The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-II ■ Assignment 3

Q1. Show that the space of all real (respectively complex) matrices is a vector space over $\mathbb{R}$ (respectively $\mathbb{C}$) with respect to the usual addition and scalar multiplication.

Q2. Which of the following are the subspaces of $\mathbb{R}^3$:

$$(a)\ \{(x,y,z)|x \geq 0\}.\ (b)\ \{(x,y,z)|x+y=z\}.\ (c)\ \{(x,y,z)|x=y^2\}.$$

Q3. Find the conditions on real numbers a, b, c, d so that the set $\{(x,y,z)|ax+by+cz=d\}$ is a subspace of $\mathbb{R}^3$.

Q4. Show that $W = \{(x_1,x_2,x_3,x_4)|x_4 - x_3 = x_2 - x_1\}$ is a subspace of $\mathbb{R}^4$, spanned by $(1,0,0,-1), (0,1,0,1)$ and $(0,0,1,1)$.

Q5. Determine whether the following sets of vectors are linearly independent or not
(a) $S = \{(1,0,2,1), (1,3,2,1), (4,1,2,2)\}$ of $\mathbb{R}^4$,
(b) $S = \{(1,2,6), (-1,3,4), (-1,-4,-2)\}$ of $\mathbb{R}^3$,
(c) $S = \{u+v, v+w, w+u\}$ in a vector space V given that $\{u,v,w\}$ is linearly independent.

Q6. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$. Let $C(A)$ denote the column space for the matrix A.

(a) Find all the sets of linearly independent column vectors in $C(A)$.

(b) Find a basis for the column space $C(A)$ and row space $R(A)$.

(c) Find the null space $N(A)$ of A and a basis for $N(A)$.

(d) Find column rank, row rank and nullity of A.

Q7. Let $(1,4,0,2), (1,3,2,0), (2,7,2,2) \in \mathbb{R}^4$. Verify linear dependence/linear independence of those vectors. Extend these vectors to a basis for $\mathbb{R}^4$.

Q8. Find the dimension of the following vector spaces
(a) $\{A : A \text{ is } m \times n \text{ real matrices}\}$.
(b) $\{A : A \text{ is } n \times n \text{ real upper - triangular matrices}\}$.
(c) $\{A : A \text{ is } n \times n \text{ real symmetric matrices}\}$

Q9. Let W be a subspace of V
(a) Show that there is a subspace U of V such that $W \cap U = \{0\}$ and $U + W = V$.
(b) Show that there is no subspace U such that $U \cap W = \{0\}$ and $\dim U + \dim W > \dim V$.

Q10. Let $P_n(\mathbb{R})$ = The vector space of polynomials with real coefficients and degree less or equal to n. Show that the set $\{x+1, x^2+x-1, x^2-x+1\}$ is a basis for $P_2(\mathbb{R})$. Hence, determine the coordinates of the following elements: $2x-1, 1+x^2, x^2+5x-1$ with respect to the above basis.

Q11. Let $W_1 = L\{(1,1,0), (-1,1,0)\}$ and $W_2 = L\{(1,0,2), (-1,0,4)\}$. Show that $W_1 + W_2 = \mathbb{R}^3$. Give an example of a vector $v \in \mathbb{R}^3$ such that v can be written in two different ways in the form $v = v_1 + v_2$, where $v_1 \in W_1, v_2 \in W_2$.

Q12. Describe all possible ways in which two planes (passing through origin) in $\mathbb{R}^3$ could intersect.