Assignment 5.

(1) (i) $y'' + x^2 y' = \sin y \sqrt{y} \cos x$

$(y'' + x^2 y')^2 = \sin^2 y (y) \cos^2 x$

Order = 2

Degree = 2

(ii) $(y')^{3/2} = 2x(ye^{-x^2} - y - 3x)$

$(y')^3 = \left[2x(ye^{-x^2} - y - 3x)\right]^2$

Order = 1

Degree = 3.

(iii) $y''' + 2xy' + x^2 = \sqrt{y} \cos x$

Order = 3

Degree = 2.

(2) (i), (iv)

(3) (i) $\frac{dy}{dx} = e^{2x+3y} = e^{2x} \cdot e^{3y}$

$\int \frac{dy}{e^{3y}} = \int e^{2x} dx$

$\int e^{-3y} dy = \int e^{2x} dx$

$= \frac{e^{-3y}}{-3} = \frac{e^{2x}}{2} + C.$

(ii) $\frac{dy}{dx} = e^x . e^y + x^2 e^{x^3} . e^y$

$\int e^{-y} dy = \int \left(e^x + \frac{3x^2 e^{x^3}}{3}\right) dx$

$-e^{-y} = e^x + \frac{e^{x^3}}{3} + C.$

(4) (i) $4x + y + 1 = t$

$$4 + \frac{dy}{dx} = \frac{dt}{dx}.$$

$$\frac{dt}{dx} - 4 = t^2$$

$$\int \frac{dt}{t^2+4} = \int dx$$

$$\frac{1}{2} \tan^{-1} t/2 = x + c.$$

$$\Rightarrow \frac{1}{2} \tan^{-1} \frac{(4x+y+1)}{2} = x + c.$$

(ii) $x + y = t.$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}.$$

$$(t^2)\left(\frac{dt}{dx} - 1\right) = a^2$$

$$\frac{dt}{dx} = \frac{a^2+t^2}{t^2}.$$

$$\frac{t^2 dt}{a^2+t^2} = dx$$

$$dt - \frac{a^2 dt}{a^2+t^2} = dx$$

$$t - \frac{a^2}{a} \tan^{-1} \frac{t}{a} = x + c.$$

$$t - a \tan^{-1}\left(\frac{x+y}{a}\right) = x + c$$

$$y - a\tan^{-1}\left(\frac{x+y}{a}\right) = c.$$

(iii) $$\frac{dy}{dx} = \cancel{\sec}\ \frac{1}{\cos(x+y)}$$

$$x + y = t$$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = \sec t.$$

$$\frac{dt}{dx} = 1 + \sec t.$$

$$\Rightarrow \frac{dt}{1+\sec t} = dx.$$

$$\cancel{\Rightarrow \left(\frac{\sec^2 t - 1}{\tan^2 t}\right) dt = dx}$$

$$\Rightarrow \frac{\cos t\, dt}{1 + \cos t} = dx$$

$$\Rightarrow dt - \frac{1}{2\cos^2 t/2} dt = dx$$

$$\Rightarrow dt - \frac{1}{2} \sec^2 t/2\, dt = dx$$

$$\Rightarrow t - \frac{1}{2} \tan t/2 \times \frac{2}{1} = x + c$$

$$\Rightarrow t - \tan t/2 = x + c \Rightarrow y - \tan\frac{(x+y)}{2} = c$$

$$\Rightarrow t -$$

$= \frac{\cos t}{\sin t}$

$\frac{\frac{1}{\cos t}}{\frac{\sin^2 t}{\cos^2 t}}$

$\sec^2 t - 1 = \tan^2 t$

$\sec^2 t =$

Teacher Signature

Scanned by CamScanner

5) (i) $y\,dx + x\,dy = \frac{y}{x}\frac{\sin y/x}{\cos y/x}(x\,dy - y\,dx)$

$$dy + \frac{y}{x}dx = \frac{1}{x}\left[\frac{y}{x}\right]\tan y/x\left(dy - \frac{y}{x}dx\right)$$

$$\left(\frac{dy}{dx} + \frac{y}{x}\right) = \left(\frac{y}{x}\tan y/x\right)\left(\frac{dy}{dx} - \frac{y}{x}\right)$$

Put $y/x = v \Rightarrow y = xv$

$dy/dx = v + x\,dv/dx$.

$$v + \frac{x\,dv}{dx} + v = v\tan v\left(\frac{x\,dv}{dx}\right)$$

$$2v = x\,v\tan v$$

$$\frac{x\,dv}{dx}\left(1 - v\tan v\right) = -2v.$$

$$\frac{2dx}{x} = \frac{(v\tan v - 1)}{v}dv$$

$$2\log x = \frac{\log|\sec v|}{2} - \frac{1}{2v}\log v + \log c$$

$$x^2 = c\frac{|\sec v|}{v}$$

$$yx^2 = c\,|\sec y/x|$$

(ii) $\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$

$y/x = v$

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx}$$

$$\cancel{v} + \frac{x\,dv}{dx} = v\log v + \cancel{v}$$

$$\frac{dx}{x} = \frac{dv}{v\log v}$$

$\log x = \log(\log v) + \log c$

Teacher Signature

$x = c \log v$

$x = c \log(y/x)$

3.) $(\lambda x^2 + \lambda e^y)\,dx + (2xe^y + 3y^2)\,dy = 0$

$\frac{\partial M}{\partial y} = \lambda e^y \qquad \frac{\partial N}{\partial x} = 2e^y$

$\lambda = 2.$

$\int (3x^2 + 2e^y)\,dx + \int 3y^2\,dy = c$

$x^3 + 2xe^y + y^3 = c.$

Rough

$\cancel{3x^2 + 2xe^y \frac{dy}{dx} + 2e^y + 3y^2 \frac{dy}{dx} = 0.}$

2.) (i) $M = yx + y^2$ $\qquad yx^2 + y^2 x \qquad y^2 x + y^3$

$N = x + 2y - 1$ $\qquad x^2 + 2xy - x \qquad xy + 2y^2 - y$

$\partial M/\partial y = x + 2y$ $\qquad \partial M/\partial y = 2x^2y + 2xy \qquad \partial M/$

$\partial N/\partial x = 1$ $\qquad \partial N/\partial x = 2x + 2y - 1$

$\text{I.F.} = e^{\int g(x)}$

$g(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = 1.$

$= e^{\int dx} = e^x \qquad / e^y$

(ii) $(xy + y^2 + y)\,dx + (x^2 + 3xy + 2x)\,dy$

$\frac{\partial M}{\partial y} = x + 2y + 1$

$\frac{\partial N}{\partial x} = 2x + 3y + 2.$

$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{y(x+y+1)}(x+y+1)$

$= \frac{1}{y}.$

$\text{I.F.} = e^{\int \frac{1}{y} dy} = e^{\log y} = y.$

(7) (i).

Soln ⇒

$M = y + \frac{y}{x} + \cos y$

$N = x + \log x - x\sin y$

$\frac{\partial M}{\partial y} = 1 + \frac{1}{x} - \sin y$ $\qquad$ $\frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y.$

Hence Exact.

$\cancel{\int (x + \log x - x\sin y)}$ $\qquad$ $\int \left(y + \frac{y}{x} + \cos y\right) dx$

$+ \cancel{\int} c$

$yx \cancel{+ \frac{-y}{x^2}}$

$yx + y\log x + x\cos y = c.$