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Q ① $\lambda = \dfrac{h}{p}$

$E = \dfrac{p^2}{2m} \Rightarrow P = \sqrt{2mE}$

$E = 40\,eV \times 10^3$ (Given)

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 40 \times 1.6 \times 10^{-19} \times 10^3}}$$

$= 3.588$ nm

Q 2. $v_g = \dfrac{d\omega}{dk}$ (Group velocity)

$\omega = 2\pi\nu = 2\pi \left(\dfrac{\gamma m c^2}{h} \right)$ $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

$k = \dfrac{2\pi}{\lambda} = 2\pi \left(\dfrac{P}{h} \right) = 2\pi \left(\dfrac{(\gamma m) v}{h} \right)$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega / dv}{dk / dv} = \frac{-\frac{1}{2h} \cdot \frac{2\pi m \cdot \left(-2v/c^2\right)}{\left(1 - v^2/c^2\right)^{3/2}}}{\frac{2\pi m \cdot 2v}{2hc^2 \left(1 - v^2/c^2\right)^{3/2}}}$$

$\boxed{v_g = v}$ → particle velocity

Q3

$$\boxed{n = \frac{c}{v_p} = \frac{ck}{\omega}} \qquad \lambda = 2\pi/k \qquad \left(v_p = \frac{\omega}{k}\right)$$

$$\frac{dn}{d\lambda} = \frac{dn/dk}{d\lambda/dk}$$

$$\frac{dn}{dk} = \frac{c}{\omega} - \frac{ck}{\omega^2}\frac{d\omega}{dk}$$

$$\frac{d\lambda}{dk} = -\frac{2\pi}{k^2} = -\frac{k}{\lambda} \qquad \left(\lambda = \frac{2\pi}{k}\right)$$

$$\left(v_g = \frac{d\omega}{dk}\right)$$

$$\therefore \frac{dn}{d\lambda} = -\frac{k}{\lambda}\left(\frac{c}{\omega} - \frac{ck}{\omega^2}v_g\right)$$

$$-\frac{\lambda\, dn}{k\, d\lambda} = \frac{c}{\omega} - \frac{ck}{\omega^2}v_g$$

$$= v_g\frac{kc}{\omega^2} = \frac{c}{\omega} + \frac{\lambda}{k}\cdot\frac{dn}{d\lambda}$$

$$v_g\frac{kc}{\omega^2} = \left(\frac{c}{\omega} + \frac{\lambda \cdot c}{\omega n}\frac{dn}{d\lambda}\right)$$

$$\frac{v_g}{v_p} = \left(1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right)$$

$$\boxed{\therefore v_g = v_p\left(1 + \frac{\lambda}{n}\cdot\frac{d\lambda}{dn}\right)}.$$

Q4. $\Delta x \times \Delta p \geq \frac{\hbar}{2}$ $\left(\hbar = \frac{h}{2\pi}\right)$

~~$\Delta x = 1.00$ nm~~

For e^- $\Delta x \times m_e \times \Delta V_e = \frac{h}{4\pi}$ — (1)

For p, $\Delta x \times m_p \times \Delta V_p = \frac{h}{4\pi}$ — (2)

$\therefore$ (1)/(2).

$$\frac{\Delta V_{e^-}}{\Delta V_p} = \frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}\ kg}{9.1 \times 10^{-31}\ kg}$$

$$\boxed{\frac{\Delta V_{e^-}}{\Delta V_p} = 1835}$$

Q5. Notes

1) ψ must be continuous & single valued.

2) $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be continuous & single valued.

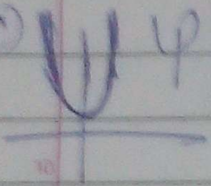
3) ψ must be normalizable which means that ψ must go to 0 as $x \to \pm\infty$. In order to that $\int |\psi|^2 dV$ over all the space be in finite constant.

~~4) One can multiply $\psi(x,t)$ by a constant say N, so~~

Q6

(a). $\psi = A\sec x$.
Not a wave f^n (because it is discontinuous)

(b) $\psi = A\tan x$
Not a wave f^n (discont.)

(c)  $\psi = Ae^{x^2}$
It is not a wave f^n (does not satisfies the conditions that at $x \to \infty$ $\psi \to 0$ ~~[illegible]~~)

(d) $\psi = Ae^{-x^2}$
It is a wave f^n (it satisfies condition that at $x \to \infty$, $\psi = 0$

(e) $\psi = A\sin x$.
It is a wave f^n (satisfies cont. & diff.)

Q7. TDSE

For $\psi = Ae^{-\frac{i}{\hbar}(Et - px)}$ function only

$$\hookrightarrow \quad i\hbar \frac{\partial \psi(x)}{\partial t} = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x)$$

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$$\hookrightarrow \quad E\psi(x) = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x)$$

Q10 (a). As $x \to \infty$, the defined $\psi(x)$ is 0. (given)

also $\int |\psi|^2 dv = \int_0^L Nx(L-x)\,dx$ would be a finite constant.

$\frac{x}{12.24 \times 5}$ $\frac{10}{29} - \frac{2}{5}$

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(b) $\langle x \rangle = \int_0^L x \cdot |\psi|^2 dx$

$= \int_0^L x \cdot N^2 x^2 (L-x)^2 dx$

$= N^2 \int_0^L x^3 (L^2 + x^2 - 2Lx) dx$

$= N^2 \left[\frac{L^2 x^7}{7} + \frac{x^6}{6} - \frac{2Lx^5}{5} \right]_0^L$

$= \frac{N^2}{60}$

$\langle x^2 \rangle = \int_0^L x^2 |\psi|^2 dx$

= Solve it.

$\Delta x = \sqrt{(\langle x^2 \rangle - \langle x \rangle^2)}$

(c) $p_x = -i\hbar \frac{\partial \psi}{\partial x}$

$\cancel{\langle p_x \rangle} =$

$\langle p_x \rangle = \int_0^L \psi^* p \psi dx$

$= \int_0^L \left[Nx(L-x) \cdot \left(-\frac{i}{\hbar} \partial \frac{(Nx(L-x))}{\partial x} \right) \right] dx$

$= \frac{-i}{\hbar} \int_0^L Nx(L-x)(NL - 2Nx) dx$

$= \frac{-iN^2}{\hbar} \int_0^L (L^2 x - 2x^2 L - x^2 L + 2x^3) dx$

$$= \frac{-\hat{i}N^2}{\hbar}\left[\frac{L^2x^2}{2} - \frac{2x^3}{3}L - \frac{x^3}{3}L + \frac{2x^4}{4}\right]_0^L$$

$$= \frac{-\hat{i}N^2 L^4}{\hbar}(1-1) = 0$$

$$\langle p_x^2 \rangle = \int_0^L \psi^* \, p^2 \, \psi \, dx$$

$$\Delta p = \sqrt{\langle p_x^2 \rangle - \langle p_x \rangle^2}$$

(d) $$\langle E \rangle = \int_0^L \psi^* \left(\frac{-\hbar^2}{2m}\hat{p}^2\right)(\psi(x))\,dx$$

$$= \int_0^L \psi^* \left(\frac{-\hbar^2}{2m}\right)\left(\frac{\partial^2}{\partial x^2}(\psi(x))\right)dx$$

$$= \int_0^L Nx(L-x)\left(\frac{-\hbar^2}{2m}\right)\left[-2N\right]dx$$

$$= \frac{+\hbar^2 N^2}{m}\int_0^L x(L-x)\,dx$$

$$= \frac{\hbar^2 N^2}{m}\left[\frac{x^2 L}{2} - \frac{x^3}{3}\right]_0^L$$

$$= \frac{\hbar^2}{m} N^2 \frac{L^3}{6}$$

~~Or~~ ~~step~~ ~~simply~~ by using $\langle E \rangle = \frac{\langle p^2 \rangle}{2m}$

$NL = 2Nx$

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Q.(11)

$$\psi(x) = Ae^{(iax - ibt)}$$

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) \cdot \frac{\hbar}{i}\left(\frac{\partial}{\partial x}\psi(x,t)\right) dx.$$

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) \frac{\hbar}{i}\left(\frac{\partial^2}{\partial x^2}\psi(x,t)\right) dx$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \_\_\_\_$$

(Lazy enough to solve another ques with the same method)

Q(12).

$x = 0.1\,nm$

$$E = \frac{2\pi^2 m k^2 z^2 e^4}{n^2 h^2} = \frac{h^2 n^2}{8 m r^2}$$

$$v = \frac{2\pi k z e^2}{nh}, \quad r = \frac{n^2 h^2}{4\pi^2 m z e^2 k}, \quad r = x = 0.1\,nm$$

(a). $\underset{(\Delta E)}{E_2 - E_1} = \frac{h^2}{8 m r^2}\left((2)^2 - (1)^2\right) = \frac{3h^2}{8 m r^2}$

(b).

$$\Delta E = h\nu$$

$$\nu = \frac{\Delta E}{h}$$

$$\& \quad \lambda = \frac{hc}{\Delta E}$$