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DMS

Assignment - 3

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01. ????1 0

????? 0

Since we can place 0 after 1 or 0 thus we can place 0 after a_{n-1} words similarly 1 can be placed after 0 only. Since we know that a can be placed a_{n-2} times by previous logic thus total number of words are $a_{n-1} + a_{n-2}$

Thus $a_n = a_{n-1} + a_{n-2}$;

02. $L_1 = 1$;
$L_2 = 3$;
$L_3 = 4$
$L_4 = 7$
$L_5 = 11$
$L_6 = 18$
$L_7 = 29$
$L_8 = 47$

03- gcd {28, 18} = gcd {18, 28 % 18}
= gcd {18, 10}
= gcd {10, 8}
= gcd {8, 2}
= gcd {2, 0}
= 2;

Premium

Q4. a) $a_n = a_{n-1} + 3$

$a_1 = 1;$

b) $a_n = (a_{n-1})^2 + 1$

$a_1 = 1;$

Q5.

a) $x = 99$

$f(f(110))$ $\quad f(110) = 100$

$f(100) = f(f(111))$

$= f(101)$

$= 91$

b) $f(91) = f(f(102)) = f(92) = f(f(103)) = f(93)$

$= f(94) = f(95) = f(96) = f(97)$

$= f(98) = f(99) = f(f(110))$

$= f(100)$

$= f(101) = 91;$

Q8. b) $A(4,0) = A(3,1) = A(2, A(3,0)) = A(2,5)$

$A(3,0) = A(2,1) = A(1, A(2,0)) = A(1,3) = 5$

$A(2,0) = A(1,1) = A(0, A(1,0)) = A(0,2) = 3$

$A(1,0) = A(0,1) = 2$

$A(2,2) = A(1, A(2,1)) = A(1,2)$

$A(1,2) = A(0, A(1,1))$

$A(1,1) = A(0, A(1,0)) = A(0,2) = 3$

$= A(0,3)$

$= 4$

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$A(2,4) = A(1, A(2,3))$
$A(2,3) = A(1, A(2,2)) = A(1,7)$
$A(2,2) = A(1, A(2,1)) = A(1,5) = 7$
$A(2,1) = A(1, A(2,0)) = A(1,3) = 5$
$A(1,4) = A(0, A(1,3)) = 6$
$A(1,3) = 5$
Thus $A(4,0) = 13$

a) $A(0,7) = n+1 = 8$
$n = 7$
$m = 0$

$\boxed{18}$

Q6 a) $a_{n-1} + n = a_n \qquad a_1 = 1;$
b) $a_{n-1} + n^2 = a_n$
c) $a_n = \begin{cases} a_{n-1} + \frac{n(n-1)}{2} & n \Rightarrow \text{odd} \\ a_{n-1} + \frac{n^2}{2} & n \Rightarrow \text{even.} \end{cases}$

Q7. $S(2,2) = 1;$

Q9

a) $a_0 = 0$
$a_n = a_{n-1} + 4n \qquad n \geq 1$
$= (a_{n-2} + 4(n-1)) + 4n$
$= (a_{n-3} + 4(n-2)) + 4(n-1) + 4(n)$
$\vdots$
$= a_{n-n} + (4(n-(n-1)) + \ldots + 4(n))$
$= a_0 + 4(1 + 2 + \ldots n)$
$= 2(n)(n+1) = 2n(n+1)$
$a_n = 2n^2 + 2n$

b) $S_1 = 1$

$$S_n = S_{n-1} + n^3$$
$$= S_{n-2} + (n-1)^3 + n^3$$
$$= S_{n-3} + (n-2)^3 + (n-1)^3 + n^3$$
$$\vdots$$
$$= S_{n-(n-1)} + ((n-(n-2))^3 + \dots n^3)$$
$$= S_1 + (2^3 + 3^3 + \dots n^3)$$
$$= \frac{n^2(n+1)^2}{4}$$

$$S_n = \frac{n^2(n+1)^2}{4}$$

c) $a_1 = 1$

$$a_n = 2 + 3\cdot 2^n - 4 - 1$$
$$= 3\cdot 2^n - 3$$
$$a_n = 3(2^n - 1)$$

d) $a_1 = 1$

$$a_n = 2a_{n-1} + (2^n - 1) \quad \forall\ n \geq 2$$
$$= 2[2a_{n-2} + (2^{n-1} - 1)] + (2^n - 1)$$
$$= 2^2 a_{n-2} + (2^n - 2) + (2^n - 1)$$
$$= 2^2[2a_{n-3} + (2^{n-2} - 1)] + 2\cdot 2^n - (1+2)$$
$$\vdots$$
$$= 2^{n-1} a_1 + (n-1)2^n - 1(2^{n-1} - 1)$$
$$a_n = (n-1)2^n + 1$$