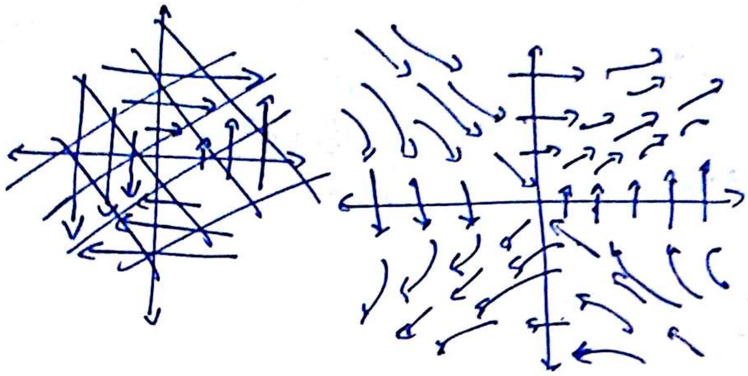
RAJAT GARGA 16UCS152

Physics 2 - Electrodynamics Assignment 1.

1.

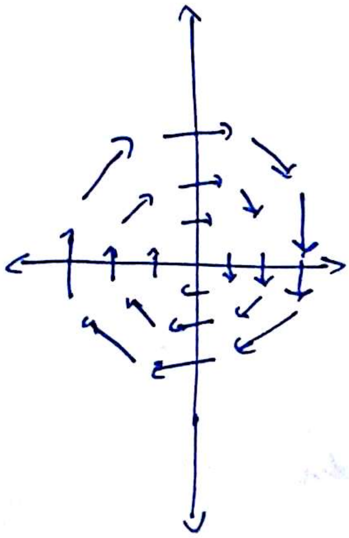
a) $\vec{v} = y\hat{i} + x\hat{j}$



divergence : $\frac{dy}{dx} + \frac{dx}{dy} = 0$

Curl : $0\hat{i} + 0\hat{k} + 0\hat{j}$

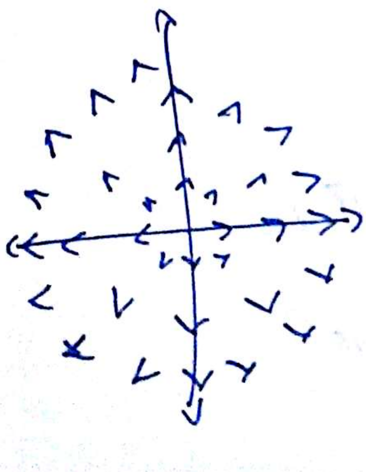
b)



divergence $= 0$

curl $= \vec{0}$

c)



divergence : $\frac{1}{\sqrt{x^2+y^2}} + \left(\frac{-1}{2}\right)\frac{x}{\sqrt{(x^2+y^2)^3}} \cdot 2x$

$+ \frac{1}{\sqrt{x^2+y^2}} + \left(\frac{-1}{2}\right)\frac{y}{\sqrt{(x^2+y^2)}} \cdot 2y$

2.

potential function $(t) = \frac{x^3}{3}\hat{i} + 2xy^2\hat{j} + \frac{1}{2}yz^4\hat{k}$

$$\int_{(0,0,0)}^{(1,1,1)} T\cdot dT = \left(\frac{1}{3}+2+\frac{1}{2}\right) - (0+0+0)$$

$$= \frac{2+12+3}{6} \cancel{\ast} = 17/6$$

a) $$\int_{(0,0,0)}^{(1,0,0)} \left(\frac{x^3}{3}+2x^2y\right)\cdot dx + \int_{(1,0,0)}^{(1,1,0)} (2xy^2+y^2z^3)\cdot dy + \int_{(1,1,0)}^{(1,1,1)} (1/2\, yz^3)\cdot dz$$

$$= \frac{1}{3}+2+\frac{1}{2} = 17/6$$

b) can be done on similar lines as (a)

c)

$$\left.\begin{array}{l} x = x \\ y = x \\ z = x^2 \end{array}\right\} \text{ parametrize the path}$$

$$\int T\cdot ds = \int T\cdot \sqrt{1+1+4x^2}\cdot dx$$

$$= \int_0^1 (x^2+4x^2+2x^7)\sqrt{1+1+4x^2}\cdot dx$$

$$= \int_0^1 (5x^2+2x^7)\sqrt{2+4x^2}\cdot dx$$

$$\int_0^1 \sqrt{50x^4+100x^6}\, dt \quad \int_0^1 \sqrt{8x^{14}+16x^{16}}\, dx$$

$$\frac{1}{2}\frac{200x^3+600x^5}{\sqrt{50x^4+100x^6}} + \frac{1}{2}\frac{112x^{13}+256x^{15}}{\sqrt{8x^{14}+16x^{16}}}$$

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3.

$a = (x+y)\hat{i} + (y-x)\hat{j}$

Part 1)

$\cancel{x = x}$

$x = y^2$

$y = y$

$\cancel{\therefore ds = \sqrt{(2y)^2 + (1)^2}}$

$\cancel{= \sqrt{1+4y^2}}$

$\cancel{a = (y+y^2)\hat{i} + (y-y^2)\hat{j}}$

$\vec{s} = y^2\hat{i} + y\hat{j}$

$d\vec{s} = (2y\hat{i} + \hat{j})\cdot dy$

$$\int \vec{a}\cdot d\vec{s} = \int_0^2 ([y+y^2]\hat{i} + (y-y^2)\hat{j})\cdot(2y\hat{i}+\hat{j})\,dy$$

$$= \int_0^2 (2y^2 + 2y^3 + y - y^2)\cdot dy$$

$$= \int_0^2 (y^2 + 2y^3 + y)\cdot dy$$

$$= \frac{y^3}{3} + \frac{y^4}{2} + \frac{y^2}{2}\Big|_0^2$$

$$= \frac{8}{3} + \frac{\cancel{16}^{\,8}}{\cancel{2}} + \frac{\cancel{4}^{\,2}}{\cancel{2}}$$

$$= \frac{8+24+6}{3} = 38/3$$

<u>part 2)</u> $u=0 \longrightarrow u=1$

$$\int_0^1 \left[(3u^2+u+2)\hat{i} + (-u^2\cancel{3}-u)\hat{j}\right]\cdot\left[(2u+1)\hat{i} + (2u)\hat{j}\right]\cdot du$$

$$6u^3+2u^2+4u+3u^2+u+2-2u^3-2u^2 \Big|_0^1$$

$$6+2+4+3+1+2-2-2-2$$

$$17-6$$

$$\boxed{=11}$$

<u>part 3)</u>

$$\underbrace{\int_{(1,1)}^{(4,1)} (x+y)\cdot dx}_{y \text{ is const}} + \underbrace{\int_{(4,1)}^{(4,2)} (y-x)\cdot dy}_{x \text{ is const}}$$

$$\frac{x^2}{2}+xy\Big|_{(1,1)}^{(4,1)} + \frac{y^2}{2}-xy\Big|_{(4,1)}^{(4,2)}$$

$$\frac{16}{2} - \cancel{\frac{1}{2}} + \frac{4}{2} + \cancel{\frac{1}{2}}$$

$$\boxed{=10}$$

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