Physics II

Classical Electrodynamics

(Dr. Subhayan Biswas)

Quantum Mechanics and Semiconductor Physics

(Dr. Manish Singh)

- **Review of Mathematical Tools** 3-4 lectures
- **Electrostatics** 4 lectures
- **Special techniques** 2 lectures
- **Concepts of Dipole** 2 lectures
- **Electric Field in Materials** 2 lectures
- **Magnetostatics** 3 lectures
- **Magnetic Field in Materials** 1 lectures
- **Electrodynamics** 2 lectures
- **Maxwell's Equation** 1 lectures

Reference Books

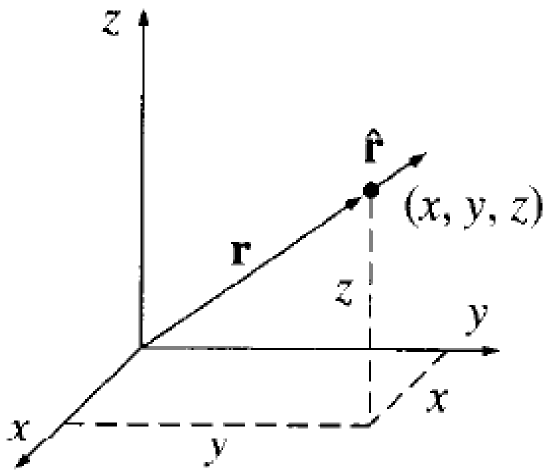
1. Introduction to Electrodynamics by David. J. Griffiths

2. Classical Electrodynamics by John David Jackson

3. Electricity and Magnetism by Edward M. Purcell

	%
1st Mid term exam	17
2nd Mid term exam.	18
Surprise quizzes and attendance and daily evaluation	15
Final exam	50

Position, Displacement, and Separation Vectors



$$\mathbf{r} \equiv x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$$

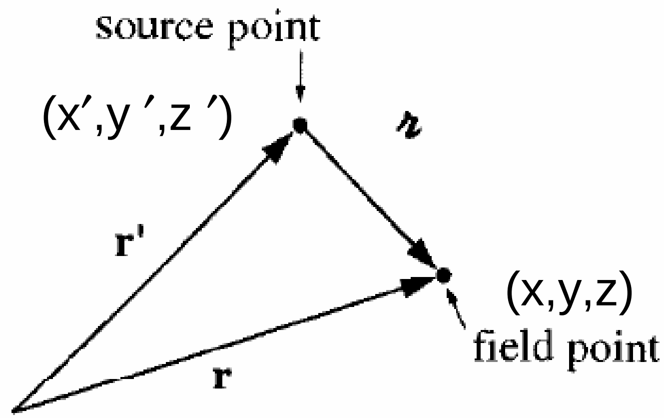
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{\sqrt{x^2 + y^2 + z^2}}$$

$$\boldsymbol{\mathscr{r}} \equiv \mathbf{r} - \mathbf{r}'$$

$$\mathscr{r} = |\mathbf{r} - \mathbf{r}'|$$

$$\hat{\boldsymbol{\mathscr{r}}} = \frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}} = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$



$$\boldsymbol{\mathscr{r}} = (x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}},$$

$$\mathscr{r} = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2},$$

$$\hat{\boldsymbol{\mathscr{r}}} = \frac{(x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}$$

infinitesimal displacement vector

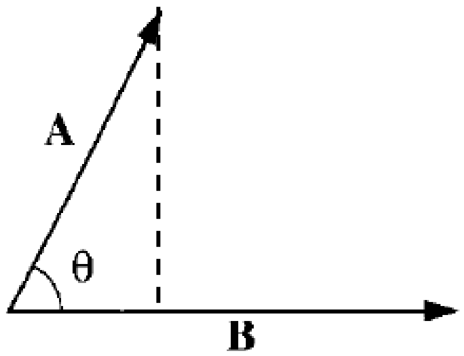
$$(x, y, z) \text{ to } (x + dx, y + dy, z + dz),$$

$$d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$

Dot product of two vectors

$$\mathbf{A} \cdot \mathbf{B} \equiv AB\cos\theta$$

$$\vec{A} \bullet \widehat{B} = \left|\vec{A}\right| \cos\theta$$

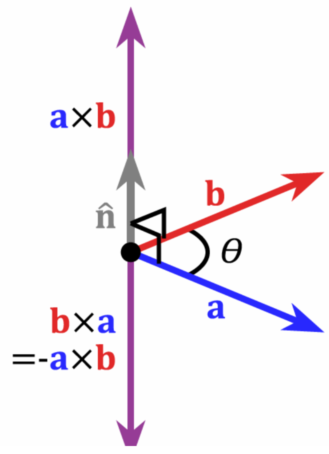


$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= (A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}) \cdot (B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}) \\ &= A_xB_x + A_yB_y + A_zB_z. \end{aligned}$$

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

$$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1; \quad \hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{x}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = 0$$

Cross product of two vectors



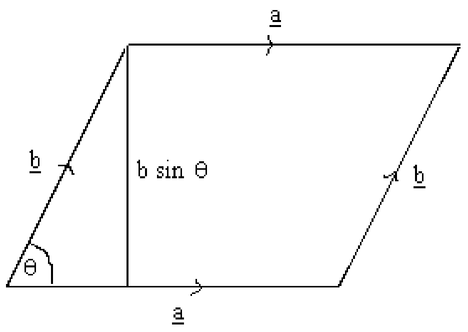
$$\mathbf{A} \times \mathbf{B} \equiv AB \sin\theta \, \hat{\mathbf{n}}$$

$$\mathbf{A} \times \mathbf{B} = (A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}) \times (B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}})$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - A_zB_y)\hat{\mathbf{x}} + (A_zB_x - A_xB_z)\hat{\mathbf{y}} + (A_xB_y - A_yB_x)\hat{\mathbf{z}}$$

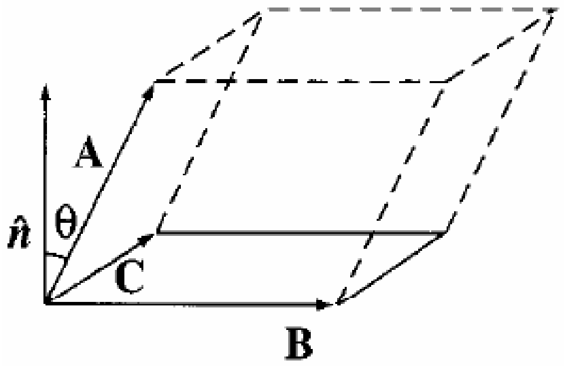
$$(\mathbf{B} \times \mathbf{A}) = -(\mathbf{A} \times \mathbf{B})$$



$$\begin{aligned} \hat{\mathbf{x}} \times \hat{\mathbf{x}} = \quad \hat{\mathbf{y}} \times \hat{\mathbf{y}} &= \hat{\mathbf{z}} \times \hat{\mathbf{z}} = 0, \\ \hat{\mathbf{x}} \times \hat{\mathbf{y}} = -\hat{\mathbf{y}} \times \hat{\mathbf{x}} &= \hat{\mathbf{z}}, \\ \hat{\mathbf{y}} \times \hat{\mathbf{z}} = -\hat{\mathbf{z}} \times \hat{\mathbf{y}} &= \hat{\mathbf{x}}, \\ \hat{\mathbf{z}} \times \hat{\mathbf{x}} = -\hat{\mathbf{x}} \times \hat{\mathbf{z}} &= \hat{\mathbf{y}}. \end{aligned}$$

Triple Products

Scalar triple product: $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$



$|\mathbf{B} \times \mathbf{C}|$ is the area of the base

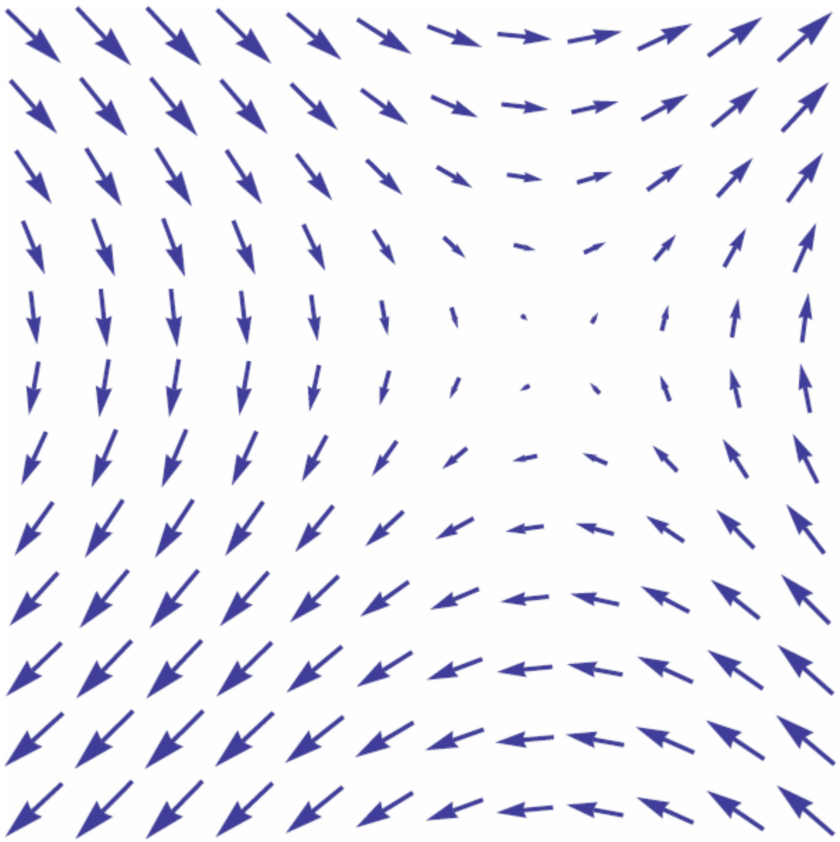
$|\mathbf{A} \cos\theta|$ is the altitude

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

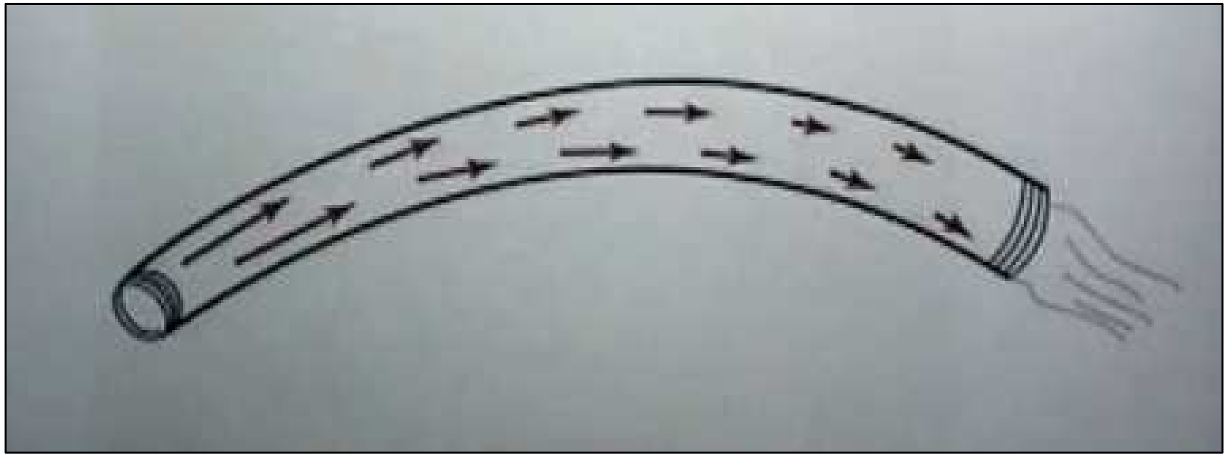
$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Vector Calculus

Vector Field / Vector Function

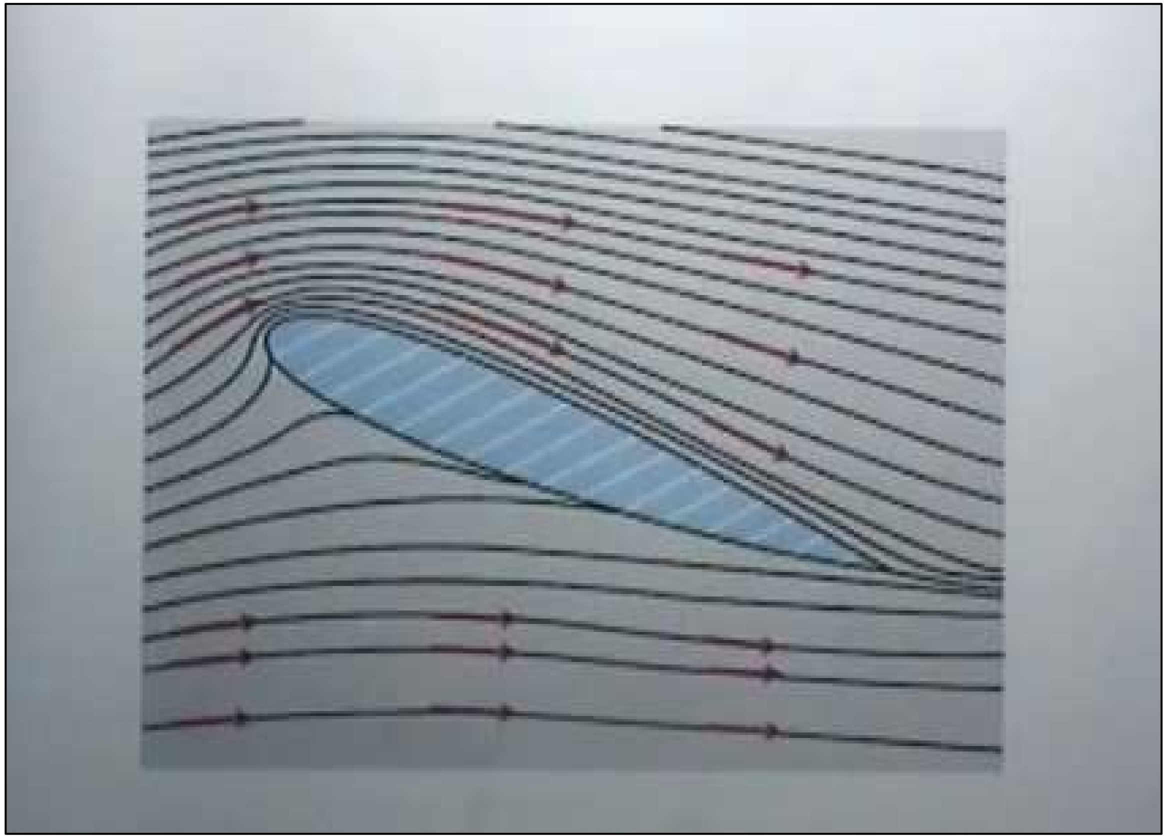


$$\vec{F} = \sin y\hat{i} + \sin x\hat{j}$$

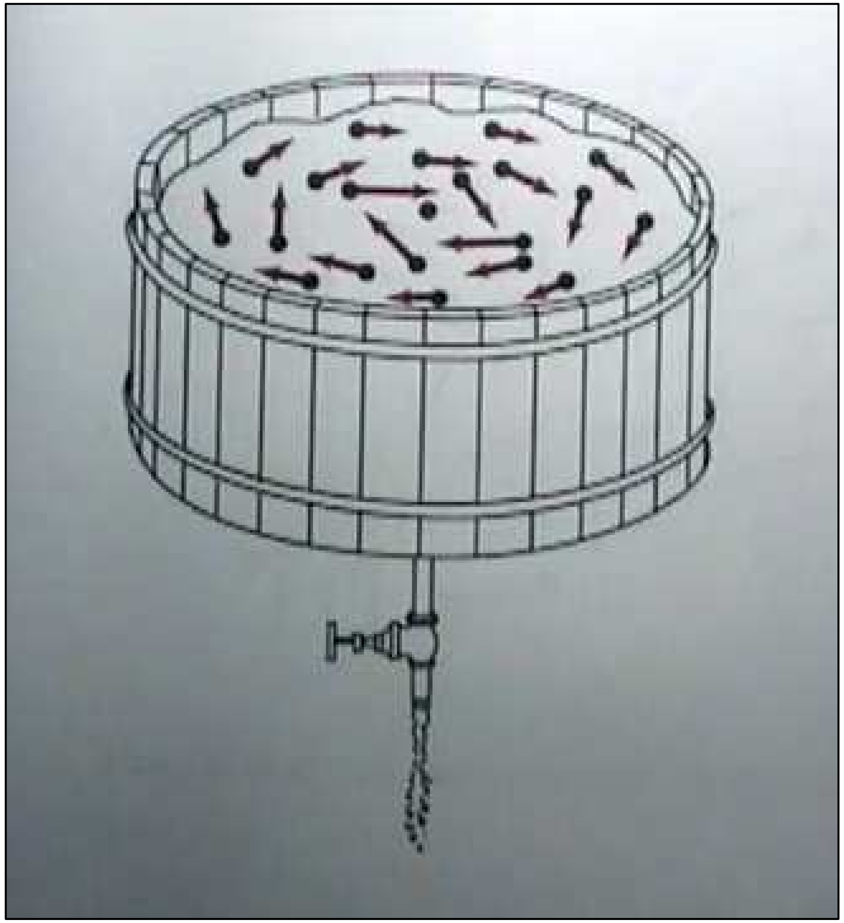


A vector field describing the velocity of a flow in a pipe

Note: All the figures of vector field have been taken from a lectures given by Dr. Chris Tisdell, UNSW

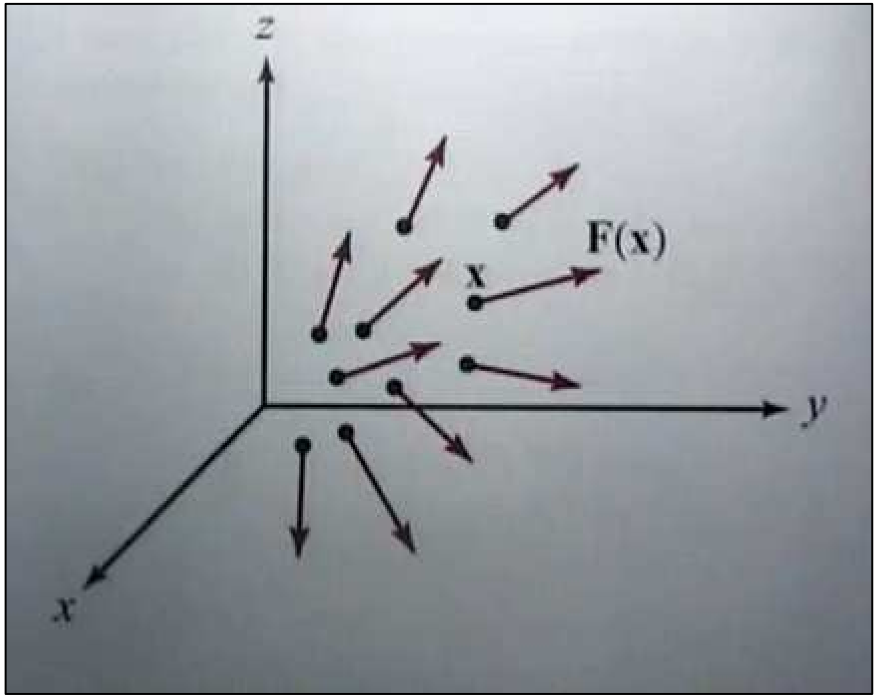


Velocity vector field of a flow around a aircraft wing



Circular flow in a tub

Vector Field or Vector function

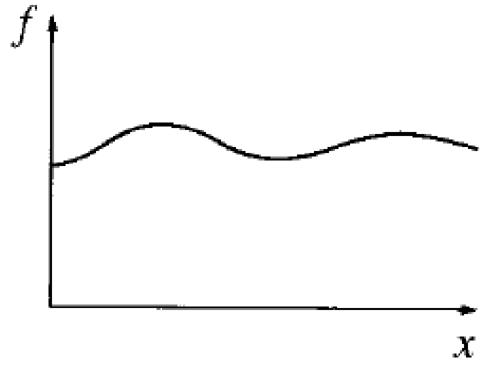


$$\vec{F}(x,y,z) = F_1(x,y,z)\hat{i} + F_2(x,y,z)\hat{j} + F_3(x,y,z)\hat{k}$$

$$\vec{F}(x,y,z) = xyz\hat{i} - x^2z^4\hat{j} + x\hat{k}$$

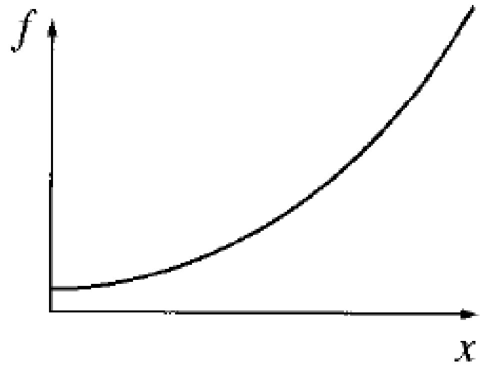
Ordinary derivatives

$$\frac{df}{dx} \qquad\qquad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$



$$df = \left(\frac{df}{dx}\right) dx$$

$T = T(x, y, z)$ Scalar field



Partial derivatives

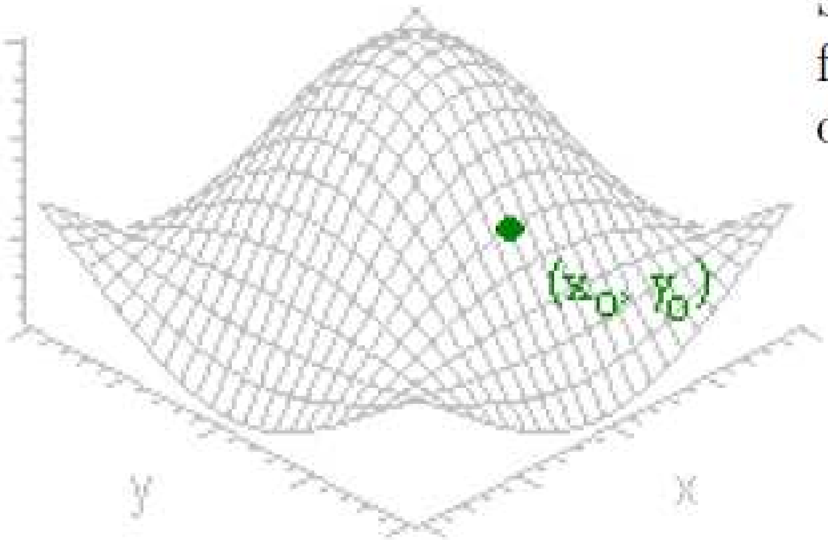
$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz.$$

$$f_x(x,y,z) = \lim_{h\to 0}\frac{f(x+h,y,z)-f(x,y,z)}{h}$$

$$f_y(x,y,z) = \lim_{h\to 0}\frac{f(x,y+h,z)-f(x,y,z)}{h}$$

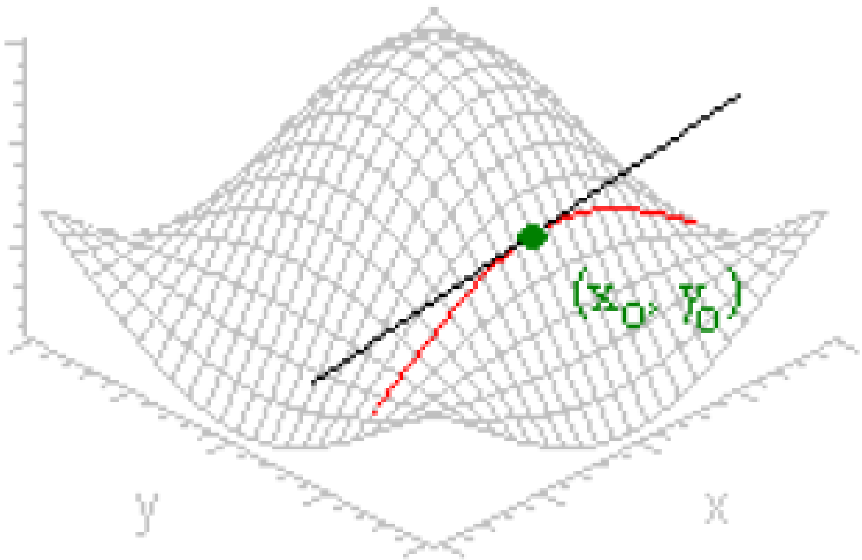
$$f_z(x,y,z) = \lim_{h\to 0}\frac{f(x,y,z+h)-f(x,y,z)}{h}$$

Geometrical Meaning



Suppose the graph of $z = f(x, y)$ is the surface shown. Consider the partial derivative of f with respect to x at a point (x_0, y_0).

$$\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)}$$



$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz.$$

$$\begin{aligned} dT &= \left(\frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}\right) \cdot (dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}) \\ &= (\boldsymbol{\nabla} T) \cdot (d\mathbf{l}), \end{aligned}$$

$$\boldsymbol{\nabla} T \equiv \frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$$

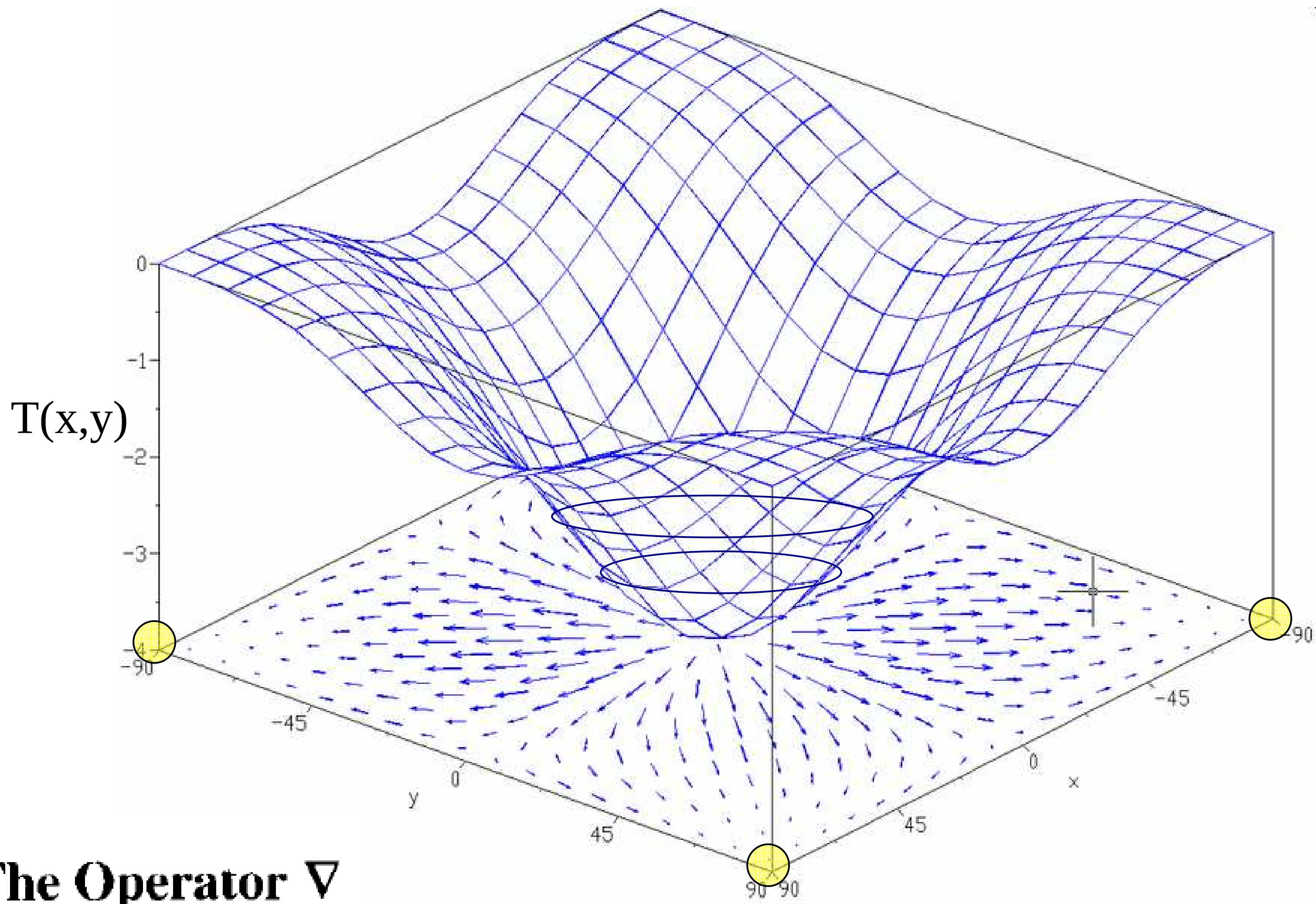
Geometrical Interpretation of the Gradient

$$dT = \boldsymbol{\nabla} T \cdot d\mathbf{l} = |\boldsymbol{\nabla} T||d\mathbf{l}| \cos\theta$$

when $\theta = 0$ (for then $\cos\theta = 1$)

The gradient $\boldsymbol{\nabla} T$ points in the direction of maximum increase of the function T.

The magnitude $|\boldsymbol{\nabla} T|$ gives the slope (rate of increase) along this maximal direction.



The Operator ∇

$$\boxed{\boldsymbol{\nabla} = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}.}$$

$$\nabla T = \underbrace{\left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right)}_{\text{Vector Field}} T$$

Scalar Field

The Divergence

$$\nabla \cdot \mathbf{v} = \left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right) \cdot (v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}})$$

$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

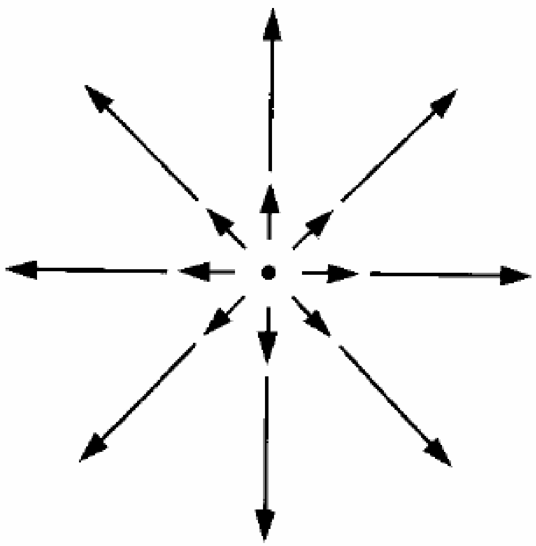
$$\nabla \cdot \vec{\boldsymbol{V}} \neq \vec{\boldsymbol{V}} \cdot \nabla$$

(a) $\mathbf{v}_a = x^2\,\hat{\mathbf{x}} + 3xz^2\,\hat{\mathbf{y}} - 2xz\,\hat{\mathbf{z}}$.

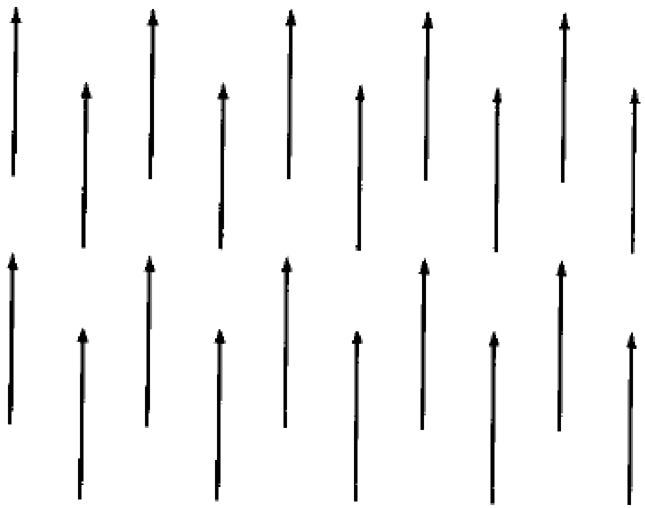
(b) $\mathbf{v}_b = xy\,\hat{\mathbf{x}} + 2yz\,\hat{\mathbf{y}} + 3zx\,\hat{\mathbf{z}}$.

(c) $\mathbf{v}_c = y^2\,\hat{\mathbf{x}} + (2xy + z^2)\,\hat{\mathbf{y}} + 2yz\,\hat{\mathbf{z}}$.

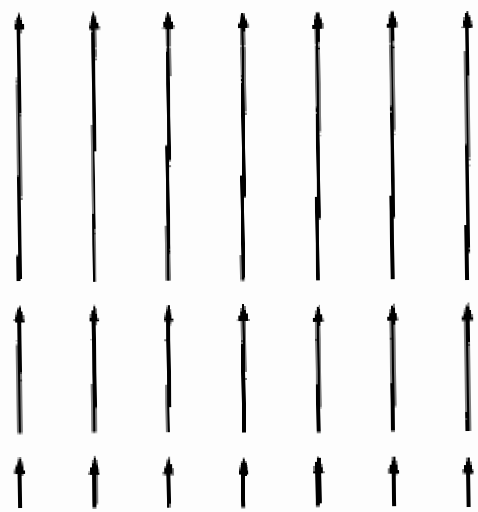
$$\mathbf{v}_a = \mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$$



$$\mathbf{v}_b = \hat{\mathbf{z}}$$

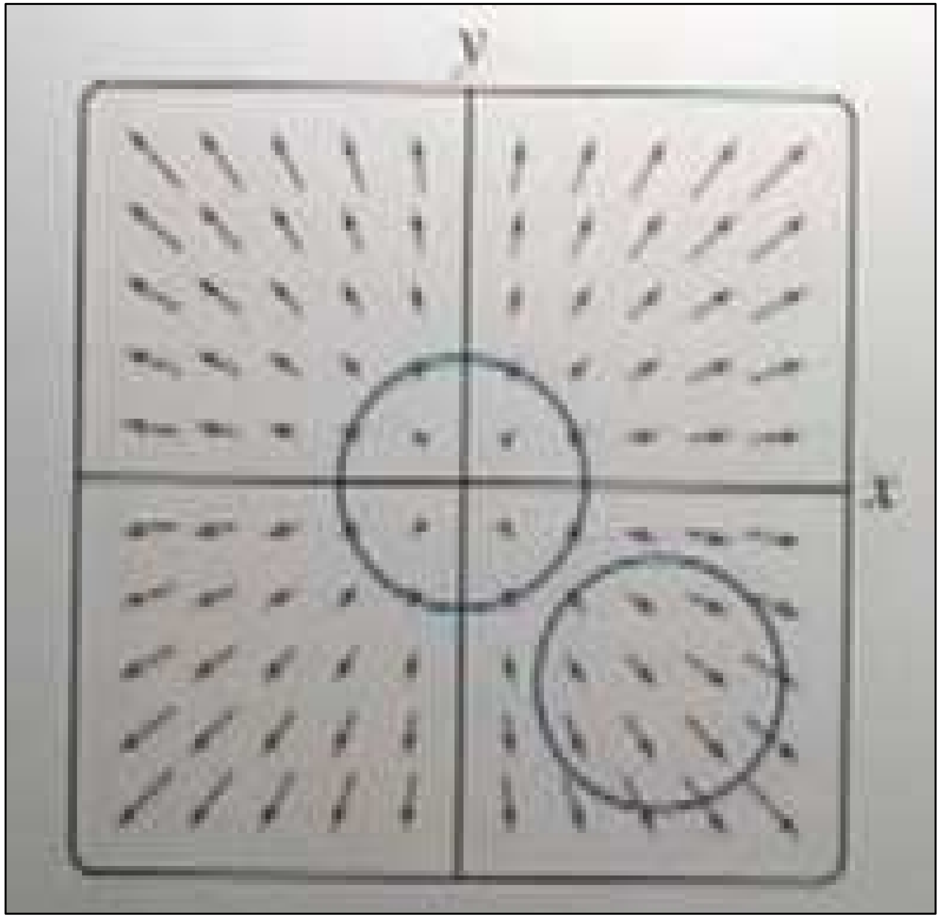


$$\mathbf{v}_c = z\,\hat{\mathbf{z}}$$



In many cases, the divergence of a vector function at point P may be predicted by considering a closed surface surrounding P and analyzing the flow over the boundary, keeping in mind that at P:

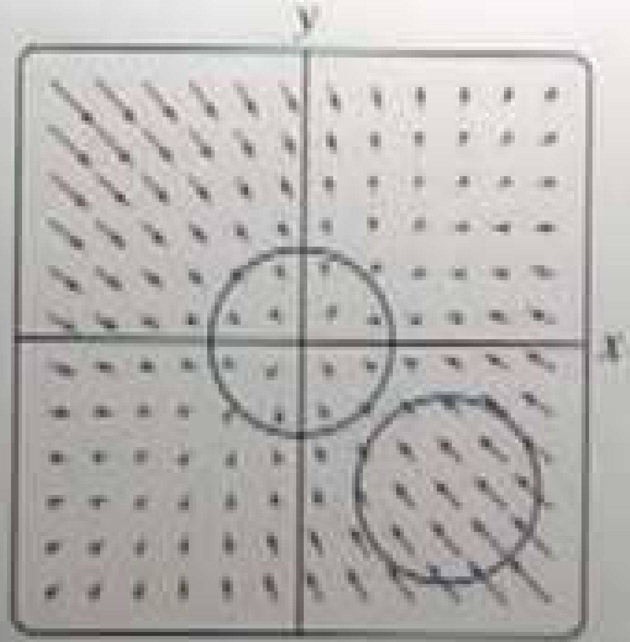
$$\nabla \cdot \vec{F} = \text{outflow} - \text{inflow}$$



$$\vec{V} = x\hat{i} + y\hat{j}$$

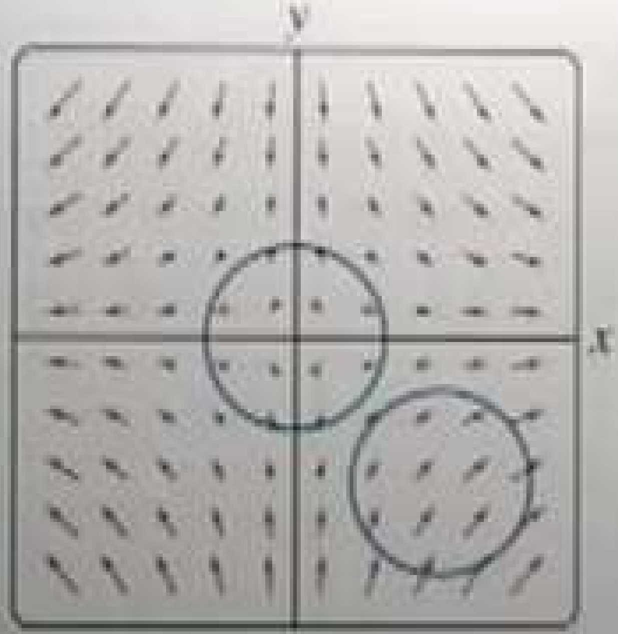
$$\vec{\nabla} \cdot \vec{V} = 2$$

In 3-D, divergence is a measure of Change of flux per unit volume



(B) The force field
$\mathbf{F} = \langle y - 2x, x - 2y \rangle$

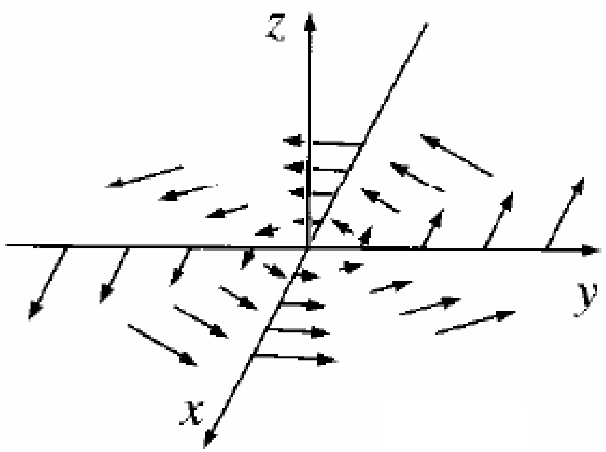
There is a net inflow into every circle.



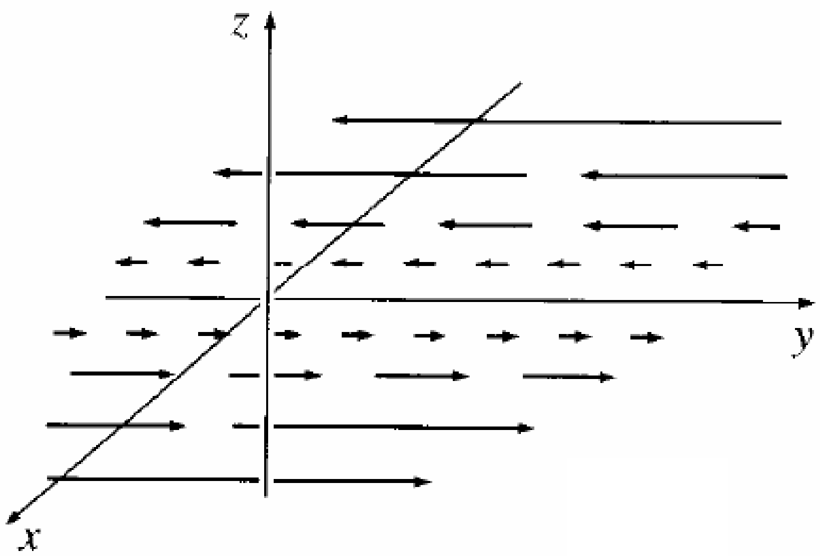
(C) The force field $\mathbf{F} = \langle x, -y \rangle$

The Curl

$$
\begin{aligned}
\nabla \times \mathbf{v} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix} \\
&= \hat{\mathbf{x}}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)
\end{aligned}
$$

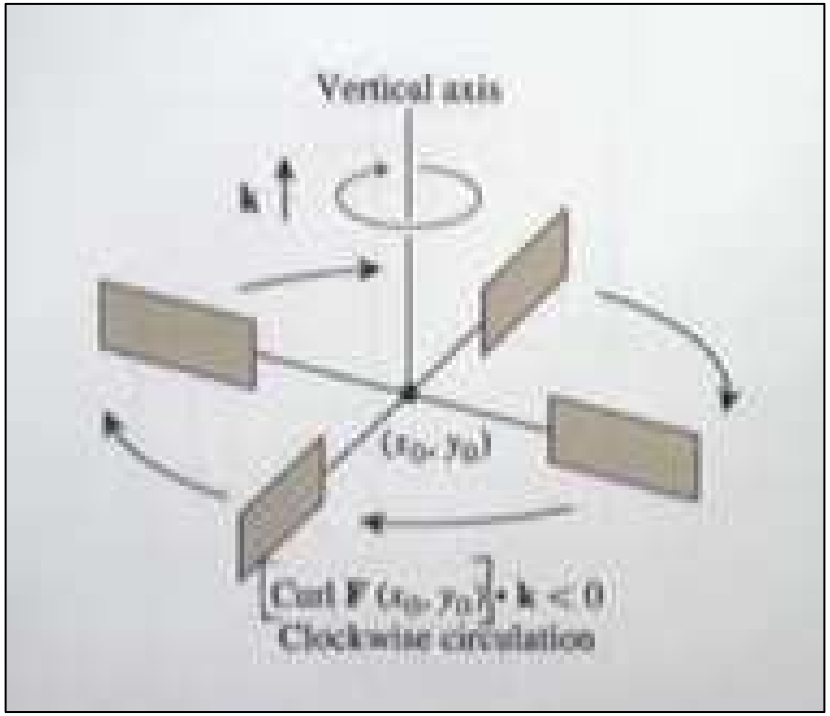


$\mathbf{v}_a = -y\hat{\mathbf{x}} + x\hat{\mathbf{y}}$



$\mathbf{v}_b = x\hat{\mathbf{y}}$

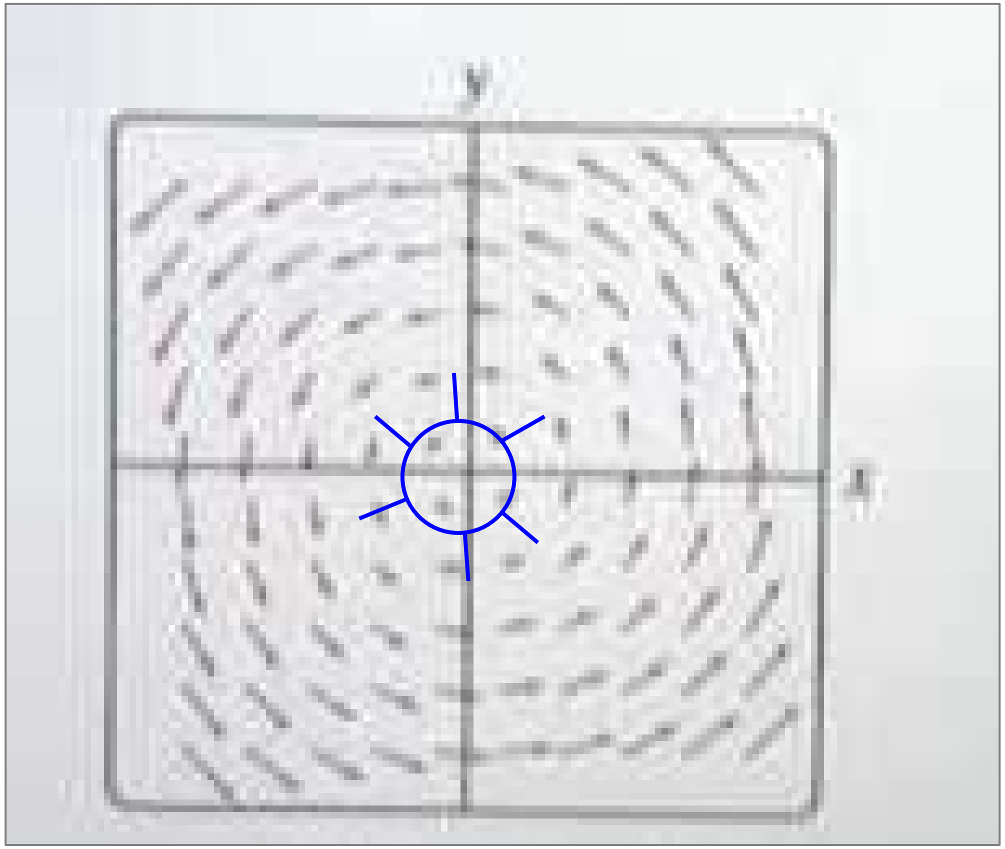
Paddle wheel analysis



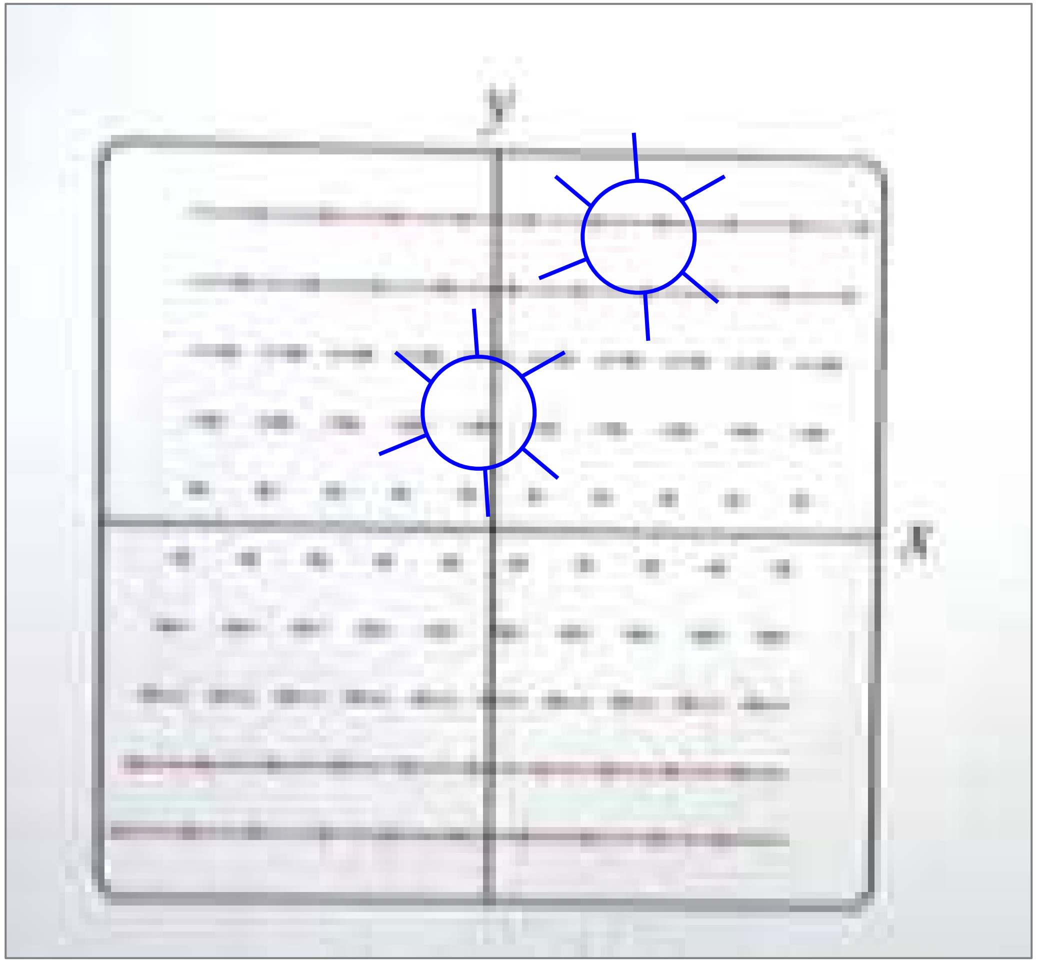
$$[\nabla \times \vec{\boldsymbol{F}}(\boldsymbol{x}_0, \boldsymbol{y}_0)] \cdot \hat{\boldsymbol{k}} < 0$$

$$\vec{F}(x, y) = -y\hat{i} + x\hat{j}$$

$$(\vec{\nabla} \times \vec{F}) \cdot \hat{k} = 2$$



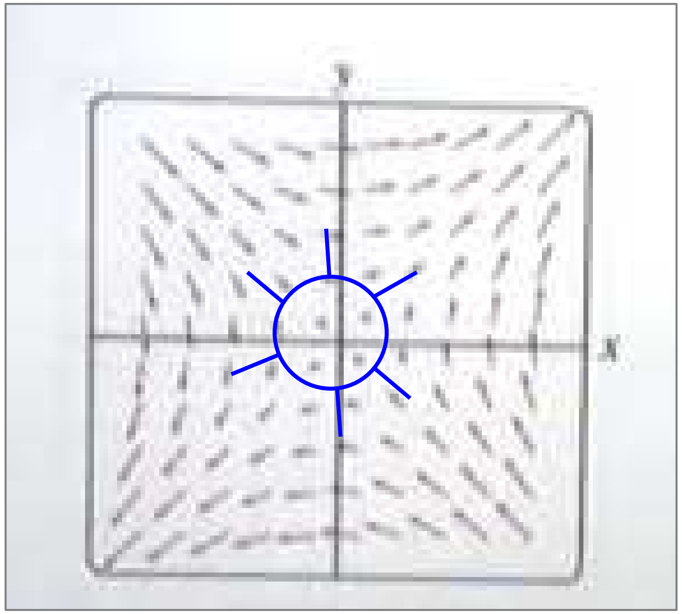
$$\vec{F}(x,y) = y\hat{i}$$

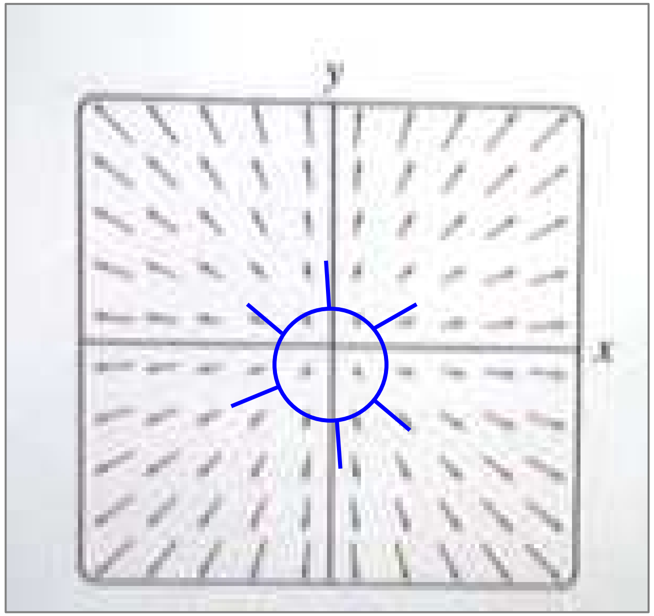


$$(\nabla \times \vec{F}) \cdot \hat{k} = -1$$

$$\vec{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{y}\hat{\boldsymbol{i}} + \boldsymbol{x}\hat{\boldsymbol{j}}$$

$$\vec{F}(x,y) = x\hat{i} + y\hat{j}$$





$$\nabla \times \vec{\boldsymbol{F}} = 0$$

$$\nabla(f+g) = \nabla f + \nabla g, \qquad \nabla \cdot (\mathbf{A}+\mathbf{B}) = (\nabla \cdot \mathbf{A}) + (\nabla \cdot \mathbf{B}),$$

$$\nabla \times (\mathbf{A}+\mathbf{B}) = (\nabla \times \mathbf{A}) + (\nabla \times \mathbf{B}),$$

$$\nabla(kf) = k\nabla f, \qquad \nabla \cdot (k\mathbf{A}) = k(\nabla \cdot \mathbf{A}), \qquad \nabla \times (k\mathbf{A}) = k(\nabla \times \mathbf{A}),$$

fg (product of two scalar functions),
$\mathbf{A} \cdot \mathbf{B}$ (dot product of two vector functions)

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

$f\mathbf{A}$	(scalar times vector),
$\mathbf{A} \times \mathbf{B}$	(cross product of two vectors)

$$\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$$

Second Derivatives

∇T is a *vector*

(1) Divergence of gradient: $\nabla \cdot (\nabla T)$

(2) Curl of gradient: $\nabla \times (\nabla T)$.

$\nabla \cdot \mathbf{v}$ is a *scalar*

Gradient of divergence: $\nabla(\nabla \cdot \mathbf{v})$

$\nabla \times \mathbf{v}$ is a *vector*

Divergence of curl: $\nabla \cdot (\nabla \times \mathbf{v})$

Curl of curl: $\nabla \times (\nabla \times \mathbf{v})$.

$$\begin{aligned} \nabla \cdot (\nabla T) &= \left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right) \cdot \left(\frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}\right) \\ &= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \\ &= \nabla^2 T \end{aligned}$$

$\nabla^2 \longleftarrow$ Laplacian oparator

Laplacian of a *vector*,

$$\nabla^2 \mathbf{v} \equiv (\nabla^2 v_x)\hat{\mathbf{x}} + (\nabla^2 v_y)\hat{\mathbf{y}} + (\nabla^2 v_z)\hat{\mathbf{z}}$$

The curl of a gradient is always *zero*. $\nabla \times (\nabla T) = 0$

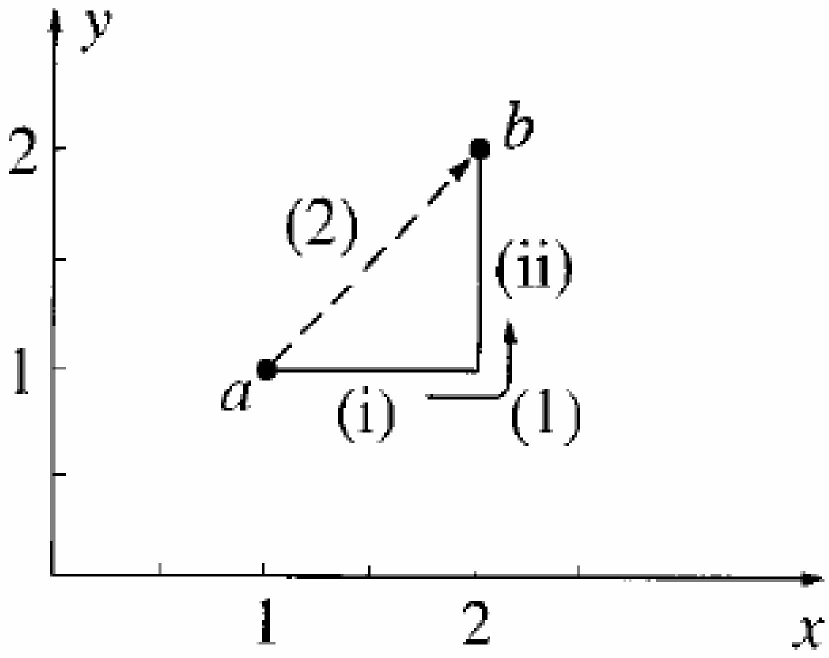
$$\nabla^2 \mathbf{v} = (\nabla \cdot \nabla)\mathbf{v} \neq \nabla(\nabla \cdot \mathbf{v})$$

The divergence of a curl, like the curl of a gradient, is *always zero* $\nabla \cdot (\nabla \times \mathbf{v}) = 0$

$$\nabla \times (\nabla \times \mathbf{v}) = \nabla(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}$$

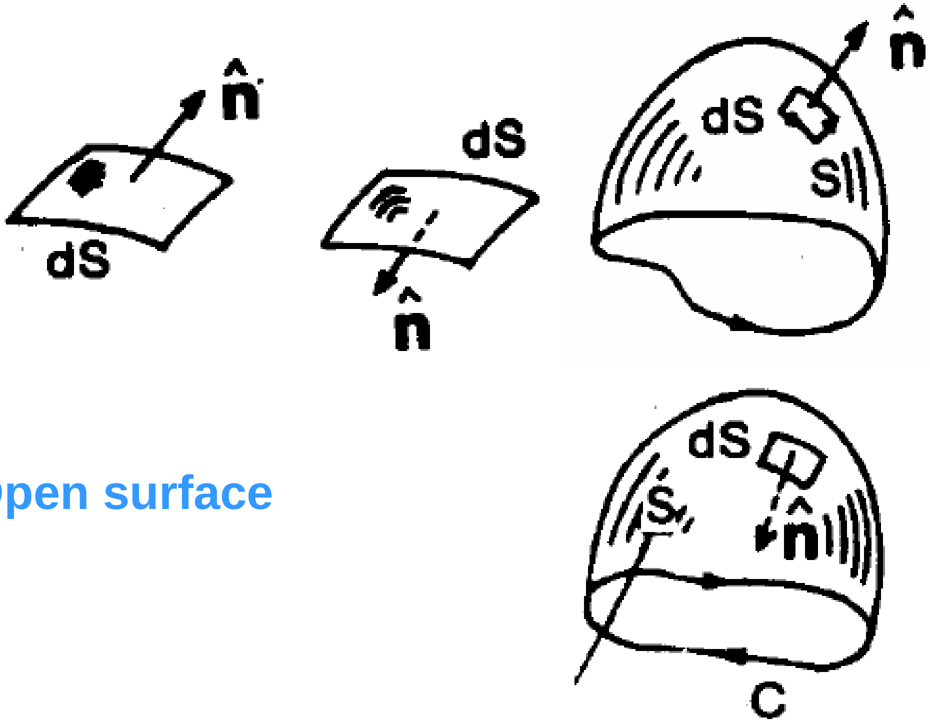
$$(i) \quad \vec{V} = -y\hat{i} + x\hat{j}$$

$$(ii) \quad \vec{V} = x\hat{i} + y\hat{j}$$

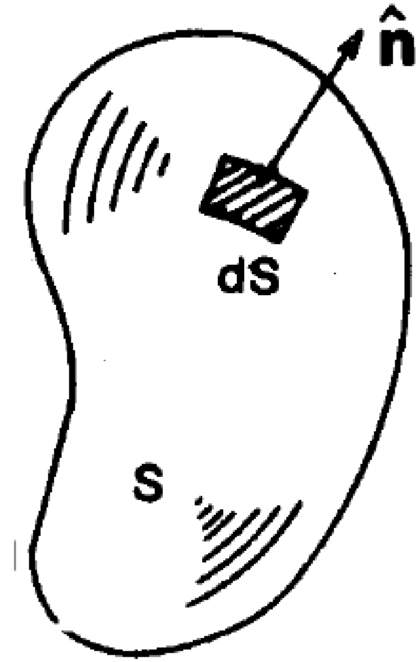


Vector Surface Integral

Vector surface Integral



Open surface

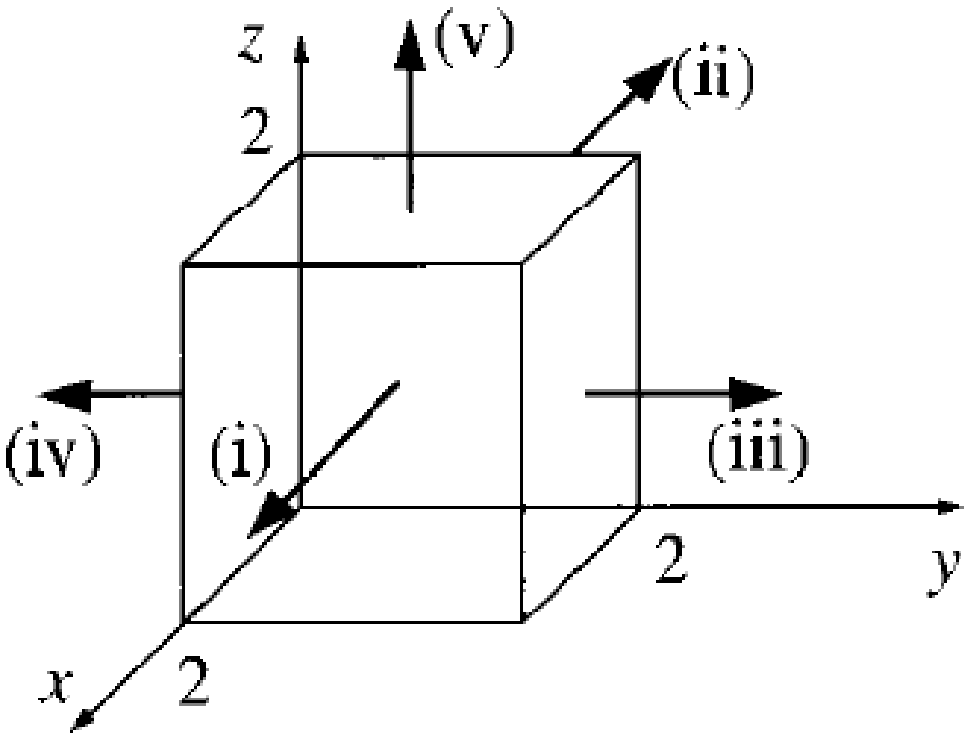


Closed surface

$$(i) \quad \vec{V}_1 = -y\hat{i} + x\hat{j}$$

$$(ii) \quad \overline{V}_2 = -y\hat{i} + xy\hat{j}$$

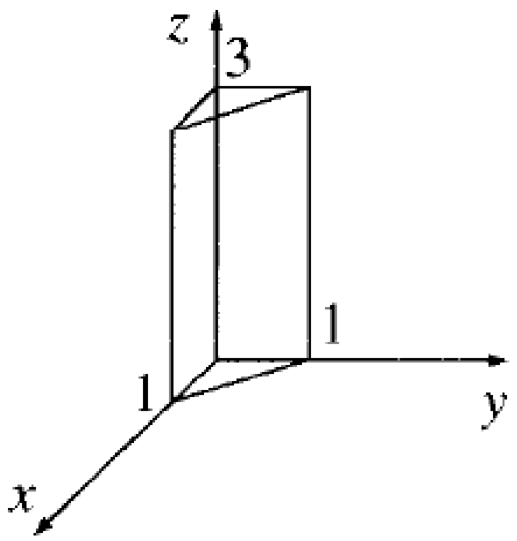
$$(iii) \quad \vec{V}_3 = x\hat{i} + y\hat{j} + z\hat{k}$$



Volume Integral

$$\int_{\mathcal{V}} T\, d\tau \qquad\qquad d\tau = dx\, dy\, dz$$

Calculate the volume integral of $T = xyz^2$ over the prism in Fig.



The Fundamental Theorem for Gradients

The integral of a derivative over a region is equal to the value of the function at the boundary

$$\boxed{\int_{\mathbf{a} \atop \mathcal{P}}^{\mathbf{b}} (\nabla T) \cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a}).}$$

$$dT = (\nabla T) \cdot d\mathbf{l}_1$$

Difference of function's value at b and a

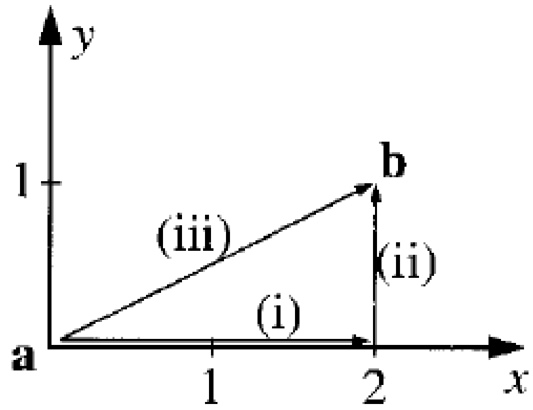
$$\vec{\nabla} \times \vec{\nabla} \boldsymbol{T} = 0 \qquad \vec{\nabla} \times \vec{\boldsymbol{F}} = 0$$

F conservative field

Corollary 1: $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T) \cdot d\mathbf{l}$ is independent of path taken from **a** to **b**.

Corollary 2: $\oint (\nabla T) \cdot d\mathbf{l} = 0$, since the beginning and end points are identical, and hence $T(\mathbf{b}) - T(\mathbf{a}) = 0$.

Let $T = xy^2$, and take point **a** to be the origin $(0, 0, 0)$ and **b** the point $(2, 1, 0)$. Check the fundamental theorem for gradients.



Divergence

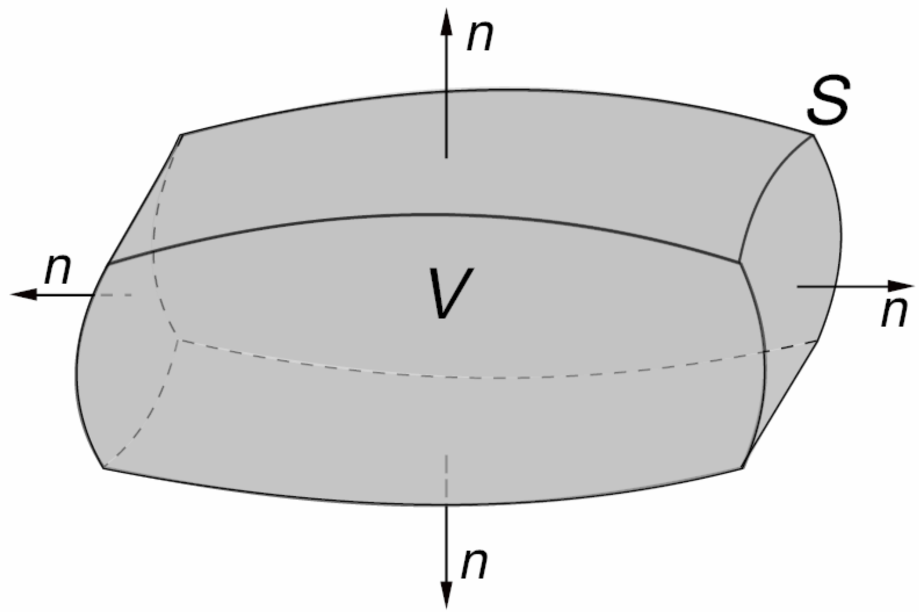
$$div\ \vec{F} = \lim_{\Delta v \to 0} \frac{\oint \vec{F} \cdot d\vec{s}}{\Delta v}$$

Curl

$$\lim_{\Delta S \to 0} \frac{\oint \vec{F} \cdot d\vec{l}}{\Delta S} = (curl\, F)\hat{n}$$

The Fundamental Theorem for Divergences

$$\boxed{\int_{\mathcal{V}} (\nabla \cdot \mathbf{v})\, d\tau = \oint_{\mathcal{S}} \mathbf{v} \cdot d\mathbf{a}.}$$



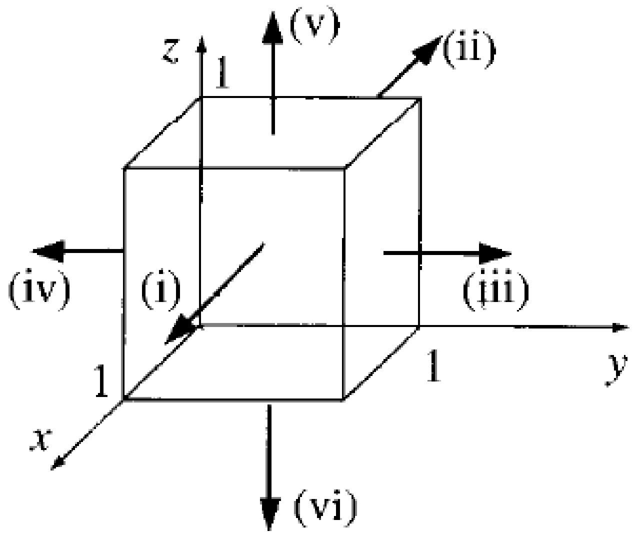
The integral of a derivative over a region is equal to the value of the function at the boundary

$$\nabla \cdot \vec{V} = \text{outflow} - \text{inflow} \longrightarrow$$

+ve (source)
-ve (sink)

$$\int (\text{faucets within the volume}) = \oint (\text{flow out through the surface})$$

Check the divergence theorem using the function $\mathbf{v} = y^2\,\hat{\mathbf{x}} + (2xy + z^2)\,\hat{\mathbf{y}} + (2yz)\,\hat{\mathbf{z}}$



(iii) $$\int \mathbf{v}\cdot d\mathbf{a} = \int_0^1\int_0^1 (2x + z^2)\,dx\,dz = \tfrac{4}{3}.$$

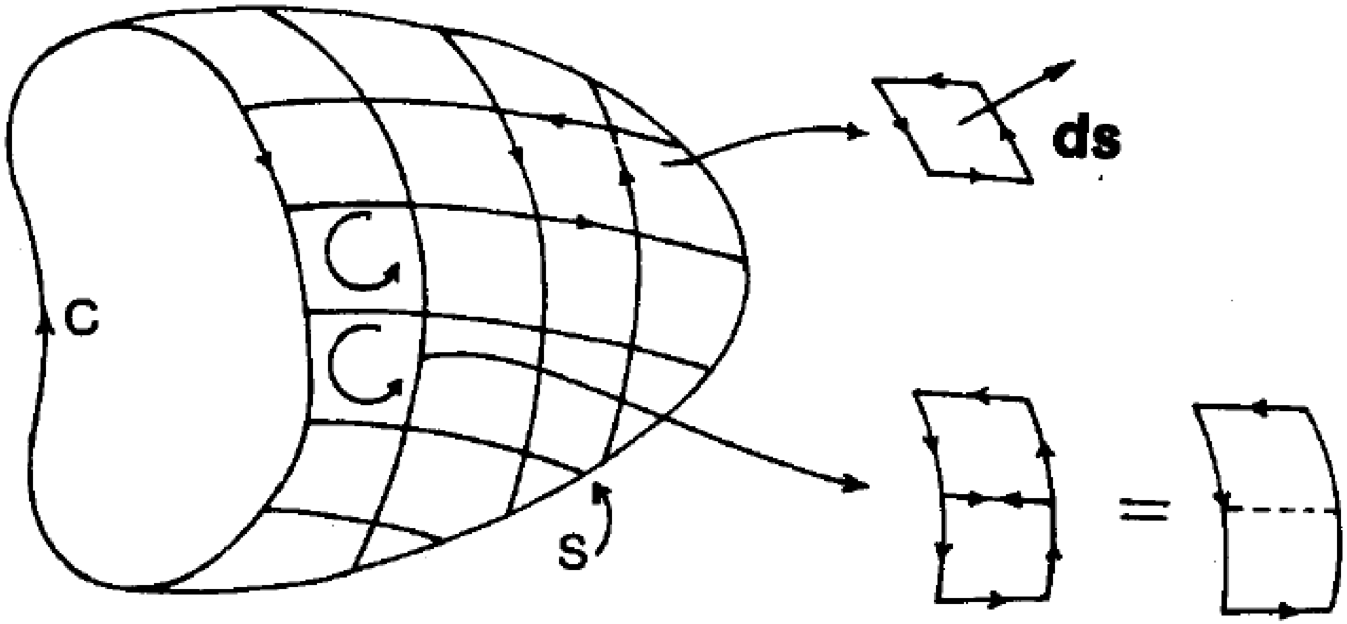
(iv) $$\int \mathbf{v}\cdot d\mathbf{a} = -\int_0^1\int_0^1 z^2\,dx\,dz = -\tfrac{1}{3}.$$

(v) $$\int \mathbf{v}\cdot d\mathbf{a} = \int_0^1\int_0^1 2y\,dx\,dy = 1.$$

The Fundamental Theorem for Curls Stokes' theorem

$$\boxed{\int_{\mathcal{S}} (\nabla \times \mathbf{v}) \cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{v} \cdot d\mathbf{l}.}$$

The integral of a derivative over a region is equal to the value of the function at the boundary

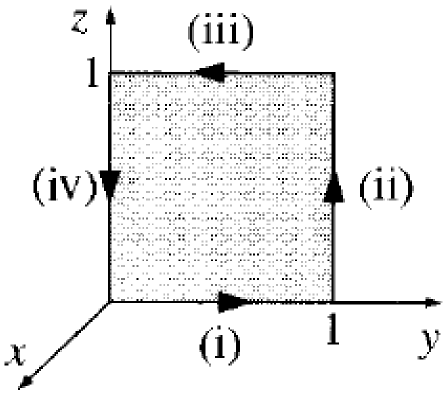


rotational force field / non-conservative force field

Corollary 1: $\int (\nabla \times \mathbf{v}) \cdot d\mathbf{a}$ depends only on the boundary line, not on the particular surface used.

Corollary 2: $\oint (\nabla \times \mathbf{v}) \cdot d\mathbf{a} = 0$ for any closed surface.

Suppose $\mathbf{v} = (2xz + 3y^2)\hat{\mathbf{y}} + (4yz^2)\hat{\mathbf{z}}$. Check Stokes' theorem for the square surface



$$\nabla \times \mathbf{v} = (4z^2 - 2x)\,\hat{\mathbf{x}} + 2z\,\hat{\mathbf{z}} \quad \text{and} \quad d\mathbf{a} = dy\,dz\,\hat{\mathbf{x}}.$$

$$\int (\nabla \times \mathbf{v}) \cdot d\mathbf{a} = \int_0^1 \int_0^1 4z^2\,dy\,dz = \tfrac{4}{3}.$$

(i) $\quad x = 0, \quad z = 0, \quad \mathbf{v} \cdot d\mathbf{l} = 3y^2\,dy, \quad \int \mathbf{v} \cdot d\mathbf{l} = \int_0^1 3y^2\,dy = 1,$

(ii) $\quad x = 0, \quad y = 1, \quad \mathbf{v} \cdot d\mathbf{l} = 4z^2\,dz, \quad \int \mathbf{v} \cdot d\mathbf{l} = \int_0^1 4z^2\,dz = \tfrac{4}{3},$

(iii) $\quad x = 0, \quad z = 1, \quad \mathbf{v} \cdot d\mathbf{l} = 3y^2\,dy, \quad \int \mathbf{v} \cdot d\mathbf{l} = \int_1^0 3y^2\,dy = -1,$

(iv) $\quad x = 0, \quad y = 0, \quad \mathbf{v} \cdot d\mathbf{l} = 0, \quad \int \mathbf{v} \cdot d\mathbf{l} = \int_1^0 0\,dz = 0.$

$$\oint \mathbf{v} \cdot d\mathbf{l} = 1 + \tfrac{4}{3} - 1 + 0 = \tfrac{4}{3}$$

- Concepts of Vector Field
- Gradient [converts scalar field to a vector field]
- Divergence [A measure of change of flux per unit volume]
- Curl [measure of rotational nature of a vector field]
- Line integration
- Surface integration
- Volume integration
- Fundamental theorem for Gradient
- Fundamental theorem for Divergence
- Fundamental theorem for Curl