The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-II ■ Assignment 1

Q1. Let A, B be 2×2 real matrices such that $A[x \;\; y]^T = B[x \;\; y]^T$ for all $(x, y) \in \mathbb{R}^2$. Prove that $A = B$.

Q2. Show that the matrix multiplication is associative and distributive over addition of matrices.

Q3. Given $A = (a_{ij})$ define $A^T = (b_{ij})$ where $b_{ij} = a_{ji}$

(a) For two matrices A and B show that $(A+B)^T = A^T + B^T$ if $A+B$ is defined.

(b) $(AB)^T = B^T A^T$ if AB is defined.

Q4. Show that every square matrix can be written as a sum of a symmetric and a skew symmetric matrices. Further, show that if A and B are symmetric then AB is symmetric if and only if $AB = BA$.

Q5. Let A and B be two $n \times n$ invertible matrices. Show that $(AB)^{-1} = B^{-1}A^{-1}$.

Q6. If a $n \times n$ real matrix A satisfies the relation $AA^T = 0$ then show that $A = 0$. Is the same true if A is a complex matrix? Show that if $n \times n$ complex matrix and $A\bar{A}^T = 0$ then $A = 0$.

Q7. A real matrix A is said to be orthogonal if $AA^T = I$. Show that if A is orthogonal then $|A| = \pm 1$.

Q8. Let A and B be two $n \times n$.

(a) If $AB = BA$ then show that $(A+B)^m = \sum_{i=1}^{m} \binom{m}{i} A^{m-i} B^i$.

(b) Show by an example that if $AB \neq BA$ then (a) need not hold.

(c) If Tr $(A) = \sum_{i=1}^{n} a_{ij}$ then show that Tr $(AB) =$ Tr (BA). Hence show that if A is invertible then Tr $(ABA^{-1}) =$ Tr (B).

Q9. Give example of 3×3 nonzero matrices A and B such that $A^n = 0$ for some $n > 1$.

Q10. Explain geometrically, why
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Q11. Let A be a 2×2 real invertible matrices. Show that the image under A of

(a) any straight line is a straight line.

(b) any straight line passing through origin is a straight line passing through origin.

(c) any two parallel straight lines are parallel straight lines.

Q12. Let A be a nilpotent ($A^m = 0$, for some $m \geq 1$) matrix. Show that $I + A$ is invertible.