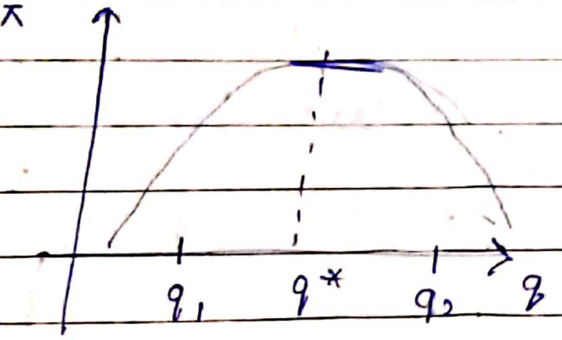
1. Neccessary & sufficient condition for profit max :-
2. Implicit function
3. ~~[illegible]~~ Constrained Maxmiz.
   a) Interpret lagragian Multiplier
4. Homogenous fn. & euler theorem.

π = Profit.

$\pi = \pi(q)$.



more q so more revenue beyond certain limit after some optimal amount then profit fall.

at $q_1 = \frac{d\pi}{dq} > 0$.

at $q^* = \frac{d\pi}{dq} = 0$.

necessary condition this is not ~~[illegible]~~ sufficient because we expect to be global maxima.

at $q_2 = \frac{d\pi}{dq} < 0$.

sufficient cond. $\frac{d^2\pi}{dq^2} < 0$.

Implicit function $\quad y = u(J, T)$

Satisfication

they are dependent to each other

$Q = f(K, L)$ labour

$y = f(x_1, x_2) \longrightarrow$ explicit

$y = f(x_1, x_2(x_1))$

$$dy = \frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2} \cdot \frac{dx_2}{dx_1}$$

$$dy = 0$$

$$\frac{dx_2}{dx_1} = -\frac{\partial f / \partial x_1}{\partial f / \partial x_2}$$

if x_1 changes by 1 unit then what change is there in x_2.

degree of substitution in input in production.

(3)

$y = f(x_1, x_2)$ $\qquad f_1 = \frac{\partial f}{\partial x_1}$

$dy = f_1 dx_1 + f_2 dx_2$

$$d^2y = \frac{\partial}{\partial x_1}(f_1 dx_1 + f_2 dx_2) \qquad f_2 = \frac{\partial f}{\partial x_2}$$

$$+ \frac{\partial}{\partial x_2}(f_1 dx_1 + f_2 dx_2)$$

$$= f_{11} dx_1^2 + f_{22} dx_2^2 + 2 f_{12} dx_1 dx_2$$

Hessian Determinant (unconstraint)

$$\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} \quad \begin{matrix} > 0 & = \text{concave fn} \\ < 0 & (\text{convex fn}) \end{matrix}$$

$$\frac{\partial^2 f}{\partial x^2} = f_{11} \qquad \frac{\partial^2 f}{\partial x_2^2} = f_{22}$$

$$f_{21} = \frac{\partial^2 f}{\partial x_1 \partial x_2}$$

$$f_{12} = \frac{\partial^2 f}{\partial x_2 \partial x_1}$$

$f_{21} = f_{12}$

= Young theorem

5uy

5&5

5uy

dz =

Constraint Maximisation & Minimisation

objective func $z = f(x_1, x_2)$ (satisfaction level)

budget

goal is to maximise z but constraints are there $b = c_1x_1 + c_2x_2$

Consumer $M = P_1x_1 + P_2x_2$

money income

total expenditure of x_1

objective func = $Q = f(K, L)$

Producer constraint

$C = WL + rK$

cost

Lagrangian Method

$$L = f(x_1, x_2) + \lambda [b - c_1x_1 - c_2x_2], \quad \lambda > 0$$

Optimal amount of x_1, x_2.

(FOC) First order condition

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow \frac{\partial f}{\partial x_1} - \lambda c_1 = 0 \quad \text{--- (1)}$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow \frac{\partial f}{\partial x_2} - \lambda c_2 = 0 \quad \text{--- (2)}$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow b - c_1x_1 - c_2x_2 = 0 \quad \text{--- (3)}$$

$$\frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}} = \frac{c_1}{c_2}$$

$$c_1 = c_2 \frac{\partial f}{\partial x_1} \Big/ \frac{\partial f}{\partial x_2}$$

$\frac{c_1}{c_2} = \frac{f_1}{f_2}$.

$$c_2 = \frac{c_1 \times \partial f/\partial x_2}{\frac{\partial f}{\partial x_1}}$$

put c_2/c_1 in eqn - 3 and find value x_1^*, x_2^*

$\lambda = \frac{\partial f/\partial x_1}{c_1}$ from eqn 1

if $x_1 \uparrow$ = by one amount then how much f change.

if I include one more constraint then what changes is there.

$\lambda = \frac{\text{marginal benefit}}{\text{Margenal Cost}}$.

we have to see whether this is optimised or not

$y = f(x_1, x_2)$

$d^2y = f_{11} dx_1^2 + f_{22} dx_2^2 + 2 f_{12} dx_1 dx_2$.

$b = c_1 x_1 + c_2 x_2$

this part is added.

$0 = c_1 dx_1 + c_2 dx_2$

we put the value of dx_1, dx_2 in

$$d^2y = f_{11} dx_1^2 - 2 f_{12} \frac{f_1}{f_2} dx_1^2 + f_{22} \frac{f_1^2}{f_2^2} dx_1^2$$

$$= \frac{f_{11} f_2^2 - 2 f_{12} f_1 f_2 + f_{22} f_1^2}{f_2^2}$$

$d^2y < 0$

Bordered form

Constrained form

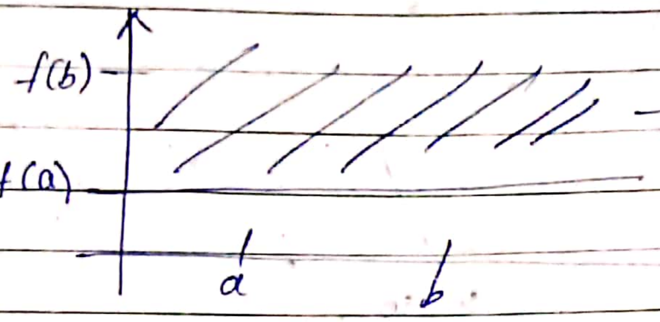
$$\begin{Vmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_2 & f_{22} \end{Vmatrix} < 0$$

Quasi concave

objective function is quasiconcave fn

when it is $f[(1-\lambda)a+\lambda b] \geq \min[f(a), f(b)]$

condition of quasi concave function



→ so means we are putting some constraint that our utility func can't below

$x_1, \quad x_2$

$(1-\lambda)x_1 + \lambda x_2$

$\boxed{0 \leq \lambda \leq 1}$

Bordered Hessian form

> 0

quasiconvex $f[(1-\lambda)a+\lambda b] \leq \max[f(a), f(b)]$

Unit-1

- Isoquant ISOQ
- shapes of ISOQ
- Different types of P.d
- Elasticity of substitution

Isoquant — It is locus of a combination b/w 2 or many I/P. Such that production level remain fixed.

Only 2 i/P (in class)

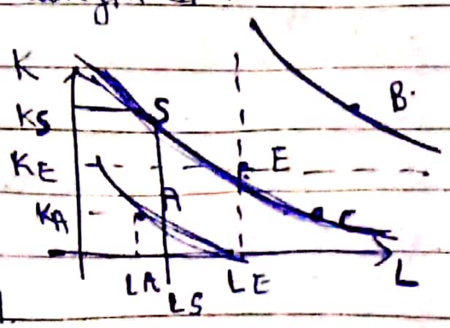
$Q = f(K, L)$

under all circumstance.

KA, LA is always less than KE, LE

so in all case E is preferred over A.

(E >p A)



Ep < B

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$Q = KL - 0.8K^2 - 0.2L^2$

if $K = 10$ $\quad AP_L$ max $= ?$

$$\left(AP_L = \frac{Q}{L} \right)$$

$$Q = K - \frac{0.8K^2}{L} - \frac{0.2L}{Q}$$

$$\frac{dQ}{dL} = 0 + \frac{0.8K^2}{L^2} + Q \quad \frac{Q2}{LQ} \quad -0.2$$

$$0.8K^2 = 0.2L^2$$

$$0.8 \times 100 = 0.2L^2$$

$$\frac{\cancel{8}4}{\cancel{10}} \times 100 = \frac{\cancel{2}}{\cancel{10}} L^2$$

$$L = 20$$

at what L ($MP_L = 0$) $\quad \frac{\partial Q}{\partial L}$

$$\frac{\partial Q}{\partial L} = K - 0.2 \times 2L$$

$$\frac{K}{0.2 \times 2} = L$$

$$\frac{5\ \cancel{10} \times \cancel{10}\ 5}{\cancel{2} \times \cancel{2}} = L \qquad L = 25$$

Elasticity of a ~~substitu~~ substitution (degree

$$y = f(x_1, x_2)$$
$$= f(x_1, g(x_1))$$

$$\sigma = \frac{\%\ \text{change in}\ (K/L)}{\%\ \text{change in MRTS}}$$

$$\sigma = \frac{d(K/L) \,/\, (K/L)}{d(\text{MRTS}) \,/\, \text{MRTS}}$$

$Q = AK^{\alpha} L^{\beta}$ what is σ.

MRTS

$$\sigma = \frac{d(\cancel{K/L})\ (\cancel{\beta/\alpha})(\cancel{K/L})}{(\cancel{K/L})\ (\cancel{\beta/\alpha})\ d(\cancel{K/L})} = 1.$$

rate at which it is substituted 1 unit of labour then 1 unit of capital.

→ What kind of curve it following
→ then find σ. (Cobb douglass curve)

$Q = aK + bL$

K, L are perfectly substitutes.

$W \to L$	if wage rate (↑)
$r \to K$	then Labour is replace by capital

$$\frac{-f_L}{f_K} = \frac{dK}{dL} = \frac{-b}{a}.$$

$MRTS = -b/a$

$dMRTS = 0$

$$\sigma = \frac{d(K/L)}{K/L} \times \frac{-b/a}{0} = \infty.\ (\text{undefined})$$

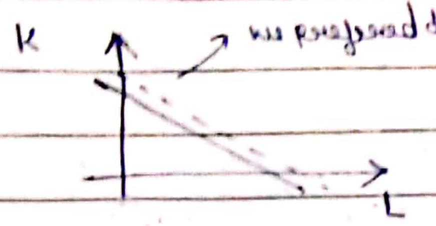
(Physical interpretation is important) → (infinitely substituted)

Perfect Complement $Q = \min\{\alpha K, \beta L\}$

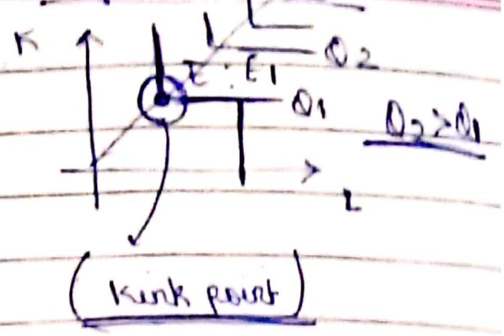
Shoe is example of Perfect complement. we can't buy right shoe or left shoe only. so basically we need capital and labour in ratio of α, β.	input are used in fixed proportion. $\frac{K}{L}$ is constant.

$\sigma = 0$ because $d\left(\frac{K}{L}\right)$ = is constant.

$Q = aK + bL$



Perfect complements



only labour change at E_1 no change in production

Euler theorem

$$z = f(x, y)$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nz$$

n is degree of homogeneity

$$z = x^2 + y^2 + xy.$$

$$= x(2x+y) + y(2y+x)$$

$$= 2(x^2+y^2+xy)$$

$(n = 2)$ (so degree is 2)

Product Exhaustion theorem: if a production function is homogenous of degree 1 then factors of production are rewarded equal to their marginal product

$$Q = f(K, L)$$

Homogenous of degree (HOD) → 1

$$K\frac{\partial f}{\partial K} + L\frac{\partial f}{\partial L} = W \quad ①$$

wage rate + rental rate of capital

if HOD > 1 Total payment > Total O/P

HOD < 1 $\quad n < "$

whatever the earning it is equally distributed b/w labour and capital.

1 lec missing

$$\frac{3 - \frac{1}{2}K^{-1/2}}{12 - \frac{1}{2}L^{-1/2}}$$

$$K^* \to \frac{\alpha\beta}{} \left(\frac{w\alpha}{r\beta}\right)^{\frac{\beta}{\alpha+\beta}} \bar{Q}^{\frac{1}{\alpha+\beta}}$$

$$\frac{3 = \frac{1}{2}\frac{1}{\sqrt{K}} L^{1/2}}{12 = \frac{1}{2} K^{1/2} \times \frac{1}{\sqrt{L}}}$$

$$\frac{3}{12} = \frac{1}{\sqrt{K} \times K^{1/2}} \times L^{1/2} \times \sqrt{L}$$

$$\boxed{\frac{1}{4} = \frac{L}{K}} \qquad \frac{K}{4} = L.$$

$$\bar{Q} = K^{1/2} L^{1/2}$$

$$40 = K^{1/2}\left(\frac{K}{4}\right)^{1/2}.$$

$$40 = \frac{K}{2} \qquad K = 80$$

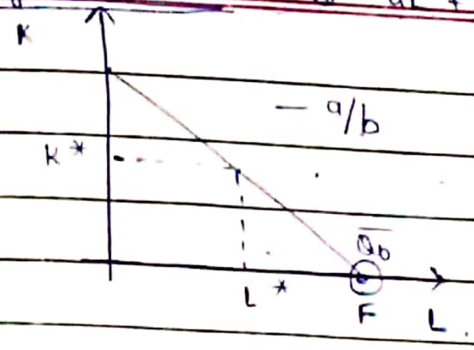
$$\frac{40}{} \times \qquad \frac{80}{4} = L = (20)$$

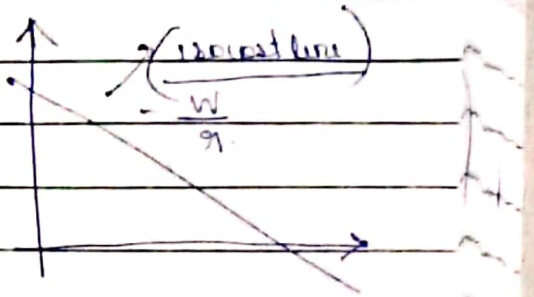
We will check whether the solution we get is optimum or not by borderd Hessian

$$\begin{vmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{21} & f_{22} \end{vmatrix} < 0$$

→ I can substitute labour with capital and vice-versa

Perfect substitute $Q = aL + bK$ $C = WL + rK$

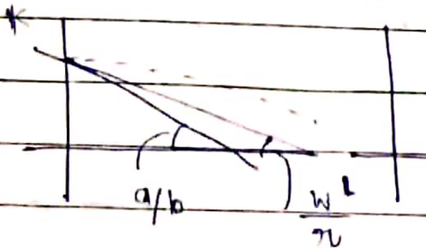




(only using labour and not capital)

if $\frac{a}{b} = \frac{W}{r}$ — then the same line is iso-cost + iso-quant.
(infinitely many solution)

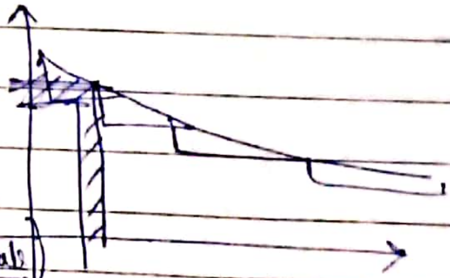
if $\frac{a}{b} < \frac{W}{r}$



$C^* = 0$

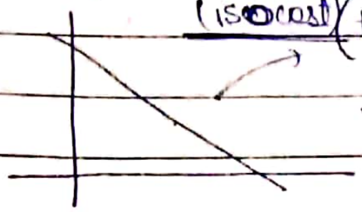
$K^* = \frac{C}{r}$

if $\frac{a}{b} > \frac{W}{r}$ $K^* = 0$ $L^* = \frac{C}{W}$

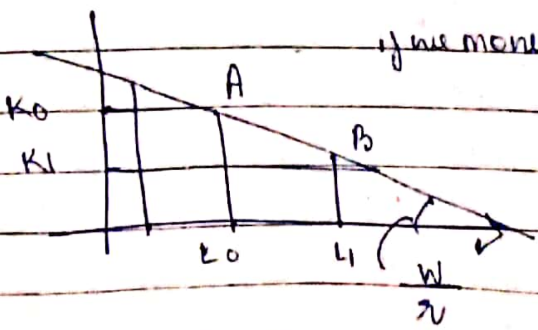


Slope of iso-quant ⇒ $MRTS = \frac{MP_L}{MP_K}$

Locus of combination



(isocost) (this shape is because they are perfect substitute)

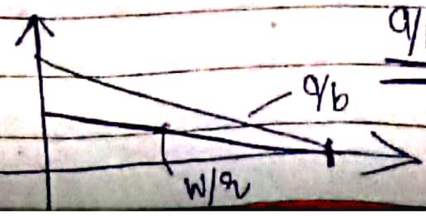


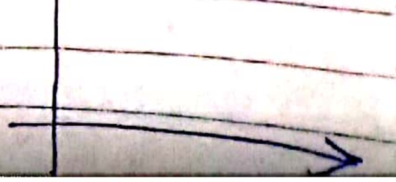
if we move $A \to B$

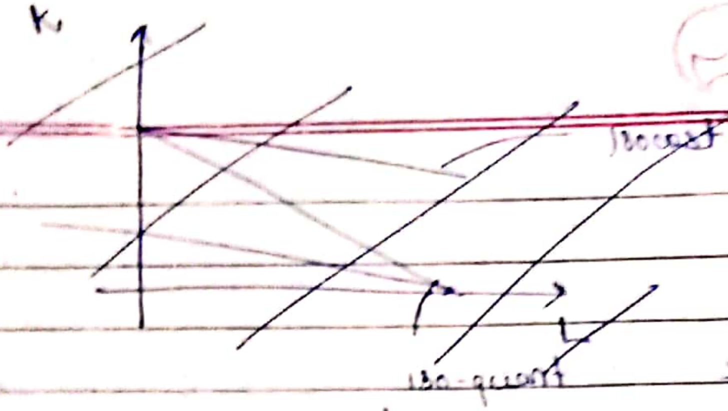
$W \uparrow$ $r \downarrow$

amount of sacrifice in order to get a labour

$a/b > W/r$

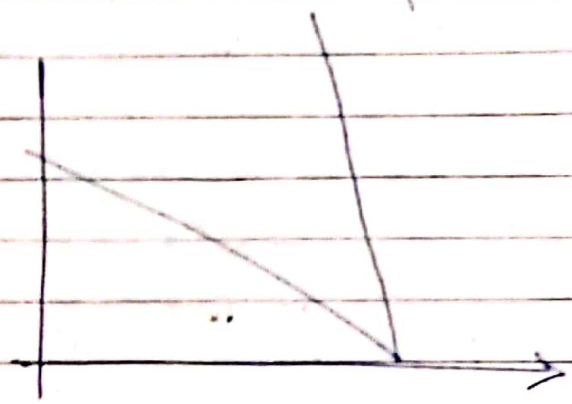






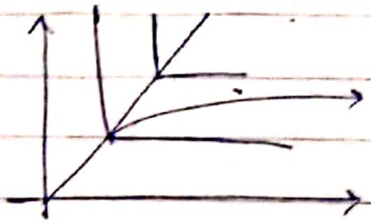
slope of isoquant

$MP = \frac{MP_L}{MP_K}$



Perfect Complement

$Q = \min\left\{\frac{K}{a}, \frac{L}{b}\right\}$



at kink point

$\frac{K}{a} = \frac{L}{b}$

$bK = aL$

$K = \frac{a}{b}L$

$C = wL + rK$

$= wL + r(a/b)L$

$C = \frac{wbL + raL}{b}$

$L^* = \frac{bc}{wb + ra}$

$K^* = \frac{ac}{wb + ra}$

Q

$q = \min\{5K, 10L\}$ $\quad a = \frac{1}{5} \quad b = \frac{1}{10}$

$C = wL + rK$

find K^*, L^*

$K = 2L$

$C = wL + r\left(\frac{.10}{.5}\right)L$

~~$L^* = 1/10 \times (wL + r(2L))$~~

$C = wL + r2L$

$C = L(w + 2r)$

$\frac{C}{w+2r} = L$ (1)

(2) (3)

$K = \frac{2C}{w+2r}$

$Ql = 7Q\, W\,Q\,tQ$ now find the total cost. $TC = f(Q)$

$$K = \frac{q}{5} \qquad l = \frac{q}{10}$$

$$\underline{C = C(Q)}$$

$$C = wL + rK$$

$$C = \frac{w \times q}{10} + r\left(\frac{q}{5}\right)$$

$$\boxed{C = q\left[\frac{w}{10} + \frac{r}{5}\right]} \longrightarrow$$

now from this if we need to find ACost then it is $\frac{TC}{q}$

$$AC = \frac{TC}{q} \Rightarrow \frac{w}{10} + \frac{r}{5}$$

$$MC = \frac{dTC}{dq} = \frac{w}{10} + \frac{r}{5}$$

$\underline{AC = MC}$

→ Producer using 1 i/p and produces 2 o/p such as farming.

$$x = A\left(q_1^{\alpha} + q_2^{\beta}\right) \quad , (\alpha, \beta > 1)$$

$P_1, P_2 \rightarrow$ 2 products.

(what will be profit maximising as the function of price?)

$$\pi = TR - C$$

$$\pi = P_1 q_1 + P_2 q_2 - Cx.$$

$$\frac{d\pi}{dq_1} = 0, \quad \frac{d\pi}{dq_2} = 0 \longrightarrow \left(P_2 - C\beta A q_2^{\beta - 1} = 0\right)$$

$$\left(P_1 - C\alpha A q_1^{\alpha-1} = 0\right)$$

$$q_1 = \left(\frac{P_1}{C\alpha A}\right)^{1/\alpha - 1}$$

$$P_2 = C\beta A\, q_2^{\beta - 1}$$

$$\left(\frac{P_2}{C\beta A}\right)^{1/\beta - 1} = q_2.$$

$\pi = PQ - C$
$= Pq - WL - rK$
$= PAK^{\alpha}L^{1-\alpha} - WL - rK.$
$q = f(K, L)$
$= AK^{\alpha}L^{1-\alpha}$

(Now find L, K)

$\frac{\partial \pi}{\partial L} = 0$

choose L and K such that profit maximisation.

$\frac{\partial \pi}{\partial K} = 0$

(there is no constraint) in this

$$\frac{d\pi}{dL} \Rightarrow PAK^{\alpha}(1-\alpha)L^{-\alpha} - W = 0$$

$$\frac{d\pi}{dK} = PA\alpha K^{\alpha-1}L^{1-\alpha} - r = 0$$

(Take $1-\alpha = \beta$)

$\Rightarrow PAK^{\alpha}\beta L^{-\alpha} - W = 0$ — (1)
$\Rightarrow PA\alpha K^{-\beta} \cdot L^{\beta} - r = 0$ — (2)

$$L^* = \left(\frac{\alpha}{W}\right)^{\frac{1-\beta}{r}} \left(\frac{\beta}{r}\right)^{\beta/r} (AP)^{1/r}.$$

$$K^* = \left(\frac{\alpha}{W}\right)^{\frac{\alpha}{r}} \left(\frac{\beta}{r}\right)^{(1-\alpha/r)} (AP)^{\frac{1}{r}}$$

previously L^* and K^* are not the function of P but now they are so they are called unconditional demand function.

($\gamma = 1-\alpha-\beta$)

(initially L, K are not function of price)

$\pi = P \cdot Q - C$
$Q = L^{\alpha}K^{\beta}$
$\pi = P(L^{\alpha}K^{\beta}) - WL - rK.$

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authors

$\frac{d\pi}{dL} = 0 = \alpha P L^{\alpha-1} K^{\beta} = W$ — (1)

$\frac{d\pi}{dK} = 0 = \beta P L^{\alpha} K^{\beta-1} = r$ — (2)

$\frac{eqn1}{eqn2} - \frac{\alpha}{\beta}\frac{K}{L} = \frac{W}{r}$

$\alpha K r = \beta L W$

$K = \frac{\beta}{\alpha}\frac{W}{r}\cdot L$ — (3)

substitute in eqn (3) in eqn (1)

$\alpha P L^{\alpha-1}\left[\frac{\beta}{\alpha}\frac{W}{r}\cdot L\right]^{\beta} = W$

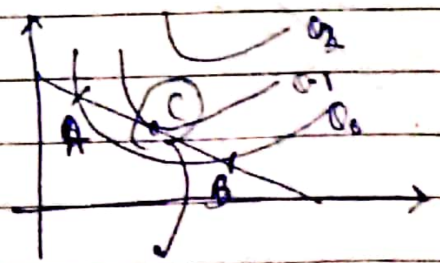
$L = f(W, r, P)$ so now L, K are function of price

$L^{\alpha+\beta-1} = \frac{W}{P\alpha\left(\frac{\beta W}{\alpha r}\right)^{\beta}}$

Q $MP_L = 10$ $W = 4$ explain why this data proof
$MPK = 15$ $r = 3$ that a corporation is done inefficiently.

isoquant $= \frac{MP_L}{MP_K} = \frac{2}{3}$ $\frac{4}{3}$ = isocost

explaination is must in exam.



$(Q_0 < Q_1 < Q_2)$

mean checking for this tangency condition.

if slope equal ⇒ efficient $\frac{2}{3} \neq \frac{4}{3}$
if slope not equal = inefficient

Problem → Sey authous

$\frac{d\pi}{dL} = 0 = \alpha P L^{\alpha-1} K^{\beta} = W$ — (1)

$\frac{d\pi}{dK} = 0 = \beta P L^{\alpha} K^{\beta-1} = r$ — (2)

$\frac{eqn1}{eqn2} - \frac{\alpha}{\beta} \frac{K}{L} = \frac{W}{r}$

$\alpha K r = \beta L w$

$\therefore K = \frac{\beta}{\alpha} \frac{w}{r} \cdot L$ — (3)

substitute in eqn (3) in eqn (1)

$\alpha P L^{\alpha-1} \left[\frac{\beta}{\alpha} \frac{W}{r} \cdot L\right]^{\beta} = W$

$L = f(W, r, P)$ so now L, K are function of price

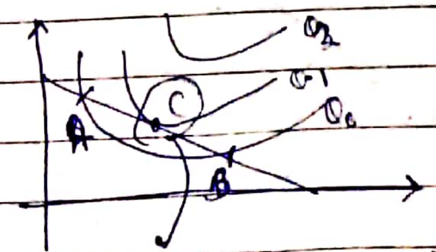
$L^{\alpha+\beta-1} = \frac{W}{P\alpha\left(\frac{\beta W}{\alpha r}\right)^{\beta}}$

Q $MP_L = 10$ $\quad W = 4.$ $\quad$ explain why this data proof that corporations done ineffeciently.

$MPK = 15$ $\quad r = 3.$

Isoquant $= \frac{MP_L}{MP_K} = \frac{2}{3}$ $\quad$ as $\frac{4}{3}$ = isocost

explaination Ensure in exam.



$(Q_0 < Q_1 < Q_2)$

we are checking for this tangency condition:

if slope equal ⇒ efficient $\quad$ $2/3 \neq 4/3$

& if slope not equal = inefficient

$\frac{MP_L}{MP_K} = \frac{w}{r}$

↑ increase the MP_L. because we need to inc. the ratio MP_L = (Productivity).

Q $Q = L^{0.5} K^{0.5}$

TVC, TFC

$Q = L^{0.5} \; K^{0.5}$ $(K = 2)$ $r = 5$.

$Q^2 = LK$.

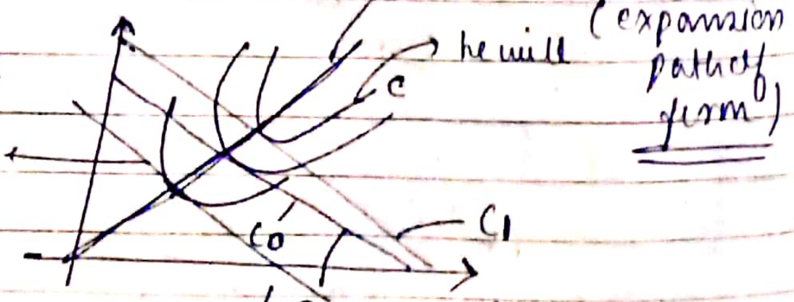
$L = \frac{Q^2}{K}$ → $C = wL + rK$

$w \cdot \frac{Q^2}{K} + rK$

$C = \underbrace{w \cdot \frac{Q^2}{2}}_{\text{V cost}} + 2r$ → fixed cost.

$$\boxed{TC = TVC + TFC}$$

Expansion Path



$(C_0 = wL_0 + rK_0)$

$C_1 = wL_1 + rK_1$ $(C_1 > C_0)$

Cost↑ then choose higher isoquant

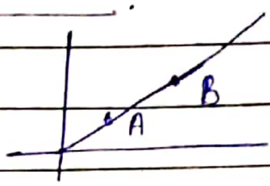
Cost↓ then choose lower isoquant

expanding through this path.

Expansion path $y = mx$.

(Tangency condition)

How to calculate expansion path?



$$y = mx$$

$$K = mL$$

$$m = \frac{fL}{fK} \quad \text{or} \quad \frac{w}{r}$$

$$\frac{K}{L} = \frac{w}{r} \qquad Lw = Kr$$

$$L = \frac{Kr}{w}$$

have many interpretation.

eqn 1 / eqn 2

1. Tangency condition.
2. Join the points we get expansion path.

Q Exam

Firm 20 workers and only variable I/P wages rate is for 60. Average Product of Labour is 30. Last worker added to 12 units to total O/P. TFC = 3600.

MC = ?

ATC = ?

AVC = ?

$L = 20$. $APL = 30$.

$W = 60$.

20×60

$C = wL$

$C = 20 \times 60 + 3600$.

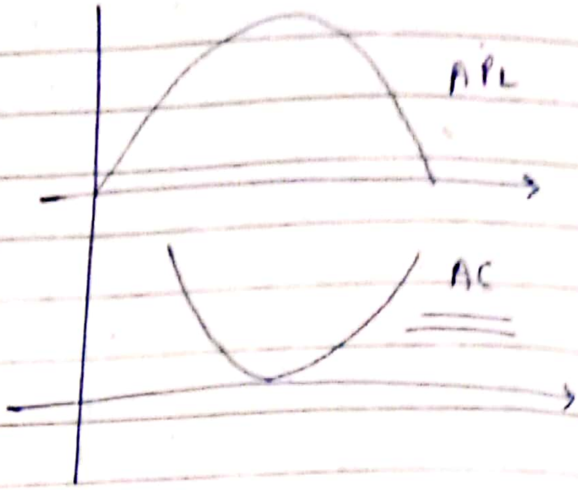
Sol

$$APL = \frac{Q}{L} \qquad MPL = \frac{dQ}{dL}$$

$AC = \frac{1}{APL}$

$AC = \frac{TC}{Q}$

$MC = \frac{dTC}{dQ}$



$MC = \frac{W}{MPL}$

$AC = \frac{W}{APL}$

$\frac{1}{MP_L}$ = Amount of labour required to get one unit of O/P.

$W \cdot \frac{1}{MP_L} =$

MP_L = 1 unit of L ↑ change Q.

$AVC \Rightarrow \frac{W}{AP_L} \cdot \frac{TVC}{Q}$ $\left(2 \rightarrow \frac{60}{30} \right)$

$AVC = \frac{W}{APL} = \frac{TVC}{Q}$

$2 = \frac{60}{30} = \frac{20 \times 60}{Q}$

$(Q = 600)$

$TFC = 3600$

$AFC = \frac{\cancel{3600}}{\cancel{600}} = 6$

$AC = 6 + 2 = 8$

$C = q\, w^{2/3}\, r^{1/3}.$

find L^*, K^*

whenever these question come apply shephards lemma.

isoquant
isocost
shape of

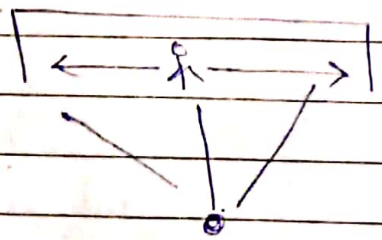
Perfect Competition

we need to study ideal so that we can study other three
1) Monopoly
2) Monopolistic
3) Oligopolistic (game theory)

similar quantity of diff variety eg Pepsi, Coca

→ Price discrimination
1) first
2) second
3) third

game theory = how 2 person interact with each other



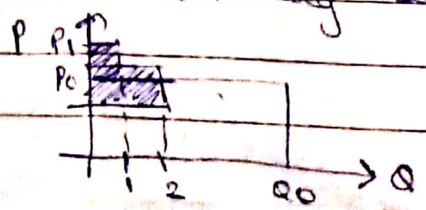
→ oligopolistic also have game theory
quantity, price.

(PC) **Perfect Competition** Consumer surplus [CS]

in PC → CS is maximum

What is CS?

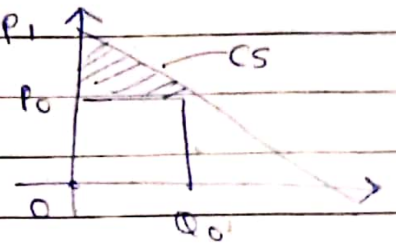
we buy good becoz it bring happiness / utility



I not have to pay P_1 amount of price than.

we will buy that product untill CS = 0

if pair of jeans (G1) at 4500 in march
G1 at 1500 in june
=> 4500 − 1500 = 3000 (CS)

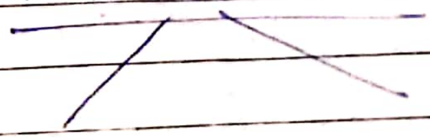


$\frac{1}{2}$ 1000 × (OP1 − OP0)

Assumptions → 1) large no. of buyer & seller
large means seller or buyer both have insignificant Share in mat.
(all seller and buyer not have any share)

Product is sold in market is homogenous.
(quality of product are identical), (no varieties)

(1) n (2) → Price taker becoz



if firm 1 ↓ Price then all buyer will go there so cost of prod ↑ so Price ↑

if firm 1 ↑ Price then no one will buy so it has to decrease price.

thus you whatever the price given by industry firm need to maintain no change in them.

$TR = \bar{P} \cdot Q$ ← fixed

$AR = TR/Q = \bar{P}$

$MR = \bar{P}$