KS

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16UCS095

DMS ASSIGNMENT $\boxed{1}$

SOLU^n

1. $A = \{1\}$ and $B = \{1, \{1\}\}$.

Since, $P(B) = \{\{1\}, \{\{1\}\}, \{1, \{1\}\}, \phi\}$

a) Since $A \in P(B) \Rightarrow A \subseteq B$.

b) Since $A \neq B$, A is proper subset of B.

2. $A = \{m \in Z \mid m = 2a$ for some integer $a\}$

$B = \{n \in Z \mid n = 2b - 2$ for some integer $b\}$

Solu → Let $a = b - 1 \Rightarrow A = B$.

Alt, Roster form of both eq^{n}s are same.

~~A = B = { φ,~~

$\Rightarrow A = B$

Proof: ① $\boxed{A \subseteq B}$

suppose x is a particular but arbitararily choosen element of A. By definition of A, there is an integer a such that $x = 2a$.

Let $b = a + 1$, then b is an integer because it is a sum of integer.

Also $2b - 2 = 2(a+1) - 2 = 2a + 2 - 2 = 2a = x$

Thus by definition of B, x is an element of B.

② $\boxed{B \subseteq A}$, Suppose y is a particular but arbitarily choosen element of B. By definition of B, there is an integer b such that $y = 2b - 2$.

Let $a = b - 1$, a is an integer because it is diff. of integers.

~~Thus~~, Also ~~2b - 2 = 2(~~ $2a = 2a + 2 - 2 = 2(a+1) - 2 = 2b - 2 = y$

Thus, by definition of A, y is an element of A.

from ① & ② we can say that $A = B$.

3. $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{b, c, e\}$

ⓐ $A \cup (B \cap C) = \{a, b, c\}$

$(A \cup B) \cap C = \{b, c\}$

& $(A \cup B) \cap (A \cup C) = \{a, b, c\}$

from above, we can see that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = \{b, c\}$

$(A \cap B) \cup C = \{b, c, e\}$

$(A \cap B) \cup (A \cap C) = \{b, c\}$

from above, we can see, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(c) $(A - B) - C = (A \cap B') - C = A \cap B' \cap C' = A \cap (B \cup C)'$
$= \{a\}$.

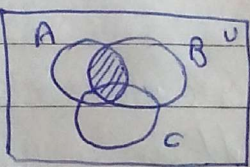
$A - (B - C) = A - (B \cap C') = A \cap (B' \cup C) = (A \cap B') \cup (A \cap C)$
$= \{a, b, c\}$.

Clearly, we are able to see,

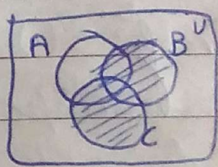
$(A - B) - C \neq (A - (B - C))$

since $\underbrace{[(A - B) - C]}_{bc} \neq \underbrace{[A - (B - C)]}_{bc}$.

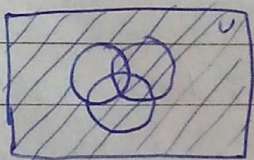
4. (a) $A \cap B$



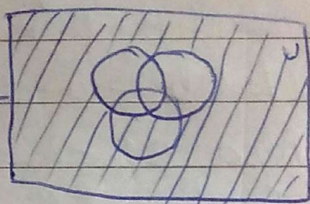
(b) $B \cup C$



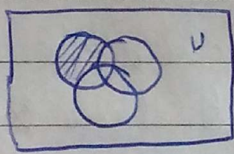
(c) A^c



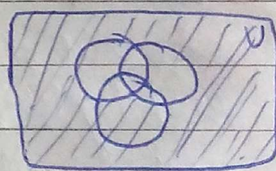
(e) ~~A-(B∪C)~~ $(A \cup B)^c$



(d) $\cancel{(A \cup B)^c}$ $A - (B \cup C)$



(f) $A^c \cap B^c$



5. T/P $A - (A \cap B) = A - B$

LHS $A - (A \cap B) = A \cap (A \cap B)' = A \cap (A' \cup B')$

$= (A \cap A') \cup (A \cap B') = \phi \cup (A \cap B') = A \cap B'$

using set laws $\rightarrow$ $= A - B$ = RHS H/P.

6. LHS: $(X - Y) - Z = (X \cap Y') - Z$

$= (X \cap Y') \cap Z' = X \cap (Y \cup Z)'$

RHS: $X - (Y \cap Z') = X \cap (Y \cap Z')' = X \cap (Y' \cup Z)$.

$= (X \cap Y') \cup (X \cap Z) \neq LHS$

$LHS \neq RHS$

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7 $(A\cap B')\cup(A'\cap B)\cup(A'\cap B')$

$= (A^c\cap B)\cup(A'\cap B^c)\cup(A\cap B^c)$

$= (A^c\cap(B\cup B'))\cup(A\cap B^c)$

$= (A^c\cap U)\cup(A\cap B^c)$

$= A^c\cup(A\cap B^c) = (A^c\cup A)\cap(A^c\cup B^c)$

$= A^c\cup B^c = U - A\cap B = (A\cap B)^c$

Simplified expression

8. $A = \{$Angelo 0.4, Bart 0.7, Cathy 0.6$\}$

$B = \{$Dan 0.3, Elsie 0.8, Frank 0.4$\}$

(a) $A\cup B = \{$Angelo 0.4, Bart 0.7, Cathy 0.6, Dan 0.3, Elsie 0.8, Frank 0.4$\}$

(b) $A\cap B = \{$Angelo 0, Bart 0, Cathy 0, Dan 0, Elsie 0, Frank 0$\}$
$= \{\}$

(c) $A^c = \{$Angelo 0.6, Bart 0.3, Cathy 0.4$\}$

(d) $A\cup B^c = \{$Angelo 0.4, Bart 0.7, Cathy 0.6, Dan 0.7, Elsie 0.2, Frank 0.6$\}$

(e) $A\cap B^c = \{$Angelo 0, Bart 0, Cathy 0, Dan 0, Elsie 0, Frank 0$\}$

(f) $A\cap A' = \{$Angelo 0.4, Bart 0.3, Cathy 0.4$\}$.

9. $A = \{a_1, a_2, \text{---} a_n\}$, $P(n)$: 2^n subsets for n elements.

Basics step $(n=0)$ $\Rightarrow A=\phi \Rightarrow$ No. of subset $=1$

also $2^0 = 1 \Rightarrow P(0)$ is true.

Inductive step:

$P(k)$ is true $\Rightarrow$ set containing k elements have 2^k subsets

Suppose $|B| = k$, 2^k

$|A| = k+1$, ?

Let $x\in A$ & $x\notin B$ $\Rightarrow A = B\cup\{x\}$

All the subsets of B are also subset of $A = 2^k$ now adding x element to all these subsets, so New ~~set with x subsets~~ having x or not having x are 2^k & 2^k.

$\Rightarrow$ Total subsets $= 2^k + 2^k = 2.2^k = 2^{k+1}$

$\Rightarrow P(k+1)$ is true.

$\Rightarrow$ By PMI $P(n)$ is true.

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10.

(a) $n(S) = 26$, $S = \{a, b, c, \_\_ z\}$

(b) $n(L) = 5$, $L = \{T, W, E, D, L\}$

(c) $n(M) = 7$, M = { January, March, May, July, August, October, December }

(d) 0 , since identifiers cannot start from numeric value.

11.

1, 2, ___ 499

(a) $\left[\frac{499}{2}\right] = 249$, $\left[\frac{499}{3}\right] = 166$, $\left[\frac{499}{6}\right] = 83$.

Ans $= 249 + 166 - 83 = 332$

(b) no. of divisible by 2 or 5 $= 40$, $5 \Rightarrow \left[\frac{499}{5}\right] = 99$.

no. o ______ 3 or 5 $= 33$.

$249 + 166 + 99 - 83 - 40 - 33 = 358$.

(c) $332 - 83 = 249$.

(d) $499 - 358 = 141$

12.

Total $\rightarrow 700$

220 → T no S

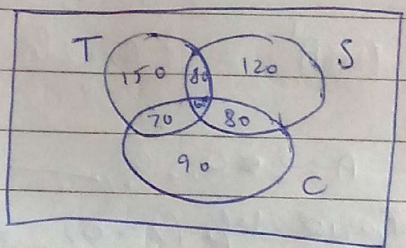
200 → S no C

170 C no T

80 → T, S no C

80 → S, C no T

70 → C, T no S.



(a) $150 + 120 + 90 = 360$

(b) $80 + 80 + 70 = 230$

(c) $150 + 80 + 120 + 70 + 60 + 80 + 90 = 650$

(d) 60.

13. $2 \in S$ if $x \in S$ then $x^2 \in S$.

Set by the listing method $= \{2, 2^2, 2^{2^2}, 2^{2^{2^2}}, \_\_\}$

14. (a) Domain (Z, Z), Range $= Z$ $\{f : Z \times Z \rightarrow Z\}$

(b) Domain Z^+ , Range $= \{x \in Z^+ | x < 10\} = R$ $\{f : Z^+ \rightarrow R\}$

(c) Domain: a set of all bit strings, Range: Z.

(d) Domain: Z^+ , Range Z^+

(e) Domain: a set of all bit strings.

Range: bit string containing only ones and the empty string.

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DMS Assignment 1)

Kshitiz Sharma

Continuing →

Q 15

$f: Z \times Z \to Z$.

(a) $f(m,n) = 2m-n$

Let $m = K$ where $K \in R$.

$f(K,n) = \begin{cases} 2K & \text{if } n=0 \\ 2K-1 & \text{if } n=1 \\ --- & \text{otherwise} \end{cases}$ ⇒ All even nos. + All odd nos. = Set of Integers.

⇒ Onto.

(b) $f(m,n) = m^2-n^2$.

$f(m,n) = 2 = m^2-n^2$

Since $2 \in Z$, but on solving, we are unable to get integers m & n. ⇒ Not Onto.

(c) $f(m,n) = m+n+1$.

Since m is integer, n & 1 are also.

& addition of integers is Integer ⇒ $f(m,n) \in Z$.

⇒ Onto.

(d) $f(m,n) = |m|-|n|$.

Since m & n are integers. & we can create all Integers using m & n that's why Onto.

eg. ~~f(m,~~ $f(m,n) = \begin{cases} |m| & n=0, \to \text{All +ive Integer} \\ -|n| & m=0, \to \text{All negative integers.} \end{cases}$

(e) $f(m,n) = m^2-4$.

Since, $m^2 \geq 0 \Rightarrow f(m,n) \geq -4$

⇒ $-5 = f(m,n)$ ⇒ Not possible for any ~~Real~~ integer value of m, n. ⇒ Not Onto.

Q 16.

(a) $f(x) = -3x+4$.

One-one onto.

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \to$ one-one

$x = \frac{y-4}{-3}$ ⇒ Range R

(b) $f(x) = -3x^2+7$

$f(x_1) = f(x_2) \Rightarrow x_1+x_2=0$ or $x_1-x_2=0$

Since $x_1+x_2=0$ is also ~~one-to~~ part of Sol.n ⇒ Not One

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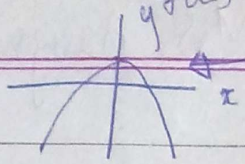
Not oneone $\Rightarrow$ Not bijective
also Not onto.



graph

ⓒ $f(x) = \frac{x+1}{x+2} \Rightarrow y = \frac{x+1}{x+2} \Rightarrow x = \frac{1-2y}{y-1}$ & $y \neq 1$.

$x \neq 2$. $\Rightarrow$ Not bijective

ⓓ $f(x) = x^5 + 1$

$x_1^5 + 1 = x_2^5 + 1 \Rightarrow x_1 = x_2 \Rightarrow$ one one

$y = x^5 + 1 \Rightarrow x = (y-1)^{1/5} \Rightarrow$ onto

$\Rightarrow$ Bijective.

Q 17. $g : A \to B$ & $f : B \to C$

ⓐ To prove $f \circ g : A \to C$ is oneone (injective),
we need that if $f \circ g(x) = f \circ g(y)$ then $x = y$.
Suppose $f \circ g(x) = f \circ g(y) = c \in C$.
This means that $f(g(x)) = f(g(y))$.
Let $g(x) = a$, $g(y) = b$, so $f(a) = f(b)$.
Since $f : B \to C$ is injective and $f(a) = f(b)$, we know that
$a = b$. This means that $g(a) = g(b)$.
Since $g : A \to B$ is injective and $g(x) = g(y)$, we know that $x = y$
we have shown that if $f \circ g(x) = f \circ g(y)$ then $x = y$.

H/P

ⓑ $g : A \to B$ & $f : B \to C$ are onto.
T/p: $f \circ g : A \to C$ is onto, we need to prove that $\forall c \in C\ \exists a \in A$ such that $f \circ g(a) = c$.
Let c be any element of C.
Since $f : B \to C$ is onto, $\exists b \in B$ such that $f(b) = c$.
Since $g : A \to B$ is onto, $\exists a \in A$ such that $g(a) = b$.
So, $(f \circ g)(a) = g(f(a)) = g(b) = c$.
This completes the proof.

H/P.

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18. Let $f, g, h : Z \to Z$

$$f(x) = x-1 \;,\; g(x) = 3x$$

$$h(x) = \begin{cases} 1 \;, & x \text{ odd} \\ 0 \;, & x \text{ even} \end{cases}$$

(a) $f \circ g(x) = g(x) - 1 = 3x - 1.$

$g \circ f(x) = 3x - 3$

$g \circ h(x) = \begin{cases} 3 & x \text{ odd} \\ \cancel{0} & \cancel{x \text{ even}} \end{cases}$

$h \circ g(x) = \begin{cases} 1 & x \text{ odd} \\ 0 & x \text{ even} \end{cases}$

$f \circ (g \circ h) = \begin{cases} 2 & x \text{ odd} \\ -1 & x \text{ even} \end{cases}$

$(f \circ g) \circ h = \begin{cases} 1 & 3x \text{ odd} \\ 0 & \cancel{3x-1} \text{ even} \end{cases}$

(b) $f^2 = (x-1)^2,$

$f^3 = (x-1)^3, \; g^2 = (3x)^2, \; g^3 = (3x)^3, \; h^2 = h^3 = h^{500} = \begin{cases} 1 & x \text{ odd} \\ 0 & x \text{ even} \end{cases}$

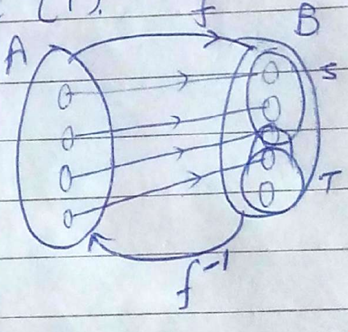
19. f be a function from A to B. Let S & T be subsets of B.

$f : A \to B.$

(a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$

(b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T).$

This can be explained via diagram



20. $S \subseteq U ; \quad f_S : U \to \{0, 1\}.$

$$f_S(x) = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{if } x \notin S \end{cases}$$

(a) $f_{A \cap B}(x) = f_A(x) . f_B(x)$ because Both A&B are independent.

(b) $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_{A \cap B}(x) = f_A(x) + f_B(x) - f_A(x) . f_B(x).$

(c) $f_{\overline{A}}(x) = 1 - f_A(x)$

(d) $f_{A \otimes B}(x) = f_A(x) + f_B(x) - 2 f_{A \cap B}(x) = f_A(x) + f_B(x) - 2 f_A(x) . f_B(x).$

Can be easily verified using laws of sets.

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