Identities related to Gradient

$$\nabla(f + g) = \nabla f + \nabla g, \qquad \nabla \cdot (\mathbf{A} + \mathbf{B}) = (\nabla \cdot \mathbf{A}) + (\nabla \cdot \mathbf{B}),$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) = (\nabla \times \mathbf{A}) + (\nabla \times \mathbf{B}),$$

$$\nabla(kf) = k\nabla f, \qquad \nabla \cdot (k\mathbf{A}) = k(\nabla \cdot \mathbf{A}), \qquad \nabla \times (k\mathbf{A}) = k(\nabla \times \mathbf{A}),$$

fg (product of two scalar functions),
$\mathbf{A} \cdot \mathbf{B}$ (dot product of two vector functions)

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

Identities related to Divergence & Curl

$f\mathbf{A}$ (scalar times vector),

$\mathbf{A} \times \mathbf{B}$ (cross product of two vectors)

$$\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$$

Second Derivatives

∇T is a *vector*

(1) Divergence of gradient: $\nabla \cdot (\nabla T)$

(2) Curl of gradient: $\nabla \times (\nabla T)$.

$\nabla \cdot \mathbf{v}$ is a *scalar*

Gradient of divergence: $\nabla (\nabla \cdot \mathbf{v})$

$\nabla \times \mathbf{v}$ is a *vector*

Divergence of curl: $\nabla \cdot (\nabla \times \mathbf{v})$

Curl of curl: $\nabla \times (\nabla \times \mathbf{v})$.

$$\nabla \cdot (\nabla T) = \left(\hat{\mathbf{x}} \frac{\partial}{\partial x} + \hat{\mathbf{y}} \frac{\partial}{\partial y} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \right) \cdot \left(\frac{\partial T}{\partial x} \hat{\mathbf{x}} + \frac{\partial T}{\partial y} \hat{\mathbf{y}} + \frac{\partial T}{\partial z} \hat{\mathbf{z}} \right)$$

$$= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$= \nabla^2 T$$

$\nabla^2 \longleftarrow$ Laplacian oparator

Laplacian of a *vector*,

$$\nabla^2 \mathbf{v} \equiv (\nabla^2 v_x)\hat{\mathbf{x}} + (\nabla^2 v_y)\hat{\mathbf{y}} + (\nabla^2 v_z)\hat{\mathbf{z}}$$

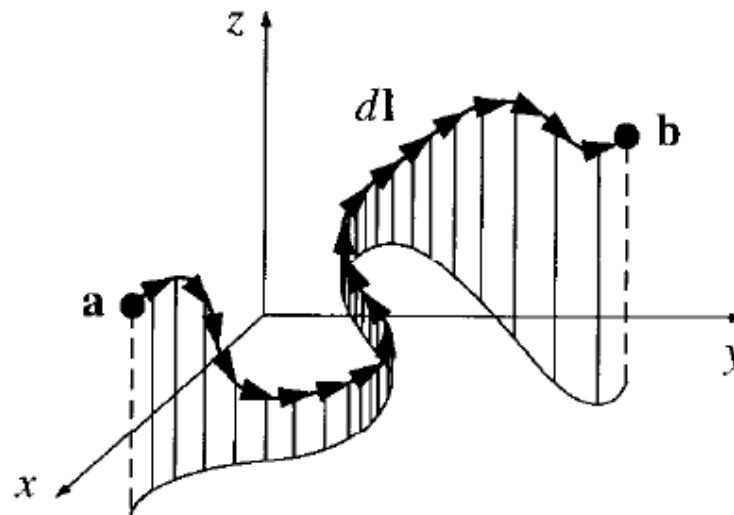The curl of a gradient is always *zero*. $\quad \nabla \times (\nabla T) = 0$

$$\nabla^2 \mathbf{v} = (\nabla \cdot \nabla)\mathbf{v} \neq \nabla(\nabla \cdot \mathbf{v})$$

The divergence of a curl, like the curl of a gradient, is *always zero* $\quad \nabla \cdot (\nabla \times \mathbf{v}) = 0$

$$\nabla \times (\nabla \times \mathbf{v}) = \nabla(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}$$

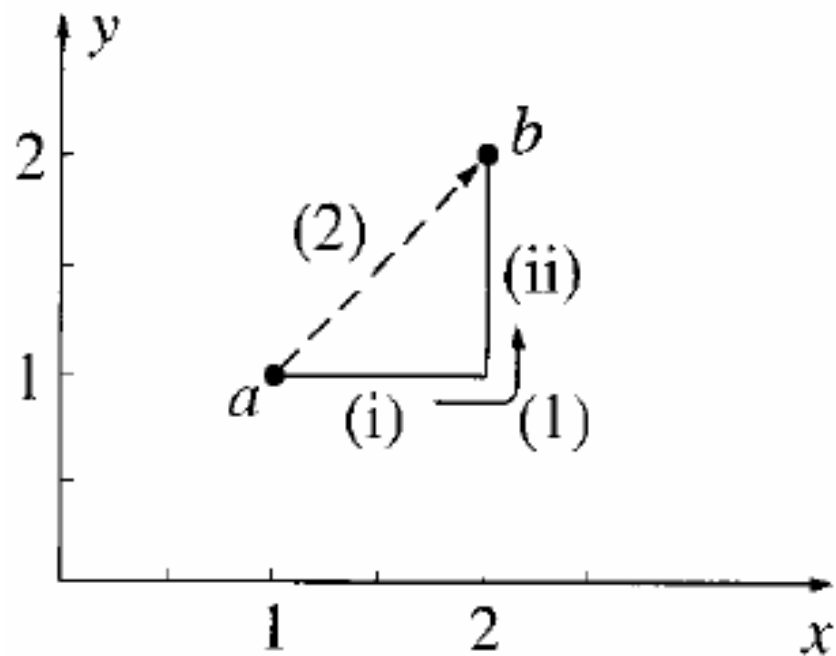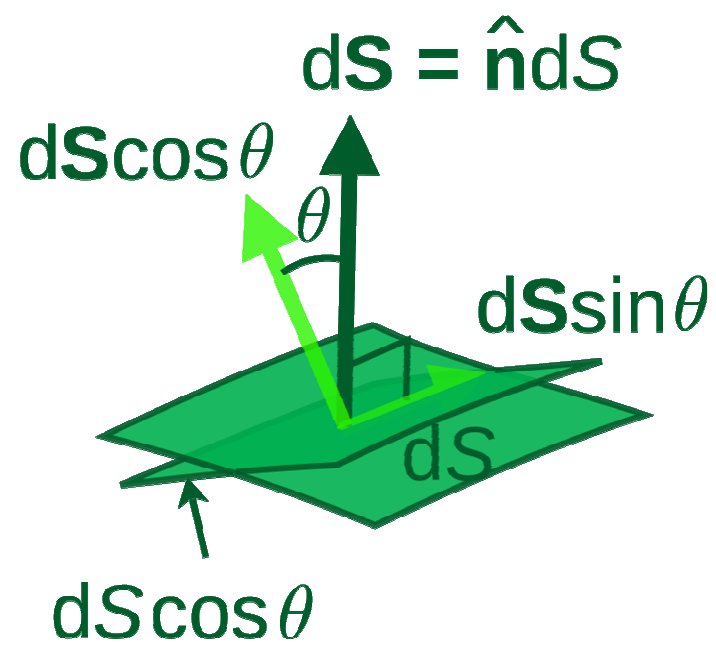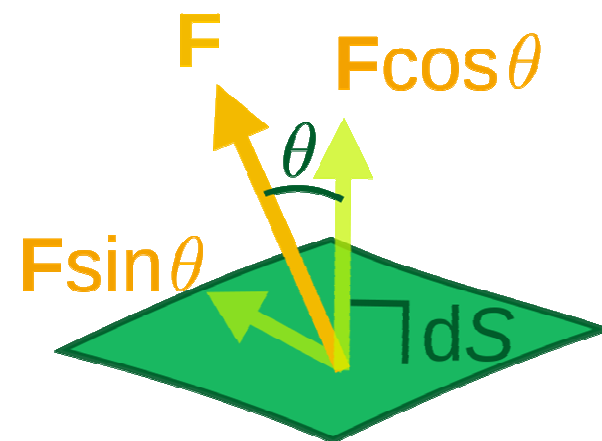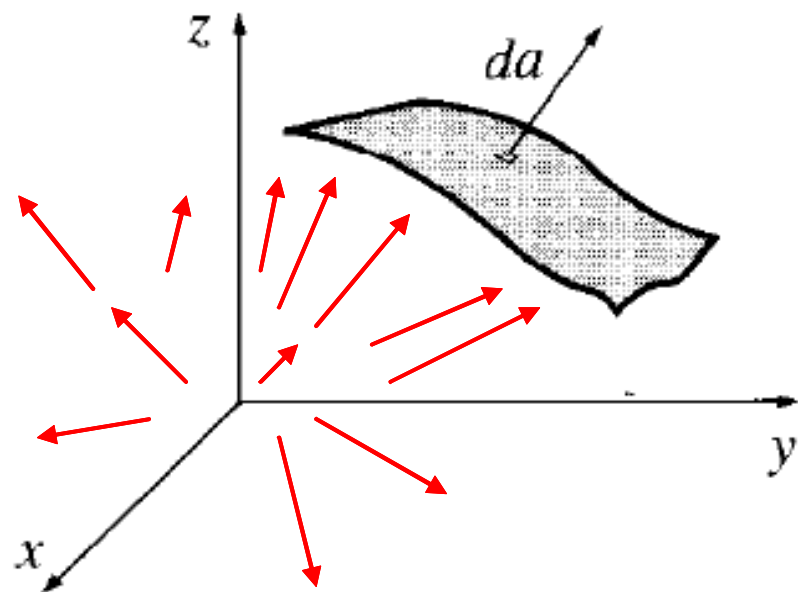# Vector Line Integration

$$(i) \quad \vec{V} = -y\hat{i} + x\hat{j}$$

$$(ii) \quad \vec{V} = x\hat{i} + y\hat{j}$$

Vector Surface Integral

da

z

y

x

F

Fcosθ

θ

Fsinθ

$\ulcorner$dS

d**S** = **n̂**dS

d**S**cosθ

θ

d**S**sinθ

dS

dScosθ

Vector surface Integral

Open surface

Closed surface
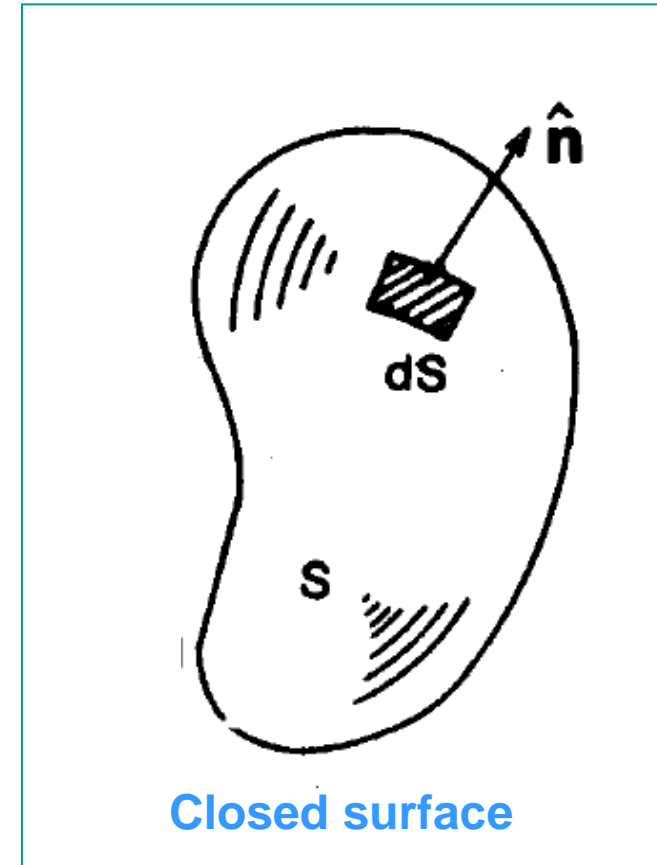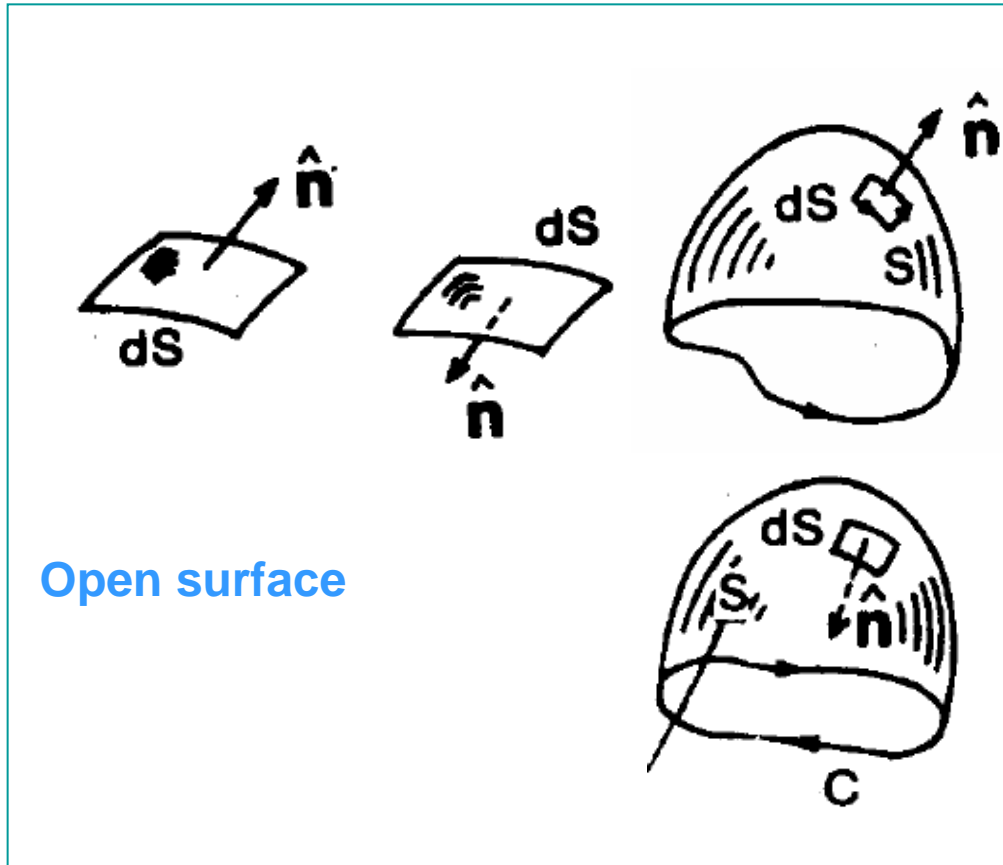
(i) $\quad \vec{V_1} = -y\hat{i} + x\hat{j}$

(ii) $\quad \overline{V_2} = -y\hat{i} + xy\hat{j}$

(iii) $\quad \vec{V_3} = xy\hat{i} + yz\hat{j} + zx\hat{k}$