$1 \cdot W$
$- \cdot 25$
$0.25 \times 6 = 6.25$

A2

The LNM Institute of Information Technology, Jaipur
Quiz-1 (January 30, 2018) SET: C
Sub: Probability and Statistics (P&S)

Duration:30 mins. Max.Marks: 10

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Instructions:

- Total six questions. Attempt all questions. All the questions should be answered on this question paper. The question paper consists of two type of question (Multiple choice and descriptive). **Answers will be rejected if there is any overwriting or cutting.**
- **0.25 marks will be deducted for each wrong answer in Multiple choice questions.**

1. Suppose that 100 men hats are mixed up, and then each man randomly select a hat. What is the probability that 50 men select their own hat? [1 M]

   $(A)\frac{50}{100}$ $(B)\frac{50!}{100!}$ $(C)\frac{1}{50!}$ (D) none of these.

2. The probability that an Instructor will take unannounced quiz during any class hour is $\frac{1}{4}$. If a student is twice absent, what is the probability that he/she will miss at leat one quiz? [1 M]

   $(A)\frac{1}{2}$ $(B)\frac{1}{16}$ $(C)\frac{7}{16}$ (D) none of these.

3. Punit and Puru appear in an interview for two vacancies of the same post. The probability of Punit's selection is $\frac{1}{7}$ and Puru's selection is $\frac{1}{5}$. What is the probability that only one of them will be selected? [1 M]

   $(A)\frac{4}{35}$ $(B)\frac{24}{35}$ $(C)\frac{6}{35}$ $(D)\frac{2}{7}$.

4. Which of the following condition does not include in the **axiomatic definition of probability**, where S is a sample space, $E_1, E_2, E_3, \cdots$ is sequence of events. [1 M]

   (A) $0 \le P(E_i) \le 1$ (B) $P(S) = 1$ (C) $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$ (D) none of these.

5. If E and F are independent events, then prove that E^c and F are independent, where E^c is the complement of E.

   Proof:

   Since E and F are independent events $P(E/F) = P(E)$
   $P(E \cap F) = P(E)P(F)$

   Now To Prove $P(E^c/F) = P(E^c)$

   $P(E^c \cap F) = P((E \cup F^c)^c) = 1 - P(E \cup F^c) = 1 - (P(E) + P(F^c) - P(E \cap F^c))$

   $P(E^c \cap F) = 1 - (P(E) + 1 - P(F) - P(E \cap F^c))$

   $P(E^c \cap F) = P(F) - P(E) + P(E \cap F^c)$

   $P(E) + P(E^c \cap F) = P(F) + P(E \cap F^c)$

   $\boxed{\begin{aligned} P(E \cap F) &= P((E \cup F)^c) = 1 - P(E \cup F) \\ &= 1 - (P(E) + P(F) - P(E \cap F)) \\ &= 1 - (P(E) + P(F) - P(E)P(F)) \\ &= P(E)P(F) \end{aligned}}$

   ~~P(E)+P(F)~~

   Thus $P(E^c \cap F) = P(E^c)P(F)$

   3

Total Marks	$14\frac{1}{2}$

6. If $\{E_n, n \geq 1\}$ is an increasing sequence of events, then prove that

$$\lim_{n\to\infty} P(E_n) = P\left(\lim_{n\to\infty} E_n\right)$$

Proof:

Considering RHS

$P(\lim_{n\to\infty} E_n) = P\left(\bigcup_{i=1}^{\infty} E_i\right)$

Let $F_i = E_i \cap E_{i-1}^c$ For $i \geq 2$ that is Event F_i are mutually exclusive

$F_2 = E_2 \cap E_1^c$

$F_1 = E_1$

Thus $P\left(\bigcup_{i=1}^{\infty} E_i\right) = P\left(\bigcup_{i=1}^{\infty} F_i\right) = \sum_{i=1}^{\infty} P(F_i) = \lim_{n\to\infty} \sum_{i=1}^{n} P(F_i) = \lim_{n\to\infty} \sum_{i=1}^{n} P(E_i)$

$P(\lim E_n) =$

$\lim_{n\to\infty} \sum_{i=1}^{n} P(E_i) = \lim_{n\to\infty} P\left(\bigcup_{i=1}^{n} E_i\right) = \lim_{n\to\infty} P(E_n)$

Total Marks $14\frac{1}{2}$