- Rule:1: If α is not the root of A LHRRWCC (not characteristic root)

then, $a_n^P = [c_k n^k + c_{k-1} n^{k-1} + \dots + c_1 n + c_0]\alpha^n$

- Rule:2: If α is the root of A LHRRWCC [characteristic multiplicity m

then, $a_n^P = n^m [\alpha_k n^k + \alpha_{k-1} n^{k-1} + \dots + \alpha_1 n + \alpha_0]$

Q $\underline{a_n = 5a_{n-1} - 6a_{n-2}} + 8n^2$ ——— (1) $a_0 = 4, a_1 = 7$

$\longrightarrow$ Its associated LHRRWCC is $a_n = 5a_{n-1} - 6a_{n-2}$

$a_n - 5a_{n-1} + 6a_{n-2} = 0$

Solⁿ: $a_n = a_n^{(h)} + a_n^P$ = general solⁿ

$\longrightarrow$ 1 particular solⁿ of given eqⁿ

∴ Its charac. eqⁿ = $x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$

$(2)^n, (3)^n$ = basic solⁿ

∴ $a_n^{(h)} = A(2)^n + B(3)^n$ ——— (A)

Now,

for particular solⁿ,

$f(n) = 8n^2 = \underbrace{[c_k n^k + c_{k-1} n^{k-1} + \dots + c_0].\alpha^n}$ } Rough part

$8n^2 . 1^n$ (Here, $\alpha = 1$)

$= [8n^2 + 0n + 0] . 1^n$

Applying Rule 1:

Let us assume a particular solⁿ of eq(1) is $a_n^P = [d_2 n^2 + d_1 n + d_0] \times 1^n$

put in (1)

$a_n^P = a_n = [d_2 n^2 + d_1 n + d_0] = 5[d_2(n-1)^2 + d_1(n-1) + d_0]$

$- 6[d_2(n-2)^2 + d_1(n-2) + d_0] + 8n^2$

Comparing LHS & RHS coeff:

$d_2 = 5d_2 - 6d_2 + 8 \Rightarrow d_2 = 4, d_1 = 28, d_0 = 60$

∴ $a_n^P = [d_2 n^2 + d_1 n + d_0] = [4n^2 + 28n + 60]$

Now, general solⁿ of eqⁿ (1) is:

$a_n = a_n^h + a_n^p = A \cdot 2^n + B \cdot 3^n + 4n^2 + 28n + 60$

$A = -83,\ B = 27$

8 $a_n = 6a_{n-1} - 9a_{n-2} + 4(n+1) \cdot 3^n$ ——— (1), $a_0 = 2,\ a_1 = 3$

Its associated LHRRWCC is $a_n - 6a_{n-1} + 9a_{n-2} = 0$ ——— (2)

Its characteristic eqⁿ is:

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0 \Rightarrow x = 3, 3 \ (m = 2)$

$\therefore$

Basic solⁿ = $3^n, n \cdot 3^n$

$\therefore a_n^{(h)} = A \cdot 3^n + B \cdot n \cdot 3^n$ ——— (3)

$f(n) = 4[n+1] \cdot 3^n$ $\therefore C_K = 4, K = 1, C_0 = 4$

Lets assume a particular solⁿ:

$a_n^p = n^2 [d_1 n + d_0] 3^n$

Ans = $d_1 = 2/3,\ d_0 = 4$

* generating functions:

Ex: $1 + x + x^2 + x^3 + x^4 + x^5 = \left(\frac{x^6 - 1}{x - 1}\right)$ is a generating fⁿ for coeff: 1, 1, 1, 1, 1, 1

Ex: $1 + 2x + 3x^2 + 4x^3 + \dots$ is generating fⁿ for natural no

Date / /
Page

Ex: $g(x) = a_0 + a_1 x + a_2 x^2 + \cdots$

$$= \sum_{n=0}^{\infty} a_n x^n$$

$$f(x) = b_0 + b_1 x + b_2 x^2 + \cdots$$

$$= \sum_{n=0}^{\infty} b_n x^n$$

$$f(x) + g(x) = \sum_{n=0}^{\infty} [a_n + b_n] x^n$$

$$f(x) \cdot g(x) = \sum_{n=0}^{\infty} \left[\sum_{i=0}^{n} a_i b_{n-i} \right] \cdot x^n$$

Ex: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ (∞ GP)

$$= \sum_{n=0}^{\infty} x^n$$

Now, $\frac{1}{(1-x)^2} = \frac{1}{(1-x)} \cdot \frac{1}{(1-x)} = \left[\sum_{n=0}^{\infty} x^n \right] \cdot \left[\sum_{n=0}^{\infty} x^n \right]$

$$= \sum_{n=0}^{\infty} \left[\sum_{i=0}^{n} 1 \times 1 \right] x^n = \sum_{n=0}^{\infty} (n+1) x^n$$

$$= 1 + 2x + 3x^2 + 4x^3 + \cdots + (n+1)x^n$$

Ex: $\frac{1}{(1-x)^3} = \frac{1}{(1-x)} \times \frac{1}{(1-x^2)}$

$$= \left[\sum_{n=0}^{\infty} x^n \right] \left[\sum_{n=0}^{\infty} (n+1) x^n \right] = \sum_{n=0}^{\infty} \left[\sum_{i=0}^{n} 1 \times (n+1-i) \right] \cdot x^n$$

Q Use generating f^n to solve tower of Hanoi.
$b_n = 2b_{n-1} + 1$, $b_1 = 1$

Sol^n: Let $g(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \dots + b_n x^n + \dots$
$= \sum_{n=0}^{\infty} b_n x^n$

put $n=1$ in $b_n \Rightarrow \cancel{b_1}^{1} = 2b_0 + 1 \Rightarrow b_0 = 0$

$2x\,g(x) = +2b_0 x + 2b_1 x^2 + 2b_2 x^3 + \dots + 2b_{n-1} x^n + 2b_n x^{n+1} + \dots$

$\frac{1}{(1-x)} = 1 + x + x^2 + \dots$

$\frac{1}{(1-ax)} = 1 + ax + a^2x^2 + \dots$

$g(x) - 2x\,g(x) - \frac{1}{1-x} = -1 + (b_1 - 2b_0 - 1)x + (b_2 - 2b_1 - 1)x^2 + \dots$
$(b_n - 2b_{n-1} - 1)x^n + \dots$

$(1-2x)\,g(x) = \frac{x}{1-x}$

$\therefore g(x) = \frac{x}{(1-x)(1-2x)} = \frac{A}{(1-x)} + \frac{B}{(1-2x)}$

$\therefore x = A(1-2x) + B(1-x) \Rightarrow A = -1, B = 1$

$\therefore g(x) = \frac{-1}{(1-x)} + \frac{1}{(1-2x)}$

$= -\sum_{n=0}^{\infty} x^n + \sum_{n=0}^{\infty} (2x)^n$

$= \sum_{n=0}^{\infty} (2^n - 1) x^n$

$\therefore b_n = 2^n - 1$; $n \geq 0$

Q $F_n = F_{n-1} + F_{n-2}$; $F_1 = F_2 = 1$, $F_0 = 0$

Solⁿ: $g(x) = F_0 + F_1 x + F_2 x^2 + \dots + F_n x^n$

$$x g(x) = F_0 x + F_1 x^2 + \dots + F_{n-1} x^n$$
$$x^2 g(x) = F_0 x^2 + F_1 x^3 + \dots + F_{n-2} x^n$$

$$(1 - x - x^2) g(x) = (F_1 - F_0) x + (F_2 - F_1 - F_0) x^2 + \dots$$
$$\underbrace{\dots + (F_n - F_{n-1} - F_{n-2}) x^n + \dots}_{\text{each term } 0}$$

$$\therefore g(x) = \frac{x}{1 - x - x^2} = \frac{x}{(1 - \alpha x)(1 - \beta x)} = \frac{A}{(1 - \alpha x)} + \frac{B}{(1 - \beta x)}$$

$1 - x - x^2 \to$ roots $= \alpha, \beta$

$$x = A(1 - \beta x) + B(1 - \alpha x)$$
$$= A + B - (A\beta + B\alpha) x$$

$\therefore A + B = 0$

$A\beta + B\alpha = -1$

$B(\cancel{\alpha} \beta - \alpha) = 1$

$= B = \left(\frac{1}{\beta - \alpha}\right)$

$$\therefore \alpha = \frac{1 + \sqrt{5}}{2}, \quad \beta = \frac{1 - \sqrt{5}}{2}$$

$$\therefore g(x) = \frac{-1}{\sqrt{5}} \left[\sum_{n=0}^{\infty} (\alpha x)^n - \sum_{n=0}^{\infty} (\beta x)^n \right]$$
$$= \frac{-1}{\sqrt{5}} \sum_{n=0}^{\infty} (\alpha^n - \beta^n) x^n$$

Q1 Write a recursive Algo for computing factorial of a non-(-ve) no. and check it correctiveness.

Q2. Write a recursive Algo for tower of Henoi & check correctness

Date / /
Page

Q3. Write recursive algo for hand-shaking problem & check its correctness.

(1) Algo:

If $(n=0)$ — True $fact(n) = 1;$
False

If $(n \neq 0)$
→ True ⇒ $fact(n) = n \times fact(n-1);$
False ⇒ End

Q Find the no. of ways for drawing a Red Queen or Black King from deck of playing cards.

Sol^n: No. of Red Queens $= 2 = m$
No. of Black Kings $= 2 = n$
∴ Total ways $= m+n = (2+2) = 4$

- Addit^n principle: $(m+n)$ = either task to be done
- Inclusion/exclusion principle:
- Multiplicat^n principle: $(m \times n)$ = both tasks to be done

Q Find no of 2 letter words that begin with vowel.
Sol^n: (26×5) words.

Q 1 type of automobile license plate no. has 1 letter & 5 digits. Compute a no. of such license plates possible.
Sol^n: ${}^6C_1 \times {}^{26}C_1 \times ({}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1)$
$= 6 \times 26 \times 10^5$

Q In programing lang, identifier consists of a le[tter] followed by alpha-numeric. Find out all the legal identifiers atmost 10.

Q How many bytes are possible having 2^{nd} place as 0 or 3^{rd} place as 1. (1 byte = 8 bits)

Sol^n: [_ _ _ _ _ _ _ _] + [_ _ _ _ _ _ _ _] − [_ _ _ _ _ _ _ _]

$2^7 + 2^7 - 2^6$

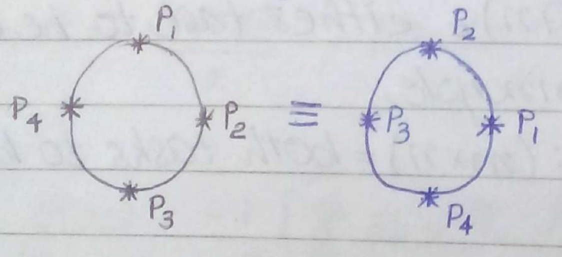
$= 2\cdot 2^7 - 2^6 = 2^8 - 2^6 = 3\cdot 2^6$

- n distinct objects: $= n! =$ ways

$$P(n,r) = \underset{1^{st}\,pos^n}{n}(n-1)(n-2)\cdots\underset{r^{th}\,pos^n}{(n-r-1)}$$

$$= \frac{n!}{(n-r)!} = {}^nP_r = r!\,{}^nC_r$$

- Cyclic $= (n-1)! =$ ways to arrange n distinct in a circle



- Recurrence rel^n for permutation:

$$P(n,r) = r\,P(n-1, r-1) + \cancel{P(n-1, r-1)} \quad ; \quad 0 < r < n$$
$$+ P(n-1, r)$$

Pf: A = collect^n of all those permutat^n having an element
B = " " " not " that element

∴ A = for 1 element r ways & rem. $(n-1)$ in $(r-1)$
∴ $r\cdot P(n-1, r-1)$; $B = P(n-1, r)$

Scanned by CamScanner

* Dearrangements:

Suppose, D_n is total no. of derrangement of n distinct objects.

$D_0 = 1$

$D_1 = 0$

$D_2 = 1$

Recurrence Rel^n: $D_n = (n-1)[D_{n-1} + D_{n-2}]$

$D_n = (n-1)D_{n-1} + (n-1)D_{n-2}$

$D_n - nD_{n-1} = D_{n-1} + (n-1)D_{n-2}$

$\underbrace{D_n - nD_{n-1}} = -[D_{n-1} - (n-1)D_{n-2}]$

$d_n = D_n - nD_{n-1}$ ✓

$d_n = (-1)\,d_{n-1}$

sol^n: $d_n = (-1)^n$ $\boxed{D_n = (-1)^n + nD_{n-1}}$

Sol^n of above recurrence rel^n gives:

$\rightarrow D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \cdots\cdots \frac{(-1)^n}{n!}\right]$

These are no of ways to derrange n-letters in n envelopes such that no letter gets into correct envelope.

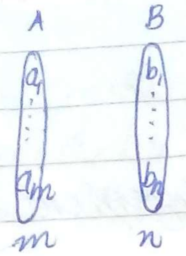
Q Given 2 sets: A, B with $|A| = m$, $|B| = n$. Find the no. of functions that can be defined from $A \rightarrow B$?

No. of Onto = $\cancel{\sum_{r=0}^{n}}$ ${}^nC_0(n-0)^m - \sum_{r=1}^{n}(-1)^{r+1}\,{}^nC_r(n-r)^m$

Onto f^n = $\boxed{\sum_{r=0}^{n}(-1)^r \cdot {}^nC_r(n-r)^m}$

Date / /
Page

Sol^n:



for 1st element $(a_1) \longrightarrow n$

$(a_2) \longrightarrow n$

$\vdots$

$(a_m) \longrightarrow n$ ways

$\therefore$ By multiplicative inverse: $n \times n \times n \times \cdots$ (m times)
$= \boxed{n^m}$

Q How many one-one functions are possible?

Sol^n: $\boxed{{}^nP_m} \longrightarrow$ true for only $n \geq m$.

Q Prove that there doesn't exist any one-one mapping when $m > n$.

Sol^n: Apply pigeon-hall principle, where
m = elements of domain = pigeons
n = elements of co-domain = holes

Q How many onto functions are possible?

Sol^n: $|A| = m$; $|B| = n$

Let, $A = \{a_1, a_2, \ldots, a_m\}$; Total fun^n = n^m
$B = \{b_1, b_2, \ldots, b_n\}$

Let A_i is the collec^n of f^n from A to B in which b_i is not an image of any domain's element.

- $\sum_{i=1}^{n}|A_i| = \sum_{i=1}^{n}(n-1)^m = n(n-1)^m = {}^nC_1(n-1)^m$
- $\sum_{i<j}^{n}|A_i \cap A_j| = \sum_{i<j}(n-2)^m = {}^nC_2(n-2)^m$ ------ ${}^nC_{n-1}(n-(n-1))^m = n$
$+ {}^nC_n(n-n)^m = 0$

$\therefore$, Total no. of Into $f^n = [{}^nC_1(n-1)^m + {}^nC_2(n-2)^m + \ldots + {}^nC_2 2^m + {}^nC_1 1^m]$

$\therefore$, Total Onto f^n: $= [n^m] - [{}^nC_1(n-1)^m + {}^nC_2(n-2)^m + \ldots + {}^nC_{n-2}2^m$

Scanned by CamScanner

Date / /
Page

* Combinations:-

Q How many committee's with 3 boys & 4 girls can be formed from a group of 6 boys & 5 girls?

Sol^n → $^6C_3 \times {}^5C_4$

Q If A be a 10-element subset of {1, 2, 3, ---, 15}. Let A_k be a subset of A containing 3 elements where A' denotes the sum of the elements. Determine if each subset of A can be identified by a unique name A_k?

Sol^n → (1, 3, 4), (1, 2, 5) have same name, so no uniqueness

$\{1, 2, 3, \ldots, 15\}$
$\downarrow$
$|A| = 10$
$\downarrow$
$|A_k| = 3$

Sum = 6, 7, 8, ----, 42
Total no. = 42 − 6 + 1 = 37

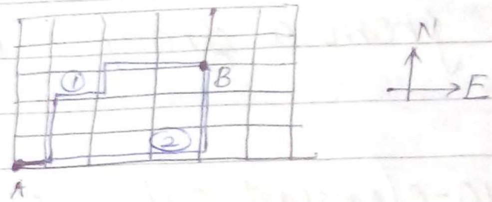
$^{10}C_3$: No. of subset = No. of pigeons = m
37 : No. of holes = n

Q How many words can be formed from "ALLAHABAD"?

Sol^n → $\dfrac{9!}{4!\,2!}$

Scanned by CamScanner

Q To move from A to B covering 8 blocks & using only N & E, Find no. of paths possible.



1 ENNENEEE

2 EEEEENNN

No. of paths = $\frac{8!}{3!\,5!}$

Q k=0 Find final value of k.

for $i_1 = 0$ to n_1
    k = k+1;
} executes (n_1+1) times
$\therefore$ at end value of $k = (n_1+1)$

for $i_2 = 0$ to n_2
    k = k+1;
} similarly, (n_2+1)

⋮

for $i_m = 0$ to n_m
    k = k+1;

Q Each user of comp. sys. has a password which is 6-8 charac. long. Each char. is an upercase letter & digit. Each password shouldve atleast 1 digit. How many passwords are possible.

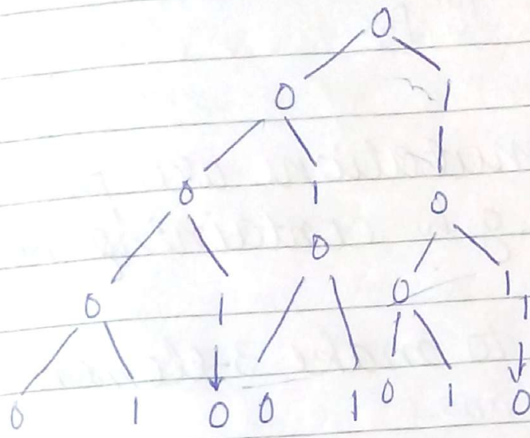
Soln. 8 character:

$^8C_1 \times {}^{10}C_1 \times {}^{26}C_7 \times 7! + {}^8C_2 \times {}^{10}C_2 \times {}^{26}C_6 \times 2! \times 6! + \ldots + {}^8C_8 \times {}^{10}C_8 \times {}^{26}C_0 \times 8!$

Q Tree str. is helpful in counting problems while dealing ~~with~~ while using exhaustive enumartion

Date / /
Page

Q How many bit string of length 4 do not have 2 consecutive 1's?



Q How many students must be there in class to guarantee that atleast 2 students receive the same score on the final exam if the exam is graded on a scale from 0 ~~to~~ 100 pts.

Sol^n No. of grades = 101
∴ Answer = 102.

Q In a group of 6 people, each pair of individual consists of 2 friends / 2 enemies. show that there are either 3 mutual friends / mutual enemies in the group?

Sol^n:
B C D ⟶ friends class of A.
A

B, D, C = friends ⟹ A, B, C = mutual friend

* Ramsay Number:
It denotes the min. no. of people at a party s.t. there are either m mutual friends or n mutual enemies assuming that every pair of people at ... enemies.

Date / /
Page

Q How many ways are there to select a 1st prize, 2nd prize / 3rd price winner from 100 diff. people.

Sol^n $^{100}C_1 \times {}^{99}C_1 \times {}^{98}C_1 = {}^{100}P_3$

Q How many permutations are possible of letter a,b,c,d,e,f,g,h containing pattern A,B,C

Q How many ways to make 3-element subset from the set {a,b}. Ans=

a, a, a
a, a, b
b, b, b
b, b, a

4 possibilites to select 3 out of 2, (repetition case) either a repeated twice or b repeated two

Q 5 person go to restaurant that serves only 3 things. Find possible orders?

Sol^n

	Tea	Coffeee	Soup
(i)	xx	xx	x
(ii)	—	—	xxxxx
(iii)	x	xxx	x

$n = 3$

$r = 5$

$\therefore \boxed{^{n+r-1}C_r} = {}^{7}C_5 = 21$

↓

if combination is done in repetition form.

Scanned by CamScanner

$T_{r+1} = {}^{15}C_r \cdot (x^2)^{n-r} \cdot (2x)^r \quad ; (x^2+2x)^{15}$

Date / /
Page

*Q Let $x_1, x_2, \dots, x_n$ are n non-(-ve) var. & r be a non-(-ve) integer. Then eq^n: $x_1 + x_2 + \dots + x_n = r$ has $C(n+r-1, r)$ integer sol^n.

→ $(x+y)^n = \sum_{r=0}^{n} {}^nC_r \, x^{n-r} \cdot y^r$ —— $(n+1)$ elements in expans^n

B.Q

Q Find coeff. of x^4 in exp. $(2x+3y)^5$

Q Compute coeff. of x^7 in $(1+x)^{11}$

→ $(1+x+x^2+x^3+x^4+x^5) = \sum_{i=0}^{5} x^i$

Q Find possible int. sol^n of $x_1 + x_2 + x_3 = 7$ if

$x_1 \geq 3$, $\quad 1 \leq x_2 \leq 3$, $\quad 0 \leq x_3 \leq 2$

$x_1 = 3,4,5,6,7 \quad x_2 = 1,2,3 \quad x_3 = 0,1,2$

coeff of x^7 in: $(x^3+x^4+x^5+x^6+x^7)(x+x^2+x^3)(1+x+x^2)$

x^7 in: $x^4(1+x+x^2+x^3+x^4)(1+x+x^2)(1+x+x^2)$

x^3 in: (")(")(")

* Generalized Inclusion - Exclusion principle:

$$\left| \bigcup_{i=1}^{n} A_i \right| = \sum_{i=1}^{n} |A_i| - \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<k} |A_i \cap A_j \cap A_k| + \dots$$

$$+ (-1)^{n+1} \left| \bigcap_{i=1}^{n} A_i \right|$$

Q Using inclusion-exclusion principle, find no. of derrangements?

D_n is collection of those permutations of A, having each element of A not at its right position.

$A = \{b_1, b_2, \dots, b_n\}$

Total permutations of $A = n!$

A_i is the collection of all those permutations of A where, b_i should be in ith position

$\therefore |A_1| = (n-1)! = |A_2| = \dots = |A_n|$

$\therefore \sum_{i=1}^{n} (n-1)! = {}^nC_1 (n-1)!$

$\sum_{i<j} |A_i \cap A_j| = {}^nC_2 (n-2)!$

Last term $= {}^nC_n (n-n)! = {}^nC_n$

$\therefore D_n = n! \left[1 - \frac{1}{1!} + \frac{1}{2!} + \dots \frac{(-1)^n}{n!} \right]$

Q Prove that: $\sum_{r=0}^{n} (-1)^r \, {}^nC_r = ?$

Sol^n: $A = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots (-1)^n \, {}^nC_n$

$= ({}^nC_0 + {}^nC_2 + {}^nC_4 + \dots) - ({}^nC_1 + {}^nC_3 + \dots)$

$A = {}^nC_n (-1)^n - {}^nC_{n-1} + \dots$

$2A = ({}^nC_0 + {}^nC_n) + ({}^nC_1 + {}^nC_{n-1}) + \dots$

* Relations:

$R \subseteq A \times B$ or $R \subseteq A \times A$

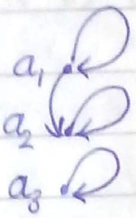
$|A| = m$, $|B| = n$, then $\longrightarrow$ $|A \times B| = m \times n = mn$

$|R \subseteq A \times B| = 2^{mn}$

$|R \subseteq A \times A| = 2^{n^2}$

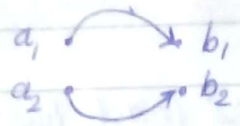
Ex: $A = \{a_1, a_2, a_3\}$

$R_1 = \{(a_1, a_1), (a_2, a_2), (a_3, a_3), (a_1, a_2)\} \longrightarrow a_1 R_1 a_1$



= Diagram of R_1 = Diag. representⁿ

Ex: $R_2 = \{(a_1, b_1), (a_2, b_2)\}$



Ex: $R_3 = \{(a_1, a_2), (a_2, a_2), (a_3, a_3)(a_3, a_1)\}$ Here, $A = \{a_1, a_2, a_3\}$

matrix representⁿ =

	a_1	a_2	a_3
a_1	0	1	0
a_2	0	1	0
a_3	1	0	1

* Types of Relations:

Ex: Write relⁿ on $A = \{\{a_1, a_2\}\} = \{a_1, a_2\}$

$A \times A = \{(a_1, a_1), (a_1, a_2), (a_2, a_1), (a_2, a_2)\}$

subsets of $A \times A$ = Relⁿ $= 2^4 = 16$

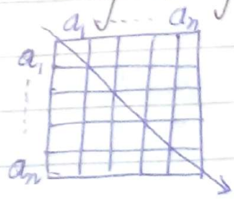
Identity : Every element of A is associated with itself o[nly]

$a<b$ or $b<a$ = non sy[mm]

✓(1) Reflexive → Every element of A is associated with itself
(2) Irreflexive → No element of A should be associ. with its[elf]
✓(3) Symmetric → If $(a,b) \in R \Rightarrow (b,a) \in R$ also
(4) Antisymmetric → (i) $(a,b) \in R \not\Rightarrow (b,a) \notin R$ & (ii) $(a,b) \in R \Rightarrow (b,a) \in R$ if $b=a$
✓(5) Transitive → If $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$
✓(6) Equivalence → If rel^n is (1), (3) & (5) ⇒ It is eqv.
(7) Partial order

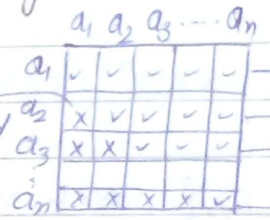
Q No. of Reflexive rel^n on A.

2^{n^2-n} = No. of irreflexive rel^n on A



→ all have 1 choice of to be part of r[el]

Q No. of symmetric rel^n on A



$2 \times 2 \times 2 \times \cdots \times 2 = 2^n$ (each has 2 choices)

$\rightarrow 2^{n-1}$

$\rightarrow 2^{n-2}$

$\rightarrow 2^{n-(n-1)} = 2^1$

already considered in pr. row

∴, No. of symm. rel^n on $A = 2^1 \times 2^2 \times \cdots \times 2^n = 2^{\Sigma n} = 2^{\frac{n(n+1)}{2}}$

Q No. of Assymmetric rel^n on A

All elements of principal diagonal → 2 choices → 2^n ways

Total elements (or order pairs : (a,b)) = ${}^nC_2$

∴, Total no. = $3^{{}^nC_2} \times 2^n$

	(a,b)	(b,a)
3 cases	×	×
	✓	×
	×	✓
(∴ ×) symm.	✓	✓

Date / /
Page

* Compositions of Relations:

$R \subseteq A \times B$, $S \subseteq B \times C$

$\Rightarrow R \circ S \subseteq A \times C$

Ex: $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$, $C = \{c_1, c_2\}$
$R = \{(a_1, b_1), (a_1, b_2), (a_2, b_1)\}$ Let
$S = \{(b_1, c_1), (b_1, c_2), (b_2, c_1)\}$

then, from (a, b_1) & (b, c_1) ∵ b_1 = same & not $(a, b_1), (b_2 c_1)$ ∵ $b_1 \neq b_2$

$R \circ S = \{(a_1, c_1), (a_1, c_2), (a_2, c_1), (a_2, c_2)\}$

$M_{R \circ S} = M_R \circ M_S$ (M = matrix repres^n of given set)
→ (x) multi:

$$M_R = \begin{array}{c c} & \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

* Recursive definition of R^n:

$$R^n = \begin{cases} R & ; n = 1 \\ R^{n-1} \circ R & ; n \geq 2 \end{cases}$$

* Connective Relation of R:

$R \subseteq A \times A$ then $R^\infty = \bigcup_{i=1}^{\infty} R^i = R_1 \cup R_2 \cup \dots \cup R^\infty$

Ex: Find connective rel^n of R if $R = \{(a,a), (a,b), (a,c), (b,c)\}$ on set $A = \{a, b, c\}$

Sol^n: $R^\infty = ?$ $R^2 = \{(a,a), (a,b), (a,c), \cancel{(b,a), (b,b), (b,c)}\} = R \circ R = R^3 \cancel{= R^\infty}$
$= R^4$

∴, $R^\infty = R \cup R^2 = R$

Every eqv. rel^n creates partition. Vice versa is also true.

Date / /
Page

Ex: $R \subseteq A \times A$ where $A = \{a, b, c\}$
$R = \{(a,b)\}$ —— Trivial Case ∵ sufficient elements aren't available ∴ all rel^n will be true for which elements ~~are of which are~~ insufficient.

Reflexive — ✗
Symm. — ✓
Transi. — ✓
Eqv. — ✗

Q. Prove that congruence modulo operator defined relation on set of integers is an equivalence rel^n

Sol^n: $Z = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$
$R = \{(z_1, z_2) \mid z_1 \equiv z_2 \bmod m, \text{ where } z_1, z_2 \in Z\}$

Reflexivity: $m \mid (z_1 - z_1)$ i.e $m \mid (z_n - z_n) \Rightarrow m \mid 0$ ∴ ✓ reflexive
or $z \equiv z \bmod m$

Symmetry: If $z_1 R z_2$ where $z_1, z_2 \in Z$
$\Rightarrow z_1 \equiv z_2 \bmod m \Rightarrow m \mid (z_1 - z_2)$
$z_1 = mq_1 + z_2$ where $q_1 \in Z$
$z_2 = m(-q_1) + z_1 \Rightarrow z_2 \equiv z_1 \bmod m$
where, $-q_1 \in Z$ ✓

∴ Symmetric ✓

Transitive: If $z_1 R z_2$ and $z_2 R z_3$
$\Downarrow$ $\downarrow$
$z_1 = mq_1 + z_2$ —(i) $z_2 = mq_2 + z_3$ —(ii) ; $q_1, q_2 \in Z$
put z_2 from (ii) into (i)
$z_1 = m(q_1 + q_2) + z_3$ where $q_1 + q_2 = q \in Z$
∴ $z_1 = mq + z_3 \Rightarrow z_1 \equiv z_3 \bmod m$
(by closure prop. of add^n)
∴ $z_1 R z_3 \Rightarrow$ Transitive ✓

Equivalence ✓

No. of partit^n on $|A| = n$

$$\sum_{r=1}^{n} S(n,r)$$

where, $S(n,1) = S(n,n) = 1$

$\therefore$

$$S(n,r) = S(n-1, r-1) + rS(n-1, r)$$

Q No. of Eqv. ~~on~~ rel^n that exist on A with $|A| = n = 3$

Sol^n: $\sum_{r=1}^{3} S(3,r) = S(3,1) + S(3,2) + S(3,3)$