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Assgn # 5.

Q.2. i) $f(x^2+y^2)$ $\quad u = x^2+y^2$

$$\frac{f'(u)}{dx} = \cancel{2x + 2y\frac{dy}{dx}}$$

$$z = xy + f(x^2+y^2) \qquad x^2+y^2 = u$$

$$p = y + 2x f'(u)$$

$$q = x + 2y f'(u)$$

$$\Rightarrow \frac{p-xy}{2x} = \frac{q-x}{xy}$$

$$\Rightarrow py - xy^2 = qx - x^2$$

$$\Rightarrow py - qx + x^2 - y^2 = 0.$$

ii) $z = f\left(\frac{x}{y}\right)$

$$\Rightarrow p = \frac{1}{y} f'(u) , \quad q = \frac{-x}{y^2} f'(u)$$

$$py = -\frac{qy^2}{x}$$

$$pxy + qy^2 = 0.$$

iii) $u = x - z$, $v = y - z$

$du = \partial x - \partial z$, $dv = \partial y - \partial z$

$dF(u,v) = 0.$

$\partial x - \partial z = 0$, $\partial y - \partial z = 0.$

$$\frac{\partial z}{dx} = 1 , \quad \frac{\partial z}{\partial y} = 1$$

$$\frac{dz}{dx} = \quad p = q$$

4. (b)

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Q. 3 a) $z = (x+a)(y+b)$

$\cancel{p} z_x = (y+b)$

$q \quad z_y = (x+a)$

$z = pq$

b) $z = ax + by$

$p = a$

$q = b$

$z = px + qy$

c) $z^2(1+a^3) = 8(x+ay+b)^3$

$2zp(1+a^3) = 8 \times 3(x+ay+b)^2$

$2zq(1+a^3) = 8 \times 3(x+ay+b)^2(1+a)$

Q. 4 a) $x^2 + y^2 + (z-c)^2 - a^2 = 0$

$\cancel{2}x + \cancel{2}(z-c)p = 0$

$\cancel{2}y + \cancel{2}(z-c)q = 0$

$\frac{x}{p} = \frac{y}{q}$

$py - qx = 0.$

b) $(x^2+y^2)\cos^2\theta - (z-c)^2\sin^2\theta = 0$

Q.5.(a)

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Q.5. (a) $u(x,y) = f(v(x,y))$

~~u(x,y)~~ $\Rightarrow u_x = f_x(v)\, v_x$

(b) $z = u(x,y) = f(x-ay) + g(x+ay)$

$p = f'(x-ay) + g'(x-ay)$

$q = -a f'(x-ay) + a g'(x+ay)$

$p'q = f''(x-ay) + g''(x-ay)$

$q' = a^2 (f''(x-ay) + g''(x+ay))$

$q' = a^2 p'$

Q.6. (a) $xp + yq = z$

$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$

$\frac{dx}{x} = \frac{dy}{y} \Rightarrow \frac{x}{y} = c_1$

$\frac{x}{z} = c_2$

$F\left(\frac{x}{y}, \frac{x}{z}\right) = 0$

(b) $x^2p + y^2q = (x+y)z$

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{xz+yz} = \frac{\frac{dx}{x} + \frac{dy}{y} - \frac{dz}{z}}{\cancel{x} + y - \cancel{x} - \cancel{y}}$$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$\Longrightarrow$ $xyz = c_1$

$$-\frac{1}{x} = -\frac{1}{y} + c_2$$

$\Longrightarrow$ $\frac{1}{x} - \frac{1}{y} = c_2.$

(c) $$\frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{xy} = \frac{xdx + ydy - 2zdz}{xyz + xyz - 2xyz}$$

$\Rightarrow$ $xdx + ydy - 2zdz = 0.$

$\Rightarrow$ $\frac{x^2}{2} + \frac{y^2}{2} - 2\frac{z^2}{2} = c_1$

$\Longrightarrow$ $x^2 + y^2 - 2z^2 = c_1.$

$xdx = ydy$

$\Longrightarrow$ $x^2 - y^2 = c_2$

(d) $$\frac{dx}{z^2 - 2yz - y^2} = \frac{dy}{x(y+z)} = \frac{dz}{xy - xz}$$

$$= \frac{xdx + ydy + zdz}{\cancel{xz^2} - \cancel{2xyz} - \cancel{xy^2} + \cancel{xy^2} + \cancel{xyz} + \cancel{xyz} - \cancel{xz^2}} = 0$$

$\rightarrow$ $x^2 + y^2 + z^2 = 0.$

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$$\frac{dy}{x(z+y)z} = \frac{dz}{x(y-z)}$$

$\Rightarrow y\,dy - z\,dy = z\,dz + y\,dz$

$\Rightarrow \frac{y^2}{2} - \cancel{zy} = \frac{z^2}{2} - \cancel{yz} + c$

$\Rightarrow y^2 - z^2 = c_1$

Q. 7. $\frac{dx}{2} = \frac{dy}{3} = \frac{dz}{-8z}$

$3x - 2y = c_1$, $\cancel{4x - 3z = c_2}$ $ze^{4x} = c_2$

$F(3x-2y, ze^{4x}) = 0$

(a) $z = 1-3x$, $y = 0$,

$\cancel{x} = s$, $\cancel{z} = 1-3s$

$3s - 2\times 0 = c_1$, $(1-3s)e^{4s} = c_2$

$\Rightarrow 3s = 3x - 2y$

$s = \frac{3x-2y}{3}$

$1 - \frac{3x-2y}{\cancel{3}} \times \cancel{3} \quad e^{4\frac{3x-2y}{3}} = ze^{4x}$

$\Rightarrow (1-3x-2y)\,e^{-\frac{8y}{3}}$

(b) $z = x^2$, $2y = 1+3x$.

$x = s$, $z = s^2$, $y = (1+3s)/2$

$\cancel{3s} - 1 - \cancel{3s} = c_1 \Rightarrow c_1 = -1$

$s^2 e^{4s} = c_2$

No solⁿ.

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c) $x = s, \quad z = e^{-4s}, \quad y = \frac{3}{2}s$

$3x - 2y = c_1$

$3s - 3s = c_1$

$\Rightarrow c_1 = 0.$

$\rightarrow 3x = 2y$

$ze^{4x} = c_2$

$e^{-4s} e^{4s} = c_2$ no solution

$\Rightarrow c_2 = 1.$

Q. 8. (a) $\frac{dx}{2xy-1} = \frac{dy}{z-2x^2} = \frac{dz}{2x-2yz}$

$= \frac{z\,dx + dy + x\,dz}{2xyz - \cancel{z} + \cancel{z} - \cancel{2x^2} + \cancel{2x^2} - \cancel{2xyz}}$

$\Rightarrow zx + y + xz = c_1$

$\Rightarrow 2xz + y = c_1$

$\frac{dx}{2xy-1} = \frac{dz}{2x-2yz}$

$\Rightarrow 2x\,dx - 2yz\,dx = 2xy\,dz - dz$

$\Rightarrow x^2 - 2xyz = 2xyz - z$

$\Rightarrow x^2 - 4xyz + z = c_2$

$x_0 = 1, \quad y_0 = 0, \quad z_0 = s$

$1 + s = c_2$

$s = c_2 - 1$

$\cancel{x^2}$ $2s = c_1$

$s = \frac{c_1}{2}$

$2c_2 - 2 = c_1$

(b) $\frac{dx}{x^3} = \frac{dy}{3x^2y + y^2} = \frac{dz}{2x^2z + yz} = \frac{\frac{dx}{x} - \frac{dy}{y} + \frac{dz}{z}}{x^2 - \cancel{3x^2} - \cancel{y} + \cancel{2x^2} + \cancel{y}}$

$\frac{xz}{y} = c_1$

$\frac{dx}{x^3} = \frac{dy}{(3x^2+y)y}$

$3x^2y\,dx + y^2\,dx = dy\,x^3$

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Assgn #6

Q.1 a) $y u_{xx} - x u_{yy} = 0$

$A = y, \quad B = 0, \quad C = x.$

$B^2 - 4AC = -4xy$

b) $u_{yy} - x u_{xy} + \dots = 0$

$A = 0, \quad B = -x, \quad C = 1$

$B^2 - 4AC = x^2 - 4$

Q.2. a) $u_{xx} - x^2 y u_{yy} = 0.$

$A = 1, \quad B = 0, \quad C = -x^2 y$

$B^2 - 4AC = 4x^2 y > 0 \Rightarrow$ hyperbolic

$\alpha^2 - x^2 y = 0.$

$\alpha = \pm x\sqrt{y}$

Choose ξ & η s.t

$\xi_x = \lambda_1 \xi_y, \quad \eta_x = \lambda_2 \eta_y$

$\frac{dy}{dx} + \lambda_1 = 0, \quad \frac{dy}{dx} + \lambda_2 = 0.$

$\frac{dy}{dx} + x = 0 \qquad \frac{dy}{dx} - x = 0$

$f_1: \; y + \frac{x^2}{2} = c_1 \qquad f_2: \; y - \frac{x^2}{2} = c_2$

$\xi = y + \frac{x^2}{2}$

$\eta = y - \frac{x^2}{2}$

$u = u$

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(b) $e^{2x}u_{xx} + 2e^{x+y}u_{xy} + e^{2y}u_{yy}$

$A = e^{2x}$, $B = 2e^{x+y}$, $C = e^{2y}$

$B^2 - 4AC = 4e^{2(x+y)} - 4e^{2(x+y)} = 0$ ∴ parabolic.

$(e^{2x}\alpha + e^{2y})^2 = 0$

$\alpha = -e^{2(y-x)}$

$\lambda = -e^{2(y-x)}$

$\cancel{\frac{\partial y}{\partial x} - e^{2(y-x)} = 0}$

Choose $\xi(x,y)$ s.t. $\frac{\partial \xi}{\partial x} = \lambda \frac{\partial \xi}{\partial y}$

s.t. $\xi_x + \xi_y = 0$

$\frac{dy}{dx} + \lambda = 0$

$\Rightarrow \frac{dy}{dx} - \frac{e^{2y}}{e^{2x}} = 0$

$\Rightarrow e^{-2y}dy = e^{-2x}dx$

$\Rightarrow e^{-2y} - e^{-2x} = c_1 = \xi$

Let $\eta = e^{-2y} + e^{-2x} = c_2$

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