Theory of Computing

Assignment − 1

September 1, 2016

1. Let f be a relations from $\mathcal{R}+$ to $\mathcal{R}$ with f defined as $f(x) = \sqrt{x}$ where $\mathcal{R}+$ is the set of all positive real numbers and $\mathcal{R}$ is the set of all real number. Is f a function? If yes, prove it. If not give a counter example to disprove it.

2. **Theorem:** All horses are of same color

   We will prove this by induction!

   Base: One horse. There is only one horse so the statement is trivially true.

   Hypothesis: Let us assume the statement is true for n horses.

   Inductive Proof: First, exclude the last horse and look only at the first n horses; all these are the same color since n horses always are the same color. Likewise, exclude the first horse and look only at the last n horses. These too, must also be of the same color. Therefore, the first horse in the group is of the same color as the horses in the middle, who in turn are of the same color as the last horse. Hence the first horse, middle horses, and last horse are all of the same color, and we have proven that:

   If n horses have the same color, then $n + 1$ horses will also have the same color.

   But obviously in reality all horses are not of same color. So where is the fallacy here in this proof by induction?

3. Let R be a relation on the set of all positive integers $\mathcal{Z}+$. R is defined as follows: aRb if and only if a and b when divided by 5 leaves the same reminder. Prove that this relation is an equivalence relation. Also show what are the equivalence classes for this relation.

4. Suppose n people attend a party and some shake hands with others (but not with themselves). Can you show that at the end, there are at least two people who have shaken hands with the same number of people.

5. Let R be relation on the set A such that R is both symmetric and transitive. Then, aRb and bRa implies aRa. This shows R is also reflexive. This implies reflexive definition is redundant. Is this so? If not, why?

6. Take any 4 numbers from the set of all natural numbers. Now show that one can find 2 numbers so that their difference is divisible by 3. (Prove this in general by using Pigeonhole Principle!)