Quantum Mechanics

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Regular old phy.

Newtonian physics — when things get fast → Special relativity

Newtonian physics — when things get small → Quantum physics

Newtonian physics — when things get small & fast → Quantum field theory

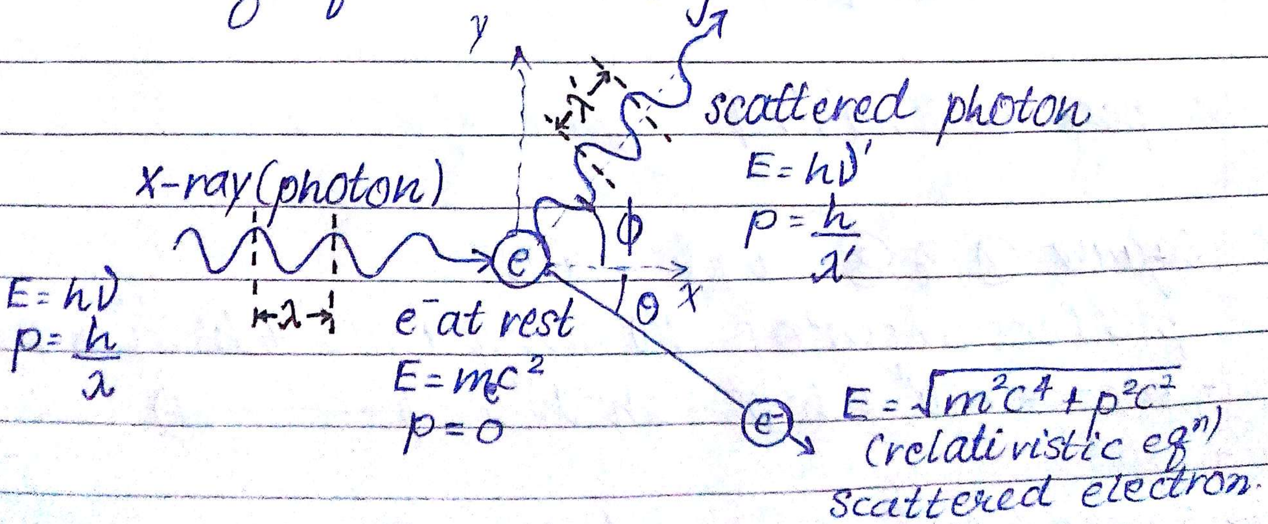
- Wein's Law:

$\lambda_{max} \cdot T = b =$ wein's constant $\Rightarrow \lambda_m \propto \frac{1}{T}$
$= 2898\,\mu m$ Kelvin

Classically, avg. energy of atom $= kT$
(for Rayleigh-Jeans)

* Compton Effect:

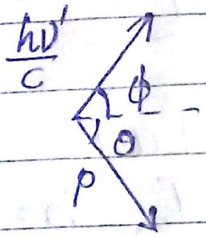
Objective: ① To show particle nature of light.
② Scattering of radiation from an electron.



Loss in energy by x-ray photon = Gain in e^- energy
since, its Elastic collision.

$\therefore, (h\nu - h\nu') = KE$ ——— ①

→ Energy and momentum (being vector is conserved in direct^n of motion as well as $\perp$ dir^n) are conserved.



$x = \frac{h\nu'}{c}\cos\phi$
$Y = \frac{h\nu'}{c}\sin\phi$

$x = p\cos\theta$
$Y = p\sin\theta$

- Case-1 : For dir^n of motion:

Initial momentum = final mom.

$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta$

$\therefore, pc = \frac{h\nu - h\nu'\cos\phi}{\cos\theta}$ ——— ②

- Case-2: In $\perp$ direct^n:

Initial p = Final p

$0 = \frac{h\nu'}{c}\sin\phi \mp p\sin\theta$ —

$\therefore, pc\sin\theta = h\nu'\sin\phi$

Square ② & ③ and add,

$p^2c^2(\cos^2\theta + \sin^2\theta) = h^2\nu'^2\sin^2\phi + h^2\nu^2 + h^2\nu'^2\cos^2\phi - 2h^2\nu\nu'\cos$

$= p^2c^2 = h^2\nu'^2 + h^2\nu^2 - 2h^2\nu\nu'\cos\phi$ ——— ④

→ Energy is conserved (Relativistic):

= E before collis^n = E after collis^n

= $E_{photon} + E_{e^-} = (E_{photon} + E_{e^-})_{scattered}$

$= h\nu + mc^2 = h\nu' + \sqrt{m^2c^4 + p^2c^2}$

from ①,

$KE + mc^2 = \sqrt{m^2c^4 + p^2c^2} \Rightarrow KE^2 + \cancel{m^2c^4} + 2mc^2(KE) = \cancel{m^2c^4} + p^2c^2$

$\therefore p^2c^2 = KE^2 + 2mc^2(KE)$.

$p^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + 2h\nu mc^2 - 2h\nu' mc^2$ —— ⑤

Substitute p^2c^2 from ④ to ⑤, we get

$= 2h(\nu - \nu')mc^2 - 2h^2\nu\nu' = -2h^2\nu\nu'\cos\phi$

$= 2h mc^2(\nu - \nu') = \cancel{2}h^2\nu\nu'(1-\cos\phi)$

$= mc^2(\nu - \nu') = h\nu\nu'(1-\cos\phi)$; $\nu = \frac{c}{\lambda}$

$\therefore mc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{h}{\lambda\lambda'}(1-\cos\phi) \Rightarrow mc(\lambda' - \lambda) = h(1-\cos\phi)$

$\therefore \boxed{\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)} \Rightarrow$ Compton Equation.

λ' = wavelength of scattered photon depends on ϕ

$\because \lambda$ and λ' are diff., Compton effect shows that light behaves as a particle (i.e particle nature) and not as wave. If it behaved as wave, there would've been no scattering.

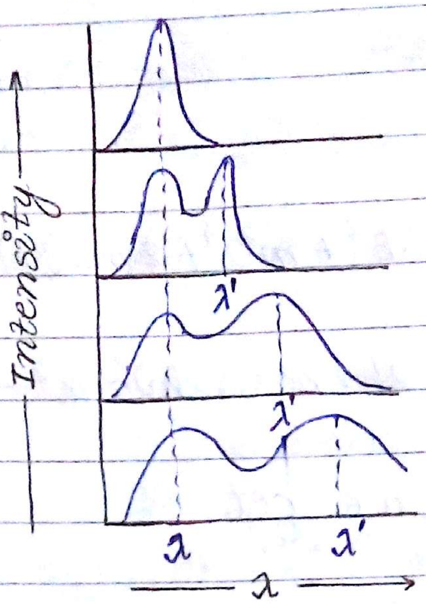
$\boxed{\lambda_c = \frac{h}{mc}} \Rightarrow$ Compton constant or Compton wavelength

$= 2.426 \times 10^{-12} m$

$= 2426 pm$

$\lambda = \frac{h}{\gamma m v}$ = Relativistic De-broglie wavelength

$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$



Q A free e^- cannot absorb a photon. Show this mathematically.
(Hint: It violates energy-momentum conservation)

- 2-slit exp. with:

(i) Bullets:

$P_{12}(\text{bullet}) = P_1 + P_2$ (NO interference pattern obtained)

(ii) e^--gun:

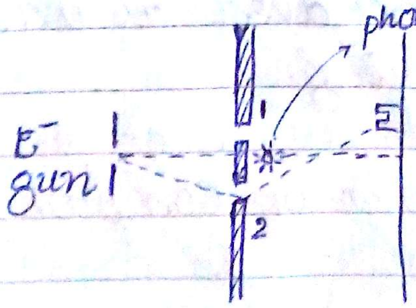
$P_{12} = P_1 + P_2 + $ (extra term) due to interference

$= |\phi_1 + \phi_2|^2$ wave fun^n

$= (\phi + \phi_2)^* (\phi_1^* + \phi_2^*)$; $P_1 = |\phi_1|^2 = \phi_1^* + \phi_1$

$= |\phi_1|^2 + |\phi_2|^2 + \phi_2\phi_1^* + \phi_1\phi_2^*$

$= P_1 + P_2 + (\qquad\qquad)$



$P_{12}' = P_1' + P_2'$

When photon & e^- interact a flash is seen and at screen click is heard. Flash is seen near a parti. slit

In bullet case interf. occurs but max., minima can't be differentiated due to large size of bullet.

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- <u>Case: I</u> (Intesity very high–say 10,000 e^- & 10,000 photons)

Max. e^- scatter and reach screen, click is heard.
No. interference pattern got due to high intensity.
Col-I : e^- passing through slit-1
Col-II : e^- passing through slit-2

Col-I	Col-II
click & flash (1)	click & flash (1)
⋮	⋮
N'	N

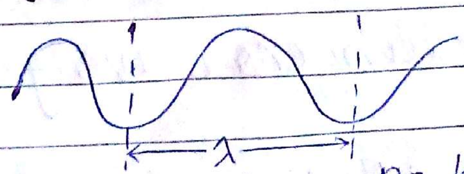
- <u>Case-II</u> (Intensity very low–say 10,000 e^- & 1,000 photons)

Interference observed

Col-I	Col-II	Rem. e^-
click & flash	click & flash	click but No flash

- Heisenberg's uncertainity principle:

$$\boxed{\Delta x \cdot \Delta p \geq \frac{h}{4\pi}}$$



$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
permitivity of free space

$a_0 = 0.53 \times 10^{-10}$ m $= 0.53$ Å

Q Find λ:

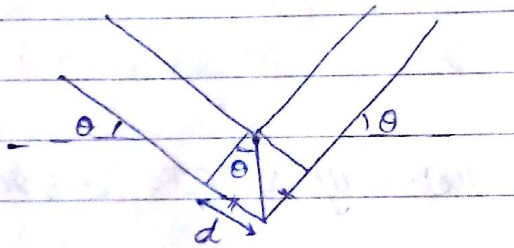
(a) $m = 46$ gm , $v = 30$ m/s

$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{46 \times 10^{-3} \times 30} = \frac{6.63 \times 10^{-32}}{138} = ---- 10^{-34}$ order

(b) $m = 9.1 \times 10^{-31}$ kg, $v = 2 \times 10^{8}$ m/s

$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 10^{8}} = ------ 10^{-12}$ order

- Bragg's law:



$\boxed{2d\sin\theta = n\lambda}$

n = order ← due to const. interference

if say $n=2$, then in $2d$ portion 2 max occurs
say $n=1$, then in $2d$ portion 1 max occurs.

→ sound wave = pressure varies
→ water waves = height/amplitude
→ light wave = intensity of electric and magnetic field

} varies periodically.

- Schrodinger's wave funⁿ:

In matter waves,

ψ = wave funⁿ

$|\psi|^2$ = probability amplitude → Dirac notation

$\psi(x,y,z,t)$ = wave funⁿ

The probability of experimentally finding the body described by wave function Ψ at point x, y, z at time t is proportional to the value of $|\Psi|^2$ there at t

- Bra|ket notation

$\langle \; | \; \rangle$ or $| \rangle$ ket

$\langle |$

Bra

initial state of obj.

$| \rangle$ = ket

what is known part in ket $(| \rangle)$

$|x=3m\rangle$ would be the state of particle known to be at posit^n $x=3cm$

$|p\rangle$ particle has momentum p

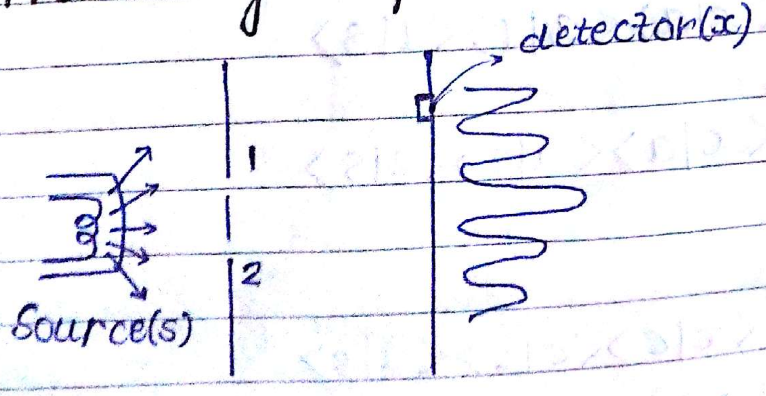
$|x=3, x=5\rangle$ particle is at $x=3$ & $x=5$ — $\times$ not possible

$|\Psi\rangle$ system in state Ψ & is therefore called the state vector

$\langle |$ = Bra

represents the final state or lang. in which you wish to express content of set.

Probability amplitude = $\langle$ Final state | Ini. state $\rangle$



$\rightarrow y = A\sin(kx-\omega t) \\ = Ae^{i(kx-\omega t)}$ } = wave eq^n

Ex: A particle with definite energy is moving from known position r_1 to r_2. Find probability amplitude?

Sol^n: Definite energy ⇒ it has momentum (say $\vec{p}$)

r_1 $\qquad\qquad$ r_2

$$\langle r_2 | r_1 \rangle = \frac{e^{i\vec{p}r_{12}/\hbar}}{r_{12}} \quad ; \hbar = \frac{h}{2\pi}$$

where, $r_{12} = r_2 - r_1$

$\langle r_2 | r_1 \rangle$ = prob. to find particle at r_2.

$p^2c^2 = E^2 - m^2c^4$ is also valid here ∵ def. energy.

- How fast de-broglie wave travels?

$v_p = \nu\lambda$ —— ①

$\boxed{v_p = \frac{c^2}{v}}$ —— ② for matter waves if $v \neq c$

phase velocity

∴ $\boxed{v_p > c}$ but v_p can't be $> c$.

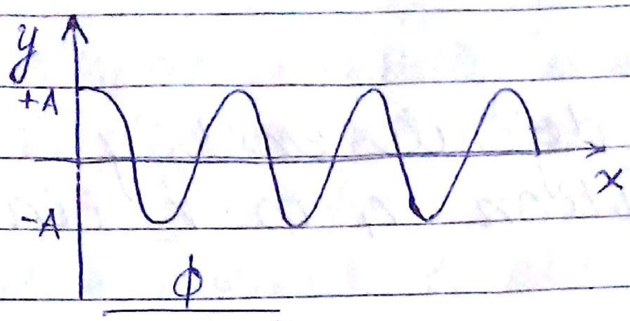
$\lambda = \frac{h}{p} = \frac{h}{\gamma m v}$ —— ③ $\qquad E = h\nu = \gamma m c^2$ —— ④

$\nu = \frac{\gamma m c^2}{h}$

from ①, ③, ④,

∴ $v_p = \frac{\cancel{\gamma m}c^2}{\cancel{h}} \times \frac{\cancel{h}}{\cancel{\gamma m}v} = \frac{c^2}{v} = ②$

- Phase velocity:



(Assump: wave in -x-dir^n

$y = A\cos(\omega t - kx)$
$\psi = A\cos(\omega t - kx)$
$\psi = Ae^{i(kx-\omega t}$

$\omega t - kx = \text{constant}$ $\therefore$, $kx = \omega t - \text{const.}$

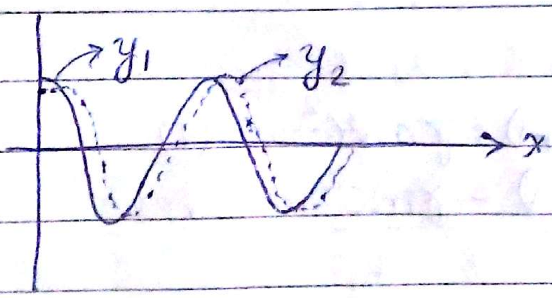
$\therefore x = \frac{\omega t}{k} - \frac{\text{const.}}{k}$

diff. w.r.t t

$$\boxed{\frac{dx}{dt} = \frac{\omega}{k} = V_p} = \nu\lambda$$

- Group velocity:

$y_1 = A\cos(\omega t - kx)$
$y_2 = A\cos((\omega + \Delta\omega)t - (k+\Delta k)x)$



$$y = y_1 + y_2 = A(\cos\alpha + \cos\beta) = 2A\cos\left(\frac{(2\omega + \Delta\omega)t}{2} - \frac{(2k+\Delta k)x}{2}\right)$$

$$\times \cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right)$$

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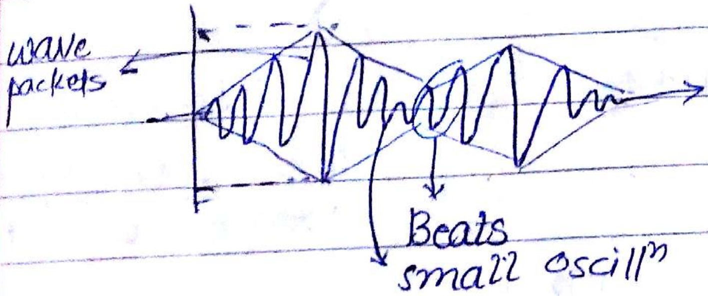
$2\omega + \Delta\omega \simeq 2\omega$ & $2k + \Delta k = 2k$

$$\therefore \boxed{y = 2A\cos(\omega t - kx)\underbrace{\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right)}_{\text{large scale oscill}^n} = \psi}$$

$$\boxed{A(x,t) = 2A\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right)}$$

superimposed waves

Inside waves = large f
Outside waves = small f



$$\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2} = \text{const.} \Rightarrow \boxed{\frac{dx}{dt} = \frac{\Delta\omega}{\Delta k} = v_g}$$ group velocity

- Rel^n b/w v_p & v_g:

$$v_g = \frac{d\omega}{dk} = \frac{d(v_p k)}{dk}$$

$$\boxed{v_g = v_p + k\frac{dv_p}{dk}}$$

→ in terms of λ,

$$\boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}}$$ → in dispersive medium.

medium in which v_p is wavelength ~~is~~ ~~wave~~ dependent

for const v_p,

$$\boxed{\frac{dv_p}{d\lambda} = 0} \Rightarrow v_g = v_p \longrightarrow \text{in non-dispersive medium.}$$

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what is group velocity for matter waves?

$$v_g = \frac{d\omega}{dk} \qquad \omega = 2\pi\nu \qquad k = \frac{2\pi}{\lambda}$$

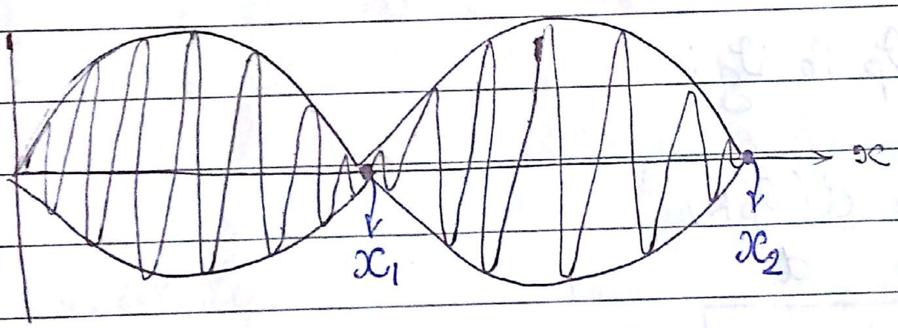
$$h\nu = \gamma mc^2 \quad ; \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\nu = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}} \times h} \qquad \lambda = \frac{h}{\gamma mv} \qquad \nu = \frac{mc^3}{h\sqrt{c^2-v^2}}$$

$$\frac{d\omega}{dv} = \frac{2\pi mc^3}{h}\left(\frac{d}{dv}\left(\frac{1}{\sqrt{c^2-v^2}}\right)\right)$$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = \frac{\quad}{\left(\frac{2\pi\gamma m}{h}\right)} \dots$$

$$\boxed{v_g = v = \text{partic. veloc.}}$$



Eqn:

$$\psi = y = 2A\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right)\cos(kx-\omega t) \text{ —— (1)}$$

$$\underbrace{\qquad\qquad\qquad}_{\text{Amplitude modulation}}$$

for $t = t_0\ (t=0)$, $A(t,k) = 0$

$\therefore$

Amplitude for x_1 : $2A\cos\left(\frac{\Delta kx_1}{2} - \frac{\Delta\omega t_0}{2}\right) = 0$

Amplitude for x_2 : $2A\cos\left(\frac{\Delta kx_2}{2} - \frac{\Delta\omega t_0}{2}\right) = 0$

phase diff. b/w x_1 & x_2,
$= \frac{\Delta k}{2}(x_2 - x_1) = \pi \Rightarrow \Delta k(x_2 - x_1) = 2\pi \Rightarrow \Delta k \Delta x = 2\pi$

Now multiply $\hbar$ both sides,
$\hbar \Delta k \Delta x = \hbar 2\pi$

$\boxed{p = \hbar k} \longrightarrow$ we know it

$\boxed{\Delta p \Delta x = h}$; $\hbar = \frac{h}{2\pi}$

$\langle\,\rangle$ = expectatⁿ

$$\left.\begin{aligned}\Delta x &= [\langle x^2 \rangle - \langle x \rangle^2]^{1/2} \\ \Delta p &= [\langle p_x^2 \rangle - \langle p_x \rangle^2]^{1/2}\end{aligned}\right\}$$ uncertainities in positⁿ & momentum

Wave function:

Wave fⁿ has no significance
→ Well behaved wave funⁿ

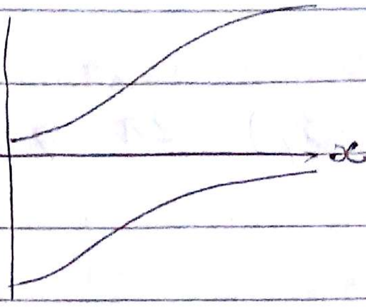
1. ψ must be cont. and single valued.
2. $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be continuous and single valued.
3. ψ must be normalizable which means ψ must go to zero as $x \to \pm\infty, y \to \pm\infty, z \to \pm\infty$ in order that ~~the~~ $\int |\psi|^2 dV$ overall space be a finite constant.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\psi_{(x,y,z)}|^2 dV = 1 \quad ; \; dV = dx\,dy\,dz$$

4. One can multiply $\psi(r,t)$ by a constant "N" so that $N\psi$ satisfies the normalizable condition.

$$|N|^2 \int_{-\infty}^{\infty} |\psi(r,t)|^2 dV = 1$$

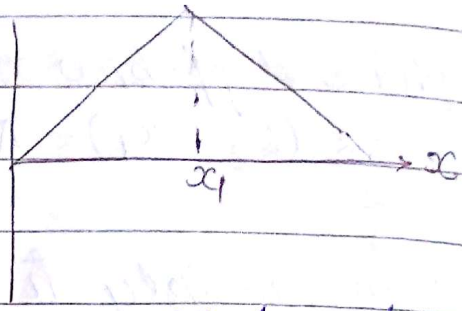
Q (i)



Not wave fuⁿ (well behaved)

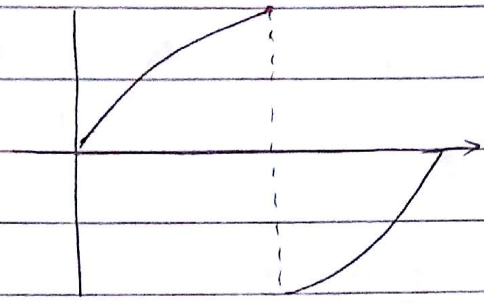
∵ Not single valued

(ii)



× well behaved due to condtⁿ 2

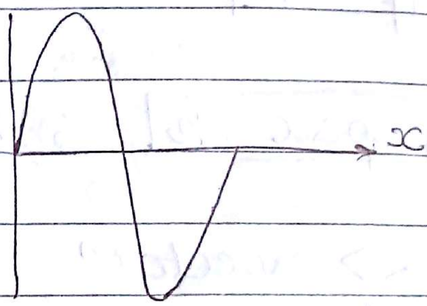
(iii)



× well behaved

∵ not continuous

(iv)



✓ well behaved wave function

(ii) $\psi = A\sec x$

$\psi = A\tan x$

$\psi = Ae^{x^2}$

× well-behaved wave f^n

$\psi = Ae^{-x^2}$ ✓ well-behaved

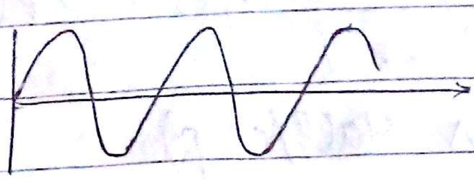
Q Normalize the wave f^n: $\psi = Ae^{-ax^2}$ (or find A)?

Solⁿ: Given that: $\int_{-\infty}^{\infty} e^{-2ax^2}dx = \sqrt{\frac{\pi}{2a}}$

$\therefore |A|^2 \int_{-\infty}^{\infty} e^{-2ax^2}dx = 1 \Rightarrow |A|^2\left(\sqrt{\frac{\pi}{2a}}\right) = 1 \Rightarrow |A| = \left(\frac{2a}{\pi}\right)^{1/4} = A$

$\therefore$ Normalized wave f^n: $\left(\frac{2a}{\pi}\right)^{1/4} e^{-ax^2}$

Time dependent Schrödinger Equation (TDSE):



$$\psi = A e^{-i(-kx+\omega t)}$$

$E = \hbar\omega$ and $p = \hbar k$.

$$\psi = A e^{-\frac{i}{\hbar}(Et - px)} \quad \text{—— (1)}$$

$$\frac{\partial \psi}{\partial x} = A e^{-\frac{i}{\hbar}(Et - px)} \cdot \frac{pi}{\hbar}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2}{\hbar^2}\psi \quad \text{—— (3)}$$

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar}\psi \quad \text{—— (2)}$$

Total energy $\rightarrow$ $E = KE + PE$

$$E = \frac{p^2}{2m} + U$$

Multiply ψ,

$$E\psi = \frac{p^2\psi}{2m} + U\psi \quad \text{—— (4)}$$

from 2 and 3,

$$-\frac{\hbar}{i}\frac{\partial \psi}{\partial t} = \frac{-\hbar^2 \partial^2 \psi}{\partial x^2 (2m)} + U\psi \qquad (\text{Now, } -1 = i^2)$$

$$\therefore, \boxed{i\hbar\frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2}\right) + U\psi}$$

In 3D:

$$i\hbar\frac{\partial\psi(r,t)}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(r,t) + U\psi(r,t)$$

$\hat{E}\psi(r,t) = \hat{H}\psi(r,t)$ ($\wedge$ = Hall/cap)

where, $\hat{E} = i\hbar\frac{\partial}{\partial t}$

$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + U$ (in 3D) or $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U$ (in x-dirn)

$\boxed{\hat{E}\psi = \hat{H}\psi}$; $\hat{E} = i\hbar\frac{\partial}{\partial t}$

Time-independent Schrödinger Eqn (T.I.S.E):

$\psi(x,t) = Ae^{-i(\omega t - kx)}$

$\psi(x,t) = Ae^{-\frac{i}{\hbar}(Et - px)}$ —— (i)

By using separation of variable technique we can write eqn (i) as:

$\psi(r,t) = \psi(x)\,\phi(t)$

$\psi(r,t) = \psi(x)e^{-iE(t)/\hbar}$ —— (ii)

If we substitute eqn (ii) in TDSE:

$$\Rightarrow i\hbar\frac{\partial}{\partial t}\left(\psi(x)e^{-iEt/\hbar}\right) = -\frac{\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\left(\psi(x)e^{-\frac{iEt}{\hbar}}\right) + U(x)\psi(x)e^{-iEt/\hbar}$$

$$= i\hbar\left(-\frac{iE}{\hbar}\right)\psi(x)e^{-iEt/\hbar} = -\frac{\hbar^2}{2m}e^{-iEt/\hbar}\frac{\partial^2}{\partial x^2}(\psi(x)) + U(x)\psi(x)e^{-i}$$

$$E\psi(x)e^{-iEt/\hbar} = \quad '' \quad + \quad ''$$

divide above eqn by, $e^{-iEt/\hbar}$

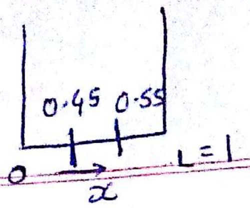
we get:

$$\therefore \boxed{E\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\psi(x)}$$

$$\therefore \boxed{E\psi(x) = \hat{H}\psi(x)}$$

T.I.S.E.

Total energy of particle



$\langle | \rangle$ = Dirac notⁿ

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For position of a particle:
Expectation value = $\langle x \rangle$

$$\therefore \langle x \rangle = \frac{\int_{-\infty}^{\infty} x|\psi|^2 dx}{\int_{-\infty}^{\infty} |\psi|^2 dx}$$ — in 1D (or x-dirⁿ)

$\int_{-\infty}^{\infty} |\psi|^2 dx \longrightarrow$ from 3rd condⁿ of well-behaved wave fⁿ, denominator = 1.

$$\therefore \langle x \rangle = \int_{-\infty}^{\infty} \cancel{x}|\psi|^2 dx$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \psi dx$$

$$\psi(x) = \begin{cases} ax & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$ is a well-defined wave;

(i) Find out the probability that the particle can be found between $x = 0.45$ to $x = 0.55$.
(ii) Find out expectation value.

; $\cancel{\langle x \rangle =}$

(i) $$\int_{0.45}^{0.55} |\psi|^2 dx = \int_{0.45}^{0.55} a^2x^2 dx = \left(\frac{a^2x^3}{3}\right)\Bigg|_{0.45}^{0.55} = 0.025a^2$$

(ii) $$\langle x \rangle = \int_{0.45}^{0.55} (x)\times(a^2x^2 dx) = \int_{0.45}^{0.55} a^2x^3 dx = \frac{a^2}{4}\left(x^4\right)\Bigg|_{0.45}^{0.55} = 0.0126a^2$$ ✗

$$\int_0^1 a^2x^3 dx = \frac{a^2}{4}$$ (here $\psi^* = ax$)

How to determine p?

Operator ($\hat{O}$): → denoting operator

$\hat{O}\psi = a\psi$

$\hat{O}$ → Operator
ψ → Eigen funⁿ
a → Eigen value

Function	Operator
① position	$\hat{x}$
② momentum	$\hat{p}_x = -i\hbar\frac{\partial}{\partial x} = \frac{\hbar}{i}\frac{\partial}{\partial x}$ (along x-axis)
	$\hat{p}_y = -i\hbar\frac{\partial}{\partial y} = \frac{\hbar}{i}\frac{\partial}{\partial y}$
	$\hat{p}_z = -i\hbar\frac{\partial}{\partial z} = \frac{\hbar}{i}\frac{\partial}{\partial z}$
③ Total Energy (Time dep.)	$\hat{E} = i\hbar\frac{\partial}{\partial t}$
④ Total Energy (Time indep.)	$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{U}(x)$ (Hamiltonian operator)
⑤ ~~Total~~ Potential Energy	$\hat{U}$
⑥ Angular momentum	$\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$
	$\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$
	$\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$

Product of operators:

Let $\hat{A}$ and $\hat{B}$ are two operators, and let us operate on a eigen/wave function ψ, then the expression $\rightarrow \hat{A}\hat{B}\psi$.

We can denote $\hat{A}\hat{B}$ by $\hat{C}$. $\hat{C}$ is product of operators $\hat{A}$ & $\hat{B}$.

Commutation:

If $[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ then this is called commutator of $\hat{A}$ & $\hat{B}$. If $[\hat{A},\hat{B}] \neq 0$, then one says that $\hat{A},\hat{B}$ do not commute & if $[\hat{A},\hat{B}] = 0$, then $\hat{A},\hat{B}$ commute with each other.

HW Find value of: $[\hat{x}, \hat{y}, \hat{z}, \hat{p}_x]$
$[\hat{x}\hat{y}\hat{z}, \hat{p}_y]$
$[\hat{x}\hat{y}\hat{z}, \hat{p}_z]$

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An operator equator of the form $[\hat{A}, \hat{B}]$ = something is called commutation relation.

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$$

Ex: $\psi(x,t) = e^{i(kx-\omega t)}$

$$\hat{p}_x \psi = -i\hbar \frac{\partial}{\partial x} e^{i(kx-\omega t)}$$
$$= -i\hbar \cdot ik \cdot e^{i(kx-\omega t)}$$
$$= -i^2 \hbar k \psi$$
$$\hat{p}_x \psi = \hbar k \psi$$

$\downarrow \hat{O}$ $\quad \downarrow$ eigen value

Ex: For $\hat{E} = +i\hbar \frac{\partial}{\partial t}$

$$\hat{E}\psi = i\hbar \frac{\partial}{\partial t} e^{i(kx-\omega t)}$$
$$= -i\hbar\omega e^{i(kx-\omega t)} \times i$$
$$= -i^2 \hbar\omega\psi$$
$$\hat{E}\psi = \hbar\omega\psi$$

$\downarrow \hat{O}$ $\quad \downarrow$ Eigen value

Q Determine the value of $[\hat{x}, \hat{p}]$

Q Determine the value of $[\hat{y}, \hat{p}_x]$
Ans, is zero.

Q Determine the value of $[\hat{x}^2, \hat{p}_x^2]$ — Ans = 0

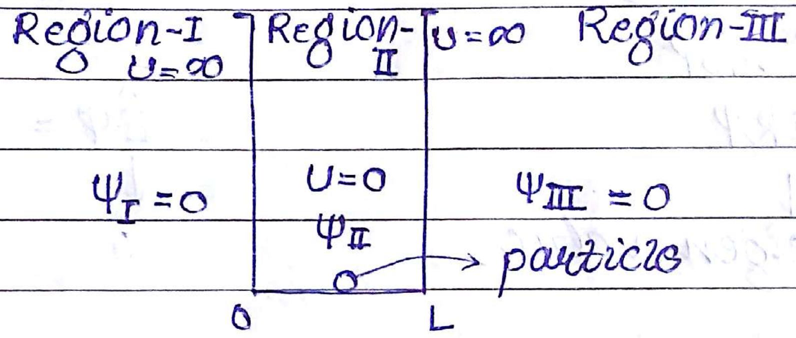
$[\hat{x}^2, \hat{p}_x^2]$

$$\hat{p}_x^2 = (-i\hbar)^2 \times \frac{\partial^2 \psi}{\partial x^2} = -\hbar^2$$

Scanned by CamScanner

* Particle in a rigid box (1-D):

1. A particle of mass 'm' tapped in a box with
2. A particle travelling along x-axis between $x=0$ an $x=L$
3. A particle does not loose energy when it collides with such walls, so that its energy stays constant.
4. No external force is applied.



ψ_i = wave funⁿ for respective regions, for finding particle in that region.

Writing, Time-independent SE for ①

$$-\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi_I}{\partial x^2} + U\psi_I(x) = E\psi_1 \quad \text{——} ①$$

since $U=\infty$, $\psi_I = 0$

for region ②

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi_{II}}{\partial x^2} = E\psi(x) \quad \text{——} ②$$

$$\frac{\partial^2 \psi_{II}}{\partial x^2} = -\frac{2mE}{\hbar^2} \psi(x) \quad \text{——} ③$$

for region ③,

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi_{III}}{\partial x^2} + U\psi_{III}(x) = E\psi_{III} \quad \text{——} ④$$

$\because U=\infty$, $\psi_{III} = 0$

$\therefore$, Only 1 sol^n exists for region-II,

$$\frac{\partial^2 \psi_{II}}{\partial x^2} = -k^2 \psi_{II}(x) \text{ ——— } (5) \quad \text{where } k^2 = \frac{2mE}{\hbar^2}$$

A general sol^n for eq^n (5),

$$\psi_{II}(x) = A\cos kx + B\sin kx \text{ ——— } (6)$$

where, $A, B =$ constants

Now, applying boundary cond^n, i.e wave fu^n must be zero at end pts $= 0$ and L.

$\therefore$,

For $x=0$, $\psi_{II}(x=0) = A = 0$

For $x=L$, $\psi_{II}(x=L) = 0$ -

$\therefore$, $A\cos Lk + B\sin Lk = 0$ $\quad \therefore$, $\tan Lk = \frac{-A}{B} = 0$

$\therefore$, $B\sin kx = \psi_{II}(L)$

for, $kx = n\pi$, $\psi_{II} = 0$ $\therefore$, $x = \frac{n\pi}{k}$ $\therefore$, for $x = L$

$$k = \frac{n\pi}{L}$$

$$\psi_{II}(x) = B\sin\frac{n\pi x}{L}$$

According to normalization condit^n, $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

$$\int_0^L B^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1 \Rightarrow B = \sqrt{\frac{2}{L}}$$

wave f^n:

$$\therefore, \psi(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), & 0<x<L \text{ (or inside the box)} \\ 0, & \text{otherwise (or outside the box)} \end{cases}$$

For energy,

$kL = n\pi$

$k^2L^2 = n^2\pi^2$

$\frac{2mE}{\hbar^2}L^2 = n^2\pi^2$

$\therefore \boxed{E = \frac{n^2\pi^2\hbar^2}{2mL^2}} \longrightarrow$ Energy is quantized.