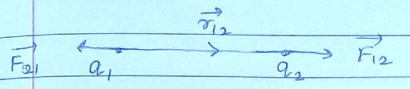
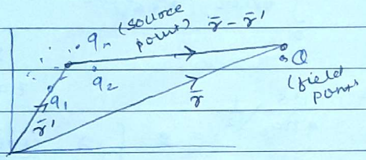
ELECTROSTATICS.

Coulomb's law :-



$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\,\hat{r}_{12}$$

→ Superposition Principle : True for certain cases
(Not true for dielectric medium)



$$F_Q = \bar{F}_{q_1 Q} + \bar{F}_{q_2 Q} + - - + \bar{F}_{q_n Q}$$

$$\bar{F}_Q(r) = \frac{q_1 Q}{4\pi\epsilon_0 |\bar{r} - \bar{r}_1'|^3}(\bar{r} - \bar{r}_1') + \frac{q_2 Q\,(\bar{r} - \bar{r}_2')}{4\pi\epsilon_0 |\bar{r} - \bar{r}_2'|^3} \dots$$

$$\boxed{\bar{F}(r) = \frac{Q}{4\pi\epsilon_0} \sum_{i=0}^{n} \frac{q_i (\bar{r} - \bar{r}_i')}{|\bar{r} - \bar{r}_i'|^3}}$$

$$\bar{F}(r) = Q\,\bar{E}(r)$$

⇒ For continuous charges :-

$$\bar{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2}\,\hat{r}$$

1.) λ, $\vec{dl}'$

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda\, dl'\, (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$$

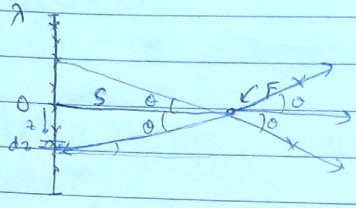
2.) Surface charge Density (σ) :

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \iint \frac{\sigma\, da'\, (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$$

* E is always in terms of r, not r'.

3.) Volume charge : $\bar{E}(r) = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho \, dz' \, (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$

Ques



1.) $d\bar{E} = \frac{dz \, \lambda \cos\theta}{4\pi\epsilon_0 (z^2+s^2)} = 2\int_0^L \frac{dz \, \lambda \, s}{4\pi\epsilon_0 (z^2+s^2)^{3/2}}$

$z^2+s^2 = t^2$

$2dz = 2t\,dt$

$z = s\tan\theta$

$= 2\int_0^L \frac{\lambda s \, dt}{4\pi\epsilon_0 t^3} = \frac{\lambda s}{4\pi\epsilon_0} \frac{1}{t^2} = 2\int \frac{\lambda}{s \, 4\pi\epsilon_0} \frac{\cos\theta \, d\theta}{\sec^2\theta}$

$E = \frac{2\lambda L}{4\pi\epsilon_0 s\sqrt{s^2+L^2}}$

2.) $\bar{r} = s\hat{s}$ $\quad \bar{r}' = z\hat{z}$

$\bar{r} - \bar{r}' = s\hat{s} - z\hat{z}$

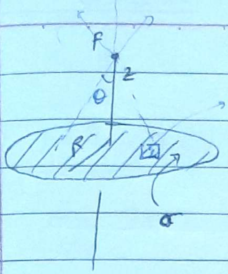
$|\bar{r} - \bar{r}'| = \sqrt{s^2+z^2}$

substitute in formula

$\int d\bar{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \, dz \, (s\hat{s} - z\hat{z})}{(s^2+z^2)^{3/2}}$

$= \frac{\lambda}{4\pi\epsilon_0} \left[\int_{-L/2}^{L/2} \frac{dz \, s\hat{s}}{(s^2+z^2)^{3/2}} - \cancel{\int_{-L}^{+L} \frac{z \, dz \, \hat{z}}{(s^2+z^2)^{3/2}}}^{\,0} \right]$

Ans



$da' = s\,ds\,d\phi$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$$2\int dE \cos\theta = \frac{2}{4\pi\epsilon_0}\int\int_{\phi=0}^{\pi} \frac{\sigma\, s\, ds\, d\phi \cos\theta}{(s^2+z^2)}$$

$s = z\tan\theta$

$ds = z\sec^2\theta\, d\theta$

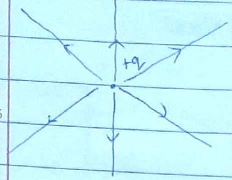
$$= \frac{\sigma}{2\epsilon_0}\int \frac{(z\tan\theta)(z\sec^2\theta\, d\theta)\cos\theta}{z^2\sec^2\theta}$$

$$= \frac{\sigma}{2\epsilon_0}\int \frac{\sin\theta}{\cancel{\cos\theta}}\, d\theta$$

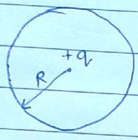
$$= -\frac{\sigma}{2\epsilon_0}\Big[\cos\theta\Big]_0^{\cos^{-1}\left(\frac{z}{\sqrt{R^2+z^2}}\right)}$$

$$\boxed{\bar{E} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]}$$

Field line :



Flux :



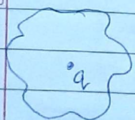
$$\phi = \oiint \bar{E} \cdot d\bar{a}$$

$$= \oiint \left(\frac{q}{4\pi\epsilon_0 r^2} \hat{r} \right) \cdot \left(r^2 \sin\theta \, d\theta \, d\phi \, \hat{r} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \, d\phi$$

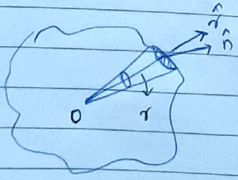
$$= \frac{q}{4\pi\epsilon_0} (2\pi)(2) = \boxed{\frac{q}{\epsilon_0}}$$

$$\boxed{\phi = \oiint \bar{E} \cdot d\bar{a} = \frac{q}{\epsilon_0}}$$: Gauss's law



Here also, $\phi = \frac{q}{\epsilon_0}$

Proof :-



$$d\Omega = \frac{\hat{n} \cdot \hat{r} \, da}{r^2}$$ (solid angle)

Total solid angle $= 4\pi$

$$\oiint d\Omega = 4\pi$$

In Gauss's law,

$$\oiint \frac{q}{4\pi\epsilon_0} \underbrace{\left(\frac{\hat{r}}{r^2} \cdot \hat{n} \, da \right)}_{d\Omega} = \frac{4\pi}{4\pi} \frac{q}{\epsilon_0} = \frac{q}{\epsilon_0}.$$: (Independent of shape of closed surface)

→ $Q_1 \quad Q_2 \quad Q_3$

$q_1 \quad q_2 \quad q_3 \quad q_4$ → I

$$E_I = \frac{q_1 + q_2 + q_3 + q_4}{\epsilon_0}$$

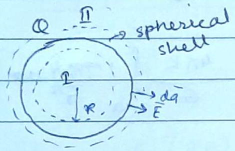
$Q_4 \quad Q_5$

* Charges which are outwards don't affect flux (all going inside also comes out, net flux is zero)

* This formula is not mostly used

Bcoz of formula, $\bar{E} \parallel d\bar{a}$ & $\bar{E}$ should be uniform everywhere on the surface.

(i.)



~~$\Phi_I = 0 \Rightarrow \bar{E} \cdot d\bar{a} = Q$~~

~~$E_{\varnothing} \quad \Rightarrow (\hat{E})($~~

(I) : $\oiint \bar{E}_1 \cdot d\bar{a} = 0$

$$\oiint E_I \, da = 0$$

$$E_1 \oiint da = 0$$

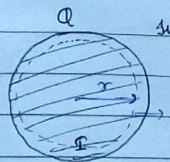
$$E_1 (4\pi R^2) = 0 \Rightarrow \boxed{E_1 = 0}$$

(II) : $\oiint \bar{E}_2 \cdot d\bar{a} = \frac{Q}{\epsilon_0}$

$$E_2 (4\pi R^2) = Q/\epsilon_0$$

$$\boxed{E_2 = \frac{Q}{4\pi\epsilon_0 R^2} \hat{R}}$$

(ii) Insulated solid sphere



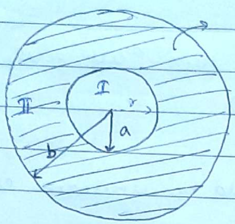
(I) : $\oiint \bar{E}_I \cdot d\bar{a} = \frac{Q \times \frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3 \epsilon_0} = \frac{Q r^3}{\epsilon_0 R^3}$

$$\bar{E}_I (4\pi r^2) = \frac{Q r^3 / R^3}{\epsilon_0}$$

$$\boxed{\bar{E}_1 = \frac{Q r}{4\pi\epsilon_0 R^3} \hat{r}}$$

$$\boxed{\bar{E}_2 = \frac{Q}{4\pi\epsilon_0 R^2}}$$

(iii)



$\rho = \frac{K}{r^2} \quad a \le r \le b$

$\int \rho$ (we can use $4\pi r^2 dr$ if ρ is not a func^n of θ & ϕ)

$$\oint \bar{E}_I \cdot d\bar{a} = 0 \Rightarrow \bar{E}_I (4\pi r^2) = 0 \Rightarrow \bar{E}_I = 0$$

$$\oint \bar{E}_{II} \cdot d\bar{a} = \int \frac{\rho \cdot 4\pi (r^2) \, dr}{\epsilon_0} = \frac{4\pi K}{\epsilon_0} \int_{r=a}^{r} 1 \, dr$$

$$\bar{E}_{II} (4\pi r^2) = \frac{4\pi K}{\epsilon_0} (r-a)$$

$$\boxed{E_{II} = \frac{K}{\epsilon_0} \frac{(r-a)}{r^2}}$$

~~$\bar{E}_{III} (4\pi r^2) =$~~

$$E_{II} (4\pi r^2) = \iiint_{r=a}^{r} \frac{K}{r^2} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

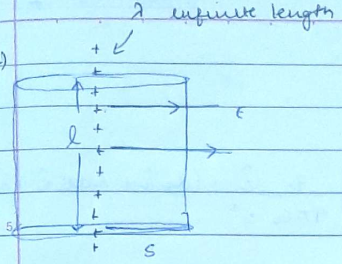
$$= \frac{K (4\pi)(r-a)}{\epsilon_0}$$

$$\bar{E}_{III} (4\pi r^2) = \iiint_{r=b}^{a} \rho \, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$\boxed{E_{III} = \frac{K(b-a)}{\epsilon_0 r^2}}$$

Quiz : 29^{th} March

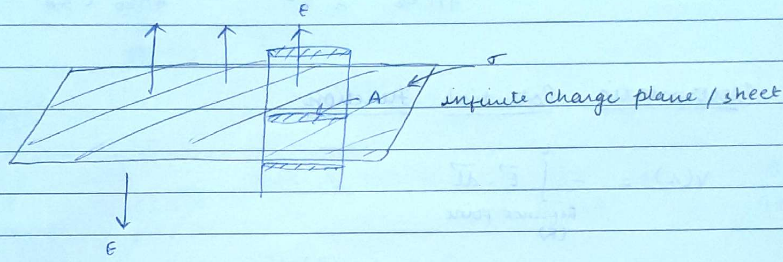
(iv.)



$$(E)(2\pi s l) = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi s \epsilon_0}\hat{s}}$$

(v.)



$$E(2A) = \frac{\sigma A}{\epsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}\hat{n}}$$

⇒

$$\oiint \bar{E}\cdot d\bar{a} = \frac{q}{\epsilon_0}$$

Using divergence theorem

$$\iiint (\bar{\nabla}\cdot\bar{E})\, d\tau = \frac{\iiint \rho(r)\, d\tau}{\epsilon_0}$$

$$\boxed{\bar{\nabla}\cdot\bar{E}(r) = \frac{\rho(r)}{\epsilon_0}}$$ E(r) : +ive 4 +ive charge

→

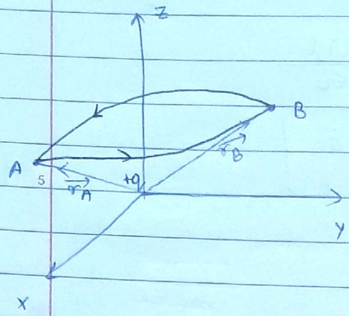
+q → E here ≠ 0 ($\rho(r) \neq 0$)

→ $\bar{\nabla}\cdot\bar{E}$ here would be 0 ($\rho(r) = 0$)

but E won't be 0.

Any conservative vector field can be written as gradient of a scalar func^n.

Date 24 / 03 / 17



$$\oint \bar{E}\cdot d\bar{l}$$

$$\int_A^B \bar{E}\cdot d\bar{l} = \int_A^B \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\cdot(dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi})$$

$$= \frac{q}{4\pi\epsilon_0}\int_A^B \frac{1}{r^2}\,dr = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

<u>Electrostatic Potential Function</u>

$$V(r) = -\int_{\text{Refrence point (R)}}^{r} \vec{E}\cdot\vec{dl}$$

$$V(A) = -\int_R^A \vec{E}\cdot\vec{dr}$$

$$V(B) = -\int_R^B \vec{E}\cdot\vec{dr}$$

$$V(B) - V(A) = -\int_A^B \vec{E}\cdot\vec{dr}$$

$$\int_A^B \nabla V\cdot\vec{dr} = V(B) - V(A)$$

$$\boxed{\vec{E} = -\vec{\nabla} V}$$

scalar function

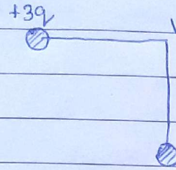
$$V(r) = -\int_R^r \bar{E}\cdot d\bar{l}$$

$R \to R'$

$$V(r') = -\int_{R'}^{r} \bar{E}\cdot d\bar{l} = -\int_R^r \bar{E}\cdot d\bar{l} + \int_{R'}^{R} \bar{E}\cdot d\bar{l} = V(r) + C$$

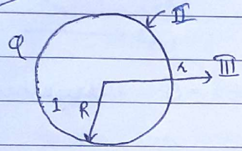
$$= \int_R^r \vec{F}_{me} \cdot \vec{dl} = -\int_R^r \vec{F}_{elec} \cdot \vec{dl}$$

$$= -\int_R^r q_2 \vec{E}\, \vec{dl} = q_2 \left\{ -\int_R^r \vec{E} \cdot \vec{dl} \right\} = \underline{q_2 V(r)}$$



$V(3q) + V(-5q)$

Hollow spheree

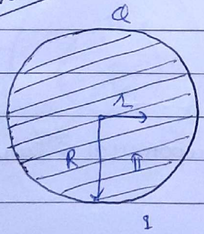


VKIN $V_{III}(r) = -\int_\infty^r \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0 r}$

$V_{II}(R) = \frac{Q}{4\pi\epsilon_0 R}$

$$V_I(r) = -\int_\infty^r \bar{E} \cdot d\bar{r} = -\int_\infty^R \bar{E} \cdot dr - \int_R^r \cancel{\bar{E}\, dr} = \frac{Q}{4\pi\epsilon_0 R}$$

solid sphere.



$dq = \frac{Q}{\frac{4}{3}\pi R^3} \times 4\pi r^2 dr = \frac{3Qr^2 dr}{R^3}$

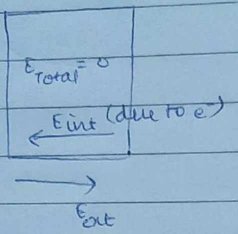
$E = \frac{dQ}{4\pi\epsilon_0}$

$V_I = \frac{Q}{4\pi\epsilon_0 R}$

$V_{II} = \int \vec{E} \cdot \vec{dl} \qquad = \frac{kQ}{2R^3}\left[3R^2 - r^2\right]$

Conductors

Properties :

(i) 

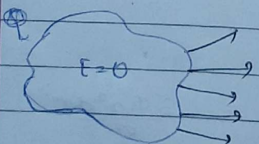
If conductor is kept in uniform external E, e^- adjust in such a way that $\bar{E}_{net}$ inside

(ii) We know that

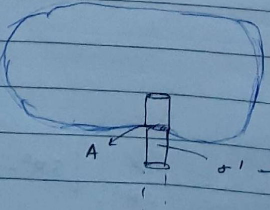
$$\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon_0}$$

if $\bar{E} = 0 \Rightarrow \rho = 0 \Rightarrow$ charge density (net) $= 0$ everywhere

(iii) If we've some excess charge, it will reside on surface. It will be distributed in such a way that $E = 0$ inside (max at sharper edges)



(iv) Everywhere, potential difference is zero. (Equipotential surface)
→ E is always $\perp$ outside. ($E_{\parallel} = 0$)

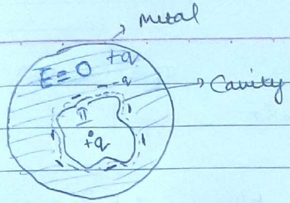
(v) 

$$EA = \frac{\sigma' A}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\sigma'}{\epsilon_0}\hat{n}}$$

σ' → local surface charge density.

electric field close to the surface.

* Electrostatic field is conservative field



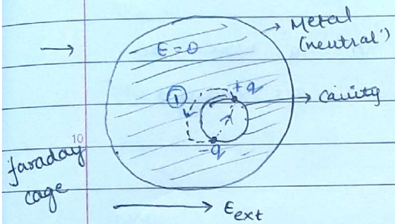
→ Gaussian surface

$\oiint \bar{E}\cdot d\bar{a} = \frac{q_{encl}}{\epsilon_0}$

↓ E = 0 in conductors

$0 = +q + (-q) \Rightarrow$ -q will be induced ~~on inner surface~~

⇒ +q will be induced on outer metal surface.



→ Let cavity induce some charge ($q_{net} = 0$ as $E = 0$)

Gaussian says that :

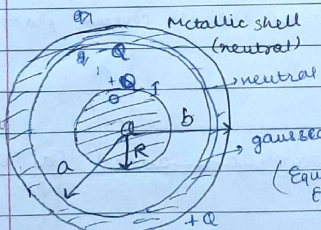
$0 \neq \oint \bar{E}\cdot d\bar{l} = 0$

We can take a path like ①

inside the cavity. $\oint \bar{E}\cdot d\bar{l} \neq 0$ ($\exists$ $\bar{E}$ due to +q, -q)

Hence, the cavity has no affect of external electrostatic field.

Eg.



Find surface charge density on different surfaces. Also find potential at the centre. $\sigma_R = ?$ $\sigma_a = ?$ $\sigma_b = ?$

(Equipotential)

$E_I = 0 \Rightarrow \frac{KQ}{R} = \frac{-Kq}{R} - \frac{Kq}{a}$ $\qquad \frac{2KQ}{R} = \frac{Kq}{a}$

$q = -2Qa/R$

$\sigma_R = \frac{Q}{4\pi R^2}$ $\qquad \sigma_a = \frac{-Q}{4\pi a^2}$ $\qquad \sigma_b = \frac{Q}{4\pi b^2}$

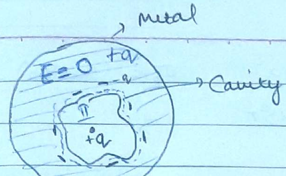
Potential at centre : $-\int_{\infty}^{0} \bar{E}\cdot d\bar{l} = -\int_{\infty}^{b} \bar{E}\cdot d\bar{l} - \int_{b}^{a} \cancel{\bar{E}}\cdot d\bar{l} - \int_{a}^{R} \cancel{\bar{E}}\cdot d\bar{l} - \underbrace{\int_{R}^{} \bar{E}\cdot d\bar{l}}_{0}$

$= \cancel{\frac{KQ}{2R^3}[3R^2] = \frac{3Q}{2\cdot 4\pi\epsilon_0 R}}$

$= -\int_{\infty}^{b} \bar{E}\cdot d\bar{l} = -\int_{\infty}^{b} \frac{KQ\cdot dr}{r^2} = \frac{KQ}{b}$

$-\int^{R} \bar{E}\cdot d\bar{l} = -\int^{R} \frac{KQ}{r^2}dr = \frac{-KQ}{a} + \frac{KQ}{R}$

* Electrostatic field is conservative field

→ 

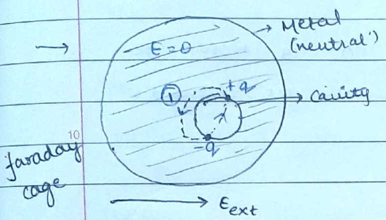
Gaussian surface:

$\oiint \bar{E} \cdot d\bar{a} = \frac{q_{encl}}{\epsilon_0}$

$\downarrow E=0$ in conductors

$0 = +q + (-q) \Rightarrow -q$ will be induced on inner surface

$\Rightarrow$ +q will be induced on outer metal surface.

→ 

Let cavity induce some charge
($q_{net} = 0$ as $E = 0$)

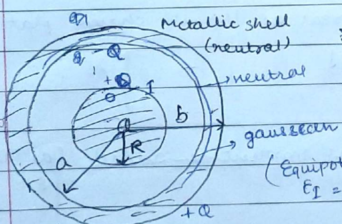
Gaussian says that:

$0 \neq \oint \bar{E} \cdot d\bar{l} = 0$

We can take a path like 1

inside the cavity, $\oint \bar{E} \cdot d\bar{l} \neq 0$ ($\exists$ $\bar{E}$ due to +q, -q)

Hence, the cavity has no affect of external electrostatic field.

Eg. 

Find surface charge density on different surfaces. Also find potential at the centre. $\sigma_R = ?$ $\sigma_a = ?$ $\sigma_b = ?$

$E_I = 0 \Rightarrow \frac{KQ}{R} = \frac{-Kq}{R} - \frac{Kq}{a}$ $\quad \frac{2KQ}{R} = \frac{Kq}{a}$

$q = -2aQ/R$

$\sigma_R = \frac{Q}{4\pi R^2}$ $\quad \sigma_a = \frac{-Q}{4\pi a^2}$ $\quad \sigma_b = \frac{Q}{4\pi b^2}$

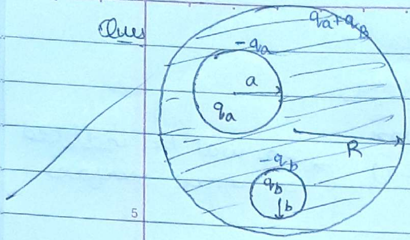
Potential at centre: $-\int_{\infty}^{0} \bar{E} \cdot d\bar{l} = -\int_{\infty}^{b} \bar{E} \cdot d\bar{l} - \int_{b}^{a} \cancel{\bar{E}} \cdot d\bar{l} - \int_{a}^{R} \bar{E} \cdot d\bar{l} - \underbrace{\int_{R}^{0} \bar{E} \cdot d\bar{l}}_{0}$

$= \cancel{\frac{KQ}{2R^3}[3R^2] = \frac{3Q}{2 \cdot 4\pi\epsilon_0 R}}$

$= -\int_{\infty}^{b} \bar{E} \cdot d\bar{l} = -\int_{\infty}^{b} \frac{KQ}{r^2} \cdot dr = \frac{KQ}{b}$

$-\int_{a}^{R} \bar{E} \cdot d\bar{l} = -\int_{a}^{R} \frac{KQ}{r^2} dr = \frac{-KQ}{a} + \frac{KQ}{R}$

Ques



$\sigma_a = -q_a/4\pi a^2$ $\qquad \sigma_R = (q_a+q_b)/4\pi R^2$

$\sigma_b = -q_b/4b^2\pi$

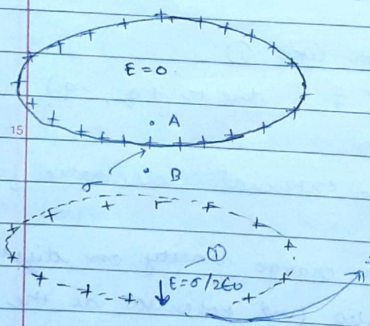
What is force b/w a & b?

0 (a & b are isolated as E b/w them is 0)

Q(c)

If bring $+Q_c$ near the conductor, $q_a + q_B$ won't be distributed uniformely, hence, σ_R will get changed.

Electrostatic Pressure



A and B are very close to surface

$\vec{E}_B = \dfrac{\sigma}{\epsilon_0}\hat{n}$ $\qquad \vec{E}_A = 0$

$E_{above} = \sigma/2\epsilon_0 \uparrow$

(treat as infinite charge plate)

$E_{below} = \sigma/2\epsilon_0 \downarrow$

① Here $E = \dfrac{\sigma}{2\epsilon_0} \downarrow$ such that when that segment is added (as in previous figure), the $E = 0$.

$$\boxed{|\bar{E}|\sigma = \frac{\sigma^2}{2\epsilon_0}}$$: Electrostatic pressure

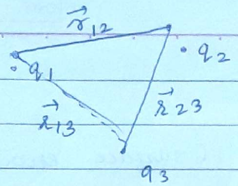
Electric field × charge per unit area = ~~fro~~ force per unit are = pressure

28-03-17

Electrostatic Potential Energy of a charged system (U)

→ The amount of work done to assemble charges.

→ Discrete charge :-



$$W_1 = 0$$

$$W_2 = \left(\frac{q_1}{4\pi\epsilon_0 r_{12}}\right) q_2$$

$$W_3 = q_3 \left[\frac{q_1}{4\pi\epsilon_0 r_{13}} + \frac{q_2}{4\pi\epsilon_0 r_{23}}\right]$$

The general expression will be :

$$W_n = q_n \left[\frac{q_1}{4\pi\epsilon_0 r_{1n}} + \frac{q_2}{4\pi\epsilon_0 r_{2n}} \dots + \frac{q_{n-1}}{4\pi\epsilon_0 r_{n-1\,n}}\right]$$

Now, U = sum of all work done

$$\Rightarrow \quad U = \sum_{i=1}^{N} q_i \sum_{j>i} \frac{q_j}{4\pi\epsilon_0 r_{ij}}$$

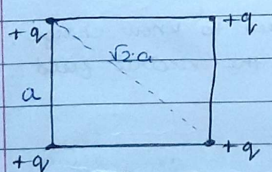
potential of all other charges at the position where q_i is located

$$= \frac{1}{2} \sum_{i=1}^{N} q_i \sum_{j \neq i} \frac{q_j}{4\pi\epsilon_0 r_{ij}}$$

↳ we are counting all the terms two times

$$\Rightarrow \boxed{U = \frac{1}{2} \sum_{i=1}^{N} q_i V(r_i)}$$

Ques.



Using expression mentioned above,

$$U = \frac{1}{2}\left[q_1 V(r_1) + q_2 V(r_2) + q_3 V(r_3) + q_4 V(r_4)\right]$$

$$= \frac{1}{2}\left\{4q\left(\frac{Kq}{a} + \frac{Kq}{a} + \frac{Kq}{\sqrt{2}a}\right)\right\}$$

$$= 2q\left(\frac{2Kq}{a} + \frac{Kq}{\sqrt{2}a}\right) = \frac{2q^2}{4\pi\epsilon_0 a}\left(2 + \frac{1}{\sqrt{2}}\right)$$

* Using the above expression, we can obtain U easily

$\boxed{\text{Electrostatic P.E. density} = \frac{\epsilon_0 E^2}{2} = u}$

→ Continuous charge distribution :

$$U = \frac{1}{2}\iiint \rho\, V\, d\tau$$

or $U = \frac{1}{2}\iint \sigma\, V\, da$ (surface energy) — ②

We know, $\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}$

$$\Rightarrow U = \frac{\epsilon_0}{2}\iiint (\bar{\nabla}\cdot\bar{E})\, V\, d\tau$$

Vector Identity : $\bar{\nabla}\cdot(\bar{F}\phi) = \phi(\bar{\nabla}\cdot\bar{F}) + \bar{F}\cdot(\bar{\nabla}\phi)$

ϕ: scalar $\bar{F}$: vector

use $\bar{F} =$

$$\Rightarrow U = \frac{\epsilon_0}{2}\iiint \bar{\nabla}\cdot(\bar{E}V)\,d\tau + \frac{\epsilon_0}{2}\iiint \bar{E}\cdot(\bar{\nabla}V)\,d\tau$$

$$\Rightarrow \boxed{U = \frac{\epsilon_0}{2}\oiint V\bar{E}\cdot d\bar{a} + \frac{\epsilon_0}{2}\iiint E^2\, d\tau}$$ → For finite val of R

For

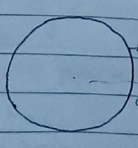
* If we put limit of 2nd term 0 to ∞, 1st term will be zero

1.) U has finite value but as r ↑es, 2nd term → ∞ so, ~~2nd~~ 1st term = 0

2.) Also, we know that potential at $\infty = 0 \Rightarrow V(\infty) = 0 \Rightarrow$ 1st term = 0

$$\Rightarrow \boxed{U = \frac{\epsilon_0}{2}\iiint_{\text{all space}} E^2\, d\tau}$$ don't need to know charge, rather the electric field. ③

Ques (i)



$\sigma = \frac{Q}{4\pi R^2}$

dr rsinθ

$da_R = R^2\sin\theta$

$$\rightarrow U = \frac{1}{2}\iiint \rho V d\tau = \frac{1}{2}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{Q}{4\pi R^2}\cdot\frac{KQ}{R} R^2\sin\theta\, d\phi\, d\theta$$

$$= \frac{1}{\cancel{2}} \times \frac{KQ^2}{4\cancel{\pi}R} \times \cancel{2} \times 2\cancel{\pi}$$

$$= \frac{Q^2}{8\pi\epsilon_0 R}$$

$OR = \frac{Q}{2 \times 4\pi\epsilon_0 R} \iint$ $V \rightarrow$ potential on choosen surface, $\vec{E}$ also on the choosen Gaussean surface

$\rightarrow U = \frac{\epsilon_0}{2} \iiint E^2 dz$

$= \frac{\epsilon_0}{2} \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{0}^{\infty} \frac{K^2 Q^2}{R^4} \cdot R^2 \sin\theta \, d\theta \, d\phi \, dr$

$= \frac{K^2 Q^2 \epsilon_0}{\cancel{2}} \times \cancel{2} \times 2\pi \int_R^\infty \frac{1}{R^2} dr$

$= \cancel{K^2} \cdot \frac{Q^2}{16\pi^2\epsilon_0^2} \times \epsilon_0 \times 2\pi \times \frac{1}{R}$

$= \frac{Q^2}{8\pi\epsilon_0 R}$

$\rightarrow$ ~~$U = \frac{\epsilon_0}{2} \iiint (\nabla \cdot E) V dz = \frac{\epsilon_0}{2}$~~

$U = \frac{\epsilon_0}{2} \oiint V\vec{E} \cdot d\bar{a} + \frac{\epsilon_0}{2} \iiint E^2 dz$

or $r = 0$ to a

~~$= \frac{\epsilon_0}{2} \oiint_{R=a}^{\infty} \frac{Kq}{a}\left(\frac{Kq}{R^2}\right) R^2 \sin\theta \, d\phi \, d\theta + \frac{\epsilon_0}{2} \iiint_R^a \frac{K^2 Q^2}{R^4} R^2 \sin\theta \, d\phi \, d\theta \, dr$~~

$= \frac{K^2 \epsilon_0 q^2}{2} \times 2 \times 2\pi \times$

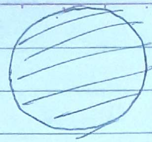
$\frac{\epsilon_0}{2} \oiint V\vec{E} \cdot d\bar{a} + \underbrace{\frac{\epsilon_0}{2} \iiint E^2 dz}_{\left(\frac{1}{a} - \frac{1}{R}\right)} \rightarrow \left(\frac{1}{R} - \frac{1}{a}\right)\frac{Q^2}{8\pi\epsilon_0}$ — (1)

·) $\oiint V\vec{E} \cdot d\bar{a} = \oiint \left(\frac{Kq}{a}\right)\left(\frac{Kq}{a^2}\right) \cdot a^2 \sin\theta \, d\phi \, d\theta$

$= \frac{\epsilon_0}{2} \frac{Kq^2}{a} \times 4\pi = \frac{q^2}{16\pi^2\epsilon_0^2} \times \frac{\epsilon_0}{2} \times \frac{4\pi}{a} = \frac{Q^2}{8\pi\epsilon_0 a}$ — (2)

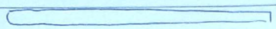
$\Rightarrow$ Total $= \frac{Q^2}{8\pi\epsilon_0 R}$ $\{ (1) + (2) \}$

(ii)



29-3-1

$\Rightarrow$



$|\delta l| = \sigma$

Rel^n b/w eq^{n}s (2) & (3)

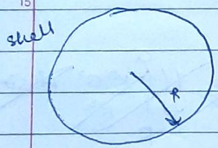
If we put above statement in

$$U = \frac{1}{2}\iint \sigma V \, da$$

we will get same exp^n as $U = \frac{\epsilon_0}{2} \underbrace{\iiint}_{\text{all space}} E^2 \, d\tau$

CAPACITANCE

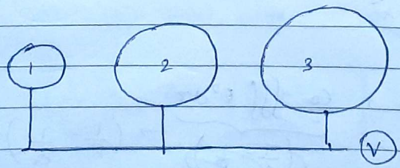
→ Capacitance : ability to store charge.



$$V = \frac{Q}{4\pi\epsilon_0 R}$$

$$V = \left(\frac{1}{4\pi\epsilon_0 R}\right) Q \Rightarrow V \propto Q \quad : V = \frac{Q}{C}$$

$$\boxed{C = 4\pi\epsilon_0 R}$$



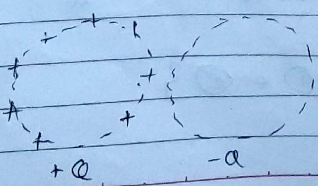
$\Rightarrow Q_1 < Q_2 < Q_3$

Q metallic sphere, $C = 1\,\mu F$

$$1 \times 10^{-6} = \frac{R}{9 \times 10^9}$$

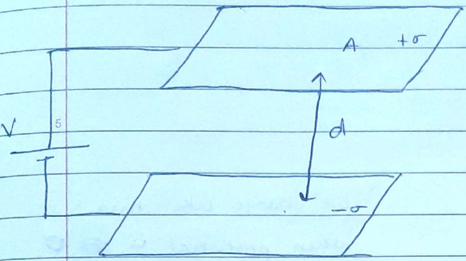
$$R = 9 \times 10^3 = 9 \text{ km}$$

→



Here, V ↓es but Q remains same (+Q). Hence, C ↑es. (-Q charge is brought closer to +Q)

Parallel Plate Capacitor



$$C = \frac{Q}{\Delta V}$$

$$\Delta V = Ed = \frac{\sigma d}{\epsilon_0} = \frac{Q}{C}$$

$$C = \frac{Q\epsilon_0}{\sigma d} = \frac{\sigma A \epsilon_0}{\sigma d}$$

$$\Rightarrow \boxed{C = \frac{\epsilon_0 A}{d}}$$

→ if $d = d/2$ keeping battery connected, ΔV = same = V, but more amount of charge will be found on plate.

→ if not connected to battery, $d = d/2$ ⇒ Q : same, V will decrease.

•) Electrostatic potential energy :- Electric field is confined b/w two plates

Method 1.

$$U = \frac{\epsilon_0}{2} \underset{\substack{\text{~~all space~~} \\ \text{volume b/w plates}}}{\iiint} \overset{\text{const}}{E^2}\, d\tau$$

$$= \frac{\epsilon_0}{2} \times E^2 \times Ad = \frac{\cancel{\epsilon_0}}{2} \times \frac{\sigma^2}{\epsilon_0^2} \times Ad$$

$$= \frac{1}{2} \frac{Q^2 d}{\epsilon_0 A} \Rightarrow \boxed{U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}}$$

Method 2 : Initially, both plates are neutral. e^- will move from 1st plate to 2nd, ΔV will be created.

$$U = \int_0^Q V(q)\, dq = \int_0^Q \frac{q}{C} dq =$$

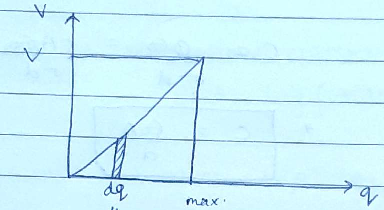
$$\Rightarrow \boxed{U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}}$$

•) If P.D. = V , Q = Q

$W_{Total} = QV$ (we know)

but here, $\therefore U = \frac{1}{2} VQ$

If we plot the graph

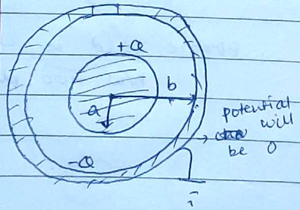


⇓

amount of charge transferred from 1 place to another

We charge like this:
when potential is 0, 0,
charge transferred is small
when V = V, total charge transferred is Q
Hence, area under curve = $\frac{1}{2}VQ$, not VQ

<u>Spherical</u> <u>Capacitor</u> :-



potential will be 0

$$C = \frac{Q}{\Delta V}$$

$$V(a) - V(b) = -\int_b^a \vec{E} \cdot dr$$

$$= -\int_b^a \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} dr$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{(b-a)Q}{4\pi\epsilon_0 ab}$$

$$\Rightarrow \boxed{C = \frac{4\pi\epsilon_0 ab}{b-a}}$$

$$U = \cancel{\frac{1}{2}QV} = \cancel{\frac{1}{2}} \frac{\epsilon_0}{2} \iiint E^2 d\tau$$

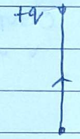
$$= \frac{\epsilon_0}{2} \iiint_{r=a}^{b} \frac{Q^2}{(4\pi\epsilon_0)^2 r^4} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$\frac{-1}{r^2}$ $\left(\frac{-1}{r}\right)$

$$= \frac{\epsilon_0}{2} \times \cancel{4\pi} \times \frac{Q^2}{\cancel{16}_4 \pi^2 \epsilon_0^2} \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\boxed{U = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{Q^2}{2C}$$

DIPOLE

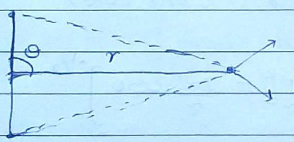


dipole moment $\vec{p} = q\vec{d}$

here, $V \propto \frac{1}{r^2}$

$|\vec{E}| \propto \frac{1}{r^3}$

31-3-17



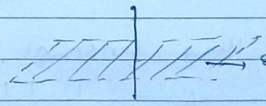
$$\boxed{V(r) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} \quad r >>> d}$$

$|\vec{p}| = q|\vec{d}|$

⇒ if $\theta < 90° \Rightarrow$ positive potential

$\theta = 90° \Rightarrow V(r) = 0$ ⟶

$\theta > 90° \Rightarrow$ -ve potential.



$\vec{E} = -\nabla . \bar{V}$ ⇒ spherical coordinate system

$\cancel{\bar{E} = + \frac{2r}{4\pi\epsilon_0}\frac{p\cos\theta}{r^3}\hat{r}}$

$$\vec{E} = -\left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi}\right)\cdot\left(\bar{V}\right)$$

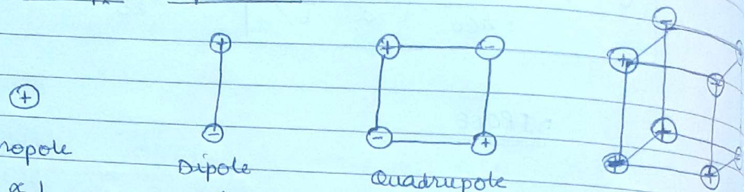
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{2p\cos\theta}{r^3}\hat{r} + \frac{p}{r^3}\sin\theta\,\hat{\theta}\right]$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

at same ϕ every where, value will be same

$$\boxed{V(r) = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2}}$$

29 **Multiple Expansion**

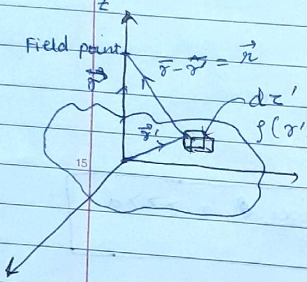


Monopole $V \propto \frac{1}{r}$

Dipole $V \propto \frac{1}{r^2}$

Quadrupole $V \propto \frac{1}{r^3}$

Octopole $V \propto \frac{1}{r^4}$



$$V_A = \frac{1}{4\pi\epsilon_0}\iiint \frac{\rho(r')\,d\tau'}{\mathscr{r}}$$

$$\mathscr{r} = (r^2 + r'^2 - 2rr'\cos\theta)^{1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 + \frac{r'^2}{r^2} - \frac{2r'}{r}\cos\theta\right]^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 + \left(\frac{r'^2}{r^2} - \frac{2r'}{r}\cos\theta\right)\right]^{-1/2}$$

Using binomial expansion,

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left\{1 + \left(\frac{-1}{2}\right)\left(\frac{r'^2}{r^2} - \frac{2r'}{r}\cos\theta\right) + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{2}\left(\frac{r'^2}{r^2} - \frac{2r'\cos}{r}\right.\right.$$

$$= \frac{1}{r}\left[1 - \frac{1}{2}\frac{r'^2}{r^2} + \frac{r'}{r}\cos\theta + \frac{3}{8}\frac{r'^4}{r^2} + \frac{3}{2}\frac{r'^2}{r^2}\cos^2\theta - \frac{3}{2}\frac{r'^3}{r^3}\cos\right.$$

we can neglect higher values of r'

$$\frac{1}{\mathscr{r}} = \frac{1}{r} + \frac{r'\cos\theta}{r^2} + \frac{r'^2}{r^3}\left[\frac{3\cos^2\theta - 1}{2}\right] + \dots$$

$$V_A = \frac{1}{4\pi\epsilon_0}\underbrace{\iiint\frac{\rho(r')\,d\tau'}{r}}_{K_0} + \frac{1}{4\pi\epsilon_0}\underbrace{\iiint\frac{r'\cos\theta\,\rho(r')\,d\tau'}{r^2}}_{K_1} + \dots$$

(potential when whole charge is concentrated at same point)

monopole

dipole

→ The 1st term is $\propto \frac{1}{r}$, 2nd term is $\propto \frac{1}{r^2}$ ... and so on.

So, 1st term is dominant over other terms.

→ If ∃ some arbitary confgⁿ of charges, we can find V due to monopole & dipole contribution for V.

Now, 2nd term is : $K_1 = \frac{1}{4\pi\epsilon_0} \frac{\iiint r'\cos\theta \, \rho(r') \, d\tau'}{r^2}$

$$= \frac{1}{4\pi\epsilon_0} \frac{\iiint \vec{r}' \cdot \hat{r} \, \rho(r') \, d\tau'}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \, \hat{r} \cdot \frac{\iiint \vec{r}' \rho(r') \, d\tau'}{r^2}$$

Earlier, $V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$

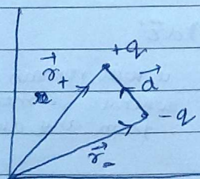
$$\boxed{\vec{p} = \iiint \vec{r}' \rho(r') \, d\tau'}$$ general form of dipole

$\Rightarrow K_1 = \frac{1}{4\pi\epsilon_0} \frac{\hat{r} \cdot \vec{p}}{r^2}$ = potential of dipole.

⇒ For distinct charge distribution,

$$\boxed{\vec{p} = \sum_{i=1}^{N} r_i' Q_i} \quad \text{— ①}$$

Eg.



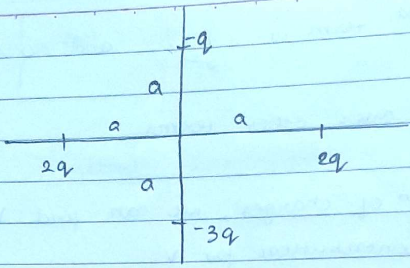
$\vec{p} = \vec{r}_+(+q) + \vec{r}_-(-q)$

$= q(\vec{r}_+ - \vec{r}_-)$

$= q\vec{d}$: special case when equal and opp. charges exist.

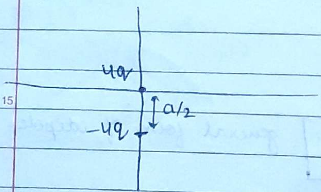
Eg. For multiple charges : use ①.

Ques. Find dipole moment of this system wrt O



$$\vec{p} = \sum_{i=1}^{4} r_i Q_i = 2qa(\hat{i}) - 2q(a\hat{i}) + -q(a\hat{j}) - 3q(a)$$
$$= 2qa\hat{j}$$

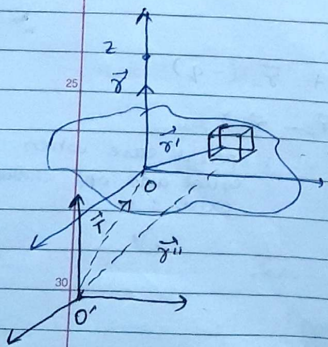
Here, total amount of +ive Q = 4q
-ive Q = -4q



⇒ dipole moment
$$\vec{p} = q(\vec{d})$$
$$= 2aq\,\hat{j}$$

* If total Q = 0 (like above), we can shift origin anywhere. The answer will always remain same.

PROOF: $\vec{p} = \iiint \vec{r}' \rho(\vec{r}')\, d\tau'$



$$\bar{r}'' = \bar{r}' + \bar{T}$$

$$= \iiint (\bar{r}' + \bar{T})\, \rho(\bar{r}'+\bar{T})\, d\tau'$$

↳ it will remain same whether we measure from any distance

$$= \iiint \bar{r}' \rho(\bar{r}')\, d\tau + \iiint \bar{T} \rho(\bar{r}')\, d\tau'$$

$$= \iiint \bar{r}' \rho(\bar{r}')\, d\tau + \bar{T} \iiint \rho(\bar{r}')\, d\tau$$

Total Charge = 0

⇒ again, we get same expression.

Multipole exp^n don't give exact value, but it is approximately zero.

Q. Find out potential at $(30a, 40a, 0)$ of previous charge $config^n$:

1^{st} term : monopole contribution $= 0$ (Total $Q = 0$)

2^{nd} term : dipole contribution $= \dfrac{1}{4\pi\epsilon_0} \dfrac{\vec{p}\cdot(\vec{r})}{r^2}$

$$= \frac{1}{4\pi\epsilon_0} \frac{2qa(4\cancel{0}\cancel{a})}{5\cancel{0}\,a} = \frac{2qa}{5\pi\epsilon_0}$$

3-4-17

→ when ρ/σ is not known and we have to find potential $func^n$:

$$\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0} \quad \text{(using Gauss's law)}$$

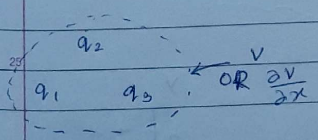
$$\bar{\nabla}\cdot(-\bar{\nabla}.V) = \frac{\rho}{\epsilon_0}$$

$$\boxed{\nabla^2 V = \frac{-\rho}{\epsilon_0}}$$ ← Poisson's Equation

if $\rho = 0$ $$\boxed{\nabla^2 V = 0}$$ ← Laplace equation —— (1)

Uniqueness theorem

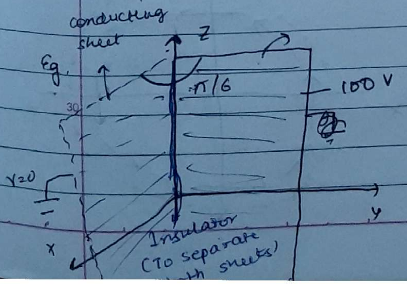
If we've known charges within a certain boundary. And,
(location & magnitude) OR $\partial V/\partial n$
at each point of boundary, V is defined, then sol^n of Poisson's equation is unique for enclosed region.



if $\nabla^2 V_A = \dfrac{\rho}{\epsilon_0}$; $\nabla^2 V_B = \dfrac{\rho}{\epsilon_0}$

Then, $V_A = V_B$

Eg.



Semi-infinite sheets :
within these two planes, ∃ no charge
⇒ using (1),
$$\nabla^2 V = 0$$

We can use cylindrical ~~s~~ coordinate (Here, f is not known)

$s \quad \phi \quad z$

$-\infty < z < \infty$: won't be func^n of z

semi infinite goes to $-\infty/\infty$ ⇒ won't be func^n of s

$$\nabla^2 V = \cancel{\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial v}{\partial s}\right)} + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} + \cancel{\frac{\partial^2 V}{\partial z^2}} = 0$$

$$= \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$V(\phi) = A\phi + B$$

$$V(\phi = 0) = B = 0$$

$$V(\phi = \pi/6) = \frac{A\pi}{6} = 100 \Rightarrow A = \frac{600}{\pi}$$

$$\Rightarrow \boxed{V(\phi) = \frac{600\,\phi}{\pi}}$$

$$E = -\nabla \cdot V$$

$$E = -\left[\cancel{\frac{1}{s}\frac{\partial}{\partial s}(sV_s)} + \frac{1}{s}\frac{\partial V_\phi}{\partial \phi} + \cancel{\frac{\partial V_z}{\partial z}}\right]$$

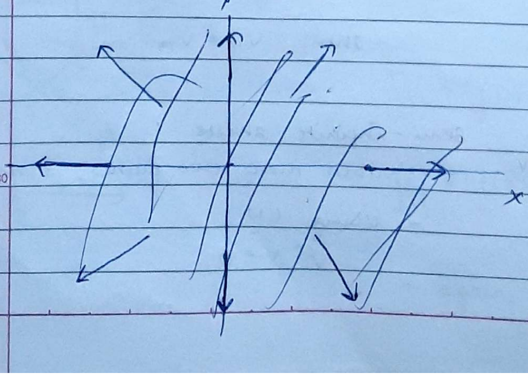
$$\Rightarrow \cancel{E = -\left[\frac{1}{s} \times\right.}$$

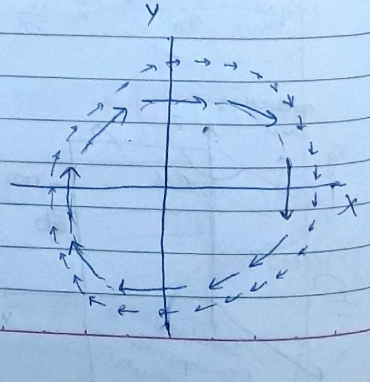
$$\bar{\nabla} = \frac{\partial}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}$$

$$\bar{E} = -\bar{\nabla}\cdot V = -\left(\frac{1}{s} \times \frac{600}{\pi}\hat{\phi}\right)$$

$s \rightarrow \sqrt{x^2+y^2}$

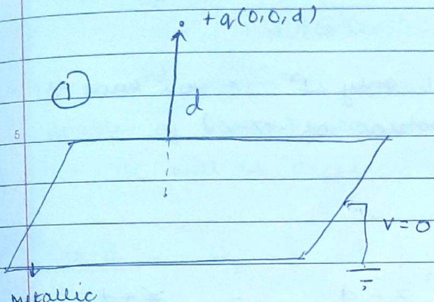
vector field :





⇒ The Method of Image :

①



Electrostatic funcⁿ in region

The charge +q will induce charge on sheet whose distribution we can't find.

2

* Instead of this, we will solve following problem :

② A +q (0,0,d)

T (x,y,z)

→ dipole

→ potential everywhere on plane = 0

Boundary (infinity)

B −q (0,0,−d)

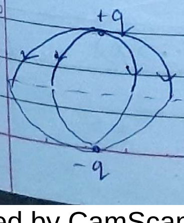
By uniqueness theorem, solⁿ of Poisson's theorem is unique

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{AT} - \frac{1}{BT}\right] = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{\left(x^2+y^2+(z-d)^2\right)^{1/2}} - \frac{1}{\left(x^2+y^2+(z+d)^2\right)^{1/2}}\right\}$$

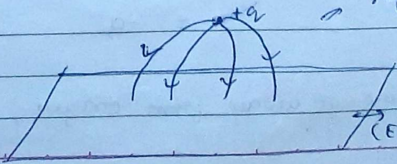
This potential is valid in case of sheet also.

Electric field :

dipole



sheet :



Only this is region of interest.

boundary (E won't be below the boundary)

Electric field just outside sheet :

$$E = \frac{\sigma}{\epsilon_0}$$

Every time, $\bar{E}$ near sheet is only $\perp^r$ ⇒ you have to consider only normal component of $(-\nabla \cdot V)$

$$\Rightarrow \quad \sigma = -\epsilon_0 \left.\frac{\partial V}{\partial z}\right|_{z=0}$$

Here, $$\frac{\partial V}{\partial z} = \frac{q}{4\pi\epsilon_0}\left[\frac{z-d}{(x^2+y^2+(z-d)^2)^{3/2}} - \frac{\cancel{z}+d}{(x^2+y^2+(z+d)^2)^{3/2}}\right]$$

$(z=0)$ $$\sigma = \frac{-qd}{2\pi}\left[\frac{1}{(s^2+d^2)^{3/2}}\right]$$

$$= -\frac{q}{2\pi}\left[\frac{d}{(s^2+d^2)^{3/2}}\right]$$

5-4-17

$$Q_{Total} = \iint \sigma \, da$$
$$= \iint \sigma \, s \, d\phi \, ds$$

$$= -\frac{q}{2\pi}\iint s \, d\phi \, ds \cdot \frac{d}{(s^2+d^2)^{3/2}}$$

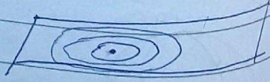
$$= -\frac{qd}{2\pi}\iint \frac{s \, ds \, d\phi}{(s^2+d^2)^{3/2}}$$ let $s^2+d^2 =$ $2s ds =$

$$= -\frac{qd}{2\pi}\iint \frac{t \, dt}{t^3} d\phi$$

$$= -\frac{qd}{2\pi}$$ $\frac{1}{t^2}$

$$= -q$$

As we go away from origin, charge ↓es.



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⇒ force experienced in both the cases will be same :

$$F = \frac{-q^2}{4\pi\epsilon_0 (2d)^2}$$

* E in 2nd case ~~region~~ is spread over all region, so U will be double of value obtained in case 1st

⇒ U will be different in both cases

Case-II : $U_B = -q\left[\frac{1}{4\pi\epsilon_0}\frac{q}{2d}\right]$

Case-I : we know force applied.

$$F = \frac{-q^2}{4\pi\epsilon_0 (2d)^2}$$

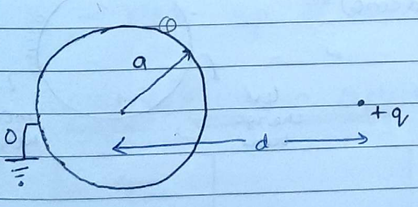
$d = z$

$$U_P = -\left(-\frac{q^2}{4\pi\epsilon_0 4}\int_{\infty}^{d}\frac{1}{z^2}\,dz\right) \qquad -\frac{1}{z} \qquad -F\cdot dr = U$$

$$= \frac{-q^2}{4\pi\epsilon_0} \times \frac{-1}{4d} = \frac{q^2}{16\pi\epsilon_0 d}$$

⇒

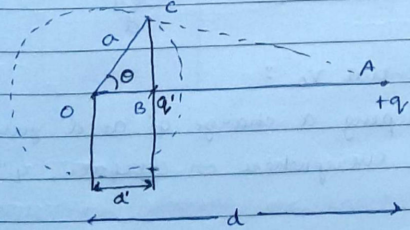
A:



conducting sphere

$$\frac{KQ}{a} + \frac{Kq}{(d-a)} = 0$$

B:



$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{AC} + \frac{q'}{BC}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{(a^2+d^2-2ad\cos\theta)^{1/2}} + \frac{q'}{(a^2+d'^2-2ad'\cos\theta)^{1/2}}\right] = 0$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q/a}{\left(1+\frac{d^2}{a^2}-\frac{2d}{a}\cos\theta\right)^{1/2}} + \frac{q'/d'}{\left(1+\frac{a^2}{d'^2}-\frac{2a}{d'}\cos\theta\right)^{1/2}}\right] = 0$$

$$\Rightarrow \quad V = 0$$

$$\frac{q}{a} = -\frac{q'}{d'} \qquad \frac{d}{a} = \frac{a}{d'}$$

$$\boxed{q' = -\frac{qa}{d}} \qquad \boxed{d' = a^2/d}$$

→ V at any point $P = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_1} + \frac{q'}{r_2}\right]$

$$V(r) = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{(r^2+d^2-2rd\cos\theta)^{1/2}} - \frac{a}{(r^2d^2+a^4-2rda\ldots}\right.$$

$$E_r = -\frac{\partial V}{\partial r}, \quad E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

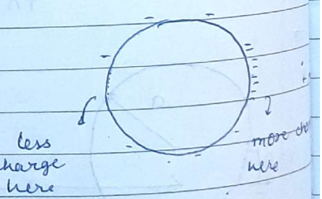
⇒ near the surface, $\vec{E}$ is always normal to surface

$$\sigma = -\epsilon_0 \left.\frac{\partial V}{\partial r}\right|_{r=a}$$

$$\sigma = -\frac{q\,(d^2-a^2)}{4\pi a(a^2+d^2-2ad\cos\theta)^{3/2}}$$

→ $\theta = 0^\circ \Rightarrow \sigma = \sigma_{max}$

$\theta = \pi \Rightarrow \sigma = \sigma_{min}$



$$Q_{Total} = \iint \sigma\, da$$

$$= \iint \sigma\, a^2 \sin\theta\, d\theta\, d\phi = -\frac{qa}{d}$$

•) If sphere is kept at a potential V_0 :

so, we already know keeping a charge q' at d' gives potential 0, to make potential V_0 everywhere on surface, q'' should be kept at centre

$$V_0 = \frac{q''}{4\pi\epsilon_0 a}$$

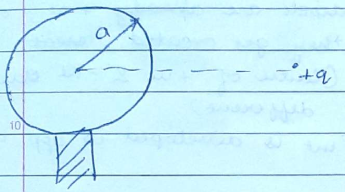
$V(r)$: we will just add potential due to q''

$$V(r) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{(r^2+d^2-2rd\cos\theta)^{1/2}} - \frac{q'}{(r^2+d'^2-2rd'\cos\theta)^{1/2}}\right] + \frac{V_0(4\pi\epsilon_0 a)}{4\pi\epsilon_0 r}$$

If we put q", same amount of charge will be induced

$$\Rightarrow \sigma = \frac{-q(d^2-a^2)}{4\pi a\,(a^2+d^2-2ad\cos\theta)^{3/2}} + \left(\frac{(4\pi\varepsilon_0 a)V_0}{4\pi a^2}\right)$$

•) If sphere is placed on insulating material



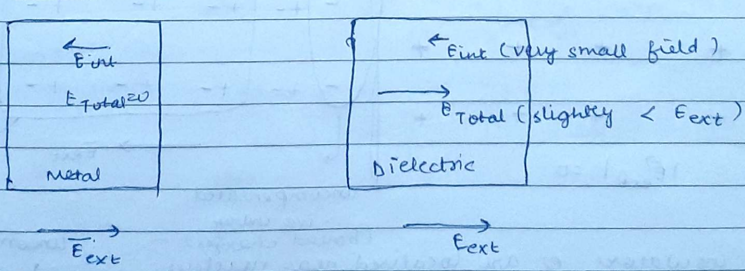
Total charge inside sphere = 0

→ B : sphere with total charge = 0
equipotential ⇒ potential at every point on surface is equal

we know if we place $-\frac{qa}{d}$ at d', $-\frac{qa}{d}$ is induced

⇒ If we place $\frac{qa}{d}$ at centre ⇒ Total charge = 0

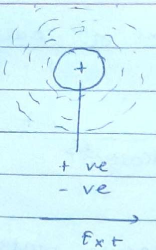
Electric Field in Dielectric Material



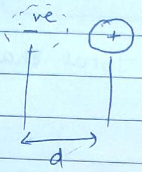
* dielectric material are Insulators

Sources of E_{int} :
in metal : free e^-
in dielectric : dipoles : when apply E_{ext}, polarisation takes place

Polarize :



(centre of +ive & -ive are same)



when E_{ext} is applied, either dipole are already +nt, or they get created, ~~creat~~ (centre of +ve & -ve are different)

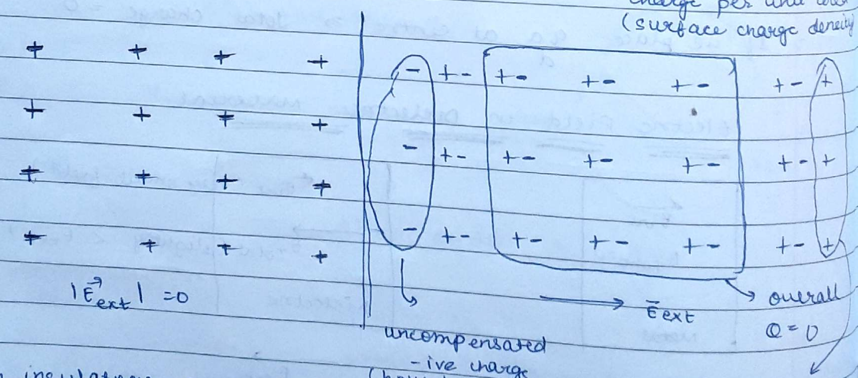
E_{int} is developed in opp. dir

→ **Polarization Vector** :- $(\vec{P})$

$$\vec{P} = \frac{\sum \vec{p_i}}{\Delta V}$$ (This gives us macroscopic property)

∘) dimension : $\frac{C \times m}{m^3} = C/m^2 = Q\,L^{-2}$

↓ Charge per unit area (surface charge density)



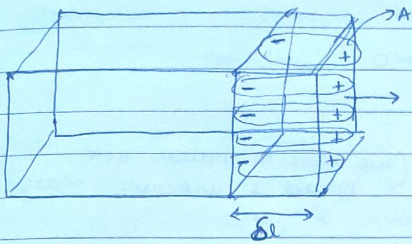
(bound charges)

* In insulators, e^- are localised near nucleus. When $\vec{E}_{ext}$ is applied, they get shifted a little but still remain bounded to the nucleus.

① $\sigma_b = \vec{P}\cdot\hat{n}$

② $\rho_b = -\vec{\nabla}\cdot\vec{P}$

$\xrightarrow{\quad} \bar{E}_{ext}$



$$\vec{P} = \frac{\sum \vec{p}_i}{V} = \frac{\vec{\delta l}\,\sum Q_i}{\vec{A}\cdot\vec{\delta l}}$$

$$\vec{P}\cdot\hat{n} = \vec{P}\cdot\frac{\vec{A}}{|\vec{A}|}$$

$$= \frac{\sum Q_i}{|\vec{A}|} = \sigma_b$$

σ_b ↓ bound surface charge density

Total surface charge ~~density~~ $\oiint \sigma_b\, da = \oiint \vec{P}\cdot\hat{n}\, da = \oiint \vec{P}\cdot\vec{da}$

Volume charge = − surface charge [so that total Q = 0]

$= -\oiint \bar{P}\cdot\bar{da}$ → Bound volume charge

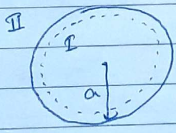
$= -\iiint_V (\vec{\nabla}\cdot\vec{P})\,d\tau$ → charge per unit volume

$$\Rightarrow \boxed{\rho_b = -\vec{\nabla}\cdot\vec{P}}$$ → Bound volume charge density

Electret

•) without applying any $\bar{E}$, it is polar (has permanent Electric dipole moment)

→ Electret :



$\xrightarrow{\text{given}}$ $\vec{P} = K\vec{r} = Kr\hat{r}$ (has volume)

$\rho_b = -\nabla\cdot\vec{P}$

$= -\frac{1}{r^2}\left(\frac{\partial}{\partial r}(r^2 P_r)\right) = -3K$, $P_r = Kr$

Ⓘ: Gaussean's law :

$$E \times 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0} = \frac{-3K \times 4/3\,\pi r^3}{\epsilon_0}$$

$$\Rightarrow \boxed{E_I = \frac{-Kr}{\epsilon_0}\hat{r}}$$

Ⓘⓘ : Total charge = 0 , ∴ By symmetry, $E_{II} = 0$

Total charge within sphere $= -3K \times \frac{4}{3}\pi a^3 = -4\pi K a^3$ — ①

Total surface charge :

$\sigma_b = \vec{P}\cdot\hat{n} = ~~K\alpha~~\ Ka$ — ②

Surface charge : $\oiint Ka \times 4\pi a^2 = 4\pi K a^3$

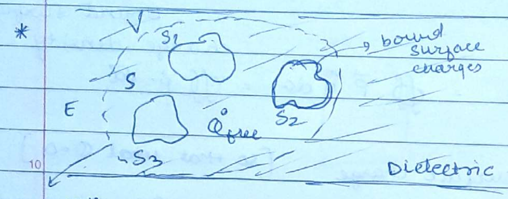
Total charge = ① + ②

$\Rightarrow$ Total charge = 0

$\Rightarrow$ $\bar{E}_{II} = 0$

$\oint\!\!\oint \bar{E} \cdot da = \frac{Q_{encl}}{\epsilon_0}$: (we have to include both bound & unbound charge here)

*



Gaussean surface

S_1, S_2, S_3 : cavities

$$\oint\!\!\oint \epsilon_0 \bar{E} \cdot d\bar{a} = Q_{free} + \iint_{S_1+S_2+S_3} \bar{P} \cdot d\bar{a} + \iiint_V \rho_b \, d\tau$$

$$= Q_{free} + \iint_{S_1+S_2+S_3} \bar{P} \cdot d\bar{a} - \iiint (\vec{\nabla} \cdot \bar{P}) d\tau$$

$$= Q_{free} + \iint_{S_1+S_2+S_3} \bar{P} \cdot da - \oint\!\!\oint \bar{P} \cdot d\bar{a}$$

$V = S + S_1 + S_2 + S_3$

$$\boxed{\oint\!\!\oint_S (\epsilon_0 \bar{E} + \bar{P}) \cdot d\bar{a} = Q_{free}}$$

$$\boxed{\oint\!\!\oint_S \bar{D} \cdot d\bar{a} = Q_{free}}$$

$\bar{D}$: electric displacement vector

$\bar{D} = \epsilon_0 \bar{E} + \bar{P}$: same unit & dimension as $\bar{P}$.

$\bar{E} = \frac{\bar{D} - \bar{P}}{\epsilon_0}$

$\bar{E} = \frac{\bar{D}}{\epsilon_0} - \frac{\bar{P}}{\epsilon_0}$ → E due to polarisation

E due to free charge

* In earlier problem {sphere}

$\oint\!\!\oint \vec{D} \cdot d\bar{a} = Q_{free}$

$\oint\!\!\oint D(4\pi r^2) = 0 \Rightarrow D = 0$.

$\epsilon_0 \bar{E} + \bar{P} = 0$

$\Rightarrow \bar{E} = -\frac{\bar{P}}{\epsilon_0}$

$= \frac{-Kr}{\epsilon_0} \hat{r}$

→ $\bar{P}_{outside} = 0$ * Electric field is always conservative field

⇒ $\bar{E} = 0$

* If polarisation : uniform ⇒ we will get bound surface charge density

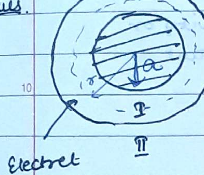
non- " ⇒ " " volume charge density

* curl of $\bar{E} = 0$ but curl of $\bar{P} \neq 0$, so curl of $D \neq 0$

⇒ D may be rotational vector field.

$\oint \bar{D} \cdot d\bar{l} = 0$ ✗ (not possible)

Ques.



$\vec{P} = \frac{K}{r}\hat{r}$ Find out E_I & E_{II}

$\rho_b = -\nabla \cdot \bar{P}$

$\sigma_b = \bar{P} \cdot \hat{n}$

$= \frac{K}{r}\hat{r} \cdot (-\hat{r})$

⇓

area inside electret [directed towards centre]

$= \frac{K}{a}$

Using gaussean's law

$\oiint \bar{E} \cdot d\bar{a} = \frac{q_{encl}}{\epsilon_0}$

⇒ $E \times 4\pi r^2 = \frac{\text{all charges enclosed}}{\epsilon_0} =$

$\rho_b = -\bar{\nabla} \cdot \bar{P}$

$= -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \vec{P}_r)$

$= \frac{-1}{r^2} \times K = \frac{-K}{r^2}$

⇒ $E \times 4\pi r^2 = \frac{-K}{a} \times 4\pi a^2 + \iiint_{r=a}^{r} \frac{-K}{r^2} r^2 \sin\theta \, d\theta \, d\phi \, dr$

$= \cancel{-4\pi a k}$

$= -4\pi r k/\epsilon_0$

⇒ $\boxed{E_I = -\frac{C}{\epsilon_0 r}}$

⇒ Using $\bar{D}$:→ $Q_{free} = 0$

$D(4\pi r^2) = 0$

$\epsilon_0 \bar{E} + \bar{P} = 0$

⇒ $\bar{E}_I = \frac{-K}{r\epsilon_0}$

* linear dielectric material : $\bar{P} \propto \bar{E}$

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Different Dielectric Materials :

for linear isotropic homogeneous medium

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

$\hookrightarrow$ Electric Susceptibility

$$\vec{D} = \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E}$$

$$= \epsilon_0 (1+\chi)\vec{E} = \epsilon_0 \epsilon_r \vec{E} \leftarrow \text{Electric field inside dielectric}$$

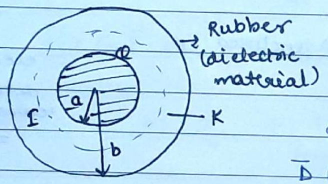
$$= \epsilon \vec{E}$$

$\hookrightarrow$ ~~Permeab~~ Permittivity

$\rightarrow$ $\frac{\epsilon}{\epsilon_0} = 1+\chi = \epsilon_r \leftarrow$ Relative permittivity or dielectric constant

* in vaccum, $\chi = 0 \Rightarrow \epsilon = \epsilon_0$

Ques



$E_I = ?$ $\quad E_{II} = ?$ $\quad \vec{D} = K\epsilon_0 \vec{E}$

$$\oint\oint \vec{D} \cdot d\vec{a} = \frac{Q_{free}}{\epsilon_0}$$

$$\vec{D}(4\pi r^2)$$

$$\cancel{(K\epsilon_0)\vec{E}(4\pi r^2)} = Q$$

$$\vec{D}_I = \frac{Q}{4\pi r^2} \qquad \vec{D}_{II} = \frac{Q}{4\pi r^2}$$

$$E_I = \frac{Q}{4\pi K\epsilon_0 r^2}\hat{r} \qquad E_{II} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

$\rightarrow$ $\vec{P} = \epsilon_0 (K-1)\vec{E}_I$

$$= \frac{(K-1)Q}{4\pi K r^2}\hat{r}$$

On inner surface of dielectric

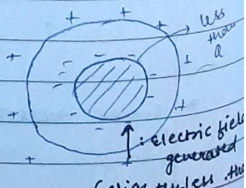
$$\sigma_b = \vec{P}\cdot\hat{n} = \frac{(K-1)Q}{4\pi K r^2}\hat{r}\cdot(-\hat{r})\Big|_{r=a} = -\frac{(K-1)Q}{4\pi K a^2}$$

$$Q_b = \sigma_b \times 4\pi a^2 = \frac{-(K-1)Q}{K}$$

* Usually, $K > 1$

$$Q_b\Big|_{surface\ r=a} = -\frac{(K-1)Q}{K} \Rightarrow Q_b < Q$$

$$Q_b\Big|_{r=b} = \frac{(K-1)Q}{K} \Rightarrow Q_b > Q$$



$\uparrow$: electric field generated (slightly less than II or in sphere)

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Potential at the centre = ?

$$\bar{V} = -\int_{\infty}^{r} \bar{E}\cdot d\bar{r}$$

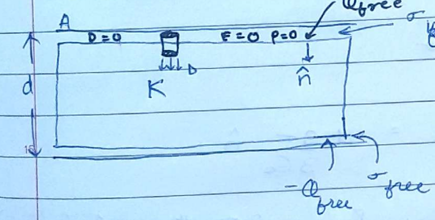
$$= -\int_{\infty}^{b} \frac{Q}{4\pi\epsilon_0 r^2} - \int_{b}^{a} \frac{Q}{4\pi K \epsilon_0 r^2} - \int_{a}^{0} \bar{E}\cdot d\bar{r}$$

$\bar{E} = 0$ (conductor)

$$\rho_b = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \bar{P}_r) = 0$$

⇒ NO bound volume charge is present here.

→ **Parallel Plate Capacitor :**



Using $\oint\!\!\oint \bar{D}\cdot d\bar{a} = Q_{free}$

Metallic plate ⇒ $E=0, P=0$
⇒ D is only in downwards dir^n.

$\bar{D} A = \sigma_{free} A$

$|\bar{D}| = \sigma_{free}$

$\bar{E} = \frac{|\bar{D}|}{\epsilon_0 K} = \frac{\sigma_{free}}{\epsilon_0 K}$

⇒ $\nabla V = |\bar{E}| d = \frac{\sigma_{free} d}{\epsilon_0 K}$

$C = \frac{Q}{\nabla V} = \left(\frac{\epsilon_0 A}{d}\right)K$

* free $\bar{E}$: σ/ϵ_0 polarised dielectric slab ⇒ bound charges produced which in turn produce $\bar{E}$ in opp. dir^n so, $\bar{E}$ ↓es

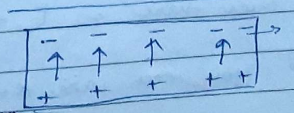
* Bound surface charges :

$\bar{P} = \epsilon_0 (K-1)\frac{\sigma_{free}}{\epsilon_0 K}\hat{n} = \frac{(K-1)\sigma_{free}}{K}\hat{n}$

dir^n of upper surface

→ σ_b (upper surface) $= \bar{P}\cdot\hat{n} = \frac{(K-1)\sigma_{free}}{K}\hat{n}\cdot(-\hat{n}) = \frac{(1-K)}{K}\sigma_{free}$

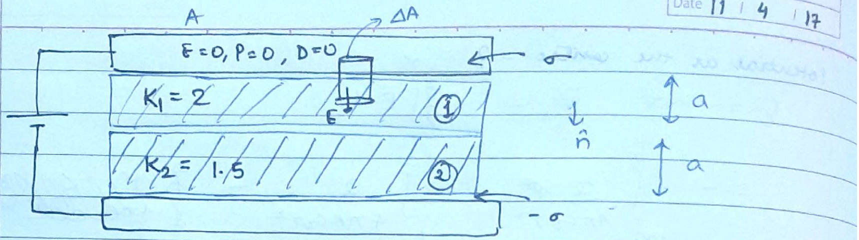
$Q_b = \sigma_b \times A$



→ $\bar{E}_D = \frac{\bar{D}}{\epsilon_0} - \frac{\bar{P}}{\epsilon_0}$ → due to bound charges, due to free charge

$= \frac{\sigma_{free}}{\epsilon_0} - \left(\frac{1-K}{K}\right)\frac{\sigma_{free}}{\epsilon_0} = \sigma_{free}/\epsilon_0 K$

Ques.



(i.) $E_1 , E_2 = ?$ (ii.) $P_1, P_2 = ?$ (iii.) σ_{b_1} (top), σ_{b_1} (bottom)

(iv.) σ_{b_2} (Top), σ_{b_2} (bottom) (iv.) ∇V

(i) $\oint \bar{D} \cdot d\bar{a} = Q_{free}$

$$\bar{D} \cdot \Delta A = \sigma \, \Delta A$$

$$D = \sigma$$

$$\vec{E} = \frac{\sigma}{\epsilon_0 K} \sim \sigma$$

$$\vec{E}_1 = \frac{\sigma}{2\epsilon_0} \hat{n} \qquad \vec{E}_2 = + \frac{2\sigma}{3\epsilon_0} \hat{n}$$

(ii.) $P_1 = \epsilon_0 (1) E = \frac{\sigma}{2} \hat{n}$, $P_2 = \frac{\sigma}{3} \hat{n}$ $\epsilon_0 (0.5)$

(iii.) $\sigma_{b_1} (top) = \vec{P}_1 \cdot \hat{n} = \left(\frac{\sigma}{2} \hat{n}\right)(-\hat{n}) = -\frac{\sigma}{2}$

$\sigma_{b_1} (bottom) = \vec{P}_1 \cdot \hat{n} = \left(\frac{\sigma}{2} \hat{n}\right)(\hat{n}) = \frac{\sigma}{2}$

(iv.) $\sigma_{b_2} (top) = \vec{P}_2 \cdot \hat{n} = \left(\frac{\sigma}{3} \hat{n}\right)(-\hat{n}) = -\frac{\sigma}{3}$

$\sigma_{b_2} (bottom) = \vec{P}_2 \cdot \hat{n} = \left(\frac{\sigma}{3} \hat{n}\right)(\hat{n}) = \frac{\sigma}{3}$

(v.) $\nabla V = E_1 a + E_2 a = \frac{7a\sigma}{3\epsilon_0}$

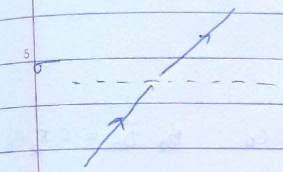
⇒ Now, if slab 2 is removed,

E over there = E due to free charges = $\frac{\sigma}{\epsilon_0}$

(or, you can take K = 1)

Boundary condition of Electric Displacement Vector

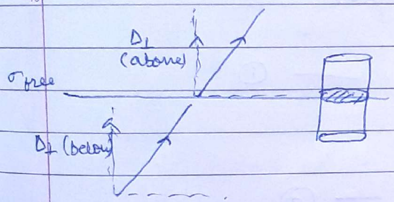
→ Boundary condⁿ of $\vec{E}$



$$E_{\perp}(\text{above}) - E_{\perp}(\text{below}) = \frac{\sigma}{\epsilon_0}$$

$$E_{\parallel}(\text{above}) = E_{\parallel}(\text{below})$$

→ Boundary condⁿ of $\bar{D}$

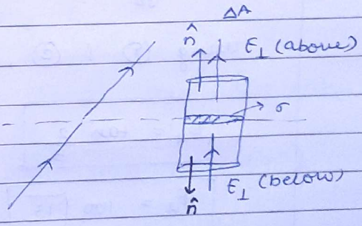


$$D_{\perp}(\text{above}) - D_{\perp}(\text{below}) = \sigma_{free}$$

$$\oiint \bar{D} \cdot d\bar{a} = Q_{free}$$

$\vec{E}$ → : satisfies 2 condⁿ :

1.) Gauss's law :

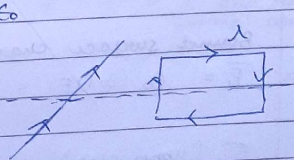


$$E_{\perp(\text{above})}\,\Delta A - \underset{\downarrow}{}\, E_{\perp(\text{below})}\,\Delta A = \frac{\Delta A \sigma}{\epsilon_0}$$

dirⁿ of surface opp. to $\bar{E}$

⇒ $E_{\perp}(\text{above}) - E_{\perp}(\text{below}) = \sigma/\epsilon_0$

2.) Conservative field ($\oint \bar{E} \cdot d\bar{l} = 0$)



Parallel component :

$E_{\parallel}(\text{above}) - E_{\parallel}(\text{below}) = 0$

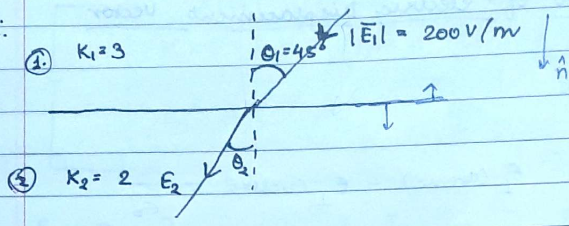
⇒ $E_{\parallel}(\text{above}) = E_{\parallel}(\text{below})$

* can't comment on $D_{\parallel}$ (not conservative field)

we can only infer that

$$\vec{D}_{\parallel(\text{above})} - \vec{P}_{\parallel(\text{above})} = \vec{D}_{\parallel(\text{below})} - \vec{P}_{\parallel(\text{below})}$$

Ex.



Find out $|\vec{E_2}|$ & θ_2.

$\vec{E_1} = \dfrac{\vec{D_1}}{\epsilon_0 K} \Rightarrow \vec{D_1} = 200 \times 3\epsilon_0 = 600\epsilon_0 \qquad \vec{D_2} = 2E_2\epsilon_0$

$\perp^r$:→ $\sigma_{free} = 0 \qquad \bar{D}_\perp \text{(above)} = \bar{D}_\perp \text{(below)}$

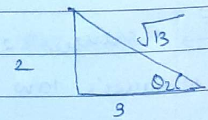
$\Rightarrow \dfrac{1}{\sqrt{2}} \times \cancel{200}^{100} \times 3\,\epsilon_0 = \cancel{2}E_2\,\epsilon_0 \cos\theta_2$

$\Rightarrow E_2 \cos\theta_2 = \dfrac{300}{\sqrt{2}}$ —— ①

→ $\bar{E}_{\parallel}\text{(above)} = \bar{E}_{\parallel}\text{(below)}$

$\Rightarrow 200 \times \dfrac{1}{\sqrt{2}} = E_2 \sin\theta_2$ —— ②

using ① & ②, $\tan\theta_2 = \dfrac{2}{3}$



$\boxed{\theta_2 = \tan^{-1}\dfrac{2}{3}}$ $\qquad E_2 \times \dfrac{2}{\sqrt{13}} = \dfrac{\cancel{200}^{100}}{\sqrt{2}}$

$\boxed{E_2 = 100\sqrt{\dfrac{13}{2}}}$ $\qquad \Rightarrow |\vec{E_2}| = 262.5\,V/m$

$\theta_2 = 33.61°$

⇒ Bound surface charge density produced at interface = ?

$\bar{P}_1 = \epsilon_0(3-1)E = 2\epsilon_0 E_\perp \hat{n}, \quad \bar{P}_2 = \epsilon_0(2-1)E_\perp = \epsilon_0 E_\perp \hat{n}$

$\downarrow \hat{n}$ $\quad \sigma_{b_1}\text{(upper slab)} = 2\epsilon_0 E_\perp \hat{n}\cdot(+\hat{n}) = +2\epsilon_0 E_1 \cos 45°$

$\uparrow \hat{n}$ $\quad \sigma_{b_2}\text{(lower slab)} = \epsilon_0 E_\perp \hat{n}\cdot(-\hat{n}) = -\epsilon_0 E_2 \cos 33.61°$

$\sigma_{b_1} = +2\epsilon_0 \times 200 \times \dfrac{1}{\sqrt{2}} = +200\sqrt{2}\,\epsilon_0$

$\sigma_{b_2} = -\epsilon_0 \times 262.5 \times \dfrac{3}{\sqrt{13}} \qquad \sigma_b = \sigma_{b_1} + \sigma_{b_2}$

$= 375.6 \times 10^{-12}\ C/m^2$

$$\boxed{\overline{\nabla}\cdot\overline{D} = \rho_{free}}$$ (Using Gauss's law)

$\overline{D} = \epsilon\vec{E}$

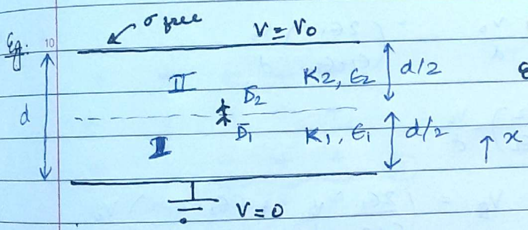
$$\overline{\nabla}\cdot\overline{E} = \frac{\rho_{free}}{\epsilon_0}$$

$\uparrow$ $-\overline{\nabla}V$

$$\Rightarrow \boxed{\nabla^2 V = -\frac{\rho_{free}}{\epsilon_0}} \Longrightarrow \text{Poission's } \cancel{\text{law}} \text{ eq}^n$$

$$\rho_{free} = 0 \Rightarrow \boxed{\nabla^2 V = 0} \Longrightarrow \text{Laplace's eq}^n$$

Eg.



Parallel plates capacitor

Electrostatic potential func^n = ?

Electric field = ?

Bounded surface charge = ?

Sol^n :-

We don't know ρ_{free} $\Rightarrow$ can't apply $\overline{D}$.

In this whole region, $\nexists$ no free charge, they exist only on plates surface

Applying laplace's eq^n, (V varies only in 1 dir^n)

$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0$$

$\vec{E}$: $\perp^r$ $\Rightarrow$ $\vec{D}$ also has only normal component

$\Rightarrow \overline{D}_1 = \overline{D}_2$

$$V = Ax + B$$

$V_I = A_1 x + \cancel{B_1}^{\,0} = A_1 x$

$V_{II} = A_2 x + B_2$ —— ①

$\rightarrow V_I(x=0) = B_1 = 0$ —— ②

$\rightarrow V_{II}(x=d) = A_2 d + B_2 = V_0$

V is always continuous

$V_I(d/2) = V_{II}(d/2)$

$\Rightarrow A_1 d/2 = A_2 d/2 + B_2$

$\Rightarrow B_2 = (A_1 - A_2)\, d/2$ —— ④

$\Rightarrow B_2 = \left(\frac{A_2\epsilon_2}{\epsilon_1} - A_2\right) d/2$

$\rightarrow \overline{D}_1 = \overline{D}_2$

$\epsilon_0 E_1 K_1 = K_2 E_2 \epsilon_0$

$E_1\epsilon_1 = E_2\epsilon_2$

$-\nabla V_1$

$(-A_1)\epsilon_1 = (-A_2)\epsilon_2$

$A_1\epsilon_1 = A_2\epsilon_2$ —— ③

Scanned by CamScanner

* For ∥ plate capacitor,
$D = \sigma_{free}$ (always)

$$\Rightarrow B_2 = A_2\left(\frac{\epsilon_2-\epsilon_1}{\epsilon_1}\right)\frac{d}{2} \qquad A_2 d + B_2 = V_0$$

$$\Rightarrow \frac{2\epsilon_1 B_2}{(\epsilon_2-\epsilon_1)\cancel{d}} = \frac{V_0 - B_2}{\cancel{d}}$$

$$\Rightarrow 2\epsilon_1 B_2 = (\epsilon_2-\epsilon_1)V_0 - (\epsilon_2-\epsilon_1)B_2$$

$$\Rightarrow (2\epsilon_1 + \epsilon_2 - \epsilon_1)B_2 = (\epsilon_2-\epsilon_1)V_0$$

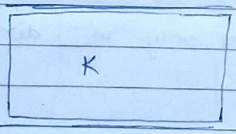
$$\Rightarrow \boxed{B_2 = \frac{(\epsilon_2-\epsilon_1)V_0}{(\epsilon_2+\epsilon_1)}}$$

$$\Rightarrow A_2 = \frac{2\epsilon_1}{\cancel{\epsilon_2-\epsilon_1}} \times \frac{\cancel{\epsilon_2-\epsilon_1}}{\epsilon_2+\epsilon_1}\frac{V_0}{d} = \left(\frac{2\epsilon_1}{\epsilon_1+\epsilon_2}\right)\frac{V_0}{d}$$

$$A_1 = \frac{\epsilon_2}{\epsilon_1}A_2 = \left(\frac{2\epsilon_2}{\epsilon_1+\epsilon_2}\right)\frac{V_0}{d}$$

$$\Rightarrow V_I = \left(\frac{2\epsilon_2}{\epsilon_1+\epsilon_2}\right)\frac{V_0}{d}x \qquad V_{II} = \left(\frac{2\epsilon_1}{\epsilon_1+\epsilon_2}\right)\frac{V_0}{d} + \left(\frac{\epsilon_2-\epsilon_1}{\epsilon_1+\epsilon_2}\right)V_0$$

12-4-17 / Energy stored in dielectric slab ~~containing~~ in ∥ plate capacitor



capacitance after inserting slab

$$U = \frac{1}{2}C_d V^2$$

$$= \frac{1}{2}\left(\frac{\epsilon_0 A K}{d}\right)(E_d d)^2$$

$$= \frac{1}{2}\frac{\epsilon_0 A K}{d} \times d^2 \times \frac{\sigma^2}{\epsilon_0^2 K^2}$$

$$= \frac{\sigma^2 A d}{2K\epsilon_0} = \frac{1}{2}\sigma\left(\frac{\sigma}{\epsilon_0 K}\right) \times dA$$

D, $\bar{E}_d$, volume

$$U = \frac{1}{2}DE_d \times \text{Volume}$$

$$\boxed{\text{Energy density}(u) = \frac{1}{2}\bar{D}\cdot\bar{E}}$$

$\vec{p}\cdot\vec{E} = Q(2a)\frac{KQ}{a^2} \Rightarrow$ Potential Energy

* So far, $u = \frac{1}{2}\epsilon_0 |E^2|$

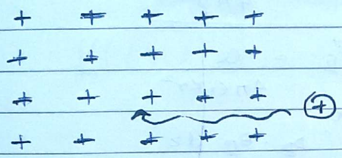
In case of dielectric, $u = \frac{1}{2}(\bar{D}\cdot E) = \frac{1}{2}(\epsilon_0 \bar{E} + \bar{P}\cdot\bar{E})$

$$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\bar{P}\cdot\bar{E}$$

Due to dielectric medium, e^- is bound in nucleus. Due to strecth, this energy gets developed.

$\vec{p}\cdot\vec{E}$: P.E. of dipole

Proof :



$U = \frac{1}{2}\epsilon_0 E^2 = 0$

(Electric field = 0)

Now, 1 extra amount of charge is brought near it so it will become polarised '−' ~~which~~ will shift towards it

Work done to bring that ⊕ charge :

$\Delta W = \iiint (\Delta \rho_{free}) V\, d\tau$

$\Delta\cdot\bar{D} = \rho_{free}$ $\quad\quad \Delta\cdot(\Delta\bar{D}) = \Delta\rho_{free}$

$\Rightarrow \Delta W = \iiint \Delta\cdot(\Delta\bar{D})\, V\, d\tau$

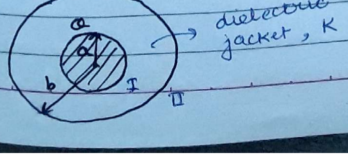
$$\Rightarrow \boxed{U = W = \frac{1}{2}\iiint_{\text{all space}} (\bar{D}\cdot\bar{E})\, d\tau}$$

use this formula in case of dielectric medium

(Read proof in Griffith)

* In case of vaccum, $\bar{P} = 0$

Ques. Find U of following system



$E_I \quad D_I = ?$

$E_{II} \quad D_{II} = ?$

$U = \frac{1}{2} \vec{D} \cdot \vec{E}$ $\qquad\qquad$ $\underline{\vec{E} = KQ}$

$$U = \frac{1}{2} \iiint_{\text{all space}} (\vec{D} \cdot \vec{E})\, d\tau$$

→ Inside metallic sphere, $E = 0$, $P = 0$, $D = 0$

→ $\underline{a \le r \le b}$:

$$\oiint \vec{D} \cdot d\vec{a} = Q_{free} = Q$$

$$\Rightarrow \vec{D} \cdot (4\pi r^2) = Q$$

$$\Rightarrow \boxed{\vec{D}_I = \frac{Q}{4\pi r^2}} \qquad \boxed{\vec{D}_{II} = \frac{Q}{4\pi r^2}}$$

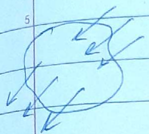
$$\vec{E}_I = \frac{Q}{4\pi \epsilon_0 K r^2} \qquad \vec{E}_{II} = \frac{Q}{4\pi \epsilon_0 K r^2}$$

$$U = \frac{1}{2} \iiint_{r=a}^{b} \vec{D}_1 \cdot \vec{E}_1\, d\tau + \frac{1}{2} \iiint_{r=b}^{\infty} D_{II} \cdot E_{II}\, d\tau$$

MAGNETOSTATICS

* Field is generated due to a steady current
* steady current :

$\boxed{\nabla \cdot \vec{J} = 0}$ ⇒ No. of charges arriving at a place = No. of charges leaving that place (No accumulation / No depletion)



Total current crossing surface : $\oiint \vec{J} \cdot d\bar{a}$

$\oiint \bar{J} \cdot d\bar{a} = \frac{-\partial}{\partial t} \iiint \rho \, d\tau$: change of volume charge density

going out, charge density ↓es inside region

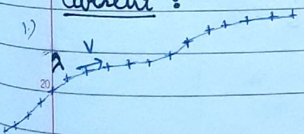
$\Rightarrow \iiint (\bar{\nabla} \cdot \bar{J}) d\tau + \iiint \frac{\partial \rho}{\partial t} d\tau = 0$

$\bar{\nabla} \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0$

→ if steady current, $\nabla \cdot \bar{J} = 0$

Current :

1.)



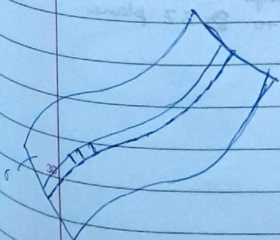
line current :

$\vec{I} = \frac{dQ}{dt} = \lambda \vec{v}$ (in free space)

length $= v \Delta t = \Delta l$

$I = \frac{\Delta Q}{\Delta t} = \frac{\lambda \Delta l}{\Delta t} = \frac{\lambda v \Delta t}{\Delta t} = \lambda v$

2.) Surface current :

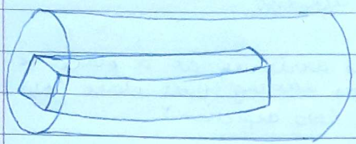


surface current : current per unit length $\perp^r$ to it

$\vec{K} = \sigma \vec{V}$

* When in free space, charges can move in any dir^n, I is vector

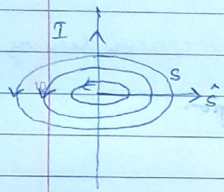
3.) volume current



Volume current $\vec{J} = \rho \vec{V}$

current per unit area $\perp^r$ to it.

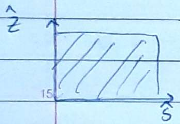
Biot - savart Law



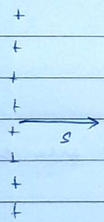
① $|\vec{B}| \propto I$

② $|\vec{B}| \propto \frac{1}{S}$

③ $\vec{B} \perp^r$ to ZS plane



Earlier →

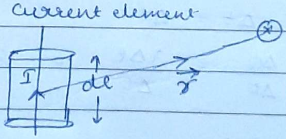


$\bar{E} = \frac{\lambda}{2\pi\epsilon_0 S} \Rightarrow \bar{E} \propto \frac{1}{S}$

⇒ Similar to point charges above

$dl\, I$: current element

Current element



$$d\vec{B} = C \frac{(I\, d\vec{l} \times \hat{r})}{r^2}$$

always $\perp^r$ to ~~B~~ SZ plane

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{(I\, d\vec{l} \times \hat{r})}{r^2}}$$

* $I\, d\vec{l}$ is not smallest source of Magnetostatic field while single point charge is smallest source of Electrostatic field.

Date 17/4/17

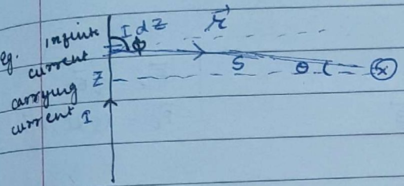
For continuous charge system :

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{r}}{r^2} \, dl' \quad : \text{ line charge distribution}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \hat{r}}{r^2} \, da' \quad : \text{ surface " "}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \hat{r}}{r^2} \, d\tau' \quad : \text{ volume " "}$$

eg. infinite current carrying current I



{ only when it is of infinite length, we can say that current is steady }

$$\int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\,dz \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \int \frac{I\,dz \sin\phi}{r^2}$$

$$= \frac{\mu_0}{4\pi} \int \frac{I\,dz \sin(90^\circ + \theta)}{r^2} = \frac{\mu_0}{4\pi} \int \frac{I\,dz \cos\theta}{r^2}$$

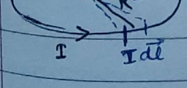
$$= \frac{\mu_0}{4\pi} \int \frac{I\,dz\cos\theta}{z^2 + r^2} = \frac{\mu_0 I}{4\pi} \int \frac{s \quad dz}{(z^2+s^2)^{3/2}} \qquad z = s\tan\theta, \quad dz = s\sec^2\theta d\theta$$

$$= \int \frac{\cancel{s}\,(\cancel{s}\sec^2\theta\,d\theta)}{s^{\cancel{3}}\sec^3\theta} = \int \frac{1}{s}\cos\theta\,d\theta$$

$$= \frac{\mu_0 I}{4\pi s} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} = \frac{\mu_0 I}{4\pi s} \lim_{z\to\infty} \left[\frac{2\cancel{z}}{\sqrt{s^2+z^2}}\right] = \boxed{\frac{\mu_0 I}{2\pi s} \hat{\phi}}$$

Eg. In a loop :

$$d\bar{B} = \frac{\mu_0}{4\pi} \int \frac{I\,d\bar{l} \times \hat{r}}{r^2} \qquad [\text{here, angle b/w } \hat{r} \,\&\, d\bar{l} = 90^\circ]$$



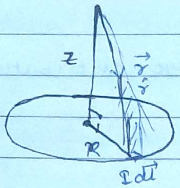
$$= \frac{\mu_0}{4\pi} \int \frac{I\,dl}{r^2}$$

$$= \frac{\mu_0 I}{4\pi r^2} \int_0^{2\pi r} dl$$

is Radius

$$= \frac{\mu_0 I}{4\pi R^2} (2\pi R) = \frac{\mu_0 I}{2r} \hat{z}$$

Ques



magnetic field will always be $\perp^r$ to the plane

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\,dl}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \int_0^{2\pi R} \frac{dl}{R^2+Z^2} = \frac{\mu_0 I\,(R)}{2(R^2+z^2)}\,\hat{r}$$

Lorentz force :

$$\vec{F}_L = q\,(\vec{V} \times \vec{B})$$

$$= I\,(d\vec{l} \times \vec{B}) = (I\,d\vec{l}) \times \vec{B}$$

$$\vec{F}_e = q\vec{E}$$

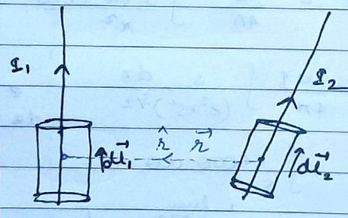
$$\vec{F}_L = (I\,d\vec{l}) \times \vec{B}$$

Properties :-

→ Work done by Lorentz force :

$$\vec{F}_L \cdot d\vec{l} = I\,(d\vec{l} \times \vec{B}) \cdot d\vec{l} = \underline{0}$$

•)



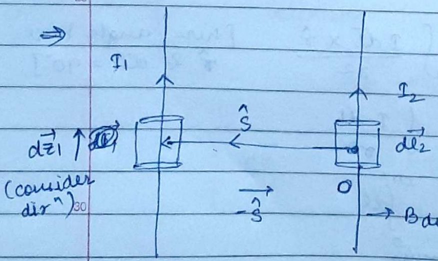
$\vec{B}_{at\ I}$

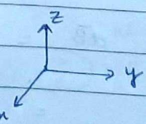
$$F\ (\text{at } I_1 \text{ due to } I_2) = (I_1\,dl_1) \times \frac{\mu_0}{4\pi} \frac{(I_2\,d\vec{l}_2 \times \hat{r})}{r^2}$$

$$\Rightarrow F_L = \frac{\mu_0}{4\pi} \frac{(I_1\,dl_1) \times (I_2\,d\vec{l}_2) \times \hat{r}}{r^2}$$

similar to $\vec{F}_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}\,\hat{r}$

⇒





→ B due to this $= \frac{\mu_0 I_2}{2\pi s}\,\hat{\phi}$

$$F_L = (I_1\,d\vec{z}_1) \times \left(\frac{\mu_0 I_2}{2\pi s}\right)\hat{\phi} = \frac{\mu_0 I_1 I_2\,dz}{2\pi s}\,(-\hat{s})$$

Camlin Page

Date / /

→ Force b/w the two wires will be attractive type of force.

→ **Electrostatics** | **Magnetostatics**

- $d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$ | $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$

- $\vec{F_e} = q\vec{E}$ | $\vec{F_L} = (I\,d\vec{l})\times\vec{B}$

- $E = \frac{1}{4\pi\epsilon_0}\iiint\frac{\rho\,d\tau'}{r^2}\hat{r}$ | $\vec{B} = \frac{\mu_0}{4\pi}\iiint\frac{\vec{J}\times\hat{r}}{r^2}\,d\tau'$

- $\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}$ | $\bar{\nabla}\cdot\bar{B} = 0$

- $E = -\nabla V$

- $\bar{\nabla}\times\bar{E} = 0$ | $\bar{\nabla}\times\bar{B} = \mu_0\vec{J}$

⇒ calc^n of $\bar{\nabla}\cdot B$

$$B(r) = \frac{\mu_0}{4\pi}\iiint\frac{J(r')\times r'}{r^2}\,d\tau'$$

$\left\{\bar{\nabla}\cdot(\vec{A}\times\vec{B}) = \bar{B}\cdot(\bar{\nabla}\times\bar{A}) - \vec{A}\cdot(\bar{\nabla}\times\bar{B})\right\}$

0 (it is in terms of r' while $\bar{\nabla}$ is wrt r)

$$\bar{\nabla}\cdot B(r) = \frac{\mu_0}{4\pi}\left[\iiint\frac{r'}{r^2}\cdot\left(\bar{\nabla}\times J(r')\right)d\tau' - \iiint J(r')\cdot\left(\bar{\nabla}\times\frac{r'}{r^2}\right)d\tau'\right.$$

$\boxed{\vec{\nabla}\times \text{conservative vector field} = 0}$

Electrostatic vector field (conservative vector field)

$\boxed{\bar{\nabla}\cdot\bar{B} \equiv 0}$ ⇒ change of flux per unit volume = 0

→ $\oint\!\!\oint \vec{B}\cdot d\bar{a} \equiv 0$ $\left\{\oint\!\!\oint\bar{E}\cdot d\bar{a} = \frac{q_{encl}}{\epsilon_0}\right\}$

→ $\oint\!\!\oint\bar{E}\cdot d\bar{a} = 0$ when $\bar{E}=0$ or $\vec{E}\neq 0$ & $q_{encl} = 0$ (equal & opp. charges)

Similarly, here, $\oint\!\!\oint\vec{B}\cdot d\bar{a} = 0$ ⇒ two types of sources with opp. types, they always coexist

Imp.

* Magnetic monopole does not exist.

$\Rightarrow$ Calc^n of $\bar{\nabla} \times \bar{B}$

$$\bar{B}(r) = \frac{\mu_0}{4\pi} \frac{I\,d\bar{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \iiint \frac{J(r') \times \hat{\imath}}{\imath^2}\, d\tau'$$

Using $\nabla \times (\bar{A} \times \bar{B}) = (\bar{B}\cdot\bar{\nabla})\bar{A} - (\bar{A}\cdot\bar{\nabla})\bar{B} + \bar{A}(\bar{\nabla}\cdot\bar{B}) - \bar{B}(\bar{\nabla}\cdot\bar{A})$

$\Rightarrow \bar{\nabla} \times \bar{B}(r) = \bar{\nabla} \times \{ \qquad \}$

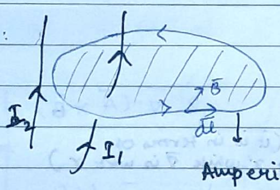
$\Rightarrow \boxed{\bar{\nabla} \times \bar{B} = \mu_0 J(r)}$ $\Rightarrow$ magnetic field is non-conservative vector field.

using stoke's theorem

$$\iint (\bar{\nabla} \times \bar{B})\, d\bar{a} = \mu_0 \iint J(r)\, da$$

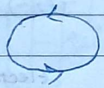
$\boxed{\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{encl}}$ $\rightarrow$ Ampere's law (Integral form)

Interpretation :



~~Ite~~ I_{encl} : penetrates the ~~enclosed~~ open surface

$I_{encl} = I_1$ (not I_2)



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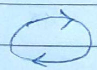

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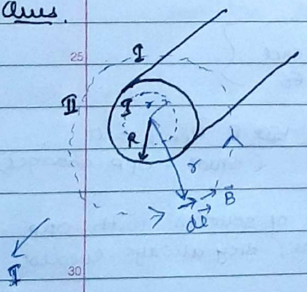
* ~~Ap~~ Ampere's law is analog to Gauss's law.

Ques.



Find B_I and B_{II}

I : current flowing in the ~~wire~~ wire.

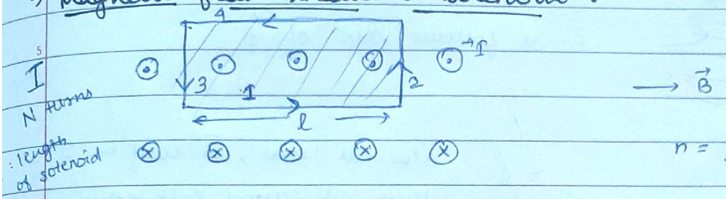
II : $\oint B_{II}\cdot d\bar{l} = B_{II}(2\pi r) = \mu_0 I_{encl}$

$\Rightarrow \boxed{|\vec{B}_{II}| = \frac{\mu_0 I}{2\pi r}}$ Ans.

I : $\oint \vec{B}_I \cdot d\vec{l} = \mu_0 I_{encl}$ $\quad B_I(2\pi r) = \mu_0 \frac{I}{\pi R^2} \times \pi r^2$

$$\boxed{|\vec{B_T}| = \frac{\mu_0 I r}{2R^2}}$$ Ans.

→ **Magnetic field inside a solenoid :**



$n = \frac{N}{L}$: no. of turns per unit length

* Always takes $\vec{B} \parallel d\vec{l} \Rightarrow$ Ampere's loop.

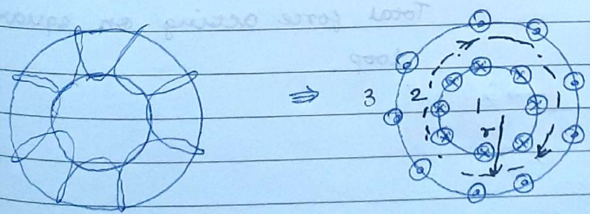
For 4^{th} Region (outside loop), magnetic field = 0

For 2nd ($\perp^r$ $d\vec{l}$ & $\vec{B}$) $\Rightarrow$ $\vec{B} \cdot d\vec{l} = 0$

For 3^{rd} : $B\,l = \mu_0 \left[I \times \frac{N}{L} \times l \right]$

$$\Rightarrow \boxed{|\vec{B}| = \frac{\mu_0 N I}{L}} \boxed{= \mu_0 n I}$$

→ **Toroid :**



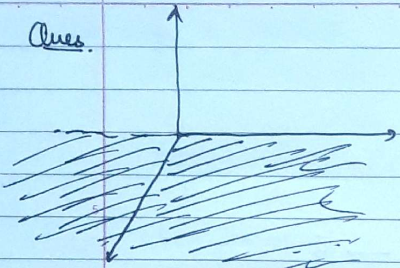
$\vec{B_1} = 0$ [$I_{encl} = 0$]

$\vec{B_2}(2\pi r) = \mu_0 I_{encl} = \mu_0 (IN)$

$$\boxed{B_2 = \frac{\mu_0 N I}{2\pi r}}$$

$\vec{B_3} = 0$ [$I_{encl} = I^+ + I^- = 0$]

Ques.

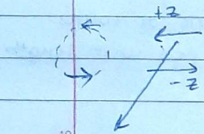


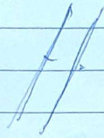
$\vec{K} = c\hat{x}$ $\vec{B} = ?$

⇓

surface current

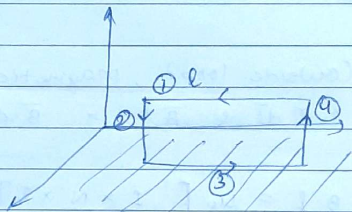
$\bar{B}$: it follows dir^n of $\hat{\phi}$





B/w all lines, $\bar{B}$ will be 0 as they will cancel each other. $\bar{B}$ will only be in ← & → dir

Amperean loop :

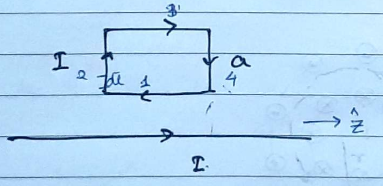


① + ③

$\bar{B}(2l) = \mu_0 (KL) = \mu_0 KL$

$$\boxed{\bar{B} = \frac{\mu_0 K}{2}}$$

Ques.



Total force acting on square loop.

$\bar{F}_B = q(\bar{v} \times \bar{B}) = I d\bar{l} \times \bar{B}$

$B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$

$\hat{z} \to \hat{s} \to \hat{\phi}$

$\bar{F}_{B_1} = I a(-\hat{z}) \times \frac{\mu_0 I}{2\pi s}(\hat{\phi})$

$= \frac{\mu_0 I^2 a}{2\pi s}\hat{s}$

$\hat{s} \times \hat{\phi}$

$\bar{F}_2 = 0 \quad \bar{F}_4 = 0$

$$\bar{F}_2 = \int I \, ds \cdot \frac{\mu_0 I}{2\pi s} = \frac{\mu_0 I^2}{2\pi} \int_{s=s}^{s+a} \frac{ds}{s} \, \hat{z}$$

$$= \frac{\mu_0 I^2}{2\pi} \ln \left| \frac{s+a}{s} \right| \hat{z}$$

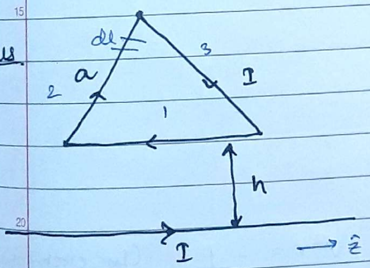
$$\bar{F}_3 = \int I(-ds) \frac{\mu_0 I}{2\pi s} \left[-\hat{s} \times \hat{\phi} \right] = \int_{s+a}^{s} \frac{\mu_0 I^2}{2\pi} \frac{ds}{s} (-\hat{z})$$

$$= \frac{\mu_0 I^2}{2\pi} \ln \left| \frac{s}{s+a} \right| (\hat{z})$$

$$\bar{F}_2 + \bar{F}_3 = 0$$

$$\bar{F}_4 = \int I \, (\cdot l)$$

Ans.



$$d\bar{l}_2 = ds\,\hat{s} + dz\,\hat{z}$$

$$d\bar{l}_3 = -ds\,\hat{s} + dz\,\hat{z}$$

we will see that $\hat{z}$ component in both will cancel each other

19-4-17 Electrostatics :

$$\vec{\nabla} \times \bar{E} = 0$$

also, we know that $\bar{\nabla} \times \bar{\nabla}\psi = 0$

$\Rightarrow \quad \vec{E} = -\nabla V$ ↳ scalar potential

Magnetostatics :

$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J} \Rightarrow$ non-conservative

Hence, $\bar{B}$ can't be written in form of scalar potential.

But we know, $\bar{\nabla} \cdot \bar{B} = 0$

$$\bar{\nabla} \cdot (\bar{\nabla} \times \bar{F}) \equiv 0$$

also,

$\Rightarrow \quad \bar{B} = \bar{\nabla} \times \bar{A}$

↳ Magnetic vector potential

* Here, $\bar{A}$ doesn't have any physical meaning
But it is imp. to study as it is analogous to V in Electros

Electrostatics	Magnetostatics
$\bar{E} = -\nabla V$	$\bar{B} = \bar{\nabla} \times \bar{A}$
$V \longrightarrow V + C$	$\bar{A} \longrightarrow \bar{A} + \bar{\nabla}\psi$ (scalar funcⁿ)
$\bar{E} = -\nabla V$	$\Rightarrow \bar{B} = \bar{\nabla} \times \bar{A}$

⇒ In calculating $\nabla \bar{A}$, we add $\bar{A} \rightarrow \bar{A} + \bar{\nabla}\psi$

$$\bar{\nabla} \cdot (\bar{A} + \bar{\nabla}\psi) = 0$$

$$\bar{\nabla} \cdot \bar{A} + \nabla^2 \psi = 0$$

$$\boxed{\nabla^2 \psi = -\bar{\nabla} \cdot \bar{A}}$$

Hence, $\boxed{\bar{\nabla} \cdot \bar{A} = 0}$ $\boxed{\bar{\nabla} \times \bar{A} = \bar{B}}$

Ampere's law :

$$\bar{\nabla} \times \bar{B} = \mu_0 \vec{J}$$

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{A}) = \mu_0 \vec{J}$$

$$\Rightarrow \bar{\nabla}(\underbrace{\bar{\nabla} \cdot \bar{A}}_{0}) - \nabla^2 A = \mu_0 \bar{J}$$

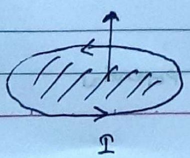
$$\Rightarrow \boxed{\nabla^2 A = -\mu_0 \vec{J}}, \quad \nabla^2 V = -\frac{\rho}{\epsilon_0} \text{ (In electrostatics)}$$

solⁿ of V : $\dfrac{1}{4\pi\epsilon_0} \iiint \dfrac{\rho \, d\tau'}{r}$

solⁿ of A : $A = \dfrac{\mu_0}{4\pi} \iiint \dfrac{\bar{J} \, d\tau'}{r}$

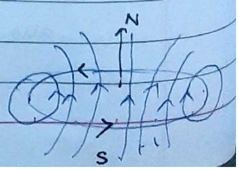
→ We earlier used multipole expⁿ to find V due to charge system. Similarly, it can be used in magnetostatics.

Eg.

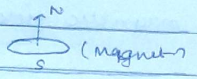


$\vec{A}$

$\vec{\mu}_m = \vec{A} I$
(magnetic dipole moment)



N



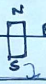
S

S ⟶ N (it will allign itself in dir^n of Magnetic field)

$$V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}\cdot\hat{r}}{r^2}$$

$$\vec{A}\text{ (here)} = \frac{\mu_0}{4\pi} \frac{\vec{\mu}_m \times \hat{r}}{r^2}$$

Ques.

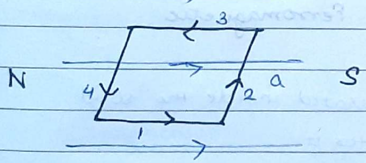


Bar magnet (Ferromagnetic material)

N S

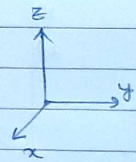
[S ⟶ N] gets alligned (why?)

→



$\vec{F} = (I\,d\vec{l} \times \vec{B})$

$\bar{F}_1 = \bar{F}_3 = 0$

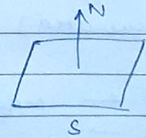


$\bar{F}_2 = \cancel{I\,dl}\ Ia(-\hat{i}) \times B(\hat{j}) = -IaB\,\hat{k}$

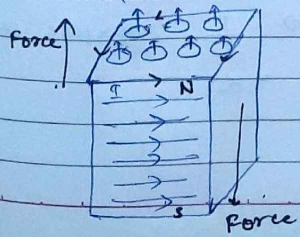
$\bar{F}_4 = IaB\,\hat{k}$

It experiences a torque

It will rotate s.t. its top is in the dir^n of S

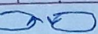


→ In above question (Bar Magnet)



all atoms show same magnetic dipole moment. Hence, some current exists due to motion of $e^{\ominus}$

Force



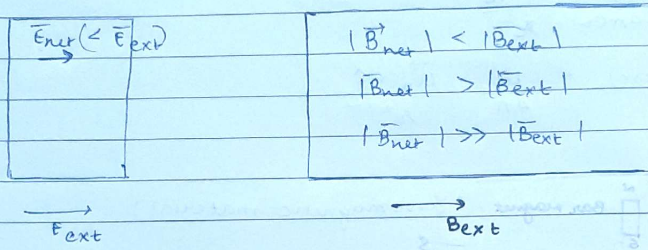
Net current = 0

But for boundary, current exists (Bound Current)

Due to the force, it will experience torque which will result ~~show~~ its allignment in dirn of magnetic field.

Magnetic field inside a Magnetic Material

Dielectric Material | Magnetic Material



3 cases possible :

1.) $|\vec{B}_{net}| < |\vec{B}_{ext}|$ $\longrightarrow$ Diamagnetic

2.) $|\vec{B}_{net}| > |\vec{B}_{ext}|$ $\longrightarrow$ Paramagnetic

3.) $|\vec{B}_{net}| >> |\vec{B}_{ext}|$ $\longrightarrow$ Ferromagnetic

1.) & 2.) $\Rightarrow$ some field is generated inside the solid some I must be responsible for it.

The magnetic material gets magnetised (Magnetic dipole gets generated) after applying $\vec{B}_{ext}$.

$\vec{M} \leftarrow$ Magnetization vector

$$\vec{M} = \frac{\sum \vec{\mu}_m}{\Delta V} = \text{Magnetic dipole moment per unit volume}$$

Dimension : $\dfrac{A I}{\Delta V}$ (area) $= \dfrac{L^2 A}{L^3} \Rightarrow \dfrac{I}{L}$; surface current $= K$

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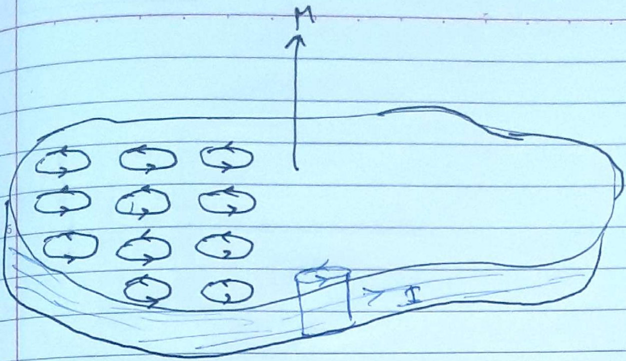
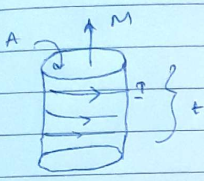


NO I_{net} inside, only I exist on boundary

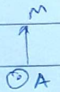


$|\vec{\mu}_m| = |\vec{M}| At$ $\Rightarrow$ $|\vec{M}| = \frac{I}{t}$

$|\vec{\mu}_m| = AI$

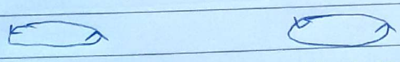
$|\vec{M}| = K$

$\boxed{\vec{M} \times \hat{n} = K_b \text{ : Bound surface current}}$



$\Rightarrow I \perp^r$ to M & A

* If magnetisation is non-uniform,



less magnetisation
$\Rightarrow$ less I

More magnetisation
$\Rightarrow$ more I

$\Rightarrow I_{net}$ inside $\neq 0$

* Bound volume charge Density

$$\boxed{\vec{J}_b = \vec{\nabla} \times \vec{M}}$$

we can find $\vec{B}$ using Ampere's law.

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