Assignment 3

Q1(a) $P(x) = |\psi(x)|^2$

$$|\psi(x)|^2 = 4\alpha^3 x^2 e^{-2\alpha x}$$

$$\frac{d|\psi(x)|^2}{dx} = 4\alpha^3 \left(x^2 e^{-2\alpha x}(-2\alpha) + e^{-2\alpha x} 2x\right) = 0$$

$$x^2 e^{-2\alpha x}(-2\alpha) = -2x e^{-2\alpha x}$$

$$x = \frac{1}{\alpha}$$

(b) $$\langle x \rangle = \int_0^\infty x |\psi x|^2 dx$$

$$= 4\alpha^3 \int_0^\infty x^3 e^{-2\alpha x} dx$$

$$= 4\alpha^3 \left[\left(\frac{x^3 e^{-2\alpha x}}{-2\alpha} + \frac{3}{4\alpha^2}\left(x^2 e^{-2\alpha x}\right) + \frac{3}{2\alpha^2}\left(\frac{x e^{-2\alpha x}}{-2\alpha} + \frac{1}{4\alpha^2} e^{-2\alpha x}\right)\right)\right]_0^\infty$$

$$= 4\alpha^3 \left[0 + 0 + 0 + \frac{3}{8\alpha^4} \times 1\right]$$

$$= 4\alpha^3 \times \frac{3}{8\alpha^4} = \frac{3}{2\alpha}$$

$$\langle x^2 \rangle = \int_0^\infty x^2 |\psi(x)|^2 dx = 4\alpha^3 \int_0^\infty x^4 e^{-2\alpha x} dx$$

$$= 4\alpha^3 \left[\frac{x^4 e^{-2\alpha x}}{-2\alpha} + \frac{4}{4\alpha^2}\left(x^3 e^{-2\alpha x}\right) + \cdots \frac{3 e^{-2\alpha x}}{4\alpha^5}\right]_0^\infty$$

$$= 4\alpha^3 \left[0 + 0 + \frac{3e^0}{4\alpha^5}\right]$$

$$= 4\alpha^3 \times \frac{3}{4\alpha^5} = \frac{3}{2\alpha^2}$$

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C $\int_0^{1/\alpha} P(x)\,dx = 4\int_0^{1/\alpha} \alpha^3 x^2 e^{-2\alpha x}\,dx$

$= 4\alpha^3 \left[\left(\frac{x^2 e^{-2\alpha x}}{-2\alpha}\right)_0^{1/\alpha} + \left(\frac{2x\,e^{-2\alpha x}}{-2\alpha}\right)\right]$

$= 4\alpha^3 \left[\frac{1/\alpha^2\, e^{-2}}{-2\alpha} - \frac{1}{2}\left[\left(\frac{x e^{-2\alpha x}}{-2\alpha}\right)_0^{1/\alpha} + \left(\frac{1}{4\alpha^2} e^{-2\alpha x}\right)\right]_0^{1/\alpha}\right.$

$= 4\alpha^3 \left[\frac{e^{-2}}{-2\alpha^3} - \frac{1}{\alpha}\left[\frac{-1}{2\alpha^2} e^{-2} + \frac{1}{4\alpha^2} e^{-2}\right]\right]$

$= \frac{4\alpha^3}{4\alpha^3} e^{-2} = 0.032$

Q3 $\int_{-1}^{1} |\psi(x)|^2 dx = 1 \Rightarrow A^2 \int_{-1}^{1} \sin^2 \pi x\, e^{-2i\omega t}\,dx = 1$

$\frac{A^2}{2} \times 2 = 1 \Rightarrow A = \frac{1}{\sqrt{2}} \times \sqrt{2} = \pm 1$

Q4 $\langle x \rangle = L/2$

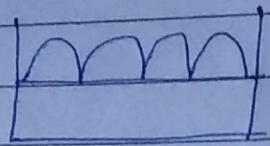
$\langle p_x \rangle = \int \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi x\,dx$

$= \frac{2}{L} \int_0^L \sin \frac{m\pi x}{L} \left(-i\hbar\left(\frac{m\pi}{L}\right) \cos \frac{m\pi x}{L}\,dx\right.$

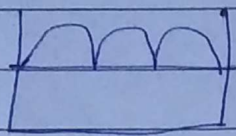
$= \frac{2}{L} (-i\hbar) \left(\frac{m\pi}{L}\right) \frac{1}{2} \int_0^L 2\sin \frac{m\pi x}{L} \cos \frac{m\pi x}{2}\,dx$

$\int \sin \frac{2m\pi x}{x} = 0$

Q5 for $n=4$ energy level we have 4 peaks

 → probability function

Q6 for $n=3$ we have 3 peaks

 → probability function

Q7 $\int_{L/4}^{L/2} |\psi|^2 dx = \frac{2}{L} \int_{L/4}^{L/2} \sin^2 \frac{n\pi x}{L} dx$

$= \frac{1}{4} + \frac{1}{2n\pi} \times n$

$= \frac{1}{4} + \frac{1}{2\pi}$

Q8 $E_4 = \frac{\pi^2 h^2 (4)^2}{8ma^2}$ $E_2 = \frac{\pi^2 h^2 (2)^2}{8ma^2}$

$E_4 - E_2 = \frac{\pi^2 h^2 16}{8ma^2} - \frac{\pi^2 h^2 (2)^2}{8ma^2}$

$h\nu = 3.243 \times 10^{14}$

width of box $= 2a$

$a = 8.92 \times 10^{-10} m$

Q2 $\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$ $0 < x < L$

$\frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi = E\psi$ ~~from I and III~~

for I & III $U = 0$ for II

$\frac{d^2\psi_{II}}{dx^2} + k^2 \psi_{II}(x) = 0$ $k^2 = \frac{2mE}{\hbar^2}$

Gen solution $\psi x = A\sin kx + B\cos kx$ $\psi x = 0$ at $x = -a$

$0 = A\sin k(-\cancel{a}) + B \sin k(-a)$

$0 = -A\sin ka + B\cos ka$ — (3)

now at $x = a$ $\psi x = 0$

$0 = A\sin ka + B\cos ka$ — (4)

$A = \frac{1}{\sqrt{a}}$ $B = \frac{1}{\sqrt{a}}$

case I $A = 0$ & $B \neq 0$

case II $A \neq 0$ & $B = 0$

$B\cos ka = 0$ if $B \neq 0$ $ka = \frac{n\pi}{2}$

$n = 1, 3, 5, 7, 9$

$A\sin ka = 0$ $A \neq 0$ $\sin ka = 0$ $ka = \frac{n\pi}{2}$ $n = 0, 2, 4, 6,$

$$\psi(x) = \begin{cases} A\sin \frac{n\pi x}{2a} & n = 2, 4, 6 \\ B\cos \frac{n\pi x}{2a} & n = 1, 3, 5, \ldots \end{cases}$$

$A = \frac{1}{\sqrt{\pi}}$ $B = \frac{1}{\sqrt{\pi}}$

(10) $T = 16\left(\frac{E}{U}\right)\left(1 - \frac{E}{U}\right) e^{-2k_1 L}$ where $k_1 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$