

16VCS126

The LNM Institute of Information Technology
 Department: CSE
 Introduction to complexity theory
 Exam Type: Mid Term

Time: 90 minutes

Date: 26 February 2019

Max. Marks: 30

Instruction: All the questions are compulsory. please read the question paper carefully and then try to answer.

Q.1 Answer the following questions and write your detailed explanation of your answer. Answer without clear explanation carry zero marks.

- ✓(a) (2 points) Is $f(m/n)=m+n$ where m, n are natural numbers a function?
- ✓(b) (2 points) Let $T=\{(i, j, k) \mid i, j, k \text{ are natural numbers}\}$. Is T countable?
- (c) (2 points) Let $w \in A \Leftrightarrow w' \in B$ and B is regular language. Then does this imply that A is regular language?
- ✓(d) (2 points) Let A and B are decidable languages. Then the language $AB = \{xy \mid x \in A, y \in B\}$ is decidable. Prove or disprove.
- (e) (2 points) If L_1 is a decidable language and L_2 is Turing recognizable language then the language $L_1 - L_2$ must be Turing-recognizable. Prove or disprove.

Q.2 Let $E_{TM} = \{\langle M \rangle \mid L(M) = \emptyset\}$ and $EO = \{\langle M \rangle \mid w \in L(M) \Leftrightarrow |w| \pmod{2} = 0\}$.

- ✓(a) (5 points) Prove that E_{TM} is undecidable.
- (b) (5 points) Prove that EO is undecidable.

Q.3 (10 points) Let $M=(Q, \Gamma, \Sigma, q_0, q_{\text{accept}}, q_{\text{reject}}, \lambda, \delta)$ where $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, \text{RESET}\}$ and λ is blank symbol be a Turing machine. If in the transition function of this Turing machine the letter R has specified then read write head will take one step right on the tape. But if in the transition function the word RESET has mentioned then our read write head will move to the left end of the tape. Prove that the languages recognized by this Turing machine is same as our standard Turing machine.