PROBABILITY AND STATISTICS

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→ Sheldon M. Ross : A 1st Course in Probability
→ " " : Introductory Statistics

→ Relative frequency : If we flip the coin a large no. of times probability of getting heads ~~its~~ is $\frac{1}{2}$.

→ ϕ : Improper set $(A \subseteq S)$
Proper set : $A \subset S$
Superset : $A \supseteq B$

Universal Set : S or Ω

→ Partition : disjoint & union is A.

Infinite set

Countable : finite set
Eg N, Z, Q
If we can place each ele in a set, we say it is countable

Uncountable :
Eg. R

→ Sample Space : ω
⇓
all possible outcomes

→ Event : Subset of a sample space.

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→ Any small interval over real line is uncountable because irrational no are uncountable

Countability

→ Multiplication Principle :
$|A \times B| = |A||B|$

~~Eg~~

Eg. $(26)^3 \times (10)^4$

Sampling : choosing an element randomly from a set

With Replacement : Repetition is possible : Direct Multiplication

Ordered : sampling in which ordering matters

Ordered : Use Permutation (without Replacement)

Unordered : Use Combination

Theorem → No. of permutations of n distinct objects is n!

→ " " taken k at a time : $^nP_k = \frac{n!}{(n-k)!}$

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Eg. How many even 3-digit no. using 1, 2, 5, 6, 9 if each digit can be used only once ?

$4 \times 3 \times 2 = \underline{\underline{24}}$

Theorem → No of permutations of n distinct objects arranged in a circle : $(n-1)!$

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→ No. of ways of partitioning n objects into r cells of sizes $n_1, n_2, \ldots, n_r$: $\dfrac{n!}{n_1! \cdot n_2! \cdots n_r!}$

$P(\text{top card is an Ace}) = \dfrac{4 * 51!}{52!} = \dfrac{4}{52} = \dfrac{1}{13}$

P(same suit next to each other)

$\boxed{}$ $\boxed{}$ $\boxed{}$ $\boxed{}$ $= \dfrac{4! \cdot 13!^4}{52!}$

$P(\text{hearts are together}) = \dfrac{40! \cdot 13!}{52!}$

↳ Unordered sampling without replacement : Combination

→ Difference b/w ${}^nC_k$ & ${}^nP_k$ is in the ordering

ordering is considered

$${}^nP_k = {}^nC_k * k!$$

→ $^5C_2 * {}^5C_3 + {}^5C_2 * {}^5C_2 + {}^5C_2 * {}^5C_2$

$\dfrac{5\times4}{2} * \left(\dfrac{5\times4}{2}\right) * 2 + \dfrac{5\times4}{2} * \dfrac{5\times4}{2} = \underline{\underline{300}}$ $\quad 7 \longleftrightarrow 3$

Binomial Theorem :

$$(x+y)^n = \sum_{k=0}^{n} {}^nC_k \, x^k y^{n-k}$$

Proof: Using Mathematical Induction

Let $(x+y)^1 = {}^1C_0 x^0 y^{1-0} + {}^1C_1 x^1 y^{1-1} = \sum_{k=0}^{1} {}^1C_k x^k y^{1-k}$

Let ~~&~~ it be true for $n-1$ Then,

$$(x+y)^{n-1} = \sum_{k=0}^{n-1} {}^{n-1}C_k \, x^k y^{n-1-k}$$

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Then, prove for $n = n$

$$(x+y)^n = (x+y)(x+y)^{n-1}$$

$$= (x+y) \sum_{k=0}^{n-1} {}^{n-1}C_k \, x^k \, y^{(n-1)-k}$$

$$= \sum_{k=0}^{n-1} {}^{n-1}C_k \, x^{k+1} \, y^{n-1-k} + \sum_{k=0}^{n-1} {}^{n-1}C_k \, x^k \, y^{n-k}$$

Let us put $k = i-1$ in 1^{st} term and $k = i$ in 2^{nd} term.

$$= \sum_{i=1}^{n} {}^{n-1}C_{i-1} \, x^i \, y^{n-i} + \sum_{i=0}^{n-1} {}^{n-1}C_i \, x^i \, y^{n-i}$$

$$= \underbrace{x^n}_{\text{last term of } 1^{st} \Sigma} + \sum_{i=1}^{n-1} {}^{n-1}C_{i-1} \, x^i \, y^{n-i} + \underbrace{y^n}_{1^{st} \text{ term of } 2^{nd} \Sigma} + \sum_{i=1}^{n-1} {}^{n-1}C_i \, x^i \, y^{n-i}$$

$$= x^n + y^n + \sum_{i=1}^{n-1} \left(\left({}^{n-1}C_{i-1} + {}^{n-1}C_i \right) x^i \, y^{n-i} \right)$$

$$= x^n + y^n + \sum_{i=1}^{n-1} {}^{n}C_i \, x^i \, y^{n-i}$$

$$= \sum_{i=0}^{n} {}^{n}C_i \, x^i \, y^{n-i}$$

Hence Proved

10/1/18 **Relative Frequency:** (Axiome / assumptions)

We suppose that an experiment, whose sample space is S is repeatedly performed under exactly the same condn and if $n(E)$ to be the no. of times in the first n repetition of the exp. that E occurs

$$\lim_{n \to \infty} \frac{n(E)}{n}$$

$E_1 \cap E_2 = E_1 E_2$

Kolmogorov's Axioms

Axiom 1: $0 \le P(E) \le 1$

Axiom 2: $P(S) = 1$

Axiom 3: For any sequence of mutually exclusive events $E_1, E_2, E_3, \ldots$ we have

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

ie, $E_i E_j = \phi \quad \forall\, i, j$

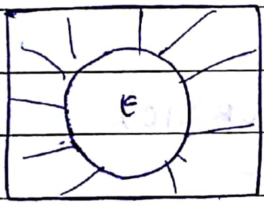
Eg. $P[\{1,3,5\}] = \frac{1}{2}$ (dice)

all are mutually exclusive

$\Rightarrow P[\{1,3,5\}] = P(1) + P(3) + P(5)$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

P1 →



$P(S) = P(E \cup E^c)$

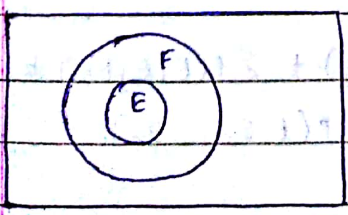
$= P(E) + P(E^c)$ (by Axiom-3)

$S - E = E^c$ $\qquad P(S) = 1$ (by Axiom 2)

$S = E \cup E^c$ $\qquad \Rightarrow 1 = P(E) + P(E^c)$

$\Rightarrow \underline{P(E^c) = 1 - P(E)}$

P2 If $E \subseteq F$, then $P(E) \le P(F)$



$E^c F$

$F = E \cup (E^c \cap F)$ → mutually exclusive

$\Rightarrow P(F) = P(E) + \underline{P(E^c F)}$ (by Axiom 3)

$\Rightarrow \underline{\underline{P(F) \ge P(E)}}$ $\quad [P(E^c F) \ge 0]$ ↳ by Axiom 1

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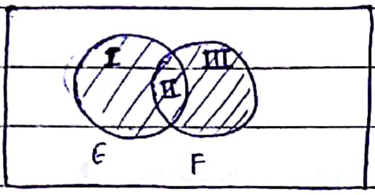
$$P(E \cup F) = P(E) + P(F) - P(EF)$$



$$P(\mathrm{I}) + P(\mathrm{II}) = P(E)$$
$$P(\mathrm{II}) + P(\mathrm{III}) = P(F)$$
$$P(\mathrm{I}) + 2P(\mathrm{II}) + P(\mathrm{III}) = P(E) + P(F)$$
$$P(E \cup F) + P(\mathrm{II}) = P(E) + P(F)$$

mutually exclusive

formal proof :-

$$E \cup F = E \cup E^cF \quad \text{→ mutually exclusive → Axiom 3}$$

$$P(E \cup F) = P(E) + P(E^cF) \quad \text{— ①}$$

$$F = EF \cup E^cF \quad \text{→ mutually exclusive (Axiom-3)}$$

$$\Rightarrow P(F) = P(EF) + P(E^cF) \quad \text{— ②}$$

Using ① & ②,

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

Hence Proved.

$$P(\underbrace{A \cup B} \cup \underbrace{C}) = \cancel{P(A) + P(B) + P(C)}$$
$$= P(A \cup B) + P(C) - P((A \cup B) \cap C)$$
$$= P(A) + P(B) - P(A \cap B) + P(C) -$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Inclusion - Exclusion Identity

$$P(E_1 \cup E_2 \cup \ldots \cup E_n) = \sum P(E_i) - \sum P(E_iE_j) + \sum P(E_iE_jE_k) \pm \ldots + (-1)^{n+1} P(E_1E_2\ldots E_n)$$

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Sample Space with equally likely Outcomes

If in a sample space each event is equally likely to occur, then

$$P(E) = \frac{\text{no. of outcomes in } E}{\text{no. of outcomes in } S}$$

If 3 balls are "randomly drawn" from a bowl containing 6 white and 5 black balls, what is the probability that one of the ball is white and other 2 are black.

$$= \frac{6}{11} * \frac{5}{10} * \frac{4}{9} * \frac{3!}{2!}$$

If order is considered:

$$S = {}^{11}P_3$$

$$= \frac{({}^6P_1 \cdot {}^5P_2) \cdot 3}{{}^{11}P_3} = \frac{4}{11}$$

~~Without~~

Probability of getting Head in exactly n^{th} ~~3rd~~ trial for 1st time:

$$P(A_1) = \frac{1}{2}$$

$$P(A_2) = P(TH) = \left(1-\frac{1}{2}\right) * \frac{1}{2}$$

$$P(A_3) = P(TTH) = \left(1-\frac{1}{2}\right) * \left(1-\frac{1}{2}\right) * \frac{1}{2} = \frac{1}{8}$$

~~P(getting at least 1 head)~~ =

P(getting a head sometime) =

$$S = \frac{1}{2} + \frac{1}{2^2} + \dots \frac{1}{2^4}$$

(addⁿ of all we got in previous quesⁿ → mutually exclusive)

$$= \frac{\frac{1}{2}\left(\cancel{1-\left(\frac{1}{2}\right)^n}\right)}{\frac{1}{2}} = \boxed{1}$$

Probability that a Projector stops on the n^{th} trial

Let us assume the probability of stopping the system = p

(We don't know exactly)

success in 1^{st} trial : p

2^{nd} trial = $(1-p) * p$

3^{rd} = $(1-p)^2 * p$

⋮

n^{th} trial = $(1-p)^{n-1} * p$

mutually exclusive

$$P(\text{stopping sometime}) = p + (1-p) * p + (1-p)^2 * p \dots = S$$

$$= p\left[1 + (1-p) + (1-p)^2 + \dots\right]$$

$$= p\left[\frac{1}{1-(1-p)}\right] = \frac{p}{p} = \boxed{1} \quad (p \neq 0)$$

$$S : p\left(1 + (1-p) + (1-p)^2 + \dots\right)$$

$$S = p + (1-p)p\left[1 + (1-p) + (1-p)^2 + \dots\right]$$

Inclusion - Exclusion Identity

comb^n

$$P(E_1 \cup E_2 \cup E_3 \cup \dots E_n) = \sum P(E_i) - \sum_{i<j} P(E_i E_j) \dots + (-1)^{n+1} P(E_1 E_2 \dots E_n) \quad (1)$$

atleast 1 of the n-Events is happening

The summation $\sum_{i_1<i_2<\dots<i_r} P(E_{i_1} E_{i_2} \dots E_{i_r})$ is taken over all of the ^nC_r possible subsets of size r of the set $\{1, 2, \dots, n\}$

The Matching Problem

Suppose that each of N men at a party throws ~~hit~~ his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat.

(1) What is the probability that none of the men selects his own hat?

$P(E_1 \cup E_2 \dots \cup E_n) \rightarrow$ atleast 1 gets his hat

Let E_i, $i = 1, 2, 3, \dots N$ denote the event that i^{th} man selects his own hat.

To address this problem, we will first calculate the probabilit that atleast 1 man gets correct hat.

Applying (1), we get: $P(\quad) = \_\_\_\_$

If we regard the outcomes of this experiment as a vector of ~~N~~ numbers where the i^{th} element is the no. of the hat drawn by the i^{th} man. Now, if $E_1, E_2, \dots, E_r$ ~~are~~ the events that each of the $1, 2, \dots, r$ selects his own hat can occur, so possible outcomes: $(N-r)!$

$(N-r)(N-\overline{r+1})(N-\overline{r+2})$

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One gets his own hat

then $E_i = (N-1)!$

$$P(E_i) = \frac{(N-1)!}{N!} = \frac{1}{N}$$

If 2 get their own hat

$$P(E_{i_1} E_{i_2}) = \frac{(N-2)!}{N!}$$

If r get their own hat

$$P(E_1 E_2 \ldots E_r) = \frac{(N-r)!}{N!}$$

Using Inclusion - Exclusion Principle,

$$P\left(\bigcup_{i=1}^{N} E_i\right) = {}^{N}C_1 \cdot \frac{1}{N} - {}^{N}C_2 \frac{(N-2)!}{N!} + \ldots + {}^{N}C_r \frac{(N-r)!}{N!} \ldots + {}^{N}C_N \frac{1}{N!}$$

$$= 1 - \frac{1}{2!} + \frac{1}{3!} \ldots + (-1)^{N+1} \frac{1}{N!}$$

$$P(\text{none gets matched}) = P\left(\bigcap E_i^c\right)$$

$$= P\left(\left(\bigcup E_i\right)\right)^c$$

$$= 1 - P\left(\bigcup_{i=1}^{N} E_i\right)$$

$$= 1 - \left(1 - \frac{1}{2!} + \ldots + (-1)^{N+1} \frac{1}{N!}\right)$$

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Q. Compute proba[bility] ... random at a ... to her husba[nd] ...

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Compute probability that 10 married couples are selected at random at a round table, ~~then~~ such that no wife sits next to her husband. (solve yourself - in book) getting exactly 1 head) $\subseteq$ (getting atmost 2 heads) : ↑ng sequence of events

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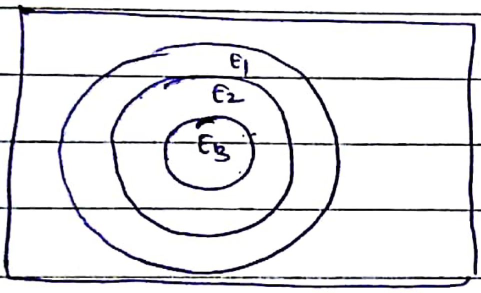
Continuous set functions

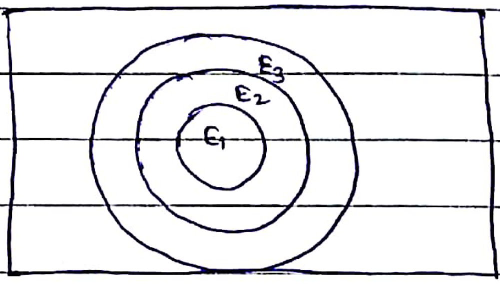
A sequence of events $\{E_n : n \geq 1\}$ is said to be increasing sequence if

$$E_1 \subseteq E_2 \subset E_3 \subseteq \ldots \subseteq E_{r+1} \subset \ldots$$

whereas it is said to be decreasing if

$$E_1 \supseteq E_2 \supseteq E_3 \supseteq \ldots \supseteq E_{r+1} \supseteq$$





↓ng sequence

$E_1 \cap E_2 \cap E_3 = E_3$

↑ng sequence

$E_1 \cup E_2 \cup E_3 = E_3$

If $\{E_n, n \geq 1\}$ is an increasing sequence of events, we denote $\lim_{n \to \infty} E_n$ by

$$\lim_{n \to \infty} E_n = \bigcup_{i=1}^{\infty} E_i$$

If $\{E_n, n \geq 1\}$ is a decreasing sequence of events, we define

proposition :

Proof :

RHS =

If we convert set into mutually exclusive sets, it is easy to get probability of union of sets.

If $\{E_n, n \geq 1\}$ is either an increasing sequence or a decreasing sequence of events, then

$$\lim_{n\to\infty} P(E_n) = P\left(\lim_{n\to\infty} E_n\right)$$

If $\{E_n, n \geq 1\}$ is an increasing sequence of events

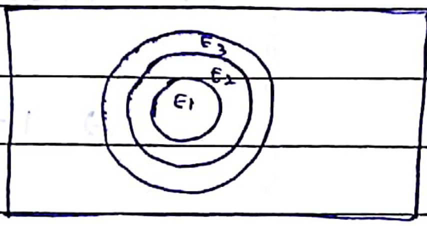
Let $F_1 = E_1$

$F_2 = E_2 \cap E_1^c$

$F_3 = E_3 \cap (E_1 \cup E_2)^c$

$\vdots$

$F_n = E_n \cap \left(\bigcup_{i=1}^{n-1} E_i\right)^c$.



~~Now, given~~ This gives :

we have constructed our event s.t. this condn is true

$$\bigcup_{i=1}^{\infty} E_i = \bigcup_{i=1}^{\infty} F_i \quad \text{and} \quad \bigcup_{i=1}^{n} E_i = \bigcup_{i=1}^{n} F_i$$

F_n consists of those outcomes in E_n which are not in any of earlier E_i, $i = 1, 2, \ldots, n-1$.

$$P\left(\lim_{n\to\infty} E_n\right) = P\left(\bigcup_{i=1}^{\infty} E_i\right)$$

$$= P\left(\bigcup_{i=1}^{\infty} F_i\right) \longrightarrow \text{mutually exclusive events}$$

$$= \sum_{i=1}^{\infty} P(F_i) = \lim_{n\to\infty} \sum_{i=1}^{n} P(F_i)$$

$$= \lim_{n\to\infty} P\left(\bigcup_{i=1}^{n} F_i\right) = \lim_{n\to\infty} P\left(\bigcup_{i=1}^{n} E_i\right) = \lim_{n\to\infty} P(E_n)$$

finite

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Even if we have 70 people , P (~~atp~~ atleast 2 have same birthday) is 99.9 %

23 $\rightarrow$ 50 %

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If $\{E_n, n \geq 1\}$ is decreasing sequence

then $\lim_{n\to\infty} P(E_n) = P(\lim_{n\to\infty} E_n)$

then $\{E_n^c, n \geq 1\}$ becomes increasing sequence

then

$$\lim_{n\to\infty} P(E_n^c) = P(\lim_{n\to\infty} E_n^c) = P\left(\bigcup_{i=1}^{\infty} E_i^c\right) = P\left[\left(\bigcap E_i\right)^c\right]$$

$$\Rightarrow 1 - \lim_{n\to\infty} P(E_n) = 1 - P\left(\bigcap_{i=1}^{\infty} E_i\right)$$

$$\Rightarrow \lim_{n\to\infty} P(E_n) = P\left(\bigcap_{i=1}^{\infty} E_i\right)$$

$$= P(\lim_{n\to\infty} E_n)$$

Birthday Paradox :

At least two people have same birthday.

No two people have same birthday.

for $n = 1$ $\quad P(E_1) = \frac{365}{365}$ $\quad P(E_1^c) = 1 - 1 = 0$

for $n = 2$ $\quad P(E_2) = \left(\frac{365}{365}\right) \times \left(\frac{364}{365}\right)$ $\quad P(E_2^c) = 1 - \frac{364}{365} = \frac{1}{365}$

for $n = 3$ $\quad P(E_3) = \left(\frac{365}{365}\right) \times \left(\frac{364}{365}\right) \times \left(\frac{363}{365}\right)$ $\quad (n-r)$

for $n = r+1$ $\quad P(E_{r+1}) = \left(\frac{365}{365}\right) \times \left(\frac{364}{365}\right) \times \cdots \times \left(\frac{365-r}{365}\right)$

$$= \frac{{}^{365}P_{r+1}}{(365)^{r+1}}$$

$(365)^{r+1}$

→ for $n = 23$

$P(E_{23}) = 0.507$

→ for $n = 41$

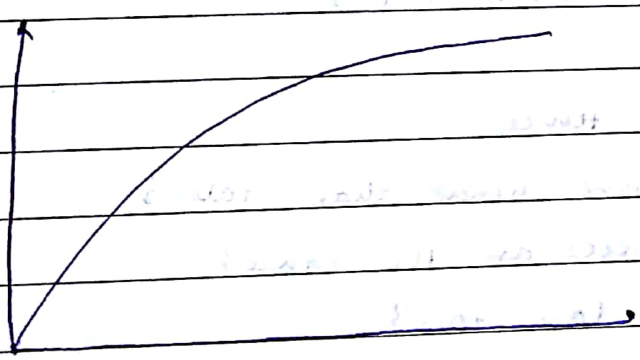
$P(E_{41}) = 0.903$

→ for $n = 70$

$P(E_{70}) = 0.999$

→ for $n = 366$

$P(366) = 1$



Independence of Event

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→ 2 R, 1 G, 1 B, 1 Y

$P(E_1) = 6/10$ $\quad$ $P(E_2) = 4/10$

1 Red $\quad$ 1 Green

$P(E_1 \cap E_2) = \frac{2}{10}$

$\boxed{P(E_1) \cdot P(E_2) \neq P(E_1 \cap E_2)}$ → Here

⇒ Just by adding 1 pen, the event no longer remains independence

Eg. Throwing a Dice:

E_1 : In first throw, get 4

E_2 : 2nd, get 5

$P(E_1) = \frac{1}{6}$ $\quad$ $P(E_2) = \frac{1}{6}$

$P(E_1 \cap E_2) = \frac{1}{36} = P(E_1) \cdot P(E_2)$

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Ex. Toss a fair coin 3 times

E_1 = { there are more heads than tales }

E_2 = { first 2 tosses are the same }

E_3 = { heads on last toss }

HHH	HHT	TTH	TTT
	HTH	THT	
	THH	HTT	

$P(E_1) = 1/2$ $\quad$ $P(E_2) = 1/2$ $\quad$ $P(E_3) = 1/2$

$P(E_1 \cap E_2) = 1/4 = P(E_1) \cdot P(E_2) = 1/4$ ✓ Independent

$P(E_1 \cap E_3) = 3/8 \neq P(E_1) \cdot P(E_3)$ ✗ Dependent

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$P(E_2 \cap E_3) = \frac{1}{4} = P(E_2) \cdot P(E_3)$ ✓ → Independent

Independence :

Two events A and B are said to be independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

Mutual Independence :

For 3 events are mutual independent if

$$P(A \cap B \cap C) = P\{A\} . P\{B\} . P\{C\}$$

Let $E_1, E_2, \ldots, E_n$ ~~are~~ be n events. They are mutually independent if

$$P(E_1 \cap E_2 \ldots \cap E_n) = P(E_1) \cdot P(E_2) \cdots P(E_n)$$

If E_1, E_2, E_3 are independent events iff — 1

$$\left.\begin{aligned} P(E_1 \cap E_2) &= P(E_1) . P(E_2) \\ P(E_1 \cap E_3) &= P(E_1) \cdot P(E_3) \\ P(E_2 \cap E_3) &= P(E_2) \cdot P(E_3) \end{aligned}\right\} \quad 2$$

$$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2) \cdot P(E_3) \longrightarrow 3$$

1 is true ⇒ 3 is true

3 is true ⇏ 1 is true

2 + 3 is true ⇒ 1 is true

In previous eg. :

$$P(E_1 \cap E_2 \cap E_3) = 1/8$$

$$= \frac{1}{2} * \frac{1}{2} * \frac{1}{2}$$

$$= P(E_1) \cdot P(E_2) \cdot P(E_3)$$

But we can't say E_1, E_2, E_3 are mutually independent because

$$P(E_1 \cap E_3) \neq P(E_1) . P(E_3)$$

$2^n - (n+1)$

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n events $E_1, E_2, \ldots, E_n$ are mutually independent iff

$P(E_i E_j) = P(E_i) \cdot P(E_j) \quad i \neq j \quad i,j = 1,2,\ldots,n$

$P(E_i E_j E_k) = P(E_i) P(E_j) P(E_k) \quad i \neq j \neq k \quad i,j,k = 1,2,\ldots,n$

⋮

$P(E_1 E_2 \ldots E_n) = P(E_1) \cdot P(E_2) \ldots P(E_n)$

Events	No. of trials
2	1 $({}^2C_2)$
3	4 $({}^3C_2 + {}^3C_3)$
⋮	
n	${}^nC_2 + {}^nC_3 + {}^nC_4 + \ldots + {}^nC_n$
	$= 2^n - (n+1)$

⇒ we must check $2^n - n - 1$ relations

→ If E_1 and E_2 are ~~mutually~~ independent, then:

i.) E_1^c & E_2 are also independent

ii.) E_1 & E_2^c " " "

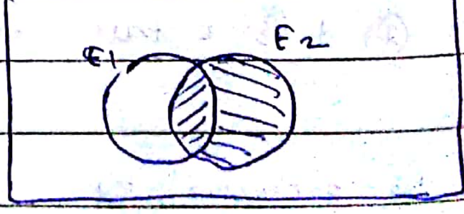
iii.) E_1^c & E_2^c " " "

i.) $E_2 = (E_2 \cap E_1) \cup (E_2 \cap E_1^c)$

ii.) $E_1 = (E_1 \cap E_2) \cup (E_1 \cap E_2^c)$

$E_2 = \underbrace{(E_2 \cap E_1) \cup (E_2 \cap E_1^c)}_{\text{mutually exclusive}}$

~~already~~ independent



To prove : $P(E_1^c \cap E_2^c) = P(E_1^c) \cdot P(E_2^c)$

$(E_1 \cup E_2)^c = E_1^c \cap E_2^c$

$\Rightarrow P(E_1 \cup E_2)^c = P(E_1^c \cap E_2^c)$

$\Rightarrow 1 - P(E_1 \cup E_2) = P(E_1^c \cap E_2^c)$

independent

$\Rightarrow P(E_1^c \cap E_2^c) = 1 - [P(E_1) + P(E_2) - P(E_1 \cap E_2)]$

$= 1 - [P(E_1) + P(E_2) - P(E_1) \cdot P(E_2)]$

$= 1 - P(E_1) - P(E_2)[1 - P(E_1)]$

$= [1 - P(E_2)][1 - P(E_1)]$

$= P(E_2^c) \cdot P(E_1^c)$

Hence Proved

If the result of the 1st toss is head, then E_1: wins the game if 2 heads occur out of 3 trials. E_2: If this doesn't happen. (need two tails)

$E_1 : \{HHT, HTH, HHH\}$: $P(E_1) = 3/4$

$E_2 : \{HTT\}$ $P(E_2) = 1/4$

$S = \{HTH, HHT, HHH, HTT\}$

Teacher's Signature..............

Conditional Probability

OR

→ $S = \{ \quad \}_8$ (coin tossed 3 times)

$E_1 = \{HHH, HHT, HTH, THH\}$

$A = \{HHH, HHT, HTH, HTT\}$ — when 1st is fixed

E_1, E_2, A are all ~~separ.~~ events

$A \cap E_1 = \{HHH, HTH, HHT\}$

$$\boxed{P(E_1 | A) = \cancel{P(E_1 \cap A)}\ \frac{P(E_1 \cap A)}{P(A)}} = \frac{3/8}{4/8} = \frac{3}{4}$$

$(P(A) \neq 0)$

→ In case of independent,

$P(E_1|A) = P(E_1)$

$\Rightarrow P(E_1) \cdot P(A) = P(E_1 \cap A)$

* A is going to influence $P(E_1)$ ⇒ dependent
  A isn't " " $P(E_1) = P(E_1|A) = P(E_1)$ ⇒ independent

Proposition → Let A and B be events with $P(B) \neq 0$. Then A & B are independent iff (Have to prove from both sides)

$P(A|B) = P(A)$

Proof: Let A and B be two independent events. By def^n of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} \quad [\because \text{independent}]$$

$$= P(A)$$

Now, let us assume

$P(A|B) = P(A)$ (have to prove independent)

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

⇒ A & B are independent events. Hence Proved.

Teacher's Signature.........

Throwing a dice

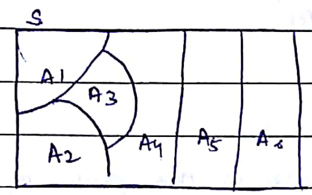
E_1 : getting even no. $P(E_1) = 1/2$

E_2 : getting 2. $P(E_2) = 1/6$

P(getting 2 given even no. has come) : $\frac{1}{3}$

$P(E_2 | E_1) = 1/3$

Partition of a set



$A_i \cap A_j = \phi$

$A_1 \cup A_2 \cup A_3 \dots \cup A_6 = S$

Partition of a Sample Space

The events $A_1, A_2, \dots, A_n$ form the partition of the sample space if the following conditions hold :

The events are pairwise disjoint

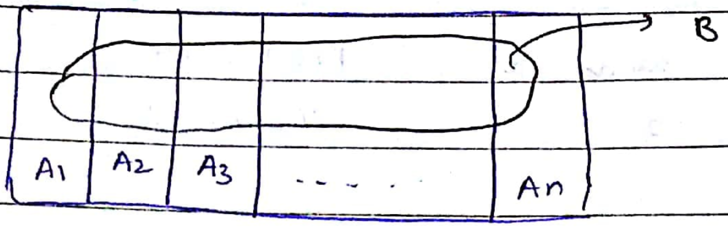
$A_i \cap A_j = \phi$ $i \neq j$

$A_1 \cup A_2 \cup \dots \cup A_n = S$

Total Probability Theorem :

Let $A_1, A_2, \dots, A_n$ form a partition of the sample space with $P(A_i) \neq 0$ for all i and B is any event.

Then,



mutually exclusive

$$B = (B \cap A_1) \cup (B \cap A_2) \cup \cdots \cup (B \cap A_n)$$

$$= \bigcup_{i=1}^{n} (B \cap A_i)$$

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i)$$

$$= \sum_{i=1}^{n} P(B|A_i) \cdot P(A_i)$$

Two players, A and B, participate in the game of throwing 2 dices. The first player who gets a sum of 7 is awarded the winner (Assume A throws 1st)

S(1, 6)

$P(A \text{ wins}) = p + q^2p + q^4p + \cdots + q^{n-1}p + \cdots$

$= p\left[1 + q^2 + \cdots\right]$

$= \dfrac{p}{1-q^2}$

$= \dfrac{1/6}{1 - \frac{25}{36}} = \dfrac{6}{11}$

$P(B \text{ wins}) = \dfrac{5}{11}$

$\Rightarrow P(A \text{ wins}) > P(B \text{ wins})$

An ice cream seller has to decide whether he has to order the stock or not. The chances of selling is 90% on sunny day, 60% on cloudy day, 20% on rainy day

Wheather forecast :

chance of sunshine is 30%. $P(E_1) = 3/10 = 0.3$

chance of cloud is 45%. $P(E_2) = 0.45$

chance of rain is 25%. $P(E_3) = 0.25$

Mutually disjoint

(Apply Total Probability theorem)

Let A : event that you completely sell all the ice-creams

$P(A|E_1) = 0.9$

$P(A|E_2) = 0.6$

$P(A|E_3) = 0.2$

$P(A) = \sum_{i=1}^{3} P(A|E_i)\,P(E_i)$

$= \left(\frac{3}{10}\right)\left(\frac{9}{10}\right) + \left(\frac{45}{100}\right)\left(\frac{6}{10}\right) + \left(\frac{25}{100}\right)\left(\frac{2}{10}\right) = \dfrac{270 + 270 + 50}{1000} = \dfrac{590}{1000}$

$= \boxed{0.59}$

Can you predict weather if sale is 100% ? → Posterior Probability

(reverse of what we solved)

↓

Using Baye's Theorem

↓

Relⁿ b/w $P(A|B)$ & $P(B|A)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\cancel{P(A \cap B)}\ P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow \boxed{P(A|B) \cdot P(B) = P(B|A) \cdot P(A)}$$

$$\Rightarrow P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

$$\boxed{P(A|B) = \frac{P(A) \cdot P(B|A)}{\sum_{i=1}^{n} P(B|A_i) \cdot P(A_i)}}$$ → Baye's Theorem

Baye's Theorem :

Let $B_1, B_2, \ldots, B_n$ be a set of mutually exclusive events of the sample ~~set~~ space S with $P(B_k) \neq 0$, $k = 1, 2, \ldots, n$ and A be any event of S with $P(A) \neq 0$

then

Priori Probability

$$\boxed{P(B_k|A) = \frac{P(B_k)\, P(A|B_k)}{\sum_{i=1}^{n} P(B_k)\, P(A|B_k)}}$$

Posterior prob.

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→ If sale is 100%, prob. that it is sunny day

$$P(\quad) = \frac{0.3 * 0.9}{0.59}$$

$P(\text{cloudy day}) =$