16UCS095

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DMS Assignment #2

1 Soln {By Induction}:

Let x be any element in Σ^*. Let $P(y)$ denote the predicate that $||xy|| = ||x|| + ||y||$, where $y \in \Sigma^*$. Since $y \in \Sigma^*$, y can be the null word λ or a non empty word.

Basic step: To show that $P(\lambda)$ is true; ~~i.e. P(λ) is true.~~

$||\lambda x|| = ||x|| + ||\lambda||$; since $x\lambda = x$, $||x|| = ||x|| = ||x|| + 0$
$= ||x|| + ||\lambda||$

So $P(\lambda)$ is true.

Induction step: Assume $P(y)$ is true, that is $||xy|| = ||x|| + ||y||$. (inductive hypothesis). We must show that $P(ys)$ is true, that is $||xys|| = ||x|| + ||ys||$.

$xys = (xy)s$ associative property of concatenation

Then $||xys|| = ||(xy)s|| = ||xy|| + 1$. Length of function.

$= (||x|| + ||y||) + 1 = ||x|| + (||y|| + 1)$

$= ||x|| + ||ys||$ → Recursive def.

Therefore, $P(ys)$ is true. Thus $P(y)$ implies $P(ys)$.
Therefore, by induction, $P(y)$ is true for every $y \in \Sigma^*$; that is, $||xy|| = ||x|| + ||y||$ for every $x, y \in \Sigma^*$.

2 Soln Suppose S has the property that every pairs of distinct $a, b \in S$ a does not divide b & b does not divide a.
Since we have $(n+1)$ elements, We partition $\{1, 2, \ldots 2n\}$ into n sets $T_1, T_2, \ldots T_n$. The pigeonhole principle says that there should be atleast two element for some i. This should be useful to ~~calc~~ conclude that there should be $a, b \in S$ such that $a|b$.

3 Soln C++ identifier contain 37 alphanumeric character.
~~26+10~~ 36 → total alphanumeric Nos So, By Pigeon hole principle, there should be atleast two characters which are same.

Teacher's Signature

Q.4 Total cost of 13 refrigerator at store is \$ 12305.

cost of 1 refrig. $= \frac{12305}{13} = 946.\ldots = A$

$\lceil A \rceil = 946$

so one refrig must cost atleast \$ $946 + 1$ = \$947

5. Partition the unit square into four square each with side 1/2. Each of the five points lie in one of these four square. Therefore by the pigeonhole principle there should be atleast two points in the same square which the maximum distance is $1/\sqrt{2}$.

6. partition the unit side triangle into 3 equilateral triangle of side 1/2. Each of the 5 points lies in one of these triangle therefore by pigeonhole principle there should be atleast two point the distance atleast two of them is no more 1/2.

7. f: ASCII → W defined by f(C) = ordinal no. of char C. so f(C) can be represented so it can be invertible

8. $\sum_{i=m}^{n} i \qquad i \to n+m-i$

$$= \sum_{i=n}^{m} (n+m-i) \neq \sum_{i=m}^{n} \cancel{(n+m-i)}$$

FALSE

9. (a) $S = \sum_{i=m+1}^{n} (a_i - a_{i-1}) = a_{m+1} - a_m + a_{m+2} - a_{m+1} + \_\_ + a_n - a_{n-1}$

$= a_n - a_m$.

(b) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \sum_{i=1}^{n} \left(\frac{1}{i} - \frac{1}{i+1}\right) = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \_\_ + \frac{1}{n} - \frac{1}{n+1}$

$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$

10. $\sum_{1 \le i \le j < 3} (a_i + a_j)$

$= a_1 + a_1 + a_1 + a_2 + a_2 + a_2 = 3(a_1 + a_2)$

11. (a) $m = p$, $n = q$, for $A+B$ to be defined.

(b) $p = r$, $q = s$, for $B-C$ to be defined

(c) $q = r$, for BC to be defined.

(d) $m = n$, for A^2 or AA to be defined.

Teacher's Signature

12. (a) No. of grams of each type of insulin needed.

$$\begin{array}{l} \text{Semi lente} \\ \text{Lente} \\ \text{ultra} \end{array}\begin{bmatrix} 25\times7+40\times14+35\times21 \\ 20\times7+15\times21+15\times28 \\ 20\times7+30\times21+40\times28 \end{bmatrix} = \begin{bmatrix} 1,470 \\ 1,190 \\ 1,890 \end{bmatrix} \text{gm}$$

(b) Total cost of insulin.

$$= \begin{bmatrix} 10 & 11 & 12 \end{bmatrix}\begin{bmatrix} 1470 \\ 1190 \\ 1890 \end{bmatrix} = \begin{bmatrix} 50,470 \end{bmatrix} \text{cent}$$

(c) insulin requirement if the guests decide to stay ~~...~~ days.

$$= \begin{bmatrix} 25\times3+40\times8+35\times8 \\ 20\times3+15\times8+15\times13 \\ 20\times3+30\times8+40\times13 \end{bmatrix} = \begin{bmatrix} 555 \\ 375 \\ 820 \end{bmatrix}$$

d) $$3\begin{bmatrix} 1470 \\ 1190 \\ 1890 \end{bmatrix} = \begin{bmatrix} 4,410 \\ 3,570 \\ 5,670 \end{bmatrix}$$

13. Euclidean Algorithm.

$\gcd\{2076, 1024\}$

$2076 = 1024 \times 2 + 28$ ($2076 - 2048 = 28$)

$1024 = 28 \times 36 + 16$ ($1024 - 84 = 184$, $184 - 168 = 16$)

$28 = 16 \times 1 + 12$ ($28 - 16 = 12$)

$16 = 12 \times 1 + 4$ ($16 - 12 = 4$)

$12 = \boxed{4} \times 3$ ($12 - 12 = 0$)

$\boxed{GCD = 4}$

14. (a) $1+2+\_\_\ 12$ gifts on 12^{th} day $= \frac{12\times13}{2} = 78$

(b) $1+(1+2)+(1+2+3)+\_\_+(1+2+3+\_\_12)$

$= 1\times12+2\times11+3\times10+\_\_+12\times1 = \sum_{i=1}^{12} i.(13-i)$

$= 13\left(\frac{12\times13}{2}\right) - \frac{12\times13\times25}{6} = 1014 - 650$

$= 364$

15. (a) $P(1)$: is true as $2(1)-1 = 1^2 = 1$

$P(k)$: Let $P(k)$ be true ⇒ $\sum_{i=1}^{k}(2i-1) = k^2$

Let $P(K)$ be true,

$$P(K+1) = \sum_{i=1}^{K+1} (2i-1) = \sum_{i=1}^{K}(2i-1) + 2K+1$$

$$= K^2 + 2K + 1 = (K+1)^2 \Rightarrow \text{By PMI } P(n) \text{ is true}$$

(b) $n^4 + 2n^3 + n^2$ is divisible by 4 : $P(n)$.

$P(1)$: $4 = 4(1)$ so $P(1)$ is divisible by 4 $\Rightarrow$ True

Let $P(n)$ is true then $P(n+1)$

$$(n+1)^4 + 2(n+1)^3 + (n+1)^2 = (n+1)^2(n^2+2n+1+2n+2+1)$$

$$= (n+1)^2(n^2+4n+4)$$

$$n^4 + 2n^3 + n^2 = n^2(n^2+2n+1)$$

$$4\lambda = n^2(n+1)^2$$

$\lambda \in \mathbb{Z}$

$$= \frac{4\lambda}{n^2}(n^2+4n+4)$$

$$= 4\lambda' \Rightarrow \text{divisible by } 4.$$

$\Rightarrow$ By PMI $\Rightarrow$ $P(N)$ is true

16. (a) $x \leftarrow 0$

for $i = 1$ to n do

$x \leftarrow x + (2i-1)$

$$x = 1 + 3 + 5 + \ldots\ 2n-1 = n^2$$

(b) $x \leftarrow 0$.

for $i = 1$ to n do

for $j = 1$ to i do

$x{+}{+}$;

$$1 + 2 + 3 + \ldots + n$$

$$= \frac{n(n+1)}{2}$$