Image Representation

Reference:
Digital Image Processing
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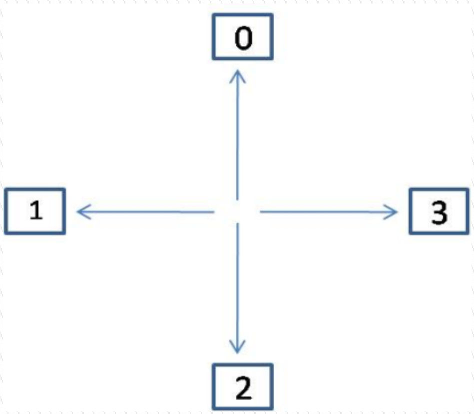
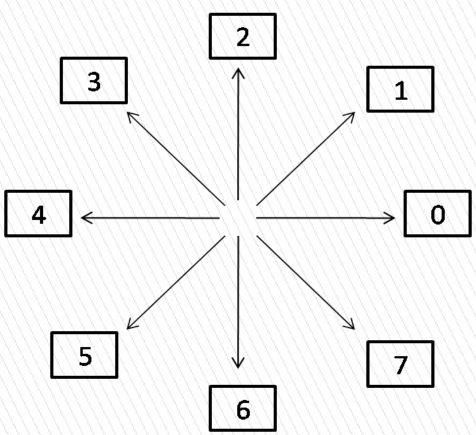
Representation

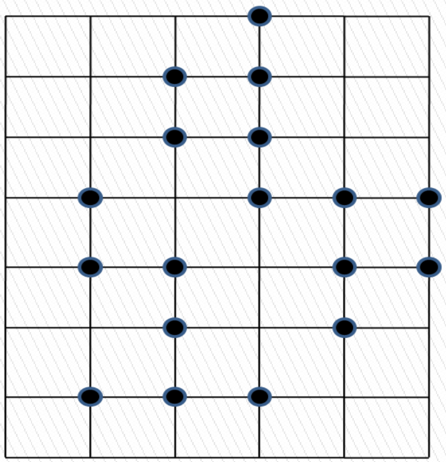
- Image regions (including segments) can be represented by either the border or the pixels of the region. These can be viewed as external or internal characteristics, respectively.
- Chose a representation scheme:
  - Chain codes (4- and 8-directional chain codes)
  - Signature (1-D functional representation)
  - Skeleton of a region – morphological operator
- Describe the region based on the scheme:
  - Boundary descriptors
    - Length, diameter, shape numbers, Fourier descriptors
    - Moments of signatures
  - Region descriptors
    - Area, compactness, principal axes
    - Texture
    - Moment

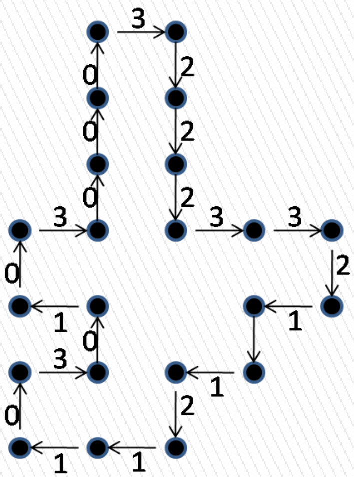
Chain Code

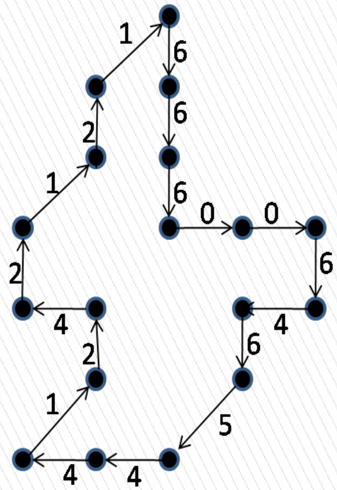
- Chain codes: represent the boundary of a connected region.











Chain Code

- Chain codes can be based on either 4-connectedness or 8-connectedness.
- The first difference of the chain code:
  - This difference is obtained by counting the number of direction changes (in a counterclockwise direction)
  - For example, the first difference of the 4-direction chain code 10103322 is 3133030.
- Assuming the first difference code represent a closed path, rotation normalization can be achieved by circularly shifting the number of the code so that the list of numbers forms the smallest possible integer.
- Size normalization can be achieved by adjusting the size of the resampling grid.

Polygonal Approximations

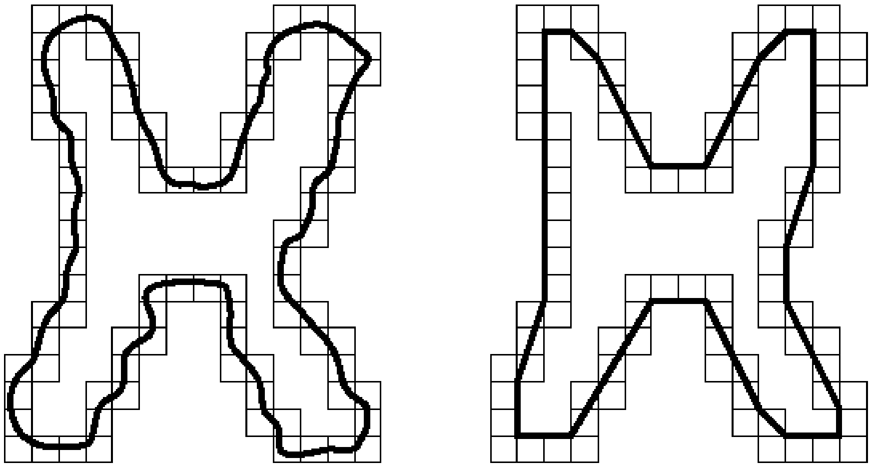
- Polygonal approximations: to represent a boundary by straight line segments, and a closed path becomes a polygon.
- The number of straight line segments used determines the accuracy of the approximation.
- Only the minimum required number of sides necessary to preserve the needed shape information should be used.
- A larger number of sides will only add noise to the model.

Polygonal Approximations: Minimum Perimeter Polygon

- Enclose a boundary by a set of concatenated cells.
- Consider object boundary as a rubber band between outside and inside strip of cells.
- If the rubber band is allowed to shrink, it produces a polygon of minimum perimeter.

a b

FIGURE 11.3 (a) Object boundary enclosed by cells. (b) Minimum perimeter polygon.

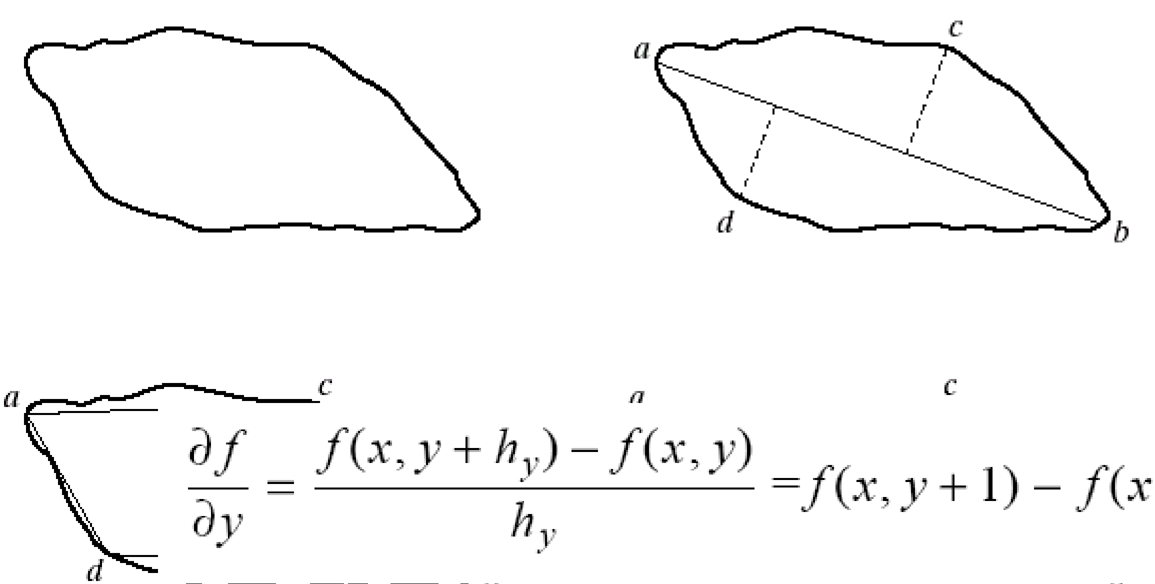


Polygonal Approximations: Merging

- One approach is to merge points along a boundary until the least square error line fit of the points merged so far exceeds a preset threshold.
- When this happens, parameters of the line are stored, error set to zero and procedure repeated.
- At end of procedure, intersection of adjacent line segments form the vertices of the polygon.

Polygonal Approximations: Splitting

- Subdivide a segment successively into two parts until a specified criterion is satisfied.
  - For ex., a criterion might be that the maximum perpendicular distance from a boundary segment to the line joining its two end points not exceed a preset threshold.
  - If it does, farthest point from the line becomes a vertex.



a	b
c	d

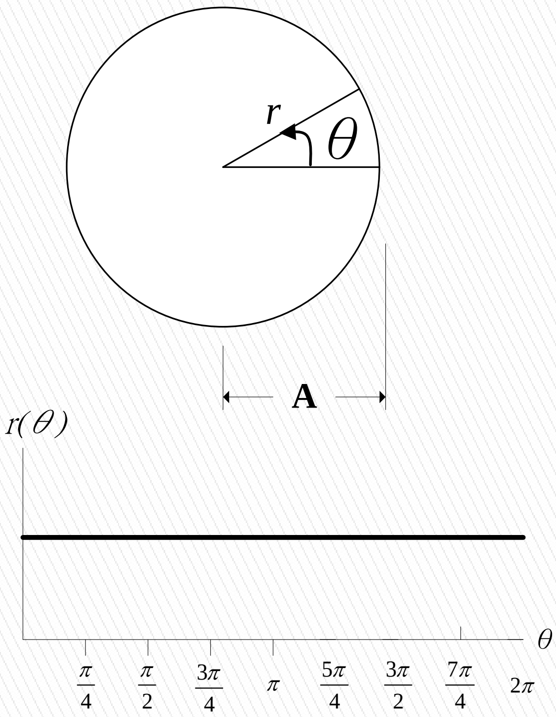
FIGURE 11.4
(a) Original boundary.
(b) Boundary divided into segments based on extreme points. (c) Joining
s.
ting

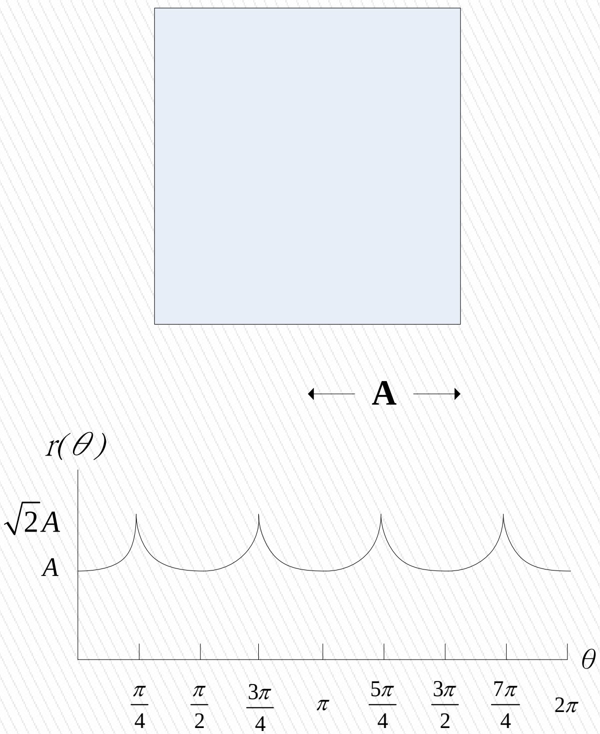
$$\frac{\partial f}{\partial y} = \frac{f(x, y + h_y) - f(x, y)}{h_y} = f(x, y+1) - f(x, y), \quad (h_y = 1)$$

Signature

- The idea behind a signature is to convert a two dimensional boundary into a representative one dimensional function.
- Signatures are invariant to location, but will depend on rotation and scaling.
  - Starting at the point farthest from the reference point or using the major axis of the region, can be used to decrease dependence on rotation.
  - Scale invariance can be achieved by either scaling the signature function to fixed amplitude or by dividing the function values by the standard deviation of the function.

Signature





Distance signature of circle shapes

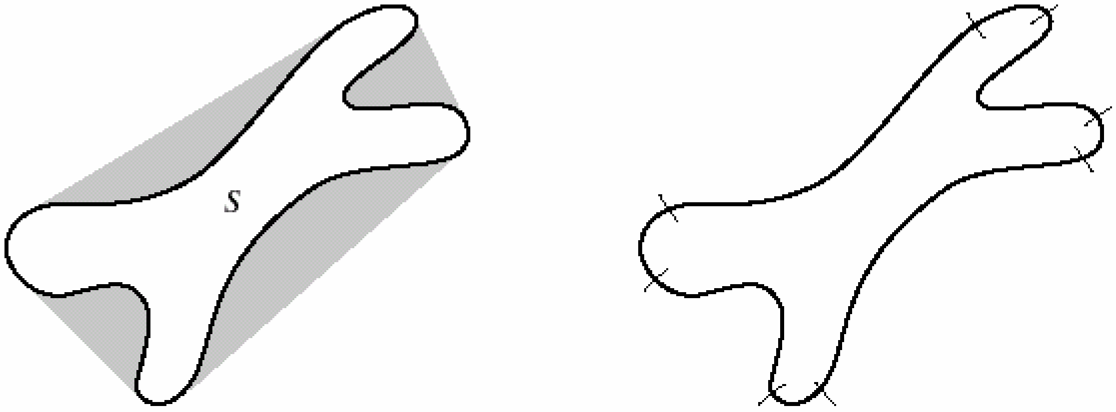
Distance signature of rectangular shapes

Boundary Segments
Convex Hull

- Decomposing a boundary into segments.
- It can be useful when boundary contains one or more significant concavities that carry shape information.
- Convex hull *H* of an arbitrary set *S* is the smallest convex set containing *S*.
- The set difference *H*–*S* is called the convex deficiency *D* of the set *S*.
  - In principle, this scheme is independent of region size and orientation.

Boundary Descriptors
Convex Hull

- The region boundary is partitioned by following the contour and marking the points at which a transition is made into or out of a component of the convex deficiency.

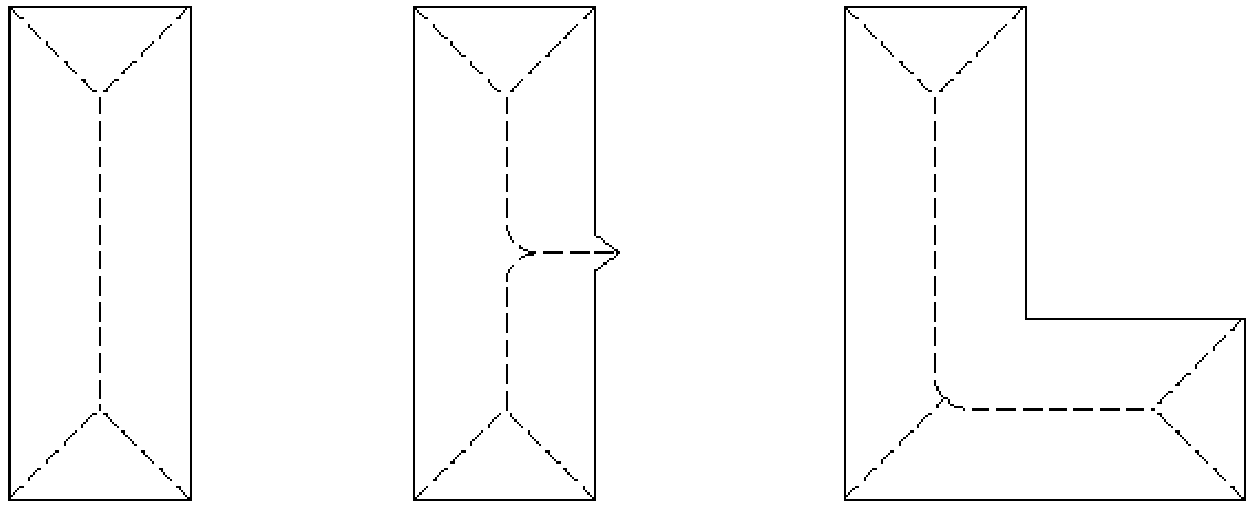


a b

FIGURE 11.6 (a) A region, S, and its convex deficiency (shaded). (b) Partitioned boundary.

Skeletons

- Skeletons: produce a one pixel wide graph that has the same basic shape of the region, like a stick figure of a human. It can be used to analyze the geometric structure of a region which has bumps and “arms”.



a b c

FIGURE 11.7 Medial axes (dashed) of three simple regions.

Skeletons

- Thinning algorithm iteratively deletes edge points of a region subject to the constraints that deletion of these points:
  - Does not remove end points
  - Does not break connectivity
  - Does not cause excessive erosion of the region

Skeletons

- Medial Axis Transformation: A thinning algorithm:
  - A contour point is any pixel with value 1 and having at least one 8-neighbor valued 0.
  - Let,

$$N(p_1) = p_2 + p_3 + \ldots + p_8 + p_9$$

$T(p_1)$: the number of 0-1 transitions in the ordered sequence

$$p_2, p_3, \ldots, p_8, p_9, p_2$$

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

Skeletons

- Step 1: Flag a contour point p_1 for deletion if the following conditions are satisfied

(a) $2 \le N(p_1) \le 6$

(b) $T(p_1) = 1$

(c) $p_2 \cdot p_4 \cdot p_6 = 0$

(d) $p_4 \cdot p_6 \cdot p_8 = 0$

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

0	0	1
1	p_1	0
1	0	1

FIGURE 11.9 Illustration of conditions (a) and (b) in Eq. (11.1-1). In this case $N(p_1) = 4$ and $T(p_1) = 3$.

Skeletons

- Step 2: Flag a contour point p_1 for deletion again. However, conditions (a) and (b) remain the same, but conditions (c) and (d) are changed to

$$(c')\quad p_2 \cdot p_4 \cdot p_8 = 0$$

$$(d')\quad p_2 \cdot p_6 \cdot p_8 = 0$$

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

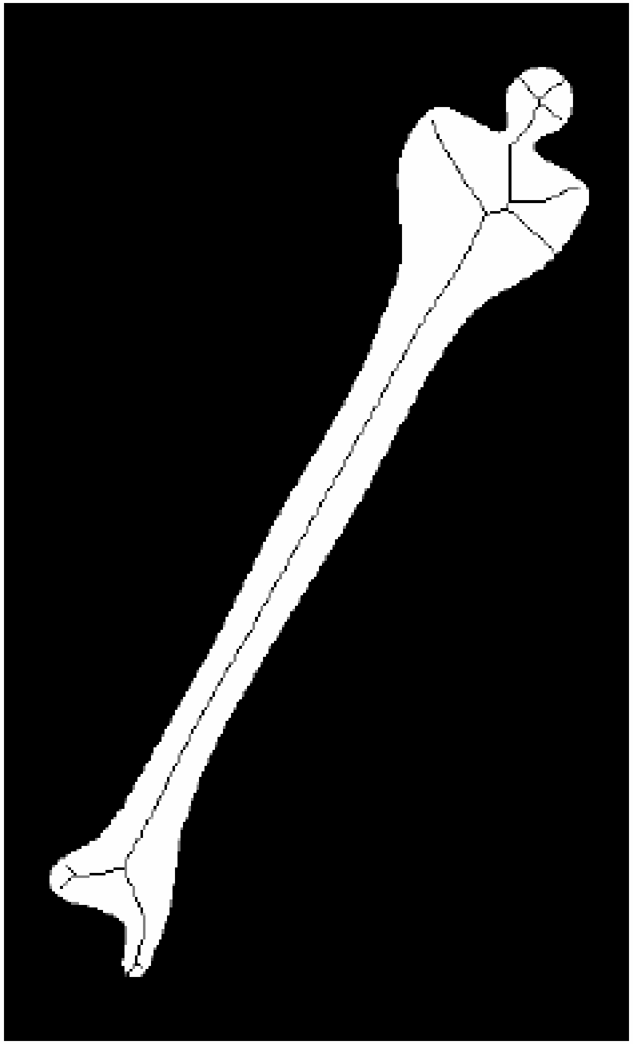
Skeletons

- A thinning algorithm:
  1. Apply Step 1 to every border pixel.
  2. Flag border points for deletion
  3. Delete the flagged points only when all border points have been processed
  4. Apply Step 2 to flag the remaining border points for deletion
  5. Delete the flagged points
  6. Apply procedure iteratively until no further points are deleted.

Skeletons

- One application of skeletonization is for character recognition.
- A letter or character is determined by the center-line of its strokes, and is unrelated to the width of the stroke lines.

FIGURE 11.10 Human leg bone and skeleton of the region shown superimposed.



Boundary Descriptors

- There are several simple geometric measures that can be useful for describing a boundary.
  - The length of a boundary: the number of pixels along a boundary gives a rough approximation of its length.
  - Curvature: the rate of change of slope.
    - To measure a curvature accurately at a point in a digital boundary is difficult.
    - The difference between the slopes of adjacent boundary segments is used as a descriptor of curvature at the point of intersection of segments.

Boundary Descriptors

- *Length* of boundary is simplest descriptor.
  - For a chain-coded curve with unit spacing in both directions, the number of vertical and horizontal components plus √2 times the number of diagonal components gives exact length.
  - Diameter of boundary B is defined as:

    $$\mathrm{Diam}(B) = \max[D(p_i, p_j)]$$

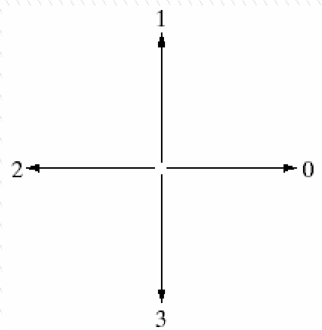
    where, D is a distance measure and p_i, p_j are points on boundary.
  - Line segment connecting the two extreme points of diameter is called major axis.
  - Line perpendicular to major axis is called minor axis.

Boundary Descriptors

- A box passing through the outer four points of intersection of boundary with major and minor axes, completely encloses the boundary. This box is called basic rectangle.
- Ratio of major to minor axis is called eccentricity of boundary.
- Rate of change of slope is curvature.
  - Difference between slopes of adjacent boundary segments can also be used.

Boundary Descriptors
Shape Numbers

- The shape number of a boundary is defined as the first difference of smallest magnitude.
- The order *n* of a shape number is defined as the number of digits in its representation.



First difference

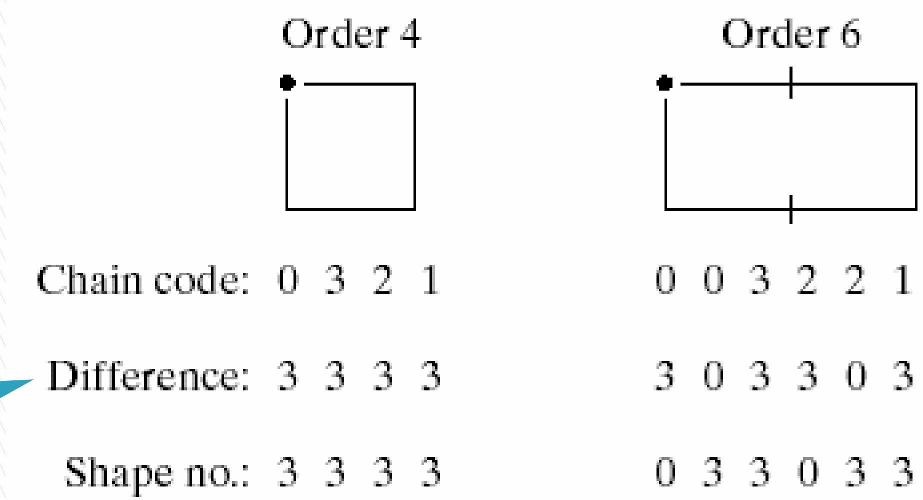
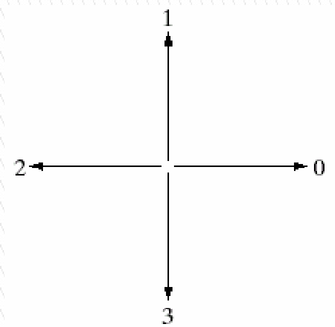
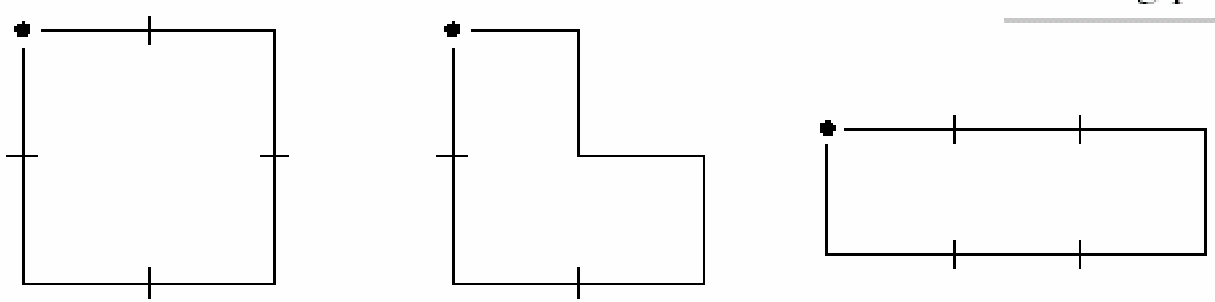


FIGURE 11.11 All shapes of order 4, 6, and 8. The directions are from Fig. 11.1(a), and the dot indicates the starting point.

Boundary Descriptors
Shape Numbers



Order 8

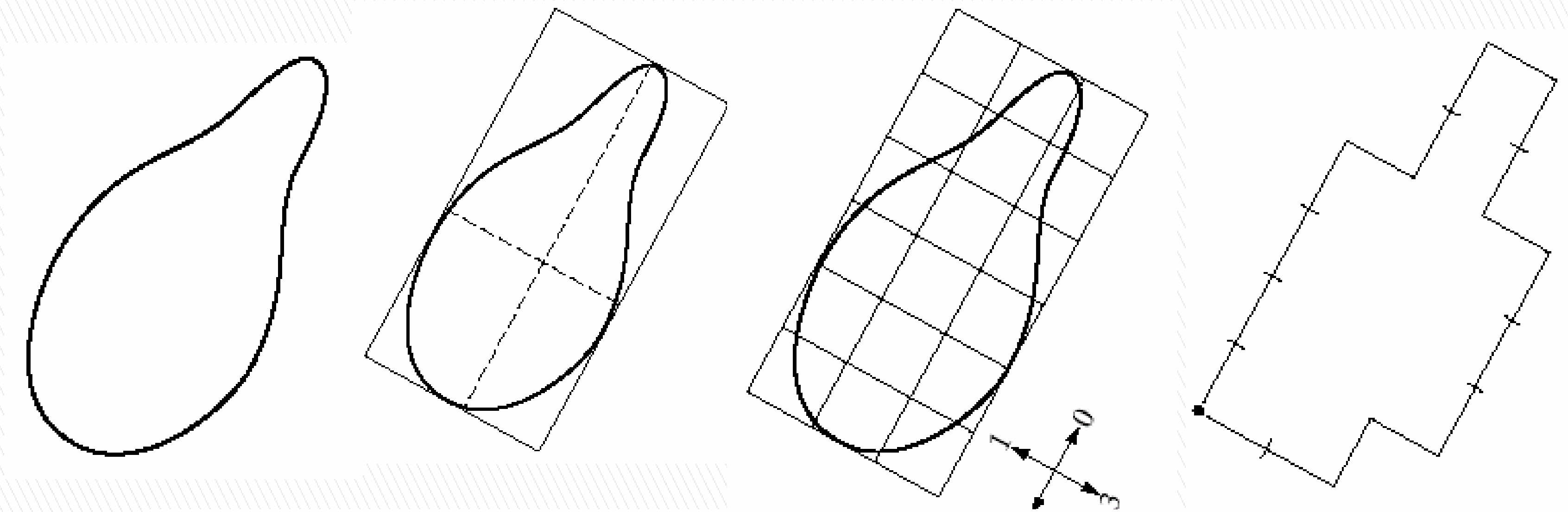


Chain code:	0 0 3 3 2 2 1 1	0 3 0 3 2 2 1 1	0 0 0 3 2 2 2 1
Difference:	3 0 3 0 3 0 3 0	3 3 1 3 3 0 3 0	3 0 0 3 3 0 0 3
Shape no.:	0 3 0 3 0 3 0 3	0 3 0 3 3 1 3 3	0 0 3 3 0 0 3 3

FIGURE 11.11 All shapes of order 4, 6, and 8. The directions are from Fig. 11.1(a), and the dot indicates the starting point.

Boundary Descriptors
Shape Numbers

- Ex., n=18 is specified for the boundary shown.



Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1

Difference: 3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0

Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

Boundary Descriptors
Fourier descriptors

- Using the Fourier transform to analyze the shape of a boundary.
  - The $x-y$ coordinates of the boundary are treated as the real and imaginary parts of a complex number:

    $$s(k) = x(k) + jy(k)$$

  - Then the list of coordinates is Fourier transformed using the DFT.

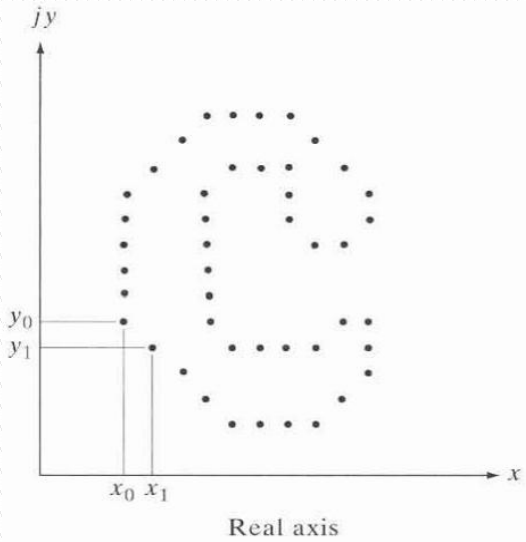
    $$a(u) = \frac{1}{N}\sum_{k=0}^{N-1} s(k)\exp\left[-j2\pi uk/N\right]$$

  - The Fourier coefficients are called the Fourier descriptors.
  - The basic shape of the region is determined by the first several coefficients, which represent lower frequencies:

    $$(k = 0 : M,\ M < N)$$

    - Higher frequency terms provide information on the fine detail of the boundary.
    - Lower frequency terms provide global shape.

Boundary Descriptors
Fourier descriptors



- Step1: $s(k) = x(k) + jy(k)$
- Step2: (DFT)

$$a(u) = \frac{1}{K}\sum_{k=0}^{K-1} s(k)e^{-j2\pi uk/K}$$

- Step3: (reconstruction)

if a(u)=0 for u>P-1

$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u)e^{j2\pi uk/K}$$

Boundary Descriptors
Fourier descriptors

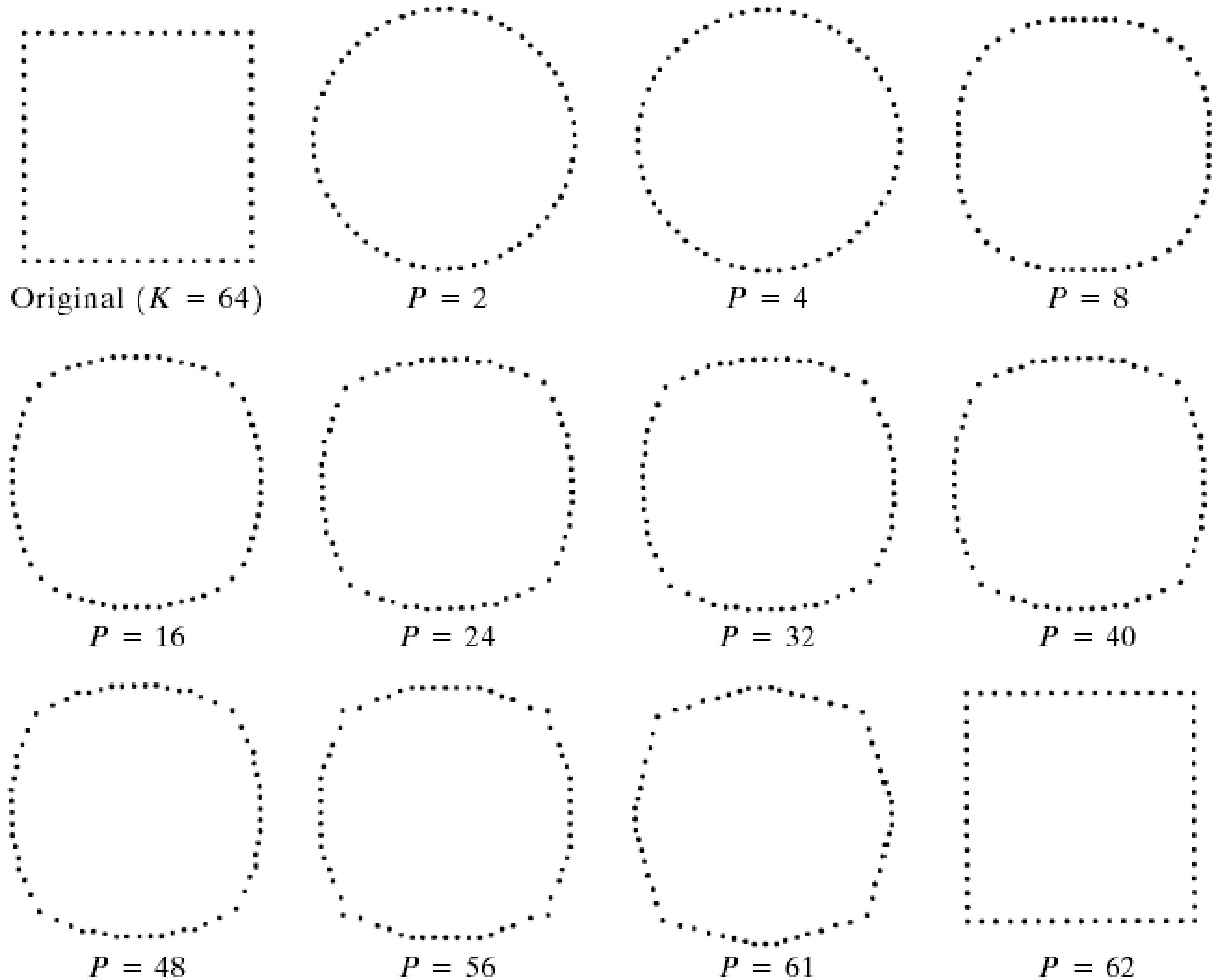


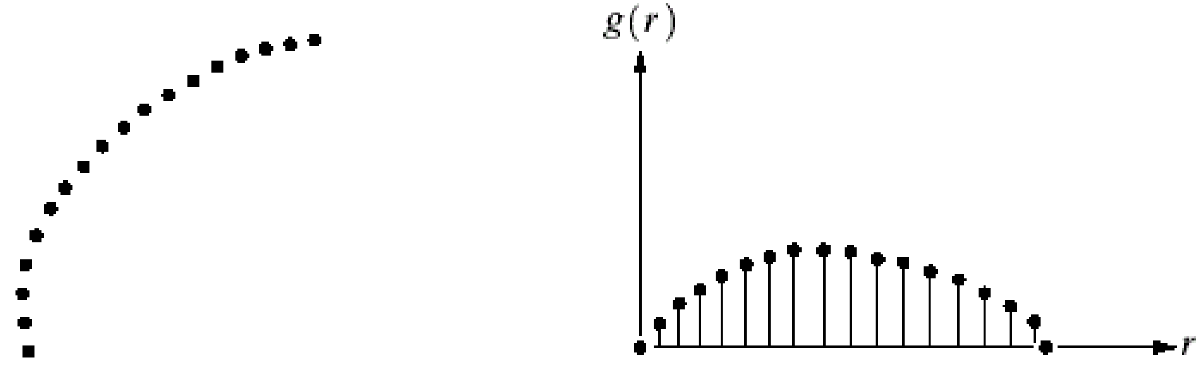
FIGURE 11.14 Examples of reconstruction from Fourier descriptors. P is the number of Fourier coefficients used in the reconstruction of the boundary.

Boundary Descriptors
Statistical Moments

- Shape of boundary segments can be described by statistical moments
  - Eg., mean, variance, higher-order moments
- Let *r* be a random variable, and *g*(*r*) be it's 1-D representation.

a b

FIGURE 11.15
(a) Boundary segment.
(b) Representation as a 1-D function.



Boundary Descriptors
Statistical Moments

- Let amplitude of *g* be a discrete random variable *v*.
  - Form an amplitude histogram $p(v_i)$, $i=0,1,2\ldots A-1$, where A is the number of discrete amplitude increments.
  - If $p(v_i)$ is the estimate of the probability of value v_i occuring, then, *n*th moment of *v* about its mean is:

$$\mu_n(r) = \sum_{k=0}^{K-1} (r_i - m)^n g(r_i)$$

$$\text{where } m = \sum_{i=0}^{K-1} r_i g(r_i)$$

  - m is the average value of v and μ_2 is its variance.
- Generally, only first few moments are required to differentiate between distinct shapes.

Boundary Descriptors
Statistical Moments

- Moments come in integer orders.
  - Order 0: number of points in the data.
  - Order 1: used to find the average.
  - Order 2: related to the variance; can be used to describe the smoothness.
  - Order 3: skew of the data; measures skewness of histogram.
  - Order 4 (kurtosis): measures the flatness of the histogram.
  - Higher orders can also be used, but don’t have simple meanings.

Regional Descriptors

- Some simple descriptors
  - The area of a region: the number of pixels in the region
  - The perimeter of a region: the length of its boundary
  - The compactness of a region: $(\text{perimeter})^2/\text{area}$
  - The mean and median of the gray levels
  - The minimum and maximum gray-level values
  - The number of pixels with values above and below the mean

Regional Descriptors
Topological Descriptors

- Useful for global descriptions of regions in the image plane, for ex.:
  - Number of holes
  - Number of connected components

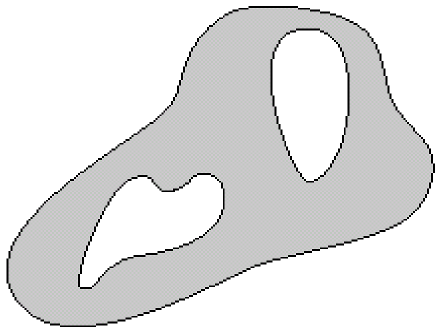


FIGURE 11.17 A region with two holes.

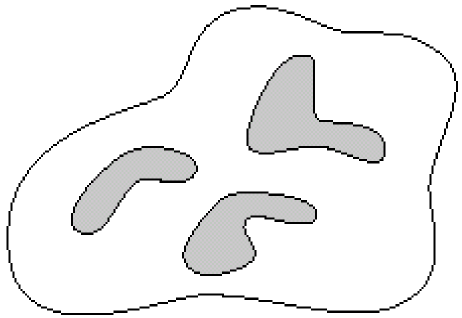
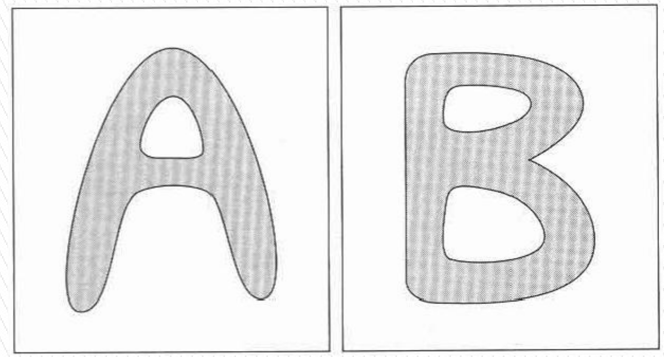
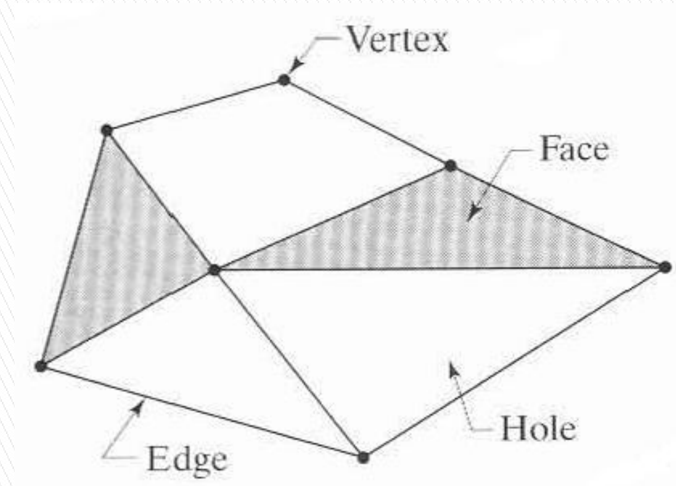


FIGURE 11.18 A region with three connected components.

Regional Descriptors
Topological

- $E = V - Q + F = C - H$
  - E: Euler number
  - V: the number of vertices
  - Q: the number of edges
  - F: the number of faces
  - C: the number of connected component
  - H: the number of holes





Regional Descriptors
Texture

- Texture is usually defined as the smoothness or roughness of a surface.
- In computer vision, it is the visual appearance of the uniformity or lack of uniformity of brightness and color.
- Basic elements that describe the texture are sometimes called texels.
- Three principal approaches to describe texture of a region are:
  - Statistical – characterizations of textures as smooth, coarse, grainy etc.
  - Structural – description of texture based on regularly spaced parallel lines
  - Spectral – based on properties of the Fourier spectrum; used to detect global periodicity in an image by identifying high energy, narrow peaks in the spectrum.

Regional Descriptors
Texture

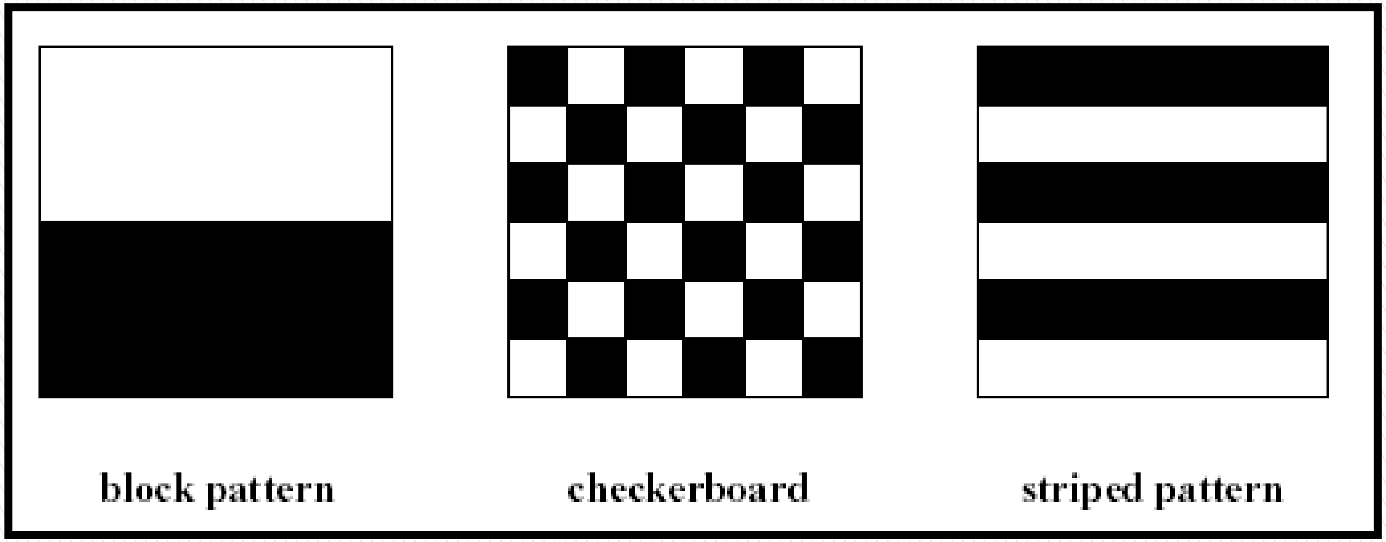
- Broadly, there are two types of texture: random and regular.
  - Random texture cannot be exactly described by words or equations; it must be described statistically. The surface of a pile of dirt or rocks of many sizes would be random.
    - Random texture is analyzed by statistical methods.
  - Regular texture can be described by words or equations or repeating pattern primitives. Clothes are frequently made with regularly repeating patterns.
    - Regular texture is analyzed by structural or spectral (Fourier) methods.

Regional Descriptors
Texture

- An image containing several different regions, each having a distinct texture.

Regional Descriptors
Texture

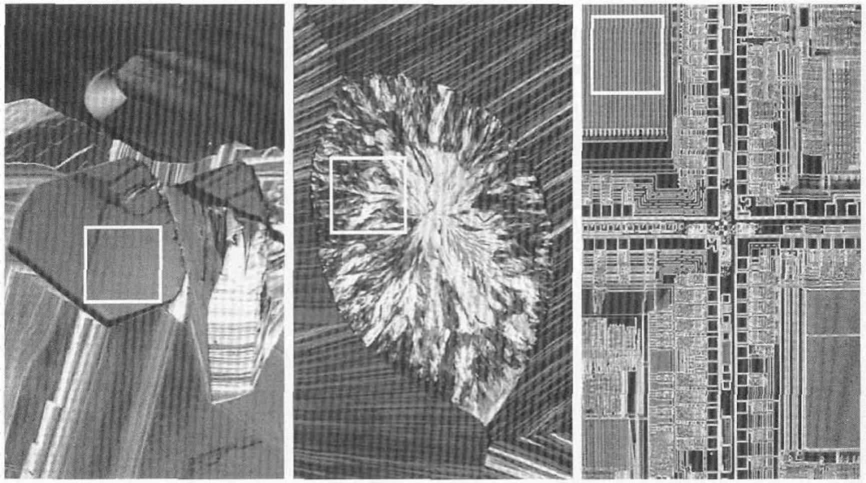
- Three different textures with the same distribution of black and white.



The statistics of the distribution of pixel values cannot describe texture well. The spatial relationship among pixels should be used to describe texture.

Regional Descriptors
Texture

- Statistical approaches
  - smooth, coarse, regular



- nth moment:

$$u_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

$$m = \sum_{i=0}^{L-1} z_i p(z_i)$$

where, z is a random variable denoting gray levels and $p(z_i)$, i=0,1,2…,L−1, be the corresponding histogram.

  - 2th moment: measure of gray level contrast (relative smoothness)
  - 3th moment: measure of the skewness of the histogram
  - 4th moment: measure of its relative flatness
  - 5th and higher moments: not so easily related to histogram shape

Regional Descriptors
Texture

- Local Binary Pattern

b_1	b_2	b_3
b_8	p	b_4
b_7	b_6	b_5

$$b_i = \begin{cases} 0 & \text{if the gray value of the ith neighbor} \le \text{the gray value of p} \\ 1 & \text{otherwise} \end{cases}$$

- A histogram of the eight-bit number $b_1b_2b_3b_4b_5b_6b_7b_8$ is used to represent the texture of the image.

Regional Descriptors
Texture

- Gray level co-occurrence matrix (GLCM)
  - Texture computed using histograms carry no information regarding relative position of pixels with respect to each other.
  - Gray-level co-occurrence matrix considers distributions of intensities and positions of pixels with equal or nearly equal intensity values.
    - Let P be a position operator.
    - Let **A** be a k x k matrix whose element a_{ij} is the number of times, that points with gray level z_i occur (in the position specified by P) relative to points with gray levels z_j, with $1 \leq i, j \leq k$

Regional Descriptors
Texture

- GLCM example:
  - Three gray levels: $z_1=0$, $z_2=1$, $z_3=2$
  - Position vector *P* is "one pixel to the right"
  - Matrix **A** is then a 3 x 3 matrix given below:

0	0	0	1	2
1	1	0	1	1
2	2	1	0	0
1	1	0	2	0
0	0	1	0	1

4	4	1
4	3	1
1	1	1

  - Position vector *P* is "one pixel to the right and one pixel below", matrix **A**:

4	2	1
2	3	2
0	2	0

Regional Descriptors
Texture

- Let *n* be the total number of point pairs that satisfy *P*.
  - If matrix **C** is formed by dividing every element of **A** by *n*, then c_{ij} is an estimate of the joint probability that a pair of points satisfying *P* will have values (z_i, z_j).
  - Matrix **C** is called the gray-level co-occurrence matrix.
  - **C** can be analyzed to determine texture of region.

Regional Descriptors
Texture

- GLCM:
  - Maximum probability: $\max(c_{ij})$
  - Element difference moment of order *k*: $\sum_i \sum_j (i-j)^k c_{ij}$
  - Contrast (k=2): $\sum_i \sum_j (i-j)^2 c_{ij}$
  - Uniformity: $\sum_i \sum_j c^2{}_{ij}$
  - Entropy: $-\sum_i \sum_j c_{ij} \log_2 c_{ij}$
  - Homogeneity: $\sum_i \sum_j \frac{c_{ij}}{1+|i-j|}$

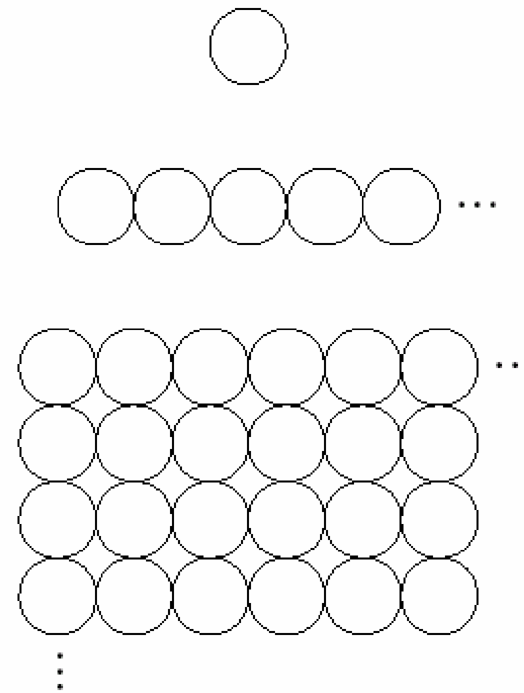
Regional Descriptors

Texture

- Structural approaches
  - Suppose that we have a rule of the form $S \rightarrow aS$, which indicates that the symbol S may be rewritten as aS.
  - If a represents a circle and the meaning of "circle to the right" is assigned to a string of the form *aaaa*…

a
b
c

FIGURE 11.23
(a) Texture primitive.
(b) Pattern generated by the rule $S \rightarrow aS$.
(c) 2-D texture pattern generated by this and other rules.



Regional Descriptors
Texture

- Spectral approaches
  - For non-random primitive spatial patterns, the 2-D Fourier transform allows the patterns to be analyzed in terms of spatial frequency components and direction.
  - It may be more useful to express the spectrum in terms of polar coordinates, which directly give direction as well as frequency.
  - Let $S(r,\theta)$ is the spectrum function, and r and θ are the variables in this coordinate system.
    - For each direction θ, $S(r,\theta)$ may be considered a 1-D function .
  - For each frequency r, $S_r(\theta)$ is a 1-D function.
  - A global description:

$$S(r) = \sum_{\theta=0}^{\pi} S_\theta(r) \qquad S(\theta) = \sum_{r=1}^{R_0} S_r(\theta)$$

Regional Descriptors
Quad-tree

- Region Quadtree:
  - a tree representation which represents recursive subdivisions of an image.

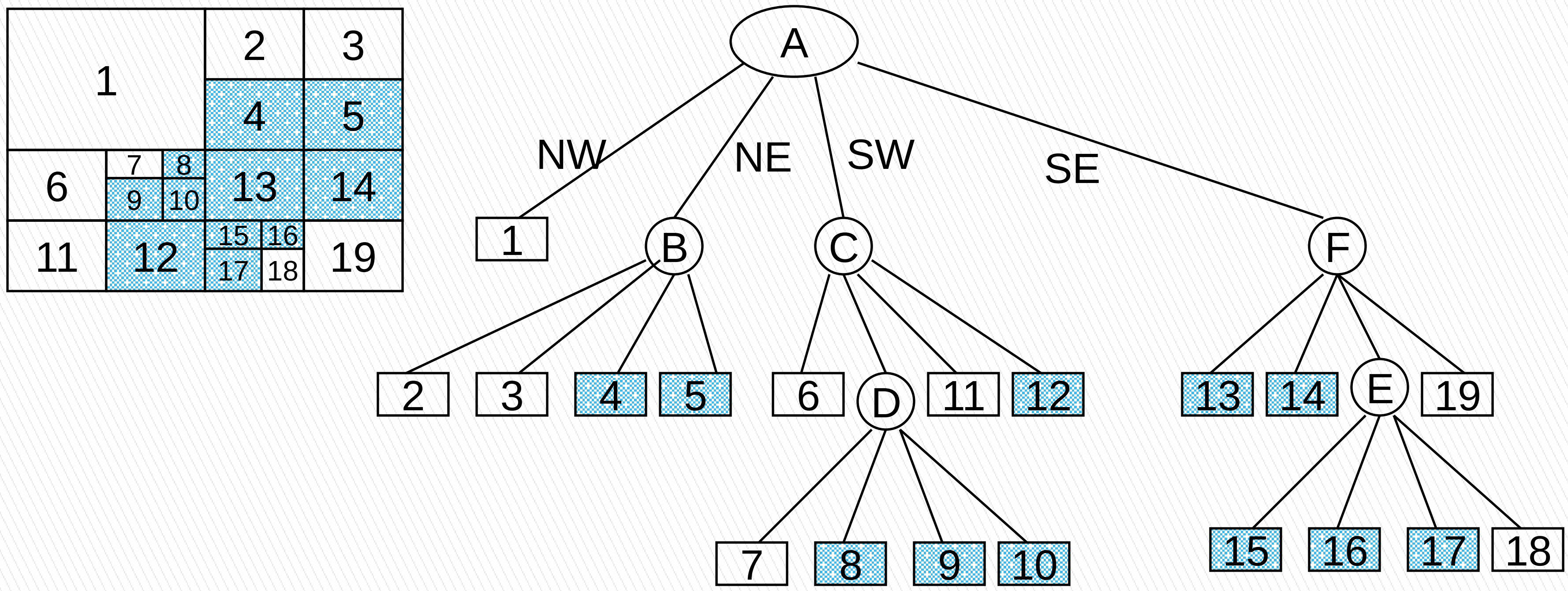
    – Whose blocks are required to be disjoint.

    – To have standard sizes (squares whose sides are power of two).

    – To be at standard locations.

    – Based on successive subdivision of image array into four equal size quadrants.

Regional Descriptors
Quad-tree



THE END