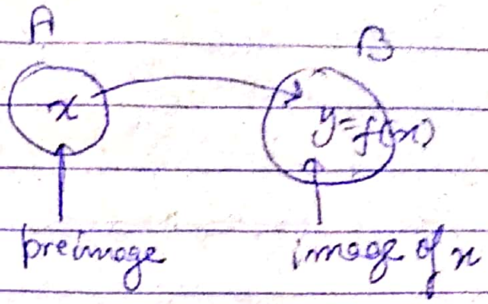
1.1SE.

Set,

Function, ~~x~~ function defined from A to B.

~~If~~ $f : A \to B$

$y = f(x)$

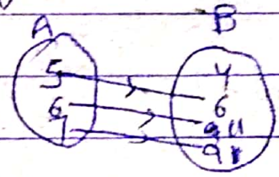


Domain, Codomain, Range
A, B, $\subseteq B$

Graph $\Rightarrow$ If $f : A \to B$ is a function then subset $\{x ; f(x) : x \in A\}$ of $A \times B$ is called a graph

Q. Let $A = \{5, 6, 7\}$ & $B = \{4, 6, 9, 11\}$
find the range and graph which is defined by.

Range $= \{6, 9, 11\} \subseteq \{4, 6, 9, 11\} = B$.

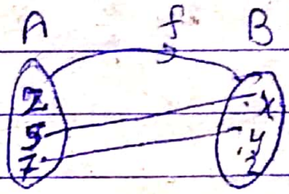


$f(5) = 6$, $f(6) = 11$, $f(7) = 9$

graph $= \{(5,6), (6,11), (7,9)\}$

Q. Let $A = \{2, 5, 7\}$, $B = \{x, y, z\}$. Is it a function?

There has to be image of set A.
here 2 ~~is~~ don't have image.
$\therefore$ Can't define function.



Real function: $f : A \to B$. $f : \mathbb{R} \to \mathbb{R}$ $\leftarrow$ Real function.
eg. $f(x) = 2x^2 + x + 8$, $x \in \mathbb{R}$

Equal function, Let f & g are two functions such that
$$\left.\begin{array}{l} f : A \to B \\ g : A \to B \end{array}\right\} \Rightarrow f(x) = g(x) \ \forall x \in A.$$

Constant function $\Rightarrow$ $f : A \to B$
If $f(x) = c \ \forall x \in A$ then it is a constant function
$c \in B$.

Identity function : Suppose $f : A \to A$. → Range & domain are same.

$f(x) = x \ \forall x \in A$.

Absolute value function :→ $f : A \to B$ Domain : R

$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ Range $[0, \infty)$

Polynomial function :→

$f(x) = a_0 x^0 + a_1 x + a_2 x^2 + \dots a_n x^n$ where $a_n \neq 0$ → n degree

$a_1, a_2, \dots a_n$ are real nos.

Let $f(x) = 3x^3 + 4x^2 + 2x + 5$. $a_0 \neq 0$

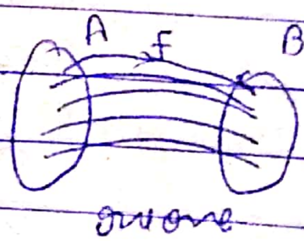
Rational function :→ Let $f(x)$ & $g(x)$ be two polynomials and $X \subseteq R$. $\frac{f(x)}{g(x)}, g(x) \neq 0$.

$-f(-x) = f(x)$. → odd function.

$f(-x) = f(x)$ → even

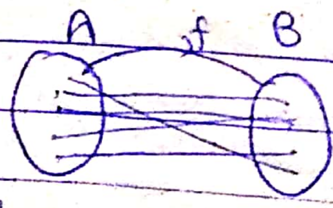
$f(x) = x^3 + x + 2$ → Odd function

One one. → for every element in A, ~~or~~ exactly one image in B.



~~one one~~

onto function.



for ~~B~~ element in B, there should be atleast one ~~four~~ pre image in A.

Identity function : Suppose $f: A \to B$. → Range & domain are same.

$f(x) = x \; \forall x \in A$.

Absolute value function :- $f: R \to R$ Domain : R

$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ Range $[0, \infty)$

Polynomial function :-

$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots a_n x^n$ where $a_n \neq 1$ → n degree

$a_1, a_2, \ldots a_n$ are real nos.

let $f(x) = 3x^3 + 4x^2 + 2x + 5$. $a_0 \neq 0$

Rational function :- Let $f(x)$ & $g(x)$ be two polynomials and $X \subseteq R$. $\frac{f(x)}{g(x)}, \; g(x) \neq 0$.

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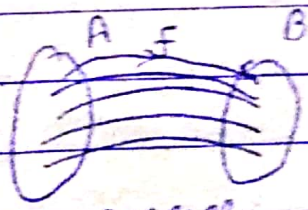
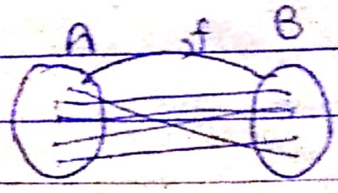


one one

onto function.



for ~~B~~ element in B, there should be atleast one ~~[illegible]~~ pre image in A.

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Probability & Statistics

Experiment

Deterministic
↳ whose outcome is sure.

Non deterministic → Random Experiment, Sochastic Exp.
↳ outcome is not fixed & we know all the possibilities

Trial: Single performance of an experiment.

Sample Space :→ Collection of all possible outcomes of a Random Ex.

→ Discrete → finite or countable infinite.

→ Continuous

Event → Any subset of Sample space

Elementary event (Simple) → compound event

↓

only one sample point

Favourable Event ↳ can come as output
eg. getting sum 5 {(1,4),(4,1),(2,3),(3,2)} Sp. space
→ in throwing dice

Equally likely event.

Compliment of an event : E' or E^c or $\bar{E} = S-E$.

Mutually exclusive events :→

$$A \cap B = \phi$$

Impossible event → Never occur/happen

Exhaustive events :→ total no. of possible outcomes in a trial.

Odd in favour of an event $= m/n$

and odd against the events $= n/m$

m is no. of outcomes favourable to a certain events

n is no. of outcomes not in favour.

Permutation: arrangement of n things in r in particular order.
⇓
$$^nP_r = \frac{n!}{(n-r)!}$$

Combination: selection

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

Definition of probability:→

1. Classical definition of probability

$$P(A) = \frac{m}{n} = p$$

$$P(B) = \frac{n-m}{n} = 1-\frac{m}{n} = 1-p = q$$

$$\Rightarrow p+q=1.$$

2. Statistical or Empirical definition of probability

$$\lim_{n\to\infty} \frac{m}{n} = P(A)$$

3. Axiomatic definition of probability: Let S be sample space & A be any event $0 \le P(A) \le 1$ — A_1

Matter of belief.

$A_2 \to$ $P(S) = 1$

$A_3 \to$ if A_1 & A_2 are mutually exclusive events then $P(A_1 \cup A_2) = P(A_1) + P(A_2)$.

Extension of $A_3 \to$

Let $A_1, A_2, \ldots A_n$ be mutually exclusive events

then $P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$.

$$= P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

Q. distribution of the Blood type in the US is roughly 41% type A, 9% type B, 4% type AB & 46% type O. An individual brought in Emergency room and is to be one of the above blood type. What is the probability that the type will be A, B or AB.

$$P(E_A \cup E_B \cup E_{AB}) = P(E_{S-O}) = 1 - \frac{46}{100}$$

$$= \frac{54}{100}$$

Let A, B, C and D be the events of blood type A, B, AB and O respectively. Given $P(A)=0.41$, $P(B)=0.09$, $P(C)=0.04$, $P(O)=0.46$.

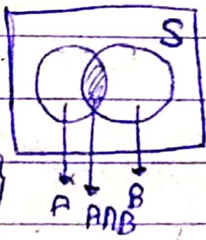
$$P(\text{Blood type A, B or AB}) = P(A\cup B\cup C)$$
$$= P(A)+P(B)+P(C)$$
$$=0.41+0.09+0.04$$
$$=0.54$$

{$\because$ A, B, C are mutually exclusive events so by A_3} Axiom 3.

Additive law of probability or Theorem of total probability :→

Let A and B two events then show that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

Proof :→



$$A\cup B = A\cup(\bar{A}\cap B)$$

{A & $(\bar{A}\cap B)$ are m.e.e}

$$\Rightarrow P(A\cup B)=P(A)+P(\bar{A}\cap B)$$
$$=P(A)+P(A\cap B)+P(\bar{A}\cap B)-P(A\cap B)$$
$$=P(A)+P(B)-P(A\cap B).$$

$B = (A\cap B)\cup(\bar{A}\cap B)$ (both are m.e.e)

$$\Rightarrow P(B) = P(A\cap B)+P(\bar{A}\cap B).$$

If A and B are m.e.e. then $A\cap B=\emptyset$.

$$\Rightarrow P(A\cup B)=P(A)+P(B).$$

• Let A, B and C be three events then show that

Proof :→ $P(A\cup \underbrace{B\cup C}_{E}) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

⇒ using reduction

Prove that (i) $P(\emptyset)=0$. ~~$\Rightarrow P(S\cup\emptyset)=P($~~

(ii) $P(A')=1-P(A)$

$S\cup\emptyset = S$ $\Rightarrow$ By A_3 $P(S\cup\emptyset)=P(S)+P(\emptyset)=P(S)$

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Q find a probability of drawing a king or a heart or both, from a deck of cards.

Let ~~A~~ A be the event of drawing a king.
Let ~~E~~ B be the event of drawing a heart.

$$P(A) = \cancel{4/52}\ {}^{4}C_1 / {}^{52}C_1 = 4/52 = 1/13$$

$$P(B) = {}^{13}C_1 / {}^{52}C_1 = \frac{13}{52} = 1/4$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$= \frac{1}{13} + 1/4 - {}^{1}C_1 / {}^{52}C_1$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

• Conditional Probability :→
Let A and B be two events then $P(A|B) = \frac{P(A\cap B)}{P(B)}$
$P(B) \neq 0$.

Theorem ~~of~~ on compound probability or Multiplication law of probability.
Let A and B be any two events in sample space S.

$$P(A\cap B) = P(B).\ P(A|B)\ ,\ P(B) \neq 0.$$
$$= P(A).\ P(B|A)\ ;\ P(A) \neq 0.$$

Let A, B and C be three events in S then

$$P(A\cap B\cap C) = P(A).\ P(C/A\cap B).\ P(B/A)$$

$$P(A_1 \cap A_2 \text{---} A_n) = P(A_1).P(A_2/A_1).P(A_3/A_1 A_2) \ldots P(A_m/A_1\cap A_2 \cap \ldots \cap A_n)$$

If $P(A/B) = P(A)$
and $P(B/A) = P(B)$

$\Rightarrow \boxed{P(A\cap B) = P(A).P(B)}$

$\Rightarrow$ A and B are independent events.

Q. If A and B be independent events then prove that.
i) A' and B' are also independent.
ii) A' and B ———
iii) A and B' ———

Proof 1). $\overline{A \cup B} = A' \cap B'$ {using De Morgan's law}.

$P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= P(A' \cap B')$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [P(A) + P(B) - P(A)P(B)]$ {∵ A & B are independent}

$= 1 - P(A) - P(B) + P(A)P(B)$

$= (1 - P(A))(1 - P(B))$

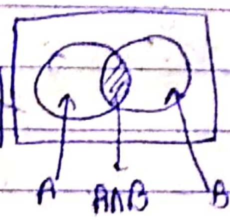
$= P(A')P(B')$

⇒ A' & B' are also independent.

2) $A' \cap B = \overline{A \cup B'}$

$P(A' \cap B) = 1 - P(A \cup B')$?

$= 1 - [P(A) + P(B') - P(A \cap B')]$



$B = (A \cap B) \cup (A' \cap B)$

$P(B) = P(A \cap B) + P(A' \cap B)$

$P(A' \cap B) = P(B) - P(A \cap B)$ {A∩B & A'∩B are mutually exclusive}

$= P(B) - P(A)P(B)$

$= P(B)(1 - P(A))$

$P(A' \cap B) = P(B)(P(A'))$

A' & B are independent.

3) $A = (A \cap B') \cup (A \cap B)$

$P(A) = P(A \cap B') + P(A \cap B)$

$P(A \cap B') = P(A) - P(A \cap B)$

$= P(A) - P(A)P(B)$ (∵ A & B are independent)

$= P(A)(1 - P(B))$

$P(A \cap B') = P(B')P(A)$

⇒ A & B' are independent.

Q. If $P(A)$ is $\frac{1}{2}$, $P(B) = \frac{1}{3}$ & $P(A \cup B) = \frac{2}{3}$.
Are A' & B' Independent?

$$\cancel{P(A\cap B) = -(P(A\cup B) - P(A) - P(B))}$$

$$\cancel{= -\tfrac{2}{3} + \tfrac{1}{2} + \tfrac{1}{3}}$$

$$\cancel{= \tfrac{5}{6} - \tfrac{4}{6} = \boxed{\tfrac{1}{6} = P(A\cap B)}}$$

$$P(A'\cap B') = 1 - P(A\cup B) = \tfrac{1}{3} \quad \text{--- (1)}$$

$$P(A') = \tfrac{1}{2} \ \& \ P(B') = \tfrac{2}{3}$$

$$P(A')\,P(B') = \tfrac{1}{2}\times\tfrac{2}{3} = \tfrac{1}{3} \quad \text{--- (2)}$$

~~(1)~~ ⇒ Both are same.

Ram & Rahul appear in an interview

(a) both of them will be selected.
(b) only one of them will be selected.
(c) none of them will be selected.

Solution :- Let A be the event that Ram will be selected ⇒ $P(A) = \frac{1}{7}$

Let B " — Rahul — " ⇒ $P(B) = \frac{1}{5}$

(a) P(both Ram and Rahul will be selected) $= P(A\cap B)$

$= P(A)P(B)$ {∵ A & B are independent

$= \frac{1}{7}\times\frac{1}{5} = \frac{1}{35}$

(b) P(only one of them will be selected)

$= P(A\cap\bar{B} \cup \bar{A}\cap B)$ { $A\cap\bar{B}$ & $\bar{A}\cap B$ are mutually exclusive events.

$= P(A\cap\bar{B}) + P(\bar{A}\cap B)$

$= P(A)P(\bar{B}) + P(\bar{A})\cdot P(B)$ { A & B are independent ⇒ $\bar{A}$ & $\bar{B}$ are also

$= \frac{1}{7}\times\frac{4}{5} + \frac{6}{7}\times\frac{1}{5} = \frac{10}{35} = \frac{2}{7}$

(c) P(None of them get selected)

$= P(\bar{A}\cap\bar{B}) = P(\bar{A})\cdot P(\bar{B})$ { $\bar{A}$ & $\bar{B}$ are independent

$= \frac{6}{7}\times\frac{4}{5} = \frac{24}{35}$

$\bar{A}\cap\bar{B} = \overline{(A\cup B)}$ Demorgan's theorem.

Pairwise independent :→ Let $A_1, A_2, \ldots A_n$ defined on sample space S & $P(A_i) > 0$ $i = 1, 2, \ldots n$ then these events are said to be pairwise independent if $P(A_i \cap A_j) = P(A_i) \cdot P(A_j)$, $i \neq j = 1, 2, \ldots n$

Mutually independent events: Let S denote the sample space for a no. of events. The events are said to be mutually independent events if the probability of simultaneous occurrence of any finite no. of them is equal to product of their seperate probabilities.

If $A_1, A_2, \ldots A_n$ be n events ~~are~~ in a sample space S, then these are said to be mutually independent if

$$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_n}) = P(A_{i_1}) P(A_{i_2}) \ldots P(A_{i_n})$$

$i = 1, 2, \ldots n.$

Q Two fair dice are thrown independently.

Three events A, B & C are defined as follows

A) ~~All~~ odd face with the first dice

B) odd face with the ~~third~~ second dice

c) ~~Same So~~ Sum of the no. from two dice are odd. → A, B, C Mutually independent ?

$S = \{(x, y) \mid x, y \in \{1, 2, 3, 4, 5, 6\}\}$

(iii) $P(A) = \frac{{}^3C_1 \times {}^6C_1}{{}^6C_1 \times {}^6C_1} = \frac{3 \times 6}{6 \times 6} = \frac{1}{2}$

$P(B) = \frac{18}{36} = \frac{1}{2}$

$P(C) = \frac{{}^3C_1 \times {}^3C_1}{{}^6C_1 \times {}^6C_1} + \frac{{}^3C_1 \times {}^3C_1}{{}^6C_1 \times {}^6C_1} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(A \cap B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(B \cap C) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{4}$

$P(A \cap B \cap C) = 0 \neq P(A) P(B) P(C) = \frac{1}{8}$

⇓

A, B & C are NOT Mutually independent.

Bayes Theorem

Rule of Elimination or Theorem of Total Probability :→

If the event A can occur with event E and let event E occur in n mutually exclusive events $E_1, E_2, \ldots E_n$.

then $P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)$ provided $P(E_i) > 0$.

Proof :→ $A = (A \cap E_1) \cup (A \cap E_2) \cup \ldots \cup (A \cap E_n)$ { $A \cap E_i$ are mutually exclusive events.

$$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$

$\because P(A/E_i) = \frac{P(A \cap E_i)}{P(E_i)} \Rightarrow P(A \cap E_i) = P(E_i) P(A/E_i)$

$$\Rightarrow P(A) = \sum_{i=1}^{n} P(E_i) P(A/E_i)$$

Baye's Theorem :→ Let $E_1, E_2 \ldots E_n$ be n mutually exclusive events of which one of them must occur. Let A be any event then

$$P(E_i/A) = \frac{P(A \cap E_i)}{P(A)}$$

$$= \frac{P(E_i) P(A/E_i)}{\sum_{i=1} P(E_i) P(A/E_i)}$$

Q A consulting firm rents car from three Agencies A_1, A_2, A_3. 20% of the cars are rented from A_1, 20% are from A_2 & Remaining 60% from A_3. If 20% of the cars are rented from A_1, 10% of cars rented from A_2 & 2% of cars are rented from A_3 have bad tyres. (A) What is the probability that a car rented from consulting firms will have bad tyres.

(B) what is the probability that the car ~~rented~~ which has a bad tyre was rented from firm A_1.

Solution :→ Let A be the events that rented cars have bad tyres.

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& let E_1, E_2 & E_3 be the events that ~~rented~~ car ~~which has bad type~~ is rented from firm A_1, A_2 & E_3 respectively.

Given: $P(E_1) = 0.20$

$P(E_2) = 0.20$

$P(E_3) = 0.60$

$P(A|E_1) = 0.20$, $P(A|E_2) = 0.10$ & $P(A|E_3) = 0.02$

(a) $P(A) = \sum_{i=1}^{3} P(E_i) P(A/E_i) = 0.20 \times 0.20 + 0.20 \times 0.10 + 0.60 \times 0.2$

$= 0.072$

(b) $P(E_1/A) = \cancel{P(A) P(E_i)}$

$= \frac{P(A/E_i)\, P(E_i)}{P(A)} = \frac{.20 \times 0.20}{0.72} = 7/18.$

Random Variable :→

is A function which is associated with an outcome of an sample space.

$s \in S$

$X(s)$ is R.V.

→ Discrete R.V.

→ Continuous R.V.

Discrete Probability distributed

Probability Mass function :→ pmf

or Point prob. function ppf

Probability density fn.

or

density function

$p(x_i) = P(X = x_i)$, $i = 1, 2, \ldots n$

$= f(x_i)$ $i = 1, 2, \ldots n$

(1) $p(x_i)$ or $f(x_i) \geq 0$ $\forall i = 1, 2, \ldots n$.

$\sum_{i=1}^{n} p(x_i) = 1.$

Distribution function or Cumulative Distribution function.

$F(x) = P(X \leq x)$

0 —— $F(x)$ x —— ∞

tabular → form.

X	x_1	x_2	x_3	—	x_n
$p(x)$	p_1	p_2	p_3	—	p_n
$F(x)$	p_1	p_1+p_2	$p_1+p_2+p_3$	—	$p_1+p_2+p_3 - p_n = 1.$

$$F(x) = 0 \;; \; x < x_1$$
$$= p_1 \;\; \cancel{x_1} \le x < x_2$$
} → gross form

Domain of $F(x) = (-\infty, \infty)$

Range of $F(x) = [0, 1]$

$0 \le F(x) \le 1$ $\qquad F(-\infty) = \lim_{x \to -\infty} F(x) = 0$

$F(x_j) = P[X \le x_j] = \sum_{i=1}^{j} p(x_i)$ $\qquad F(\infty) = \lim_{x \to \infty} F(x) = 1$

$F(x_{j-1}) = P(X \le x_{j-1}) = \sum_{i=1}^{j-1} p(x_i)$

$F(x_j) - \cancel{P(X} F(x_{j-1}) = p(x_j)$

Q If a pair of fair dice is rolled. Find the probability distribution & distribution function for getting their sum = 2, 3, 4, ... 12.

Soln

If a pair of fair dice is rolled.

$$S = \left\{ \begin{array}{l} (1,1), (1,2), \ldots (1,6) \\ (2,1), (2,2) \ldots (2,6) \\ \vdots \\ (6,1) \ldots\ldots (6,6) \end{array} \right\}$$

X = r.v. which shows sum of dice.

$P(X=2) = 1/36$

$P(X=3) = \cancel{P\{(1,2),(2,1)\}}\; P(\{(1,2),(2,1)\}) = 2/36$

$P(X=4) = P\{(1,3),(3,1),(2,2)\} = 3/36$

$\vdots$

$P(X=12) = P(6,6) = 1/36$.

X	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36
$F(x)$	1/36	3/36	6/36	10/36	15/36	21/36	26/36	30/36	33/36	35/36	36/36

→ Make sure this is 1

A r.v. X has a following distribution ~~and~~

$X=x$	-2	-1	0	1	2	3
$p(x)$	0.1	k	0.2	$3k$	$2k$	0.3

1) Determine k & hence compute $P(X<2)$, $P(X\geq 2)$
2)
3) Find the minimum value of k such that $P(X\leq 1)\leq 0.32$

Solu. To find k, $\sum_{\forall x} p(x)=1$

1

$$6k+0.6=1 \Rightarrow k=1/15$$

2

$$P(X<2) = P(X\leq 1)$$

$$= F(1)$$

$$= 0.1+\frac{1}{15}+0.2+\frac{3}{15} = \frac{17}{30}$$

$$P(X\geq 2) = 1-P(X\leq 1)$$

$$= \frac{13}{30}$$

3 $P(X\leq 1) > 0.32$

$$.3+4k > 0.32$$

$$4k > 0.02 \Rightarrow k > \frac{0.02}{4} = 0.005$$

Continuous Probability distribution.

$f(x)$ is said to be pdf if

(i) $f(x)\geq 0 \;\forall x$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

iii) $\int_a^b f(x)\,dx = P(a<x<b)$

Cumulative Distribution Function (CDF): $F(x)=P(X\leq x)$

$$= \int_{-\infty}^{x} f(t)\,dt$$

Properties

(i) Domain is $(-\infty,\infty)$ & Range $[0,1]$

(ii) $F(x)$ is non decreasing fn of x in the right.

i.e. $F'(x)=f(x) \;\forall x\geq 0$

iii) $F(x)$ is continuous on the right.

iv) $F(-\infty) = 0 \Rightarrow \int_{-\infty}^{-\infty} f(t)\,dt = 0 \Rightarrow$ P(one point) is 0.

$F(\infty) = 1 \Rightarrow \int_{-\infty}^{\infty} f(t)\,dt$

$$P(a \le X < b) = P(a < X \le b) = P(a \le X \le b) = P(a < X < b)$$

$$= \int_a^b f(x)\,dx$$

→ Exponential Distribution.

Ex. $f(x) = ke^{-3x}, \; x \ge 0$

$0 \quad (x<0)$ elsewhere.

determine k & hence compute cdf of x.

$$\int_{-\infty}^{\infty} f(x) = 1 \Rightarrow \int_0^{\infty} ke^{-3x}dx + \int_{-\infty}^{0} 0\,dx = 1$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_0^{\infty} f(x)\,dx \qquad k\int_0^{\infty} e^{-3x}dx = 1$$

$$\frac{k}{(-3)}\left[e^{-3x}\right]_0^{\infty} = 1$$

$$-\frac{k}{3}\left(0 - e^{0}\right) = 1$$

$$-\frac{k}{3} \times -1 = 1 \Rightarrow \boxed{k = 3}$$

$$\Rightarrow f(x) = 3e^{-3x}, \; x \ge 0$$

0 elsewhere.

$$F(x) = P(X \le x) = \int_{-\infty}^{x} 3e^{-3t}dt = \int_{-\infty}^{0} 3e^{-3t}dt + \int_0^{x} 3e^{-3t}dt$$

$$= \begin{cases} 1 - e^{-3x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$\cancel{t\left[e^{-3t}\right]_0^{x}} = \cancel{t\left(xe^{-3x}\right)}$$

$$\cancel{F(x) = t(1-e^{-3x})}$$

$$\cancel{F(x) = xe^{-3x}}$$

$$F(2) = P(X \le 2) = 1 - e^{-3\times 2} = 1 - e^{-6}$$

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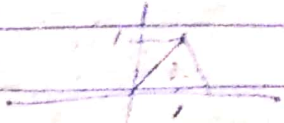
Q The pdf of r.v. is x. $f(x) = x \quad 0 \le x < 1$
$2-x \quad 1 \le x < 2$
$0, \quad x \ge 2$

compute cdf of F(x) and hence calculate (i) $P(-1 \le X \le 3)$
(ii) $P(1 \le X \le 1.5)$



Ans. $F(x) = P(X \le x) = 0$ if $x < 0$

$= \int_{-\infty}^{x} t\,dt = \frac{x^2}{2}, \quad 0 \le x < 1$

For $1 \le X < 2$

$F(x) = \int_{-\infty}^{0} f(x)dx + \int_0^1 f(x)dx + \int_1^x f(t)dt$

$= 0 + \frac{1}{2} + \int_1^x (2-t)\,dt = 2x - \frac{x^2}{2} - 1, \quad 1 \le x < 2$

$= 1 \quad x \ge 2$

$F(x) = 0, \quad x < 0$

$= \frac{x^2}{2}, \quad 0 \le x < 1$

$= 2x - \frac{x^2}{2} - 1, \quad 1 \le x < 2$

$1, \quad x \ge 2.$

$P(-1 \le X \le 3) = F(3) - F(-1)$

$= 1 - 0 = 1$

$P(1 \le X \le 1.5) = F(1.5) - F(1)$

$= \left(2 \times \frac{3}{2} - \frac{(1.5)^2}{2} - 1\right) - \left(2 - \frac{1}{2} - 1\right)$

$= 2 - \frac{9}{8} - \frac{1}{2}$

$=$

Expectation: Suppose X be a r.v.

$\mu = E(X) = \sum_{\forall x} x\,p(x)$, if X is discrete

$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$, if X is continuous

→ Provided integral exists.

$$E[g(x)] = \sum_{\forall x} g(x)p(x) \rightarrow \text{discrete}$$

$$X = g(x) \qquad = \int_{-\infty}^{\infty} g(x) f(x)\,dx \rightarrow \text{continuous.}$$

Variance:→

$$\sigma^2 = E\left[(x-\mu)^2\right] = \sum_{\forall x} (x-\mu^2)\,p(x)$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

Standard Deviation:→

Positive square root of variance

$$S.D. = +\sqrt{E((x-\mu^2))} = \sigma$$

Computational formula for σ^2:

$$\sigma^2 = \sum x^2 p(x) - \left[\sum x p(x)\right]^2$$

$$\sigma^2 = E\left[(x-\mu)^2\right] = \sum_{\forall x} (x-\mu)^2 p(x)$$

$$= \sum \left[(x^2 - 2\mu x + \mu^2)\,p(x)\right]$$

$$= \sum \left(x^2 p(x) - 2\mu x p x + \mu^2 p(x)\right)$$

$$= \sum_{\forall x} x^2 p(x) - 2\mu \sum_{\forall x} x p(x) + \mu^2 \sum_{\forall x} p(x)$$

$$= \sum x^2 p(x) - 2\mu\mu + \mu^2 \cdot 1$$

$$= \sum x^2 p(x) - [\underbrace{\sum x\, p(x)}_{\mu}]^2$$

& for continuous
Replace $\sum_{\forall x}$ with $\int_{-\infty}^{\infty}$

Properties of Expectation & Variance ⇒

i) $E(c) = c$ $\quad = \sum_{\forall x} c\,p(x) = c \sum_{\forall x} p(x) = c \cdot 1 = c$

ii) $E(cx) = cE(x)$ $\quad = \sum_{\forall x} (cx)p(x) = c \sum_{\forall x} x\,p(x) = cE(x)$

iii) $E[c+x] = c + E(x)$

iv) $Var(c) = 0$ $\quad = E((X-\mu)^2) = E(X^2) - (E(X))^2$

v) $Var(cx) = c^2 Var(x)$ $\quad = E(c^2) - (E(c))^2$

$= c^2 - (c)^2 = 0.$

Q If $E(x) = 5$, $Var(x) = 2$ Find $E(X-5)$, $Var(-2X)$

$$E(X-5) = E(x) - 5 = 0$$

$$Var(-2X) = (-2)^2 Var(X) = 4 \times 2 = 8.$$

Variance of any distribution can never be negative.
Variance doesn't have unit.
→ Very important statistical constant

The r^{th} movement about origin.

$$\mu_r' = E(X^r) = \sum_{\forall x} x^r p(x) \text{ if } X \text{ is discrete}$$

$$= \int_{-\infty}^{\infty} x^r f(x)\,dx \text{ if } X \text{ is continuous}$$

$r = 1$

$\mu_1' = E(X) = \mu$ ⇒ first movement around origin.

r^{th} movement about mean

$$\mu_r = E((X-\mu)^r) = \sum_{\forall x} (x-\mu)^r p(x)$$

$$= \int_{-\infty}^{\infty} (x-\mu)^r f(x)\,dx$$

If $r = 2$

$$\mu_2 = E((X-\mu)^2) = \sigma^2$$

Second movement around mean is variance

$$\sigma^2 = \mu_2' - \mu^2$$

Proof: $E(x^r) = \mu_r'$ {# proved previously.}

for $r = 2$

$$E(x^2) = \mu_2'$$

$$E(x) = \mu$$

⊙ Movement generating function :→ MGF

represented by $M_x(t) = E(e^{tx})$

$$= \sum_{\forall x} e^{tx} p(x)$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$\mu_r' = \frac{d^r}{dt^r} \left[M_x(t) \right]_{t=0}$$

⊛ Mean deviation about mean :-

$$M.D = \begin{cases} \sum_{\forall x} |x-\mu| p(x) & \text{discrete} \\ \int_{-\infty}^{\infty} |x-\mu| f(x)\, dx & \text{continuous.} \end{cases}$$

⊛ Characterstic function :-

$$\phi_x(t) = E[e^{itx}] = \sum_{\forall x} e^{itx} p(x)$$

$i = \sqrt{-1}$

$$= \int_{-\infty}^{\infty} e^{itx} f(x)\, dx.$$

⊛ Bernoulli's distribution :→

$P(X=1) = p$ — success

$P(X=0) = q$ — failure

$$p(x) = \begin{cases} p^x q^{1-x}, & x = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

→ Most imp line

$E(x) = p$

$Var(x) = pq$

$M_x(t) = q + pe^t$