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Discrete Mathematical Structure

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16UCS128

01 $A = \{1\}$ $B = \{1, \{1\}\}$

a) Each element of A belongs to B so $A \subseteq B$.
b) On the other hand, $\{1\} \in B$ but $\{1\} \notin A$. Hence $A \neq B$
Therefor A is a proper subset of B.

02. $A = \{ m \in Z \mid m = 2a$ for some integer $a \}$
$B = \{ n \in Z \mid n = 2b-2$ for some integer $b \}$

$n = 2b-2 = 2(b-1)$ here $a \in I$
$b \in I \Leftrightarrow b-1 \in I$

Thus $m \in$ even integer
and $n \in$ even integer

Thus A is a set of even integers
B is a set of even integers
Thus $A = B$;

03.a) i) $A \cup (B \cap C)$ $A = \{a, b, c\}$
ii) $(A \cup B) \cap C$ $B = \{b, c, d\}$
iii) $(A \cup B) \cap (A \cup C)$ $C = \{b, c, e\}$

$B \cap C = \{b, c\}$
i) $A \cup (B \cap C) = \{a, b, c\}$ $A \cup C = \{a, b, c, e\}$
$A \cup B = \{a, b, c, d\}$ iii) $(A \cup B) \cap (A \cup C) = \{a, b, c\}$
ii) $(A \cup B) \cap C = \{b, c\}$ Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$;

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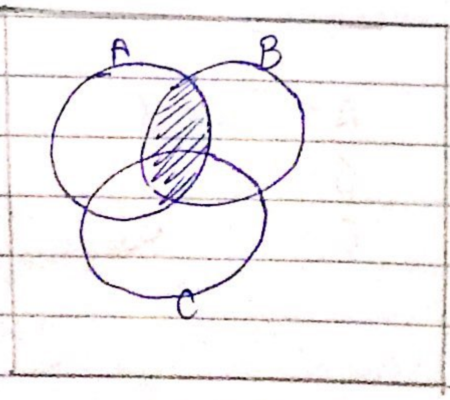
b) i) $A \cap (B \cup C)$ $\quad A = \{a, b, c\}$
ii) $(A \cap B) \cup C$ $\quad B = \{b, c, d\}$
iii) $(A \cap B) \cup (A \cap C)$ $\quad C = \{b, c, e\}$

$B \cup C = \{b, c, d, e\}$
i) $A \cap (B \cup C) = \{b, c\}$
ii) $(A \cap B) \cup C = \{b, c, e\}$
$A \cap B = \{b, c\}$
iii) $(A \cap C) = \{b, c\}$
$(A \cap B) \cup (A \cap C) = \{b, c\}$
Thus
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;

c) $(A-B)-C$ and $A-(B-C)$
$A = \{a, b, c\}$
$B = \{b, c, d\}$
$C = \{b, c, e\}$

$(A-B) = \{a\}$ $\quad (B-C) = \{d\}$
$(A-B)-C = \{a\}$ $\quad A-(B-C) = \{a, b, c\}$
Thus
$(A-B)-C$ is not equal to $A-(B-C)$;

Q4.
a)



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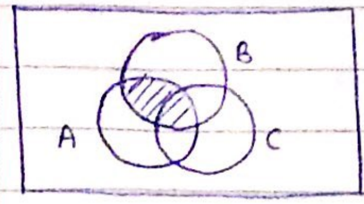
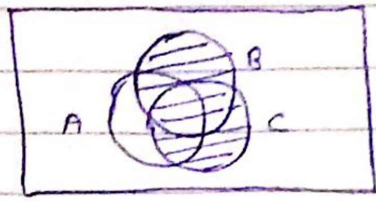
4.(a) $A \cap B$

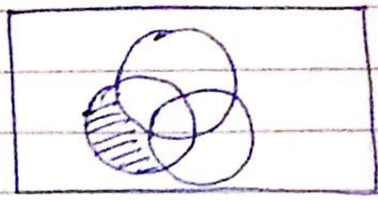


(b) $B \cup C$

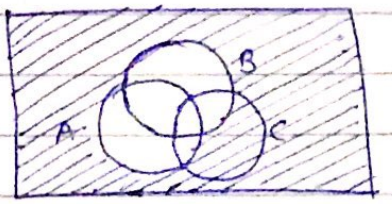


(c) A^c

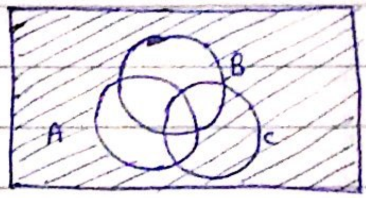
(d) $A - (B \cup C)$



(e) $(A \cup B)^c$



(f) $(A^c \cap B^c)$



Q5. LHS $A - (A \cap B)$

$A \cap (A \cap B)'$

$(A \cap A') \cup (A \cap B')$

$A \cap B'$

RHS $A - B$

$A \cap B'$

LHS = RHS

Thus $A - (A \cap B) = A - B$

Property Used

$A - B = A \cap B'$

$A \cap A' = \phi$

$\phi \cup A = A$;

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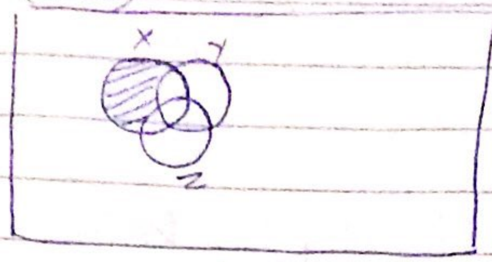
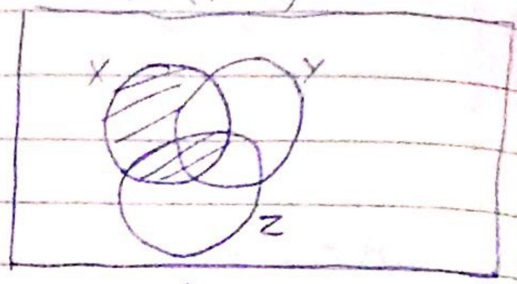
Q6. $(X-Y)-Z \neq X-(Y-Z)$

$(X-Y)-Z$ $\qquad\qquad$ $X-(Y-Z)$





Thus by Ven diagram these are not equal.

$(X-Y)-Z$

$(X \cap Y') \cap Z$

$X-(Y-Z)$

$X \cap (Y \cap Z')'$

$X \cap (Y' \cup Z)$

$(X \cap Y') \cup (X \cap Z)$

Q7. $(A \cap B') \cup (A' \cap B) \cup (A' \cap B')$

$(A \cap B') \cup (A' \cap (B \cup B'))$

$(A \cap B') \cup (A' \cap U)$

$(A \cap B') \cup A'$

$(A \cup A') \cap (B' \cup A')$

$U \cap (B' \cup A')$

$\cancel{(A \cup B)'}$ $(A \cap B)'$

Property Used

$A \cup A' = U$

$A \cap A' = \Phi$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A' \cup B' = (A \cap B)'$ demorgan

Q8. A = { Angelo 0.4, Bart 0.7, Cathy 0.6 }

B = { Dan 0.3, Elsie 0.8, Frank 0.4 }

a) $A \cup B$ = { Angelo 0.4, Bart 0.7, Cathy 0.6, Dan 0.3, Elsie 0.8, Frank 0.4 }

b) $A \cap B = \Phi$

c) A'

$A' = \{$Angelo 0.6, Bart 0.3, Cathy 0.4, Dan 1, Elsie 1, Frank 1$\}$;

d) $A \cup B'$

~~$B' = \{$Dan 0.7, Elsie 0.2, Frank 0.6$\}$~~

~~$A \cup B' = \{$Angelo 0.4, Bart 0.7, Cathy 0.6, Dan 0.7, Elsie 0.2, Frank 0.6$\}$~~

$A \cup B' = \{$Angelo 1, Bart 1, Cathy 1, Dan 0.7, Elsie 0.2, Frank 0.6$\}$

e) $A \cap B' = \phi$

f) $A \cap A' = \{$Angelo 0.4, Bart 0.3, Cathy 0.4$\}$

Q9. Basis Step

For $n = 0$

$A = \phi$

$2^0 = 1$ thus there is only one subset i.e the set itself.

Inductive step

We assume above statement is true for $n = k$;

$B = \{a_1, a_2 \ldots a_k\}$

Thus the no. of subsets are 2^k i.e

$\phi, \{a_1\}, \{a_2\} \ldots \{A\}$

For $n = k+1$

$A = \{a_1, a_2, \ldots a_k, a_{k+1}\}$

Let $x \in A$ $\quad x \in B$ $\quad A = B \cup \{x\}$;

$A = B \cup \{x\}$

All the subsets of B are also subsets of A. Now I am adding 'n' element to all those subsets
⇒ These new subsets are the subsets of $A = 2^n$

Q10
a) 26
b) 5
c) 7
d) 0

Q11. a) A = Divisible by 2
B = Divisible by 3.

~~$A = \frac{500}{2} = 250$~~

$\frac{498}{2} = 249 = A$

$\frac{498}{3} = 166 = B$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = \frac{498}{6} = 83$

$n(A \cup B) = 249 + 166 - 83 = 332$;

b) C = Divisible by 5 $= \frac{495}{5} = 99$

$n(A \cap B \cap C) = \frac{480}{30} = 16$

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$n(A\cap C) = \frac{490}{10} = 49$

$n(B\cap C) = \frac{495}{15} = 33$

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C)$
$\quad + n(A\cap B\cap C)$

$= 249 + 166 + 99 - 49 - 33 - 83 + 16$

$= 365$

$15\overline{)500}$
490

c) $n(A\cup B) - n(A\cap B) = 249$

$n(A\cup B) = 332$

$n(A\cap B) = 83$

d) ~~$(A'\cap B'\cap C') = (A\cup B\cup C)' = (A\cup B\cup C)$~~

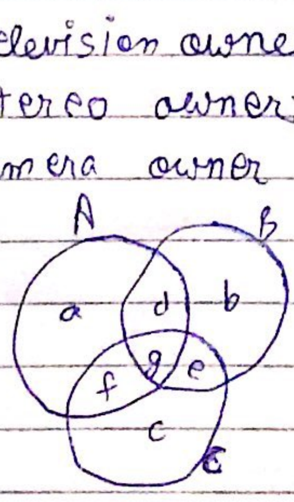
~~$n(A'\cap B')$~~

$n(A')\cap n(B')\cap n(C') = (n(A)\cup n(B)\cup n(C))'$

$U - (n(A)\cup n(B)\cup n(C))$

$499 - 365 = 134$

Q12. A = Television owner
B = Stereo owner
C = Camera owner



$n(U) - (a+b+c+d+e+f+g) = 50$

$a+f = 220$

$d+b = 200$

$c+e = 170$

$a+f+c = 270$

$b+e+c = 80$

$a+d+b = 80$

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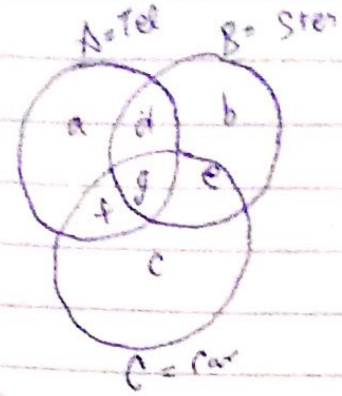

$a+d+b+f+g+e+c+50 = 750$

$a+d+b+f+g+e+c = 650;$

$a+f = 220$

$c+e = 170$

$d+b = 200$

$d = 80.$

$e = 80$

$f = 70$

$a = 150$

$b = 120$

$c = 90$

i) $a+b+c = 360;$

ii) $d+e+f = 230;$

iii) 650

iv) $g = 650-360-230 = 60;$

Q13.

$2 \in S \Rightarrow 4 \in S$

$4 \in S \Rightarrow 16 \in S$

Set becomes

$S = \{2, 2^2, 2^{2^2}, 2^{2^{2^2}}, - - \};$

Premium

Q14

a) ~~$f: Z^+ \times Z^+ \to Z^+$~~

Domain (Z^+, Z^+)
Range (Z^+)

b) Domain Z^+
Range: $\{x \in Z^+ \mid x < 10\}$

c) Domain: a set of all bit string
Range: Z

d) Domain: Z^+
Range: Z^+

e) Domain: a set of all bit strings
Range: bit string containing only ones, and the empty string.

Q15.

a) $f(m, n) = 2m - n$

$$f(m,n) = \begin{cases} 2m & n=0 \\ 2m-1 & n=1 \\ \vdots & \end{cases}$$

Since $2m \longrightarrow$ even
$2m-1 \longrightarrow$ odd
~~to~~ so both combinely form integer
thus
onto function

Q15

b) $f(m,n) = m^2 - n^2$;

Since Z contains 2;
And there exists no $m, n \in Z$ for which $m^2 - n^2 = 2$

<u>into</u>

c) $f(m, n) = m + n + 1$;

$m \in Z$
$n \in Z$
$m + n \in Z$
$m + n + 1 \in Z$
Thus $f(m, n) = m + n + 1$ is onto

d) $f(m, n) = |m| - |n|$

$$f(m, n) = \begin{cases} m - n & m > 0 \quad n > 0 \\ m + n & m > 0 \quad n < 0 \\ -m + n & m < 0 \quad n < 0 \\ -m - n & m < 0 \quad n > 0 \end{cases}$$

$m - n \in Z$
$m + n \in Z$
$-m + n \in Z$ Thus $f(m, n)$ is onto;
$-m - n \in Z$

e) $$f(m, n) = \begin{cases} |m| & n = 0 \rightarrow \text{set of all +ve int,} \\ -|n| & m = 0 \rightarrow \text{set of all -ve int,} \end{cases}$$

onto func;

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e) Let $f(m, n) = -5$

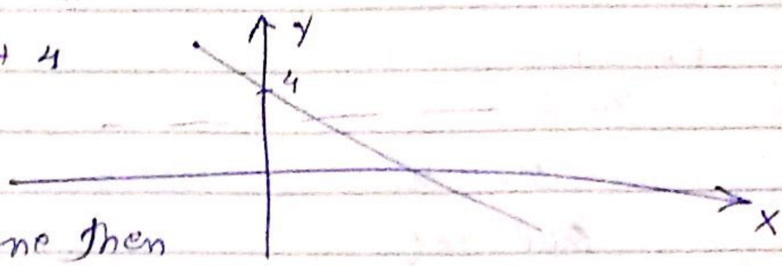
Since $-m^2 - 4 \neq -5$ for any int m

Thus

$f(m,n)$ is an into function;

Q16

a) $f(x) = -3x + 4$



Let many one then

$f(x_1) = f(x_2)$

$-3x_1 + 4 = -3x_2 + 4$

$x_1 = x_2$ Thus by contradiction

$f(x)$ is one-one

$y = -3x + 4$

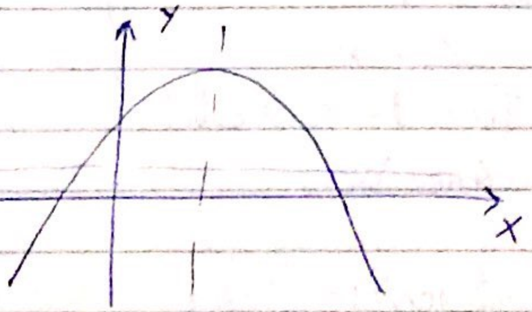
$x = \frac{4-y}{3}$

$x \in R \Rightarrow y \in \mathbb{R}$

$f(x)$: onto;

Bijective.

b) $f(x) = -3x^2 + 7$



Let many one then

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$x_1 = x_2$ or $x_1 = -x_2$

Not one-one.

$y = -3x^2 + 7$

$x^2 = \frac{7-y}{3}$

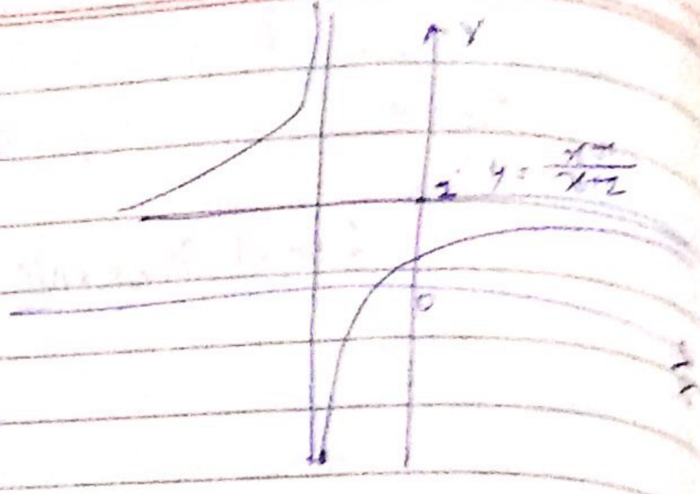
$x = \pm\sqrt{\frac{7-y}{3}}$

$\therefore y < 7/3$

Not one-one

Not bijective

c) $f(x) = \frac{x+1}{x+2}$



Let many one

$f(x_1) = f(x_2)$

$\frac{x_1+1}{x_1+2} = \frac{x_2+1}{x_2+2}$

$1 - \frac{1}{x_1+2} = 1 - \frac{1}{x_2+2}$

$x_1 = x_2$

One-one

$x = \frac{1-2y}{y-1}$

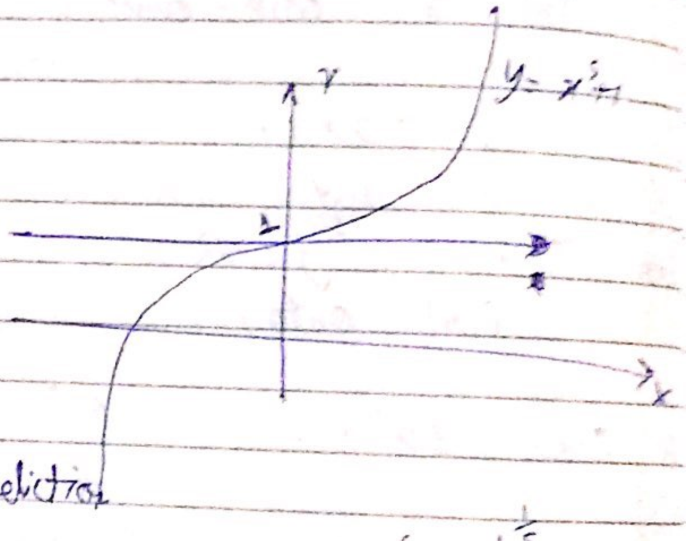
Not bijective

$y \neq 1$

Not onto

Thus into

d) $f(x) = x^5+1$



Let many one

$f(x_1) = f(x_2)$

$x_1^5+1 = x_2^5+1$

$x_1 = x_2$

Thus by contradiction

f(x) is One-One

$x = (y-1)^{\frac{1}{5}}$

Bijective

$x \in R$

$y \in R$

Onto.

Q18.

$$f(x) = x - 1$$
$$g(x) = 3x$$
$$h(x) = \begin{cases} 1 & x \text{ odd}; \\ 0 & x \text{ even}; \end{cases}$$

a) i) $f \circ g = f(g(x)) = gx - 1 = 3x - 1$

ii) $g \circ f = g(f(x)) = 3f(x) = 3x - 3$

iii) $g \circ h = g(h(x)) = 3h(x) = \begin{cases} 3 & x \text{ odd} \\ 0 & x \text{ even} \end{cases}$

iv) $h \circ g = h(g(x)) = \begin{cases} 1 & g(x) \text{ odd} \\ 0 & g(x) \text{ even} \end{cases} = \begin{cases} 1 & 3x \text{ odd} \\ 0 & 3x \text{ even} \end{cases}$

$$= \begin{cases} 1 & x \text{ odd} \\ 0 & x \text{ even} \end{cases}$$

v) $f \circ (g \circ h) = f(g(h(x))) = \begin{cases} 2 & x \text{ odd} \\ -1 & x \text{ even} \end{cases}$

vi) $(f \circ g) \circ h$

$(f \circ g) = \omega(x) = 3x - 1$

$\omega \circ h = \omega(h(x))$

$$= \begin{cases} 2 & x \text{ odd} \\ -1 & x \text{ even} \end{cases}$$

b) f^2

$f^2(x) = (x-1)^2 = x^2 - 2x + 1$

$f^3 = (x-1)^3 = x^3 - 3x^2 + 3x - 1$

$g^2 = 9x^2$

$$h^2 = \begin{cases} 1 & x \text{ odd;} \\ 0 & x \text{ even;} \end{cases}$$

$$h^3 = \begin{cases} 1 & x \text{ odd;} \\ 0 & x \text{ even;} \end{cases}$$

$$h^{500} = \begin{cases} 1 & x \text{ odd;} \\ 0 & x \text{ even;} \end{cases}$$

Q17 a) Given

$f(x)$, $g(x)$ one-one

To prove

$f(g(x))$ is also one one

$f(x) = f(y) \Rightarrow x = y = k$

Similarly

$g(x) = g(y) \Rightarrow x = y = k$

TPT $f(g(x)) = f(g(y))$

Since $f(k) = f(k)$ that is $k = k$

~~then~~ thus $f \circ g$ is also one-one

b) $g : A \to B$ & $f : B \to C$ are onto

TPT : ~~$f(g(x)) : A \to C$ is~~

$f \circ g : A \to C$ is onto, we need to prove that

$\forall\, c \in C \;\exists\; a \in A$ such that $f \circ g(a) = c$

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Let c be any element of C
Since $f: B \to C$ is onto, $\exists\, b \in B$ s.t $f(b) = c$
Since $g: A \to B$ is onto, $\exists\, a \in A$ s.t $g(a) = b$
so $(f \circ g)(a) = g(f(a)) = g(b) = c$
Thus proofed;

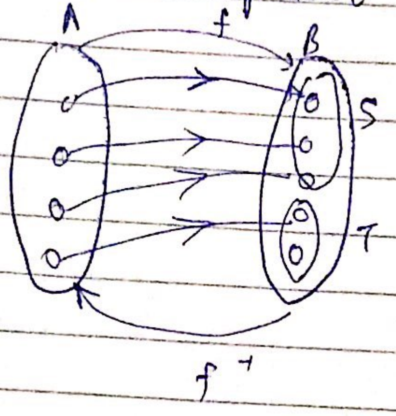
Q19. $f: A \to B$

Let S & T be subset of B

a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$

b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

This can be expanded via diagram:



Q20. $S \subseteq U$; $f_S: U \to \{0, 1\}$

$$f_S(x) = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{if } x \notin S \end{cases}$$

a) $f_{A \cap B}(x) = f_A(x) \cdot f_B(x)$ because both A & B are independent

b) $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_{A \cap B}(x)$
$= f_A(x) + f_B(x) - f_A(x) \cdot f_B(x)$

c) $f_{\bar{A}}(x) = 1 - f_B(x)$

d) $f_{A \oplus B}(x) = f_A(x) + f_B(x) - 2 f_{A \cap B}(x)$
$= f_A(x) + f_B(x) - 2 f_A(x) \cdot f_B(x)$

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