LNMIIT

# The LNM Institute of Information Technology
## Department: CSE
### Introduction to Complexity Theory
**Exam Type: End Term**

**Time Limit:** 3 hours **Exam Date:** 30 April 2019 **Max marks:** 40

You can use the results which have proven in class and have given as the reading assignmnets.

1. State the following statements whether it is True or False and give your reason too. Answer without a valid reason carries no marks.
   (a) (2 points) If a language $L \in$ NP-complete $\cap$ co-NP then co-NP=NP
   (b) (2 points) EXP $\subseteq$ PSPACE
   (c) (2 points) NL=L
   (d) (2 points) NL $\subseteq$ P
   (e) (2 points) Consider a graph $G$, deciding whether $G$ has a clique of size 50, is NP-Complete.
2. Prove that the following problems are NP-complete.
   (a) (4 points) 0/1 INTEGER-LINEAR-PROGRAM: it is a linear program in which varaibles can take values from the set $\{0, 1\}$.
   (b) (4 points) For a given graph $G = (V, E)$, the vertex cover $S \subseteq V$ such that for every edge $(u, v) \in E$ either $u \in S$ or $v \in S$. Let the language VERTEX-COVER= $\{(G, k) :$ The undirected graph $G$ has a vertex cover of size $k\}$.
3. (5 points) Prove that $TIME(2^n) \subsetneq TIME(2^{3n})$.
4. (5 points) Let $PATH = \{\langle G, s, t\rangle :$ There exist a path from $s$ to $t$ in the diagraph $G\}$. Prove that $PATH \in SPACE(\log^2 n)$ where $n$ is the size of input. Our model of Turing machine is same as we define in case of class $L$.
5. (5 points) Let $SAT = \{\langle\phi\rangle \mid \phi$ is a satisfiable logical formula$\}$ and the language $A = \{x \in SAT \mid f(|x|)$ is even$\}$ where the function $f$ is same as we have defined in the proof of ladner theorem. Prove that $A \notin P$ and A is not NP-Complete.
6. (7 points) Let $\#SAT = \{\langle\phi, k\rangle |$ $3SAT$ formula $\phi$ has precisely $k$ satisfiable assignments$\}$. Prove that if $\#SAT \in NP$ then NP=co-NP.