14/10/19 **After Mid-sem**

Mid-Term sol^n

② $Z = 15LK - 0.3L^2 - 0.5K^2 + \lambda[1500 - 60L - 74K]$

$\frac{\partial Z}{\partial L} = 0, \quad \frac{\partial Z}{\partial K} = 0, \quad \frac{\partial Z}{\partial \lambda} = 0$

③ Max Q
sub to cost const.

Min C
sub. to o/p constraint

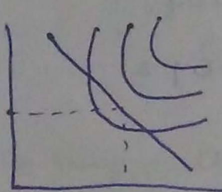
$Z = f(K,L) + \lambda_0[C - wL - rK]$

$Z = wL + rK + \lambda_1[\bar{Q} - f(K,L)]$

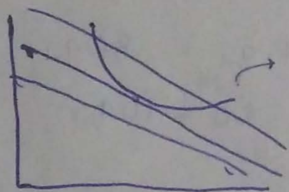
$\frac{\partial Z}{\partial K} = 0 \Rightarrow f_K = \lambda_0 r$

$\frac{\partial Z}{\partial K} = 0 \Rightarrow r = \lambda_1 f_K$

$\lambda_1 = \frac{1}{\lambda_0}$



→ isocost



→ isoquant

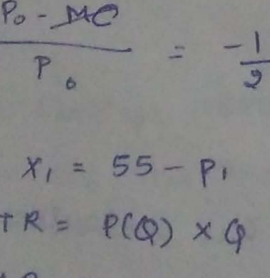
④ (a) $500 - \frac{P}{2} = P - 100 \Rightarrow P = 400, \; Q = 300$

(b) $P = MC$

$400 = 2q + 1 \Rightarrow q = 199.5$

It is perfect competition ∴ don't see fixed part

⑤ MC



This is supply curve

(b) $SS > 0$ $P \geq AVC_{min}$

$SS = 0$ $P < AVC_{min}$

⑥ $\frac{P_0 - \cancel{MC}^{20}}{P_0} = -\frac{1}{2} \Rightarrow P_0 = 40/3 \qquad P_1 = 50/3 \qquad \Delta P = 25\%$

⑦ $X_1 = 55 - P_1$ $X_2 = 70 - 2P_2$

$TR = P(Q) \times Q$

$MR_1 = 55 - 2X_1$

$MR_1 = MC$

$X_1 = 25$ $X_2 = 30$

$P_1 = 30$ $P_2 = 20$

(c) $\pi = P_1 \times X_1 + P_2 \times X_2 - 5(55)$ $X_1 + X_2 \equiv 55$

Topics post Midsem:
- Monopolistic Competⁿ
- Game Theory
- Oligopolistic Competⁿ
- Macro

Monopolistic Competition

- Introduced by → Edwin H Chamberlain (1933) The theory of M.C.
(Harvard)

· Just graphs & curves, no explaination

- Joan Robinson → The Economics of Imperfect compⁿ.
(Cambridge)

- Avinash Dixit & Joseph Stiglitz → 1977

Assumptions / Facts about Monopolistic Competition

1. Product Differentiation: goods are not perfect substitutes, but ~~so~~ closed substitutes (unlike in perfect comptⁿ)
Ex. Surf Excel & Tide

2. Non-price competition:
↳ Kingfisher used to give extra goodies to customers (better than others)
↳ Advertising, Packaging, Quality std.

3. large no. of buyers & sellers

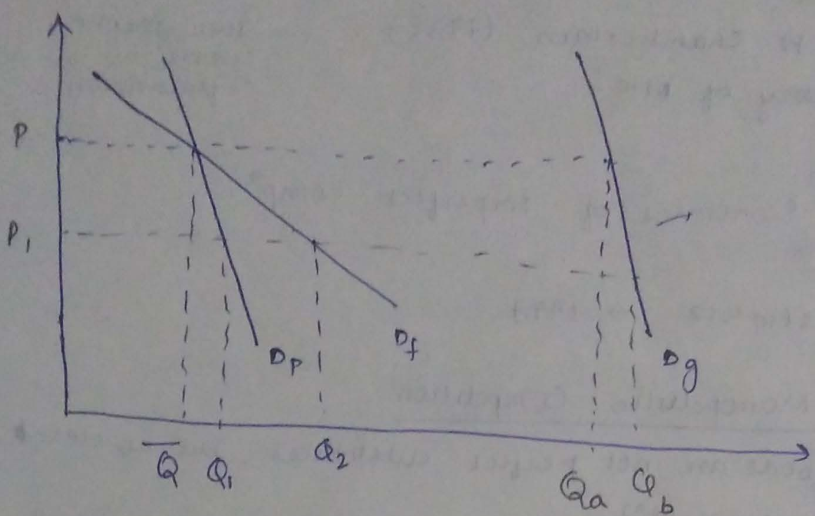
4. Firms are capable of independent behavior.
↳ since producer are producing close substitutes, their stay in market don't depend on other firms' decisions.
↳ Ex. Coca-cola → much variety Pepsi → not that much
Most stores have Coca-Cola.
How Pepsi dealt with it? → started giving commission in Restaurants
↓
People don't have choice to go to another place

Product Differentiation

- **Horizontal**
  - Different variety
  - Same Quality
- **Vertical**
  - Different variety
  - Different Quality
  - Ex. Iphone & Micromax.
  - AK47, Katta

Demand Curves faced by a firm in a Monopolistic Competition
(Not Monopolist)

* Group demand curve (D_g) (Market demand curve)
* Proportional demand curve (D_p)
* Perceived demand curve. (D_f)



$D_p = \left(\frac{D}{n}\right)$ → Total demand; → no of firms

D_g: Any industry will face
→ Firms are ~~not~~ roughly identical here. They have almost similar cost.
Hence, Demand can be equally distributed b/w all the firms
⇓
D_p → divide equally.

D_f: Firms don't consider D_p, but D_f as demand curve. — ①

Reason: Firms think that since there are large no of buyers & sellers and they are independent, their behavior would go unnoticed. (diff b/w this & Oligopolistic)
↳ Products are differentiated
↳ trying to differentiate his product

Firms think: $P \rightarrow P_1 \Rightarrow$ sales will ↑ to Q_2
That is the reason it sells Q_1.
$Q_2 - Q_1$: Overprodⁿ in Monopolistic

Q. If all firms bring P down to P_1, why demand has ↑ed?

	P		Q	TR
Rs	10	→	5	50
Now	5	→	10	50

D_g: total demand
D_f & D_p → demand curves

Curve reprentating this information : D_g

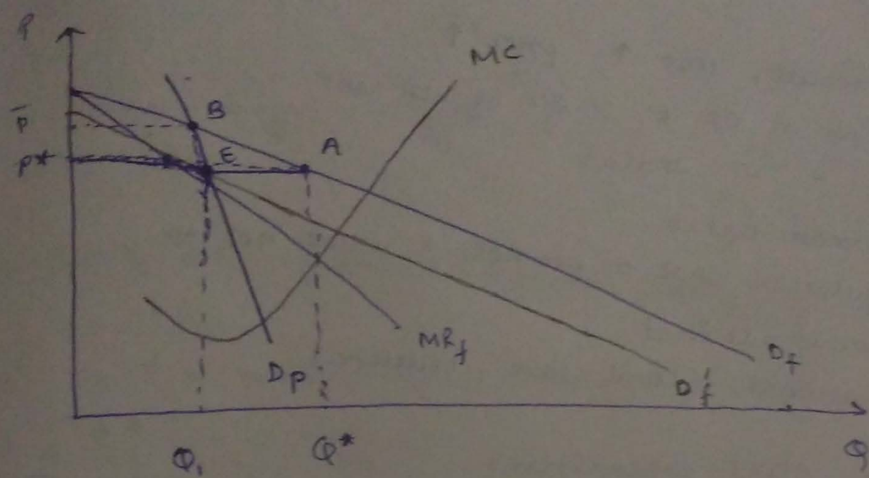
$\boxed{\text{* } D_f \text{ is more elastic than } D_p}$ → Firm thinks that since his product is differentiated, so if he cuts down cost, his demand will ↑se at a large scale

- Firms perceive elastic demand.

Reason: large no — ①

$Q_2 - Q_1 \rightarrow$ surplus production by the firm in monopolistic market.

Short Run Equilibrium



D_f' : leftward shift of D_f

$MR_f \rightarrow$ perceived

Initially $\rightarrow$ $\bar{P}$ and Q_1 (No surplus prod^n)

$MR = MC \rightarrow P^*$ & Q^*

→ Firms think they are independent, so they will charge P^* & Q_1 will be the demand.

→ Surplus prod^n will occur.

→ To gain eq^m,

D_f will shift leftwards (D_f') [down]

→ eq^m will only happen if $D_P = D_f'$ (P^* and Q_1)

⟹

1. All firms are charging $\bar{P}$
2. The i^{th} firm feels it is independent having differentiated product(s). So, it can operate independently.
3. He will operate at $MR_f = MC \rightarrow P^*$ & Q^*
4. Not only the i^{th} firm, but all n firms in a market think in the same way.
5. All firms will produce Q^*
6. But firms will actually have proportional demand (D_P)
7. At $P = P^*$, DD is Q_1
8. Excess Prod^n by firm = $Q^* Q_1$

9. Firms will re-perceive its demand to ~~get~~ obtain eq^m

10. $D_f \rightarrow D_f'$

11. Eq^m will be reached when the perception is equal to proportion.
$D_f' \equiv D_p$ $(P^* \,\&\, Q_1)$ (E)

<u>Why People Advertise</u> ?

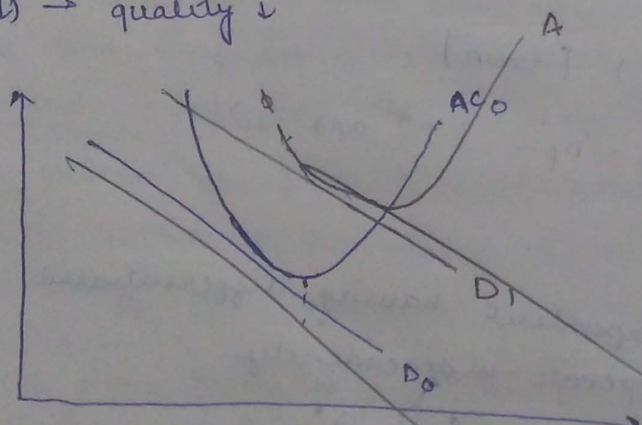
→ To get more DD. As a result, cost ↑, price ↑
→ Advertisement expenditure in US is ~ 2% of its GDP
(more than education " in India)

→ ~~take~~ Licker ads never promote licker.
→ Ads always have regulation what to promote & what not to.
→ Products are ~~trans~~ trans-national.
Ads are not trans-national. (Local stars, culture)

Suppose a firm has an ADVT expenditure
↳ Cost (↑)
↳ P_n (↑) or (↓) ?
↓
~~By~~ my ↑ DD due to ADVT
may go away due to ↑ P_n (Price)

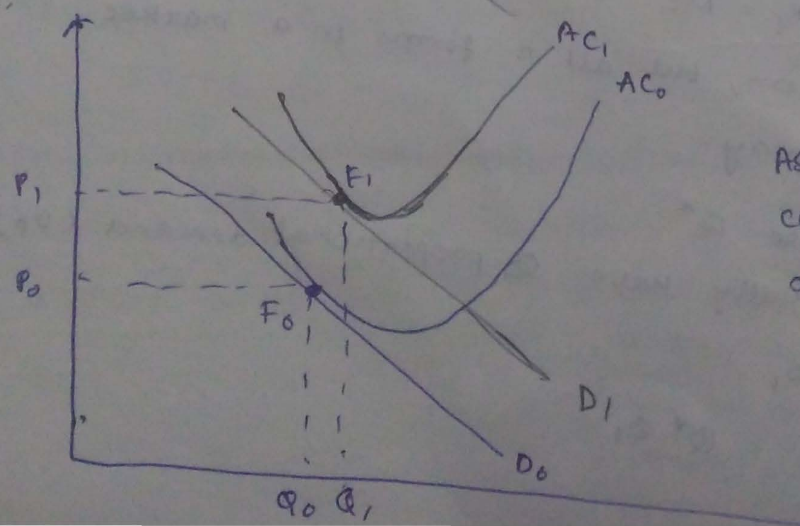
→ If I ~~do~~ ADVT & it is better quality → people will buy at ↑ Price
→ If bad quality → P ↓, will fade out of market soon.

P_n(↑) → quality ↑
P_n(↓) → quality ↓



~~After~~ Because of ADVT, cost ↑, DD also shifts

Case-I

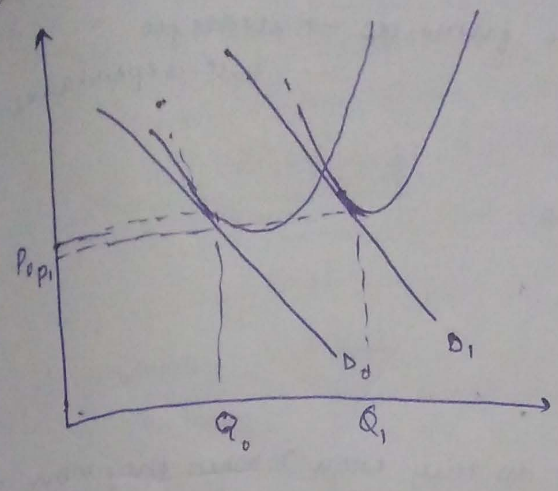


Due to advt., cost ↑, DD also shifts

Assuming consumer care about quality of products.

(Rel)^n b/w AC & D comes from LR)

Case-II



$P_0 > P_1$

To ↑ demand, P will be ↓

→ consumers will only pay a higher price for an advertised product if that product is of better quality. → Case-I

Case-II: If it is of lower / inferior quality, then P_n (↓).

Quesⁿ. If P is replaced by % of Marks in +2.

ultimately, % will ↑ due to advt. → fb / insta

So, advt. makes people foolish. They believe they'll get job sooner.

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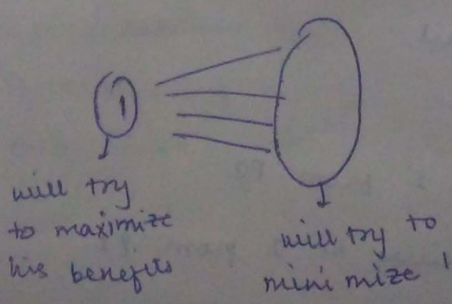
Game Theory

Books: 1. Microeconomic Theory by Mascollel, Whinston & Green (Ch. - 7 & 8)

2. Game Theory for Applied Economists by Robert Gibbons (Ch. - 1,2)

↳ Game: You try to understand someone's psychology & vice-versa. The process of settling down to 1 point (eq^m) is what is involved in game theory.

↳ Formally, a game is a formal representation of a situation in which a number of individuals interact in a setting of strategic interdependance.



game theory tries to find best possible outcomes for individuals.

Ex. Multiple firms :

F1 F2 - - Fn

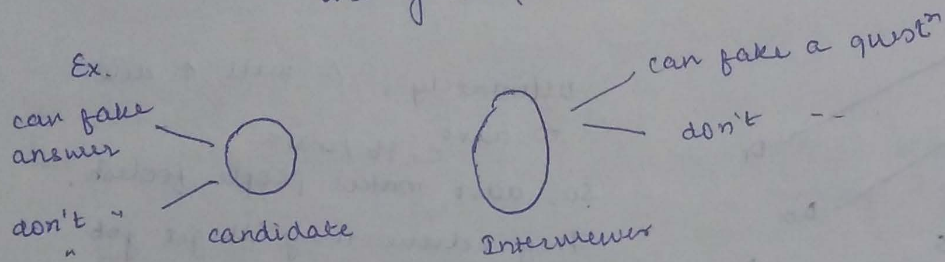
If F1 sets P (↓), what will other firms do → strategic interdependance.

↳ we'll generally have 2 individuals.

Q. What needs to be understood ?

1. **Players** → who is involved ?
2. **Rules** → Who moves when ? what do they know ? When they move ?
3. **Outcomes** → For each possible set of actions what is the outcome of the game ?
4. **Payoff** → what benefit a ~~person~~ player gets from the outcome of the game ?

Ex.



If get selected → get ~~sat~~ job as payoff (salary)

Game
- Non-cooperative games
  - simultaneous move games
  - sequential move games
  - (Players are non-cooperative with each other)
- Cooperative games (X) ↳ won't talk about it

Game of Matching Coins / Pennies (Simultaneous move)

1. 2 Players (P1, P2)
2. Rules :
   - Both players ~~[illegible]~~ simultaneously puts a penny down, either player gets a head or a tail.
3. Outcome :

   If the pennies match, P1 receives Re 1 from P2.

   If the pennies don't match, P2 receives Re 1 from P1.

Normal Form Representation:

		P2	
		h	t
P1	H	(1, -1)	(-1, 1)
	T	(-1, 1)	(1, -1)

→ Ex: Goalkeeper & Player

		P	
		l	r
G	L	(1, -1)	(-1, 1)
	R	(-1, 1)	(1, -1)

⇒ How should G & P move to maximize their benefits (G → save goal, P → make goal)

Games of common Interests / Coordination games

↳ Battles of sexes.

		G	
		Joker	Sky is Pink
B	Joker	(2, 1)	(0, 0)
	Sky is Pink	(0, 0)	(1, 2)

→ Normal Form Representation

Boy likes → J
Girl → SP
They want to go to date.
↓
must go to same movie

• If both go to J → girl won't be satisfied
SP → boy — — — — —

What is best possible solⁿ ?

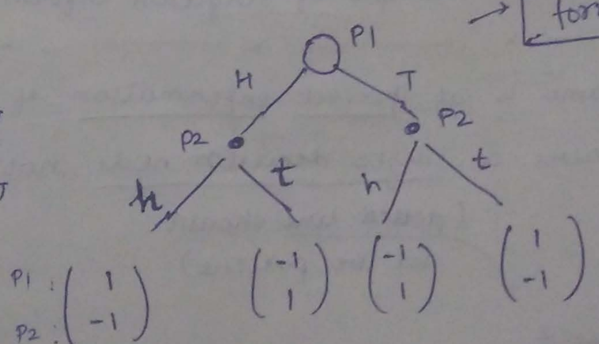
Sequential :→

P1 tosses a coin.

P1 : H → P2 : → H, → T

: T → P2 : → H, → T

Payoff : ⟶



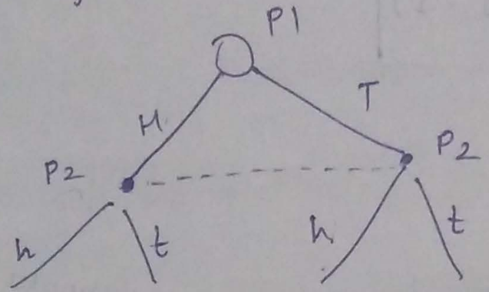
→ Extensive form

→ Payoff

* Initial decision node is represented through an open circle.
* After that, if we have any other decision nodes, we represent them with a closed circle.

→ If outcome of coin tossed by P1 is hidden from P2, P2 won't be a decision node (he doesn't know whether he has to move from left or right).

Represented by :



→ Imperfect Information

→ singleton Information Set

significance of Singleton Information Set :

↳ Player 2 knows Player 1 has moved but does not know where exactly he is.

Maket for layman :

Buying 2nd hand car — 1/2 good quality — P → Rs. 2 lakh; 1/2 bad quality — P → Rs. 1.5 lakhs

Seller is willing to sell at Rs. 2 lakh

Buyer will try to reduce this price as he doesn't know whether he will get good or bad quality

$2 \times \frac{1}{2} + 1.5 \times \frac{1}{2} = 1.75$ → try to reduce to 1.75. (buyer)

If good quality → seller will never sell at 1.75

bad → " sell at 1.75

So, bad quality cars are sold at ↑ price.

This is an example of singleton information set. (Asymmetric Information)

↳ A game is of perfect information if each information set contains a single decision node, not otherwise

(dotted line should not be present)

has 2 decision nodes

- Perfect Information : One information set & One decision node
- Imperfect Information : One information set & two decision nodes

1. Normal form
2. extensive form
3. strategic form

- strategy of P1 : $S_1 = \{H, T\}$
- P2 has 2 information sets. She has to plan one action each for 2 different situations → (it can follow 1st or 2nd path)

P2 has the following possible strategies :

i.) play h if P1 plays H) hh (A)
play h if " " T (E)

ii.) play h if P1 plays H) ht (B)
play t if " " T (F)

iii.) play t if P1 plays H) th (C)
play h " " T (G)

iv.) play t if P1 plays H) tt (D)
play t " " T (H)

$S_2 = \{hh, ht, th, tt\}$ → possible set of strategies

P2

		hh	ht	th	tt
P1	H	(1,−1) A	(1,−1) B	(−1,1) C	(−1,1) D
	T	(−1,1) E	(1,−1) F	(−1,1) G	(1,−1) H

→ Representation of strategic form.

- If we have singleton information set, only 1 info. set is available

P2

		h	t
P1	H	(1,−1)	(−1,1)
	T	(−1,1)	(1,−1)

⇒ $\boxed{\text{strategic form} \equiv \text{Normal form}}$

Nash Equilibrium

P (kicker)

		l	r
G	L	1, −1	−1, 1
	R	−1, 1	1, −1

best thing to do

If player kicks L → Goalee moves L
R → R

If goalee moves L → Player kicks R
R → L

so, no eq^m is obtained.

→ max^m thing we can give ~~#~~ to these players is the probability.
(Prob. that player kicks left / right, prob. that goalee moves left / right)

↳ The Nash eq^m can not be obtained through pure strategies
↳ It can be obtained through mixed strategies

mixed strategies → probability $distrib^n$ b/w strategies

pure strategies → sticking to a particular strategy.

- Mixed strategies : These are probability distributions over pure strategies

$$P(L_K) + P(R_K) = 1 \quad ; \quad P(R_G) + P(L_G) = 1$$

If kicker has prob. to kick left $= p$
right $= 1-p$

If goalee ... .. left $= q$
right $= 1-q$

Q. Expected payoffs of kicker and goalee ?

Kicker : $p \times (1-q) \times (1) + pq(-1) + (1-p)(1-q)(-1) + (1-p)(q)(1)$

$= p(1-q) - pq - 1 - pq + p + q + q - pq$

$= p - pq - pq - 1 - pq + p + q + q - pq$

$= 2(p+q) - 4pq - 1$ ——— (1)

Goalee : $p \times (1-q) \times (-1) + pq(1) + (1-p)(1-q)(1) + (1-p)(q)(-1)$

$= -p + pq + pq + 1 - p - q + pq + pq - q$

$= 4pq - 2(p+q) + 1$ ——— (2)

Q. How to find optimal values of p & q?

$p=1, q=0$: $E[K] = 1$

$p=0, q=1$: $E[K] = 1$

$p=0.5, q=0.5$: 0

10/12 ① Pure strategy

Expected payoff of player 1 when P1 believes that P2 will play heads (h) with prob (q) and tails with prob (1-q)

		P2 H	P2 t
P1	H	1, -1	-1, 1
	t	-1, 1	1, -1

case-I P1 plays Heads (H) : $q(1) + (1-q)(-1) = 2q-1$

case-II P1 plays Tales (T) : $q(-1) + (1-q)(1) = 1-2q$

Pure strategies $q \in [0,1]$

if $q > \frac{1}{2}$ ⇒ $2q-1 > 1-2q$ → P1 will always play Head (H)

→ P1 will want to get highest payoffs.

$q < 1/2$ ⇒ $1-2q > 2q-1$ → P1 will always play Tail (T)

* ⇒ If $q > 1/2$ → P1 will play Heads, else, Tails.

* If a goalee moves right with prob. > 1/2 (→ get this from impirical data), player will ~~no~~ kick in the left. Else, he'll kick right.

↳ Based on these payoffs, analysts find critical probabilities.

- If $q = \frac{1}{2}$ → P1 is indifferent b/w strategies H and T.

Pure strategies → if this condⁿ ⇒ do this
if that ⇒ do that

② **Mixed strategy** : How to find solⁿ in this case

P1 : $(p, 1-p) \longrightarrow (H, T)$

P2 : $(q, 1-q) \longrightarrow (h, t)$

↳ For each value of $q \in [0,1]$, we now compute values of p denoted by $p^*(q)$ s.t. $(p, 1-p)$ is a best response function for P1 to $(q, 1-q)$ of P2

↳ My strategy should be s.t. my benefits should be maximized whatever move P2 makes.

↳ strategic interdependence. $(p^*(q))$

$$
\begin{aligned}
E(P1) &= 4pq - 2p - 2q + 1 \\
&= (1-2q) + p(4q-2) \\
&= (1-2q) + 2p(2q-1) \\
&= (1-2p)(1-2q)
\end{aligned}
$$

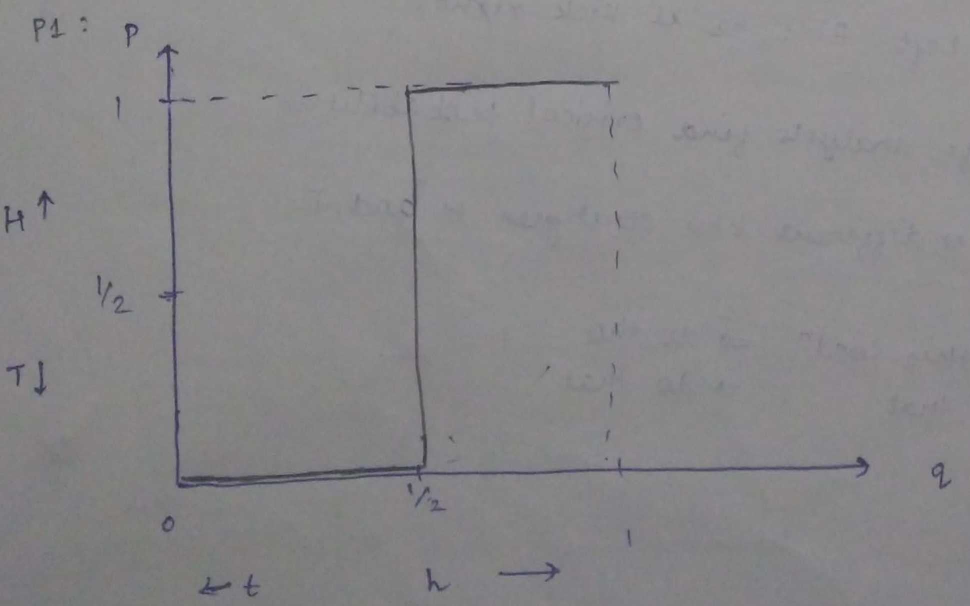
↳ $E(P1)$ (↑) if $q > 1/2$ → $E(P1)$ is max when $p = 1$ → P1 will play H certainly

$E(P1)$ (↓) if $q < 1/2$ ↳ $E(P1)$ is max when $p = 0$ → P1 will play T.

↳ getting same answer as in pure strategy.

Q. So why are we interested in mixed strategies?

Graph for best response funcⁿs :



$q > 1/2 \Rightarrow$ P1 will have $p = 1$

$q < 1/2 \Rightarrow$ P1 will have $p = 0$

Q. Draw for P2 also.

Nash equilibrium

↳ In a n-player game, the strategy of i^{th} player (s_i) is the best response to a rival strategy (s_j), $i \neq j$, if player i can not obtain a strictly higher payoff with any other possible strategies.

↳ eq^m → a point where there is no tendency to change.

Ex.

P1	P2
S1	S1
S2	S2
S3	S3

When P2 plays S2, because it gives best payoff P1 gets highest payoff with S3. So, he plays S3. and vice-versa.

↳ To solve this, Nash used correspondence ↳ Kakutani's Fixed Point Theorem

~~for~~ In correspondence → we get a band ($f(x)$) of values corresponding to single value of x

---

Simultaneous Move games

1. Prisoner's Dilemma game

2 individuals are arrested for an alleged crime and are held in separate cells. The inspector tries to extract confession from each prisoner. Each prisoner is privately told that if he is the only one to confess then he will be rewarded with a life sentence of 1 year. While the defunctant prisoner will go for jail for 10 years. However, if he is the only one not to confess, then it is him who will serve jail for 10 years. If both confess, ~~the~~ police will show some mercy & they will get jailed for 5 years. If niether confesses, it will still be possible to convict both of a lesser crime that carries a sentence of 2 years.

Prisoner's objective : Not to confess and get minimum life sentence.

		P2	
		DC	C
P1	DC	(-2, -2)	(-10, -1)
	C	(-1, -10)	(-5, -5)

C → Confessed

DC → don't confess

→ Bad eq^m : -5, -5

Good eq^m : -2, -2

P1 thinks → If I don't confess → 2

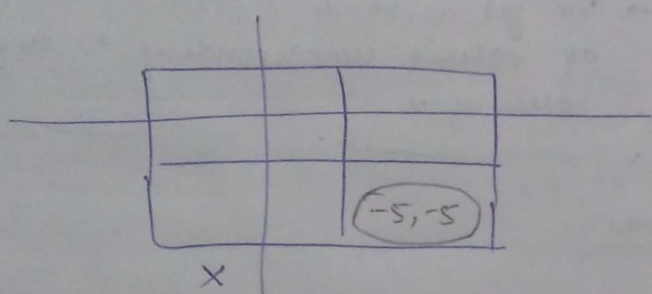
but if other prisoner confesses → 10) → dilemma

⇓

can't have good eq^m

Strictly Dominated & Weakly Dominated strategies :

For P1 : Irrespective of what P2 is playing, P1 will try to play C (less punishment (-1, -5) < (-2, -10))



X can cancel this

For P2 : Out of -10 & -5 → He will choose -5

For P1 → confess is a strictly dominated strategy.

↓

→ Optional Book: (The art of strategy by Avinash Dixit)

s_i is strictly dominated strategy for player i, if it maximizes uniquely player i's payoff for any strategy player j is playing

Mathematically,

$$U_i(s_i, s_j) > U_i(s_i', s_j)$$

Ex. suppose, payoffs are changed to :

		P2	
		DC	C
P1	DC	0, -2	-10, -1
	C	-1, -10	-5, -5

X

→ P1 can't have strictly dominated (0 > -1 but -10 > -5)

→ But P2 has ((-1, -5) > (-2, -10)) Will confess

⇓

Iterative elimination of dominant strateg[y]

Weakly dominated strategy:

Case-I:

P2

P1	Left (L)	Center (C)	Right (R)
(T) Top	1,1	1,1	0,0
(B) Bottom	0,0	1,2	1,2

P2: don't have strictly dominated strategy → similarly for P1.

But, we can ~~state~~ say L is weakly dominated by C, → can cancel L

$$U_1(s_i, s_j) \geq U_1(s_i', s_j)$$

For P1: (for remaining) → can cancel Top

Top is weakly dominated strategy by Bottom. → weakly dominant

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Remaining strategies are: BC and BR → (1,2)

Case-II:

	L	C	R
T	1,1	1,1	0,0
B	0,0	1,2	1,2

For P2: R is weakly dominated by C

For P1 (remaining): B is weakly dominated by T.

Remaining strategies: TL & TC → (1,1)

In both cases, we are getting different $eq^m s$.

* In weakly dominated strategies, game can't be solved in iterative fashion.
(we will have multiple $eq^m s$)

Dominance with Mixed strategies :- (Earlier → Pure strategy)

		P2 L	P2 R
	(1/2) U	10,1	0,2
P1	M	4,2	4,1
	(1/2) D	0,0	10,1

For P1 → no strictly dominated strategy.

But if we have prob associated with strategies

U → 1/2
D → 1/2 } → Expected payoff (U) = 5, (D) = 5, (M) = 4 } ⇒ M will be strictly dominated by mixed strategy U → 1/2 & D → 1/2

Rationalizability

→ Game theory assumes players are rational → want to maximize their payoff.

Ex. 1

		P2 L	P2 M	P2 R
P1	U	2,0	3,5	4,4
	D	0,3	2,1	5,2

Def^n :→ After eliminating all strategies iteratively, remaining strategies are rationalizable

~~In the above example~~

Ex. 2

	L	R
U	2,2	0,0
D	0,0	1,1

can't eliminate anyone here.
So, rational strategies are :

(UL) ; (UR);
(DL) ; (DR);

	L	R
U	2,2	0,0
D	0,0	1,1

we want to find Nash eq^m.

If P1 plays U → P2 will play L

D → " " R

For Nash eq^m → we ~~will~~ should see which box has both terms underlined.

so here, 2 pure strategies Nash eq^m (PSNE)

(U,L) & (D,R)

(If we talk about std. Head-Tails game → we did not have any box with 2 underlined values → we had mixed strategy Nash eq^m (MSNE) there)

↓

1,-1	-1,1
-1,1	1,-1

In Ex-3: If U → p and D → 1-p, so, MSNE will be:

L → q and R → 1-q

If P1 plays U : 2q ⇒ $q = 1/3$ & $p = 1/3$

D : 1-q

E(P1) will be maximized when :

→ if $q = 0$ E(P1, U) < E(P1, D)

$q = 1$ E(P1, U) > E(P1, D)

$E(P1) = 2pq + (1-p)(1-q)$

$= 3pq - (p+q) + 1 \equiv (1-q) + p(3q-1)$

∴ E(P1) will be maximized when $q > 1/3$.

For P2 also, we'll have same as P1. (symmetric case)

To get exact value,

$$2q = 1 - q \Rightarrow q = 1/3$$

↓

Player would want to have similar payoff in both the cases. (outcomes)

↳ Now, we'll try to relate game theory with economics Model

<u>Automobile</u> <u>Crisis</u> (demand of automobiles ↓)

ex. Maruti Suzuki cuts down its cost by 10%.
What'll happen to other companies?

Oligopoly → thinks about strategies based on game theory (unlike earlier discussed ones)

1. Cournot Model (1838)

Assumptions :

- Firms are producing <u>homogenous</u> goods.
- Each firm must <u>decide how much to produce</u> and they decide <u>simultaneously</u>.
  ⇓ strategy
- 2 firms. i & j

→ The result of this model can be translated into differentiated goods - both Horizontal & Vertical.

→ Market demand :

$$P(Q) = a - Q \quad , \quad Q = q_i + q_j$$

→ Total cost of firm $= c\,q_i$ (In multiple monopolist $= c(q_1 + q_2)$).

Firm i's profit :

$\pi_i = [a-(q_i+q_j)-c]\,q_i$

objective of firm → maximize profit.

so, it must choose a strategy s.t

$U_i(s_i^*, s_j^*) > U_i(s_i, s_j^*)$ → for i^{th} player (Condition for Nash eq^n)

what I'm playing is best strategy for me given the other player is also playing his best strategy.

Obj : Max $U_i(s_i, s_j^*)$ Strategies → quantity but his payoff is the profit obtained

Payoff : Profit

⇒ Max. $[a-(q_i+q_j^*)-c]\,q_i = \pi_i$

i believes j is playing optimal strategy ⇒ $q_j = q_j^*$

To find q_i^*,

$\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow (a-(q_i+q_j^*-c)) - 2q_i = 0$

$\Rightarrow \boxed{q_i^* = \frac{a-c-q_j^*}{2}}$ — (1)

similarly, $\frac{\partial \pi_j}{\partial q_j} = 0$

$\boxed{q_j^* = \frac{a-c-q_i^*}{2}}$ — (2)

→ Reaction func^{n}s

From eq^n (1) & (2),

$\frac{dq_i}{dq_j} < 0$ ⇒ If i^{th} firm ↑ o/p, j^{th} firm ↓ o/p ⇒ strategic dependence

Reaction func^{n}s :