LNMIIT/B.Tech/C/IC/2017-18/EVEN/MTH221/M

LNMIIT
The LNM Institute of Information Technology

16UCS126.

THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
PROBABILITY AND STATISTICS & MTH221
MID TERM

Time: 90 minutes | Date: 20/02/2018 | Maximum Marks: 25

Note: You should attempt all questions. Your writing should be legible and neat. Marks awarded are shown next to the question. Please make an index showing the question number and page number on the front page of your answer sheet in the following format, otherwise you may be penalized by the deduction of **2 marks**.

Question No.				
Page No.				

1. In the game of cards, the entire deck of 52 cards is dealt out to 4 players what is the probability that [2 + 2]
   (a) one of the players receives all 13 spades;
   (b) each player receives 1 ace?

2. Let each of N men throws his hat at a place. The hats are mixed up. and then each man randomly select from among their own N hats. What is the probability that exactly k of the N men have matches? [4]

3. State the Bayes theorem and using it solve the following problem: [4]
   Suppose that there are two websites, A and B, for renting books. The site A receives 60% of all orders. Among the orders placed on site A, 75% arrive on time. Among the orders placed on site B, 90% arrive on time. Given that an order arrived on time, find the probability that the order was placed on site B.

4. Show that Poisson random variable may be used as an approximation for a binomial random variable with parameter (n, p), when n is large, p is small enough so that $\lambda = np$ is moderate size. Compute the expected value and variance of poisson random variable. [5]

5. Let X be continuous random variable denotes the required time (in years) to develop a software. Suppose that X has the following probability density function: [4]

$$f(x) = \begin{cases} b(4x - 2x^2), & 0 < x < 2; \\ 0, & \text{otherwise.} \end{cases}$$

   (a) What is the value of b
   (b) Compute the probability that it takes more than one year to develop the software.
   (c) Compute the expected number of years it takes to develop a software.

6. Let X be an exponential random variable. Find the distribution function of X and X^2. [2 + 2]