30/08/18

Ass. → In openCV no in-built func^n

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Geometric Image Transformations (Linear) (2D - 2D)

Image: Visual Representation in terms of func^n. $f(x,y)$

$\rightarrow T(f(x,y))$

Till now: manipulating intensities keeping co-ordinates same.
Here: co-ordinates " , intensities almost ..

$\rightarrow f(T(x,y))$

→ App^n's: Noise Removal
↳ Image Enhancements.
↳ Dynamic Range
↳ segmentation.

Why G I T ?
↳ Correct defects due to camera orientation.
↳ Lenso distortions (→ More appealing)
↳ Special effects (Morphology)
↳ Relate / combine images taken at different time / by diff. cameras / sensors

→ Geometry :→
(Lines & Points & their relationships)

→ Algebraic Geometry :→ Apply Algebra on geometric primitives (adding 2 pts, finding intersection of 2 lines, etc.)

To do this, we need to
↳ establish a co-ordinate system

① Cartesian coordinate system
2-D (x,y) [Euclidean Space]

② Homogeneous Coordinate System
[Projective space]

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1.) Cartesian : represent any point as (x,y)

(x,y) in 1.) $\equiv$ $(x,y,1)$ in 2.)

2.) Homogenous :
↳ any n-dimensional point is represented by $(n+1)$ numbers / coordinates

$(2,3) \longrightarrow (2,3,1)$

$(x,y,z) \longrightarrow (x,y,z,1)$

Q.1 Why 1 ?
Q2 Why "homogenous" ?
Q3. Why it is called "homogenous" ?

1.)
$(3,2,1)$
$(6,4,2)$ → $(3,2)$
$(12,8,4)$

(divide all no by last 1 to get point in cartesian Plane)

In general:
$(kx, ky, k) \longrightarrow (x,y)$

3.) ↳ Homogenous because a set of points represent same point (are equivalent) in Cartesian.

2.) Eg. Parallel lines : 1) distance remains same
2) '∩' ~~between~~ at infinity
↓
don't know how to represent this point in Cartesian.

Can be represented in Homogenous

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* Point me : $(x, y) \rightarrow (x, y, 1)$
Line me : can't do like this.
If Hom. : $ax + by + c = 0$
$\quad\quad\quad\quad\quad \overset{\shortparallel}{k}(a\ b\ c)^T \rightarrow$ ~~correction~~ : get diff. lines for diff. values of k

- Points at infinity are called : Ideal Points
  Can be represented as :

$(x, y, 0)$ $\quad\quad (26, 5, 0) \equiv (26/0, 5/0)$
$(12, 13, 0)$ $\quad\quad\quad\quad \equiv (\infty, \infty)$
$(1, 0, 0) \rightarrow$ Ideal point in x-dir^n
$(0, 1, 0) \rightarrow$ Ideal point in y-dir^n.

Algebraic Geometry :

1.) Point :
- Use column vector to represent a point (generally)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underline{x} \equiv \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \rightarrow \text{in homogenous}$$

- Row vector $\rightarrow [x\ y]^T = \underline{x}^T$

2.) Line :

Homogenous representation of Lines :

$ax + by + c = 0$
line : $(a\ b\ c)^T$ $\quad$ ($T \rightarrow$ because we're considering column vector)

eg. $(x\ y)^T$ lies on the line $(a\ b\ c)^T$
$\Rightarrow$ This condition will be satisfied :

$$(x, y, 1)\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

or $\underset{\text{point}}{x} \cdot \underset{\text{line}}{l} = 0$

* To find intersection point of 2 lines $\rightarrow$ find cross product

$l \times l = \underset{\text{point of intersection}}{x}$

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$-x+2=0$

line 1 → $x = 2$ → $-1 \quad 0 \quad 2$ } → 2 parallel lines
line 2 → $x=1$ → $-1 \quad 0 \quad 1$

$$\begin{vmatrix} i & j & k \\ -1 & 0 & 2 \\ -1 & 0 & 1 \end{vmatrix} = i(0) - j(1) + k(0)$$

$$= \hat{0} - \hat{j} = \hat{j} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

This means they meet at ∞ ∴ || lines.

Overview / Hierarchy of Linear Transformations

Increased Generalization ↑

$\boxed{\text{Affine}} \xrightarrow{\text{+ Projective effects}} \boxed{\text{Projectives}}$

↑ shear + non-uniform scaling

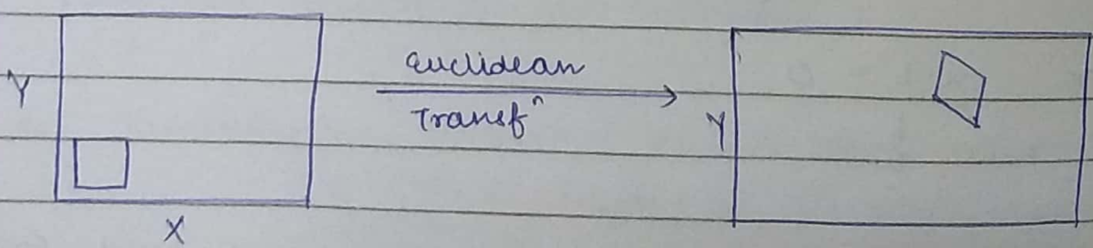
$\boxed{\text{Similarity}}$

↑ + scale

$\boxed{\text{Euclidean Transformation}}$ → Translation, → Rotation (Area, Angle b/w lines)

↳ **Invariants** : Quantities which Remain same even after applying transformation

Euclidean :-



Invariants are : 1) Area,
2.) Angle b/w the lines
3.) length

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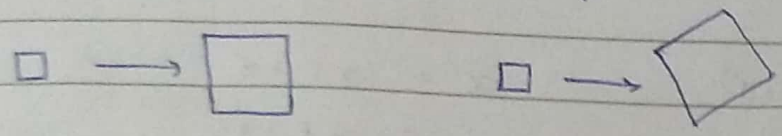
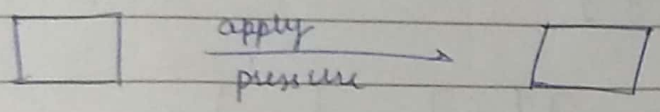
Similarity : Uniformly scaling



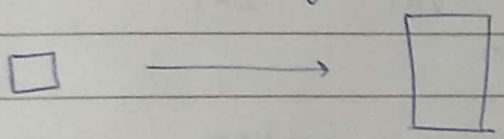
Invariants are : 1) Ratio of areas (Object : Image)
2.) Angle b/w lines
~~3.) length~~

Affine :

Shear :



Non- uniform scaling



Invariants are : 1.) Parallelism
2.) Ratio of lengths

→ Invariants for Projective are :
↳ Collinearity → straight lines remain straight

↳ ↑ Generalization → Distortion ↑

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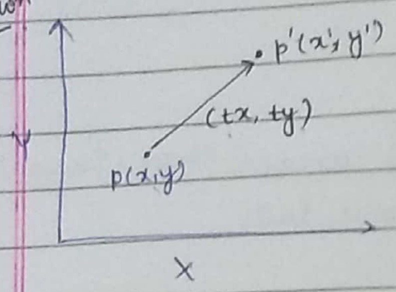
1. <u>Euclidean Transformation</u> → 3 dof

Translation



$x' = x + tx$

$y' = y + ty$

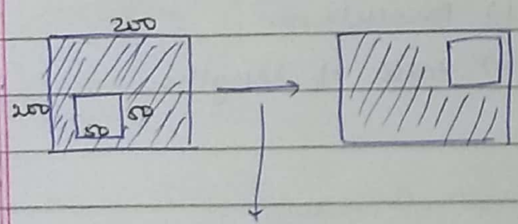
To represent them in matrix notation:

$$\cancel{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}}$$

$$\underset{2\times1}{\begin{bmatrix} x' \\ y' \end{bmatrix}} = \underbrace{\underset{2\times2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} x \\ y \end{bmatrix}}}_{2\times1} + \underset{2\times1}{\begin{bmatrix} tx \\ ty \end{bmatrix}}$$

<u>Ass.</u> :→ ↳ can translate object from 1 position to another.

Eg.



1st identify object (consider as blob)
then do transformation

$tx = 180$
$ty = 180$ } → Some part will lie outside image → have to ↑ size of image so that picture of interest doesn't get cut off.

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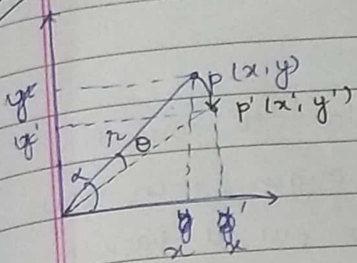
Rotation



$x = r\cos\alpha \qquad y = r\sin\alpha$

$x' = r\cos(\alpha - \theta) \qquad y = r\sin(\alpha - \theta)$

$x' = r(\cos\theta \underbrace{\cos\alpha}_{x} + \sin\theta \underbrace{\sin\alpha}_{y})$

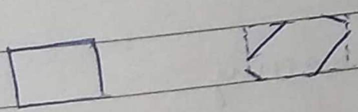
$y' = r(\underbrace{\sin\alpha}_{y}\cos\theta - \underbrace{\cos\alpha}_{x}\sin\theta)$

$\Rightarrow \quad x' = x\cos\theta + y\sin\theta$

$y' = y\cos\theta - x\sin\theta$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$ → To rotate an image by 'θ' degrees.

eg.



→ Corners are cut, so resize the ~~image accordingly~~.

↳ Euclidean = Translation + Rotation

$$\Rightarrow \underset{2\times1}{p'} = \underset{2\times2}{R}\,\underset{2\times1}{p} + \underset{2\times1}{t}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Parameters req. : θ, t_x, t_y

No. of parameters required = Degree of Freedom

↓

Here, it is $\boxed{3}$

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Ass. → Next Tuesday

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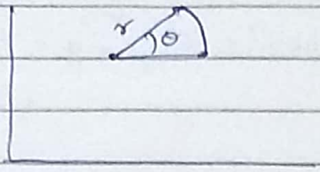
→ We have shifted P wrt origin here.

→ If we move wrt another point



→ 1^{st} translate to coincide it with origin, then rotate, then again put it back to its original position.
↓
Inverse of translation

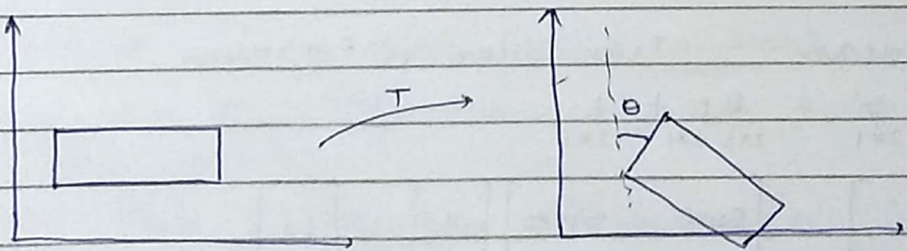
Assignment:
↳ Group details
↳ Problem statement, Observation, Code, O/p.
↓
pdf, code file

~~4-9-18~~

Assignment:
↳ 120° → Black holes : Background → black : use rotate keys &
15° → coloured holes

Forward Mapping v/s Inverse Mapping

$\boxed{\text{Forward Mapping}}$



(x,y) → integers

find target image values on the basis of transformation function applied on source image.

$f(x,y) \xrightarrow{T} g(x',y')$
integers real values

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→ Binary Interpolation
→ ~~Resize image~~ : Blocky effect

Eg $f(x,y) \xrightarrow{T} g(x',y')$ → Drawback : can't reduce holes

real values

(69.257, 135.79)

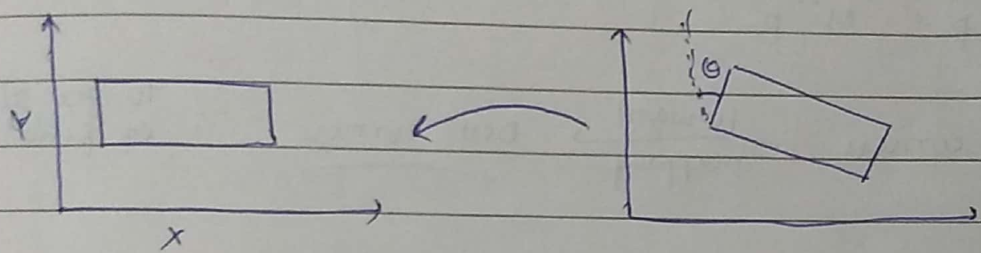
↓

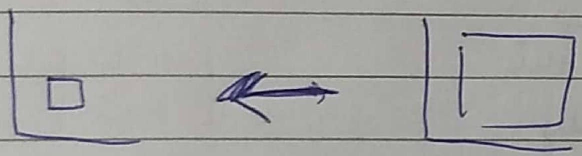
(69, 135)

maybe : don't get any value corresponding to some (x, y)

(68, 135), ... → Black holes.

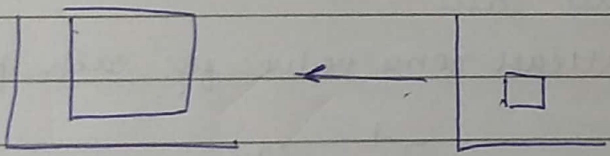
$\boxed{\text{Reverse Mapping}}$: ~~can reduce~~ Drawback : holes → can reduce holes.



→ 

S D

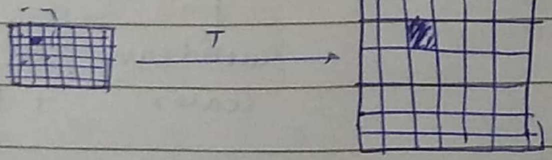
: Overlapping effect ~~(some pixel)~~

→ 

S D

; Blocking effect
↓ Remove
Image Interpolation
(not much blocking effect)

→ Without Interpolation



Blocking effect

Interpolation : nbs ko consider krke map karenge

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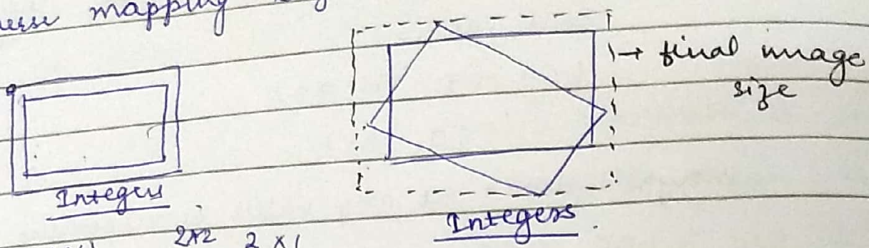
Friday-

Ass 7: Reverse mapping laga ke 120° Rotation.



$$\overset{2\times1}{p'} = \overset{2\times2}{M}\ \overset{2\times1}{p}$$

M → invertible matrix

$$p = M^{-1} p'$$

Source corners $\xrightarrow[\text{mapping}]{\text{forward}}$ Dest^n corners : To get size of final image

Use final image:

$$p = \underbrace{M^{-1} p'}_{\text{know this}}$$

get p & equate intensity

why no holes in this case:

Here, I'll get atleast some value for each pixel

$$M^{-1} : \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^{-1}$$: can Use inbuilt func^n

2. Similarity Transformation → $\boxed{4 \text{ dof}}$

↳ Uniform Scaling

Euclidean : 3

scaler : 1 (uniform in x & y dir^n)

- Invariants:
  - Ratio of areas
  - Angle b/w lines.

→ more generalised than Euclidean.

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scaling factor (uniform scaling)

$$\underset{2\times1}{p'} = \underset{1\times1}{s}\,\underset{2\times2}{R}\,\underset{2\times1}{p} + \underset{2\times1}{t}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 1 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

<u>Rotation</u>, <u>scaling</u> & <u>Translation</u> :

→ Origin will always map to origin (0, 0)

→ Angle will remain same

→ straight line

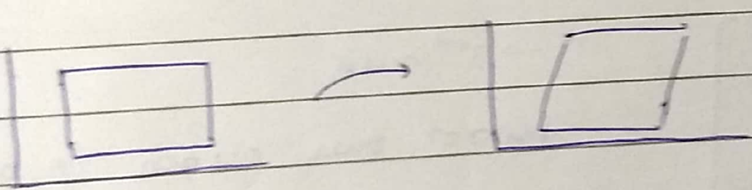
→ Parallelism.

3. <u>Affine</u> <u>Transformation</u> : $\boxed{6 \text{ dof}}$

↳ Similarity + Euclidean + shear

↳ shear

↳ Non-uniform scaling



$$p' = \overset{A}{\cancel{R}}\,p + t$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

→ a, b, c, d, t_x, t_y

↳ 6 dof

shear

↳ non-uniform scaling

→ In general form we write

$p' = Mp$ (transform $p \to p'$)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} c\cos\theta & c\sin\theta \\ -c\sin\theta & c\cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

COMM... for all transformations

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★ $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} c\cos\theta & c\sin\theta & t_x \\ -c\sin\theta & c\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ → Dimensions are incorrect ⇓ homogenous

Transl^n = $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & t_x \\ 0 & 0 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

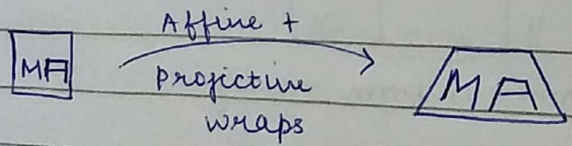
need these 6 parameters (6 dofs)

Affine: ~~Scaler~~ $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ ⟷ generalised form

Eg. $\begin{bmatrix} -1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$: Translating : converting mirror image over y-axis. (Flip over y-axis)

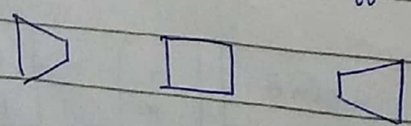
Eg. $\begin{bmatrix} -1 & 0 & t_x \\ 0 & -1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$: $1^{st} \rightarrow 3^{rd}$ quad : Mirror over origin (0, 0).

4. Projective Transformations $\boxed{8 \text{ dof}}$



: Parallel lines won't remain parallel anymore

Board : looks diff. from diff. dir^{n}s



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→ To avoid these : we need homogenous system
(need ∞ which can be represented in homogenous system only)
⇓
Only interested in ratios
↓

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \underbrace{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}}_{H} \begin{bmatrix} x \\ y \\ h \end{bmatrix} \qquad i = 1$$

(Homography) ≡ Homogenous matrix

all 9 elements

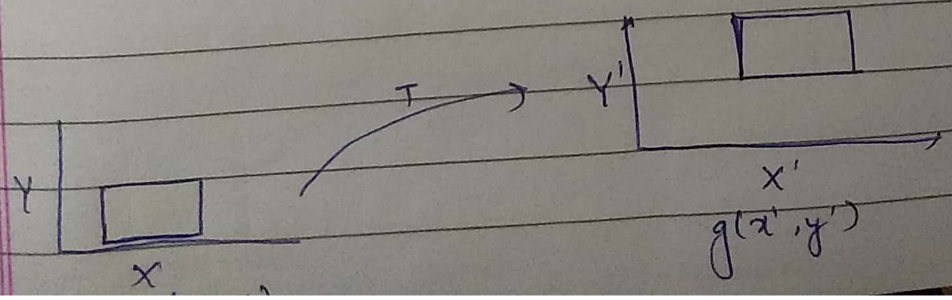
★ If I multiply H by non-zero scalar quantity , it'll still be projective in nature.

→ If we divide all elements (other 8) by i, 9th element will become 1.

→ <u>Invariant</u> : collinearity
↓
straight line will remain straight.

* <u>Affine</u> & <u>Projective</u> : Origin may not map to Origin (0,0)

↳ Source Image $\xrightarrow{T}$ Destn Image
↓
How to recover this tranformation matrix ?



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6/9/18

$p' = \boxed{M} p$ → (using) corresponding Points
?

→ How many pairs of corresponding points needed to get translation

$x' = x + t_x$

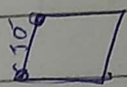
$y' = y + t_y$

Translation → 1

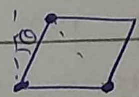
(have (x, y) & (x', y') → can find transl^n matrix using 1 pair of points

→ Rotation : → 2

(to get θ)



→ Affine → 3



Rectange $\xrightarrow{\text{Shear}}$ ||gm
need 3 ~~to~~ pts to find a ||gm.

→ Projective → 4 (non- parallel also).

Eg.

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