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Propositional Logic

Proposition :- eg:- Washington DC is the capital of America.

→ Any declarative sentence which is either true or false.

$x+1=2$ → sentence type but we don't know it is true or false so it is not a proposition.

1. p and q are two proposition.

(I) → p is a proposition, ~p (negation of p).
If p is true then, ~p is false.
" " " false " " " true.

(II) Conjuction (∧) / And. , p & q are two proposition.

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F.

(III) Disjuction (∨) / or., p & q are two proposition.

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F.

(IV) ⊕ Exclusive OR

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F.

Conditional Statement ($p \to q$)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \leftrightarrow q$ (Bi-conditional)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth table for compound proposition

$(p \vee \sim q) \to (p \wedge q)$

p	q	$\sim q$	$p \vee \sim q$	$p \wedge q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	F
F	F	T	T	F

$(p \vee \sim q) \to (p \wedge q)$
T
F
T
F

→ $p \vee q \vee r = p \vee (q \vee r)$

~~Operat~~ Operator Precedence :-

Operator	Precedence
$\sim$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

→ Exercise - 1. (60,70).

$(p \leftrightarrow q) \vee (\sim p \leftrightarrow q)$

p	q	$p \leftrightarrow q$	$\sim p$	$\sim p \leftrightarrow q$	$(p \leftrightarrow q) \vee (\sim p \leftrightarrow q)$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

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* Propositional Equivalence.

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

→ A compound proposition that is always true that occur in it is called tautology.

→ A compound proposition that is always false that is called Contradiction

→ If all not True or false then called contigency.

Example: Logical Equivalence

The compound proposition p & q are called logically equivalent if $p \leftrightarrow q$ is a tautology.

Then,

$p \equiv q$ (Symbol)

Example :- $\sim(p \vee q)$

p	q	$p \vee q$	$\sim(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

$\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$\sim(p \vee q) \leftrightarrow \sim p \wedge \sim q$

$\sim(p \vee q) \leftrightarrow \sim p \wedge \sim q$
T
T
T
T

= tautology

So, they are logically equivalent.

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De Morgan's Law :-

→ $\sim(p_1 \vee p_2 \vee \cdots \vee p_n) \equiv (\sim p_1 \wedge \sim p_2 \wedge \cdots \wedge \sim p_n)$

→ $\sim(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \equiv (\sim p_1 \vee \sim p_2 \vee \cdots \vee \sim p_n)$

$\sim(p_1 \vee p_2) \equiv \sim p_1 \wedge \sim p_2$, $\sim(p_1 \vee p_2) \leftrightarrow \sim p_1 \wedge \sim p_2$.

Example: $\neg(p \vee (\neg p \wedge q))$ & $\neg p \wedge \neg q$

$= \neg p \wedge \neg(\neg p \wedge q) \equiv \neg p \wedge (p \vee \neg q)$

$\equiv \neg p \wedge (p \vee \neg q) \equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

→ Distributive Property

$\equiv F \vee (\neg p \wedge \neg q) \equiv (\neg p \wedge \neg q) \equiv \neg p \wedge \neg q$

alternate :- By constructing the truth table.

* $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \wedge (q \vee r) \longleftrightarrow (p \wedge q) \vee (p \wedge r)$ → Tautology.

* PREDICATE :-

$\underbrace{x}_{\text{subject}}$ is $\underbrace{\text{greater than } 3}_{\text{Predicate}}$

subject → main objective of statement (domain)

$P(x)$ → Predicate → propositional function

→ subject

$P(5)$ is true, $P(1)$ is false

$x = x+1$.

Universal quantifiers :- $\forall x, P(x)$ is not true

$\underbrace{x}_{\text{subject}}$ is $\underbrace{\text{equal to } x+1}_{\text{Predicate}}$

Truth value of $P(x)$ → false $\forall x > 0$

If $P(x,y)$: truth value is True $x = y+1$
then $x = 3, y = 2$
$P(3, 2)$ is true. $P(x_1, x_2 \ldots x_n)$
$\exists x\, P(x)$ is true
is same as $P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots$.

Ex:- Truths value of $\exists x\, P(x)$
$P(x)$: $x^2 > 10$
$x = \{1, 2, 3, 4\}$
$\exists x\, P(x) \equiv P(1) \vee P(2) \vee P(3) \vee P(4)$
$P(4)$: $4^2 > 10$ $\quad \exists x\, P(x)$ is True

$\forall x\, P(x) \vee Q(x)$ } (Precedence)
$\forall x\, (P(x) \vee Q(x))$ $\rightarrow \exists x\, (P(x) \vee Q(x))$

Precedence $\rightarrow$ Quantifiers > logic

$\boxed{\neg \forall x\, P(x) \equiv \exists x \neg P(x)}$ $\rightarrow$ existential quantifier

Universal quantifier.
$P(x)$ is true
$\neg P(x)$ is false
$\neg \forall x\, P(x) \equiv \exists x \neg P(x)$
$\boxed{\neg \exists x\, Q(x) \equiv \forall x \neg Q(x)}$

$\neg \forall x\, P(x)$ $\quad \forall x\, P(x)$ is True
$\neg \forall x\, P(x)$
$\exists x \neg P(x)$

$\neg \forall x\, (x^2 > x)$
$\exists x \neg (x^2 > x)$

$\forall x\, P(x) \vee Q(x)$
$\forall x\, (P(x) \vee Q(x))$ } → both are not same.

$\underbrace{\forall x\, P(x)}_{\text{first simplify this}} \vee Q(x)$ i.e. $(\forall x\, P(x)) \vee Q(x)$.
→ then take disjunction.

similarly :-

$$\exists x\, P(x) \vee Q(x) = (\exists x\, P(x)) \vee Q(x) \neq \exists x\, (P(x) \vee Q(x))$$

example: $\neg \forall x\, (P(x) \rightarrow Q(x))$ and $\exists x\, (P(x) \wedge \neg Q(x))$.

~~$P(x) \rightarrow Q(x)$~~	$P(x)$	$Q(x)$	$P(x) \rightarrow Q(x)$	$P(x) \wedge \neg Q(x)$
	T	T	T	F
	T	F	F	T
	F	T	T	F
	F	F	T	F

so, $\neg(x \rightarrow y) \equiv x \wedge \neg y$.

$$\neg \forall x\, (P(x) \rightarrow Q(x)) \equiv \exists x\, \neg (P(x) \rightarrow Q(x))$$
$$\equiv \exists x\, (P(x) \wedge \neg Q(x))$$

→ **Nested Quantifier**

$\forall x\, \exists y\, (x + y = 0)$
$\forall x\, \exists y\, P(x)$
$P(x) : x + y = 0$
$\forall x\, \forall y\, (x + y > 0)$

example:- Translate into english
$\forall x\, (C(x) \vee \exists y\, (C(y) \wedge F(x,y)))$ → y has computer & x & y are friends.
where $F(a,b)$ means a & b are friend & the domain for
x, y & z consists of all students in your school.
$C(x)$ → computer

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Q. For every student x in your school x has computer or there is a student y such that y has computer & x & y are friends.

→ Mathematical Induction

ex:- $1 + 2 + \_\_\_ + n = \frac{n(n+1)}{2}$.

$P(n) : \frac{n(n+1)}{2}$

$n = 1$

$P(1) = 1$. → $P(1)$ is true.

$P(k) = \frac{k(k+1)}{2}$ → True.

$1 + 2 + \_\_\_ + k + k+1 = \frac{k(k+1)}{2} + (k+1)$

$= \frac{k(k+1) + 2k+2}{2} = \frac{k^2 + 3k + 2}{2}$

$= \frac{k^2 + k + 2k + 2}{2} = \frac{k(k+1) + 2(k+1)}{2}$

$= \frac{(k+1)(k+2)}{2}$

$P(k+1) = \frac{(k+1)(k+2)}{2}$ → $P(k+1)$ is true.

$P(n)$ is true.

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BOOLEAN ALGEBRA $\{0,1\}$

operation :- $1+1=1$, $1+0=1$, $0+0=0$, $0+1=1$

addition → OR operation

multiplication → AND operation

$1 \cdot 1 = 1$, $1 \cdot 0 = 0$, $0 \cdot 0 = 0$, $0 \cdot 1 = 0$

$\bar{0} = 1$, $\bar{1} = 0$

example :- $1 \cdot 0 + (\overline{0+1})$ find the value.

$0 + \bar{1} = 0 + 0 = 0$.

If $T = 1$, $F = 0$

then, $(T \wedge F) \vee \neg(T \vee F) = F$

AND $\cdot$ $\wedge$, ~~TRUE~~ OR $+$ $\vee$

negation :- $\neg$ $\bar{\ }$ (compliment)

Boolean expression	logical Expression
0	F
1	T
$\cdot$	$\wedge$ (AND)
$+$	$\vee$ (OR)
$\bar{\ }$ (complement)	$\neg$

Boolean f^n :- $B = \{0,1\}$, $B^n = \{x_1, x_2, \ldots x_n\}$ $x_n \in B$

$F : B^n \longrightarrow B$

x	y	$F(x,y)$
1	1	0
1	0	1
0	1	0
0	0	0

example. $F(x,y,z) = x\bar{y} + z$

└ Boolean f^n

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x	y	z	xy	z	xy + z
1	1	1	1	1	1
1	1	0	1	0	1
1	0	1	0	1	1
1	0	0	0	0	0
0	1	1	0	1	1
0	1	0	0	0	0
0	0	1	0	1	1
0	0	0	0	0	0

(ii) Find $f(x,y,z) = xy + z$.

$xy + z$:

1
1
0
1
0
1
0
1

Boolean Identity.

Identity	Name
$\bar{\bar{x}} = x$	
$x + x = x$	
$x \cdot x = x$	
$x + 0 = x$	
$x \cdot 1 = x$	
$x + 1 = 1$	
$x \cdot 0 = 0$	
$x + y = y + x$	
$xy = y \cdot x$	
$x + (y + z) = (x + y) + z$	
$x \cdot (yz) = (xy)z$	

$x + yz = (x+y)(x+z)$

$x(y+z) = xy + xz$.

" $\overline{(xy)} = \bar{x} + \bar{y}$, $\overline{(x+y)} = \bar{x}\,\bar{y}$

for $x(y+z) = xy + xz \longrightarrow$ verify { Distribution Law }

x	y	z	$y+z$	$x(y+z)$	xy	xz	$xy+xz$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

so, $x(y+z) = xy + xz$.

* $x(x+y) = x$ (Absorption Law)

→ Dual

(i) $x(y+0)$ and (ii) $\bar{x}\cdot 1 + (\bar{y}+z)$

↳ If $\cdot$ put $+$ & where $+$ put $\cdot$ & replace 0 by 1

so, for (i) :- $x + (y\cdot 1)$

so, dual of $x(y+0)$ is $x + (y\cdot 1)$

similarly dual of $\bar{x}\cdot 1 + (\bar{y}+z)$ is $(\bar{x}+0)\cdot(\bar{y}\cdot z)$

ex:- 694

$F(x,y,z) = (x+y)\bar{z}$

Sum of product :- $(x+y)\bar{z} = x\bar{z} + y\bar{z} = x\bar{z}\cdot 1 + y\bar{z}\cdot 1$

$= x\bar{z}(y+\bar{y}) + y\bar{z}(x+\bar{x})$

$= x\bar{z}y + x\bar{z}\bar{y} + xy\bar{z} + \bar{x}y\bar{z} \longrightarrow$ sum of product.

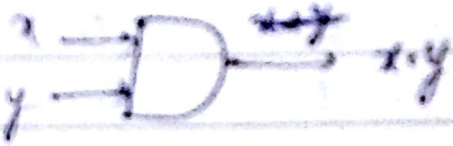
ex:- 698

Logic gates :-

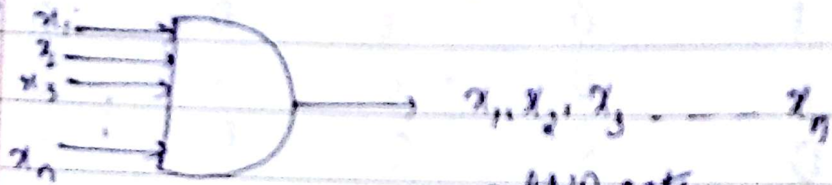
→ Inverter

→ OR gate

→ AND gate

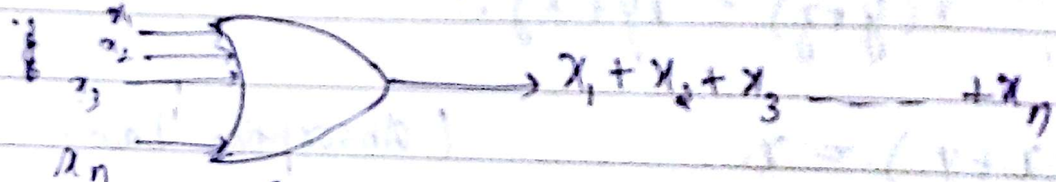


→



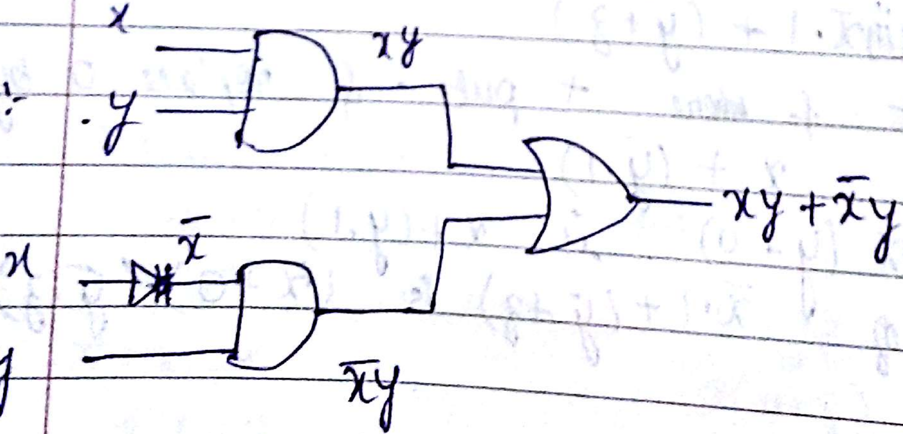
— AND gate

→



OR gate

ex:-



ex:- 704.

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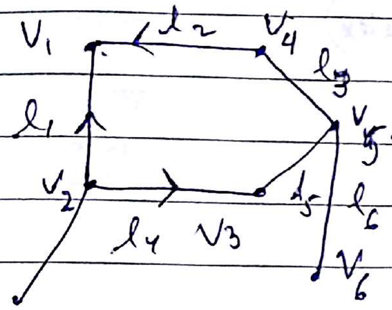
Graphs

$G = (V, E)$ V → vertices, E → edge.

$V = \{v_1, v_2, v_3 \ldots v_n\}$

$E = \{e_1, e_2 \ldots e_n\}$

All the edges must have a dirⁿ.



→ degree of vertex

$v_5 = 3$

(how many edges are incidence at that vertex)

Directed graph :- if ~~are the~~ the dirⁿ of edges is in a particular dirⁿ.

degree = 2.

$\deg(v_1) = 2$

$V = \{v_1, v_2, v_3 \ldots v_6\}$

$E = (e_1, e_2, e_3, e_4, e_5, e_6\}$

$$\boxed{2E = \sum_{v \in V} \deg(v)}$$

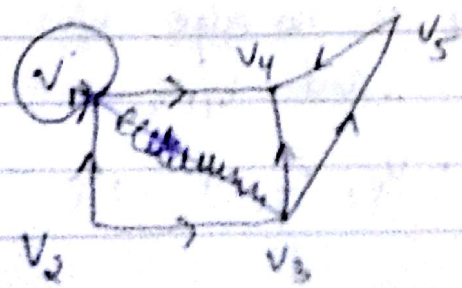
(Theorem) → sum of degree of all the vertex.

E = no. of edge.

Example How many edges are there in a graph with 10 vertices each of degree 6?

Sol: $\sum \deg(v) = 6 \times 10 = 60$

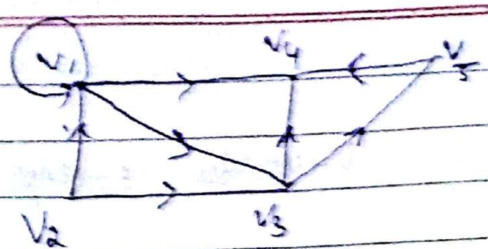
$2E = 60$, $\boxed{E = 30}$



Indegree → towards the vertex.

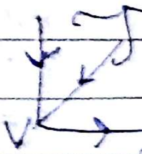
~~→ vertex~~

outdegree → out from the vertex



$deg^-(v_1) = 2$ → indegree (towards the vertex)

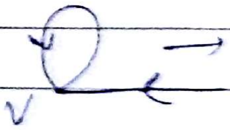
$deg^+(v_1) = 2$ → outdegree (outwards the vertex)



↳ directed towards v

↳ outwards v

so, $deg^+(v) = 1$, $deg^-(v) = 2$



→ towards.

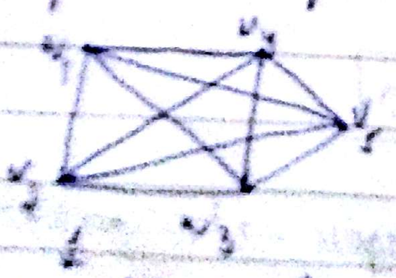
Let $G = (V, E)$ be a graph with directed edges then :

$$\sum_{v \in V} deg^-(v) = \sum_{v \in V} deg^+(v) = |E|$$

↓ Total no. of edge

→ Complete Graph

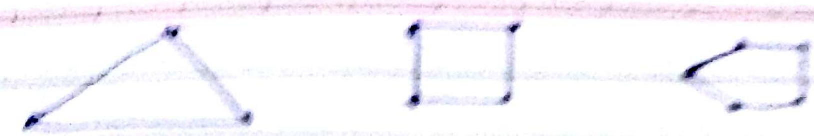
n - vertices denoted by K_n is simple graph that contains exactly one edge b/w each pair of vertices.



complete graph

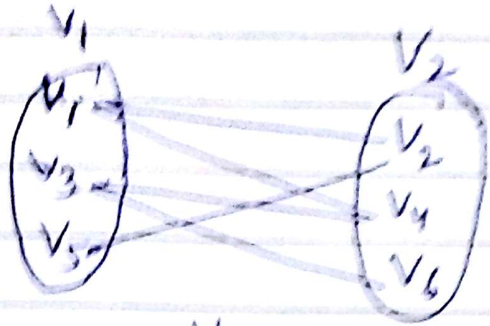
not complete graph

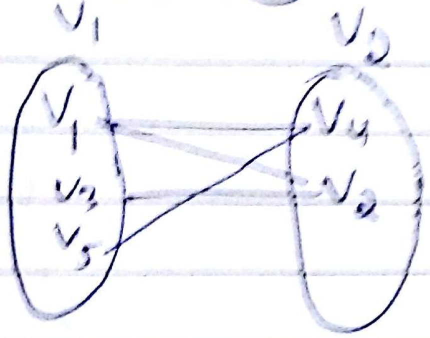
→ not a complete graph as there is no edge b/w v_2 & v_4 and b/w v_1, v_5 & v_3, v_5

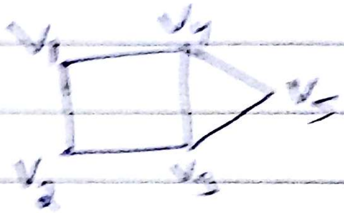
→ 

→ no. of vertices = no. of edges in a cycle.

→ Bi partite :- nothing [illegible] [illegible] b/w vertices [illegible] the set of [illegible] in the sets.







Representing Graphs and Graph Isomorphism

Q. There is an edge b/w b, c & e
ie, b, c and e are the adjacent vertices of a.

Vertices	Adjacencies
a	b, c, e
b	a
c	a e d
d	c, e
e	a, c, d.

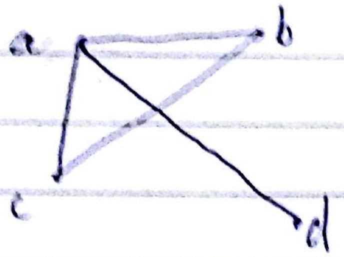
→ **Adjacency Matrix**

let $G = (V, E)$

Adjacency Matrix $A = [a_{ij}]$

$$a_{ij} = \begin{cases} 1 & \text{if } (i, j) \text{ is an edge of } G \\ 0 & \text{otherwise.} \end{cases}$$

$V = \{v_1, v_2, \ldots, v_n\}$.



$$\begin{array}{cccc} a & b & c & d \\ 1 & 2 & 3 & 4 \\ a_{11} & a_{12} & a_{13} & a_{14} \end{array}$$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$a_{11} = 0$ (~~edge b/w a & b~~)

$a_{12} = 1$ (edge b/w a & b)

$a_{13} = 1$ (edge b/w a & c).

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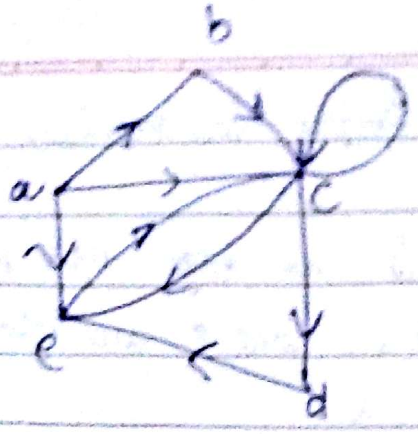
Example:



vertices	terminal vertices
a	b, c, e
b	c
c	d, e, c
d	e
e	c

Isomorphism of Graph

iso + morphe
equal form

$G_1 = (V_1, E_1)$

$G_2 = (V_2, E_2)$

One-to-one and onto functions if
$f_1 : V_1 \rightarrow V_2$ with property that a & b are adjacent in G_1 if & only if $f(a)$ & $f(b)$ are adjacent in G_2 for all a, b in V_1

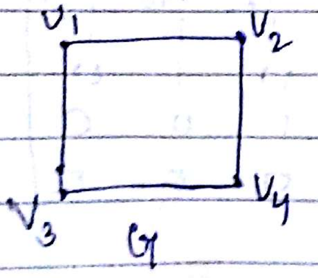
G_1 G_2

V_1 V_2

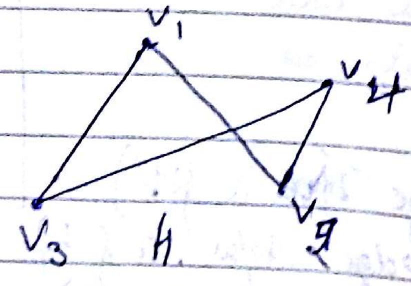
$f : V_1 \rightarrow V_2$

$a, b \in V_1$, $f(a), f(b) \in V_2$

Example:



G



H

$V_G = \{v_1, v_2, v_3, v_4\}$

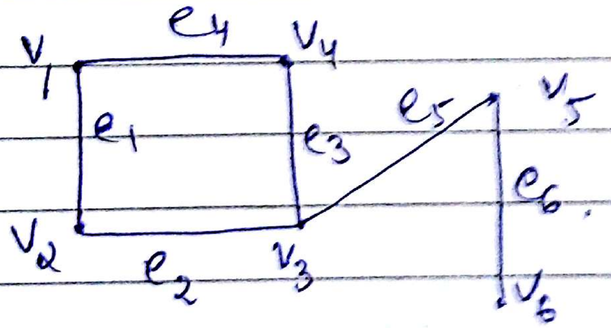
$V_H = \{v_1, v_2, v_3, v_4\}$

for G		for H	
vertex	adjacent	vertex	adjacent
v_1	v_2, v_3	v_1	v_2, v_3
v_2	v_1, v_4	v_2	v_1, v_4
v_3	v_1, v_4	v_3	v_1, v_4
v_4	v_2, v_3	v_4	v_2, v_3

so, both G & H are isomorphic.

Connectivity

Path :- A path is a sequence of edges that begin with a vertex of a graph & travel from vertex to vertex along edge of the graph.



$\{v_1, v_4, v_3, v_5, v_6\}$.
Path :-

↳ connected graph there must exist two edge /.