Nov/2017

M-III Assignment-~~3~~4

Q1.

a) $\sum \frac{z^n}{n^2}$

~~Root Test~~ Ratio Test.

$= \frac{z^{n+1}}{(n+1)^2} \cdot \frac{n^2}{z^n} = \left| \frac{z\, n^2}{(n+1)^2} \right|$

$|z| < 1$

b) $\sum \frac{z^n}{n!}$

$\left| \frac{z^{n+1}}{(n+1)!} \cdot \frac{n!}{z^n} \right| = \lim_{n\to\infty} \left| \frac{z}{n+1} \right|$

$z \in C$

c) $\sum \frac{z^n}{2^n}$

$\frac{z^{n+1}}{2^{n+1}} \times \frac{2^n}{z^n} = \left| \frac{z}{2} \right| < 1$

$|z| < 2.$

d) $\sum \frac{1}{2^n}\left(\frac{1}{z}\right)^n$

$= \frac{2^n}{2^{n+1}} \cdot \frac{z^n}{z^{n+1}} = \left| \frac{1}{2z} \right| < 1$

$\frac{1}{2} < |z|$

Q2. $a_n = \frac{(-1)^n}{\sqrt{n}} + i\frac{1}{n^2}$

$$\frac{a_{n+1}}{a_n} = \frac{\sqrt{\frac{(-1)^{2n+2}}{n+1} + \frac{1}{(n+1)^4}}}{\sqrt{\frac{(-1)^{2n}}{n} + \frac{1}{n^4}}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\sqrt{\frac{\left((n+1)^3+1\right)n^4}{(n+1)^4\left(n^3+1\right)}}$$

$$\cancel{\frac{1 + \frac{1}{(n+1)^3}}{}}$$

$$L = \lim_{n\to\infty}\sqrt{\frac{\left(1+\frac{1}{n+1}\right)}{\left(\frac{n+1}{n}\right)\left(1+\frac{1}{n^3}\right)}} = 1$$

Test Fails.

For $a_n = \frac{(-1)^n}{n} + i\frac{1}{n^2}$ ~~to be con~~

$\Sigma a_n \to$ convergent Conditionally.

⇓ Need to proof.

need to proof $\frac{(-1)^n}{\sqrt{n}} \to$ convergent

and $\frac{1}{n^2} \to$ convergent

For $\Sigma a_n \to$ Absolute convergence

⇓ Need to proof

$\frac{1}{\sqrt{n}} \to$ converges $\qquad \frac{1}{n^2} \to$ converges

$\frac{1}{n^p}$ $p>1$ convergent

$p\le 1$ diverdent thus $\frac{1}{\sqrt{n}}$ diverges $\to$ X Absolute

$\frac{1}{n} + \frac{1}{n^4}$

$$\frac{\frac{(n+1)^3 + (n+1)}{(n+1)^4}}{\frac{n^3+1}{n^4}}$$

$$\frac{n^3\left[\left(1+\frac{1}{n}\right)^3 + \left(\frac{1}{n^2}+\frac{1}{n^3}\right)\right]}{n^4\left(1+\frac{1}{n^4}\right)}$$

$n^3($

For $\frac{(-1)^n}{a_n}$

By alternating series test

* $\frac{1}{n}$ decreases * $\lim_{n\to\infty} \frac{1}{n} = 0$

Thus converges.

converges but not absolutely. $\frac{1}{n^2} \to$ converges thus

$$\left|\frac{a_n}{a_{n+1}}\right| = R$$

a) $$\lim_{n\to\infty} \left(\frac{\ln(n)}{\ln(n+1)}\right)^2 = \left(\frac{\frac{1}{n}}{\frac{1}{n+1}}\right)^2 = 1 = R$$

b) $$\lim_{n\to\infty} \left|\frac{n!}{(n+1)!}\right| = \frac{1}{n+1} = 0 = R.$$

c) $$\lim_{n\to\infty} \frac{n^2}{(n+1)^2} \frac{4^n + 3n}{4^{n+1} + 3(n+1)} = \frac{1}{(1+\frac{1}{n})^2} \frac{1 + \frac{3n}{4^n}}{4 + \frac{3(n+1)}{4^n}} = \frac{1}{4} = R$$

d) $$\lim_{n\to\infty} \frac{(n!)^3}{\cancel{3n!}} \times \frac{3(n+1)!}{((n+1)!)^3} = \frac{(3n+3)(3n+2)(3n+1)}{(n+1)^3} \; 27 = R.$$

e) $$\lim_{n\to\infty} \left(\frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n^2+n}}\right) \frac{\sqrt{(n+1)^2 + (n+1)}}{\sqrt{n+2} - \sqrt{n+1}}$$

$$\lim_{n\to\infty} \left(\frac{\sqrt{n+2} + \sqrt{n+1}}{\sqrt{n+1} + \sqrt{n}}\right)\left(\sqrt{\frac{\cancel{(n+1)}(n+2)}{n\,\cancel{(n+1)}}}\right) = 1 = R$$

Q4. $\sum_{n=0}^{\infty} (n+1)^2 z^n = \frac{1+z}{(1-z)^3}$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$$

$$\frac{1}{(1-z)^2} = \sum_{n=0}^{\infty} n z^{(n-1)}$$

$$\frac{2}{(1-z)^3} = \sum_{n=0}^{\infty} n(n-1) z^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1) z^n$$

$$\frac{2z}{(1-z)^3} = \sum_{n=0}^{\infty} (n+1)(n) z^n$$

$$\frac{2+2z}{1-z^3} = 2\sum_{n=0}^{\infty} (n+1)^2 z^n = \frac{1+z}{1-z^3}$$

Q5. a) $f(z) = \frac{1}{z^2}$

$$\frac{1}{a+z-a} = \frac{1}{a} \frac{1}{1+\left(\frac{z-a}{a}\right)} \qquad \left|\frac{z-a}{a}\right| < 1$$

$$\frac{1}{z} = \frac{1}{a} \sum_{n=0}^{\infty} (-1)^n \left(\frac{z-a}{a}\right)^n$$

$$\frac{1}{z^2} = \frac{1}{a}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{a^n}n(z-a)^{n-1}$$

$$\frac{1}{z^2} = \sum_{n=0}\frac{(-1)^{n+1}}{a^n}n(z-a)^{n-1}$$

$$f(z) = \frac{6z+8}{(2z+3)(4z+5)} \quad at \quad z=1$$

$$= \frac{1}{2z+3} + \frac{1}{4z+5}$$

$$= \frac{1}{2z-2+5} + \frac{1}{4z-4+9}$$

$$= \frac{1}{5}\frac{1}{\frac{2}{5}(z-1)+1} + \frac{1}{9}\frac{1}{\frac{4}{9}(z-1)+1}$$

$$= \frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n(-1)^n(z-1)^n + \frac{1}{9}\sum_{n=0}\left(\frac{4}{9}\right)^n(-1)^n(z-1)^n$$

c) $$f(z) = \frac{e^{z-1}}{z+1}\, e \qquad |z-1|<1$$

$$= \cancel{1+\frac{1}{z}\left(1+z+\frac{z^2}{2!}+\frac{z^3}{3!}+\ldots\right)}\frac{1}{2+(z-1)}$$

$$= \left(\frac{1}{2}\frac{1}{1+(z-1)}\right)\left(1+z-1+\frac{(z-1)^2}{2!}+\frac{(z-1)^3}{3!}+\ldots\right)$$

$$= \frac{1}{2}\left(1+(z-1)+(z-1)^2+\ldots\right)\left(1+(z-1)+\frac{(z-1)^2}{2!}+\ldots\right)$$

Q6.

a) $f(z) = \frac{1}{z-3}$ $\qquad |z| > 3$

$\frac{1}{|z|} < \frac{1}{3} < 1$

$= \left(\frac{1}{z}\right)\left(\frac{1}{1-\left(\frac{3}{z}\right)}\right)$ $\qquad \left|\frac{3}{z}\right| < 1$

$= \frac{1}{z}\left(1 + \frac{3}{z} + \left(\frac{3}{z}\right)^2 + \cdots\right)$

$= \frac{1}{z} + \frac{3}{z^2} + \frac{9}{z^3} + \cdots$

b) $f(z) = \frac{1}{z(z-1)}$ $\qquad 0 < |z| < 1$

$= \frac{-1}{z}\left(1 + z + z^2 + \cdots\right) = \frac{-1}{z} - 1 - z - z^2 + \cdots$

c) $f(z) = z^3 e^{\frac{1}{z}}$ for $|z| > 0$

case 1. $\qquad 0 < |z| < 1$

$f(z) = z^3\left(1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right)$

$= z^3 + z^2 + z + 1 + \frac{1}{z} + \cdots$

Case 2 $\qquad |z| > 1$ $\qquad \left|\frac{1}{z}\right| < 1$

$f(z) = z^3 + z^2 + z + 1 + \frac{1}{z} + \cdots$

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Q7. $f(z) = \frac{1}{z(1+z^2)}$ $|z| < 1$

$\frac{1}{z} \sum_{n=0}^{\infty} (-1)^n z^{2n} = \sum_{n=0}^{\infty} (-1)^n z^{2n-1}$

$\frac{1}{|z|^3} \left(\frac{1}{1 + \frac{1}{|z|^2}} \right)$ $|z| > 1$

$\left|\frac{1}{z}\right| < 1$

$\frac{1}{|z|^3} \left(1 + \frac{(-1)}{|z|^2} + \frac{1}{|z|^4} + \frac{(-1)}{|z|^6} + \cdots \right)$

$\frac{1}{|z|^3} - \frac{1}{|z|^5} + \frac{1}{|z|^7} - \frac{1}{|z|^9} + \cdots$

Q8. $|f(z)| - |g(z)| < 0$

$|g(z)| - |f(z)| > 0$

$\cancel{|g(z)|}$ $\cancel{|g(z) - f(z)| > |g(z)| - |f(z)|} > 0$

$\cancel{|f(z) - g(z)|} > 0$

$\left|\frac{f(z)}{g(z)}\right| < 1$ by Liouville Theorem

$|f'(z)| < |f(z)|$ $\left|\frac{f(z)}{g(z)}\right|$ is constant say λ

$f(z) = \lambda g(z)$

$f'(z) = k f(z)$ $\ln f(z) = k + c$

$f(z) = e^{(k+c)z} = e^{pz}$.

Q9.

a) $ze^{1/z}$ $\quad z=0$ Essential Singular Point.

b) $\dfrac{\sin z}{z}$ $\quad z=0$ Removable Singular Point.

c) $\dfrac{1-\cos z}{z^2}$ $\quad z=0$ Removable Singular Point.

d) $\dfrac{\pi \cot \pi z}{z^2}$

$$\pi \frac{1}{z^2}\left(\frac{1}{z} - \frac{z}{3} + \frac{z^3}{245} + \cdots\right)$$

Pole Order 2 ~~Principal part has finite no. of terms~~

If finite terms (b_n) then ~~Thus removable singularity.~~

pole of order = no of terms.

e) $\dfrac{z-\sin(z-1)}{z-1} = \dfrac{z-\left((z-1) - \dfrac{(z-1)^3}{3!} + \cdots\right)}{z-1}$

$$= \frac{1 + \frac{(z-1)^3}{3!} - \frac{(z-1)^5}{5!} + \cdots}{(z-1)}$$

Non Removable

~~Essential Singularity.~~ Pole of Order 1.

f) $\dfrac{z^2+\sin z}{\cos(z)-1} = \dfrac{z^2 + z - \frac{z^3}{3!} + \frac{z^5}{5!} + \cdots}{\cancel{1} - \frac{z^2}{2!} + \frac{z^4}{4!} + \cdots - \cancel{1}}$

Essential Singularity.

Q10. a) $\dfrac{\sin z}{z^2-\pi^2}$ $\quad z = \pm\pi$ Removable Singularity.

$$\frac{\sin(\pi - z)}{-(z+\pi)(z-\pi)} \qquad \frac{\sin(z+\pi)}{(z-\pi)(z+\pi)}$$

$z = \pi$ is pole of order 1.

ii) $\frac{z\cos z}{1-\sin z}$ $\quad z = \pi/2$

$\frac{z^2 \sin(\pi/2 - z)}{(1-\cos z)\, z}$ Removable Singularity

Q11.

a) $\frac{1}{z(z+1)} = \frac{1}{z}\left(1 - z + z^2 - z^3 + \ldots\right)$

$b_1 = 1$

b) $z\cos\frac{1}{z} = z\left(1 - \left(\frac{1}{z}\right)^2\frac{1}{2!} + \left(\frac{1}{z}\right)^4\frac{1}{4!} + \ldots\right)$

$= z - \frac{1}{2z} + \frac{1}{4!\,z^3} + \ldots$

$b_1 = \frac{-1}{2}$

c) $\frac{z-\sin z}{z} = \frac{z - \left(z - \frac{z^3}{3!} + \frac{z^5}{5!} + \ldots\right)}{z}$

$= \frac{\frac{z^3}{3!} - \frac{z^5}{5!} + \ldots}{z} = \frac{z^2}{3!} - \frac{z^4}{5!} + \ldots$

$b_1 = 0$

d) $\frac{\cot z}{z^4} = \frac{1}{z} - \frac{z^8}{3} + \frac{z^3}{245} + \frac{z^5}{(\quad)} + \ldots$

$b_1 = \frac{1}{245}$

Q2.

a) $\dfrac{e^{-z}}{z^2}$ $z=0$

$\dfrac{1}{z^2}\left(1 - z + \dfrac{z^2}{2!} + \text{---}\right)$

$b_1 = \dfrac{-1}{\cancel{2}}$

$2\pi i\left(\dfrac{1}{2}\right) = \pi i$

b) $\dfrac{e^{-z+1}}{\cancel{e}(z-1)^2}$ $\dfrac{1}{e}$

$\dfrac{1}{e}\dfrac{1}{(z-1)^2}\left(1 - (z-1) + \dfrac{(z-1)^2}{2!} + \text{---}\right)$

πi

c) $z^2 e^{\frac{1}{z}}$

$z^2\left(1 + \dfrac{1}{z} + \dfrac{1}{2z^2} + \text{---}\right)$

$2\pi i$

e) ~~#~~ $\dfrac{e^z}{(z^2-1)^2}$

$\text{Res} f(z) = \dfrac{1}{(m-1)!}\lim_{z\to z_0}\dfrac{d^{m-1}}{dz^{m-1}}\left((z-z_0)^m f(z)\right)$

$2e\pi i$

$z = e^{i\theta} = \cos\theta + i\sin\theta$

$\frac{1}{z} = e^{-i\theta} =$

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d) $\frac{z+1}{z^2-2z}$ $\quad z=0 \quad z=2$

$\underset{z=0}{\text{Res}}\, f(z) = -3/4$ $\cancel{z+1}$ $\quad \underset{z=1}{\text{Res}}\, f(z) = -1/4$

$\frac{\cancel{1}}{\cancel{z}}$ $\quad \frac{d}{dz}\Big|_{z=0} \frac{z+1}{z-2} \cdot \frac{(z-2)-(z+1)}{(z-2)^2} = \frac{-3}{(z-2)^2}$

$= \frac{-3}{4}$

$\frac{d}{dz}\Big|_{z=2} \frac{z+1}{z} = \cancel{\frac{1}{4}} \frac{-1}{z^2} = \frac{-1}{4}$

$-2\pi i$

e) $\frac{\pi \cot \pi z}{\left(z+\frac{1}{2}\right)^2}$

$\cancel{\frac{d^2\, \pi \cot \pi z}{dz^2}}$ $\quad \cancel{\frac{d}{dz} = \pi^2 \operatorname{cosec}^2 \pi z}$ $\quad m=2.$

$-\pi^2 \operatorname{cosec}^2 \pi z$ $\quad -\pi^3\, 2\operatorname{cosec}^2 \pi z \cot \pi z$

$-\pi^2$

$+ \cancel{4}2\pi i \left(\cancel{\operatorname{cosec}^2 2\pi}\ \cancel{\cot 2\pi}\right)$

$2\pi i\left(-\pi^2\right) = -2\pi^3 i$

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Q13.

a) $\displaystyle\int_0^\infty \frac{2x^2-1}{x^4+5x^2+4}\,dx$ Roots should be imaginary.

Application of Cauchy Residue theorem.

$\cancel{\dfrac{(2x^2-1)\,dx}{x^4+4x^2+x^2+4}}$

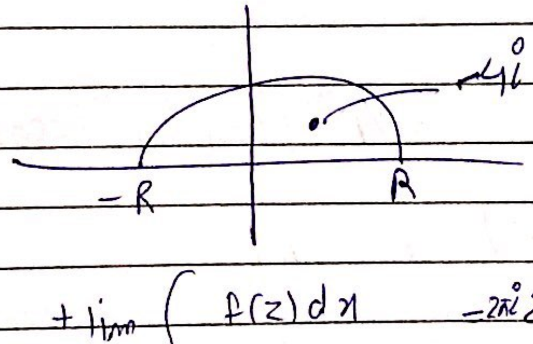
$\cancel{\dfrac{(2x^2-1)\,dx}{x^2(x^2+4)+1(x^2+4)}} = \cancel{\dfrac{(2x^2-1)\,dx}{(x^2+4)^2}}$

$\cancel{\dfrac{d(2z^2-1)\,dz}{dz} = 4z = 16i}$

$\dfrac{2x^2-1}{x^4+5x^2+4} \quad = \quad \dfrac{x^2}{x^2}\,\dfrac{(2x^2-1)}{(x^2+4)^2}$

$\Downarrow$

$\dfrac{2-\frac{1}{x^2}}{x^2+5+\frac{4}{x^2}}$ $\qquad f(z) < \dfrac{K}{|z|^2}$ Numerator 2 power less than denominator



$\displaystyle\lim_{R\to\infty}\int_{-R}^{\infty} f(z)\,dz + \lim_{R\to\infty}\int_{C_R} f(z)\,dx = 2\pi i \sum \operatorname{Res} f(z)$

$\displaystyle\lim_{R\to\infty}\int_0^\infty f(z)\,dz = \pi i \operatorname*{Res}_{z=2i} f(z) + \pi i \operatorname*{Res}_{z=i} f(z)$

$$\frac{d^i}{dx^2}\frac{(2x^2-1)(x^2+4)^2}{(x^2+x)^2} = 4$$

$$\boxed{4\pi i}$$

$$\frac{2x^2-1}{(x^2+1)(x^2+4)}$$

$$\frac{d}{dx}\frac{2x^2-1}{x^2+1} = \frac{4x(x^2+1)-(2x^2-1)2x}{(x^2+1)^2}$$

$x = 2i$

$$= \frac{8i(-3)+18i\times 2}{9}$$

$$= \frac{(-24+36)i}{9} = \frac{6i}{9} = -\frac{4i}{3}$$

$$\frac{2x^2-1}{x^2+4} = \frac{4x(x^2+4)-(2x^2-1)(2x)}{(x^2+4)^2}$$

$$= 4i \quad \frac{4i(3)+6i}{9} = \frac{18i}{9} = 2i$$

$$\frac{2x^2-1}{4x^3+10x} \qquad \frac{-9}{-32i+20i} = \frac{9}{12i} = \frac{-3i}{4}$$

$$\frac{-3}{-4i+10i} = \frac{-3}{6i} = \frac{+i}{2}$$

$$\frac{1}{4}i \times \pi i = \boxed{-\pi/4}$$

Q13 b) $\int_0^\infty \frac{dx}{x^4+1} = \pi/2\sqrt{2}$

x^4

$x^4+1 \longrightarrow \text{Roots} \longrightarrow \sqrt{i}, -\sqrt{i}, \sqrt{-i}$

$(-1)^{1/4} = (i)^{1/2}$

$= e$

$(e^{-\pi i})^{1/4} = e^{((-\pi+2k\pi)i)\frac{1}{4}}$

$e^{-i\pi/4} \quad e^{i\pi/4} \quad e^{i3\pi/4} \quad e^{5\pi i/4}$

$\cos(-\pi/4) + i\sin(-\pi/4) = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ ✓

$2\pi - \pi/4$

$\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ ✓

$\frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$ $\quad 2\pi + \pi/4$

$\frac{1}{4x^3} = \frac{\cancel{1}}{\sqrt{2}\,\cancel{4(-i)}}$

$\frac{-i}{2\sqrt{2}} \quad \pi i$

$\boxed{\frac{\pi}{2\sqrt{2}}}$

$\cancel{\sqrt{2}\,4(1+i)^3}$

$\frac{\frac{2i}{\sqrt{2}}}{4\left(\frac{i}{2} - \frac{1}{2}\right)}$

$\frac{1}{4e^{3i\pi/4}} \qquad \frac{1}{4e^{i9\pi/4}}$

$\frac{1}{4\left(\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)} \qquad \frac{1}{4\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)}$

Q14

a) $\displaystyle\int_0^{\infty} \frac{\cos ax}{x^2+1}\,dx$

Let us consider $x \to z$

$$\lim_{R\to\infty}\int_{-R}^{R} \frac{e^{iaz}}{z^2+1}\,dz$$

$z = \pm i \to z = i$

$$\frac{d}{dz}\,\frac{e^{iaz}}{z+i}$$

$$= \frac{iae^{iaz}(z+i) - e^{iaz}}{(z+i)^2}$$

$$\frac{+2a}{e^a 4} + \frac{1}{4ae^a} = \frac{a}{2e^a} + \frac{1}{4e^a}$$

$$\frac{p(x)}{q'(x)} = \frac{e^{-a}}{2z} \qquad z = i$$

$$\frac{-ie^{-a}}{2} \times \pi i = \boxed{\frac{\pi e^{-a}}{2}}$$

b) $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)}$

$$x = bi \qquad \frac{e^{ix}}{2x(\cancel{x^2+b^2}) + 2x(x^2+a^2)} = \frac{e^{-b}}{2bi(a^2-b^2)}$$

$$= \frac{e^{-a}}{2ai(b^2-a^2)} \Rightarrow \frac{\pi}{a^2-b^2}\left(\frac{e^{-b}}{b} \quad \frac{e^{-a}}{a}\right)$$

Ans
a) $\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$

$\boxed{\text{Answer should be } 0}$

$a > |b|$

$\frac{a}{b} > 1$

$\sin\theta = \frac{z - \frac{1}{z}}{2i}$

$z = e^{i\theta}$

$dz = ie^{i\theta}d\theta$

$\int_0^{2\pi} \frac{dz}{iz\left(a + \frac{b}{2i}\left(z + \frac{1}{z}\right)\right)}$

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2}$

$\int_0^{2\pi} \frac{dz}{iz\left(a + \frac{b}{2i}\left(z - \frac{1}{z}\right)\right)} = \int_0^{2\pi} \frac{2dz}{z\left(2a + bz - \frac{b}{z}\right)}$

$= \int_0^{2\pi} \frac{2dz}{(2az + bz^2 - b)}$

$bz^2 + 2azi - b$

$bz^2 + azi + azi - b$

$\frac{-2a \pm \sqrt{4a^2 - 4b^2}}{2b}$ $\qquad \frac{\sqrt{4a^2 - 4b^2} - \sqrt{4a^2}}{2b}$

$2bz + 2a$ $\qquad \frac{\pm\sqrt{a^2 - b^2} - \sqrt{a^2}}{2b}$

$2\sqrt{a^2 + b^2} \quad -2a + 2a$

$\sqrt{a^2 - b^2} + a$ $\qquad \boxed{\frac{2\pi}{\sqrt{a^2 - b^2}}}$

815. b) $\displaystyle\int_0^{2\pi} \frac{dz}{zi\left(3-\frac{2}{2}\left(z+\frac{1}{z}\right)+\frac{1}{2i}\left(z-\frac{1}{z}\right)\right)}$

$$\frac{dz}{3zi - zi\left(z+\frac{1}{z}\right)+\frac{z}{2}\left(z-\frac{1}{z}\right)}$$

$$3zi - z^2 i - i + \frac{z^2}{2} - \frac{1}{2}$$

$$\left(\frac{1}{2}-i\right)z^2 + 3zi - \left(\frac{1}{2}+i\right)$$

$$\frac{-3i \pm \sqrt{9(-1)+4\left(\frac{1}{4}+1\right)}}{2\left(\frac{1}{2}-i\right)} = z.$$

$-9 \cdot 3$ $\quad \dfrac{-3 \pm 2i}{2-2i} \qquad \cancel{\dfrac{-3 \pm i\sqrt{12}}{1-2i}} = \cancel{\dfrac{-3+2\sqrt{3}i}{1-2i}}$

$(1-2i)z + 3i \qquad \cancel{-3i} + 2i + \cancel{3i}$

$\cancel{+3+2\sqrt{3}i+3i}$ $\qquad \dfrac{2\pi i}{2i} = \boxed{\pi}$