DATE: / /
PAGE NO.:

Proof

ii) $\frac{\lambda}{x+1} p(x)$, $x = 0, 1, \ldots$

Proof $\Rightarrow P(x+1) = \frac{e^{-\lambda} \lambda^{x+1}}{\lfloor x+1}$ $= \frac{\lambda e^{-\lambda} \lambda^{x}}{(x+1) \lfloor x}$

$= \frac{\lambda}{x+1} P(x) \rightarrow$ Recurrance relation

$\Rightarrow$ Hypergeometric Dist:- Bcoz it gives you Hypergeometric series.

Out of n, $r = 1$ type, Compute the Prob. that what would be the Prob of 'x' drawn at random without replacement 'n' objects.

Out of 'x' of kind of 'r' objects

- no. of ways x can be done of $r \Rightarrow {}^{r}C_{x}$

Out of r is drawn $(n-r)$ = remaing. objects.

$(n-x) \Rightarrow$

$= {}^{r}C_{x} \times {}^{n-r}C_{n-x}$

How many way 'n' Can be drawn $= {}^{n}C_{n}$

$$\frac{{}^{r}C_{x} \times {}^{n-r}C_{n-x}}{{}^{n}C_{n}}$$

Sup^c → Binomial
Nor...
⇒ Uni...
find on your own
⇒

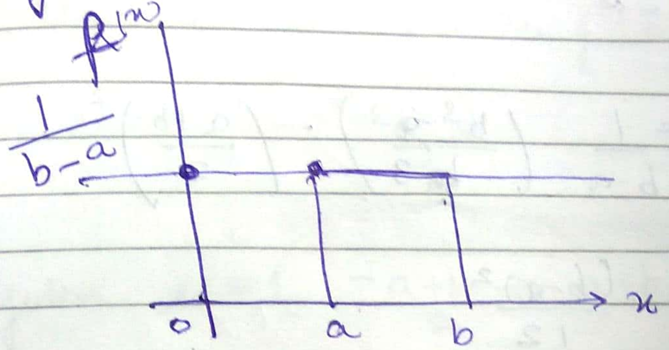
Tut/2 ~~Binomial Dist~~ Exponential Dist
Normal Dist uniform "
normal "

DATE : / /
PAGE NO. :

⇒ Uniformdist :- $x = 1 \ldots n$

Find on your own
$\begin{cases} \text{mean} = \\ \text{Var} = \end{cases}$

⇒ **Uniform distribution** :-



continuous random Variable.

$$x \sim U(a, b)$$

$$f(x) = \frac{1}{b-a}, \quad a \le x \le b$$
$$0, \quad e.w.$$

$$F(x) = P(X \le x) = 0, \quad x < a$$
$$= \frac{x-a}{b-a}, \quad a \le x < b$$
$$= 1, \quad x \ge b$$

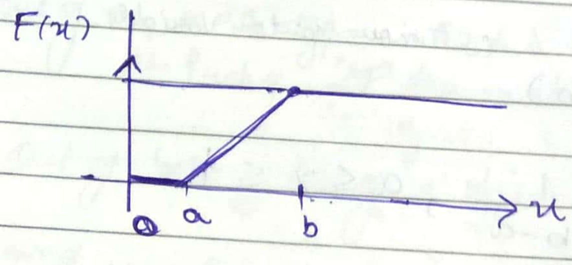
$$\mu = E[X] = \mu_1' = \int_{-\infty}^{\infty} x f(x)\, dx.$$
$$= \int_a^b x \cdot \frac{1}{b-a}\, dx = \frac{1}{b-a}\left(\frac{b^2-a^2}{2}\right)$$
$$= \frac{b+a}{2}.$$

$$\Rightarrow \sigma^2 = E[X^2] - \{E(X)\}^2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_a^b x^2\left(\frac{1}{b-a}\right)dx = \frac{1}{b-a}\left(\frac{b^3-a^3}{3}\right)$$

$$\sigma^2 = \frac{1}{b-a}\left(\frac{b^3-a^3}{3}\right) - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{(b-a)^2}{12}$$



Q// A random Variable x has unif. dist with Pdf :-

$$f(x) = \frac{1}{100}, \quad 0 < x < 100$$
$$0, \quad \text{e.w.}$$

Obtain prob. :-

i> $P(X > 60)$

ii> $P(20 \leq X \leq 40)$

Soln i> $\int_{60}^{100} f(x)\,dx = \int_{60}^{100} \frac{1}{100}\,dx = 0.40$

DATE : / /

PAGE NO. :

ii) ⇒ $\int_{20}^{40} f(x)\,dx = \frac{1}{100}\int_{20}^{40} dx = 0.20$

3) Q4 If x is uniformly distributed with mean 1 & Variance $\frac{4}{3}$, then find $P(X<0)$.

$\mu = 1$, $\sigma^2 = 4/3$

Ans Given $\mu = 1 = \frac{a+b}{2} \Rightarrow a+b = 2$

$\sigma^2 = \frac{4}{3} = \frac{(b-a)^2}{12} \Rightarrow (b-a)^2 = 16$

$\Rightarrow b-a = 4$

$\Rightarrow b = 3, \; \& \; a = -1$

$f(x) = \frac{1}{b-a} = \frac{1}{4}, \quad -1 \le X \le 3$

$P(X<0) = \int_{-1}^{0} f(x)\,dx = \frac{1}{4}\int_{-1}^{0} dx = 1/4.$

Imp ⇒ Exponential distribution :-

Memory less or forgetfulness / ~~mor~~ Korean

→ If a prop can survive a time it ~~has~~ the Prob that it will survive exponential x time.

reliability relies on bath tub property.

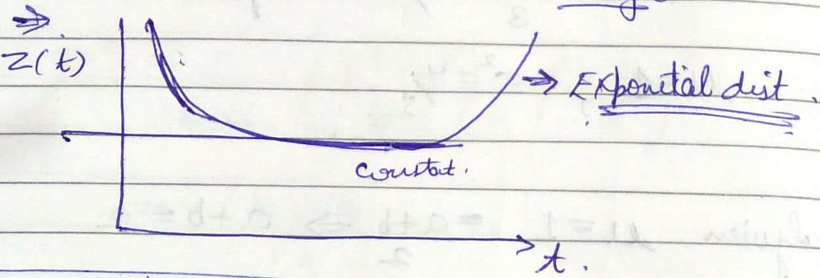
Constant → again inc from lower to higher

⇒ x = time , the infant mortality rate.

⇓

child < 6 age

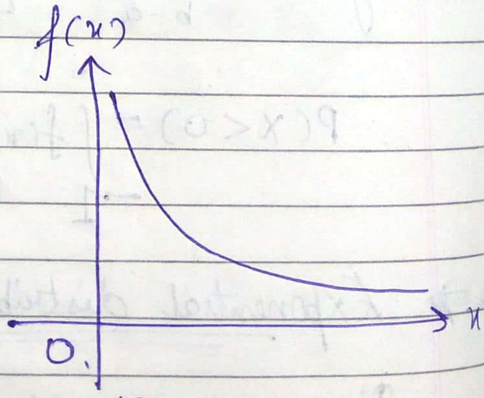
⇒



A random Exp distribution is followed when

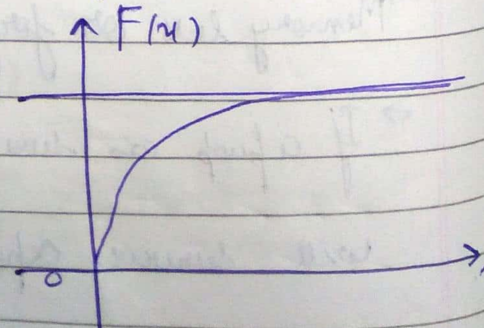
$$X \sim \exp(\lambda)$$

$$f(x) = \lambda e^{-x}, \quad x > 0$$
$$0, \quad o.w.$$



$$F(x) = P(X \le x)$$
$$= \int_0^x \lambda e^{-\lambda t}\, dt$$

$$= 1 - e^{-\lambda x}, \quad x \geq 0$$
$$0 \quad ; o.w.$$



Mean $\mu_1' = \mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

$$= \int_0^{\infty} x \lambda e^{-\lambda x} dx$$

$$= \lambda \int_0^{\infty} x e^{-\lambda x} dx$$

$$= \frac{\lambda \sqrt{2}}{(\lambda)^2} \rightarrow \text{factorial } 1 = 1$$

$$= \frac{1}{\lambda}$$

$\int_0^{\infty} \underbrace{x^{n-1} e^{\alpha x}}\, dx$

function name. of Value

⇓

Gamma func with parameter.

$$= \frac{\sqrt{n}}{\alpha^n}$$ — gamma fn

$\sqrt{n} = \underline{|n-1}$

for 2 ⇒ $\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\, dx.$

$$= \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx.$$

$$= \lambda \int_0^{\infty} x^2 e^{-\lambda x} dx.$$

$$= \frac{\lambda \sqrt{3}}{(\lambda)^3} = \frac{2}{\lambda^2}$$

$$\sigma^2 = \mu_2' - \mu^2$$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}.$$

Q eg:- A r.v. 'x' has random distribution with pdf :-

DATE : / /
PAGE NO. :

$$f(x) = 2e^{-2x}, \quad x > 0$$
$$\quad 0, \text{ e.w.}$$

What is the Prob. that X is not less than 2.

$$P(X \geq 2) = \int_2^{\infty} 2e^{-2x}\, dx$$

$$= e^{-4}$$

Study own

Gamma distribution :-

$$\mu = \frac{r}{\lambda}$$

$$\sigma^2 = \lambda$$

⇒ Normal Distribution (Gaussian distribution)

$$\underbrace{\qquad}_{\text{Same}}$$

⇒ A Continuous r.v. 'x' is said to follow normal Dist :-

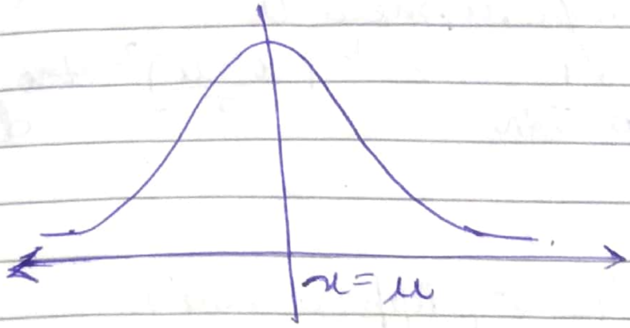
$$X \sim N(\mu, \sigma^2)$$

$$f(x; \mu, \sigma) = \cancel{\frac{1}{\sigma\sqrt{2\pi}}\, e^{-1/2} (x-\mu)^2}$$

$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$-\infty < x < \infty$$
$$-\infty < \mu < \infty$$
$$\sigma > 0$$

Scanned by CamScanner



x axis is asymptotic to the curve.

Mean = Mode = Median $\rightarrow$ median divides the distribution to two equal parts.

Properties of Normal Dist :-

$\Rightarrow$

i> Mean = mode = median.

ii> Area under Curve is unity.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \text{ (unity)}$$

Proof on our own.

iii> Curve is bell shaped and symmetric about $x = \mu$.

iv> ~~Unimodal~~ and ~~Val~~ max Value (when $x = \mu$) from $\left(\frac{x-\mu}{\sigma}\right)^2 = 1$.
Uni modal (single maximum Value.)

max Value is $\Rightarrow \dfrac{1}{\sigma\sqrt{2\pi}}$.

v> The Pt. of inflection $\mu \pm \sigma$

$\Downarrow$

get by double differentiation $= 0$.
Third " $\neq 0$.

$E(X) = \mu$

$Var(X) = \sigma^2$

DATE: / /

PAGE NO.:

VI) The mean deviation from mean μ

$$= \int_{-\infty}^{\infty} |x-\mu| \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \sqrt{\frac{2}{\pi}} \text{ or } \frac{4}{5}\sigma \text{ (Approximate Value.)}$$

$X = a_1 X_1 + \dots a_n X_n$

$X_i \sim N(\mu_i, \sigma_i^2)$

$X \sim N(a_i \sum \mu_i, a_i^2 \sum \sigma_i^2)$

$\Rightarrow \mu_{2n+1} = 0 \quad , n = 0, 1, 2 \dots$

$\mu_{2n} = 1 \cdot 3 \cdot 5 \dots (2n-1)\sigma^{2n}, \quad n = 1, 2 \dots$

if $n=1$

$\mu_2 = \sigma^2$

$\Rightarrow$ Coefficient of $\beta = \frac{\mu_3^2}{\mu_2^3} = 0$

if $n = 2$. $\quad 2(2)-1)\sigma^4 \quad 3\sigma^4$

$\Rightarrow$ Kurtosis $= \frac{\mu_4}{\mu_2^2} = 3$

(flat peak of platykurtic)

$4-3$, $3\sigma^8/\sigma^4$, $3\sigma^2$

DATE : / /

PAGE NO. :

$\Rightarrow P(a \leq x \leq b)$

$= P(a \leq x < b)$

$= P(a < x \leq b)$

$= P(a < x < b)$

$= F(b) - F(a)$

$\Rightarrow \boxed{\dfrac{X-\mu}{\sigma} = Z} \rightarrow$ Standard Normal Variable

$\Downarrow$

proof by que

$Z \sim N(0,1)$

$\mu = 0$

Mean = 0

Variance = 1

$\sigma^2 = 1$

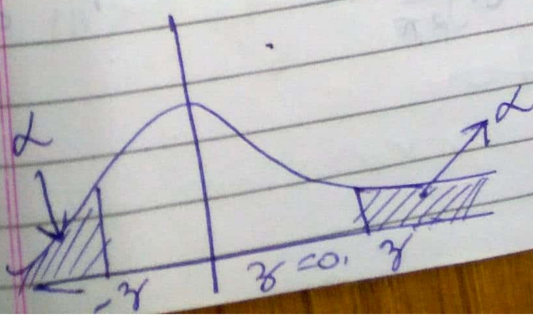
$\cancel{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)}} \Rightarrow$

$F(z) = P(Z \leq z) = \underbrace{\dfrac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz}$

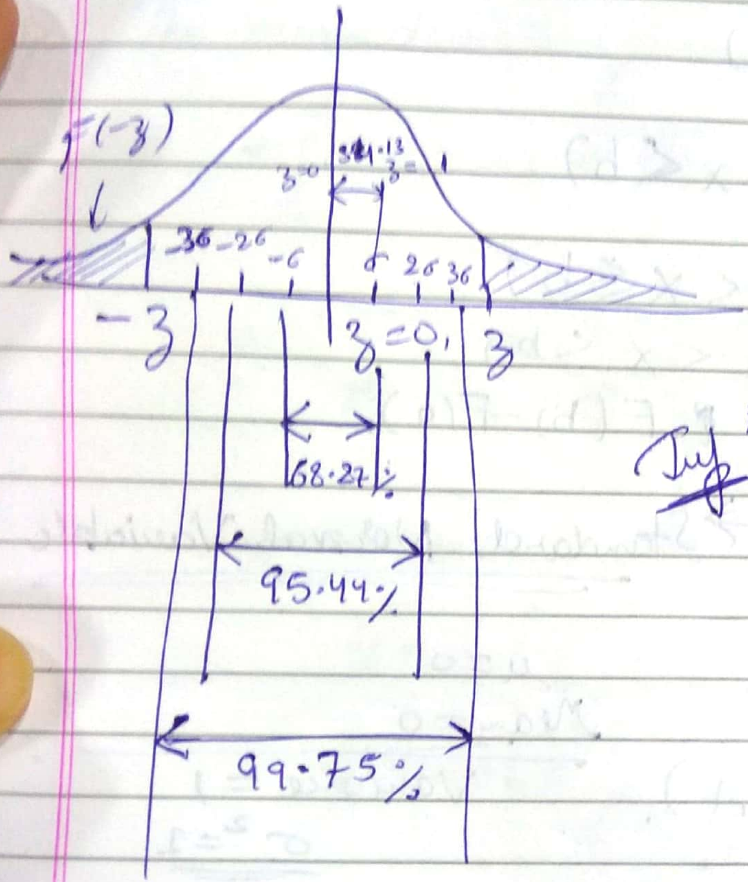
error function

(it used in theory of errors or sampling)

$\Rightarrow f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty \leq z \leq \infty$



2σ = now a days 1 errors after over 1000 millions
millions



$P(Z \geq z) = \alpha = 1 - P(Z \leq z)$
$= 1 - F(z)$

$P(Z \leq -z) = \alpha$

$\boxed{F(-z) = 1 - F(z)}$

Imp

$= 0.6827$

$\Rightarrow \mu \pm 2\sigma$, $z = (-2 \text{ to } 2)$
$\Rightarrow 95.44\%$

$= \mu \pm 3\sigma$ &, $z = (-3 \text{ to } 3)$

$\Rightarrow 99.75\%$

$\Rightarrow$ **Moment generating function :-**

$$M_x(t) = E[e^{tx}]$$

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

DATE: / /
PAGE NO.:

$= e^{\mu t + \frac{t^2 \sigma^2}{2}}$

Prob. erro = 0.654

Q4 A r.v. x has mean 10. & Stand. Deviation = 5.

$\sigma = 5$

$X \sim N(10, 25)$ Var = $\sigma^2 = 25$

find i> $P(X \leq 15)$
ii> $P(X \geq 15)$
iii> $P(10 \leq X \leq 15)$

Sol i> $P\left(\underbrace{\frac{X-\mu}{\sigma}}_{Z} \leq \frac{15-10}{5}\right)$

Stand Normal variable.

$\Rightarrow P[Z \leq 1] = F(1)$
$= 0.8413$.

$z=1.0$ → Check from table.

ii> $P(X \geq 15) \Rightarrow 1 - P(X \leq 15)$
$= 1 - 0.8413$
$= 0.1583$.

iii> $P(10 \leq X \leq 15) = P\left(\frac{10-10}{5} \leq \frac{X-10}{5} \leq \frac{15-10}{5}\right)$

$= P(0 \leq z < 1) = F(1) - F(0)$ divides into 2 equal parts
$= 0.8413 - 0.5$
$= 0.3413$

Poisson of Discrete Dist.   Normal ⇒ Continuous dist

DATE : / /

PAGE NO. :

⇒ Normal Distribution acts as Poisson Dist. when $\lambda \to \infty$

⇒ **Normal App to Binomial Dist.**

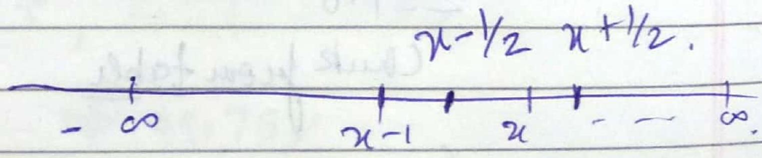
$$X \sim B(n, p), \quad E(X) = np$$
$$Var(X) = npq$$

$$P\left(\frac{X - np}{\sqrt{npq}} \leq z\right), \quad n \to \infty$$

$$P(Z \leq z) = F(z), \quad n \to \infty$$

When $p \leq 0.5$, $np > 5$
or $p > 0.5$, $nq > 5$

⇒ $P(X \leq x) = P(X < x)$
$P(X \leq x) \neq P(X < x)$



⇓

To convert discrete into continuous :-

we sub 1/2 from LHS & add 1/2 to RHS

Correction of Continuity

Shepard correction of continuity.

DATE: / /

PAGE NO.:

Q. A safety engineer feels that 30% of all industrial accidents in his plan are ~~post by~~ caused by failure of his employees to follow instructions, find the approximate probability that among 84 industrial accidents anywhere from 20 to 30 both inclusive will be able to failure of employees to follow instructions. (use normal approximation)

Ans. ~~30%~~ Let X = no. of accidents due to failure of employees to follow instruction & p be the probability

$p = 0.30$

$n = 84$

$np = \mu = 84 \times 0.30 = 25.2$

$(1-p) = (0.70)$

$npq = \sigma^2 = 17.64 = \sigma = 4.2$

$P(20 \le X \le 30) = P\left(20 - \frac{1}{2} \le X \le 30 + \frac{1}{2}\right)$

discrete to continuous.

Binomial Dist = Normal dist.

$= P(19.5 \le X \le 30.5)$

$\Rightarrow P\left(\frac{19.5 - 25.2}{4.2} \le \frac{X - np}{\sqrt{npq}} \le \frac{30.5 - 25.2}{4.2}\right)$

$= P(-1.36 \le Z \le 1.26)$

$[\because F(-3) = 1 - F(3)]$

$= F(1.26) - F(-1.36) = F(1.26) - 1 + F(1.36)$

$= 0.8962 - 1 + 0.9131$

from table $\Rightarrow$ $= 0.8093$

DATE : / /
PAGE NO. :

Q/ A safety engineer feels that 30% of all industrial accidents in his plan are ~~post by~~ caused by failure of his employees to follow instructions, find the approximate probability that among 84 industrial accidents anywhere from 20 to 30 both inclusive will be able to failure of employees to follow instructions. (use normal approximation)

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$P(20 \le X \le 30) = P\left(20 - \frac{1}{2} \le X \le 30 + \frac{1}{2}\right)$

discrete to continuous.

Binomial Dist $\frown$ Normal dist.

$= P(19.5 \le X \le 30.5)$

$\Rightarrow P\left(\frac{19.5 - 25.2}{4.2} < \frac{X - np}{\sqrt{npq}} \le \frac{30.5 - 25.2}{4.2}\right)$

$= P(-1.36 \le Z \le 1.26)$

$= F(1.26) - F(-1.36) = F(1.26) - 1 + F(1.36)$

$= 0.8962 - 1 + 0.9136$

$= 0.8093$

$[\because F(-3) = 1 - F(3)]$

from table $\Rightarrow$

DATE : / /
PAGE NO. :

→ <u>Chebychev Enquality</u> : If a prob dist has mean μ and Standard deviation σ, the prob. of getting a value which denotes from μ at least $K\sigma$ is at most $\frac{1}{K^2}$.

$$P[|X-\mu| \geq K\sigma] \leq \frac{1}{K^2}$$

<u>or</u> $$P[|X-\mu| < K\sigma] \geq 1-\frac{1}{K^2}$$

Q4 ⇒ The no. of customers to visit a Car dealer showroom in a day with random Variable where ~~μ is con~~

$\mu = 18$

& $\sigma = 2.5$, with what Prob, can we assert that there will be b/w 8 to 28 customers.

<u>Soln</u>. <u>How to compute K</u> :-

$\mu = 18$

$\sigma = 2.5$

<u>8 to 28</u>

Sir Soln.

$$K = \frac{28-18}{2.5} = 4.$$

$$\text{Prob} = 1-\frac{1}{K^2} = 1-\frac{1}{16} = \frac{15}{16}.$$

$X-\mu < K\sigma$

$\frac{X-\mu}{\sigma} = K$