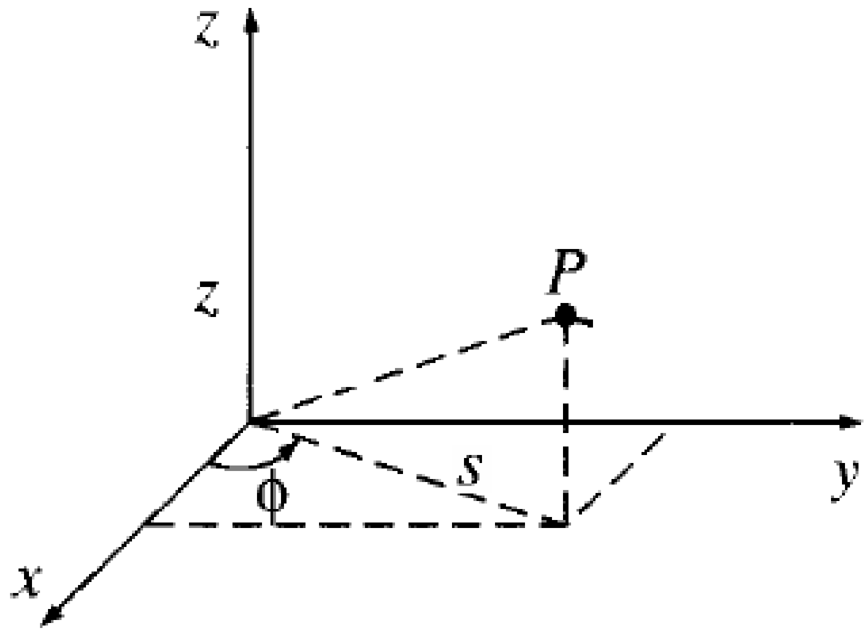
s

Cylindrical Coordinates

$0 \le s \le \infty$

$0 \le \varphi \le 2\pi$

$-\infty \le z \le \infty$



s

φ

z

$$x = s\cos\phi, \qquad y = s\sin\phi, \qquad z = z$$

$$s = \sqrt{x^2 + y^2} \qquad \varphi = \tan^{-1}\left(\frac{y}{x}\right) \qquad z = z$$

Unit Vectors

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\hat{s} = \frac{\frac{\partial \vec{r}}{\partial s}}{\left|\frac{\partial \vec{r}}{\partial s}\right|} \qquad \hat{\varphi} = \frac{\frac{\partial \vec{r}}{\partial \varphi}}{\left|\frac{\partial \vec{r}}{\partial \varphi}\right|} \qquad \hat{z} = \frac{\frac{\partial \vec{r}}{\partial z}}{\left|\frac{\partial \vec{r}}{\partial z}\right|}$$

$$\vec{r} = s\cos\varphi\,\hat{i} + s\sin\varphi\,\hat{j} + z\hat{k}$$

$$\hat{s} = \cos\varphi\,\hat{i} + \sin\varphi\,\hat{j}$$

$$\hat{\varphi} = -\sin\varphi\,\hat{i} + \cos\varphi\,\hat{j}$$

$$\hat{z} = \hat{k}$$

$$\vec{A} = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}$$

$$\vec{A} = a_s \hat{s} + a_\phi \hat{\phi} + a_z \hat{z}$$

$$a_x = a_s \cos\phi - a_\phi \sin\phi$$
$$a_y = a_s \sin\phi + a_\phi \cos\phi$$
$$a_z = a_z$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_s \\ a_\varphi \\ a_z \end{bmatrix}$$

$$a_\rho = a_x \cos\phi + a_y \sin\phi$$
$$a_\phi = -a_x \sin\phi + a_y \cos\phi$$
$$a_z = a_z$$

$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$dl_s = ds, \qquad dl_\phi = s\,d\phi, \qquad dl_z = dz$$

$$d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$$

The infinitesimal volume element $d\tau$

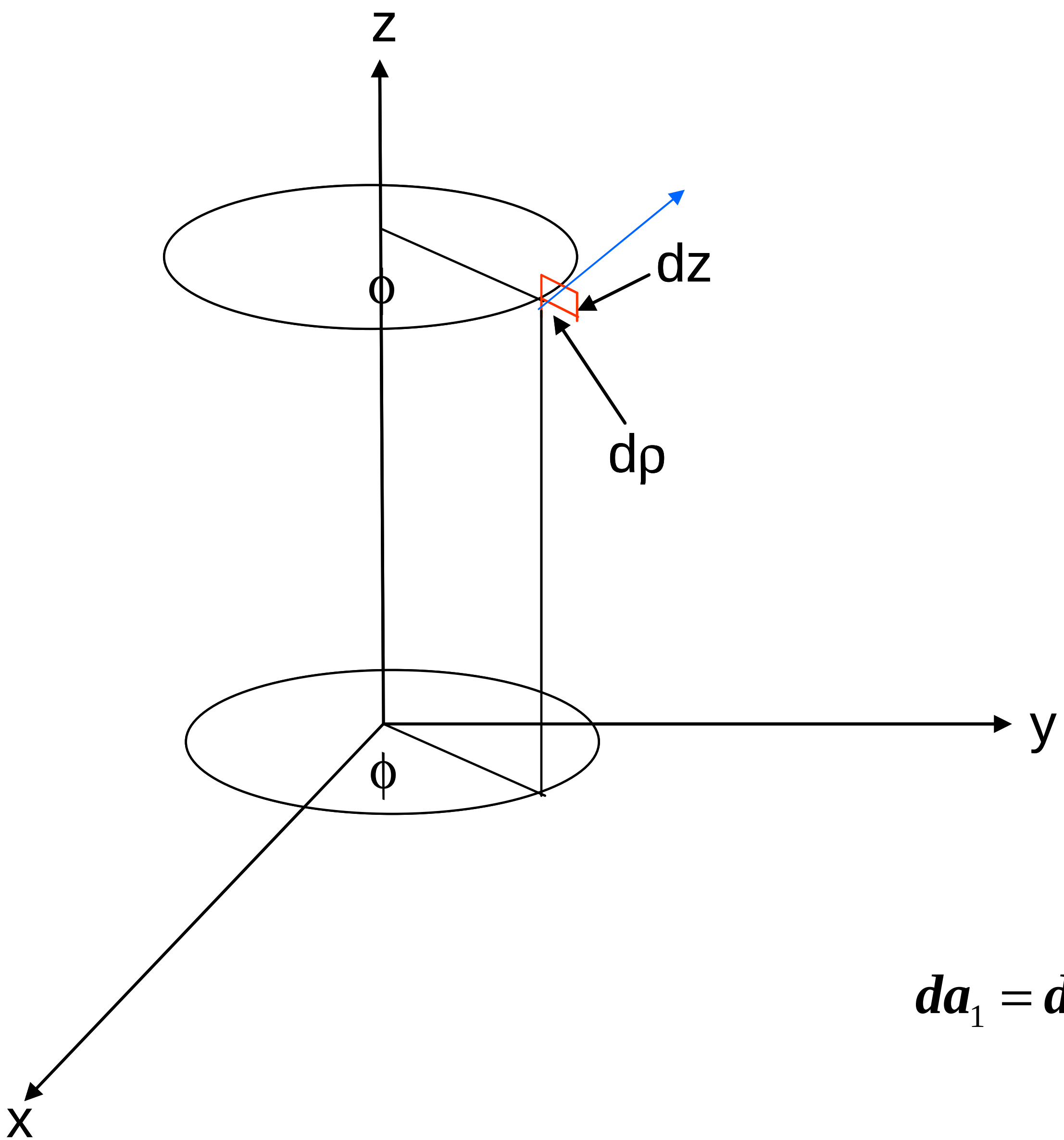
$$d\tau = s\,ds\,d\phi\,dz$$

The infinitesimal surface elements

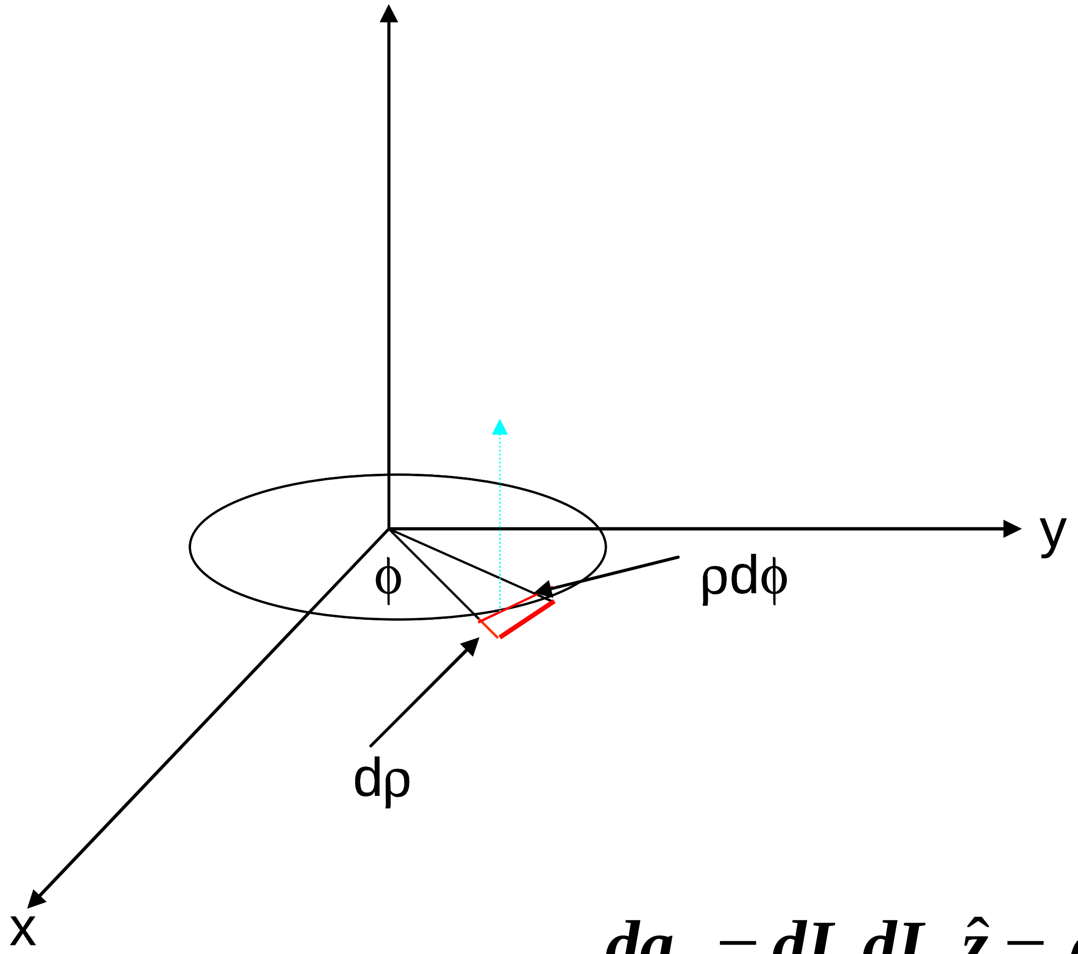
$$d\vec{a}_1 = dI_z\hat{z} \times dI_\rho\hat{\rho} = dzd\rho\hat{\phi}$$

$$d\vec{a}_2 = dI_\rho\hat{\rho} \times dI_\phi\hat{\phi} = \rho d\phi d\rho\hat{z}$$

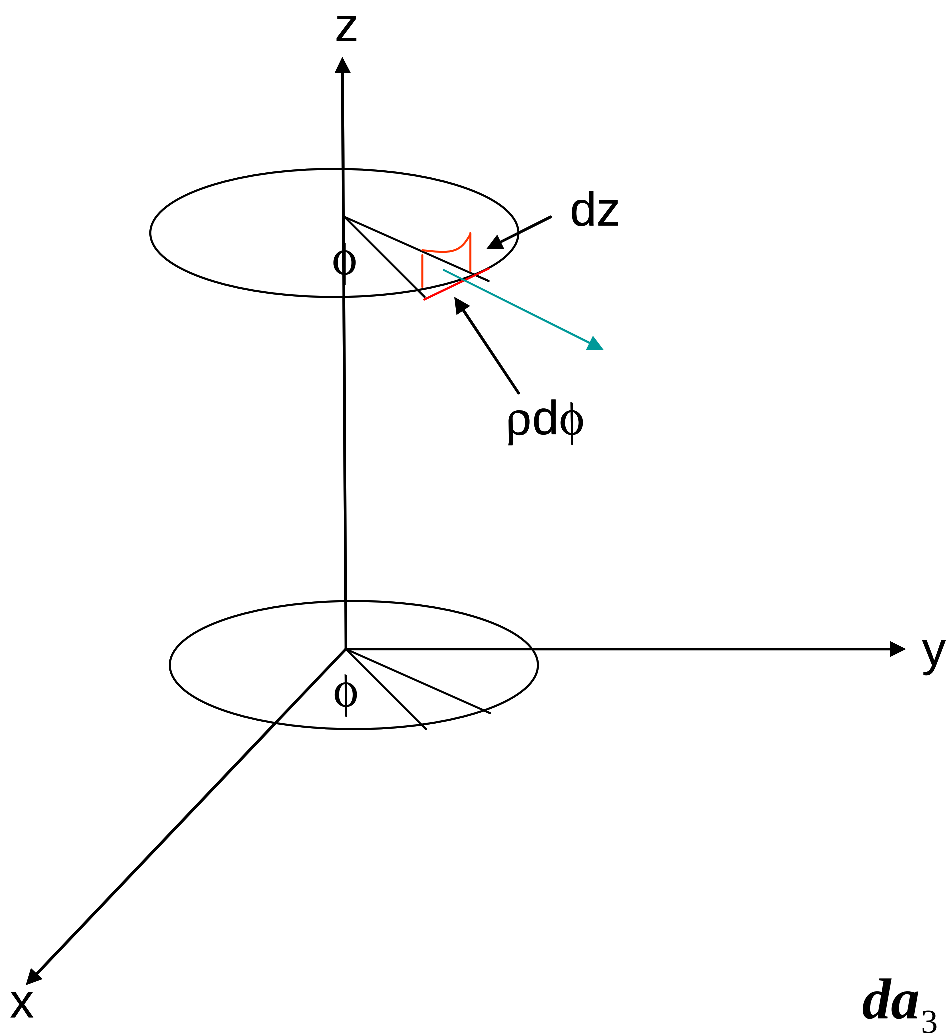
$$d\vec{a}_3 = dI_z\hat{z} \times dI_\phi\hat{\phi} = \rho d\phi dz\hat{\rho}$$



$$\boldsymbol{da}_1 = \boldsymbol{dI}_z \boldsymbol{dI}_\rho \hat{\boldsymbol{\phi}} = \boldsymbol{dz}\boldsymbol{d\rho}\hat{\boldsymbol{\phi}}$$

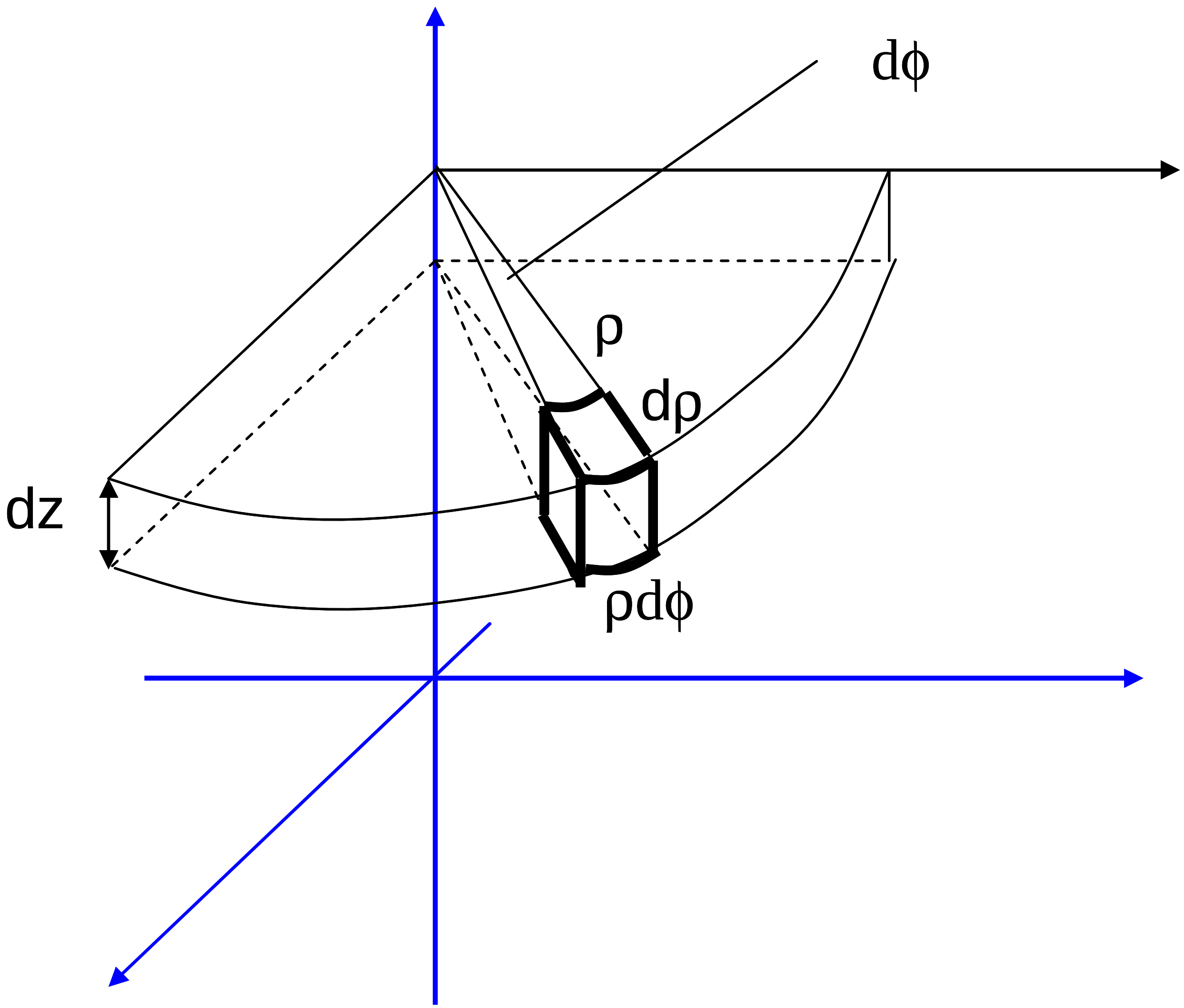


$$\boldsymbol{da}_2 = \boldsymbol{dI}_\phi \boldsymbol{dI}_\rho \hat{\boldsymbol{z}} = \rho \boldsymbol{d}\phi \boldsymbol{d}\rho \hat{\boldsymbol{z}}$$

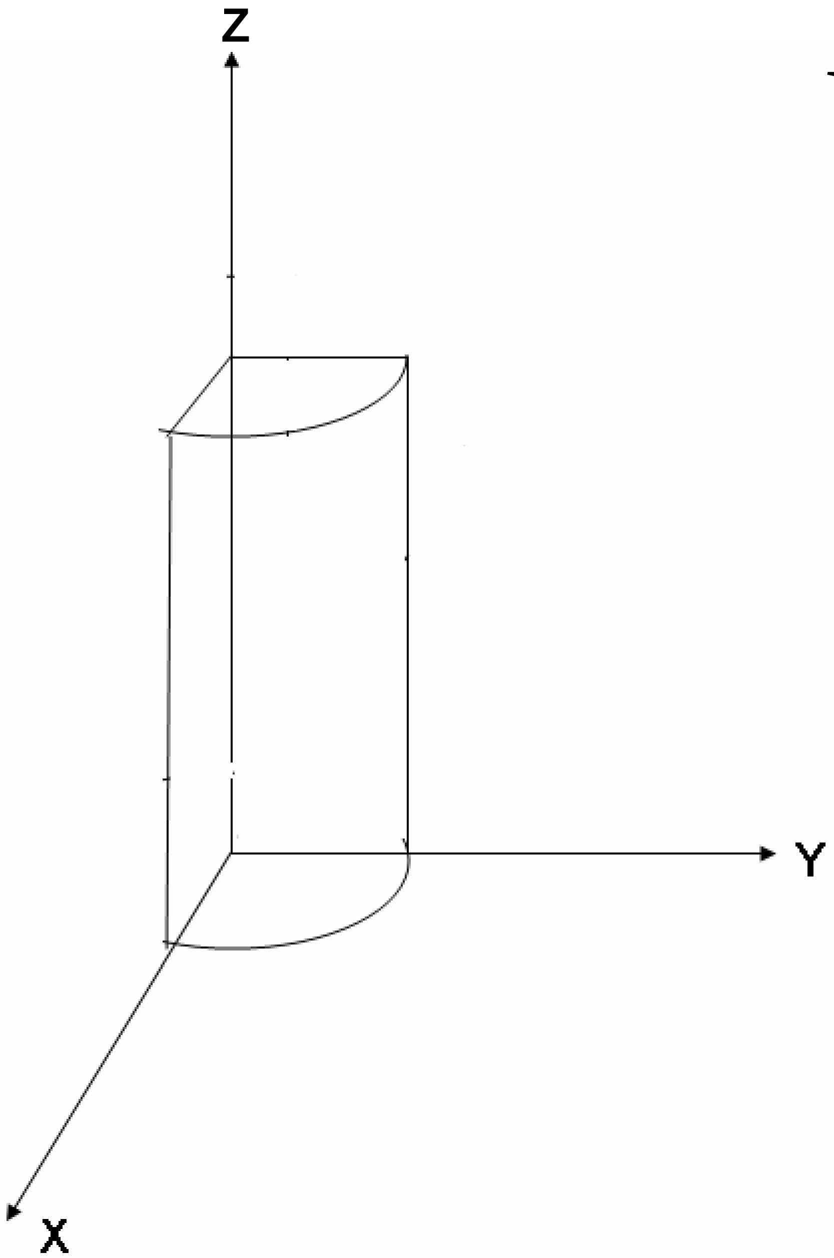


$$\boldsymbol{da}_3 = \boldsymbol{dI}_z \boldsymbol{dI}_\phi \hat{\rho} = \rho \boldsymbol{d}\phi \boldsymbol{dz} \hat{\rho}$$

Volume element in cylindrical coordinate system

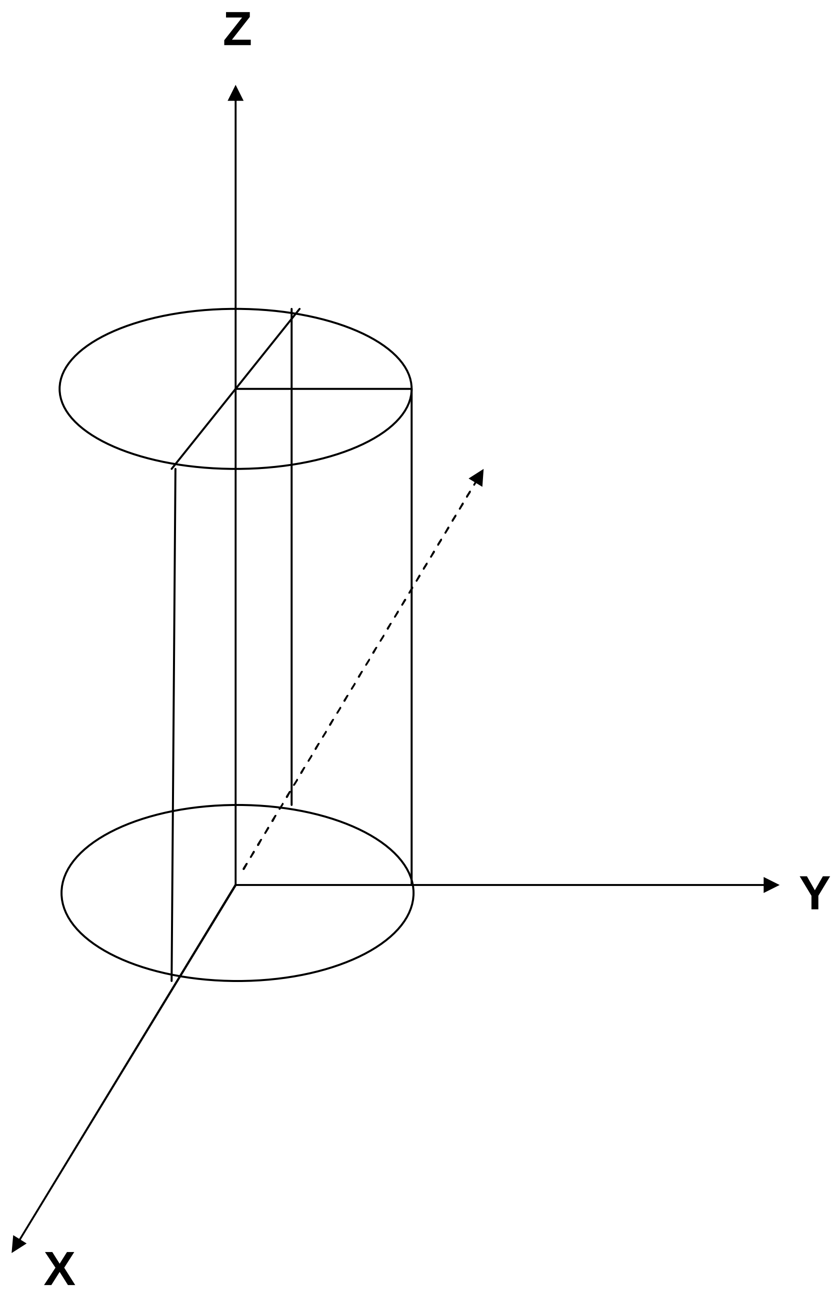


$$\vec{V} = \hat{\phi}$$



$$\vec{V} = \hat{\rho} + \sin\phi\hat{\phi} + z\hat{z}$$

$$\vec{V} = \rho\hat{\rho} + \hat{\phi} + z\hat{z}$$



$$\nabla T = \frac{\partial T}{\partial s}\,\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial \phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\,\hat{\mathbf{z}}.$$

$$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\nabla \times \mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

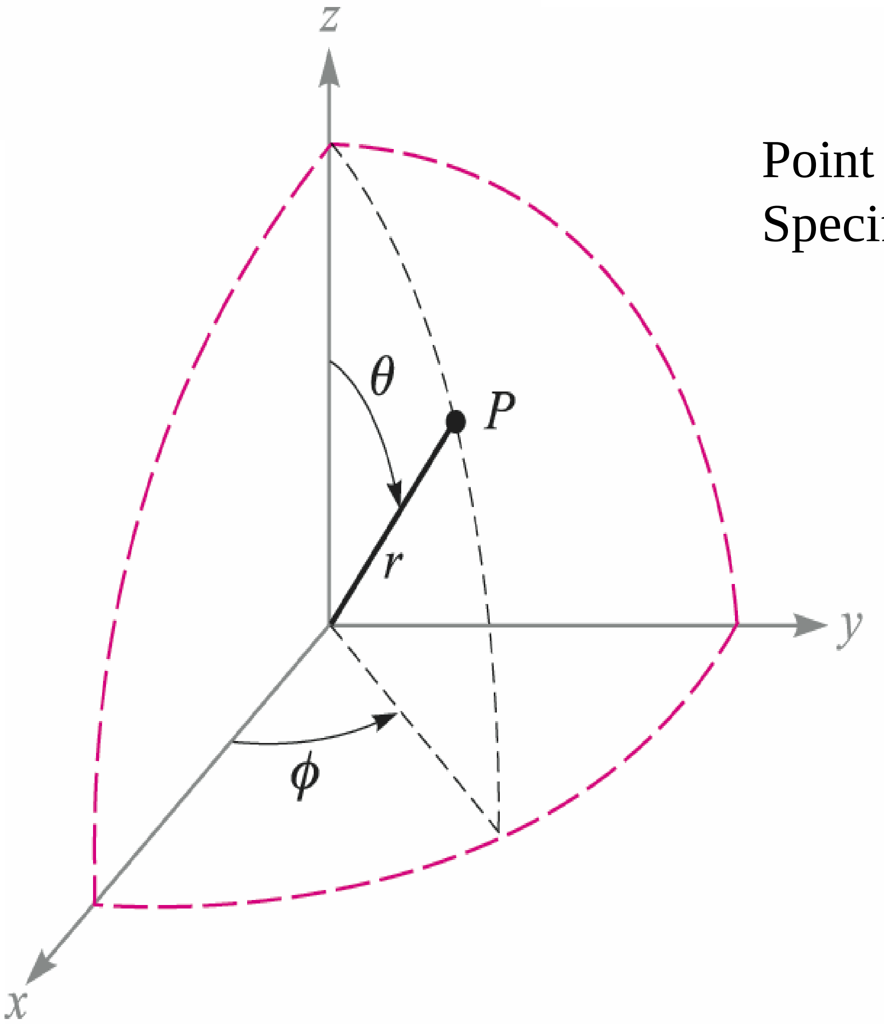
$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Spherical Co-ordinate System

$$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$

$$\phi = \tan^{-1} \frac{y}{x}$$



Point P has coordinates Specified by $P(r,\theta,\phi)$

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

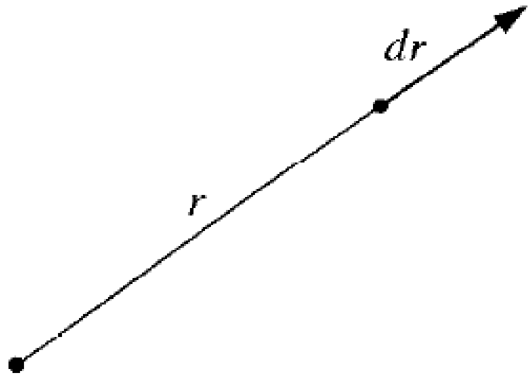
$$\vec{A} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$$

$$\vec{A} = a_r\hat{r} + a_\theta\hat{\theta} + a_\phi\hat{\phi}$$

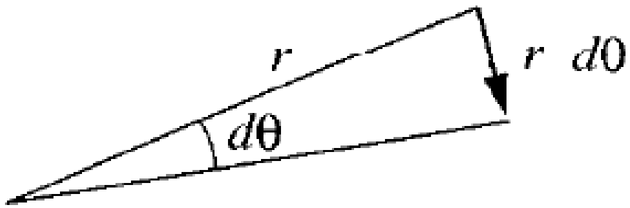
$$a_x = a_r \sin\theta\cos\phi + a_\theta \cos\theta\cos\phi - a_\phi \sin\phi$$
$$a_y = a_r \sin\theta\sin\phi + a_\theta \cos\theta\sin\phi + a_\phi \cos\phi$$
$$a_z = a_r \cos\theta - a_\theta \sin\theta$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix}$$

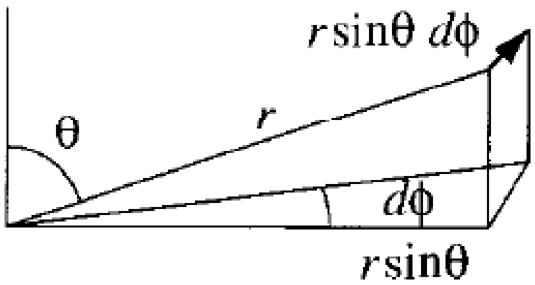
$$\begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$



$$dl_r = dr$$

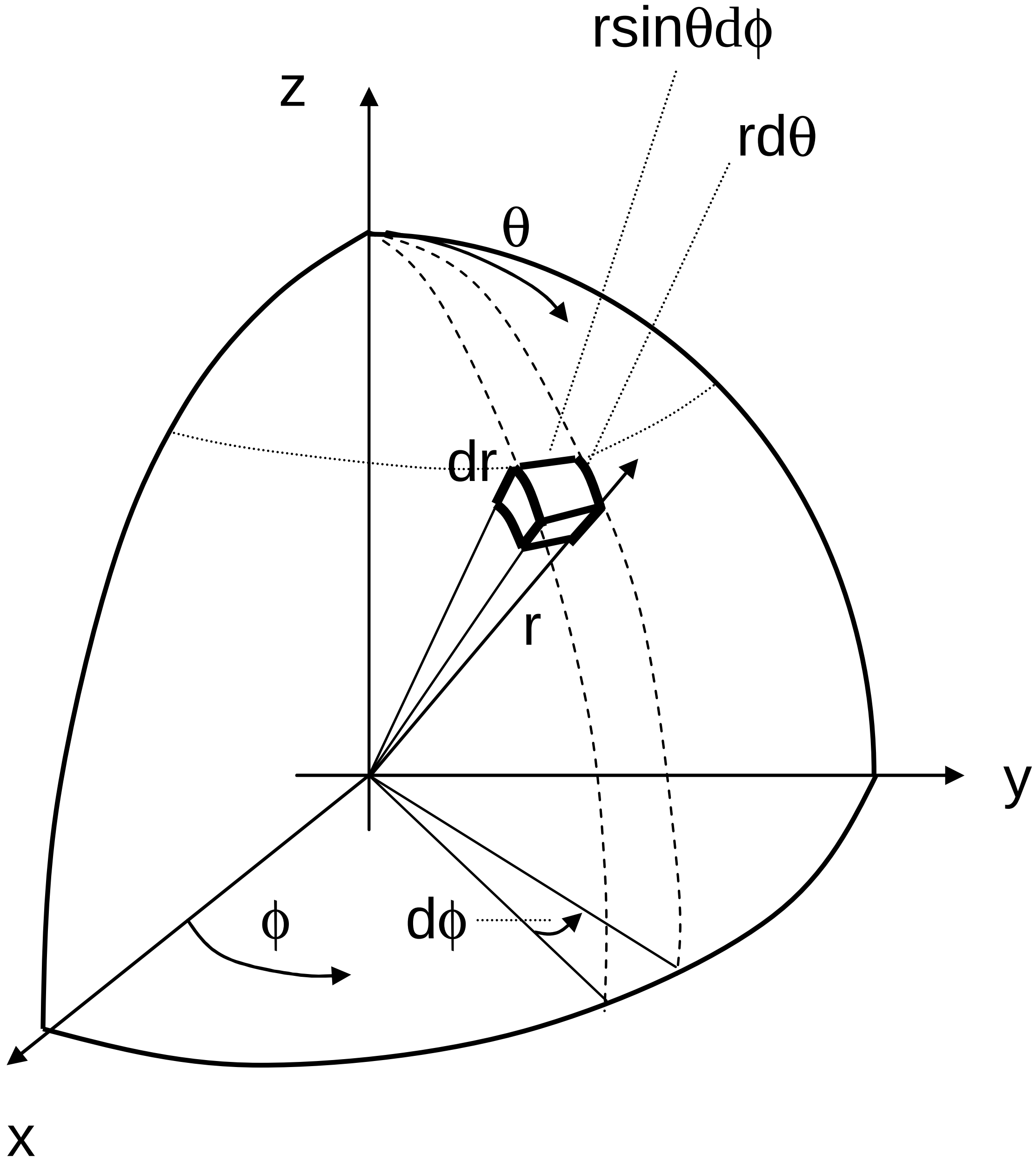


$$dl_\theta = r\,d\theta.$$



$$dl_\phi = r\sin\theta\,d\phi$$

$$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$


rsinθdϕ
rdθ
z
θ
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r
y
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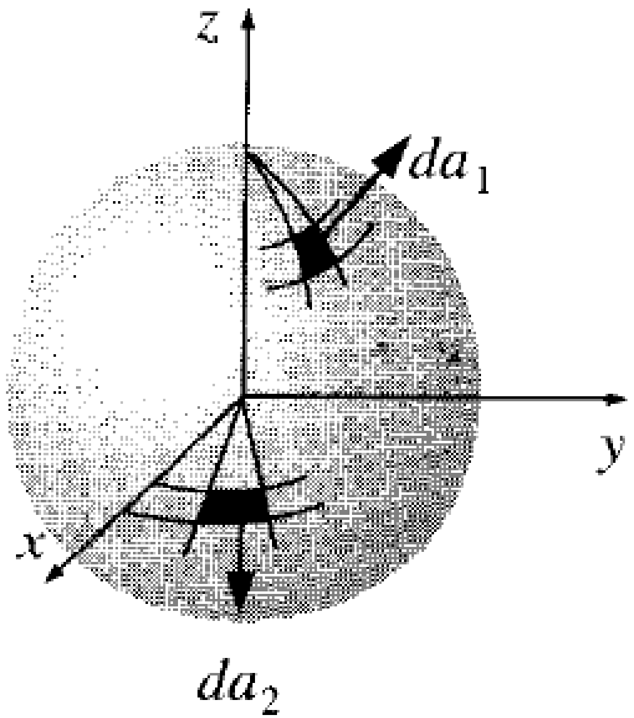
dϕ
x

The infinitesimal volume element $d\tau$

$$d\tau = dl_r\, dl_\theta\, dl_\phi = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

The infinitesimal surface elements



$$d\mathbf{a}_1 = dl_\theta\, dl_\phi\, \hat{\mathbf{r}} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{\mathbf{r}}$$

$$d\mathbf{a}_2 = dl_r\, dl_\phi\, \hat{\boldsymbol{\theta}} = r\, dr\, d\phi\, \hat{\boldsymbol{\theta}}$$

$$\boldsymbol{da}_3 = \boldsymbol{dI_r dI_\theta \hat{\phi}} = \boldsymbol{rdrd\theta\hat{\phi}}$$

$$\nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$\begin{aligned}\nabla \times \mathbf{v} \quad &= \quad \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} \\ &+ \quad \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}.\end{aligned}$$

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}$$