THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
JAIPUR, RAJASTHAN

MATHEMATICS-I
End Term Exam (Part-2)

NOVEMBER 18, 2015
TIME: 3 HOURS, MAXIMUM MARKS: 50

Note: You should attempt all questions. Your writing should be legible and neat. Make an index showing the question number and page number on the front page of your answer sheet in the following format, otherwise you may be penalized by the deduction of **2 marks**.

Question No.				
Page No.				

1. (a) Let S be a nonempty subset of $\mathbb{R}$ and $\beta \in \mathbb{R}$. If $\beta = \inf S$, then show that for every $\epsilon > 0$, there is some $x \in S$ such that $\beta + \epsilon > x$. **[2 marks]**

   (b) Let S be a nonempty subset of $\mathbb{R}$ and $\beta \in \mathbb{R}$ be a lower bound. If for every $\epsilon > 0$, there is some $x \in S$ such that $\beta + \epsilon > x$ then show that $\beta = \inf S$ **[3 marks]**

   (c) Let (a_k) be a real sequence such that (a_{2k}) and (a_{2k-1}) diverges to $+\infty$. Then Show that (a_k) diverges to $+\infty$. **[5 marks]**

2. (a) For $k \in \mathbb{N}$, let $a_{2k-1} := \frac{1}{4^k}$ and $a_{2k} := \frac{1}{9^k}$. Discuss the convergence/divergence of the series $\sum_{k=1}^{\infty} a_k$. **[3 marks]**

   (b) Let D be a non-empty subset of $\mathbb{R}$ and $f : D \to \mathbb{R}$ be uniformly continuous. Prove that f is continuous on D. **[2 marks]**

   (c) Let r be a nonnegative rational number and consider $a_n := \sum_{k=1}^{n} \frac{k^r}{n^{r+1}}$ for $n \in \mathbb{N}$. Determine the limit of the sequence (a_n) by expressing the nth term as a Riemann sum for a suitable function. **[5 marks]**

3. (a) *Viviani's curve* is the intersection of the cylinder $(x - \alpha)^2 + y^2 = \alpha^2$ and the sphere $x^2 + y^2 + z^2 = 4\alpha^2$ and has parametric equation: **[5 Marks]**

$$\alpha : [0, 4\pi] \longrightarrow \mathbb{R}^3 : t \mapsto \alpha\left(1 + \cos t, \sin t, 2\sin\frac{t}{2}\right).$$

   Show that the curvature and torsion of this curve are given by

$$\kappa(t) = \frac{\sqrt{13 + 3\cos t}}{\alpha(3 + \cos t)^{3/2}}, \quad \tau(t) = \frac{6\cos\frac{t}{2}}{\alpha(13 + 3\cos t)}.$$

   (b) Let $f(x, y, z) = x^2 + y^2 + z^2$. Prove f is differentiable at $(1, 1, 1)$ with linear transformation $T(x, y, z) = 2x + 2y + 2z$ as its derivative. **[3 Marks]**

   (c) A mosquito is flying around a room in which the temperature is given by $T(x, y, z) = x^2 + y^4 + 2z^2$. The mosquito is at the point $(1, 1, 1)$ and realizes that he's cold. In what direction should he fly to warm up most quickly? **[2 Marks]**

4. (*a*) Many airlines require that carry-on luggage have a linear distance (sum of length, width, height) of no more than 45 inches with an additional requirement of being able to slide under the seat in front of you. Assuming that the carry-on is to have the shape of a rectangular box and one dimension is half of one of the other dimensions (to insure "slide under seat" is possible), what dimensions of the carry-on will lead to maximum storage (i.e., maximum volume)? **[6 Marks]**

[Hint: *Apply Lagrange's multiplier method with more than one constraints*]

(*b*) Suppose S is a "*light-bulb-shaped region*" shown in Figure 1. Imagine a light-bulb cut

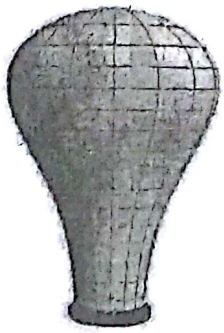

Figure 1: Bulb-shaped region S

off at the base so that its boundary is the unit circle $x^2 + y^2 = 1$, oriented with the outward-pointing normal. Suppose $F = (xe^{z^2-2z}, 1+y+\sin(xyz), e^{z^2}\sin(z^2))$. Compute the flux integral $\int\int_S curl\, F \cdot d\sigma$ using Stokes' theorem. **[4 Marks]**

5. (*a*) Compute the integral

$$\int_C \left(5y + \sqrt{1+x^5}\right) dx + \left(5x - e^{y^2}\right) dy,$$

where C is a circle of radius 5 centered at the origin. **[3 Marks]**

(*b*) In this problem S is the surface given by the quarter of the right-circular cylinder centered on the $z-$axis, of radius 2 and height 4, which lies in the first octant. A vector field $F(x,y,z) = y\hat{j}$ is defined on S.

(*i*) Compute the flux integral $\int\int_S F \cdot \hat{n}\, d\sigma$.

(*Hint: Use the normal which points outward from S, i.e. on the side away from the* $z-$*axis.*)

(*ii*) D be the solid in the first octant given by the interior of the quarter cylinder defined above. Use the divergence theorem to compute the flux of the field $F = y\hat{j}$ out of the solid D. **[4+3 Marks]**

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Electronics I
End Term Exam

Date: 17th November 2015 **Time: 180 Minutes** **Max Marks. 50**

Notes: Start every solution on fresh page.
Highlight your answers by inboxing or underlining them.
Assumptions made should be written clearly.

1. An Inverting Schmitt Trigger is shown in Figure 1. The output voltage can either be $+V_{CC}$ = 10V or $-V_{CC}$ = -10V. Find the value of switching thresholds V_{IL} and V_{IH} for the circuit if, R_2= 4kΩ and R_1 = 2k.Ω. Consider the circuit in figure 2 with same values of R_1 and R_2 and V_1 = 2V. Find the value of switching thresholds V_{IL} and V_{IH} for the circuit. **[5]**

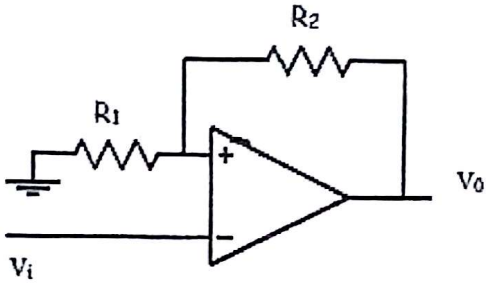

Figure 1

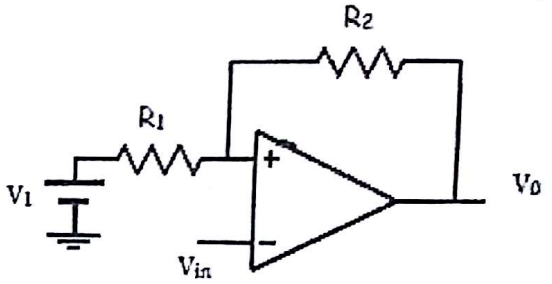

Figure 2

2. Plot the magnitude Bode plot for the system with transfer function

$$G(s) = \frac{2000(jw+0.5)}{jw(jw+10)(jw+50)}$$ **[5]**

3. In the circuit shown in figure 3, the setup and hold times of the flipflops are 5 ns and 1 ns respectively. The propagation delay of the flipflops (C1 can represent any flip flop) may vary between 4 and 7 ns while the propagation delay of the gates(& and ! can be any logic gate) may vary between 2 and 6 ns. Calculate the minimum and the maximum propagation delay T_{A-V} ~~and~~ and T_{B-V}. Hence calculate the maximum frequency of the clock, C. **[5]**

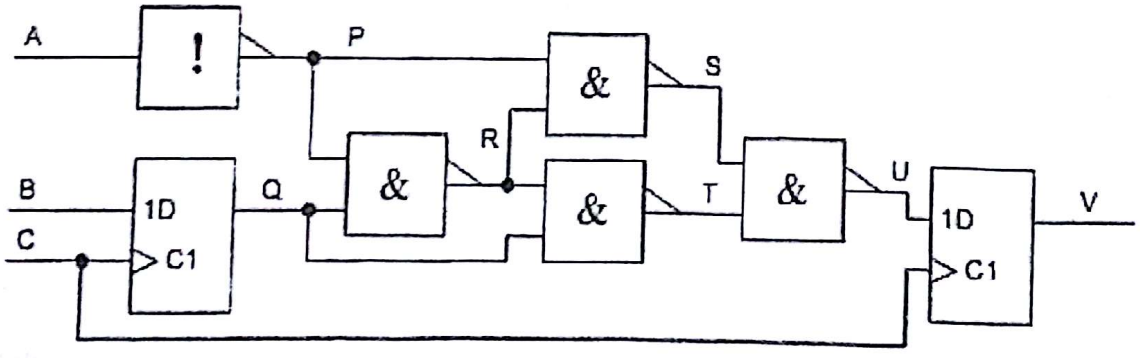

Figure 3

4. You are provided with various input signals like, ASint, BSin2t, CSin3t, ACost, BCos2t and CCos3t. Design a circuit whose output is A(Sint-Cost) + B(Sin2t-Cos2t) + C (Sin3t-Cos3t) **[5]**

5. Find out the value of Vx in figure 4 using Superposition theorem. [5]

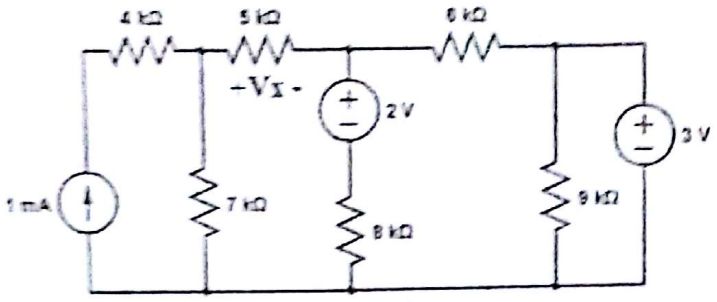

Figure 4

6. X is an integer with values from 0 to 7. Y = (X + 3) modulus 6. Design a combinational circuit that converts every value of X to corresponding value of Y. [5]

7. A new flip flop is designed with following description. It has 3 inputs A, B and C and output is Q. It holds the output if all inputs are 0 and flips the current output if all the inputs are 1. If the number of 1's in inputs are 1, then it resets the output to 0 and sets the output to 1 if the number of 1s are 2. Write down the characteristic table, characteristic function and excitation table for this flip flop. [5]

8. Simplify the expression F = AB(C + D') + A'B(C' + D) using a K-map. Write down the optimized SoP and PoS for F. Neglecting NOT gates in the design, which expression (SoP or PoS) can be implemented with less number of logic gates? [5]

9. Design a 2 bit down counter using T flip flops and logic gates. [5]

10. Observe the circuit given in Figure 5. Consider Shift ='1', then write down the truth table expressing the relation of X with Y. Write down another similar truth table when Shift = '0'. Looking at the output patterns of both the Truth tables and what can you say about the behaviour of the circuit? [5]

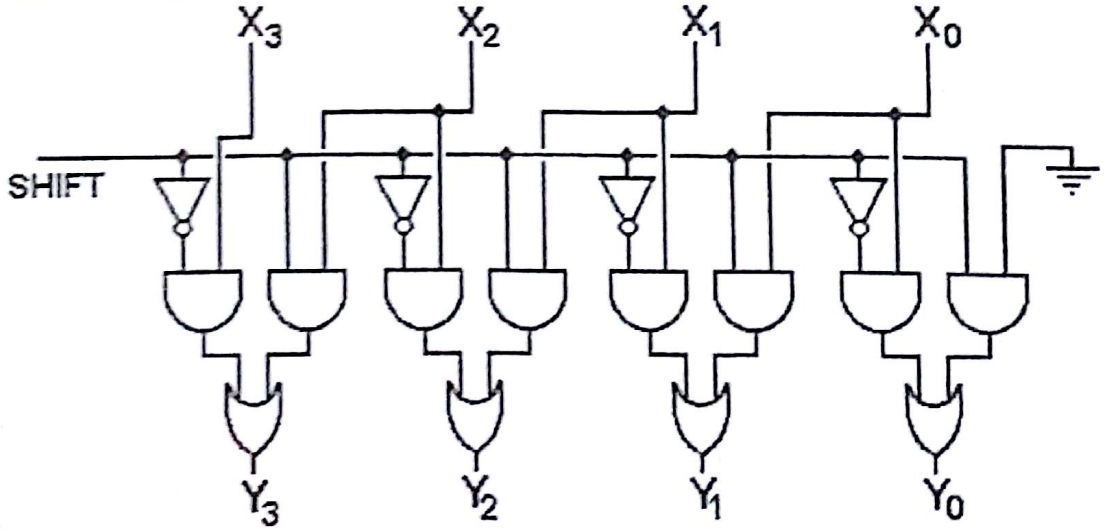

Figure 5