LNM IIT, JAIPUR, DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

# Introduction to complexity theory

## Assignment 1

## Last submission date: 27 April 2019

February 15, 2019

**Note:** Before describing the state diagram of your Turing machine, first describe how is your Turing machine works in points. Your assignment must be written in your hand writing. If I find that a submitted assignment is a photostate or some unacceptable cheating of some other assignemnt then I will happily assign zero marks to those students. Please write your name as well as roll number on your assignment. I will not accept any assignment after 27 April 2019, 5:00 PM.

1. Let $S(n) = n+1$ is sucessor function. Assume that input has given in unary form, mean to say input alphabet is $\Sigma = \{1\}$. Construct a Truring machine $M$ which compute this function.

2. Let $A_{TM} = \{\langle M, w\rangle \mid \text{Turing machine } M \text{ accepts } w\}$. Construct a Turing machine which accepts $A_{TM}$.

3. Prove that if $L$ is Turing recognizable and co-Turing recongnizable then $L$ is decidable.

4. Design a Turing machine $M$ which decides $L = \{\langle G\rangle | G \text{ is a connected and undirected graph}\}$.

5. A lazy Turing machine is defined by $M = (Q, \Gamma, \Sigma, q_0, q_{accept}, q_{reject}, \sqcup, \delta)$ where $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, S, R\}$ and the symbol $S$ means stay on same cell, in other words, no left or right movement of head. Prove that lazy Turing machine and our standard Turing machine are equivalent in terms of computation power.

6. $EQ_{CFG} = \{\langle G_1, G_2\rangle |$ For the Context free grapmmers $G_1$ and $G_2$, $L(G_1) = L(G_2)\}$. Prove that $EQ_{CFG}$ is undecidable.

7. Prove that $A_{TM}$ can not be mapping reducable in to $E_{TM}$.

8. Let $T = \{\langle M\rangle \mid TM\ M$ accepts $w^R$ whenever it accepts $w\}$. Prove that $T$ is undecidable.

9. Let a language $L$ is decidable and a language $L_1 \subseteq L$. Is $L_1$ decidable?

10. Let a language $L$ is Turing recognizable and a language $L_1 \subseteq L$. Is $L_1$ Turing recognizable?

11. Let $\Sigma_{i=0}^{n} c_{n+1-i} x^i$ be a polynomial with a root at $x = x_0$. Let $c_{max}$ be the largest absolute value of any $c_i$. Show that $|x_0| < (n+1)\frac{c_{max}}{|c1|}$.

12. Let $L_1$ and $L_2 \in NP$, then prove that $L_1 \cup L_2$ and $L_1 \cap L_2 \in NP$.

13. Show that if a language $L \in$ **NP**-complete $\cap$ **co**-**NP**, then **co**-**NP** = **NP**.

14. **P** $\subseteq$ **NP** $\cap$ **co**-**NP**

15. **P** = **NP** $\implies$ **NP** = **co**-**NP**

16. Prove that $HALT = \{\langle M, w\rangle |$ Turing machine $M$ halts on $w\}$ is **NP**-hard. Is $HALT$ in **NP**?

17. $3COLOR = \{G = (V, E) \mid$ A coloring of the vertices of $G$ by three colors then each adjacent vertices get different color $\}$. Prove that $3COLOR$ is **NP**-complete.

18. A *cut* in an undirected graph is a separation of the vertices $V$ in to disjoint subsets $S$ and $T$. The size of cut is the number of edges that have one end point in $S$ and the other in $T$. Let $MAX\text{-}CUT = \{\langle G, K\rangle | G$ has cut of size $k$ or more$\}$. Prove that $MAX\text{-}CUT$ is **NP**-complete.

19. Prove that $2SAT \in$ **NL**. Is $2SAT$ **NL**-complete?

20. Prove that the language $\{\langle G\rangle \mid G$ is strongly connectd diagraph$\}$ is **NL**-complete.

21. Prove that the function $H(n)$ defined in the proof of Ladner's theorem is computable in time polynomial in $n$.