2 Aug 17

OTA

Formulation → Objective functions → linear (of power of all variables = 1)

Objective functions → Non linear (of power of any one variable or more is greater than 1)

Eg

$$Z = f(x)$$
$$Z = a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_nx_n \quad \text{Objective } f^n$$

$$l_{11}x_1 + l_{12}x_2 + \ldots\ldots l_{1n}x_n \le c_1$$
$$l_{21}x_1 + l_{22}x_2 + \ldots\ldots l_{2n}x_n \le c_2$$
$$\vdots$$
$$l_{m1}x_1 + l_{m2}x_2 + \ldots\ldots l_{mn}x_n \le c_m$$

Q. A company sells 2 different products A and B making profit of Rs 40 & Rs 30 per unit respectively. They are both produced with the help of a common production process and sold in different markets. The production process has a total capacity of 80,000 man hours. It takes 3 hrs to produce a unit of A and 1 hr to produce a unit of B. The market has been surveyed and company officials feels that maximum number of units of A that can be sold in 8000 units and that of B is 12000 units. Subject to the limitations, production can be sold in any combination. Formulate problem as an LP model to maximize profit.

Ans Let x_1, x_2 be the no of units of A and B to be produced respectively

$$Z = 40x_1 + 30x_2 \; ($$

$$3x_1 + x_2 \leq 30{,}000 \text{ (Max hours)}$$

$$\left.\begin{array}{l} x_1 \leq 8000 \\ x_2 \leq 12000 \end{array}\right\} \text{(Marketing)}$$

Important Definitions

1. Solution: The value of the decision variable x_j ( ) that satisfy the constraints of an LP problem is said to constitute the solution to that LP problem.

2. Feasible Solution: The set of decision variable x_j ( ) that satisfy the constraints and non negativity condition of an LP problem is said to constitute the feasible solution to that LP problem.

3. Basic Solution: For a set of m smallest eq^n in n variables ($n > m$) a sol^n obtained by putting (n-m) variables equal to zero and so now only m variables are left and m eq^{n}s are there hence we can solve for these m variables easily. These m variable sol^n are called Basic sol^n.

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Magic
Date
Page

Convert to Standard form

Objective funⁿ

optimize $\Rightarrow z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$
(max/min)

Subject to

$$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n \le b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n \le b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n \le b_m$$

Min(z) $\rightarrow$ Max(z^*) $\qquad z^* = -z$

On the basis of equality in constraints we select one of the 3 variables

- → Slack variable
- → Surplus variable
- → Decision variable

$$Max(z) = c_1x_1 + c_2x_2 + \cdots c_nx_n + 0s_1 + 0s_2 \cdots 0s_n$$

Sub.

$$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n + s_1 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n + s_2 = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n + s_m = b_m$$

$$z = \sum_{j=1}^{n} c_jx_j + \sum_{i=0}^{m} 0s_i$$

Constraints

$$\sum_{j=1}^{n} a_{ij} x_j + S_i = b_i \qquad i = 1, 2, \ldots m$$

$x_j, S_i \geq 0$ for all i and j

Optimize (Max or Min) $z = Cx + 0.S$

$Ax + s = b, \quad x, s \geq 0$

Q Min $(z) = 2x_1 + 3x_2$

Sub. to $3x_1 - x_2 \leq 2$

$x_1 + x_2 \leq +1$

or $-(x_1 + x_2) \geq 1$

Max (z^*) $z^* = -z = -(2x_1 + 3x_2) + 0S_1 + 0S_2$

Sub. to $3x_1 - x_2 + S_1 = 2$

$x_1 + x_2 + S_2 = 1$

Simplex Method

Step 1 → (Formulation and convert to S.F)

Step 2 → Set up the initial solution.
In order to get the basic feasible solution
$[x_B = B^{-1}b]$

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n + 0s_1 + 0s_2 + \cdots + 0s_m$$

coefficient of basic variable	variable in Basis B	variable of basic variable $b = (x_{B_i})$	x_1	x_2	x_n	s_1	s_2	s_m
	s_0							
C_{B_1}	s_1	$x_{B_1} = b_1$	a_{11}	a_{12}	a_{1n}	1	0	0
C_{B_2}	s_2	$x_{B_2} = b_2$	a_{21}	a_{22}	a_{2n}	0	1	0
⋮	⋮	⋮						
C_{B_m}	s_m	$x_{B_m} = b_m$	a_{m1}	a_{m2}	a_{mn}	0	0	1

z_j, $c_j - z_j$

$s_1, s_2 \text{ --- } s_n \rightarrow$ slack variable

we put m variables to zero.
Then n-m variables are basic variables and their coefficients are C_{B_1}, C_{B_2} ——.

Step3 → (i) If all $c_j - z_j \leq 0$ then basic feasible solution is optimum.

(ii) If atleast one $(c_j - z_j) > 0$ and each column has atleast one positive element a_{ij} for some row, then this indicates the improvement in the value of the objective function.

(iii) If atleast one column of the coefficient (a_n) for which $(c_k - z_j) > 0$ and all other elements are negative $(a_{ik} < 0)$ then there exist unbound soln to the given problem

Step 4 → If Case III of Step 3 holds then select a variable that has the largest $(c_j - z_j)$ value to enter the next solution. that is

$$C_k Z_k = \max \{(c_j - z_j),\ c_j - z_j > 0\}$$

Then column is to be entered is called key column or pivot column.

Step 5 → $\frac{a_{Br}}{a_{rj}} = \text{Min} \left\{ \frac{a_{ri}}{a_{rj}} \quad a_{rj} > 0 \right.$

Max $z = 3x_1 + 5x_2 + 4x_3$

Sub to
$$2x_1 + 3x_2 \le 8$$
$$2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$

Max $z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$

$$2x_1 + 3x_2 + s_1 = 8$$
$$2x_2 + 5x_3 + s_2 = 10$$
$$3x_1 + 2x_2 + 4x_3 + s_3 = 15$$

$x_1 = x_2 = x_3 = 0$

$s_1 = 8,\ s_2 = 10,\ s_3 = 15$

		c_j	3	5	4	0	0	0
C_B	variable in Basic	Sol. value	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	8	2	3	0	1	0	0
0	s_2	10	0	2	5	0	1	0
0	s_3	15	3	2	4	0	0	1

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Simplex Method

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3$$

Subject to
$$2x_1 + 3x_2 \le 8$$
$$2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$

Step 1 → $\text{Max } z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$

Sub
$$2x_1 + 3x_2 + s_1 = 8$$
$$2x_2 + 5x_3 + s_2 = 10$$
$$3x_1 + 2x_2 + 4x_3 + s_3 = 15$$

$x_1 = x_2 = x_3 = 0$ } setup initial basic variable solution

$x_1 = x_2 = x_3 = 0,\ s_1 = 8,\ s_2 = 10,\ s_3 = 15$

Min ratio x_B/x_2 pivot row	Profit/unit C_B	variables in basic B.	C_j / Sol^n value $b = x_B$	3 x_1	5 x_2	4 x_3	0 s_1	0 s_2	0 s_3
8/3	0	s_1	8	2	3	0	1	0	0
10/2	0	s_2	10	0	2	5	0	1	0
15/2	0	s_3	15	3	2	4	0	0	1
			Z_j	0	0	0	0	0	0
			$C_j - Z_j$	3	5	4	0	0	0

coeff of basic variable i.e. s_1, s_2, s_3

$Z = 0$

Key column

min. of (x_B/x_2) is pivot row.

$3 - 2\left(\frac{2}{3}\right)$ $\quad \frac{3-4}{2}$

$$c_j - z_j = c_j - c_B B^{-1} a_j = c_j - c_B$$

$z_1 = 0(2) + 0(0) + 0(3) = 0$ (x_1 column)
$z_2 = 0(3) + 0(2) + 0(2) = 0$ (x_2 col^n)
$z_3 = 0(0) + 0(5) + 0(4) = 0$ (x_3 col^n)

$c_1 - z_1 = 3 - 0 = 3$
$c_2 - z_2 = 5 - 0 = 5$
$c_3 - z_3 = 4 - 0 = 4$

If $c_j - z_j \le 0$, feasible sol^n & optima
else not and proceed to next step.

Key / Pivot column : Col^n in which value of $c_j - z_j$ is maximum.

$$\text{Max } (c_j - z_j) : (c_j - z_j) \ge 0, \quad j = 1, 2, 3$$

* Intersection key row and pivot column gives key element i.e. 3

Now make key element as 1 and all others elements below it as zero.

Row operations

$R_1(\text{new}) = R_1(\text{old})/3$

$R_2(\text{new}) = R_2(\text{old}) - 2R_1(\text{new})$
$R_3(\text{new}) = R_3(\text{old}) - 2R_1(\text{new})$

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* Treat table as a determinant

Which variable is to be left and which needs to be introduced?

Not defined.

Pivot column replaces pivot row
vari variable

Min ratio	Profit/unit	variable in Basic B	Sol^n col^n c_j	3 x_1	5 x_2	4 x_3	0 s_1	0 s_2	0 s_3
—	5	x_2	8/3	2/3	1	0	1/3	0	0
$\left(\frac{14}{3}\right)/5$	0	s_2	14/3	−4/3	0	5	−2/3	1	0
$\left(\frac{29}{3}\right)/4$	0	s_3	29/3	5/3	0	4	−2/3	0	0
	$Z = 40/3$		z_j	10/3	5	0	5/3	0	0
			$c_j - z_j$	−1/3	0	4	−5/3	0	0

$s_1 = x_1 = x_3 = 0$ $\qquad$ > 0.

perform same process again till $c_j - z_j \le 0$

$$Z_1 = 5 \times \frac{2}{3} + 0 \times \left(\frac{-4}{3}\right) + 0 \times \left(\frac{5}{3}\right) = \frac{10}{3}$$

$$Z_2 = 5(1) + 0(0) + 0(0)$$

$$Z_3 = 0 \quad 5(0) + 0(5) + 0(4)$$

$$C_1 - Z_1 = 3 - \frac{10}{3} = \frac{-1}{3}$$

$$C_2 - Z_2 = 5 - 0 = 5$$

$$C_3 - Z_3 = 4 - 0 = 4$$

In min. ratio, −ve and ∞ values are not considered.

min ratio (x_B/x_1)	profit/unit C_B	variable in Basis	sol value	x_1 (3)	x_2 (5)	x_3 (4)	s_1 (0)	s_2 (0)	s_3 (0)
4	5	x_2	8/3	2/3	1	0	1/3	0	0
−7/2 —	4	x_3	14/15	−4/15	0	1	−2/15	1/5	0
89/41	0	s_3	89/15	41/15	0	0	−2/15	−4/5	1
$Z = \frac{256}{15}$		Z_j		34/15	5	4	17/15	4/5	0
		$C_j - Z_j$		11/15	0	0	−17/15	−4/5	0

$$Z = x_1 c_1 + x_2 c_2 + x_3 c_3.$$

$$= 0(3) + \frac{14}{15}(5) \quad \frac{8}{3}(5) + \frac{14}{15}(4)$$

min ratio	profit/unit	sol. value	x_1 (3)	x_2 (5)	x_3 (4)	s_1 (0)	s_2 (0)	s_3 (0)
5	x_2							
4	x_3		0	0	1	186/123	76/615	4/41
3	x_1	89/41	1	0	0	−2/41	−12/41	15/41

$$R_2 \longrightarrow R_2 + \frac{4}{15} R_3$$

1^{st} phase → eliminate A_1 and A_2.
2^{nd} phase → do exactly same as in simplex method and get optimal solⁿ.

Iteration 1

C_B	variables Basics (B)	solⁿ variable	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
-1	A_1	3	13/7	0	-1	1/7	1	-1/7	
0	x_2	1	1/7	1	0	-1/7	0	1/7	
			2i	0	1	-1/7	-1	1/7	
			z = 13/7	0	-1	-1/7	0	-8/7	

Iterations 2
To remove A1 from the solution, enter S_2 on the basic by applying row operations.

$R_1(\text{new}) = R_1(\text{old}) \times 7$
$R_2(\text{new}) = R_2(\text{old}) * \frac{1}{2} R_1(\text{new})$

C_B	B	b_i	x_1	x_2	S_1	S_2	A_1^*	A_2^*
0	S_2	21	13	0	-7	1	7	-1
0	x_2	4	2	1	-1	0	1	0
		Z_i	0	0	0	0	0	0
$Z = 0.$		$C_i - Z_i$	0	0	0	0	-1	-1

$z^* = -A_1 - A_2.$

move to phase 2.

C_B	B	b	x_1	x_2	s_1	s_2
-1	x_1	$21/13$	1	0	$-7/13$	$1/13$
-1	x_2	$4/13$	0	1	$1/13$	$-2/13$

Kajal NB

Case I — When the constraints are of $\leq$ type

$$\sum_{j=1}^{N} a_{ij} x_j \leq b_i$$ $x_j \geq 0$ but RHS constraints are -ve

$\boxed{b_i < 0}$

in this case after adding the non negative slack variable s_i ($i = 1, 2, 3 \_\_\_ n$), the initial solⁿ

$s_i = -b_i$

$(x_1 = x_2 = x_3 = \_\_\_\_ = 0)$

(This is not optimal sol.) (as it violates ideal so
($b_i < 0$) here

Case 2

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \qquad x_i \geq 0$$

$$\sum_{j=1}^{n} a_{ij} x_j - s_i = b_i \qquad x_j \geq 0, \; s_i \geq 0$$

s_n is negative slack variables

NOTATION

1. $\leq$ slack variable
2. $\geq$ surplus variables (negative slack)
3. $=$ Artificial variables

$$\sum_{i=1}^{n} a_{ij} x_j - s_i + A_i = b_i$$

Total variable = $n + m + m$

An initial basic feasible solⁿ of the new system can be obtained by equating $(n+2m-m) \Rightarrow (n+m)$ variable equal to zero. So the new solⁿ is

$A_i = b_i \quad ; \quad i = 1, 2, \ldots m$

this is not the original solⁿ as this and prev. system are not equivalent.

Q. Min $= x_1 + x_2$

Sub to

$2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7 \quad , x_1, x_2 \geq 0$

$\Rightarrow$ Total variable : $x_1, x_2, s_1, s_2, A_1, A_2$

$(\text{Max } z^*) = -A_1 - A_2$

Sub to $=$ $2x_1 + x_2 - s_1 + A_1 = 4$

$x_1 + 7x_2 - s_2 + A_2 = 7$

		c_j	0	0	0	0	-1	-1	
C_B	var. in basics B	sol. value	x_1	x_2	s_1	s_2	A_1	A_2	Min ratio
-1	A_1	4	2	1	-1	0	1	0	4/1
-1	A_2	7	1	7 key	0	-1	0	1	7/7 = 1
		z_j	-3	-8	1	1	-1	-1	
		$c_j - z_j$	3	$\boxed{8}$	-1	-1	0	0	

key col. (all $c_j - z_j$ should be ≤ 0)

$\rightarrow$ initial solⁿ

$x_1 = x_2 = 0 \qquad A_1 = 4$

$s_1 = s_2 = 0 \qquad A_2 = 7$

$\sum_{j=1}^{N} a_{ij} x_j \geq b_i$ — we then two phase method

$x_j \geq 0$

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Two Phase Method

$\sum a_{ij} x_j - S_i + A_i$

→ A_i eliminates (do elimination till A_i is removed)

→ new variable | S_i has to be introduced

phase, After having A_1 & A_2 phase, complete the sol^n, still not optimal sol^n (and introduce slack variable non base variable x_1, x_2)

Iteration-I

To remove A_1 from solution enter S_2 in the basis by applying row operation.

Profit per unit / coeff of non basic var.	variable in basic x_B	sol value (x_B)	x_1	x_2	S_1	S_2	A_1	A_2	min ratio x_B/x_1
-1	A_1	3	13/7	0	-1	1/7	1	-1/7	21
0	x_2	1	1/7	1	0	-1/7	0	1/7	-
	Z_j		-13/7	0	1	-1/7	-1	1/7	
	$c_j - z_j$		13/7	0	-1	1/7	0	-8/7	

→ key column

See max value of $c_j - z_j = 13/17$

C_B

But through this optimum solⁿ / bounded solⁿ
non come to take $1/7$ key column / value of 2

Make key element 1.

$Z^* = -13/13$

$Z = 31/13$

$x_1 = 21/13$

$x_2 = 10/13$

$s_1 = 0, \quad s_2 = 0$

$x_1 = 0, \quad x_2 = 0$.

$R_1(new) = R_1(old) \times 7$

$R_2(new) = R_2(old) + \frac{1}{7} R_1(new)$

C_B	variable in basic B	sol. value X_B	x_1	x_2	S_1	S_2	A_1^*	A_2^*
0	S_2	21	13	0	-7	1	7	-1
0	x_2	4	2	1	1	0	1	8
		z_j	0	0	0	0	0	0
		$c_j - z_j$	0	0	0	0	-1	-1

≤ 0

Phase I compute

$x_1 = 0 \qquad x_2 = 4$

$S_1 = 0 \qquad S_2 = 21$

$A_1 = 0 \qquad A_2 = 0$

Phase 2

Now we have to maximise

$$\text{Max } z^* = -x_1 - x_2$$

			-1	-1	0	0	
C_B	variable in Basis	solu b (X_B)	x_1	x_2	s_1	s_2	min ratio
0	S_2	21	13	0	-7	1	21/13
-1	x_2	4	2	1	-1	0	4/2
	z_j	-2	-1	-1	0		
	$c_j - z_j$	1	0	-1	0		

C_B	var. in basic	sol. value (x_B)	x_1	x_2	s_1	s_2
-1	x_1	21/13	1	0	-7/13	1/13
-1	x_2	4/13	0	1	1/13	-2/13
	z_j		-1	-1	6/13	1/13
	$c_j - z_j$		0	0	-6/13	-1/13

$x_1 = 21/13$, $x_2 = 4/13$

in center of the inequalities
The right constraints is -ve then either

Big M/2 phase

Simplex Method

Max Z	Min Z
sub to	sub to
	* Two phase method
	* Big M Method

The Big-M Method

* Assign a large undesirable coefficient to the artificial variable from the point of view of objective function. M (unacceptable penalty)

* If the objective function z is to be minimized then a large positive price called penalty is assigned to each artificial variable.

* Similarly, if z is to be maximized then a very large negative price is assigned to each of these variables.

* The penalty is supposed to be assigned by -M for maximisation problem and +M for minimization problem, where M>0.

Step I Express the LP problem in S.form by adding slack variable, surplus variable and artificial variables. Assign a zero coefficient to both slack & surplus variables. Then add a very large positive coefficient

$S_1, S_2 \rightarrow$ Slack / surplus variable

$f(x) = x_1 + x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

(Maximisation case.) $+M$ (for minimisation) to the artificial variable in the objective function.

Step II

Initial basic feasible sol. is obtained by assigning zero to original variables ($x_1 = x_2 = x_3 = 0$)

Step III

Calculate the value of $c_j - z_j$ in the last row of the simplex table and examine the value

(i) if all $(c_j - z_j) \geq 0$, then the current basic feasible sol. is ~~optional~~ optimal.

(ii) If for a column k $(c_k - z_k)$ is most negative and all entries of this column is -ve then the problem has unbounded optimal sol.

Imp

(iii) If one or more $(c_j - z_j) < 0$ (minimisation) then select the variable to enter into the basis with largest negative $c_j - z_j$ value,

$$c_k - z_k = \text{Min}\{c_j - z_j \quad c_j - z_j < 0$$

Step IV Find key row and then key element

Step V Make the key element one by performing the row operation and make other entries of key column zero.

Min $Z = 5x_1 + 3x_2$

Sub to

$2x_1 + 4x_2 \le 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \ge 10$, $x_1, x_2 \ge 0$

Min $Z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + MA_1 + MA_2$

$x_1 = x_2 = 0, s_2 = 0$

$2x_1 + 4x_2 + s_1 = 12$

$2x_1 + 2x_2 + A_1 = 10$ (for equality constraints A_1, A_2 are added)

$5x_1 + x_2 - s_2 + A_2 = 10$

		C_j	5	3	0	0	M	M	
Loss/unit C_B	variable in basic B	Sol value	x_1	x_2	s_1	s_2	A_1	A_2	Min ratio
0	s_1	12	2	4	1	0	0	0	$12/2 = 6$
M	A_1	10	2	2	0	0	1	0	$10/2 = 5$
M	A_2	10	5	2	0	-1	0	1	$10/5 = 2$
		Z_j	7M	4M	0	-M	M	M	
		$C_j - Z_j$	5-7M	3-4M	0	M	0	0	
		min.							

$0(2) + 2(M) + 5(M) = 7M$

$0(4) + 2(M) + 2(M) = 4M$

A_1, A_2 are not original variables

Date ____
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Big-M Method

Eg:- Continued . . .

Loss/unit C_B	variable in Basic B	Basic value b	x_1 (5)	x_2 (3)	s_1 (0)	s_2 (0)	A_1 (M)	Min ratio
0	s_1	8	0	$16/5$	1	$2/5$	0	$8/(16/5)$
M	A_1	6	0	$6/5$	0	$2/5$	1	$6/(6/5)$
5	x_1	2	1	$2/5$	0	$-1/5$	0	$2/(2/5)$
		z_j	5	$6M/5+2$	0	$2M/5-1$	M	
		$c_j - z_j$	0	$-6M/5+1$ $-M/5$	0	$-2M/5+1$ $3M/5$	0	

↑ Min

Iteration II

Loss/unit	variables in Basic B	sol. value (x_B)	x_1	x_2	s_1	s_2	A_1	min ratio
3	x_2	$5/2$	0	1	$5/16$	$1/8$	0	$5/2/(\frac{1}{8})=40$
M	A_1	3	0	0	$-3/8$	$1/4$	1	$3/(\frac{1}{4})=12$
5	x_1	1	1	0	$-1/8$	$-1/4$	0	—
		z_j	5	3	$-\frac{3M}{8}+\frac{5}{16}$	$\frac{M}{4}-\frac{7}{8}$	M.	
		$c_j - z_j$	0	0	$\frac{3M}{5}\ \frac{+5}{16}$.	$\frac{-M}{4}+\frac{7}{8}$	0	

$\frac{15}{8}-\frac{5}{8}$

$\frac{5}{10}$

$\frac{15}{16}$

Iteration - III

Cost/unit (C_B)	Variable in basic B	Sol^n value b	5 x_1	3 x_2	0 s_1	0 s_2	Min ratio
3	x_2	1	0	1	1/2	0	
0	s_2	12	0	0	-3/2	1	
5	x_1	4	1	0	-1/2	0	
		z_j	5	3	-1	0	
		$c_j - z_j$	0	0	1	0	

All $(c_j - z_j) \geq 0$.

$x_1 = 4$ $\quad s_1 = 0$ $\quad$ Min $\boxed{Z = 23}$

$x_2 = 1$ $\quad s_2 = 12$ $\quad 5(4) + 3(1)$

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Primary Model	Dual Model (Dual LP program)
Max z	Min z
min z	max z
sub to $\leq$	$\geq$
$\geq$	$\leq$
$x_1, x_2, - x_n$	$y_1, y_2 -- y_n$

Duality Theorem

If an optional solution exists to either the primal or symmetric dual program, then this other program also has an optional sol and this two objective function have some optional value.

Primal LP	Dual LP.
1. right hand side constant $b_1, b_2 - b_m$ →	coefficients in z_y of $y_1 y_2 -- y_m$
2. Coefficient of $(x_1, x_2 -- x_n)$ $c_1, c_2 -- c_n$ →	right hand side constants in this dual LP program
3. Maximisation Prime LP with constraint $\leq$ →	Minimization dual LP ← with constant $\geq$

$$z_p = \text{Max } z = c_1 z_1 + c_2 z_2 + -- c_n z_n$$

sub to

$$a_1 x_1 + a_2 x_2 \quad -- \quad + a_n x_n \leq b_1$$

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$$a_{m1}x_1 + a_{m2}x_2 \qquad + a_{mn}x_n \le b_m$$

$$x_1, x_2 \; --- \; x_n \geq 0$$

$$Z_y = b_1y_1 + b_2y_2 + -- + b_my_n$$

Duality

Max $z = x_1 - x_2 + 3x_3$

sub to $x_1 + x_2 + x_3 \le 10$

$2x_1 - x_2 - x_3 \le 2$

$2x_1 - 2x_2 - 3x_3 \le 6$ $\quad x_1, x_2, x_3 \geq 0$.

Dual LP Min

$Z_y = 10y_1 + 2y_2 + 6y_3$

sub to

$y_1 + 2y_2 + 2y_3 \geq 1$

$y_1 - y_2 - 2y_3 \geq -1$

$y_1 - y_2 - 3y_3 \geq 3$

Q Max $z = -3x_1 - \cancel{3}2x_2$

Sub to

$x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \leq 3 \quad x_1, x_2 \geq 0$

Max $z = -3x_1 - \cancel{3}2x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

Sub to

$x_1 = 0$

$x_2 = 0$

$s_1 = -1$

$s_2 = 7$

$s_3 = -10$

$s_4 = 3$

$-x_1 - x_2 + s_1 = -1$

$x_1 + x_2 + s_2 = 7$

$-x_1 - 2x_2 + s_3 = -10$

$x_2 + s_4 = 3$

(key col)

C_B	variable in basic B	sol. value b (x_B)	$c_j \rightarrow$	-3	-2	0	0	0	0
				x_1	x_2	s_1	s_2	s_3	s_4
0	s_1	-1		-1	-1	1	0	0	0
0	s_2	7		1	1	0	1	0	0
(key row) 0	s_3	-10		-1	-2	0	0	1	0
0	s_4	3		0	1	0	0	0	1
		z_j		0	0	0	0	0	0
		$c_j - z_j$		-3	-2	0	0	0	0

Since all $(c_j - z_j) \leq 0$ and all sol. ~~vatu~~ value (x_{B_i}) are not non negative

Find first key row, then key col.

Select the key rows

$$\underset{(\text{row})}{x_{B_r}} = \text{Min}\{x_{B_i} \text{ --- } x_{B_n} < 0\}$$

$$\text{Min}\{-1, -10\} = -10\,(x_{B_3})$$

key column

$$\text{Min}\left\{\frac{c_j - z_j}{c_{rj}},\ y_{rj} < 0\right\}$$

elements of key row

$$\text{Min}\left\{\left(\frac{-3}{-1}\right), \left(\frac{-2}{-2}\right)\right\} = \text{Min}(3, 1) = 1$$

Dual Simplex Method

Step I Determine the initial sol
Convert to SF by adding slack, surplus and artificial variables

Step II Test the optimality of the sol^n
If all sol^n value are +ve ($x_{B_i} \geq 0$) then no need to apply a dual simplex method because improved sol^n can be obtained by simplex method easy. Otherwise go to Step III

Step III (Test the feasibility of this sol)

If there exist a row r for which the sol^n value is -ve ($x_{B_r} < 0$) and all elements in this row r and column j are +ve (i.e. $y_{rj} \geq 0$ for all j) then the correct sol^n is infeasible sol^n.

Step IV (Obtain improved solution)

(i) Select a basic var. associated with row (called key row) that has largest -ve value i.e.

$$x_{B_r} = \min \{ x_{B_i}, x_{B_i} < 0 \}$$

(ii) Determine the minimum ratio only for those column that has a -ve element in row r. Then select a non-basic variable for entering into the basis associated with this column for which

$$\min\left\{\frac{c_j - z_j}{y_{rj}},\quad y_{rj} < 0\right\}$$

C_B	variable in Basis B	sol^n value (x_B)	x_1	x_2	s_1	s_2	s_3	s_4
0	s_1	-1	-1	-1	1	0	0	0
0	s_2	7	1	1	0	1	0	0
0	s_3	-10	-1	-2	0	0	1	0
0	s_4	3	0	1	0	0	0	1
		z_j	0	0	0	0	0	0
		$c_j - z_j$	-3	-2	0	0	0	0

Since all $(c_j - z_j) \leq 0$ and all sol. value x_B are not non negative, an optimal sol^n but infeasible sol^n has been obtained.

key row (variable is to leave the basis)

$$= \text{Min}\{x_{B_i},\quad x_{B_i} < 0\}$$

$$= \text{Min}\{-1, -10\},\ -10\ (= x_{B_3})$$

key columns (variable to enter to basis)

$$\text{Min.}\left\{\frac{c_j - z_j}{y_{rj}}\qquad y_{rj} < 0\right\}$$

$$\text{Min}\left\{\left(\frac{-3}{-1}\right), \left(\frac{-2}{-2}\right)\right\} = \text{Min}(3,1) = 1.$$

we aren't taking s_3 cox it has already lefted the table

C_B	variable in basis B	sol^n value $b = (x_B)$	x_1	x_2	s_1	s_2	s_3	s_4
0	s_1	4	$-1/2$	0	1	0	$-1/2$	0
0	s_2	2	$1/2$	0	0	1	$1/2$	0
0	x_2	5	$1/2$	1	0	0	$-1/2$	0
0	s_1	-2	$-1/2$	0	0	0	$1/2$	1
	z_j		-1	-2	0	0	1	0
	$c_j - z_j$		-2	0	0	0	-1	0

key col.

$$\min \left\{ \frac{c_j - z_j}{c_{rj}}, y_{rj} < 0 \right\}$$

C_B	variable in basic B	sol^n value (x_B)	x_1 (−3)	x_2 (−2)	s_1 (0)	s_2 (0)	s_3 (0)	s_4 (0)
0	s_1	6	0	0	1	0	-1	-1
0	s_2	0	0	0	0	1	1	1
-2	x_2	3	0	1	0	0	0	1
-3	x_1	4	1	0	0	0	-1	-2
	z_j		-3	-2	0	0	-3	4
	$c_j - z_j$		0	0	0	0	$+3$	-4

$s_1 = 6$ | $x_1 = 4$
$s_2 = 0$ | $x_2 = 3$
$s_3 = 0$

$\boxed{Max\ z = 15}$