The LNM Institute of Information Technology, Jaipur
Department of Computer Science & Engineering
Final Exam

CSE 331: Theory of Computation

Time: 180 min Nov 25, 2016

Answer all questions
Wherever proofs are required write clearly what is given, what is to be proved
This is an open notes exam
Sharing of notes is not allowed
No print materials allowed
All the best!

1. **Demonetization Turing machine:** Design a Turing machine that accepts 500 and 2000 and rejects 1000 and all others! (Assume that $\Sigma = \{0, 1, 2, 5\}$. You may use a state q_{reject} for rejecting a string) (2)

2. Any electronic computer has only finite memory size then what is the need for Turing machine model that has infinite memory. Is finite state automata enough for our study? Explain very briefly and crisply. (2)

3. The Traveling Salesman Problem (also called as Hamiltonian circuit problem) is an NP-Complete problem. Suppose a researcher finds that it is solvable by a deterministic Turing machine running in polynomial time what will be the (most appropriate) consequence and why? Explain in one or two sentence(s) only. (2)

   (a) All NP-Complete problems are solvable in polynomial time
   (b) $NP = P$
   (c) $NP \neq P$
   (d) None of the above

4. Design a Turing machine M that accepts all binary numbers that correspond to a positive even integer. For example: 100 (that correspond to 4) should be accepted but not 1011 (that correspond to 11). Assume that the input alphabet of M as $\{0, 1\}$. (4)

5. Construct a Turing machine M that accepts the language

$$\{a^n b^n c^n \mid n \geq 1\}$$

   and rejects any other string given as input. Basically M should not go into an infinite loop in any case, rather it should stop on all inputs (Hint: You may use a state called q_{reject} to reject a string not of the required form/pattern. You may also use some tape alphabets). (4)

6. Construct pushdown automata for the following languages: (10)

   (a) $\{a^i b^j c^k \mid i, j, k \geq 1, i + j = k\}$
   (b) $\{a^i b^j c^k \mid i, j, k \geq 1, i + k = j\}$
   (c) $\{a^n b^m \mid n \leq m \leq 2n\}$

7. Following are problems in context-free grammars

(a) Convert the following Context-Free Grammar into Chomsky Normal Form.

$$S \to AbA$$

$$A \to Aa \mid \epsilon$$

Note: Chomsky Normal Form will be consisting of rules of the type $A \to BC$ and $A \to a$ (4)

(b) Consider the grammar:

$$S \to aS \mid Sb \mid a \mid b$$

Describe the language accepted by this grammar. (3)

(c) Consider the grammar:

$$S \to aSbS \mid bSaS \mid \epsilon$$

Describe the language accepted by this grammar. (3)

8. Using pumping lemma prove that the language $\{ww \mid w \in \{a,b\}^*\}$ is not a context-free language. (4)

9. Three teams: Australia, India and Pakistan are playing for Australasia cup down under. There is a first round where each team play each other team once. Assume that for this tournament only a win or loss is possible – means there are no ties. For every match winning team gets 2 points and the losing team gets no points. At the end of first round, the first two best team (with respect to number of points) will go to the finals (If there are teams with equal number of points then the teams are selected according to the alphabetical order). The finals between the two best teams is a best of three matches where whoever wins two times will be declared as the winner of the Australasia cup.

   (a) Design a finite state automata for the different scenarios that can occur in this tournament. The final states of this automata will be q_A, q_I and q_P that denotes the winner as Australia, India and Pakistan, respectively. (6)

   (b) If the tie (when the points are same) is broken by tossing a coin how many new states will have to be included in your finite state automata. Discuss it briefly. (2)

   Hint: Spend some time to think about what will be the states and the input for this automata. Once you find this designing is very easy.

10. The *min* of a language L is defined as

$$min(L) = \{w \in L : \text{ there is no } u \in L, v \in \Sigma^+, \text{ such that } w = uv\}$$

Show that the family of regular languages is closed under *min*. (4)