Assignment - 3

1) $\langle f, g \rangle = \int_a^b f(t) g(t)\, dt$

a) $f(t) = t+2$ $\quad g(t) = t^2 - 3t + 4$ $\quad a = -1, b = 1$

(i) $\langle pf + g, h \rangle = \int_a^b (pf(t) + g(t))\, h(t)$

$= \int_a^b p\, f(t) h(t) + \int_a^b g(t) h(t)$

$= p \langle f, h \rangle + \langle g, h \rangle$

(ii) $\langle f, g \rangle = \int_a^b \overline{f(t)}\, g(t)\, dt$

$\overline{\langle g, f \rangle} = \int_a^b \overline{g(t)}\, f(t)$

$= \langle f, g \rangle$

a) $\langle f, g \rangle = \int_{-1}^{1} (t+2)(t^2 - 3t + 4)\, dt$

$= \int_0^1 (-2t^2 + 16)\, dt$

$= 16 - \frac{2}{3} = \frac{46}{3}$

$$\|f\| = \sqrt{\langle f, f \rangle} = \sqrt{\int_{-1}^{1} (t+2)^2 \, dt}$$

$$= \sqrt{\int_{0}^{1} -2t^2 + 8} = \sqrt{\frac{26}{3}}$$

$$\|g\| = \sqrt{\langle g, g \rangle} = \int_{-1}^{1} (t^2 - 3t + 4)^2 \, dt$$

c) $|\langle f, g \rangle| \leq \sqrt{\|f\| \, \|g\|}$

2) a) $\langle \alpha, \beta \rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + 4 x_2 y_2$

(i) $\langle a\alpha + \beta, \gamma \rangle = \langle (a x_1 + y_1, a x_2 + y_2), (z_1, z_2) \rangle$

$$= (a x_1 + y_1) z_1 - (a x_1 + y_1) z_2 - (a x_2 + y_2) z_1 + 4 (a x_2 + y_2) z_2$$

$$= a (x_1 z_1 - x_1 z_2 - x_2 z_1 + 4 x_2 z_2) + y_1 z_1 - y_1 z_2 - y_2 z_1 + 4 y_2 z_2$$

$$= a \langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle$$

$\langle \alpha, \beta \rangle = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$

$\langle \beta, \alpha \rangle = y_1x_1 - y_1x_2 - y_2x_1 + 4y_2x_2$

$\langle \alpha, \beta \rangle = \langle \beta, \alpha \rangle$

$\therefore$ $\langle \alpha, \beta \rangle$ is a inner product.

3) $\alpha = (x_1, x_2)$ $\beta = (y_1, y_2)$

$\langle \alpha, \beta \rangle = x_1y_1 - 3x_1y_2 - 3x_2y_1 + ax_2y_2$

$\langle p\alpha + \beta, \gamma \rangle = \langle (px_1 + y_1, px_2 + y_2), (z_1, z_2) \rangle$

$= (px_1 + y_1)z_1 - 3(px_1 + y_1)z_2 - 3(px_2 + y_2)z_1 + a(px_2 + y_2)z_2$

$= p(x_1z_1 - 3x_1z_2 - 3x_2z_1 + ax_2z_2) + y_1z_1 - 3y_1z_2 - 3y_2z_1 + ay_2z_2$

$= p\langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle$

$\Rightarrow \boxed{a \in R}$

5) $\{\underset{v_1}{(1,1,1,1)}, \underset{v_2}{(1,-1,2,2)}, \underset{v_3}{(1,2,-3,-4)}\}$

$w_1 = v_1$

$$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1$$

$$w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle v_3, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2$$

$w_1 = (1,1,1,1)$

$$w_2 = (1,-1,2,2) - \frac{(1-1+2+2)}{4}(1,1,1,1)$$

$$w_3 = (1,2,-3,-4) - \frac{(1+2-3-4)}{4}(1,1,1,1) - \frac{(0-4-3-4)}{4+1+1}(0,-2,1,1)$$

$$w_2 = (1,-1,2,2) - (1,1,1,1)$$
$$= (0,-2,1,1)$$

$$w_3 = (1,2,-3,-4) + (1,1,1,1) + \frac{11}{6}(0,-2,1,1)$$
$$= (2,3,-2,-3) + \left(0, -\frac{11}{3}, \frac{11}{6}, \frac{11}{6}\right)$$
$$= \left(2, -\frac{2}{3}, -\frac{1}{6}, -\frac{7}{6}\right)$$

$\frac{36}{4}$ $4+1+1$ $4+\frac{4}{6}+\frac{1}{36}+\frac{49}{36}$

$\overline{36}$

Orthogonal basis : $\left\{(1,1,1,1);\ (0,-2,1,1);\ \left(2,-\frac{2}{3},-\frac{1}{6},-\frac{7}{6}\right)\right\}$

Orthonormal basis : $\left\{\frac{1}{2}(1,1,1,1);\ \frac{1}{\sqrt{6}}(0,-2,1,1);\ \frac{1}{\sqrt{180}}\left(2,-\frac{2}{3},-\frac{1}{6},-\frac{7}{6}\right)\right\}$

6) b) $\{(1,0,1),(1,0,-1),(0,3,4)\}$

$\langle(x,y,z),(x',y',z')\rangle = xx'+2yy'+3zz'$

$w_1 = v_1 = (1,0,1)$

$$w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1$$

$$= (1,0,-1) - \left(\frac{1+0-3}{1+3}\right)(1,0,1)$$

$$= (1,0,-1) + \frac{1}{2}(1,0,1)$$

$$= \left(\frac{3}{2}, 0, -\frac{1}{2}\right)$$

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$$w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle v_3, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2$$

$$= (0,3,4) - \left(\frac{0+0+12}{4}\right)(1,0,1)$$

$$- \left(\frac{0+0-6}{3/2}\right)\left(\frac{3}{2}, 0, \frac{-1}{2}\right)$$

$$= (0,3,4) - (3,0,3) - (6,0,-2)$$

$$= (-9, 3, 3)$$

orthonormal $= \left\{ \frac{1}{2}(1,0,1) ; \sqrt{\frac{2}{3}}\left(\frac{3}{2}, 0, \frac{-1}{2}\right) ; \frac{1}{\sqrt{126}}(-9,3,3) \right\}$

7) $\{1, t, t^2\}$

$$\langle f, g \rangle = \int_0^1 f(t) g(t)\, dt$$

$w_1 = v_1 = 1$

$$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1$$

$$= t - \frac{\int_0^1 t\, dt}{\int_0^1 dt} (1) = t - \frac{1}{2} = \frac{2t-1}{2}$$

$$\omega_3 = v_3 - \frac{\langle v_3, \omega_1 \rangle}{\langle \omega_1, \omega_1 \rangle}\,\omega_1 - \frac{\langle v_3, \omega_2 \rangle}{\langle \omega_2, \omega_2 \rangle}\,\omega_2$$

$$= t^2 - \frac{\int_0^1 t^2\,dt}{\int_0^1 dt}\,(1) - \frac{\int_0^1 t^2\left(\frac{2t-1}{2}\right)dt}{\int_0^1 \left(\frac{2t-1}{2}\right)^2 dt}\left(\frac{2t-1}{2}\right)$$

$$= t^2 - \frac{1}{3} - \frac{\cancel{1/12}}{\cancel{1/12}}\left(\frac{2t-1}{2}\right)$$

$$= t^2 - \frac{1}{3} - \left(\frac{2t-1}{2}\right)$$

$$= \frac{6t^2 - 2 - 6t + 3}{6}$$

$$= \frac{6t^2 - 6t + 1}{6} = \left(t^2 - t + 1/6\right)$$

orthogonal set $= \left\{ 1,\ t - 1/2,\ t^2 - t + 1/6 \right\}$