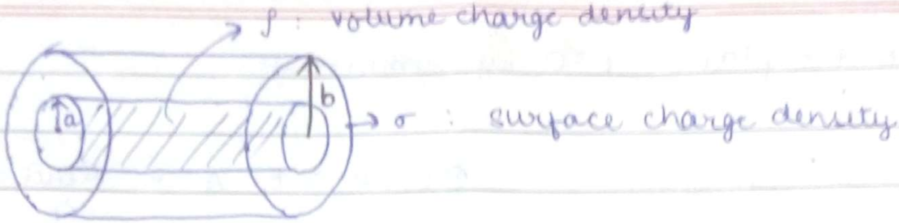
PRIYANSI SINGH 16UCS143

ASSIGNMENT - 03

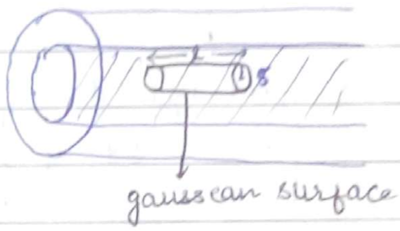
DATE: / / B+
PAGE NO.

(1)



Given, the cable as a whole is neutral

i) $\bar{E}$ inside inner cylinder :

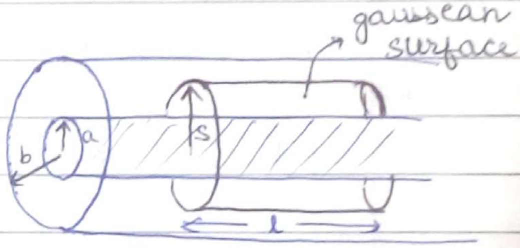


$$\oint \bar{E}\cdot d\bar{s} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Rightarrow E \cdot 2\pi s l = \frac{\rho \pi s^2 l}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\rho s}{2\epsilon_0}\hat{s}}$$

ii) $\bar{E}$ in the region $a<s<b$

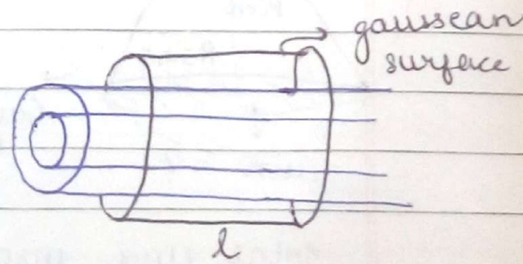


$$\oint \bar{E}\cdot d\bar{s} = \frac{Q_{encl}}{\epsilon_0}$$

$$\bar{E}\, 2\pi s l = \frac{\rho \cdot \pi a^2 l}{\epsilon_0}$$

$$\boxed{\bar{E} = \frac{\rho a^2}{2 s \epsilon_0}\hat{s}}$$

iii) $\bar{E}$ outside the cable :



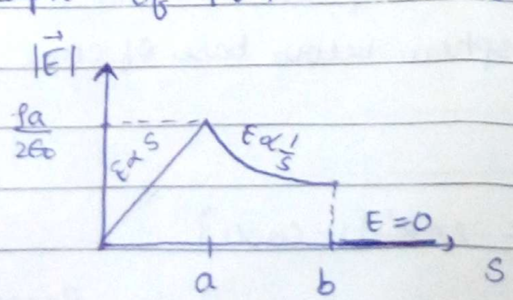
$$\oint \bar{E}\cdot d\bar{s} = \frac{Q_{encl}}{\epsilon_0} = 0$$

[cable is neutral]

$$\Rightarrow \bar{E}\cdot 2\pi s l = 0$$

$$\Rightarrow \boxed{\bar{E} = 0}$$

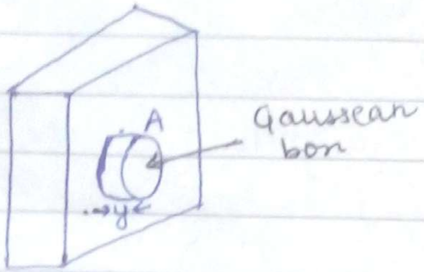
(2) Graph of $|\vec{E}|$ v/s s is :



Premium

② On the xz plane, E=0 by symmetry.

$$\oint \bar{E}\cdot da = E\cdot A = \frac{Q_{encl}}{\epsilon_0} = \frac{1}{2} A y \rho$$



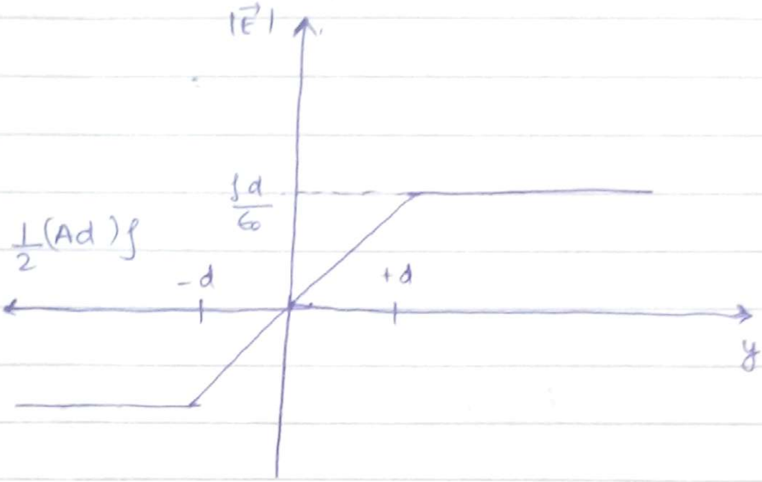
→ for $|y| < d$

$$\boxed{\bar{E} = \frac{\rho y}{\epsilon_0}\hat{y}}$$

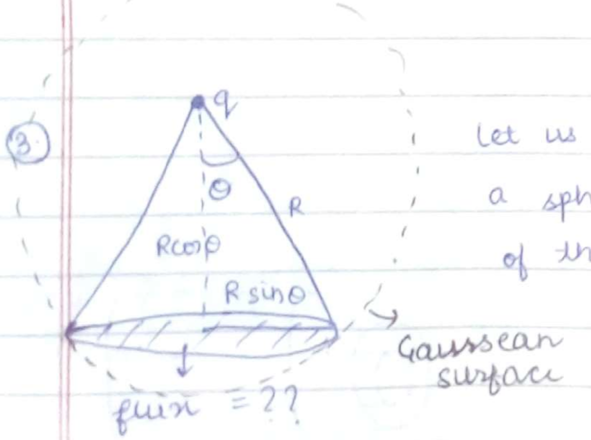
→ for $|y| > d$

$$\oint \bar{E}\cdot da = E\cdot A = \frac{1}{2}(Ad)\rho$$

$$\boxed{\bar{E} = \frac{\rho d}{\epsilon_0}\hat{y}}$$



③



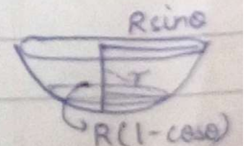
Let us consider a Gaussean surface as a sphere of radius equal to slant height of the cone with ~~sas~~ centre at top

Total flux through spherical surface $= \frac{q}{\epsilon_0}$

ϕ through base of cone $= \frac{q}{\epsilon_0}\left(\frac{S}{S_0}\right)$

where S_0 = area of whole sphere $= 4\pi R^2$

S = area of sphere below base of cone



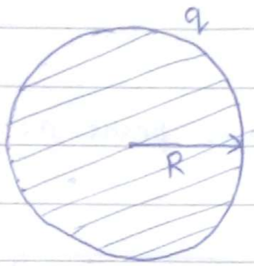
$$dS = (2\pi r)R\,d\alpha$$

$$\int dS = \int_0^\theta 2\pi R^2 \sin\alpha\, d\alpha = 2\pi R^2[1-\cos\theta]$$

DATE: / /
PAGE NO.
B+

$$\phi = \frac{S}{S_0}\left(\frac{q}{\epsilon_0}\right) = \frac{2\pi R^2(1-\cos\theta)}{4\pi R^2}\,\frac{q}{\epsilon_0} = \boxed{\frac{q}{2\epsilon_0}(1-\cos\theta)}$$ Ans

(4.)



a) For $r > R$

$$V = -\int \bar{E}\cdot d\bar{l} = -\int_{+\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr = \frac{1}{4\pi\epsilon_0}\,q\left[\frac{1}{r}\right]\Bigg|_{\infty}^{r}$$

$$\Rightarrow \boxed{V(r) = \frac{q}{4\pi\epsilon_0 r}}$$

b) For $r < R$

$$V = -\int_{\infty}^{r} \bar{E}\cdot d\bar{l} = -\int_{\infty}^{R} \bar{E}\cdot d\bar{l} - \int_{R}^{r} \bar{E}\cdot d\bar{l}$$

$$= -\int_{\infty}^{R} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr - \int_{R}^{r} \frac{q\,r}{4\pi\epsilon_0 R^3}\,dr$$

$$= -\frac{q}{4\pi\epsilon_0 R} - \frac{q}{4\pi\epsilon_0 R^3}\left[\frac{r^2-R^2}{2}\right]$$

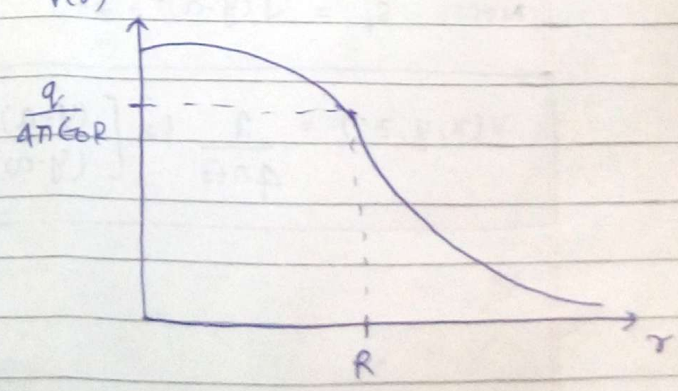
$$= +\frac{q}{4\pi\epsilon_0 R}\left[1 - \frac{r^2-R^2}{2R^2}\right]$$

$$\boxed{V(r) = \frac{q}{8\pi\epsilon_0 R^3}\left[3R^2 - r^2\right]}$$

→ (5.) For $r > R$

$\bar{E} = -\nabla\cdot V$

$\bar{E} = \frac{q}{4\pi\epsilon_0 r^2}\,\hat{r}$ ✓



→ For $r < R$

$\bar{E} = -\nabla\cdot V$

$\Rightarrow \bar{E} = \frac{q}{8\pi\epsilon_0 R^2}\left[2r\right]$

$= \frac{q\,r}{4\pi\epsilon_0 R^3}\,\hat{r}$ ✓

Premium

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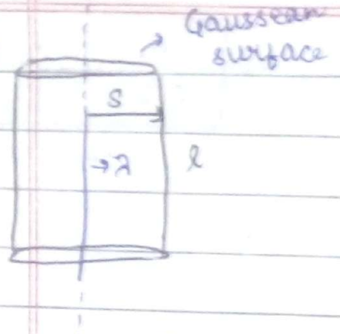
(5)



$$\oint \bar{E}\cdot d\bar{a} = \bar{E}\,(2\pi s l) = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{\bar{E} = \frac{\lambda}{2\pi s \epsilon_0}\,\hat{s}}$$

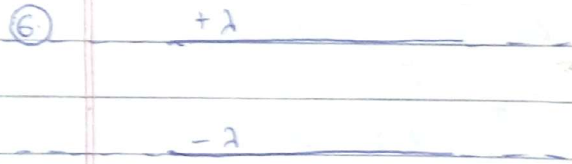
In this case, we can't take reference point at ∞, since charge ~~itsef~~ itself extends to ∞

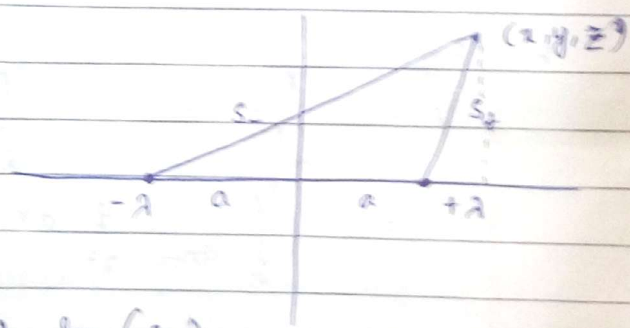
let $s = a$ be reference point

$$V(s) = -\int_a^s \bar{E}\cdot d\bar{l} = -\int_a^s \frac{\lambda}{2\pi\epsilon_0 s}\,ds = \boxed{\frac{2\lambda}{4\pi\epsilon_0}\ln\left(\frac{s}{a}\right)}$$

$$\bar{E} = \nabla V = -\frac{2\lambda}{4\pi\epsilon_0}\times\frac{1}{s}\,\hat{s} = \frac{-\lambda}{2\pi s\epsilon_0}\,\hat{s} = \bar{E} \checkmark$$

(6)





(a) Potential of $+\lambda$ is $V_+ = \frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{s_+}{a}\right)$

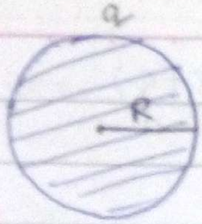
Potential of $-\lambda$ is $V_- = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{s_-}{a}\right)$

total $V = V_+ + V_-$

$$\boxed{V = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{s_-}{s_+}\right)}$$

Now, $s_+ = \sqrt{(y-a)^2 + z^2}$, $s_- = \sqrt{(y+a)^2 + z^2}$

$$\boxed{V(x,y,z) = \frac{\lambda}{4\pi\epsilon_0}\ln\left[\frac{(y+a)^2 + z^2}{(y-a)^2 + z^2}\right]}$$

7. 

i) $U = \frac{1}{2}\iiint \rho V d\tau$

$$= \frac{1}{2}\iiint_0^R \frac{q}{\frac{4}{3}\pi R^3} \times \frac{\cancel{4\pi\epsilon_0} q}{4\pi\epsilon_0 \times 2R^3}\left[3R^2 - r^2\right] dr \, . R^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{3}{2}\frac{q^2}{4\pi R^3 \times 4\pi\epsilon_0 \times 2R^3} \times 4\pi R^2 \times \left[3R^3 - \frac{R^3}{3}\right]$$

$$= \boxed{\frac{1}{4\pi\epsilon_0}\left(\frac{3}{5}\frac{q^2}{R}\right)}$$

ii) $U = \frac{\epsilon_0}{2}\int E^2 d\tau$

$$= \frac{\epsilon_0}{2}\left\{\int_{r=R}^{\infty}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{1}{(4\pi\epsilon_0)^2}\frac{q^2}{r^4}\cdot r^2 \sin\theta \, d\theta \, d\phi \, dr + \iiint \frac{1}{(4\pi\epsilon_0)^2}\frac{q^2 r^2}{R^6}\cdot r^2 \sin\theta \, d\theta \, d\phi \, dr\right.$$

$$= \frac{\epsilon_0}{2} \times \frac{q^2}{(4\pi\epsilon_0)^2} \times 4\pi\left\{\int_{r=R}^{\infty}\frac{dr}{r^2} + \int_{r=0}^{R}\frac{r^4}{R^6}dr\right\}$$

$$= \frac{q^2}{2\times 4\pi\epsilon_0}\left[\frac{1}{R}+\frac{1}{5R}\right] = \boxed{\frac{1}{4\pi\epsilon_0}\left(\frac{3}{5}\frac{q^2}{R}\right)}$$

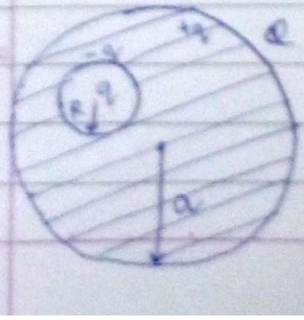
iii) $U = \frac{\epsilon_0}{2}\left[\oint_S V\bar{E}\cdot d\bar{a} + \int_V E^2 d\tau\right]$

$$= \frac{\epsilon_0}{2}\iint \frac{1}{4\pi\epsilon_0}\frac{q}{a}\cdot\frac{1}{4\pi\epsilon_0}\frac{q}{a^2}\, a^2 \sin\theta \, d\theta \, d\phi + \frac{\epsilon_0}{2}\iiint_{r=R}^{a}\frac{q^2}{(4\pi\epsilon_0)^2 r^4}\cdot r^2 \sin\theta \, d\phi \, d\theta \, dr$$

$$+ \frac{\epsilon_0}{2}\iiint_0^R \frac{q^2 r^2}{(4\pi\epsilon_0)^2 R^6}\cdot r^2 \sin\theta \, d\phi \, d\theta \, dr$$

$$= \frac{\epsilon_0}{2}\left[\frac{q^2}{4\pi\epsilon_0^2 a} + \frac{q^2}{4\pi\epsilon_0^2}\left(\frac{1}{R}-\frac{1}{a}\right) + \frac{q^2}{4\pi\epsilon_0^2}\left[\frac{1}{5R}\right]\right]$$

$$= \frac{q^2}{2\cdot 4\pi\epsilon_0}\left(\frac{1}{a}+\frac{1}{R}-\frac{1}{a}+\frac{1}{5R}\right) = \boxed{\frac{1}{4\pi\epsilon_0}\left(\frac{3}{5}\frac{q^2}{R}\right)}$$

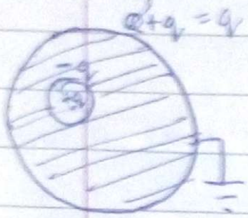
8. 

$\sigma_{cavity} = \frac{-q}{4\pi R^2}$

$\sigma_{sphere} = \frac{q+Q}{4\pi a^2} \quad \frac{(Q+q)}{4\pi a^2}$

When the conducting sphere is grounded,

$\overset{0}{Q}+q = q$



$V_{\text{outer sphere}} = 0 \Rightarrow \dfrac{-q}{4\pi\epsilon_0 a} + \dfrac{1\cdot(Q+q)}{4\pi\epsilon_0\, a} = 0$

$\Rightarrow \boxed{Q = 0}$

$\sigma_{\text{cavity}} = \dfrac{-q}{4\pi R^2} \qquad \sigma_{\text{sphere}} = \dfrac{q}{4\pi a^2}$

Electric field outside the conducting sphere $= \boxed{\dfrac{q}{4\pi\epsilon_0 r^2}}$

(9) $\vec{P} = \sum_{i=1}^{8} r_i Q_i = (-3\mu C)[2mm\,\hat{i}] + (-2\mu C)[1mm\,\hat{i}]$
$+ (-4\mu C)[3mm(-\hat{j})] + (-2\mu C)\{[1mm\,\hat{j}] + [2mm\,\hat{j}]\}$
$+ (5\mu C)[1mm\,\hat{k}] + 10\mu C\,[1mm(-\hat{k})] + (-2\mu C)(1mm\,(-\hat{i}))$

$= [(-6-2+2)\hat{i} + (12-6)\hat{j} + (5-10)\hat{k}] \times 10^{-9}$

$\vec{P} = (-6\hat{i} + 6\hat{j} - 5\hat{k}) \times 10^{-9}$ Cm

For $(0.4m, 0.3m, 0)$

Overall charge $= 0$

Monopole distribution $= 0$

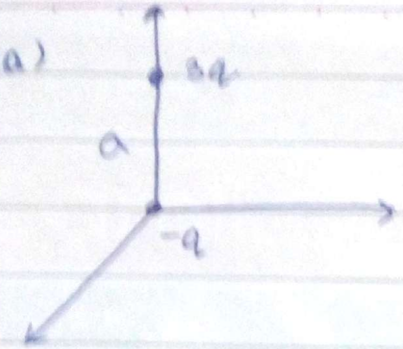
Dipole distribution $(V) = \dfrac{1}{4\pi\epsilon_0}\dfrac{\vec{P}\cdot\vec{r}}{r^3}$

$= 9\times10^{9}\left[\dfrac{(-6\hat{i}+6\hat{j}-5\hat{k})\cdot(0.4\hat{i}+0.3\hat{j})}{(0.5)^3}\right]\times10^{-9}$

$= \dfrac{9\times[-2.4+1.8]}{(0.5)^3} = \dfrac{-9\times0.6}{(0.5)^3}$

$\boxed{V = -43.2V}$

(10) (a)



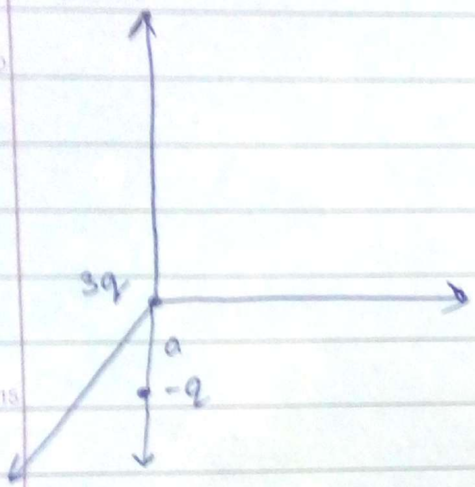
(i) Monopole moment : $\boxed{Q = 2q}$

(ii) Dipole moment : $3qa\,\hat{z}$

(iii) Multipole moment . $V \approx \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r} + \frac{\vec{p}\cdot\hat{r}}{r^2}\right]$

$$\approx \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{r} + \frac{3qa\cos\theta}{r^2}\right]$$

(b)

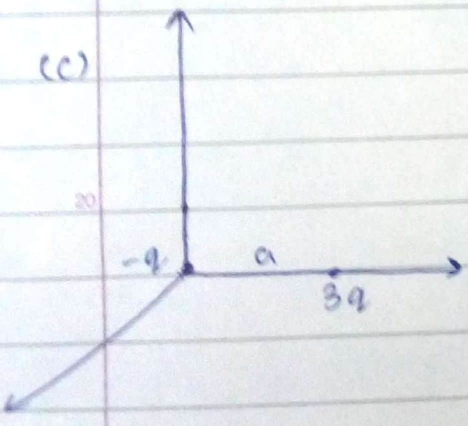


(i) Monopole : $\boxed{Q = 2q}$

(ii) Dipole moment : $qa\,\hat{z}$

(iii) $V \approx \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{r} + \frac{qa\cos\theta}{r^2}\right]$

(c)



(i) Monopole : $\boxed{Q = 2q}$

(ii) Dipole moment = $3qa\,\hat{y}$

(iii) $V \approx \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{r} + \frac{3qa\sin\theta\sin\phi}{r^2}\right]$

$\{\hat{y}\cdot\hat{r} = \sin\theta\sin\phi\}$