Assignment-2

) a) $R(Q)$ $\quad V(F)$ $\quad \alpha, \beta \in V$ $\quad a, b \in F$

(i) $\alpha, \beta \in R$ $\quad a \in Q$

$\alpha + \beta \in R$] hence vector space

also $a\alpha \in R$]

b) $C(Q)$

$\alpha, \beta \in C$ $\quad a \in Q$

$\Rightarrow \alpha + \beta \in C$] vector space

$\Rightarrow a\alpha \in C$]

c) $R(C)$

$\alpha, \beta \in C$ $\quad a \in C$

$\alpha + \beta \in R$] not a vector space

$a\alpha \notin R$]

d) $R(R)$

$\alpha, \beta \in R$ $\quad a \in R$

$\alpha + \beta \in R$] vector space.

$a\alpha \in R$]

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3) a) $W = \{(x, y, z) \mid x \geq 0\}$

(i) $(x_1, y_1, z_1) + (x_2, y_2, z_2) \in W$

$a(x_1, y, z_1) = (ax_1, ay, az_1)$

(ii) $a \in R \Rightarrow ax_1 \in R$ $\therefore \notin W$

not a subspace

b) $W = \{(x, y, z) \mid x + y = z\}$

~~(i) $(x_1, y_1, z_1) + (x_2,$~~

(i) $(x_1, y_1, x_1 + y_1) + (x_2, y_2, x_2 + y_2)$

$= (x_1 + x_2, y_2 + y_1, + x_1 + x_2 + y_1 + y_2)$

$\in W$

(ii) $a(x_1, y_1, x_1 + y_1) = (ax_1, ay_1, ax_1 + a$

$\in W$

✓ subspace

c) $W = \{(x, y, z) \mid x = y^2\}$

i) $(y_1^2, y_1, z_1) + (y_2^2, y_2, z_2)$

$= (y_1^2 + y_2^2, y_1 + y_2, z_1 + z_2)$

$\notin W$

not a subspace

d) $W = \{(x, y, z) \mid xy = 0\}$ $x_1 y_1 = 0$

$x_2 y_2 = 0$

(i) $(x_1, y_1, z_1) + (x_2, y_2, z_2)$

$= (x_1 + x_2, y_1 + y_2, z_1 + z_2)$ $\Rightarrow \neq 0$

$(x_1 + x_2)(y_1 + y_2) \neq 0$

not a subspace

4) $v = ap_1 + bp_2 + cp_3$

$1 = a + 2b$

$4 = -2a - 3b + c$

$-3 = 5a - c$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ -2 & -3 & 1 & 4 \\ 5 & 0 & -1 & -3 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow R_2 + 2R_1 \\ R_3 \rightarrow R_3 - 5R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 6 \\ 0 & -10 & -1 & -8 \end{array}\right] \quad R_3 \rightarrow R_3 + 10R_2$$

$-10+10$

$-1+10$

$-8+60$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 9 & 52 \end{array}\right]$$

$9c = 52$

$c = \frac{52}{9}$

$5a = -3 + c$

$5a = -3 + \frac{52}{9}$

$\cancel{a = 1 - 2b}$

$b = \frac{1-a}{2}$

$a = \frac{2.77}{5}$

5) a) $S = \{(1,0,2,1), (1,3,2,1), (4,1,2,2)\}$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 3 & 2 & 1 \\ 4 & 1 & 2 & 2 \end{bmatrix} \simeq \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & -6 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -6 & -2 \end{bmatrix} \Rightarrow \text{L.I.}$$

b) $$\begin{bmatrix} 1 & 2 & 6 \\ -1 & 3 & 4 \\ -1 & -4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 6 \\ 0 & 5 & 10 \\ 0 & -2 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 6 \\ 0 & 5 & 10 \\ 0 & 0 & 8 \end{bmatrix} \Rightarrow \text{L.I.}$$

c) $S = \{(\underset{\alpha}{u+v}, \underset{\beta}{v+w}, \underset{\gamma}{w+u})\}$

$a\alpha + b\beta + c\gamma = 0$

$a(u+v) + b(v+w) + c(w+u) = 0$

$u(a+c) + v(a+b) + w(b+c) = 0$

$\{u, v, w)$ are L.I.

so $\left.\begin{aligned} a+c &= 0 \\ a+b &= 0 \\ b+c &= 0 \end{aligned}\right\} \Rightarrow a = 0 = b = c$

$\therefore \alpha, \beta, \gamma$ are L.I.

d) $\begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 4 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & -7 & 1 \\ 0 & -7 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 0 \\ 0 & -7 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow$ L.D

6) $a \sin x + be^{x} + cx^{2} = 0$

only when

$a = b = c = 0$ so linearly independe

7) $\{\alpha_1, \alpha_2, \alpha_3 -- \alpha_n\} \Rightarrow$ L.D.

then $a_1\alpha_1 + a_2\alpha_2 + --- a_n\alpha_n = 0$

if $a_k \neq 0$

$\alpha_k = \frac{-1}{a_k} \{a_1\alpha_1 + a_2\alpha_2 + -- a_{k-1}\alpha_{k-1} -- \alpha_n a_n$

$\Rightarrow \alpha_k$ is linear combination of $(\alpha_1, \alpha_2 -- \alpha_n)$

8) $(1,1,1,1)$ $(0,1,1,1)$ $(0,0,1,1)$, $(0,0,0,1)$
$\quad e_1 \quad e_2 \quad e_3 \quad e_4$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

in row-echelon form no ~~no~~ zero row $\Rightarrow$ L.I.

for span

$(x, y, z, \omega) = ae_1 + be_2 + ce_3 + de_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & x \\ 0 & 1 & 1 & 1 & y \\ 0 & 0 & 1 & 1 & z \\ 0 & 0 & 0 & 1 & \omega \end{array}\right]$$

$d = \omega$

$c + d = z \qquad c = z - \omega$

$b + c + d = y \qquad b = y - z$

$a = x - y$

$\therefore$ ~~sol~~ unique solutions exist for all values of x, y, z, ω $\therefore$ span

span + L.I. $\Rightarrow$ basis.

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9) a) $\{1, x, x^2\}$

$a + bx + cx^2 = 0$

$a = b = c = 0$ L.I

also spans

$\therefore$ basis

b) $\{1+x+x^2\}$.

$\Rightarrow$ does not span $P_2(x)$

c) $\{1, x, x^2, 1+x+x^2\}$

$\Rightarrow$ L.D. so not a basis

d) $\{1, 1+x, 1+x+x^2\}$

$\Rightarrow$ L.I. also spans $P_2(x)$ $\therefore$ basis

10) a) $\{A : A$ is 2×3 real matrices$\}$

basis has 6 elements so dimension $= 6$

b) $\{A : A$ is 3×3 upper triangular$\}$

basis is

$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

$\dim = 6$

c) $\{A : A \text{ is } 3\times 3 \text{ Symmetric}\}$

$$\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \qquad \dim = 6$$

d) $\{A : A \text{ is } 2\times 2 \text{ real sq. skew-Symmetric}\}$

$$\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix} \qquad \text{basis} = \left\{ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$$

$$\dim = 1$$

11)

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 4 & 3 & 7 & 5 & 6 \\ 1 & 2 & 3 & 5 & 7 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{basis} = \left\{ \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \right\}$$

$$\dim = 2$$

12)

$$A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 2 & 2 & 0 & 1 \\ 5 & 4 & 3 & k \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -6 & -1 \\ 0 & 0 & 0 & k-3 \end{bmatrix}$$

if rank = 2 => no. of non-zero rows = 0

=> $\underline{\underline{k = 3}}$

$$B = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 3 & 0 & 9 & 3 \\ 2 & 2 & 0 & 1 \\ 4 & 2 & 6 & k \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -6 & -1 \\ 0 & 2 & -6 & k-4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -6 & -1 \\ 0 & 2 & -6 & k- \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -6 & -1 \\ 0 & 0 & 0 & k-3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{k = 3}$$