Propositional logic :-

Proposition :-

$x+1=2$

p, q

p and q are two preposition.

1) p is a preposition $\sim p$ (negation of p)

If p is true than $\sim p$ is false

p is False than $\sim p$ is True.

2) conjuction ($\wedge$), p and q are two propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3) Disjunction ($\vee$)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p, q are two proposition

Exclusive OR.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

conditional statement $(p \rightarrow q)$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
$\rightarrow$ F	T	T
F	F	T

Biconditional statement $(p \leftrightarrow q)$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
$\rightarrow$ F	T	F
F	F	T

eg:- $(p \vee \sim q) \rightarrow (p \wedge q)$.

$(p \rightarrow q)$

p	q	$p \vee \sim q$	$p \wedge q$	
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

$Eg - 60, 70.$
Ex -
$P \ \bar{P} \ \neg P$

operator precedence

$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

$(p \vee \sim q) \rightarrow (p \wedge q)$.

$(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$.

p	q	$\neg p$	$(p \leftrightarrow q)$	
T	T	F	T	T
T	F	F	F	F
F	T	T	F	
F	F	T	T	

Propositional Equivalence :-

P	$\neg P$	$P \vee \neg P$	$P \vee \neg P$
T	F	T	F
F	T	T	F

If all are true
A compound proposition this is always true occur in it is called tautology

A compound proposition this is always false is called contradiction.

either true or false $\rightarrow$ contingency.

Camlin Page
Date / /

Logical equivalence:-

The compound proposition p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology $p \equiv q$.

eg:- $\neg(p \vee q)$ and $\neg p \wedge \neg q$

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$(\neg p \vee q) \leftrightarrow (\neg p \wedge \neg q)$
T	T	F	F	T
T	F	F	F	T
F	T	F	F	T
F	F	T	T	T

Yes the two are equivalent.

Demorgan's law:-

$\sim(p_1 \vee p_2 \vee \cdots \vee p_n) \equiv (\sim p_1 \wedge \sim p_2 \wedge \sim p_3 \cdots \wedge \sim p_n)$

$\sim(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \equiv (\sim p_1 \vee \sim p_2 \cdots \vee \sim p_n)$.

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$.

$\equiv \neg p \wedge \neg(\neg p \wedge q)$

$\equiv \neg p \wedge (p \vee \neg q)$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$\equiv F \vee (\neg p \wedge \neg q)$

$\equiv (\neg p \wedge \neg q) \vee F \equiv \neg p \wedge \neg q$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Subject → (domain)

Camlin Page
Date / /

x is greater than 3

x - subject

is greater than 3 (is predicate)

→ subject

$P(x)$ → propositional function

↳ Predicate

$x = 5$

$P(5)$ is True

$P(1)$ is false

Quantifiers :-

Universal Quantifiers :-

$\forall x$ $P(x)$ is not True.

$P(x)$

$\exists x$ $P(x)$ is true

$\exists x$ $P(x)$ is false $\underline{x = 1, 2}$

There exist

eg:- $x = x + 1$

x $\underbrace{\text{is equal to } x+1}_{\text{Predicate}}$

$P(x)$: $x = x + 1$

↳ subject

$P(x)$ is false

$\forall x$ $P(x)$ is false. $x \in R$.

$x = y + 1$

$P(x, y)$ is true

$x = 3 \quad x = 2$

$P(3,2)$ is true

$P(x_1, x_2 \cdots x_n)$

$\exists x \quad P(x)$ is true is same as

$P(x_1) \vee P(x_2) \vee \cdots P(x_n)$ is true.

Eg:- Truth value of $\exists x \; P(x)$

$P(x) : x^2 > 10.$

$x = \{1, 2, 3, 4\}$

$\exists x \quad P(x) \equiv P(1) \vee P(2) \vee P(3) \vee P(4)$

$P(4) \quad 4^2 > 10 \quad \exists x \; P(x)$ is true.

$\left.\begin{array}{l} \underbrace{\forall x \, P(x)}_{\mathrm{I}} \vee \underbrace{Q(x)}_{\mathrm{II}} \\ \underbrace{\forall x}_{\mathrm{II}} \; \underbrace{(P(x) \vee Q(x))}_{\mathrm{I}} \end{array}\right\}$ (precedence)

$\exists x \; (P(x) \vee Q(x))$.

$\neg \forall x P(x) \equiv \exists x \; \neg P(x)$.

$P(x)$ is true

$\neg P(x)$ is false.

$\neg \forall \; x P(x) \qquad \forall x \; P(x)$ is true

$\neg \forall x \; P(x)$

$\exists x \; \neg P(x)$

Ex - Pg - 43.

Camlin Page

Date / /

$\neg \exists x \; Q(x) \equiv \forall x \; \neg Q(x)$

$\neg \forall x \; (x^2 > x)$

$\exists x \; \neg(x^2 > x)$

$\underline{\forall x \; P(x) \vee Q(x)} \Rightarrow (\forall x P(x)) \vee Q(x)$

$\forall x \; (P(x) \vee Q(x))$

$\exists x P(x) \vee Q(x) \Rightarrow (\exists x P(x)) \vee Q(x)$

$\exists x (P(x) \vee Q(x))$

$\neg \forall x (P(x) \rightarrow Q(x))$ and $\exists x \; (P(x) \wedge \neg Q(x))$

$\neg (P(x) \rightarrow Q(x))$

$\neg (\neg P(x) \vee Q(x))$

$P(x) \wedge \neg Q(x)$

$\neg (x \rightarrow y)$

$\equiv \underline{x \wedge \neg y}$

$\neg \forall x (P(x) \rightarrow Q(x)) \equiv \exists x \neg (P(x) \rightarrow Q(x))$

$\equiv \exists x \; P(x) \wedge \neg Q(x)$

Ex- Pg-54

Camlin Page
Date / /

Nested Quantifiers:-

$\forall x \exists y \ (x+y=0)$

$\forall x \exists y \ P(x) \qquad P(x) \ x+y=0$

$\forall x \forall y \ (x+y>0)$

eg. $\forall x(C(x) \vee \exists y (C(y) \wedge F(x,y)))$

Translate into english
where F(a, b) means a and b are friends and the domain for x, y and z consists of all students in your school.

[or all / or each]

Ans Translation For every student x in your school x has computer or there is a student y such that y has computer and x and y are friends

Mathematical Induction :-

$$1+2+\cdots\cdots n = \frac{n(n+1)}{2}$$

$$P(n) = \frac{n(n+1)}{2}$$

$$P(1) = \frac{1\times(1+1)}{2} = 1$$

$n=k$ is true

$P(k) = 1+2+3\cdots\cdots k = \frac{k(k+1)}{2}$ P(k) is true

$$P(k+1) = 1+2+\cdots\cdots k + k+1 = \frac{k(k+1)}{2} + k+1$$

$$= \frac{(k+1)(k+2)}{2}$$

Scanned by CamScanner

Boolean Algebra:-

$\{0, 1\}$

OR $1+1=1$

$0+1=1$

$1+0=1$

$0+0=0$

AND

$1 \cdot 1=1$

$1 \cdot 0=0$

$0 \cdot 1=0$

$0 \cdot 0=0$

$\overline{0}=1$

$\overline{1}=0$

Q) Find the value of

Boolean exp $1 \cdot 0+(\overline{0+1})$

$0+0=0$

$T=1$

$F=0$

Logical exp. $(T \wedge F) \vee \neg(T \vee F)=F$

AND $\cdot$ $\wedge$ (conjuction)

OR $+$ $\vee$ (disjunction)

(negation) $\neg$ $-$

Boolean Expression	Logical Expressions
0	F
1	T
+	$\vee$ (OR)
.	$\wedge$ (AND)
$-$	$\neg$ (NEGATION)

$B = \{0, 1\}$

$B^n = \{x_1, x_2 \ldots x_n\}$

$x_n \in B$.

$F : B^m \longrightarrow B$

x	y	$F(x,y)$
1	1	0
1	0	1
0	1	0
0	0	0

$F(x, y, z) = xy + z$

x	y	z	xy	z	$xy + z$	$\bar{z}$	$xy + \bar{z}$
1	1	1	1	1	1	0	
1	1	0	1	0	1	1	
1	0	1	0	1	1	0	
1	0	0	0	0	0	1	
0	1	1	0	1	1	0	
0	1	0	0	0	0	1	
0	0	1	0	1	1	0	
0	0	0	0	0	0	1	

Boolean Identities :-

Identity

$x = x$

$x + x = x$

$x \cdot x = x$

$x + 0 = x$

$x \cdot 1 = x$

$x + 1 = 1$

$x \cdot 0 = 0$

$x + y = y + x$

$x \cdot y = y \cdot x$

$x + (y + z) = (x + y) + z$

$x \cdot (y \cdot z) = (xy)\, z$

$x + (y z) = (x + y)(x + z)$

$x \cdot (y + z) = xy + xz$

$$\boxed{\begin{aligned} \overline{xy} &= \bar{x} + \bar{y} \\ (\overline{x + y}) &= \bar{x} \cdot \bar{y} \end{aligned}}$$ Demorgan's law

$x \cdot (y + z) = xy + xz$

x	y	z	$y + z$	$x(y+z)$	xy	xz	$xy + xz$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

same

$$\left\{\begin{array}{l} + \longrightarrow \cdot \\ \cdot \longrightarrow + \\ 1 \longrightarrow 0 \\ 0 \longrightarrow 1 \end{array}\right\}$$

eg - Ex 694
Ex - 698

Dual:-

$x\cdot(y+0)$ and $\bar{x}\cdot 1+(\bar{y}+z)$

Dual

$\rightarrow x+(y\cdot 1)$

$(\bar{x}+0)\cdot(\bar{y}\cdot z)$

$F(x,y,z) = (x+y)\cdot\bar{z}$

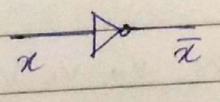
Sum of Product

$$\begin{aligned}(x+y)\bar{z} &= x\bar{z}+y\bar{z}\\ &= x\bar{z}\cdot 1+y\bar{z}\cdot 1\\ &= x\bar{z}(y+\bar{y})+y\bar{z}(x+\bar{x})\\ &= x\bar{z}y+x\bar{z}\bar{y}+y\bar{z}x+y\bar{z}\bar{x}\end{aligned}$$

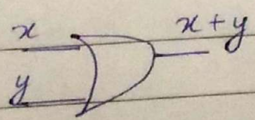
$x\cdot x\cdot y$

Logic Gates:-

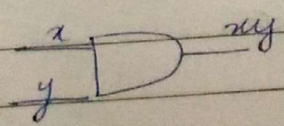
Inverter

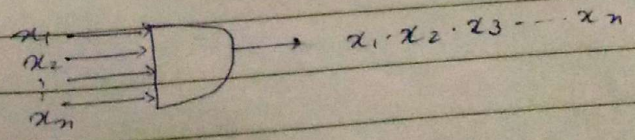


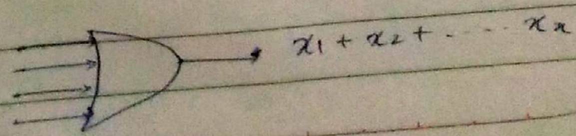
OR gate



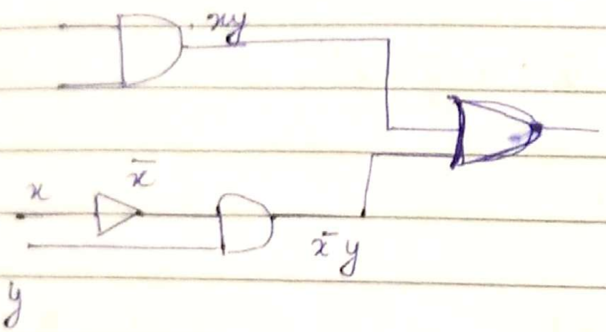
AND.







Eg-704



$xy + \bar{x}y = y$

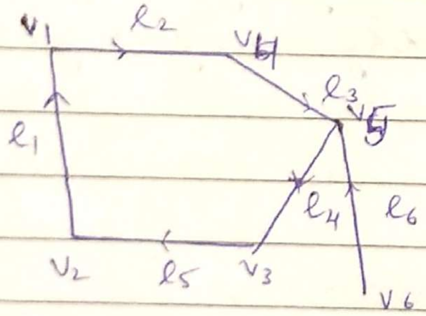
Graphs

$G = (V, E)$

$V = \{v_1, v_2, v_3 \ldots v_n\}$

$E = \{e_1, e_2 \ldots e_m\}$.

Directed graph



$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$E = \{e_1, e_2, e_3 \ldots e_6\}$.

- vertices are nodes
- edges have direction

$deg(v_1) = 2$

$deg(v_4) = 2$

$deg(v_5) = 3$

Camlin Page

Date / /

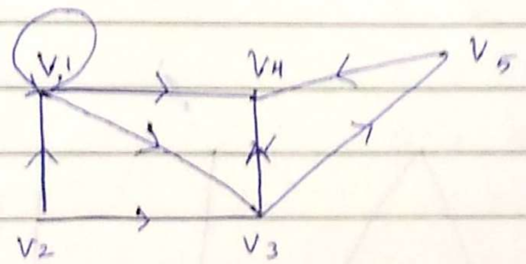
$G = (V, E)$

$$2e = \sum_{v \in V} deg(u)$$

eg: How many edges are there in a graph with 10 vertices each of degree six?

$$60 = 2e$$

$$e = 30$$



$deg^-(v_1) = 2$

(indegree) towards the vertex.

$deg^+(v_1) = 2$

out degree

Let $G = (V, E)$ be a graph with dirrected edges.

then $\sum_{u \in V} deg^-(u) = \sum_{u = v} deg^+(v) = |E|$

Total no. of edge

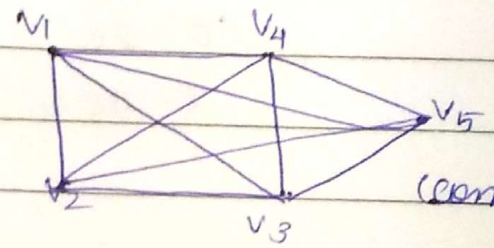
1

Camlin Page
Date / /

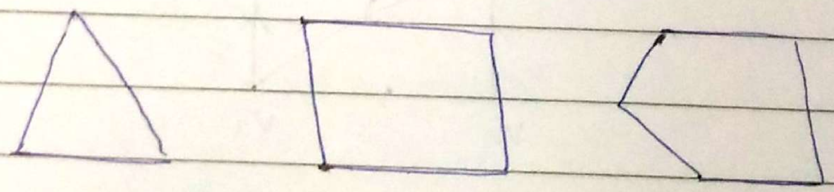
complete graph:-

n-vertices denoted by K_n is simple graph that contains exactly one edge between each pair of vertices.

$$K_n = \{v_1, v_2 \text{ ------ } v_n\}$$



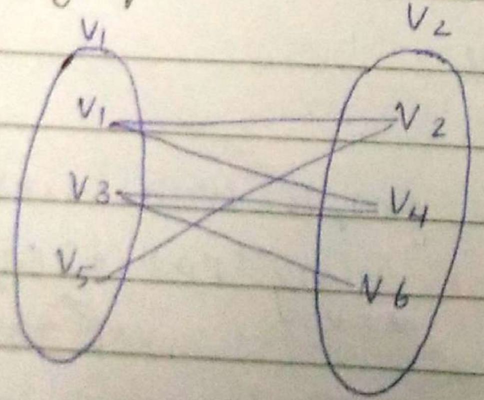
(complete graph)



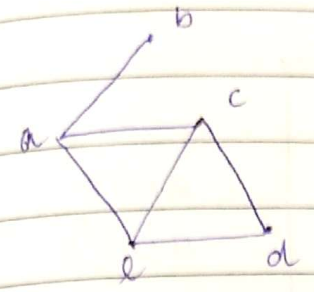
Cycle:-

No. of vertices = No. of edges.

Biparted graph:-



Representing Graphs and Graph Isomorphism:-



Vertices	Adjacency list
a	b, c, e
b	a
c	a, d, e
d	e, c
e	a, c, d .
~~f~~	

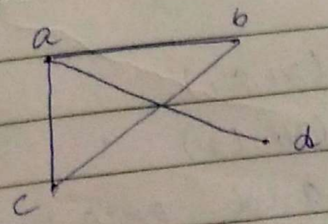
Adjanency Matrix

Let $G = (V, E)$

Adjanency matrix $A = [a_{ij}]$

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \text{ is an edge of } G) \\ 0. & \text{otherwise} \end{cases}$$

$$V = \left\{ \begin{matrix} v_1, & v_2 & \ldots & v_n \\ 1 & 2 & \ldots & n \end{matrix} \right\}$$



a , b , c , d

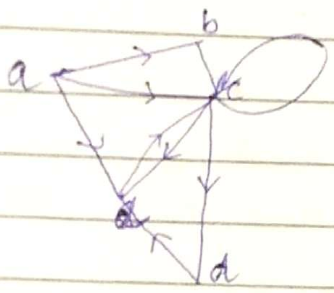
1 2 3 4

$$\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ \end{array}$$
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$a_{11} = 0$

$a_{12} = 1$

$a_{13}\ (a, c) = 1$



Vertex	terminal vertex (outward vertex).
a	b, c, e
b	c , c
c	d , e , c
d	e
e	c

Isomorphism of Graphs :-

iso + morphe
equal form

$G_1 = (V_1, E_1)$

$G_2 = (V_2, E_2)$

one-to-one and onto function f
$f: V_1 \to V_2$ with property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2 for all a, b in V_1.

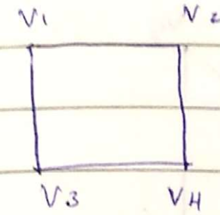
Pg-556

G_1 $\quad$ G_2

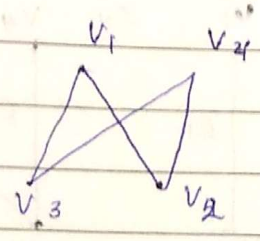
V_1 $\quad$ V_2

$a, b \in V_1$

$f(a), f(b) \in V_2$



G



H

$V_G = \{V_1, V_2, V_3, V_H\}$ $\qquad$ $V_H = \{V_1, V_2, V_3, V_4\}$.

G

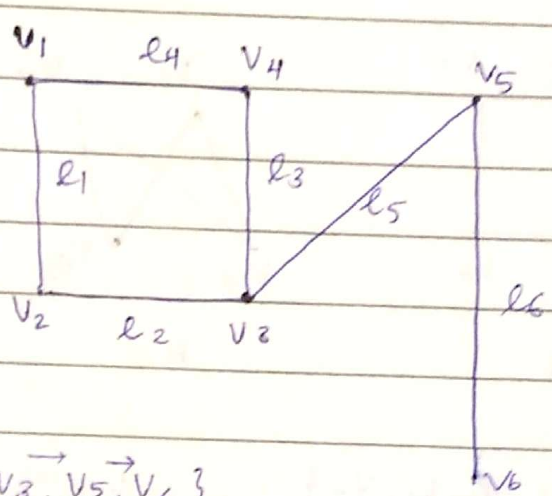
Vertex	Adjacency.
V_1	V_2, V_3
V_2	V_H, V_1
V_3	V_1, V_4
V_H	V_2, V_3

H

Vertex	Adgacency
V_1	V_2, V_3
V_2	V_H, V_1
V_3	V_1, V_H
V_H	V_3, V_2

Connectivity :-

Path :- A path is a sequence of edges that began a vertex of a graph and travel from vertex to vertex along edge of the graph.



path :- to

$\{ v_1 \overset{\rightarrow}{,} v_4 \overset{\rightarrow}{,} v_3 \overset{\rightarrow}{,} v_5 \overset{\rightarrow}{,} v_6 \}$