The LNM Institute of Information Technology, Jaipur
Mid-Term Examination September 2016
Sub: Mathematics-III

Duration : 90 minutes

Max.Marks: 14+16

Name: ANUSHKA Roll No.: 15UCS022 Date: 12-Sept-2016

Note: **Submit Part-I after 45 minutes of commencement of the exam.** Use only the last page of main answer sheet for rough work and calculation. Use only pen to write answers. Answers written by pencils will not be evaluated.

Part-II

1. Show that $u(x,y) = xy - x + y$ is harmonic. Find a conjugate harmonic $v(x,y)$ of $u(x,y)$. Find $f(z) = u + iv$ in-terms of z. [1+2+1 Marks]

2. Evaluate the integral $\int_C \frac{z+2}{z} dz$, where C is [1+1+2]

   (a) the upper half of the circle $|z| = 2$ in the counter clockwise direction.

   (b) the upper half of the circle $|z| = 1$ in the clockwise direction.

   (c) the parabola $y = x^2$ from 0 to $1 + i$.

3. Find all values of z for which $z^4 = -8 - 8\sqrt{3}i$ and locate theses values in the complex plane. [4]

4. Consider $f(z) = \frac{z}{z^2+1}$ over $C : z(\theta) = 4e^{i\theta}$, $\frac{\pi}{3} \leq \theta \leq \frac{\pi}{2}$. [2+1+1]

   (a) Find an upper bound for $f(z)$.

   (b) Find the length of the contour C.

   (c) Find an upper bound for $\int_C f(z)dz$, without evaluating the integral.