20 Aug 19.

MSE class 1.

~~H 12:30 Friday~~
~~12:30 - 2 Monday~~

10% class participat n

5% MT   5% ET

Q1 - 10%
Q2 - 10%
MT - 30%
ET - 50

Friday 8 - 9:30

Theory of Sets.

Sets are always represented by capital letters

ways to represent set

Roaster Method / Tabular form
when all the element of a set are in { } bracket

Rule Method / Set - Builder Method.
properties is P(x)

$A = \{x : x \text{ statisfies property } P\}$

ex:- Set of all natural numbers

$A = \{1, 2, 3, 4, \ldots\}$ Roaster method.

~~$A = \{x : x > 0 \text{ \& } x \in I$~~

$A = \{x : x \in N\}$ → set builder

ex:- set of all even nos.

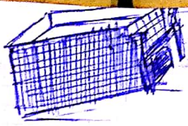
*** $A = \{\cancel{2}, 2, 4, 6, \ldots\}$ Roaster method

$A = \{x : x \text{ is an even no.}\}$

Different types of sets

1. Empty set ⟶ null set / void set

$\phi = \{\ \}$



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2. finite set
$A = \{a, e, o, u, i\}$ finite set
∞ set = eg $N = \{1, 2, \text{------}\}$ ∞ set

3. Singleton set
ex:- $A = \{x\}$

4. Universal set : U

* Subset: let A & B be 2 events. If every element of B is an element of A then B is called subset of A. $B \subseteq A \longrightarrow$ B contains A $\longrightarrow$ check.

1. If $A \subseteq B$
A is subset of A
~~if &~~ iff $x \in A \Rightarrow x \in B$

2. every set is a subset of a set. $A \subseteq A$
3. A null set is subset of each set.
4. If $A \subseteq B$ & $B \subseteq C$ then $\Rightarrow$ $A \subseteq C$

5. If $A \subseteq B$ then B is called superset of A
6. Suppose A is having n element
then no. of ~~elem~~ subsets 2^n
ex:- $A = \{1, 2\}$
subset = $\phi, \{1\}, \{2\}, \{1, 2\}$

Equal sets :-
$A = \{1, 2, 3\}$
$B = \{1, 2, 3\}$
all elements must be same.

Equivalent set
$A = \{1, 2, 3\}$
$B = \{x, y, z\}$
# of elements must be same.

[redacted]

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Proper ~~set :~~ subset :-

$A = \{1, 2, 3, 4\}$
$B = \{1, 3, 4\}$
$C = \{1, 2, 3, 4\}$

$\left.\right\}$ $B \subseteq A$, $C \subseteq A$

C is not proper subset of A whereas B is a proper subset of A

defⁿ – Let A & B be two sets & $A \subseteq B$ but $A \neq B$ then A is proper subset of B.

Cardinality :-

Let A be a finite set then
$n(A) = |A| \rightarrow$ no of elements distinct in nature.

$A = \{1, 2, 3, 4, 5, 6, 7\}$
$B = \{1, 1, 2, 2, 3, 4, 5, 6, 7\}$

$n(A) = n(B) = 7$

Operation of sets

1. Union of sets –
$A \cup B = \{x : x \in A$ or $x \in B$ or both$\}$

2. Intersection :-
$A \cap B = \{x : x \in A$ and B both$\}$
$= \{x : x \in A$ and $x \in B\}$

3. Difference :-
$A - B = x \in A$ & $x \notin B$

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4. Complement of A

$B = \{x : x \notin A\}$

5. Disjoint sets A & B

if $A \cap B = \phi$

2nd Class
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Function :-

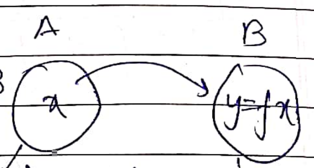
Domain

Codomain

Range:

Graph

$f : A \to B$ or $f : A \xrightarrow{f} B$



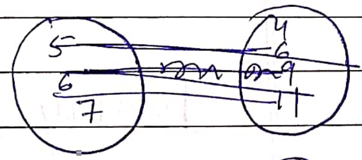
preimage   image of x

if $f : A \to B$ is a function that the subset $\{x : f(x) : x \in A\}$ of $A \times B$ is called a graph

let $A = \{5, 6, 7\}$

$B = \{4, 6, 9, 11\}$

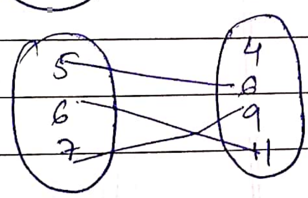
find the range and graph



Sol:- $f(5) = 6$

$f(6) = 11$

$f(7) = 9$



~~range = 11 - 6 = 5~~

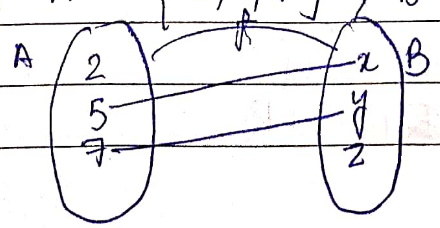
what is domain:- $\{5, 6, 7\}$

codomain: $\{4, 6, 9, 11\}$

range: $\{6, 9, 11\}$

Graph: $\{(5, 6), (6, 11), (7, 9)\}$

ex2:- let $A = \{2, 5, 7\}$, $B = (x, y, z)$



Is it a function

not a graph bcoz 2 is not mapped to B

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What is a real function?

Real function

$f : A \to B$

ex:- $f(x) = 2x^2 + x + 8$, $x \in R$

$\Rightarrow f : R \to R$ $\Rightarrow$ is a real function

Equal function

Let f & g are two function such that

$f : A \to B$

$g : A \to B$

$f(x) = g(x) \; \forall x \in A$

Codomain could be diff but the domain & range should be the same.

Constant function :-

Let $f : A \to B$ &

$f(x) = c \; \forall x \in A$ & c is a constant

Identity function :-

$f : A \to A$

$\Rightarrow f(x) = x \; \forall x \in A$

Absolute function

$f : A \to B$ &

$f(x) = |x| = \begin{cases} x \,;\, x \geq 0 \\ -x \,;\, x < 0 \end{cases}$

What will be domain $\Rightarrow R$

Range : $[0, \infty)$

What is polynomial?

Suppose a $f^n : R \to R$

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$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$ where $a_0 \neq 0$

$a_0, a_1, a_2, \dots, a_n$ are real nos

what is rational function?

$f(x), g(x)$ be two polynomials & $x \in R$ &

$\frac{f(x)}{g(x)}$, $g(x) \neq 0$

Odd function:- if $f(-x) = -f(x)$

even function:- if $f(x) = f(-x)$

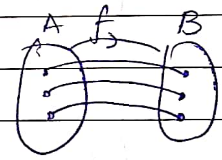
$f(x) = x^4 + x^2 + 6 \rightarrow$ even

He says $f(x) = x^3 + x + 2 \rightarrow$ ~~neither even nor odd~~

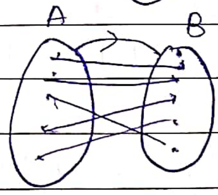
$\hookrightarrow$ odd function

He says constant functions are neither even nor odd.

One-One function



Onto function



$\Rightarrow \cancel{f(a)}$

$a \in A$

$\Rightarrow f(a) = b$

One-One onto

Probability & Statistics

Experiment : A physical process which gives an outcome

Experiment

Deterministic

nondeterministic.

When outcome not fixed but we know all the possibilities

(Random expt)

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What is trial → single performance of an expr.

What is sample space → collectⁿ of set of all possible outcome of a random expr

coin tossing $S : \{H, T\}$

H & T are sample point.

Sample space

discrete

continuous

Event – A when $A \subseteq S$

event

elementary event / simple event

compound event

$S = \{HT, HH, TT, TH\}$

favourable event :-

2 dices are thrown

$\{(1,1), (1,2), \ldots, (6,6)\}$

Equally likely event :- occurance probability same for each event

Compliment of an event : E' or E^c, $\bar{E}$

$\bar{E} = S - E$

Mutually exclusive events

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$ ⟹ $A \cap B = \Phi$

$B = \{2, 4, 6\}$

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exhaustive events:- total no of possible outcome occuring in single trial

Odd in favour of an event $= m/n$
and odd against event $= n/m$
$m \rightarrow$ # of outcomes ~~to~~ favourable to a certain event
$n \rightarrow$ # ——————— not in favour

What is permutation -
n distinct objects
r to be picked
$${}^nP_r = \frac{n!}{(n-r)!}$$
$${}^nC_r = \frac{n!}{r!(n-r)!}$$

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Definitⁿ of probability
1. Classical defⁿ of probability $\longrightarrow P(A) = \frac{m}{n}$
2. Statistical or Empirical def of probability
3. Axiomatic defⁿ of probability

→ no. of cases equally likely, no. of time executⁿ is finite.

→ $n \rightarrow \infty \quad \frac{m}{n} = P(A)$

→ let S be a sample space & A be any event
$A_1: \; 0 \leq P(A) \leq 1$
0 → Impossible event
1 → Sure event

$A_2: \; P(S) = 1$
$A_3:$ If A_1 & A_2 are mutually exclusive events then

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$P(A_1 \cup A_2) = P(A_1) + P(A_2)$

Extension of A3

Let $A_1, A_2, ---- A_n$ be mutually exclusive events

$P(A_1 \cup A_2 \cup A_3 ---- \cup A_n) = P(A_1) + P(A_2) + P(A_3) + --- + ------ + P(A_n)$

$= \sum_{i=1}^{n} P(A_i)$

The distributⁿ of Blood type in US is

41%	type A	Probability that the type will be A, B or AB
9%	type B	
4%	type AB	
46%	type O	

$P = 1 - .46 = .54$

Let A, B, C & D be the events of blood type A, B, AB and O respectively

Given $P(A) = .41$ $\quad P(C) = .04$

$P(B) = .09$ $\quad P(D) = .46$

$P(\text{Blood type A, B or AB}) = P(A \cup B \cup C)$

$= P(A) + P(B) + P(C)$

(∵ A, B, C are m.e.e. by A3)

$= .54$

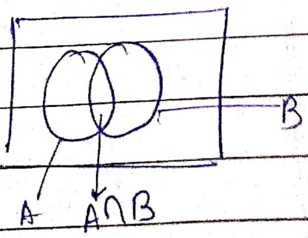
Additive Law of probability or Theorem of total probability :-

Let A & B ~~are two~~ be two events then show that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof :-

$A \cup B = A \cup (\bar{A} \cap B)$



(A & $\bar{A} \cap B$ are mutually exclusive event

using A3

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$$P(A\cup B) = P(A) + P(\bar{A}\cap B)$$
$$= P(A) + P(A\cap B) + \cancel{P(\bar{A}\cap B) - P(A\cap B)}$$
$$= P(A) + P(B) - P(A\cap B)$$

$B = (A\cap B) \cup (\bar{A}\cap B)$ (by A3

$P(B) = P(A\cap B) + P(\bar{A}\cap B)$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

Proove that

i) $P(\phi) = 0$

ii) $P(\bar{A}) = 1 - P(A)$

$S\cup\phi = S$ (S & ϕ are m.e.e)

i) $P(S\cup\phi) = P(S)$

$P(S) + P(\phi) = P(S)$

$P(\phi) = 0$

ii) $A\cup\bar{A} = S$

$P(A\cup\bar{A}) = P(S)$

$P(A) + P(\bar{A}) = 1$

$P(\bar{A}) = 1 - P(A)$

Q:- find the probability of drawing a king for a ~~hard~~ heart from a deck of cards

→ $\dfrac{4 + 13 - 1}{52} = \dfrac{16}{52}$

A is event when king is drawn $= \dfrac{^4C_1}{^{52}C_1}$

B —— " —— heart is drawn $= \dfrac{^{13}C_1}{^{52}C_1}$

$P(A\cap B) = \dfrac{^1C_1}{^{52}C_1}$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$= \frac{^4C_1 + {}^{13}C_1 + {}^1C_1}{^{52}C_1}$$

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Conditional probability

Let A & B be two events then $P(A|B) = \frac{P(A \cap B)}{P(B)}, \; P(B) \neq 0$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \; P(A) \neq 0$$

Theorem on compound probability or multiplication law of probability

Let A & B be any two events in sample space S then

$$P(A \cap B) = P(B) \cdot P(A|B), \; P(B) \neq 0$$
$$= P(A) \cdot P(B/A), \; P(A) \neq 0$$

Let A, B & C be three events in S

then $P(A \cap B \cap C) = P(A) \cdot P(C|A \cap B)$

$= \cancel{P(B) \cdot P(}$

$$P(A_1 \cap A_2 \cap A_3 \cap \text{----}) = P(A_1) \cdot P(A_2 / A_1) \cdot P(A_3 / A_2 \cap A_1) \cdot \text{-----}$$

if $P(A|B) = P(A)$

or $P(B/A) = P(B)$

$P(A \cap B) = P(A) \cdot P(B)$

$\Rightarrow$ A & B are independent events

If A & B be independent events then prove that

i) A' & B' are also independent

ii) A' & B ——— " ———

iii) A & B' ——— " ———

$\cancel{P(A \cap B) = P(A) \cdot P(B)}$

$\cancel{(1-P(A))(1-P(B))}$

$\cancel{= 1 - P(A) - P(B)}$

$P(A \cap B) = \cancel{1 - P(A) - P(B) + P(A)P(B)}$

$\cancel{P(A) \cdot P(B)}$

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proof:- $\overline{A \cup B} = A' \cap B'$ demoivers theorum

$1 - P(A \cup B) = P(A' \cap B')$

A & B are independent events

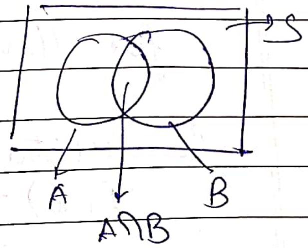
$\Rightarrow 1 - (P(A) + P(B) - P(A \cap B)) = P(A' \cap B')$

$\Rightarrow 1 - (P(A) + P(B) - P(A)P(B)) = P(A' \cap B')$ $\because$ A & B are indep.

$= (1 - P(A))(1 - P(B)) = P(A' \cap B')$

$= P(\bar{A}) P(\bar{B}) = P(\bar{A} \cap \bar{B})$

ii) A' & B are independent



$B = \underbrace{(A \cap B) \cup (A' \cap B)}_{\text{mee}}$

$P(B) = P(A \cap B) + P(A' \cap B)$

$\Rightarrow P(B) = P(A)P(B) + P(A' \cap B)$

$\Rightarrow P(B) P(A') = P(A' \cap B)$

Q:- If $P(A) = \frac{1}{2}$ $P(B) = \frac{1}{3}$ $P(A \cup B) = \frac{2}{3}$

$\frac{2}{3} = \frac{1}{2} + \frac{1}{3} - P(A \cap B)$

$-\left(\frac{2}{3} - \frac{5}{6}\right) = P(A \cap B)$

$\frac{1}{6} = P(A) \cdot P(B) = P(A \cap B)$

A & B are ~~complement~~ independent

$\Rightarrow$ A' & B' are independent as well

Sol^n $P(\overline{A \cup B}) = 1 - P(A \cup B) = P(A' \cap B')$

$P(A' \cap B') = 1 - \frac{2}{3} = \frac{1}{3}$

$P(A') = \frac{1}{2}$ $P(B') = \frac{2}{3}$

$P(A') \cdot P(B') = \frac{1}{3} = P(A' \cap B')$

$\Rightarrow$ A' & B' are independent

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last class missed.

Next week Quiz

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~~Revision~~

Continuous Probability distribution:

$f(x)$ is pdf if
cont random variable

i) $f(x) \geq 0 \quad \forall x$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

iii) $\int_a^b f(x)\,dx = P(a < x < b)$

for discrete x $\quad \sum_{\forall x} f(x) = 1$

Cumulative distributⁿ fⁿ (CDF)

$F(x) = P(X \leq x)$

$= \int_{-\infty}^{x} f(t)\,dt$

Properties of $F(x)$

i) Domain is $(-\infty, \infty)$
   Range $[0,1]$

ii) $F(x)$ is nondecreasing fⁿ of x in the right
   i.e $F'(x) = f(x) \; \forall x \geq 0$

iii) $F(x)$ is continuous on the right.

iv) $F(-\infty) = 0 \Rightarrow \int_{-\infty}^{-\infty} f(t)\,dt = 0$

v) $F(\infty) = 1 \Rightarrow \int_{-\infty}^{\infty} f(t)\,dt = 1$

$P(a \leq x < b) = P(a < x \leq b) = P(a < x < b)$

$= \int_a^b f(x)\,dx$

$F'(x) = f(x) = dF(x)$

Q:- $f(x) = ke^{-3x}, \; x \geq 0$

determine k & hence compute cdf of x

$k \int_0^{\infty} e^{-3t}\,dt = 1$

$= k \left.\frac{e^{-3t}}{-3}\right|_0^{\infty} \quad \frac{-k}{3}(0-1) \Rightarrow k = 3$

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CDF $F(x) = \int_0^x k e^{-3t} dt$

$3 \frac{e^{-3t}}{-3} = \Big|_0^x - e^{-3t}$

$1 - e^{-3x}$ $x \geq 0$
0 otherwise.

Soln To find k

$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{-\infty}^{0} f(x)dx + \int_0^{\infty} f(x)dx$

$= \int_{-\infty}^{0} 0\, dx + \int_0^{\infty} k e^{-3x} dx = 1 \Rightarrow k = 3$

Q: Pdf of a random variable x

$f(x) = x \;;\; 0 \leq x < 1$
$= 2 - x \;;\; 1 \leq x < 2$
$0 \;;\; x \geq 2$

Compute CDF of x (F(x)) & hence calculate

i) $P(-1 \leq X \leq 3)$
ii) $P(1 \leq X \leq 1.5)$

$F(x) = P(X \leq x) = 0 \;,\; x < 0$

$F(x) = \int_0^x t\,dt = \frac{x^2}{2}$ $x \geq 0$ & $x \leq 1$

$\frac{x^2}{2}\Big|_0^1 + \int_1^x 2 - t\,dt$ $1 \leq x < 2$

$= \frac{1}{2} + \left| 2t - \frac{t^2}{2} \right|_1^x$

$= \frac{1}{2} + 2x - \frac{x^2}{2} - \left(2 - \frac{1}{2}\right)$

$F(x) = 2x - 1 - \frac{x^2}{2}$

$F(x) = 4 - 1 - 2$ $x \geq 2$
$= 1$

i) 1

ii) $F(1.5) - F(1)$

$2 \times \frac{3}{2} - 1 - \frac{9}{8} - \left(2 - 1 - \frac{1}{2}\right) = 2 - \frac{9}{8} - \frac{4}{8} = 2 - \frac{13}{8} = \frac{3}{8}$

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Expectation

$\mu = E(x) = \sum_{\forall x} x p(x)$, if x is discrete.

$= \int_{-\infty}^{\infty} x\, p(x) dx$, if x is continuous.

let $g(x)$ then

$E[g(x)] = \sum_{\forall x} g(x) p(x)$

$= \int_{-\infty}^{\infty} g(x) \cdot f(x) dx$

Variance

$\sigma^2 = E[(x-\mu)^2] = \sum_{\forall x} (x-\mu)^2 p x$

$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$

$SD \equiv \sigma = + \sqrt{E(x-\mu)^2}$

Computational formulae for variance

$\sigma^2 = \sum x^2 p(x) - [\sum x p(x)]^2$

$= E(x^2) - [E(x)]^2$

$\sigma^2 = E[(x-\mu)^2] = \sum_{\forall x} (x-\mu)^2 p x$

$= \sum [x^2 - 2\mu x + \mu^2) p x$

$= \sum [x^2 p(x) - 2\mu x p(x) + \mu^2 p(x)]$

$= \sum_{\forall x} x^2 p(x) - 2\mu \sum_{\forall x} x p(x) + \mu^2 \sum p(x)$

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$= \sum x^2 p(x) - 2\mu \cdot \mu + \mu^2 \times 1$

$= \sum x^2 p(x) - \mu^2$
$= (E[X^2]) - (E[X])^2$

Properties of Expectatⁿ & variance
i) $E(C) = C$
ii) $E(cx) = c\,E(x)$
iii) $E(c+x) = c + E(x)$
iv) $var(C) = 0$
v) $var(cx) = c^2 Var(x)$

Q:- If $E(x) = 5$
$var(X) = 2$
find $E(X-5) = -5 + E(X)$
$= -5 + 5 = 0$
$var(-2x) = (-2)^2 var(x)$
$= 8$

SD → unit
variance → has no unit

The r^{th} moment about origin
$\mu_r' = E[X^r] = \sum_{\forall x} x^r p(x)$ if x is discrete
μ_r'
$= \int_{-\infty}^{\infty} x^r f(x)\,dx$ —"— cont

$r=1$
$\mu_1 = E[x]$ → first moment about origin.
$= \mu$

r^{th} moment about mean
$\mu_r = E[(X-\mu)^r] = \sum_{\forall x} (x-\mu)^r p(x)$
$= \sum_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

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if $r = 2$

$$\mu_2 = E[(x-\mu)^2] = \sigma^2$$

$$\boxed{\sigma^2 = \mu_2' - \mu^2}$$ prove it.

Moment generating f^n : MGF

$$M_x(t) = E[e^{tx}]$$

$$= \sum_{\forall x} e^{tx} p(x)$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$$

$$\mu_r' = \frac{d^r}{dt^r}\left[M_x(t)\right]_{t=0}$$

$$\mu_1' = \mu = \frac{d}{dt}\left[M_x(t)\right]_{t=0}$$

Mean deviat^n abt mean.

$$MD = \sum_{\forall x} |x-\mu|\, p_x$$

$$= \int_{-\infty}^{\infty} |x-\mu|\, f(x)\,dx$$

Characteristic function

$$\phi_x(t) = E[e^{itx}] = \sum_{\forall x} e^{itx} p(x)$$

$$= \int_{-\infty}^{\infty} e^{itx} f(x)\,dx$$

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Bernoulli's dist

$P(X=1) = p$ — success

$P(X=0) = q$ → failure

$p(x) = p^x q^{1-x}$, $x = 0,1$

$\quad\quad 0$, else where

$E(X) = p$

$V(X) = pq$

$M_x(t) = q + pe^t$

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Binomial dist

n = no. of independent bernoulli's trials

p = prob. of success in each trial const

q = —"— failure —"—

X = # of successes

$P(X=x) = {}^nC_x \, p^x q^{n-x}$ for $x = 0,1,2,\dots,n$

$E(X) = np$

$var(X) = npq$

$$E(x) = \sum_{\forall x} x\,p(x)$$

$$= \sum_{x=0}^{n} x \, {}^nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{\infty} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

$$= np(q+p)^{n-1} = np$$

$$Var(X) = \sigma^2 = \mu_2' - \mu^2$$

$$\mu_2' = E[X^2] = \sum_{\forall x} x^2 p(x)$$

$$= \sum_{x=0}^{\infty} x^2\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{\infty} (x(x-1)+x)\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{\infty} x(x-1)\, {}^nC_x\, p^x q^{n-x} + \sum x\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{x(x-1)\, n\, C_{x-1} \ldots !}{x(x-1)} + p^x q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!}\, p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 (q+p)^{n-2} + np = n^2p^2 - p^2 n + np$$

$$\sigma^2 = \mu_2' - \mu^2 = n^2p^2 - np^2 + np - (np)^2$$

$$= np(1-p)$$

$$= npq$$

Moment generating f^n (MGF)

$$M_x(t) = E[e^{tx}] = \sum_{\forall x} e^{tx} p(x)$$

$$= \sum_{x=0}^{n} e^{tx}\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x (pe^t)^x q^{n-x}$$

$$= (q + pe^t)^n$$

$$\mu = E(x) = \frac{d}{dt}(M_x(t))\Big]_{t=0} = \frac{d}{dt}(q + pe^t)^n\Big]_{t=0} = np$$

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$$Skewness = \frac{1-2p}{\sqrt{npq}}$$

Geometric distribution

p – prob of success

x – no. of Bernoulli independent trial required to get first success

$$p(X=x) = q^{x-1}p^{1} \quad , x = 1, 2, \ldots$$

$$E(x) = \frac{1}{p} \quad , Var(x) = \cancel{\frac{q}{p^2}} \; \frac{q}{p^2}$$

Q:- 6 Dices are thrown 729 times, how many times you expect atleast 3 dice to show a 5 or 6

~~X = no to show 5 or 6 in~~

X = # of dices that show 5 or 6

$$p = \text{prob of success} = P(5) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} = \frac{2}{3}$$

$M \rightarrow$ freq. of experiments

$$P(X \geq 3) = {}^{n}C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{n-3} + \cancel{{}^{n}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^{n-4} + {}^{n}C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^{n-5}}$$

$\frac{6 \times 5 \times 4}{6}$

$$= {}^{6}C_3 \times \left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + {}^{6}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + {}^{6}C_5\left(\frac{1}{3}\right)^5\frac{2}{3}$$

$$+ {}^{6}C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^0$$

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Sol^n ~~$P(X\geq 3)$~~

$n_0 = N \times P(X\geq 3)$

$= 233$ $\quad N = 729$

Mean & var of a binomial dist are 4 & 3 respectively, find the prob of getting exactly 6 successes in this distribution

$E(x) = \mu = np = 4$

$Var(x) = \sigma^2 = npq = 3$

$q = \frac{3}{4}$, $p = 1 - \frac{3}{4} = \frac{1}{4}$

$n = 16$

Prob of 6 successes $P(X=6) = {}^{16}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{10}$

If np is constant — Poisson's distribution

suppose X = random discrete variable

$p(\lambda) = P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots$

$\lambda > 0$

parameter of the distribution

$E(x) = V(x) = \lambda$

$\Rightarrow E(X) = \lambda$

$Var(X) = \lambda$

$M_x(t) = e^{\lambda(e^t - 1)}$

$E(x) = \sum_{x=0}^{\infty} x\, p(x)$

$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$

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$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \left[1 + \lambda + \frac{\lambda^2}{2!} + \cdots \right]$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda$$

$$Var(x) = \sigma^2 = \mu_2' - \mu^2$$

$$\mu_2' = \sum_{x=0}^{\infty} x^2 p x$$

$$= \sum_{x=0}^{\infty} (x(x-1)+x) \frac{e^{-\lambda}\lambda^x}{x!} + \sum x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$e^{-\lambda}\lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= \cancel{\lambda^2+\lambda\lambda^2} \quad \lambda^2 + \lambda$$

$$var = \lambda^2 + \lambda - (\lambda)^2$$

$$= \lambda$$

$$M_x(t) = E[e^{tx}] = \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^t}$$

$$= \cancel{e^{\lambda}(e^t-1)} \quad e^{\lambda(e^t-1)}$$

6 coins are tossed 6400 times what is the approximate prob. of getting 6 heads as x times

Poisson dist as an app. of binomial dist

$$\lim_{\substack{n\to\infty \\ p\to 0 \\ np\to\lambda}} {}^{n}C_x p^x q^{n-x} = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, \ldots$$

$P = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

$n = 6400$

$np = \lambda = 6400 \times \frac{1}{64} = 100$

$P(X=x) = \frac{e^{-100}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots\ldots 6400$

Show that

i) $\sum_{\forall x} p(x) = 1$

ii) $p(x+1) = \frac{\lambda}{x+1} p(x), \quad x = 0, 1, 2, \ldots\ldots$

i) $\sum_{\forall x} p(x) = \sum_{\forall x} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda} \sum_{\forall x} \frac{\lambda^x}{x!}$

$= e^{-\lambda} e^{\lambda} = 1$

Sol^n $\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}[1 + \lambda + \lambda^2 + \ldots\ldots + \infty]$

$= e^{-\lambda} \cdot e^{\lambda} = 1$

ii) $p(x+1) = \frac{\lambda}{x+1} P(x), \quad x = 0, 1, 2, \ldots\ldots$

$P(X+1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!} = \frac{\lambda}{x+1} \frac{e^{-\lambda}\lambda^x}{x!}$

$= \frac{\lambda}{x+1} P(x)$

Hypergeometric distribut^n

$N \rightarrow$ objs

$r \rightarrow$ of 1 kind

compute the prob of n out of N when drawn at random w/out replacement out of ~~w~~

${}^{r}C_x \; {}^{N-r}C_{n-x} / {}^{N}C_n$

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Continuous distribut^n

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X denote temp °C & Y denote time in min that takes diesel engine

$$f_{XY}(x,y) = C(4x+2y+1)$$

$$0 \le x \le 40 \quad 0 \le y \le 2$$

a) find value of c that makes this a density.

$\Rightarrow$ ~~$\iint f_{XY} = 1$~~ To find c

$$\int_0^\infty \int_0^\infty f_{XY}(x,y)\,dx\,dy = 1$$

$$\int_0^{40} \int_{y=0}^{2} c(4x+2y+1)\,dx$$

$$c \int_0^{40} \left[4xy + y^2 + y\right]_0^2 dx$$

$$c \int_0^{40} 8x+4+2\,dx$$

$$c\left|4x^2+6x\right|_0^{40} = 1$$

~~$4\times 40\times 40 +$~~

$$4(40)^2 + 6(40) = \frac{1}{c}$$

$$\frac{1}{6640} = c$$

b) find prob that on a randomly selected day the air temp will exceed 20°C & it will take at least 1 min for the car to be ready.

c) find marginal densities for x & Y

$$f_x = \frac{(8x+6)}{6640} \qquad f_y = \frac{1}{6640}\int_0^{40} 4x+2y+1 = \frac{80y+3240}{6640}$$

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b) $P(X \geq 20, Y \geq 1)$

$$= \int_{20}^{40} \int_{1}^{2} \frac{1}{6640} (4x + 2y + 1)\, dx\, dy$$

$$= \frac{2480}{6640}$$

d) find the prob that on a randomly selected day it will take at atleast 1 min for car to be ready to start

$$P(Y \geq 1) = \int_{1}^{2} \frac{80y + 3240}{6640}\, dy$$

e) $P(X \geq 20) = \int_{20}^{40} \frac{8x + 6}{6640}\, dx$

f) Check Independence.

$f_x \times f_y \; != f_{xy} \Rightarrow$ not independent

Q5:- non defective $= .9$

defective salvageable $= .08$

defective & nonsalvageable $= .02$

20 items are randomly selected & classified

let X_1 = nondefective items

X_2 — defective but salvageable

X_3 defective & nonsalvagiable

a) find $P[X_1 = 15, X_2 = 3, X_3 = 2]$

b) find the general formula for density for (X_1, X_2, X_3).

extention of binomial dist $\Longrightarrow$ multinomial dist

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$P(X_1 = x_1, X_2 = x_2, X_3 = x_3) = f(x_1, x_2, x_3)$

$= \frac{n!}{x_1! x_2! x_3!} p_1^{x_1} p_2^{x_2} p_3^{x_3}$

where $\sum x = n$

a) $\frac{20!}{3! 2! 15!} (.9)^{15} (.08)^3 (.02)^2$

$= .0065$

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LT-18.

Expectatn & Covariance.

Def:- Let (X,Y) be a 2D r.v e- joint density f_{XY}. Let $H(X,Y)$ be a r.v. The expected value of $H(X,Y)$, denoted by $E[H(X,Y)]$ is given by

$$E[H(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(x,y) f_{XY}\, dy\, dx$$

provided

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |H(x,y)| f_{XY}(x,y)\, dy\, dx \text{ exists}$$

for (X,Y) continuous

Q:- Let X & Y are joint density f^n

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2) & \text{if } 0<x<1, 0<y<1 \\ 0 & \text{o.w} \end{cases}$$

find $E(X^2+Y^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x^2+y^2) f(x,y)\, dy\, dx$

$\Rightarrow \int_0^1\int_0^1 (x^2+y^2)\frac{3}{2}(x^2+y^2)$

$\left(\frac{2}{5}+\frac{2}{9}\right)\frac{3}{2}$ $\quad \frac{3}{2}\int_0^1\int_0^1 x^4+y^4+2x^2y^2 = \frac{3}{2}\left(\frac{x^5}{5}+y^4x+\frac{2x^3}{3}y^2\right)\Big|_0^1$

$\frac{3}{2}\int_0^1 \frac{1}{5}+y^4+\frac{2}{3}y^2\, dy$

$\boxed{\frac{14}{15}}$ $\Rightarrow \frac{1}{5}y+\frac{y^5}{5}+\frac{2}{9}y^3\Big|_0^1 = \left(\frac{1}{5}+\frac{1}{5}+\frac{2}{9}\right)\frac{3}{2}$

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Univariate Average.

$$E[X] = \sum_{\forall x \forall y} x f_{XY}(x,y)$$

Covariance

$$Cov(X,Y) = E[(X-\mu_x)(Y-\mu_y)]$$

$E(X) = \mu_x$

$E(Y) = \mu_y$

small values of X associated with small values of Y &

large values of X associated with large values of Y $\Rightarrow$ Cov +ve

else -ve

for X, Y be independent

Cov = 0

$$Cov.(X,Y) = E[XY] - E[X]E[Y]$$

proof $E[(X - E[X])(Y - E[Y])]$

$$= E[XY - XE[Y] - YE[X] + E[X]E[Y]]$$

$$= E[XY] - E[X]E[Y] - E[X]E[Y] + E[X]E[Y]$$

$$= E[XY] - E[X]E[Y]$$

If X & Y are independent then

$$E[XY] = E[X]E[Y]$$

proof

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_{XY}(x,y)\, dx\, dy$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_X f_Y \, dx\, dy$$

$$\int_{-\infty}^{\infty} x f_X \int_{-\infty}^{\infty} y f_Y \, dy\, dx$$

$$= E[X]E[Y]$$

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Ex:- Let X be a r.v. e⁻
$p(-1) = p(0) = p(1) = 1/3$
Letting $Y = X^2$ we have
$E[X] = \frac{(-1+0+1)}{3} = 0$

$E[Y] = E[X^2] = ((-1)^2 + 0^2 + 1^2)/3 = \frac{2}{3}$

$E[XY] = E[X^3] = ((-1)^3 + 0^3 + 1^3)/3 = 0$
Thus $E[XY] = E[X]E[Y]$ while clearly X & Y are independent but covar $= 0$

⇒ X, Y indep ⇒ cov $= 0$
$\not\Leftarrow$ converse not always true.

2) ~~If X=Y then~~ $\text{Cov}(X,Y) = \text{Var}(X) = \text{Var}\,Y$
$= E[XY] - E[X]E[Y]$
$= E[X^2] - E[X]^2$
$= \text{Var}(X)$

next class → $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2\,\text{Cov}(X,Y)$

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Last class that was extra missed

Eng. Mathematics C.B. Gupta

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Discrete Joint densities

Let X & Y be discrete r.v. The ordered pair (X, Y) is called a 2D discrete r.v. A fn f_{XY} such that

$f_{XY}(x,y) = P[X=x \ \& \ Y=y]$ is called the joint Density

$f(x,y) \geq 0$

$\sum_{\forall y} \sum_{\forall x} f(x,y) = 1$

} Necessary & sufficient condition for a fn to be ~~denn~~ discrete Joint densities.

Marginal Distributions:

Discrete

$\sum_{\forall y} f(x,y) = f_X(x)$

$\sum_{\forall x} f(x,y) = f_Y(y)$

Continuous Joint Density.

1. $f_{XY}(x,y) \geq 0$; $-\infty < x < \infty$
   $-\infty < y < \infty$

2. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dy\,dx = 1$

3. $P[a \leq X \leq b \text{ and } c \leq Y \leq d] = \int_a^b \int_c^d f_{XY}(x,y)\,dy\,dx$

for a, b, c, d real, it is called joint density for (X, X)

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Continious Marginal densities.

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$$

<u>Independence</u>:

$$P(A_1 \cap A_2) = P(A_1) \times P(A_2)$$

$$P(X=x \cap Y=y) = P(X=x) \times P(Y=y)$$

$$= f_X(x) \cdot f_Y(y)$$

$$f(x_1, x_2, \ldots, x_n) = f(x_1) \cdot f(x_2) \ldots \ldots f(x_n)$$

→ then $x_1, x_2, x_3, \ldots, x_n$ are Independent random variable.

Ex:- let X denote the # of defective welds Y the # of improperly tightened bots produced per car. & Since X & Y are discrete, (X, Y) is a two dimensional discrete r.v. past data indicate

X/Y	0	1	2	3	$f_X(x)$
0	.840	.030	.02	.01	.9
1	.060	.010	.008	.002	.08
2	.010	.005	.004	.001	.02
$f_Y(y)$	.91	.048	.032	.013	1

Sol^n

X = # of defective welds

Y = # of improbuly produced bots per car.

Q:- Use tabele to find

a) The prob that exactly 2 defective welds & one improperly tighend bot → $P(X=2 \cap Y=1) = .005$

b) P atleast one defective weld &
$P(x \geq 1 \cap Y \geq 1) = .03$

c) P atmost one defective weld
$P(x \leq 1) = P(X=0) + P(X=1)$
$= .98 = .9 + .08$

Last cle ex

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9) $P(Y \geq 2) = P(Y=2) + P(Y=3)$

$.032 + .013 = .045$

8) Joint density

$$f_{XY}(x,y) = \frac{1}{n^2}$$

$x = 1, 2, ----, n \; ; \; y = 1, 2, 3, ---n;$

~~9) $\sum_{y=1}^{n} \sum_{x=1}^{n} \frac{1}{n^2}$~~

a) verify that $f_{XY}(x,y)$ satisfies the condit^n necessary to be a density

b) find marginal densities for X & Y

c) Are X & Y independent

Sol^n

a) $f_{XY} \geq 0$

$$\sum_{y=1}^{n} \sum_{x=1}^{n} f_{XY}(x,y)$$

$$= \frac{1}{n^2} \sum_{y=1}^{n} \sum_{x=1}^{n} 1$$

$$= \frac{n^2}{n^2} = 1$$

b) Marginal Densities

$\sum_{x=1}^{n} f_{XY} = f_Y(y) = \frac{1}{n} \quad n = 1, 2, ---- \; || \; 0$ elsewhere

$\sum_{y=1}^{n} f_{XY} = f_X(x) = \frac{1}{n} \quad n = 1, 2, ---- \; || \; 0$ elsewhere

c) $f_X \times f_Y = f_{XY} \Rightarrow$ Independent