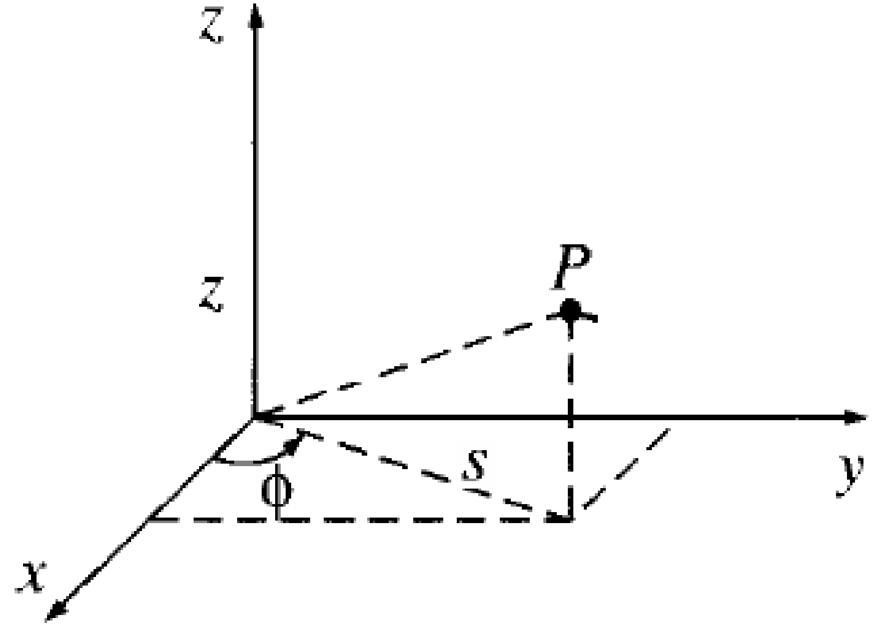
Cylindrical and spherical co-ordinate system

Cylindrical Coordinates



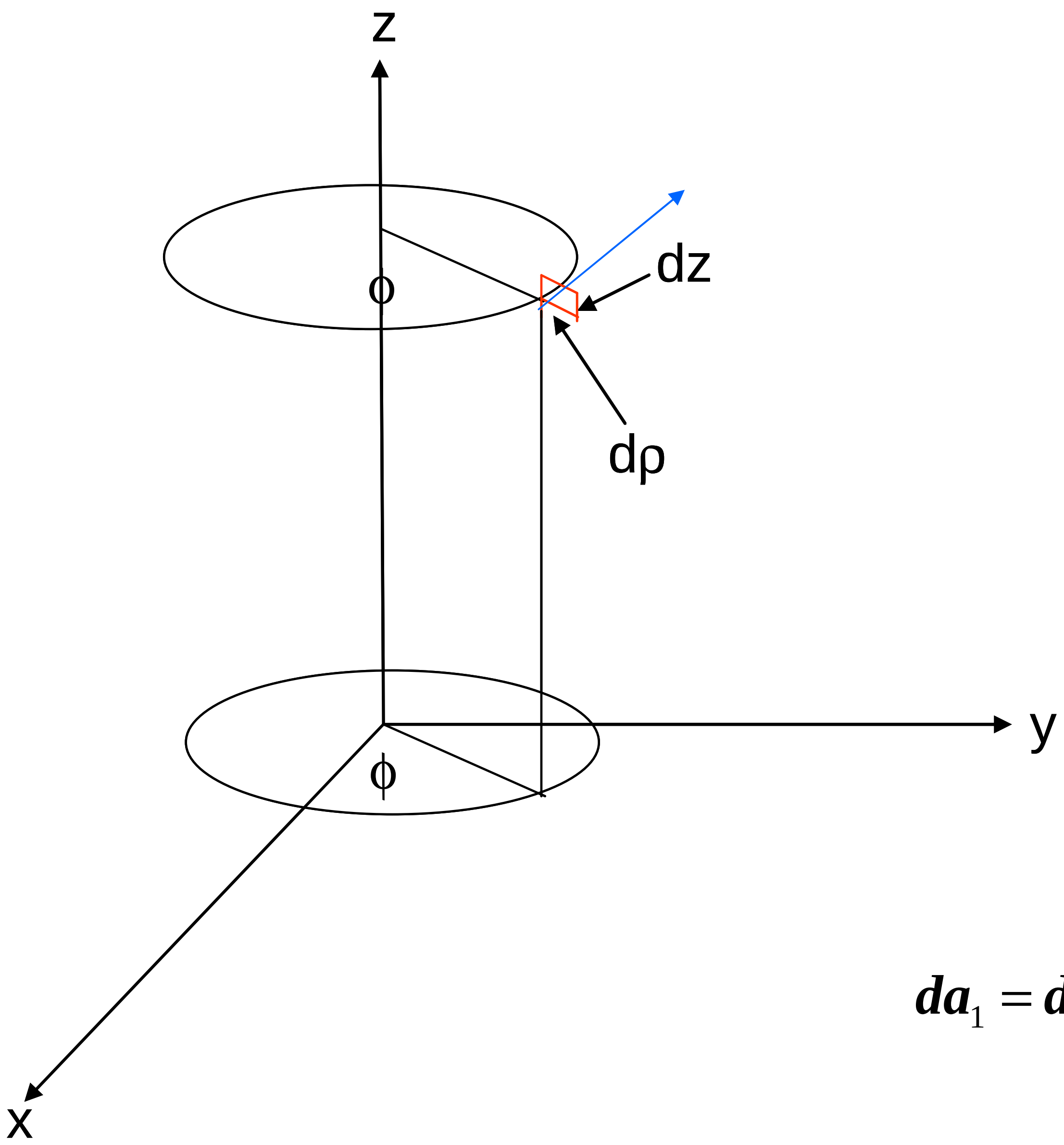
$$x = s\cos\phi, \qquad y = s\sin\phi, \qquad z = z$$

$$\vec{A} = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}$$

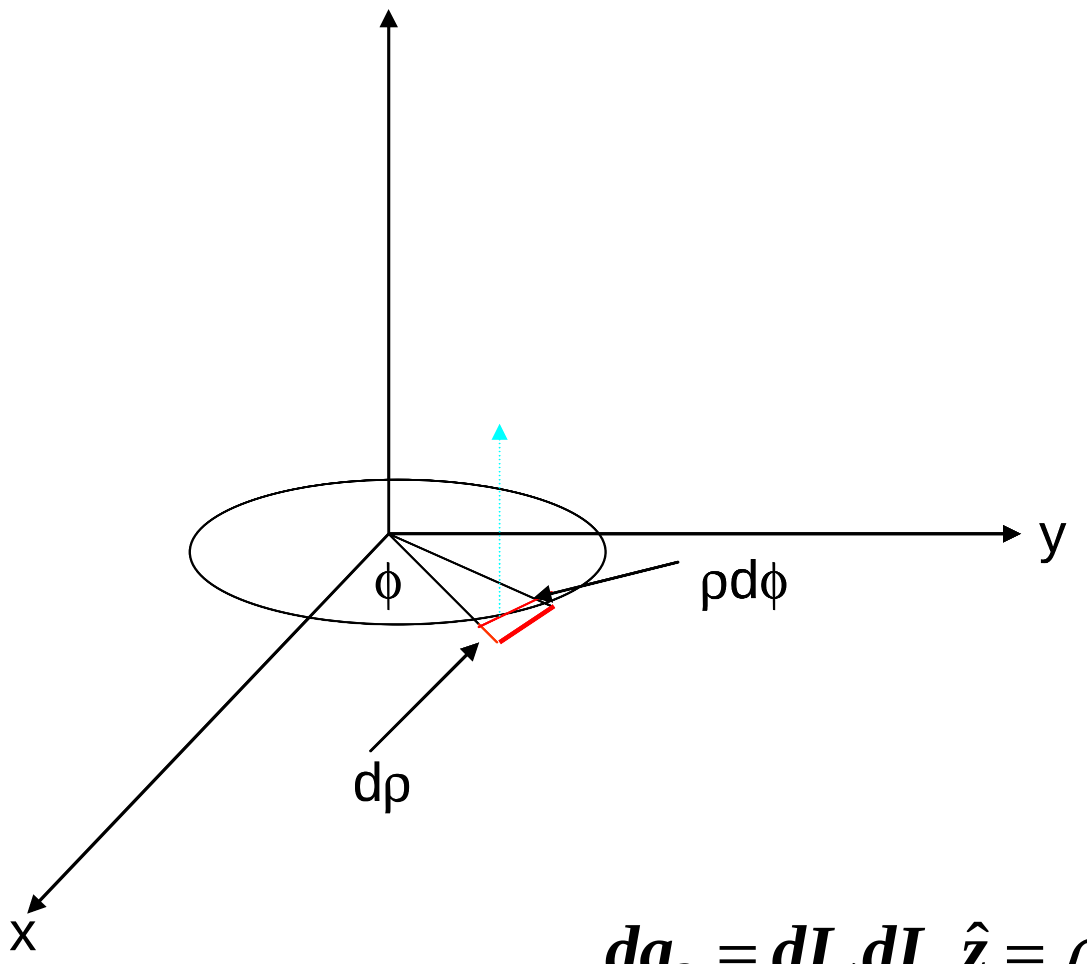
$$\vec{A} = a_\rho \hat{\rho} + a_\phi \hat{\phi} + a_z \hat{z}$$

$$\begin{aligned} a_x &= a_\rho \cos\phi - a_\phi \sin\phi \\ a_y &= a_\rho \sin\phi + a_\phi \cos\phi \\ a_z &= a_z \end{aligned} \qquad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\varphi \\ a_z \end{bmatrix}$$

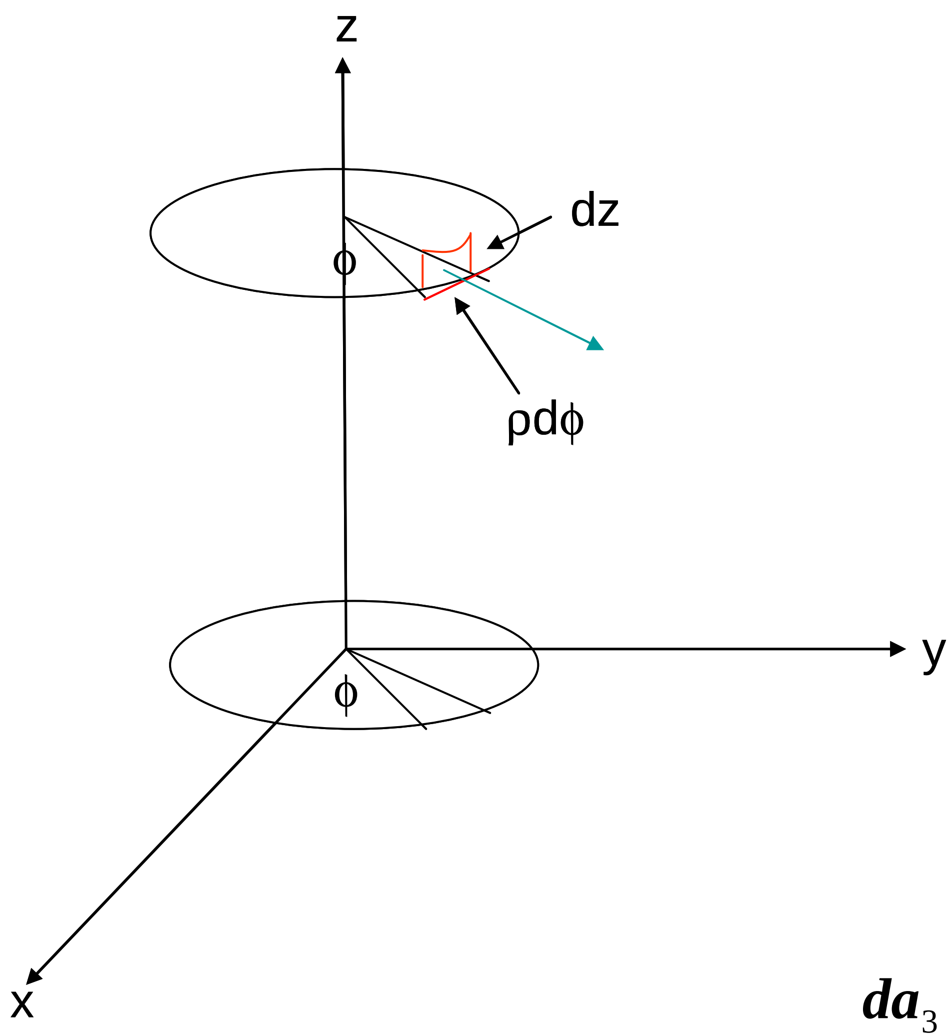
$$\begin{aligned} a_\rho &= a_x \cos\phi + a_y \sin\phi \\ a_\phi &= -a_x \sin\phi + a_y \cos\phi \\ a_z &= a_z \end{aligned} \qquad \begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$



$$da_1 = dI_z dI_\rho \hat{\phi} = dz d\rho \hat{\phi}$$

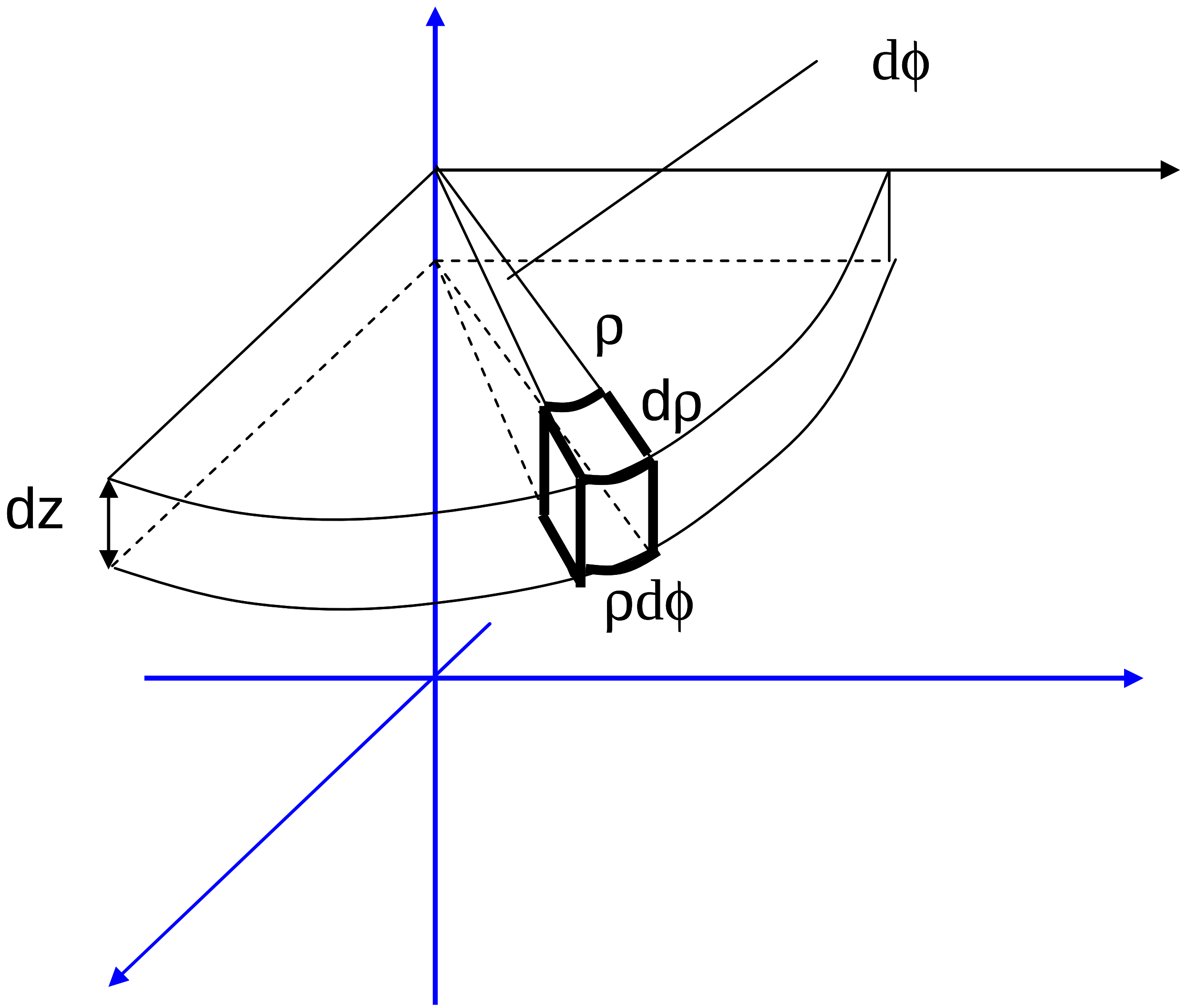


$$da_2 = dI_\phi dI_\rho \hat{z} = \rho d\phi d\rho \hat{z}$$



$$da_3 = dI_z dI_\phi \hat{\rho} = \rho d\phi dz \hat{\rho}$$

Volume element in cylindrical coordinate system



$$dl_s = ds, \qquad dl_\phi = s\,d\phi, \qquad dl_z = dz$$

$$d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$$

The infinitesimal volume element $d\tau$

$$d\tau = s\,ds\,d\phi\,dz$$

The infinitesimal surface elements

$$d\vec{a}_1 = dI_z\hat{z} \times dI_\rho\hat{\rho} = dzd\rho\hat{\phi}$$

$$d\vec{a}_2 = dI_\rho\hat{\rho} \times dI_\phi\hat{\phi} = \rho d\phi d\rho\hat{z}$$

$$d\vec{a}_3 = dI_z\hat{z} \times dI_\phi\hat{\phi} = \rho d\phi dz\hat{\rho}$$

$$\nabla T = \frac{\partial T}{\partial s}\,\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial \phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\,\hat{\mathbf{z}}.$$

$$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

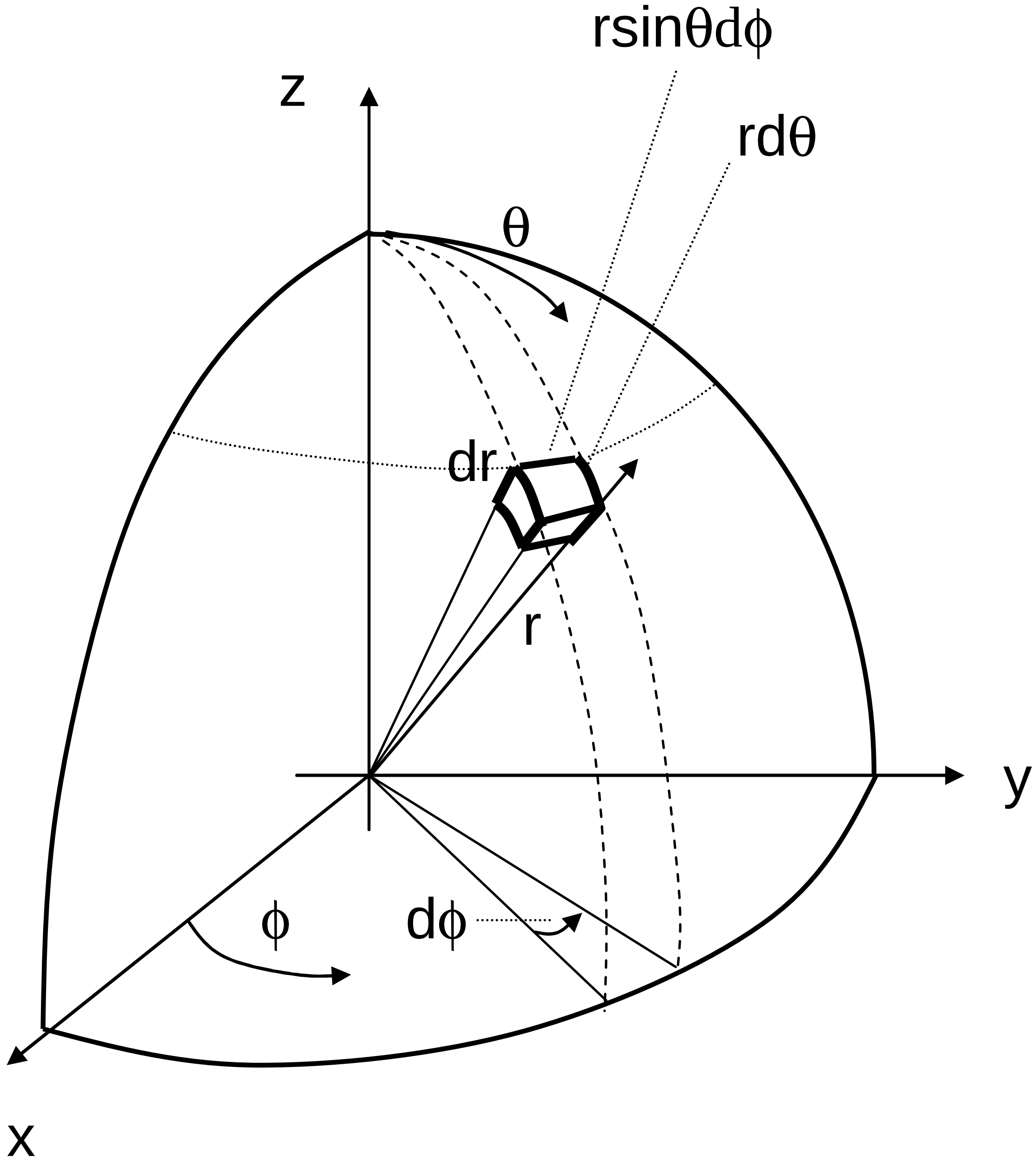
$$\vec{A} = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}$$

$$\vec{A} = a_r \hat{r} + a_\theta \hat{\theta} + a_\phi \hat{\phi}$$

$$a_x = a_r \sin\theta\cos\phi + a_\theta \cos\theta\cos\phi - a_\phi \sin\phi$$
$$a_y = a_r \sin\theta\sin\phi + a_\theta \cos\theta\sin\phi + a_\phi \cos\phi$$
$$a_z = a_r \cos\theta - a_\theta \sin\theta$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix}$$

$$\begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$


rsinθdφ
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r
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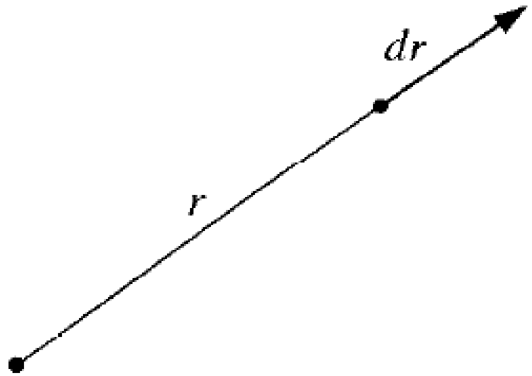
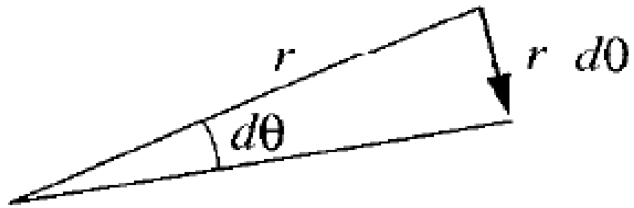
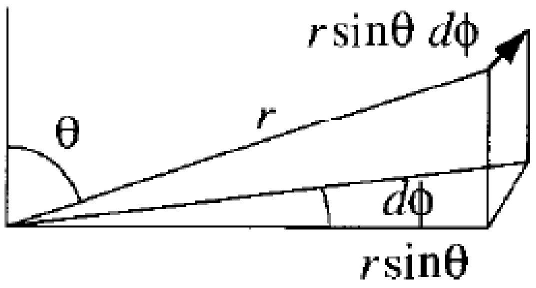
x



$$dl_r = dr$$



$$dl_\theta = r\,d\theta.$$



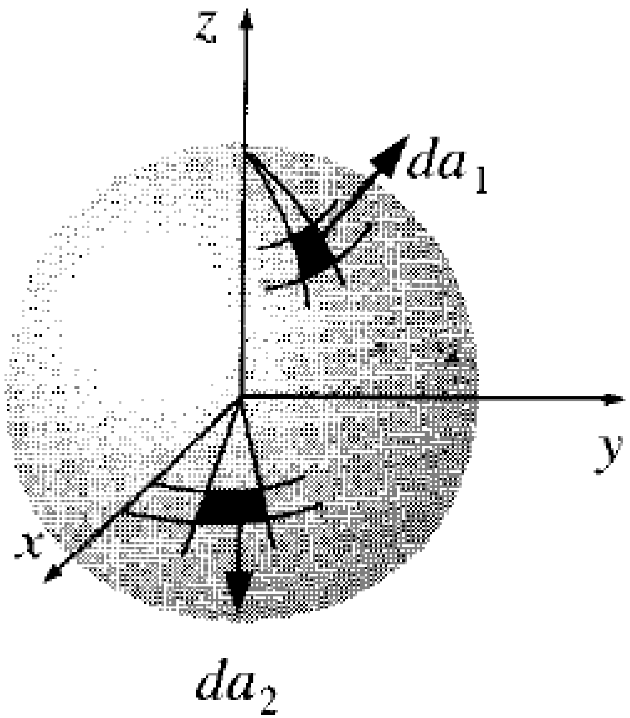
$$dl_\phi = r\sin\theta\,d\phi$$

$$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$

The infinitesimal volume element $d\tau$

$$d\tau = dl_r\, dl_\theta\, dl_\phi = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

The infinitesimal surface elements



$$d\mathbf{a}_1 = dl_\theta\, dl_\phi\, \hat{\mathbf{r}} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{\mathbf{r}}$$

$$d\mathbf{a}_2 = dl_r\, dl_\phi\, \hat{\boldsymbol{\theta}} = r\, dr\, d\phi\, \hat{\boldsymbol{\theta}}$$

$$da_3 = dI_r dI_\theta \hat{\phi} = rdrd\theta\hat{\phi}$$

$$\nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$\begin{aligned}\nabla \times \mathbf{v} \quad = \quad & \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} \\ + \quad & \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}.\end{aligned}$$

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}$$