Roll No ______________ Subject ______________

Date ______________ Signature ______________

Q.1 (a) $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent

$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$$
$$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$$
$$\equiv \neg p \wedge (p \vee \neg q)$$
$$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$$
$$\equiv f \vee (\neg p \wedge \neg q)$$
$$\equiv (\neg p \wedge \neg q) \vee = \neg p \wedge \neg q$$

(b)

p	q	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	f	f	T	T
f	T	f	T	T
f	f	f	f	T

Q.2 (i) Let $P(x)$: x be a real number

$Q(x)$: x is an integer

Some real number are integer $\exists x (P(x) \wedge Q(x))$

(b) All integers are real number

$$\forall x \,(Q(x) \to P(x))$$

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(e) For some positive integer, there is a positive integer greater than it.

Let $P(x)$: x be a real number.

$Q(x, y)$: x is greater than y

$$\exists x \left[P(x) \rightarrow \exists y \left(P(y) \wedge Q(y, x) \right) \right]$$

Q.3 (a) $\neg (\exists x \, (P(x) \wedge Q(y)))$

$$\Rightarrow \neg \exists x \, P(x) \vee \neg Q(y)$$

$$\Rightarrow \forall x \, (\neg P(x)) \vee \neg Q(y)$$

$$\Rightarrow \neg P(z) \vee \neg Q(y)$$

$$\Rightarrow P(z) \rightarrow \neg Q(y)$$

$$\Rightarrow \exists x \, P(x) \rightarrow \neg Q(y)$$

(b) $n^3 + 2n$ is divisible by 3.

$P(n)$ denotes the proposition "$n^3 + 2n$ is divisible by 3"

$P(1)$ is true because $P(1) = 1 + 2 = 3$ is divisible by 3.

Let

$P(k)$ is true so $k^3 + 2k$ is divisible by 3

$$P(k+1) : (k+1)^3 + 2(k+1)$$

$$= k^3 + 3k^2 + 3k + 1 + 2k + 2$$

$$= (k^3 + 2k) + 3k^2 + 3k + 3$$

$$= (k^3 + 2k) + 3(k^2 + k + 1)$$

is divisible by 3.

Q.4

$$f(x, y, z) = \overline{(\bar{x} + z)(y + \bar{z})}$$