Jayprakash Sir

Kenneth H. Rosen Discrete Mathematical Structure

Propositional Logic

Propositions

$x+1=2 \longrightarrow$ not proposition

$\downarrow$

don't know true or false

p q

1 p and q are two proposition

p is a proposition $\neg$(negation of p)

If p is True then $\neg p$ is false

p is False then $\neg p$ is True

2 Conjuction ($\wedge$)

p and q are two proposition

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction ($\vee$)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
T	T	T

p and q are two proposition

p	q	$p \oplus q$
T	T	F
F	T	T
T	F	T
T	T	F

Premium

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Conditional proposition $(p \to q)$

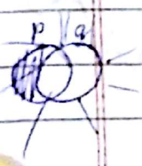
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$1 + 2 = 3 \to$ statements proposition

$p \leftrightarrow q$ ~~con~~ (on condition)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$(p \vee \neg q) \to p \wedge q$



operator	precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\to$	4
$\leftrightarrow$	5

1 Exercise

01/Mar/2019 Propositional Equivalence.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

* The compound proposition that is always true that occur in it is called tautology.

* A compound proposition that is always false is called contradiction

Logical equivalence
The compound proposition p and q are called logically equivalent if $p \leftrightarrow q$ in a tautology, $p \equiv q$

$p \leftrightarrow q$ $\neg(p \vee q)$ and $\neg p \wedge \neg q$

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

De Morgan's law

$\neg(p_1 \vee p_2 \vee \ldots \vee p_n) = (\neg p_1 \wedge \neg p_2 \wedge \ldots \wedge \neg p_n)$

$\neg(p_1 \wedge p_2 \wedge \ldots \wedge p_n) = (\neg p_1 \vee \neg p_2 \vee \ldots \vee \neg p_n)$

$\neg(p_1 \vee p_2) \equiv \neg p_1 \wedge \neg p_2$

$\neg(p_1 \vee p_2) \leftrightarrow \neg p_2 \wedge \neg p_2$

Q $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$

~~p∨~~ Don't skip the steps.

$\neg p \wedge (p \vee \neg q)$

$\neg p \wedge \neg q$.

Q $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

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15/May/2017 (Absent)

17/May/2017 Predicate logic

$x > 3$ → Predicate
subject

x is greater than, 3,
sub Predicate ← Propositional function.

$P(x)$: x is greater than 3
$P(4)$: True
$P(2)$: False

→ M 3 is functioning properly
P(M 3): ___ ___ : only true for M 3 otherwise F.
M4 is not ___
P(M 4) : False

→ Universial Quantifier
The universal quantification of $P(x)$ is a statements $P(x)$ for all values of x in the domain" : $\forall x P(x)$
" $x > 3$ "

$P(x)$
$\forall x P(x)$: universally quantifying
is false for $x = 1, 2$
$y \cdot x \neq 1, 2$
$\forall x P(x)$ is true
$\exists x P(x)$ is false
$x = 1, 2)$

Statement : When true ?
$\forall x P(x)$ $P(x)$ is true for every x $P(x)$ is false
when false

$\neg \forall x (4x^2 > 3x^2) \equiv \exists x \neg (4x^2 > 3x^2)$

$\exists x P(x) \xrightarrow{} \text{True}$ is same as
$P(x_1) \vee P(x_2) \vee \ldots P(x_n) \rightarrow$ True

Ex Truth val of $\exists x P(x)$

$P(x) : x^2 > 10$
$x : \{1, 2, 3, 4\}$ disjunction operator.
$\exists x P(x) \equiv P(1) \vee P(2) \vee P(3) \vee P(4)$
$P(4) \quad 4^2 > 10 \quad \exists x P(x)$ is true.

$\exists x P(x) \vee Q(x)$ Not Equal. $\exists x (P(x) \vee Q(x))$

$\neg \forall x P(x) \equiv \exists x \neg P(x)$
$\neg \exists x Q(x) \equiv \forall x \neg Q(x)$

$\neg \forall x (x^2 > x) \quad \exists x \neg (x^2 > x)$

20/Mar/2017 $\forall x P(x) \vee Q(x)$
$\forall x (P(x) \vee Q(x))$
$(\forall x P(x)) \vee Q(x)$

$\exists x P(x) \vee Q(x)$
$\exists (x) (P(x) \vee Q(x))$
$(\exists x P(x)) \vee Q(x)$
$\neg \forall x (P(x) \rightarrow Q(x))$ and
$\exists x (P(x) \wedge \neg Q(x))$

$\neg (x \rightarrow y) \equiv x \wedge \neg y$ $\quad$ $\neg (P(x) \rightarrow Q(x))$
$P(x) \wedge \neg Q(x)$

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$P(x_1) \vee P(x_2) \vee \ldots$ —

$\sim P(x_1) \wedge \sim P(x_2) \ldots$

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$\neg \forall x (P(x) \to Q(x)) \equiv \exists x \neg (P(x) \to Q(x))$

$\equiv \exists x \, P(x) \wedge \neg Q(x)$

Pg 43

$\neg \forall x (P(x) \to Q(x)) \equiv \exists x \neg (P(x) \to Q(x))$

$\equiv \exists x \, P(x) \wedge \neg Q(x)$

Nested Quantifier

$\forall x \exists y (x+y=0)$

$\forall x + y \; P(x) \quad P(x) \; x+y=0.$

Ex $\forall x (C(x) \vee \exists y (C(y) \wedge F(x,y)))$

where $F(a,b)$ means a and b are Predicate and the domain for x, y and z Constitute of all students is your school.

For every student x in your school x has computer or their is a student y such that y has computer and x and y are friends

Pg-54 Quiz Propositional predicate

Mathematical Induction

Boolean Algebra

~~{1,0}~~ $\{0,1\}$.

OR	AND
$1+1=1$	~~1+~~ $1\cdot 1=1$
$1+0=1$	$1\cdot 0=0$
$0+0=0$	$0\cdot 1=0$
$0+1=1$	$0\cdot 0=0$.

$\bar{0}=1$

$\bar{1}=0$

find the value of

$= 1\cdot 0+0+1$

$= 0$

$(T \wedge F) \vee \neg(T \vee F) = F$

$x+y=y+x$

$xy=yx$

$x+(y+z)=(x+y)+z$

$x(yz)=(xy)z$

Ex694

$\boxed{x+yz=(x+y)(x+z)}$

$x(y+z)=xy+xz$

$x(y+0)$ $\bar{x}\cdot 1+(\bar{y}+z)$

$x+(y\cdot 1)$

Q $(x+y)\bar{z} = x\bar{z}+y\bar{z}$

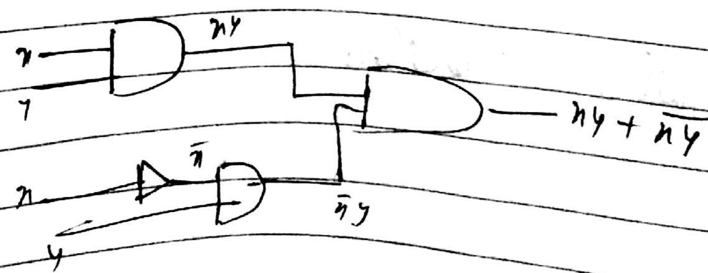
$= x\bar{z}\cdot 1 + y\bar{z}1$

$= x\bar{z}(y+\bar{y}) + y\bar{z}(x+\bar{x})$

$= x\bar{z}y + x\bar{z}\bar{y} + y\bar{z}x + y\bar{z}\bar{x}$

Ex 648.

Ex 704



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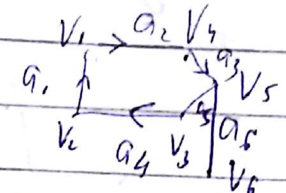
$G = (V, E)$

$V = \{V_1, V_2, V_3, \ldots V_n\}$

$E = \{a_1, a_2, \ldots a_n\}$



Directed Graph.

degree of vertex a_5 is 3

" " " a_1 is 2

$$2e = \sum_{v \in V} \deg(v)$$

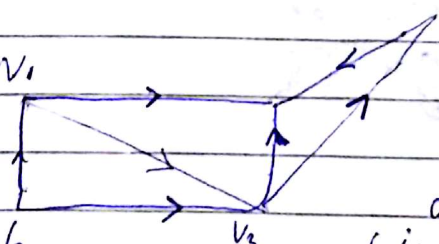
$a_1 = e_1 \quad a_2 = e_2 \sim a_n = e_n$

Ex How many edges are there in a graph with 10 vertices each of degree six?

$2e = 60$

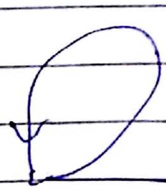
$e = 30.$

Ex

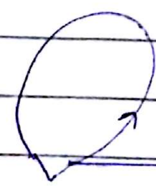


$\deg^-(V_1) = 2$
(indegree)

$\deg^+(V_1) = 2$



Inward

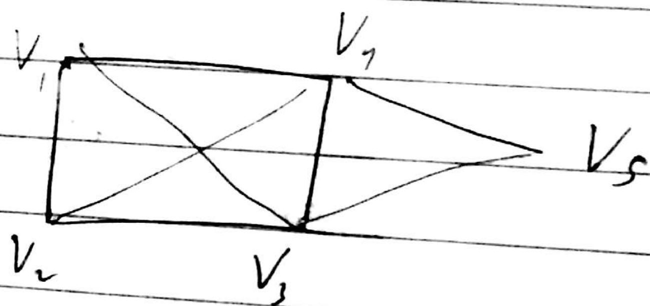


outward

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

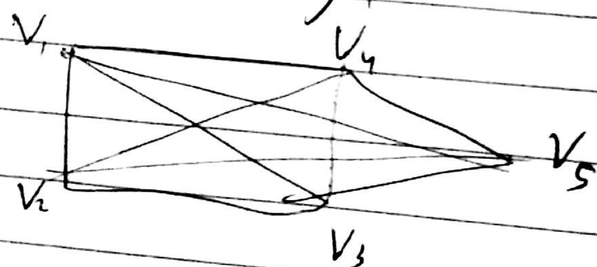
Complete graph:
n- vertices divided by K_n is simple graph that contains exactly one edge b/w each pair of vertices.

Not a complete graph



each pair must have a edge

Complete graph.



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