End Term Exam Question Paper

Subject: **Design and Analysis of Algorithms (CSE 325)**

Max Marks = **80** Time = **3 hours**

Note: Answer all parts of the question concisely at one place only. Sub part of questions which are answered elsewhere may not be corrected. ~~There are~~ Total 4 sheets

Q1. Answer the following questions concisely. [15 marks]

a) What is minimum spanning tree? State the difference between Prim's and Kruskal's algorithm for finding minimum spanning tree. [2 marks]

b) Mention the names of two algorithms used for finding single source shortest path in a given graph and their time complexity. [2 marks]

c) What is meant by topological sort and strongly connected component in a directed graph. [2 marks]

d) When a problem is said to be polynomially reducible to another problem? [2 marks]

e) Explain why the statement, "the running time of algorithm A is at least $O(n^2)$" is meaningless. [2 marks]

f) Briefly state two properties that the problems solved by greedy algorithm exhibit? [2 marks]

g) State the principle of optimality. [1 marks]

h) Define NP Complete problem and mention the first problem that was shown to be NP Complete. [2 marks]

Q2. Attempt any Five of the following questions. [5 x 5 = 25 marks]

a) Prove or disprove: If f1(n) = O(g1(n)) and f2(n) = O(g2(n)), then f1(n) x f2(n) = O(g1(n) x g2(n)).

b) For an Array A of *N* elements, if there are *K* **inversions** then prove that Insertion sort will sort the array in $\Theta(N+K)$ time. (note: Elements i and j are inverted if i<j but A[i] > A[j].)

c) Write the pseudo code of Kruskal's algorithm and analyze its time complexity?

d) Explain how Bellman-ford algorithm is used to detect the presence of negative edge cycle in the given graph?

e) Explain the algorithms for union and find operation on disjoint sets represented using rooted trees and weighted union heuristic.

f) Prove the correctness of Dijkstra's algorithm.

Part 2. Attempt any Four of the following questions (from Q3 to Q7). **[4 x 10 = 40 marks]**

Q3. (a) Explain Theta notation. Also express the lower bound for the time complexity of the below function h(n) in terms of T1, T2 and n, where T1(n) and T2(n) is the time complexity for some functions f(n) and g(n), respectively and n is the input value. [4 marks]

```
int h(int n)
  { int sum=0;
     for (int i=0; i<n-1; i++)    { sum += i; }
     if ((sum%5) > 2)  {  i= f(n);  }
     else { i=g(n); }
     return i;
}
```

(b) Consider the problem of scheduling n jobs of known durations t1, t2, ... tn for execution by a single processor. The jobs can be executed in any order, one job at a time. You want to find a schedule that minimizes the total time spent by all the jobs in the system. (The time spent by one job in the system is the sum of the time spent by this job in waiting plus the time spent on its execution.) Design a greedy algorithm for this problem and show that it will always yield an optimal solution. [6 marks]

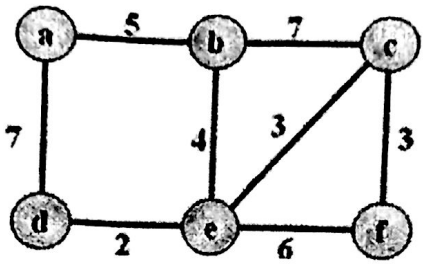
Q4 (a) Give an algorithm using dynamic programming to determine how many distinct ways there are to give 'x' cents in change using any coins from among pennies (1), nickels (5), dimes(10), and quarters(25). For example, there are 6 ways to give 16 cents change: a dime, a nickel, and a penny; a dime and 6 pennies; 3 nickels and a penny; 2 nickels and 6 pennies; one nickel and 11 pennies; and 16 pennies. Demonstrate your solution by showing a step-by-step solution for 12 cents change. [6 marks]

(b) Solve the following instance of knapsack problem where capacity W=20. The pairs (w,v) below corresponding to a task describe the weight w and its total value v. (Assume fractional knapsack.) [4 marks]

Task1: (4, 40), Task2: (7, 56), Task3: (9, 63), Task4: (5, 15), Task5: (6,54)

Q5 (a) Prove that for almost uniform character distribution (where the maximum character frequency is less than twice the minimum character frequency), Huffman encoding is no more efficient than using fixed length encoding. [5 marks]

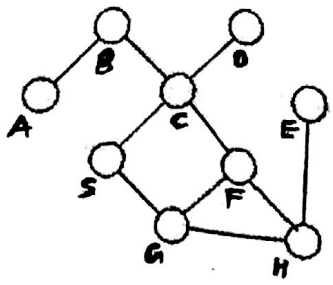
(b) Apply the Prim's algorithm on the following graph to find minimum spanning tree in given graph. Show the intermediate steps clearly [5 marks]



Q6 (a) What is transitive closure of the given graph G(V,E) and how Floyd-Warshall's algorithm is used for finding it. [4 marks]

(b) Use dynamic programming recurrence for the matrix chains problem for 4 matrices of dimension 4x5, 5x3, 3x2 and 2x7 to determine the optimum number of scalar multiplications to multiply the matrices? Also find the best way to multiply these matrices. [6 marks]

Q7 (a) Describe how DFS algorithm will work for the following graph starting at vertex A. Show the intermediate steps and mention the time stamps for each vertex. [4 marks]



(b) Use Bellman Ford's algorithm to find distance on a shortest path between the nodes s and others in the following directed graph. Show the intermediate steps clearly. [6 marks]