Attendance - 5%
Quiz - (10 + 5)%
Mid Term - 30%
End Term - 50%

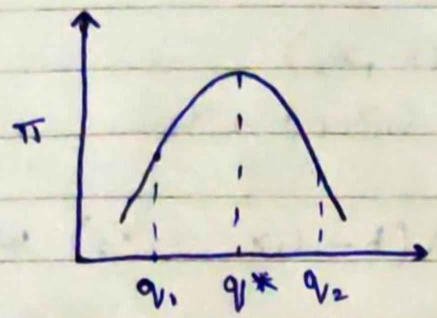
Att. ≥ 90% - 5 marks
90 > Att ≥ 80 - 4 marks
80 > Att ≥ 75 - 3 marks
75 > Att ≥ 70 - 2 marks
70 > Att ≥ 65 - 1 mark
65 > Att - 0 mark

APPECO

zwacf9z

Necessary & sufficent condition for Profit Maximisation

Profit $(\pi) = f(q)$ {quantity}



at q_1 $\frac{d\pi}{dq} > 0$

at q^* $\frac{d\pi}{dq} = 0$

at q_2 $\frac{d\pi}{dq} < 0$

Necessary

Maximum $\frac{d^2\pi}{dq^2} < 0$ concave fxn

Sufficent

Min $\frac{d^2\pi}{dq^2} > 0$ (convex Function)

Implicit Function

ex - $Q = f(K, L)$ — Consumer → Jeans & Shirt; Producer → $L = f(K)$

$y = f(x_1, x_2(x_1))$, considering y = const.

$$\frac{dy}{dx_1} = \frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2}\frac{dx_2}{dx_1}$$

Since y = const $\frac{dy}{dx_1} = 0$ $\Rightarrow$ $\boxed{\frac{dx_2}{dx_1} = -\frac{\partial f/\partial x_1}{\partial f/\partial x_2}}$

~~Constrained Maximisation~~

$y = f(x_1, x_2)$ ~~(Optimisation for Unconstrained case)~~

$dy = f_1\,dx_1 + f_2\,dx_2$ where $f_1 = \partial y/\partial x_1$

$f_2 = \partial y/\partial x_2$

$$d^2y = \frac{\partial}{\partial x_1}(f_1 dx_1 + f_2 dx_2)dx_1 + \frac{\partial}{\partial x_2}(f_1 dx_1 + f_2 dx_2)dx_2$$

$$= f_{11}dx_1^2 + f_{22}dx_2^2 + 2f_{12}dx_1 dx_2$$

for max. profit $d^2y < 0$

Hessian Determinant (for Function) (X isoquant)

$$\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} \begin{matrix} >0 \text{ (concave function)} & f_{11}<0 \text{ or } f_{22}<0 \\ \geq 0 \text{ (convex function)} & f_{11}>0 \text{ or } f_{22}>0 \end{matrix}$$

$f_{11} = \dfrac{\partial^2 f}{\partial x_1^2}$ $\quad f_{12} = \dfrac{\partial^2 f}{\partial x_1 \partial x_2} = f_{21}$

$f_{22} = \dfrac{\partial^2 f}{\partial x_2^2}$ $\quad f_{21} = \dfrac{\partial^2 f}{\partial x_2 \partial x_1} = f_{12}$

Young's theorem $\Rightarrow f_{12} = f_{21}$

Optimisation for constrained case

Objective to max. $z = f(x_1, x_2)$

subject to $\boxed{b = c_1 x_1 + c_2 x_2}$ (budget constraint)

Solve by Lagrangian method $\boxed{C = WL + rK}$ (cost constraint)

$\mathcal{L} = f(x_1, x_2) + \lambda(b - c_1 x_1 - c_2 x_2)$ where $\lambda > 0$

First order conditions:

① $\dfrac{\partial \mathcal{L}}{\partial x_1} = 0 \Rightarrow \dfrac{\partial f}{\partial x_1} - \lambda c_1 = 0$

② $\dfrac{\partial \mathcal{L}}{\partial x_2} = 0 \Rightarrow \dfrac{\partial f}{\partial x_2} - \lambda c_2 = 0$

③ $\dfrac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow b - c_1 x_1 - c_2 x_2 = 0$

① ÷ ② gives $\dfrac{c_1}{c_2} = \dfrac{\partial f / \partial x_1}{\partial f / \partial x_2}$ — ④

Using ③, $b = c_1 x_1 + c_2 x_2$ — ⑤

Solve x_1 and x_2 using ④ & ⑤

Using ① $\boxed{\lambda = \dfrac{\partial f / \partial x_1}{c_1} = \dfrac{\text{Marginal Benefit}}{\text{Marginal cost}}}$ Significance of Lagrange Multiplier

For SOC

$d^2 y = f_{11} dx_1^2 + f_{22} dx_2^2 + 2 f_{12} dx_1 dx_2$

$b = c_1 x_1 + c_2 x_2$

$0 = c_1 dx_1 + c_2 dx_2$

$$d^2y = f_{11}\,dx_1^2 - 2f_{12}\frac{f_1}{f_2}dx_1^2 + f_{22}\frac{f_1^2}{f_2^2}dx_1^2$$

$$\frac{d^2y}{dx_1^2} = \left(\frac{f_{11}f_2^2 - 2f_{12}f_1f_2 + f_{22}f_1^2}{f_2^2}\right) < 0$$

$d^2y < 0$ $\quad \begin{bmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{21} & f_{22} \end{bmatrix} > 0$ Bordered Hessian (Quasi Concave)

Objective function is quasi-concave function.

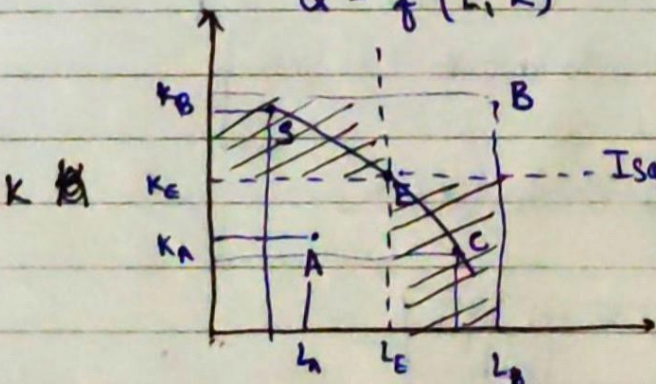
$$f((1-\lambda)a + \lambda b) \geq \min(f(a), f(b))$$

.8.19

Isoquant

Locus of combination of 2 or many inputs such that the production level remains fixed.

$Q = f(L, K)$

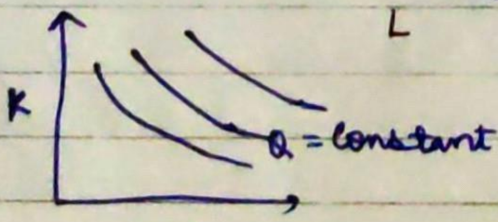


Preference Order (More Production)

$\boxed{B > E > A}$

not possible for A & B

Hence isoquant line lies in II and IV quadrant



~~More~~ K Higher the line, higher the productivity.

$dQ = 0$ (for isoquant)

$$0 = \frac{\partial f}{\partial K}dK + \frac{\partial f}{\partial L}dL = f_K\,dK + f_L\,dL$$

$$\frac{dK}{dL} = -\frac{f_L}{f_K} < 0 = \frac{MP_L}{MP_K}$$

Downward facing

$$\frac{d^2K}{dL^2} = \frac{d}{dL}\left(\frac{-f_L}{f_K}\right) = -\frac{f_K f_{LL} + f_L f_{LK}\frac{dK}{dL}}{f_K^2}$$

$$= -\frac{f_K f_{LL} - f_L^2 f_{LK}/f_K}{f_K^2}$$

$$= -\left(\frac{f_K^2 f_{LL} + f_L^2 f_{LK}}{f_K^3}\right)$$

$$+ f_L\left(f_{KL} + f_{KK}\left(-\frac{f_L}{f_K}\right)\right)$$

$$+ f_L\left(\frac{f_{KL} f_K - f_{KK} f_L}{f_K^3}\right)$$

$$= \frac{f_K^2 f_{LL} + f_L^2 f_{KK} - 2 f_L f_K f_{LK}}{f_K^3}$$

$$\frac{d^2K}{dL^2} = \frac{d\left(\frac{dK}{dL}\right)}{dL} = \frac{d\left(-f_L/f_K\right)}{dL} = 0 - \frac{d\left(f_L/f_K\right)}{dL} = -\left[\frac{\partial(f_L/f_K)}{\partial L} + \frac{\partial(f_L/f_K)}{\partial K}\frac{dK}{dL}\right]$$

$$= -\left[\frac{f_K f_{LL} - f_L f_{KK}}{f_K^2} - \frac{f_L}{f_K}\left(\frac{f_K f_{LK} - f_L f_{KK}}{f_K^2}\right)\right]$$

$$= -\left[\frac{f_K^2 f_{LL} - f_L f_K f_{LK} - f_L f_K f_{LK} + f_L^2 f_{KK}}{f_K^3}\right]$$

$$= -\left[\frac{f_L^2 f_{KK} + f_K^2 f_{LL} - 2 f_L f_K f_{LK}}{f_K^3}\right]$$

$$= -\frac{1}{f_K}\left[f_{KK}\left(\frac{f_L}{f_K}\right)^2 + f_{LL} - 2\left(\frac{f_L}{f_K}\right)f_{LK}\right]$$

(−) (+)   + (−)   + (+)

(−)   + (−)   + (−)

(−)

REFER COPY

Cobb-Douglas Function

$$Q = AK^{\alpha} L^{\beta} \qquad 1 > \alpha, \beta > 0 \qquad \alpha + \beta = 1$$

Regression Problem $\quad Q = A + BK + CL + E(\text{error})$

$$\frac{dK}{dL} = -\frac{f_L}{f_K} = \frac{-\beta K}{\alpha L}$$

$$\frac{d^2K}{dL^2} = \frac{f_K^2 f_{LL} - 2 f_K f_L f_{KL} + f_L^2 f_{KK}}{f_K^3}$$

$f_K = \alpha A K^{\alpha-1} L^{\beta} = \frac{\alpha Q}{K} > 0 \qquad f_{KK} = \alpha(\alpha-1) A K^{\alpha-2} L^{\beta} < 0$

$f_L = \beta A K^{\alpha} L^{\beta-1} = \frac{\beta Q}{L} > 0 \qquad f_{LL} = \beta(\beta-1) A K^{\alpha} L^{\beta-2} < 0$

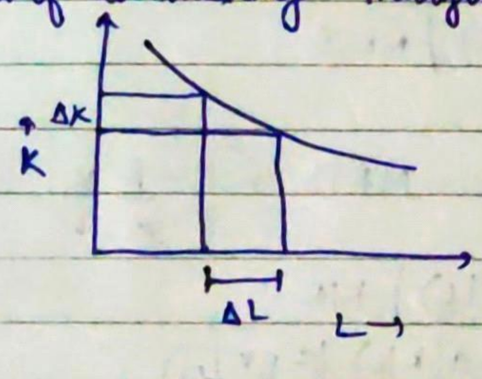
$f_{KL} = \alpha\beta A K^{\alpha-1} L^{\beta-1} = \frac{\alpha\beta}{KL} Q > 0$

$$-\frac{d^2K}{dL^2} = \frac{(+)(-) - (+)(+)(+) + (+)(-)}{} = (-) + (-) + (-) < 0$$

$\Rightarrow \frac{d^2K}{dL^2} > 0$ Convex $(+)$

Slope of isoquant = Marginal Rate of Technical ~~substitution~~ substitution (MRTS)

Law of diminishing marginal rate of technical ~~sts~~ substitution



So for increasing 1 unit of capital more than 1 unit of labour is sacrificed.

Q.) $\quad Q = KL - 0.8K^2 - 0.2L^2 \qquad\qquad AP_L = \frac{Q}{L}$

if $K = 10$ ~~at~~ at what L is AP_L maximum?

Ans $\quad AP_L = \frac{Q}{L} = K - 0.8\frac{K^2}{L} - 0.2L$

$$\frac{d\,AP_L}{dL} = \cancel{\frac{dK}{dL}}^{0} - 0.8\left[\frac{2K}{L}\cancel{\frac{dK}{dL}}^{0} - \frac{K^2}{L^2}\right] - 0.2$$

$$0 = 0.8 \times \frac{100}{L^2} - 0.2$$

$$L^2 = \frac{80}{0.2} = 400$$

$$L = 20$$

(ii) at what L, MP_L is ~~max~~ zero ?

$$\cancel{MP_L = \frac{dQ}{dL}} = \cancel{K - 0.4L} \quad \cancel{\frac{0.8K^2}{L^2} - 0.2}$$

$$\cancel{\frac{dMP_L}{dL} = 0.8K^2\left(\frac{-2}{L^3}\right)}$$

$$MP_L = \frac{dQ}{dL} = \frac{d(KL - 0.8K^2 - 0.2L^2)}{dL}$$

$$0 = K - 0.4L$$

$$\cancel{\frac{dMP_L}{dL} = 0.4}$$

$$L = \frac{K}{0.4} = 25$$

Elasticity of Substitution

$$\sigma = \frac{\%\text{ change in } K/L}{\%\text{ change in MRTS}}$$

$$= \frac{d(K/L)\,/\,(K/L)}{d(MRTS)\,/\,MRTS}$$

(A) For Cobb-Douglas Fxn, $MRTS = -\frac{\beta}{\alpha}\frac{K}{L}$, $d(MRTS) = \frac{-\beta}{\alpha}d\left(\frac{K}{L}\right)$

$$\sigma = \frac{d(K/L)\,/\,K/L}{-\beta/\alpha\, d(K/L)\,/\,(-\beta K/\alpha L)}$$

$$= \frac{\cancel{d(K/L)}\ \cancel{(-\beta/\alpha)}\ \cancel{(K/L)}}{\cancel{K/L}\ \cancel{(-\beta/\alpha)}\ \cancel{(d(K/L))}} = 1$$

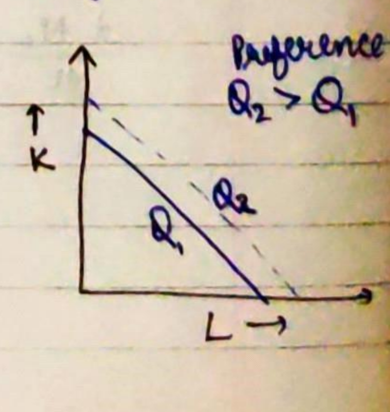
ie equal proportion of K and L can be interchanged

(B) For Production fxn where K and L are perfect substitutes

example - $Q = aK + bL$

$$0 = a\,dK + b\,dL$$

$$\frac{dK}{dL} = \frac{-b}{a}$$



give explanation for mathematics.

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Date

$$MRTS = \frac{dK}{dL} = \frac{-b}{a}$$

$$d(MRTS) = 0$$

$$\sigma = \frac{1}{0} = \text{undefined } (\infty)$$

This means we can interchange any amount of K with L ie ~~we~~ the company can change whole of K to L or vice versa.

(c) For Perfect Complements ie $Q = \min\{\alpha K, \beta L\}$

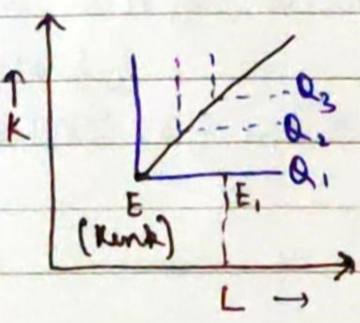
In these function some amount of K and L each are necessary for producing output, and the proportion is decided by α, β. Example - left shoe, right shoe.

Input is used in fixed proportion $= \frac{K}{L}$ which is constant

since $\frac{K}{L}$ = constant

$$d(K/L) = 0$$

$$\sigma = 0$$



$$Q_3 > Q_2 > Q_1$$

Producer is interested only at Kink point because at that point, ~~and~~ there is least input of K and L whereas at E_1, at same amount of Q, L is more.

<u>Euler's Theorem</u>

given $Z = f(x, y)$

$$\Rightarrow \quad x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$$ where n is degree of homogeneity

For example - $Z = x^2 + y^2 + xy$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x(2x+y) + y(2y+x)$$
$$= 2(x^2+y^2+xy) = nz$$

so $\boxed{n = 2}$

Product Exhaustion Theorem

If a production fxn is homogeneous of degree one, then the factors of production are rewarded equal to their marginal product.

$Q = f(K, L)$

if Homogeneous of degree $= 1$

$K \frac{\partial f}{\partial K} + L \frac{\partial f}{\partial L} = WQ$ where $W = w_1 + w_2$

if $W > 1$ Total payments $>$ Total output

if $W < 1$ Total payments $<$ Total output.

14-8-19 Concavity & Convexity through the Hessian determinant.
(for function) (x isoquant / curve)

$H = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix}$ concave function if H is -ve definite

$f_{11} < 0$ (first minor)

$f_{11} f_{22} - f_{12} f_{21} > 0$ (second minor)

* $F = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

$a =$ First minor

$\begin{vmatrix} a & b \\ d & e \end{vmatrix} =$ Second —

$F =$ Third —

Alt Minors have opposite sign

Production Fxn $y = f(x_1, x_2)$

$\frac{\partial x_2^2}{\partial x_1^2} < 0$ (convex)

$\frac{\partial x_2^2}{\partial x_1} > 0$ (concave)

Concavity / Convexity For curve

$$w = \frac{MP_K}{AP_K}$$

$$\frac{\partial Q}{\partial K} = MP_K \qquad \frac{\partial Q}{\partial L} = MP_L$$

$$K \times \frac{\partial Q}{\partial K} \qquad L \times \frac{\partial Q}{\partial L}$$ } Total contribution to the output given by labour.

Total payments = W×Total Output

Q $q = A\, x_1^{\alpha}\, x_2^{1-\alpha}$ (Cobb Douglas Fxn)

$$\frac{\partial q}{\partial x_1} = A\alpha\, x_1^{\alpha-1} x_2^{1-\alpha} = A\alpha \left(\frac{x_1}{x_2}\right)^{\alpha-1}$$

$$\frac{\partial q}{\partial x_2} = A(1-\alpha)\, x_1^{\alpha} x_2^{-\alpha} = A(1-\alpha)\left(\frac{x_1}{x_2}\right)^{\alpha}$$

$$x_1 \frac{\partial q}{\partial x_1} + x_2 \frac{\partial q}{\partial x_2} = A\alpha x_1^{\alpha} x_2^{1-\alpha} + A(1-\alpha)\, x_1^{\alpha} x_2^{1-\alpha}$$

$$= \alpha q + (1-\alpha) q$$

$$= q$$

$$\Rightarrow \boxed{n = 1}$$

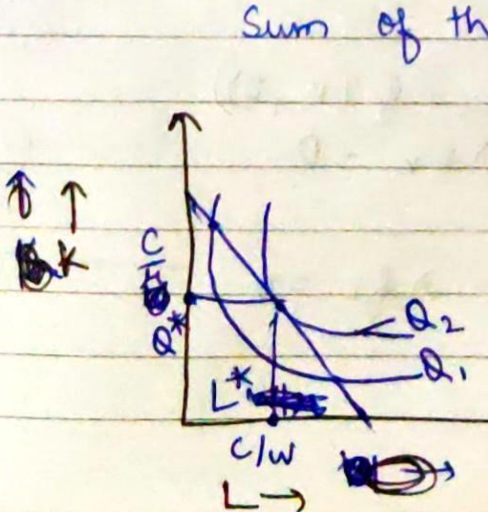
$$L\frac{\partial f}{\partial L} + K\frac{\partial f}{\partial K} = nQ$$

$$\frac{L}{Q}\frac{\partial f}{\partial L} + \frac{K}{Q}\frac{\partial f}{\partial K} = n$$

$$\boxed{\frac{MP_L}{AP_L}} + \frac{MP_K}{AP_K} = n$$

output elasticity.

Sum of these elasticities should be n



$wL + rK = C$

wages rent

slope $= \frac{w}{r}$

Since higher the isoquant, higher the production. So at this point production is max out of all intersecting pt

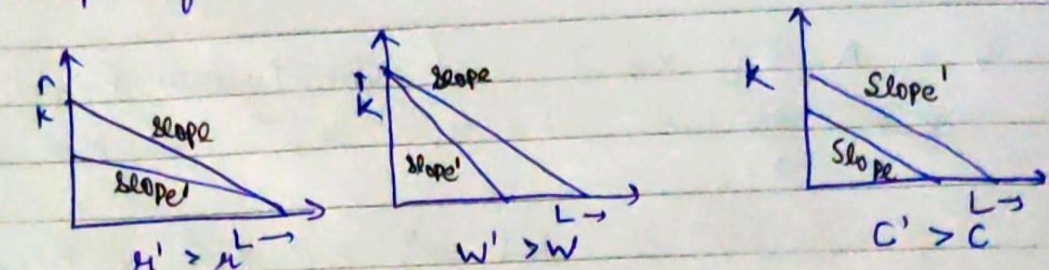
Eqm condition will be when slope of isoquant and iso cost lines are equal.

$WL + rK = C$

$rK = -\frac{W}{r} L + \frac{C}{r}$

<u>Iso-cost line</u> – ~~locat~~ locus of combination of inputs such that the cost of production remains fixed.

Shape of iso-cost line



CONSTRAINED OPTIMIZATION

(A) Max Q ~~to~~ wrt cost constraints. Primal

(B) Min C — output constraints. Dual

(A) $$Z = f(K, L) + \lambda(C - WL - rK)$$

$$\frac{\partial Z}{\partial K} = 0 \Rightarrow f_K - \lambda r = 0 \quad \text{—— (1)}$$

$$\frac{\partial Z}{\partial L} = 0 \Rightarrow f_L - \lambda W = 0 \quad \text{—— (2)}$$

$$\frac{\partial Z}{\partial \lambda} = 0 \Rightarrow C - WL - rK = 0 \quad \text{—— (3)}$$

(1) ÷ (2) $$\frac{f_K}{f_L} = \frac{r}{W} \rightarrow \left[\frac{-1}{\text{Slope of iso cost line}}\right]$$

$$\frac{f_K}{f_L} \longrightarrow \left[\frac{-1}{\text{Slope of isoquant line}}\right]$$

(B) $$L = WL + rK + \lambda(\bar{Q} - f(K, L))$$

$$\frac{\partial L}{\partial K} = 0 \Rightarrow r - \lambda f_K = 0 \quad \text{—— (1)}$$

$$\frac{\partial L}{\partial L} = 0 \Rightarrow W - \lambda f_L = 0 \quad \text{—— (2)}$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \bar{Q} = f(K, L) \quad \text{—— (3)}$$

(1) ÷ (2) $\quad \frac{f_K}{f_L} = \frac{r}{w}$

K^*, L^*

Q.) Min $C = wL + rK$ st $\bar{Q} = K^\alpha L^\beta$ Find K^*, L^*

Ans

$$\frac{\partial Z}{\partial L} = 0 \Rightarrow w = \lambda \beta K^\alpha L^{\beta-1} \quad \text{—— (1)}$$

$$\frac{\partial Z}{\partial K} = 0 \Rightarrow r = \lambda \alpha K^{\alpha-1} L^\beta \quad \text{—— (2)}$$

$$\frac{\partial Z}{\partial \lambda} = 0 \Rightarrow \bar{Q} = K^\alpha L^\beta \quad \text{—— (3)}$$

(1) ÷ (2) $\quad \frac{w}{r} = \frac{\beta}{\alpha} \frac{K}{L}$

$$L = \frac{\beta r K}{w \alpha} \quad \text{—— (4)}$$

Using (4) in (3), we get, $\bar{Q} = K^\alpha L^\beta = K^\alpha \left(\frac{r \beta K}{w \alpha}\right)^\beta$

$$K^* = \left(\frac{w\alpha}{r\beta}\right)^{\left(\frac{\beta}{\alpha+\beta}\right)} \bar{Q}^{\left(\frac{1}{\alpha+\beta}\right)}$$

Using K^* in (4)

$$L^* = \frac{\beta r}{w\alpha} \left(\frac{w\alpha}{r\beta}\right)^{(\beta/\alpha+\beta)} \bar{Q}^{\left(\frac{1}{\alpha+\beta}\right)}$$

$$= \left(\frac{\beta r}{w\alpha}\right)^{(\alpha/\alpha+\beta)} \bar{Q}^{\left(\frac{1}{\alpha+\beta}\right)}$$

$$C^* = wL^* + rK^*$$

$$= Q^{\frac{1}{\alpha+\beta}} \left[w^{\beta/\alpha+\beta} \left(\frac{\beta r}{\alpha}\right)^{\alpha/\alpha+\beta} + r^{\alpha/\alpha+\beta} \left(\frac{w\alpha}{\beta}\right)^{\beta/\alpha+\beta} \right]$$

19-8-19

Shepard's Lemma

$$\frac{\partial c^*}{\partial w} = L^* \quad , \quad \frac{\partial c^*}{\partial r} = K^*$$

Q.) $Q = K^{\frac{1}{2}} L^{1/2}$; $[w = 12 \; ; \; r = 3]$

$q_0 = 40$

Short cut

$$K^* = \left(\frac{12}{3}\right)^{\frac{1}{2}} \times 40 = 80$$

$$L^* = 40 \times \left(\frac{3}{12}\right)^{\frac{1}{2}} = 20$$

Ans $Z = 12L + 3K + \lambda(40 - \sqrt{KL})$

$$\frac{\partial Z}{\partial L} = 12 - \frac{\lambda}{2}\sqrt{\frac{K}{L}} = 0 \Rightarrow 24 = \lambda\sqrt{\frac{K}{L}}$$

$$\frac{\partial Z}{\partial K} = 3 - \frac{\lambda}{2}\sqrt{\frac{L}{K}} = 0 \qquad 6 = \lambda\sqrt{\frac{L}{K}}$$

$$\frac{\partial Z}{\partial \lambda} = 40 - \sqrt{KL} = 0 \Rightarrow 40 = \sqrt{KL}$$

① ÷ ② $\Rightarrow 4 = \frac{K}{L} \Rightarrow 4L = K$

Using in ③, $40 = \sqrt{4L \times L}$

$40 = 2L$

$L = 20$

$\Rightarrow K = 80$

To check, we solve bordered hessian

$$\begin{matrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{21} & f_{22} \end{matrix}$$

$\frac{w}{r} = k$. $\frac{f_L}{f_K} = m$

c $M > K$ $K, L \rightarrow$ Subst

$\frac{f_L}{f_K} > \frac{w}{r}$. $r \downarrow$ $w \downarrow$

$w(\downarrow)$ $r(\uparrow)$ $w(\uparrow)$; $\frac{w}{r}(\uparrow)$

$\hookrightarrow K(\uparrow)\ L(\downarrow)$

$r(\uparrow)\ \frac{w}{r}(\downarrow)$

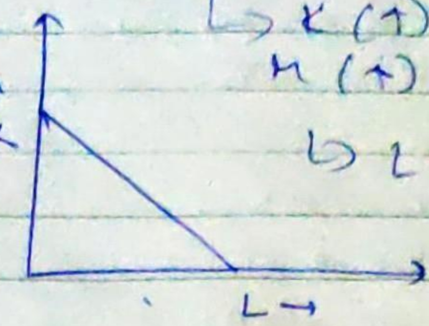
$\hookrightarrow L(\uparrow) ; K(\downarrow)$

For Perfect Substitute

$Q = aL + bK$

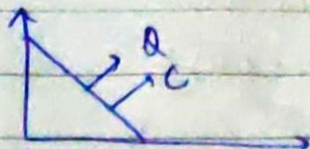
$C = wL + rK$

slope of isoquant $= \frac{-a}{b}$



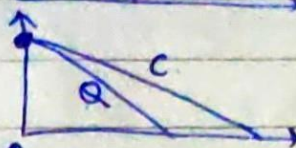
—— isocost $= \frac{-w}{r}$

If $\frac{-a}{b} = \frac{-w}{r}$

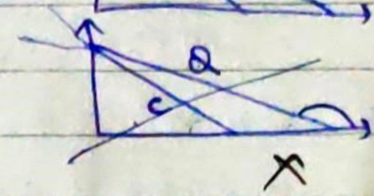


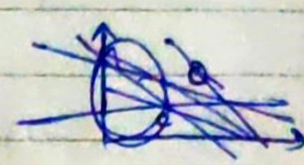
Infinitely many soln

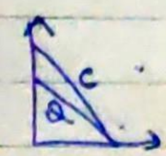
$\frac{-a}{b} < \frac{-w}{r}$



$\frac{-a}{b} > \frac{-w}{r}$



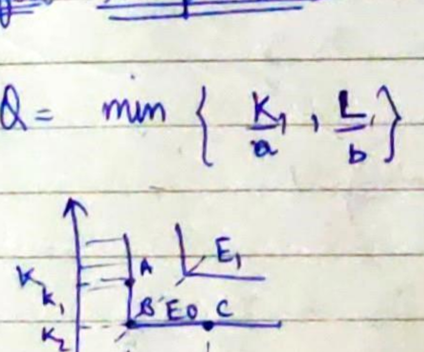




$MRTS = \frac{MP_L}{MP_K}$

Perfect Complements

$Q = \min\left\{ \frac{K_1}{a}, \frac{L_1}{b} \right\}$



$A = \min(K_1 L_1)$

$B = (K_2 L_1)$

$C = (K_2 L_2)$

$\frac{K}{a} = \frac{L}{b}$

$bK = aL$

$K = \frac{a}{b} L$

$C = wL + rK$

$C = wL + r\frac{a}{b}L = \left(w + \frac{ra}{b}\right)L = \left(\frac{wb + ra}{b}\right)L$

$L^* = \frac{Cb}{wb + ra}$

use L^* in C for $K^* = \frac{aC}{wb + ra}$

Q.) $Q = \min\{5K, 10L\}$

$C = wL + rK$ $\qquad$ ~~$w = 10$~~ ~~$r = 5$~~

$5K = 10L$

$K = 2L$

$C = wL + 2rL$

$C = (w+2r)L$

$$L^* = \frac{c}{w+2r}$$

$$K^* = \frac{2c}{w+2r}$$

$TC = wL^* + rK^*$

$$= \frac{wc + 2rc}{w+2r} = c$$

$q = 5K = 10L$

$$K = \frac{q}{5} \qquad L = \frac{q}{10}$$

$$TC = C = \frac{wq}{10} + \frac{rq}{5}$$

$$TC = \left(\frac{w+2r}{10}\right) q$$

$$AC = \frac{TC}{q} = \frac{w}{10} + \frac{r}{5}$$

$$MC = \frac{d(TC)}{dq} = \frac{w}{10} + \frac{r}{5}$$

ROBO JESS
ECE

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Q.) $x = A(q_1^\alpha + q_2^\beta)$ $\alpha, \beta > 1$

q_1 & q_2 are no of products, which we sell at prices P_1 & P_2

Ans $SP = q_1 P_1 + q_2 P_2$

$CP = Cx$

Profit, $\pi = SP - CP = q_1 P_1 + q_2 P_2 - Cx$

$$\frac{\partial \pi}{\partial q_1} = P_1 - CA\alpha q_1^{\alpha-1} = 0$$

$$\frac{\partial \pi}{\partial q_2} = P_2 - CA\beta q_2^{\beta-1} = 0$$

q_2

$$\frac{P_1}{P_2} = \frac{CA\alpha q_1^{\alpha-1}}{CA\beta q_2^{\beta-1}} \qquad q_1^{\alpha-1} = \frac{P_1}{CA\alpha}$$

$$\frac{P_1}{P_2} = \frac{\alpha}{\beta}\frac{q_1^{\alpha-1}}{q_2^{\beta-1}} \qquad q_2^{\beta-1} = \frac{P_2}{CA\beta}$$

Q.) Maximise π for Cobb douglas ~~function~~ where $\alpha+\beta<1$, $0<\alpha, \beta<1$

$\pi = Pq - C$

$\pi = PAK^\alpha L^{1-\alpha} - WL - rK$

$$\frac{\partial \pi}{\partial L} = \frac{PAK^\alpha(1-\alpha)}{L^\alpha} - W = 0$$

$$\Rightarrow PA\left(\frac{K}{L}\right)^\alpha (1-\alpha) = W \quad \text{— (1)}$$

$$\frac{\partial \pi}{\partial K} = PA\alpha\frac{K^{\alpha-1}}{L^{\alpha-1}} - r = 0$$

$$\Rightarrow PA\alpha\left(\frac{K}{L}\right)^{\alpha-1} = r$$

$$PA\alpha\left(\frac{K}{L}\right)^\alpha = r\frac{K}{L} \quad \text{— (2)}$$

Using in (1) $\quad PA\left(\frac{K}{L}\right)^\alpha - \frac{rK}{L} = W$

$$\frac{rK}{LW} = \frac{\alpha}{1-\alpha}$$

$$\cancel{\frac{1}{\alpha} = \frac{WL+rK}{rK} \Rightarrow \alpha = \frac{rK}{WL+rK}} \quad \frac{1-\alpha}{\alpha} = \frac{LW}{rK} \quad \text{— (3)}$$

Using ① $\left(\frac{K}{L}\right)^{\alpha} = \frac{W}{PA(1-\alpha)}$

$K = L\left[\frac{W}{PA(1-\alpha)}\right]^{\frac{1}{\alpha}}$

Using this in ②

$\frac{W\alpha}{1-\alpha} = r$

$\gamma = 1-\alpha-\beta$ **By Solving, we get.** Using ③ in ① and ②

$L^{*} = \left(\frac{r}{\alpha}\right)^{\frac{1-\beta}{\gamma}} \left(\frac{W}{\beta}\right)^{\beta/\gamma} A P^{1/\gamma} = \left[\frac{W}{P\beta\left(\frac{W\alpha}{r\beta}\right)^{\alpha}}\right]^{\frac{1}{\alpha+\beta-1}}$

$K^{*} = \left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\gamma}} \cdot \left(\frac{W}{\beta}\right)^{\frac{1-\alpha}{\gamma}} A P^{\frac{1}{\gamma}} =$

---

21 Aug 2019

Q.) $w = 20 \quad r = 15 \quad MP_L = 10 \quad MP_K = 15$

Explain why this data proves ~~a calculation~~ ~~corporation~~ calculation is done inefficiently.

~~$\frac{W\alpha}{1-\alpha} = r$~~

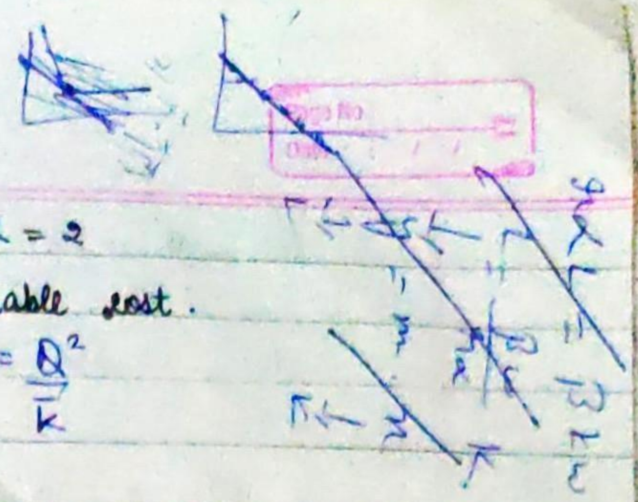
~~$W\alpha = r - r\alpha$~~

~~$(W+r)\alpha = r$~~

~~$\alpha = \frac{r}{W+r} = \frac{15}{20+15} = \frac{15}{35} = 0.42$~~

~~$TC = 20L + 15K$~~

Slope of isoquant, $\frac{10}{15} \neq \frac{20}{15}$, Slope of isocost



Q.) $Q = L^{0.5} \bar{K}^{0.5}$ (short run) $K = 2$

Calculate total fixed cost & total variable cost.

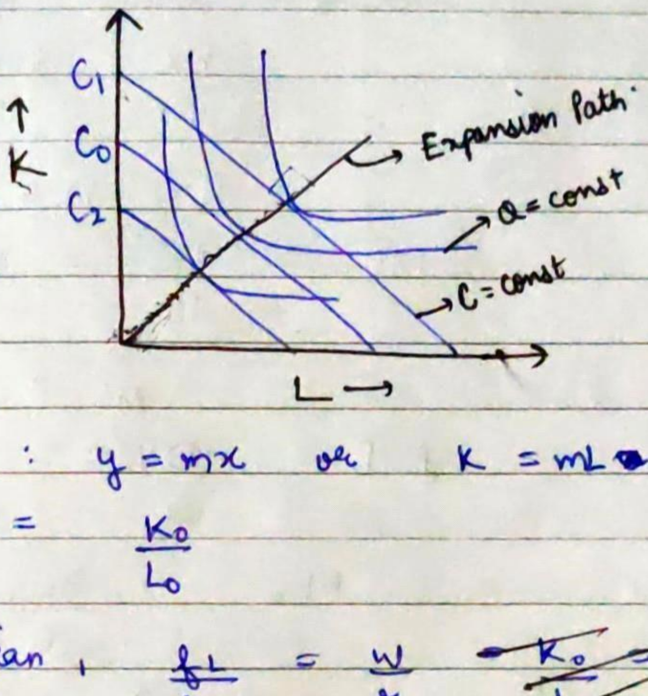
Ans $\cancel{MP_L = \frac{dQ}{dL} = 0.5 \left(\frac{\bar{K}}{L}\right)^{0.5}}$ $L = \frac{Q^2}{\bar{K}}$

$TC = wL + r\bar{K}$

$\cancel{MP_K = \frac{dQ}{dK}}$ $TC = w\frac{Q^2}{\bar{K}} + r\bar{K}$

$TC = \underbrace{\frac{wQ^2}{2}}_{\text{Total variable cost}} + \underbrace{2r}_{\text{Total Fixed cost}}$

Expansion Path



$C_2 < C_0 < C_1$

Eqn of line : $y = mx$ or $K = mL$

$m = \frac{K_0}{L_0}$

Using Lagrangian, $\frac{f_L}{f_K} = \frac{w}{r}$ $\cancel{\frac{K_0}{L_0} = m}$?

$K = \frac{w}{r} L$] eqn of expansion path

Q.) Suppose a firm is employing 20 labours and Labour is the only variable input and wage rate of 60. $AP_L = 30$. Last worker added 12 units to total output. Total Fixed cost = 3600. What is MC, Avg variable cost, Avg total cost.

Ans $C = wL + rK$

$\cancel{Q = 60L +}$ $rK = 3600$

Variable ~~Fixed~~ cost $= wL = 60 \times 20 = 1200$

Total cost $= 4800$

60

~~Avg variable cost = 1200/20 = 60~~

~~Avg fixed cost = 3600/20~~

~~$MC = \frac{dTC}{dQ} = 60$~~

$AP_L = \frac{TP}{L} \Rightarrow TP = 30 \times 20 = 600$

$MP_L = 12 = a = \frac{dTP}{dL}$

$Q = aL + bK = 12L + bK$

~~$Q = 240 + bK = 360 = 600$~~

$bK = 600 - 12L = 600 - 12 \times 20$

$= 360$

$Q = 12L + 360$

$MC = \frac{dTC}{dQ} = \frac{d(60L + 3600)}{d(12L + 360)}$

$MC = 5$

$AVC = \frac{VC}{Q} = \frac{1200}{600} = 2$

$AFC = \frac{3600}{Q} = 6$

Q.) $c = q w^{2/3} r^{1/3}$, find L^*, K^*

$L^* = \frac{\partial c}{\partial w}$ $\quad K^* = \frac{\partial c}{\partial r}$

$L^* = q r^{1/3} \times \frac{2}{3} w^{-1/3} = \frac{2q}{3}\left(\frac{r}{w}\right)^{1/3}$

$K^* = \frac{q}{3} w^{2/3} r^{-2/3} = \frac{q}{3}\left(\frac{w}{r}\right)^{2/3}$

UNIT - 2
Market

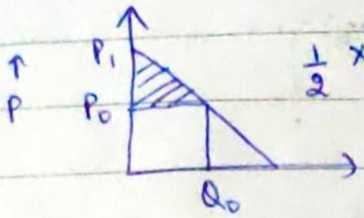
Perfect Competition
Monopoly
Monopolistic } → Similar Qty
Oligopolistic } Game Theory – Increase in Qty
– Increase in Price
} Slides

Perfect Competition (PC)

In PC, consumer surplus is max.

CS = at a ~~g~~ given price level how much is one able to save.

Consumer will buy a product until CS is 0.



$\frac{1}{2} \times OQ_0 \times (OP_1 - OP_0)$

Jeans at MRP = 3999
Jeans at sale = 1999
CS = 2000

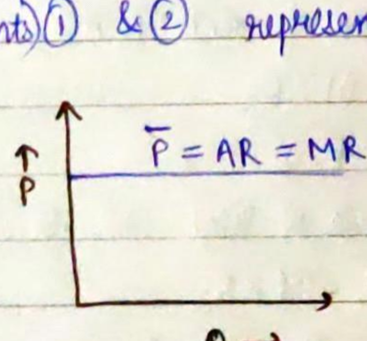
ASSUMPTIONS

① Large no of buyers & sellers. – ~~both~~ every sellers / buyers have insignificant share in the market (negligible share)

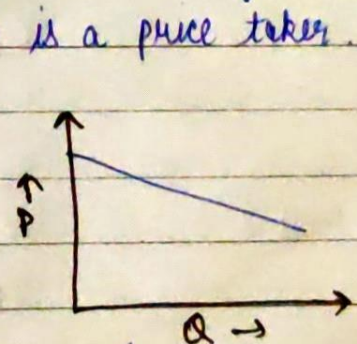
② Homogeneous Product – Quality of the product is identical i.e no. varieties

Assumptions • Seller Won't be able to survive in the market if it changes the price.

③ (Points) ① & ② represents that the firm is a price taker.



Demand curve faced by firm ~~industry~~



Demand curve faced by ~~individual seller~~ industry

$TR = \bar{P} \times Q$

$AR = \frac{TR}{Q} = \bar{P}$

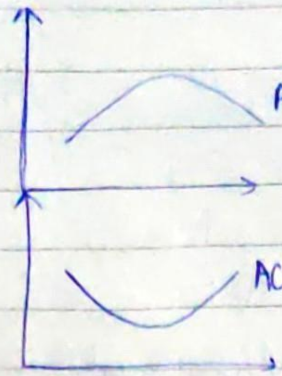
$MR = \frac{d(TR)}{dQ} = \bar{P}$

④ Free entry & exit of firms

⑤ Zero Transportation cost

$TC = WL + rK$

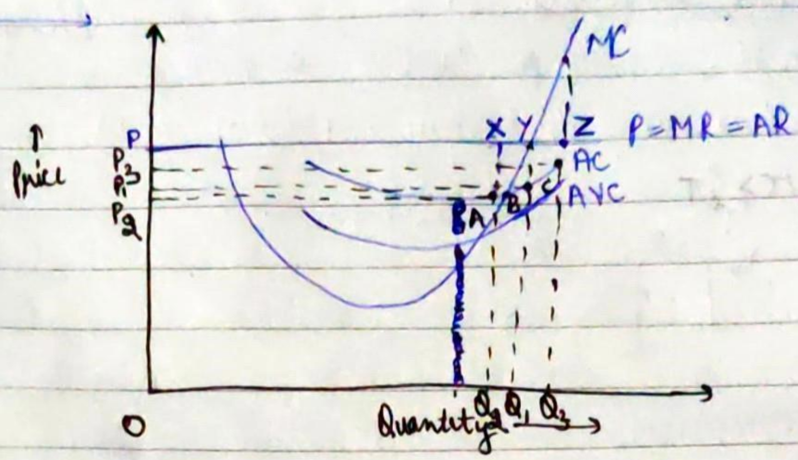
$AC = \cancel{WL}\ \frac{TC}{Q} = \frac{WL}{Q} + \frac{rK}{Q} = \boxed{\frac{W}{AP_L} + \frac{r}{AP_K}}$



$MC = \frac{dTC}{dQ} = \frac{d(WL + rK)}{dQ}$

$\boxed{MC = \frac{W}{MP_L} + \frac{r}{MP_K}}$

TR = 4



at Point P1 $\quad TC_1 = AC_1 \times Q_1$ = area of rect OQ_1BP_1

$TR_1 = AR_1 \times Q_1$ = area of rect $OPYQ_1$

$\pi_1 = TR_1 - TC_1 = P_1PYB$

at Point 2 $\quad \cancel{TC_3 = AC_3 \times Q_3 = \text{area of rect } O}$

$TC_2 = AC_2 \times Q_2$ = area of rect OP_2AQ_2

TR_2 = area of rect $OPXQ_2$

$\pi_2 = TR_2 - TC_2 = P_2PXA$

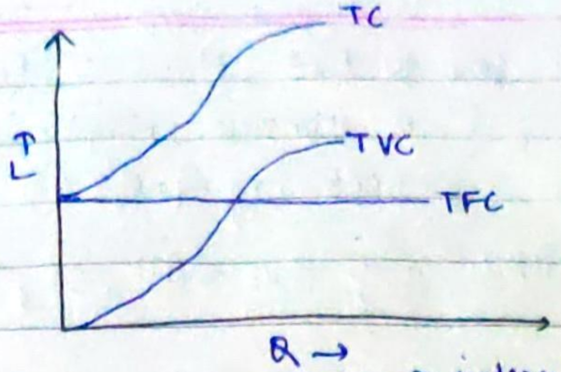
at Point 3 $\quad TC_3 = AC_3 \times Q_3$ = area of rect OP_3CQ_3

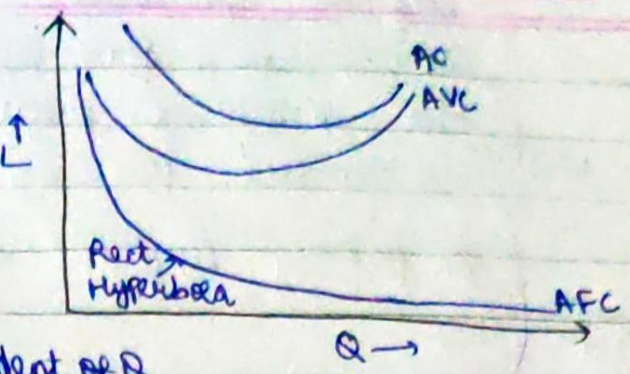
TR_3 = area of rect $OPZQ_3$

$\pi = TR_3 - TC_3$ = area of rect P_3PZC

23-8-19

SHORT RUN



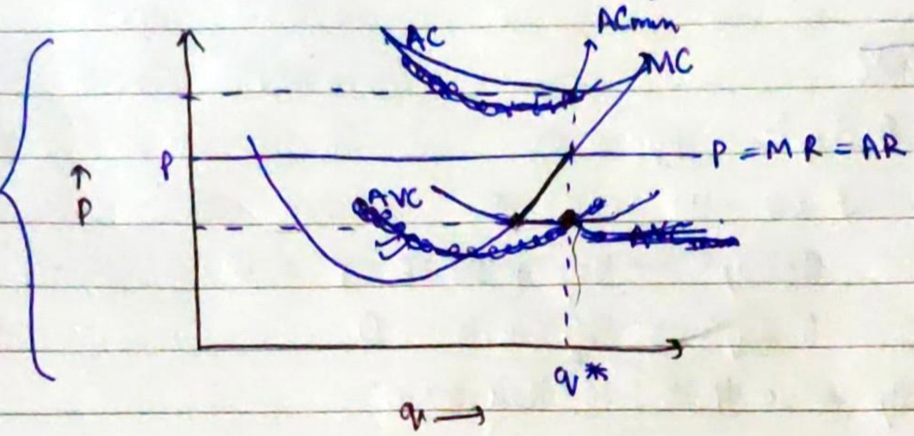


TC = TFC + TVC
(TFC → independent of Q; TVC → dependent of Q)

$$AC = ATC = \frac{TC}{Q} = \frac{TFC}{Q} + \frac{TVC}{Q} = AFC + AVC$$

* When does a firm decide to shut down?

graph

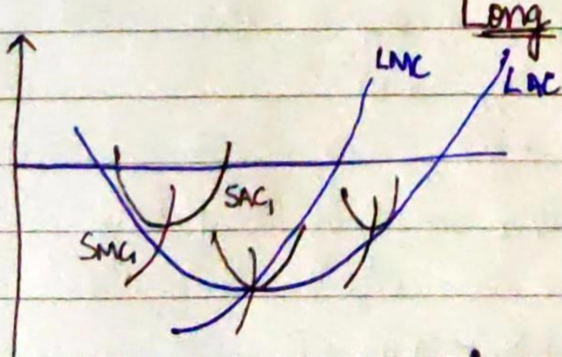


TVC < TR
TR < TC

Shut down rule :- if price < AVC$_{min}$
Firm continue operation if price ≥ AVC$_{min}$

<u>Supply Curve</u> : MC above AVC min is the supply curve of the firm because ~~it is also~~ AVC min is the shut down condⁿ and it is producing more than AVC$_{min}$ so firm would survive.

<u>Long Run</u>



The long run AC (LAC) curve is an envelope of the short run Average curves.

Long run eqm condition

More firms, price will fall to AC_{min} ~~AVC~~ ~~lco~~, profit = 0. Firm will eat each other's cost

Not accounting profit, it is economic profit.

This situation exists Economic profit = 0, Accounting profit > 0

In perfectly competitive market, $\pi = 0$ Price = Cost, CS = max, Producer surplus = 0

2/9/19

$$TC = C(q) = 0.1q^3 - 2q^2 + 15q + 10$$

$$TVC = 0.1q^3 - 2q^2 + 15q$$

$$AVC = 0.1q^2 - 2q + 15$$

AVC_{min} $$\frac{d(AVC)}{dq} = 0.2q - 2 = 0 \Rightarrow q = 10$$

$$P = MC$$

$$P = 0.3q^2 - 4q + 15$$

$$P = 30 - 40 + 15 = 5$$

Supply Fxn $\Rightarrow$ $$0.3q^2 - 4q + 15 - P = 0$$

$$0.3q^2 - 4q + 10 = 0$$

$$q = \frac{4 \pm \sqrt{16 - 4 \times 0.3 \times 10}}{2 \times 0.3}$$

$$q = \frac{4 \pm 2}{2 \times 0.3} = \frac{2 \pm 1}{0.3} = 10 \text{ or } 3.33$$

$$q = \frac{4 \pm \sqrt{1.2P - 2}}{0.6} \times n$$

$$C_{1i} = 0.04 q_{1i}^3 - 0.8 q_{1i}^2 + 10 q_{1i}$$

$$C_{2i} = 0.04 q_{2i}^3 - 0.8 q_{2i}^2 + 20 q_{2i}$$

$$TVC_i = 0.04 q_{1i}^3 - 0.8 q_{1i}^2 + 10 q_{1i}$$

$$AVC_i = 0.04 q_{1i}^2 - 0.8 q_{1i} + 10$$

$$\frac{d(AVC)_{min}}{dq} = 0.08 q_{1i} - 0.8 = 0$$

$$q_{1i} = \frac{0.80}{0.08} = 10$$

$$P = MC = 0.12 q_{1i}^2 - 1.6 q_{1i} + 10 - P$$

$$P_1 = 12 - 16 + 10 = 6$$

280 224

160 64 320 64 128 192

$$q = \frac{1.6 \pm \sqrt{1.6^2 - 4 \times 0.12 \times (10-P)}}{2 \times 0.12}$$

$$q = \frac{1.60 \pm \sqrt{(2.56 - 4.8 + 0.48P)10^4}}{0.24}$$

$$q = \frac{20 \pm \frac{10}{3}\sqrt{256 - 480 + 48P}}{3}$$

$$= \frac{20 \pm 1.25\sqrt{48P - 224}}{3}$$

480 224 704

$AVC_2 = 0.04 q_{2i}^2 - 0.8 q_{2i} + 20$

AVC_{min} is at $q_{2i} = 10$

$P = MC = 0.12 q_{2i}^2 - 1.6 q_{2i} + 20 - P$

$P = 12 - 16 + 20 = 16$

$$q = \frac{1.6 \pm \sqrt{1.6^2 - 4 \times 0.12 \times (20-P)}}{2 \times 0.12}$$

$$= \frac{20 \pm 1.25\sqrt{48P - 704}}{3}$$

Q.) The cost function, $C = q^3 - 4q^2 + 8q$
The firms will enter the industry is π are +ve and leave the industry when π are -ve.
For Part (ii) demand fxn, $D = 2000 - 100P$
(i) Supply Fxn?
(ii) Determine the price, quantity and no. of firms operating in the industry.

Ans(i) $AVC = q^2 - 4q + 8$

$(AVC)_{min} = \frac{d(AVC)}{dq} = 2q - 4 = 0 \Rightarrow q = 2$ $\boxed{\frac{d^2(AVC)}{dq} > 0}$

$P = MC = 3q^2 - 8q + 8 = 3 \times 4 - 8 \times 2 + 8 = 4$

$MC = 3q^2 - 8q + (8 - P)$

$$q = \frac{8 \pm \sqrt{64 - 4 \times 3 \times 8 + 12P}}{6}$$

$$= \frac{8 + \sqrt{12P - 32}}{6}$$

eqm price = 4

eqm ~~qty~~ qty = 2

demand at eqm price = 2000 − 100×4 = 1600

no of firms = $\frac{\text{demand}}{\text{supply qty}} = \frac{1600}{2} = 800$

4-9-19 **Industrial Long Run Supply Curve.**

The shape of the Industrial Long Run Supply Curve depends on the extent to which changes in the industry output affect the prices that firms must pay for inputs into the production process.

① Constant cost industry, Ex – unskilled labour

⇒ Increase in industry o/p, does not change the price of input

② Increasing cost industry, Ex – skilled labour

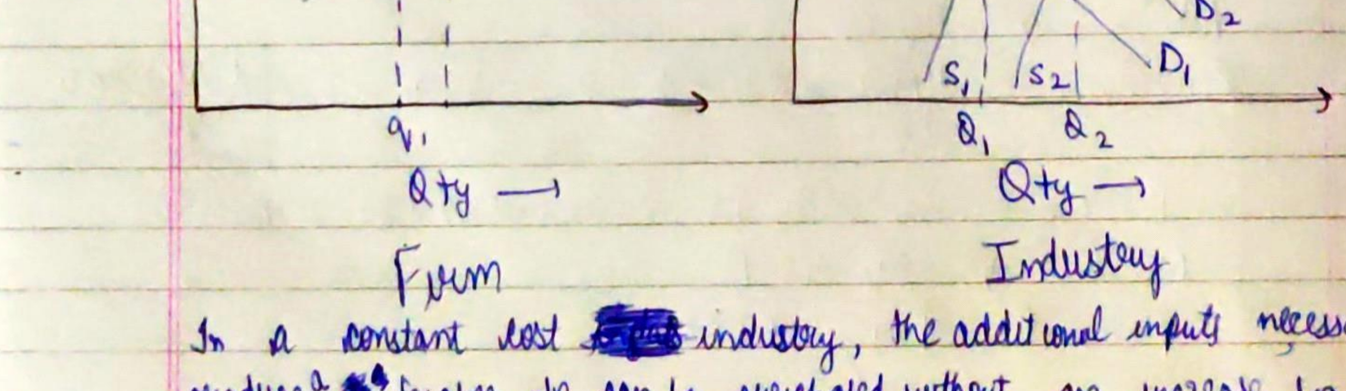
⇒ ______________, increases ______________

③ Decreasing cost industry, Ex – ?

⇒ ______________, decreases ______________

CONSTANT COST INDUSTRY

Income Tax Reduced



Firm   Industry

In a constant cost industry, the additional inputs necessary to produce a higher o/p can be purchased without an increase in per unit price.

No ____ Price

Page No.
Date

Q.) $TC = q^3 - 8q^2 + 30q + 5$. At what level of Q, MC cuts AVC.

Ans $MC = 3q^2 - 16q + 30$

$AVC = q^2 - 8q + 30$

$MC = AVC \Rightarrow \quad 3q^2 - 16q \cancel{+30} = q^2 - 8q \cancel{+30}$

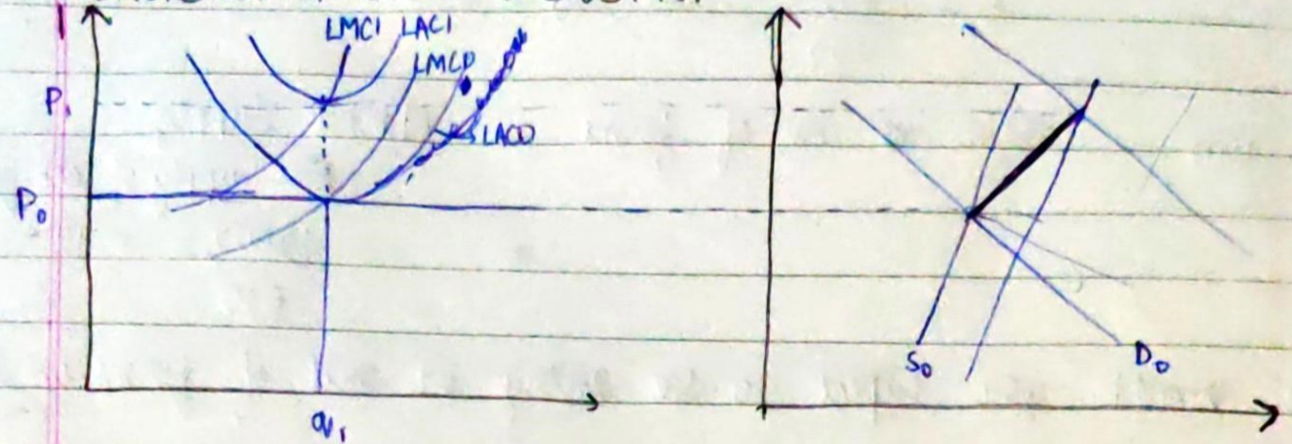
$2q^2 = 8q$

$q^2 = 4q$

$q(q-4) = 0$

$\Rightarrow \boxed{q = 0, 4}$

INCREASING COST INDUSTRY



Long run eqm is reached at that point where zero eco π prevails.

Firms will enter as long as Eco π > 0.

Q) Market D, $Q_D = 6500 - 100P$

Market S, $Q_S = 1200P$

TC, $C(Q) = 722 + \frac{Q^2}{200}$

Ans (i) For eqm price,

$$6500 - 100P = 1200P$$

$$\frac{6500}{1300} = P = 5$$

(ii)

$$P = MC$$

$$5 = \frac{2q}{200} \Rightarrow q = 500$$

(iii) Total no of firms = $\frac{\text{Market Supply}}{\text{Supply of each firm}}$

$$= \frac{1200 \times 5}{500} = 12$$

(iv) Would you expect to see entry or exit of firms in long run?

$$\pi = Pq - C$$

$$\pi = 5 \times 500 - \frac{500^2}{200}$$

$$= 500\left(5 - \frac{500}{200}\right) = 500 \times 2.5$$

$$= 1250$$