① LINEAR ALGEBRA

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Matrices.

~~Matxr~~ Matrix is a rectangular array of Numbers arranged in rows and columns.

eg.

In general $\rightarrow \in R^{m\times n}$. {Newtonian plane}

$A_{m\times n} = (a_{ij})$.

$\underbrace{A_{m\times n}}_{\text{Dimension of } A}$ $\quad (a_{ij}) \rightarrow$ element/entries of matrix

a_{ij}: i → Row index, j → column index

$A = \begin{bmatrix} & \end{bmatrix}$ ↓ columns, ← Rows

$i = 1, 2, \ldots m$

$j = 1, 2, \ldots n$.

#) Algebraic Operations.

1. Addition. (commutative)

we can add two matrices A & B if they are of the same dimension.

ie. $A = (a_{ij})_{m\times n}$ → defining Matrix → describing too → Any. $\quad B = (b_{ij})_{m\times n}$.

then.

$$A + B = (\underbrace{a_{ij}}_{\text{element}} + b_{ij})_{m\times n} = (b_{ij} + a_{ij})_{m\times n}$$

$$= B + A$$

2. Scalar ~~Multiminn~~ Multiplications.

$K \in R, \quad A = (a_{ij})_{m\times n}$

$KA = (Ka_{ij})_{m\times n}$.

$$KA = \begin{bmatrix} Ka_{11} & Ka_{12} & \cdots & Ka_{1n} \\ \vdots & & & \\ Ka_{m1} & \cdots & \cdots & Ka_{mn} \end{bmatrix}$$

3. Matrix Multiplication.

let $A = (a_{ij})_{m\times n}$ & $B = (b_{ij})_{n\times p}$

then $AB = (C_{ij})_{m\times p}$

where $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

BA is defined when $\boxed{m=p}$

$\Rightarrow BA = (d_{ij})_{n \times n}$

$\rightarrow BA \neq AB \longrightarrow$ General

Eg. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$

$AB \neq BA$.

$BA = \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix}$

$\Downarrow$

{ Commutative law Do Not Hold. }

Transposition :→

$A = (a_{ij})_{m \times n}$

$A^T = (a_{ji})_{n \times m}$

T = Transpose.

Properties → $(CA)^T = C(A)^T$ if $C \in R$.

$(A+B)^T = A^T + B^T$ $\quad (A^T)^T = A$.

Types of Matrix:

→ Square Matrix:

- Digonal Matrices: $D = (d_{ij})_{m \times n}$ * $m = n$ &

$d_{ij} = \begin{cases} 0 & \forall\, i \neq j \\ d_{ii} & \text{for } i = j \end{cases}$

where, d_{ii} with atleast one NON Zero d_{ii}.

- Triangular Matrices
  - Lower triangular Matrix: $L = \begin{cases} l_{ij} & \text{if else} \\ 0 & \text{if } j > i \end{cases}$

    for Strictly Lower tri M. $L = \begin{cases} l_{ij} & \text{if else} \\ 0 & \text{if } j \geq i \end{cases}$
  - Upper Δ Matrix:

    $U = \begin{cases} u_{ij} & \text{if } \_\_\_ \\ 0 & \text{if } i > j \end{cases}$

    for strictly $i \geq j$

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- **Scalar Matrix.** $A = (a_{ij})_{m\times m}$

$$\Rightarrow \begin{cases} a_{ij} = 0 & \text{if } i \neq j \\ a_{ij} = \underset{\text{Scalar}}{\underset{\downarrow}{K}} & \text{if } i = j \end{cases}$$

$K = 1 \Rightarrow$ Identity matrices. (I)

- **Symmetric Matrix:** T: Transpose

$$\boxed{A = A^T}$$

eg: $\begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$

- **Skew Symmetric Matrix:**

$$\boxed{A^T = -A}$$

eg: $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

- **Orthogonal Matrix:**

$$\boxed{AA^T = I}$$

* Systems of Linear Equations

A general system of linear equations with m equations in n unknowns:

$$(1)\begin{cases} a_{11}x_1 + a_{12}x_2 + \text{———} + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{21}x_2 + \text{———} + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \text{———} + a_{mn}x_n = b_n \end{cases}$$

In Matrix form $AX = B$

$A = (a_{ij})_{m\times n}$ $\quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$

$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}$

Summation form:

$$\sum_{j=1}^{n} a_{ij}x_j = b_i \qquad 1 \leq i \leq m$$

Definition (consistency)

A system (I) is said to be **consistent** if it has **at least one solution**.

① is inconsistent if $AX = B$ has no solution.

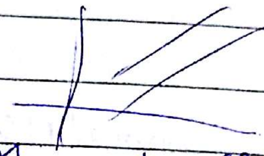
Example: (i) $-x + y = 1$ Line 1

$2x + y = 4$ Line 2

$(x, y) = (1, 2) \longrightarrow$ solution. (Unique solution.)

(ii) $-x + y = 1$ $L1$

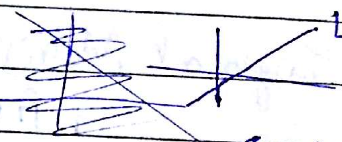
$2x - 2y = 4$ $L2$



No solution $\longrightarrow$ parallel lines.

(iii) $-x + y = 1$

$2x - 2y = -2$



$\downarrow$

Infinitely Many solutions.

Consistency	Inconsistency
(i), (iii)	(ii)

Eg →

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$(x_1, x_2, x_3) = (1, 0, 1)$

* Backward Substitution

Gaussian Elimination:

Basic Idea: Reduce the given system, by elimination to a "Simpler" form

eg. Triangular form.

→ So that the Solution at Does not change.

Example:

①

$$2y + 4z = 2$$
$$x + 2y + 2z = 3$$
$$3x + 4y + 6z = -1.$$

$\boxed{AX = B}$ Solve it by Gaussian Elimination.

$$\begin{bmatrix} 0 & 2 & 4 \\ 1 & 2 & 2 \\ 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

Elementary Row Operations:

Definition: The elementary row operations are of the following type.

i) Adding a scalar multiple of one row to another row. $R_i \rightarrow R_i + kR_j$

ii) Interchange of two rows. $R_i \leftrightarrow R_j$

iii) Multiply a row by a non zero scalar.

$R_i \rightarrow kR_i$ ~~kR_j~~ $k \neq 0$.

- ~~Arg~~ Augmented Matrix

$[A \; B]$

For this example:

$$\begin{bmatrix} 0 & 2 & 4 & 2 \\ 1 & 2 & 2 & 3 \\ 3 & 4 & 6 & -1 \end{bmatrix} \underset{\sim}{\xrightarrow{R_1 \leftrightarrow R_2}} \begin{bmatrix} \boxed{1} & 2 & 2 & 3 \\ 0 & 2 & 4 & 2 \\ 3 & 4 & 6 & -1 \end{bmatrix}$$

Pivot

Perform $R_3 \rightarrow R_3 + 3R_1$

$$\begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & 2 & 4 & 2 \\ 0 & 0 & 4 & -1 \end{bmatrix} \underset{\sim}{\xrightarrow{R_2 + R_3}} \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & 2 & 4 & 2 \\ 0 & 0 & 4 & -8 \end{bmatrix}$$

This is called row Echleon form. $\leftarrow$ $\begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & 2 & 4 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

$-2R_3 + R_2$ & $-2R_3 + R_1$

* Elementary Row Operation on a matrix A

① Multiplication of one row of a matrix A by a non-zero constant.

② Addition of a constant multiple of one row to another row.

③ Interchange any two rows.

Eg, $x_1 + 2x_2 = 1$

$3x_1 + x_2 = 0.$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$[A|B] = \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 1 & 0 \end{array}\right]$$

$$R_2 \leftarrow R_2 - 3R_1 \quad \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -5 & -3 \end{array}\right]$$

$$R_1 \leftarrow R_1 + \frac{2}{5} R_2 \quad \sim \left[\begin{array}{cc|c} 1 & 0 & -1/5 \\ 0 & -5 & -3 \end{array}\right]$$

$$R_2 \leftarrow -1/5 R_2 \quad \sim \left[\begin{array}{cc|c} 1 & 0 & -1/5 \\ 0 & 1 & 3/5 \end{array}\right]$$

$\Rightarrow x_1 + 0x_2 = -1/5 \Rightarrow x_1 = -1/5$

$0x_1 + x_2 = 3/5 \Rightarrow x_2 = 3/5$

• Row equivalent :→ If A & B are $m \times n$ matrices over the field F. We say that B is row-equivalent to A if B can be ~~obser~~ obtained from A by finite sequence of elementary row operations.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

* Echelon form ⇒

① All non zero ~~are~~ rows are above any row of all zeros.

② Each leading entry of a row is in a column to the right of the leading entry of the row above it.

③ All entities in a column below a leading entity are zero.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$ → leading entries

in the right of the leading entry of the row above it.

Eg →

$$A = \begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix} \quad R_1 \leftrightarrow R_4$$

$$\sim \begin{bmatrix} \boxed{1} & 4 & 5 & -9 & -7 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix} \quad \begin{array}{l} R_2 \leftarrow R_2 + R_1 \\ R_3 \leftarrow R_3 + 2R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & \boxed{2} & 4 & -6 & -6 \\ 0 & 5 & 10 & -15 & -15 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix} \quad \begin{array}{l} R_3 \leftarrow R_3 - \frac{5}{2}R_2 \\ R_4 \leftarrow R_4 + \frac{3}{2}R_2 \end{array}$$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 \end{bmatrix}$$

NOTE :→ Remember definition too.

→ Determine the pivot column (Q. in eg. to like) & its location.

$R_3 \leftrightarrow R_4$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & -6 \\ 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Continue

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★ Row Reduced Echelon form →

If a matrix in echelon form satisfies the following additional condition, then it is said to be row reduce echelon form.

(iv) Leading entry in each non zero row is 1.

(v) Each leading 1 is the only non zero entry in its column.

continue.

$R_2 \leftarrow R_2 * \frac{1}{2}$

$R_3 \leftarrow R_3 * \left(\frac{-1}{5}\right)$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 4 & -6 & -6 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \leftarrow R_1 - 4R_2$

$R_2 \leftarrow R_2 + 3R_3$

$$\begin{bmatrix} 1 & 0 & -3 & 3 & 5 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \leftarrow R_1 - 3R_3$

$R_2 \leftarrow R_2 + 3R_3$

Pivot columns are the columns where leading entries are there.

$$\begin{bmatrix} \boxed{1} & 0 & -3 & 0 & 5 \\ 0 & \boxed{1} & 2 & 0 & -3 \\ 0 & 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\{1, 2, 4\} \rightarrow$ Pivot column

(#) Linear System of equations in n unknowns $x_1, x_2, x_3, \ldots x_n$ is of the form

$$a_{11}x_1 + \text{———} + a_{1n}x_n = b_1$$
$$a_{21}x_1 + \text{———} + a_{2n}x_n = b_2$$
$$|$$
$$a_{m1}x_1 + \text{———} + a_{mn}x_n = b_m$$

The system is called linear because each variable x_i, $i = 1, 2, \ldots n$ appears in the first power only.

$a_{11}, a_{12}, \ldots a_{mn}$ are called the coefficient of the system

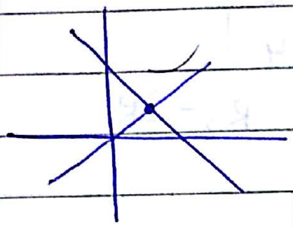
$b_1, b_2, \ldots b_m$ are given constant.

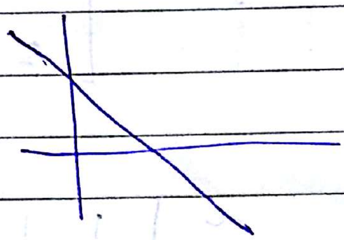
Homogenious if all $b_1, b_2, - b_m$ are zero.
$\hookrightarrow$ solution always exist.

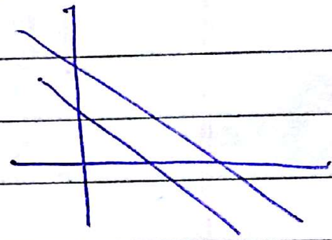
Eg $\rightarrow$
$$x_1 + 2x_2 + x_3 = 0$$
$$3x_2 + x_3 = 0$$
$$x_1 + \quad 5x_3 = 0$$

Non Homogenious linear System
If atleast one of b_j is non zero.
$$x_1 + 2x_2 + 3x_3 = 0$$
$$2x_1 \quad + x_3 = \boxed{1} \rightarrow \text{Non Zero.}$$
$$5x_1 + x_2 + x_3 = 0$$

Ex $\rightarrow$ ① $x_1 + x_2 = 2$ ② $x_1 + x_2 = 2$ ③ $x_1 + x_2 = 1$
$x_1 - x_2 = 0$ $2x_1 + 2x_2 = 4$ $x_1 + x_2 = 2.$


Unique Solution (one Solution)


Infinitely many solution


No Solution

- System of linear equations can be written in matrix form
$AX = b$

$$A = \begin{bmatrix} a_{11} & a_{12} & \text{—} & a_{1n} \\ a_{21} & a_{22} & \text{—} & a_{2n} \\ | & & & \\ a_{m1} & & \text{—} & a_{mn} \end{bmatrix} \rightarrow \text{Coefficient Matrix}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ | \\ b_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ | \\ x_n \end{bmatrix}$$

- Argumented matrix   {Merged A & b}
$[A \mid b]$
$$[A|b] = \left[\begin{array}{ccc|c} a_{11} & \text{—} & a_{1n} & b_1 \\ a_{21} & \text{—} & a_{2n} & b_2 \\ | & & & \\ a_{m1} & & a_{mn} & b_m \end{array}\right]$$

① $x_1 + x_2 = 2$
$x_1 - x_2 = 0.$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

Pivot 1

$$[A|b] = \left[\begin{array}{cc|c} \boxed{1} & 1 & 2 \\ 1 & -1 & 0 \end{array}\right] \quad R_2 \leftarrow R_2 - R_1$$

eliminate

$$\sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -2 & -2 \end{array}\right]$$

$x_1 + x_2 = 2$ Back Substitution
$-2x_2 = -2 \Rightarrow x_2 = 1$

$x_1 = 1$

② $x_1 + x_2 = 2$
$2x_1 + 2x_2 = 4$

$$[A|b] = \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 2 & 4 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$

$$\sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 + x_2 = 2. \rightarrow$ only 1 eq

③ $x_1 + x_2 = 1$
$x_1 + x_2 = 2.$

$$[A|b] = \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 1 & 2 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1$

$x_1 + x_2 = 1$
$\boxed{0 = 1}$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right]$$

$\rightarrow$ Not possible $\Rightarrow$ No Solution

* If eqs are Homogeneous then ③ case not possible.

Unique $\rho[A] = \rho[A|b] = n$ ~~(finite pts)~~

Infinite $\rho[A] = \rho[A|b] < n$

No Sol. $\rho[A] \neq \rho[A|b]$

Gauss elimination method

$x_1+x_2+x_3=2$
$2x_1+x_2+3x_3=1$
$x_1+x_2+x_3=2$
$x_2+2x_3=0$

$$A=\begin{bmatrix}1&1&1\\2&1&3\\1&1&1\\0&1&2\end{bmatrix} \quad b=\begin{bmatrix}2\\1\\2\\0\end{bmatrix}$$

Pivot value

$$[A|b]=\left[\begin{array}{ccc|c}1&1&1&2\\ \boxed{2}&1&3&1\\ \boxed{1}&1&1&2\\0&1&2&0\end{array}\right]$$

eliminate

$R_2 \leftarrow R_2-2R_1$
$R_3 \leftarrow R_3-R_1$

Pivot

$$\sim\begin{bmatrix}1&1&1&2\\0&\boxed{-1}&1&-3\\0&0&0&0\\0&1&2&0\end{bmatrix}$$

eliminate

$R_4 \leftarrow R_4+R_2$

$R_3 \leftrightarrow R_4$

$$\sim\left[\begin{array}{ccc|c}1&1&1&2\\0&-1&1&-3\\0&0&0&0\\0&0&3&-3\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&1&1&2\\0&-1&1&-3\\0&0&\boxed{3}&-3\\0&0&0&0\end{array}\right]$$

already Zero & processed here.

Back Substitution

$3x_3=-3 \Rightarrow x_3=-1$
$-x_2+x_3=-3$ $\qquad x_1+x_2+x_3=2$
$x_2=2$ $\qquad x_1=1$

~~$x_1\ x_2\ +x_3$~~ $= x+y+z=2$
$2x+y-2z=1$
$3x+2y-z=k$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 1 & -2 & 1 \\ 3 & 2 & -1 & k \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

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$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -4 & -3 \\ 0 & -1 & -4 & k-6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -4 & -3 \\ 0 & 0 & 0 & k-3 \end{array}\right]$$

$0 = k - 3 \rightarrow$ infinitely Many Solutions $\boxed{k=3}$

No solution for $\boxed{k \neq 3}$ $k \in R$.

Row Operations.

Row echelon form → Gauss elimination method. {Back substitution}

Row Reduced Echelon form → Gauss-Jordan elimination method.

$[\ \bigcirc \ |\ \ ]$ → Identity matrix

eg → $x_1 + x_2 = 1$

$x_1 + 2x_2 = 0$

$$[A|b] = \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \end{array}\right] \rightarrow \text{GEM}$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right] \rightarrow \text{GJEM}$$

No. of Non Zero Rows in Row Echlon form = Rank of a matrix.

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Vector Space

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Binary operations:

$*: S \times S \to S$

$(a, b) \in S$

$a * b = c$, $c \in S$. (for * to be Binary operation

(P1) → Closer ~~axiom~~ / law / postulate.

eg → ① $+ : N \times N \to N$ or $(N, +)$

$(2, 3) \in N$ $(N, +)$

$2 + 3 = 5$.

② $- : N \times N \to N$ $(N, -)$

↓ is not closed as $1 - 2 = -1 \notin N$.

$(I -), (N, *)$ → is ~~closed~~ defined.

(P2) → Associtive law. $a, b, c \in S$

$a * (b * c) = (a * b) * c$

① $(N, +)$

② $(I, -)$ → Does Not Satisfy Associtive law as LHS ≠ RHS.

③ $(N, \times)$

④ None.

(P3) → Identity: If ∃ an element e such that

$a + e = e + a = a \quad \forall a \in S$,

But, $e \in S$, should

(P4) → Inverse: For each $a \in S$ ∃ $b = a^{-1}$ such that

$+ \quad a + b = b + a = e$

$ab = ba = e$

(P5) → Satisfying All property.

$\{Q - \{0\}, *\}$, $(Q, +)$, $(R, +)$, $(R - \{0\}, *)$

$(C, +)$, $(C - \{0\}, *)$

(P5) Commutative law.

$a + b = b + a$

$a * b = b * a$

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Group : A ~~stracte~~ structure $(S, +)$ has P_1, P_2, P_3, P_4. $\quad + : S \times S \to S$

Eg → $(R, +)$

* Commutative Group : $P1, P2, P3, P4, P5$.

Field :

1) $(S, +)$ commutative Group.
2] $(S - \{0\}, *)$ commutative Group
3] Distributive law. $a(b+c) = ab + ac$.

eg → ~~Group~~ : $(I - \{0\}, *)$ → Not a group. as Inverse (X). $2 \times \frac{1}{2} = 1$, $\frac{1}{2} \notin I$

closer property is not satisfied.

$(IR, +, *)$, $(I, +, *)$ → Not a field.

eg →

① $(R, +, *)$ → field?
$(R, +)$ group? ?
$(R - \{0\}, *)$ group ? ?

② $(C, +, *)$ ③ $(Q, +, *)$ → field

$R^2 = \{(a, b) : a, b \in R\}$ → Real numbers. {one of the field}.

Touple

scalar, Vector.

$R^n = \{(a_1, a_2, \_\_ a_n) ; a_i \in R\}$

$M = \{[a_{ij}]_{m \times n} ; a_{ij} \in R\}$

Let $\langle F, +, \cdot \rangle$ be a Field and V is a non empty set with two operations.

key property

Vector addition : This assigns to any $\alpha, \beta \in V$ to $\alpha + \beta \in V$ (closer) similar.

Scalar Multiplication : This assigns to any $\alpha \in V$, $a \in F$ a product $a.\alpha \in V$

Then V is said to be vector space over the field if the following axioms holds.

$\boxed{A1}$ $(\alpha+\beta)+\gamma = \alpha+(\beta+\gamma)$ Associative

$\boxed{A2}$ There is a vector 0 in V such that

$\alpha+0 = 0+\alpha = \alpha \quad \forall \alpha \in V$ { identity property }

$\boxed{A3}$ For each $\alpha \in V$, there exist $-\alpha \in V$ such that

$\alpha+(-\alpha) = (-\alpha)+\alpha = 0$. { inverse

$\boxed{A4}$ $\alpha+\beta = \beta+\alpha$. commutative.

$\boxed{M1}$ $a(\alpha+\beta) = a\alpha + a\beta$, $\forall a, b \in F$

$\boxed{M2}$ $(a+b)\alpha = a\alpha + b\alpha$

$\boxed{M3}$ $(ab)\alpha = a(b\alpha)$

$\boxed{M4}$ $1.\alpha = \alpha \quad \forall \alpha, 1 \in F$. $\Rightarrow$ 10 properties

Question: Show that V is a Vector Space over R.

$\boxed{OR}$ Is V(R) a vector space ??

$$V = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2} a_{ij} \in R \right\}$$

over the field R

$F = R$.

Solution Addition is closed

① $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix} \in V$ since, each entries ~~are~~ is Real since, addition of 2 Real Nos. is Real No.

② $A+(B+C) = (A+B)+C$.

$\rightarrow$ Let Matrices then Prove LHS = RHS, by solving.

③ $I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ← Identity of V

$A+I = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = A$

Similarly $I+A = A$.

$\Rightarrow$ I is identity of V.

commutative.

(4) $A+B = B+A$.

$$A+B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$$

(from (1))

$$= \begin{bmatrix} b_{11}+a_{11} & b_{12}+a_{12} \\ b_{21}+a_{21} & b_{22}+a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= B+A.$$

Inverse:

$$-A = \begin{bmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{bmatrix}$$

$$A+[-A] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (-A)+A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Also prove M1, M2, M3, M4 properties in similar fashion.

If all 10 Properties are satisfied or (Hold), then only V is a vector space.

Q. Set of all ~~pol~~ polynomial of degree ~~les~~ 2 vector space over R.

$$V = \{a_0 + a_1 x + a_2 x^2 : a_0, a_1, a_2 \in R, a_2 \neq 0\}$$

Soln $\alpha = 1 + x + 2x^2, \quad \beta = 1 + 3x - 2x^2$

$\alpha + \beta = 2 + 4x \rightarrow$ degree 1. $\Rightarrow$ closer property is Not hold

$\Downarrow$

Not a Vector Space

Q. Set of all polynomial of degree less than and equal 3 Vector space over R.

$$V = \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 : a_1, a_2, a_3, a_0 \in R\}$$

$\rightarrow$ All property will be satisfied.

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⇒ Set of all Polynomials is a vector space over R
→ Vector Space.

Q→ Let $V = \{(a,b) : a,b \in R\}$,
be a non empty set
(1) with the two operations.
Vector addition: $(a_1,b_1) + (a_2,b_2) = (a_1+b_1,\ a_2+b_2)$.
Scalar Multiplication: $k(a_1,b_1) = (ka_1, 0)\ k \in R$.

Soln → all properties hold except last.

$1.(a_1,b_1) = (1.a_1, 0) = (a_1,0) \neq (a_1,b_1)$

eg → $1.(2,3) = (1\cdot 2, 0) = (2,0) \neq (2,3)$.

→ Pointout 1 property only, → against val.

(2) operations,

Q. $\begin{cases} (a_1,b_1) + (a_2,b_2) = (a_1+b_1;\ a_2+b_2) \\ k(a,b) = (ka, kb) \end{cases}$

→ ~~X~~ identity false → $(a,b_1) = (2,3)$ say.

$(2,3) + (0,0) = (2+3, 0+0) = (5,0) \neq (2,3)$

identity doesn't exist.

(3) $\begin{cases} (a_1,b_1) + (a_2,b_2) = (a_1,b_1) \\ k(a_1,b_1) = (ka_1, kb_1) \end{cases}$

→ Not commutative, identity, inverse.

for disproving → just 1 counter example.

Vector Subspace

$W = \{(a,0),\ a \in R\}$ $W \subset V$

Ex→ $V = \left\{ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} a_{ij} \in R \right\}$

Under matrix usual addition & Scalar multiplication.

$W_1 = \left\{ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} : A^T = A, a_{ij} \in R \right\}$

$W_2 = \left\{ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} : A^T = -A \right\}$ → skew symmetric matrix

$W_3 = \left\{ A \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} : a_{ij} \in R \right\}$ → upper triangular matrix

$W_4 = \left\{ \begin{bmatrix} a & b+ic \\ b-ic & d \end{bmatrix}, a,b,c,d \in R \right\}$ → Hermition matrix.

$W_5 = \left\{ \begin{bmatrix} ai & -b+ic \\ b+ci & di \end{bmatrix} a,b,c,d \in R \right\}$ → skew "

complex no. → $z = a+ib \Leftrightarrow \bar{z} = a-ib$

$(\bar{A})^T = A^\theta = A$ {Hermition}

$(\bar{A})^T = -A$ {skew-hermition}

$(\bar{A})^T = (\overline{A^T}) = A^\theta$

★ Is $W_i(R)$ a vector space ??
if Yes then Also $W_i \subset V$ ⇒ subset ⇒ Subspace.

Q Is $W_4(C)$ a vector space ??

★ feild change hota hai to check property m1, m2, m3, m4 ...

Soln. A Eg,

$\cancel{A = \begin{bmatrix} 1+2i & 1+i(i) \\ 1-i(i) & 2+i \end{bmatrix}}$

$\cancel{A^T =}$

if $A = \begin{bmatrix} 1 & 1+i \\ 1-i & 3 \end{bmatrix}$ & $a = 2i$

then $B = aA = \begin{bmatrix} 2i & 2i-1 \\ 2i+1 & 6i \end{bmatrix}$

$(\bar{B})^T = \begin{bmatrix} -2i & -2i+1 \\ -2i-1 & -6i \end{bmatrix}$ $(\bar{B})^T = -B$

⇒ Not a Vector Space

Not getting Hermition Matrix

Q If $W_5(C)$ a vector space??

$A = \begin{bmatrix} i & -1+i \\ 1+i & 2i \end{bmatrix}$ a

$a = i$ ⇒ $aA = \begin{bmatrix} -1 & -i-1 \\ -i-1 & -2 \end{bmatrix}$

Q→ W_4 is a vector space over the field.

✓ (a) R (b) C (c) Z (d) None

Z → Not a field.

Subspace: Let w be a subset of a Vector space V(F). Then w is said to be a subspace if w is a vector space over F under the same operations.

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ASSIGNMENT 1 [HINTS]

1. Given $A = (a_{ij})$, $B = (b_{ij})$

$(AB)^T = B^T A^T$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ \vdots & & \ddots & \\ a_{m1} & \cdots & & a_{mn} \end{bmatrix}$$

ij element of AB $= a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{im}b_{mj}$

—"— $(AB)^T = a_{im}b_{mj} + \cdots + a_{i1}b_{1j}$

Similarly,

$B^T A^T = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{im}b_{mj}$

$\Rightarrow (AB)^T = B^T A^T$

2. A & B are invertable matrix

$(AB)^{-1} = B^{-1}A^{-1}$

Lets take, $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1}$
$= AA^{-1} = I$

$\Rightarrow (AB)(B^{-1}A^{-1}) = I$

$\Rightarrow (AB)^{-1} = B^{-1}A^{-1}$

3. $A = \frac{2A}{2} = \frac{1}{2}(A+A) = \frac{1}{2}(A + A^T + A - A^T)$

$= \frac{1}{2}(A+A^{-1}) + \frac{1}{2}(A - AT)$

Here, for A be a n order matrix,

$A + A^T$ is always symmetric

$A - A^{-1}$ is always skewsymmetric.

4. A is a real orthogonal matrix

$AA^T = I$

Taking determinant on both side.

$|AA^T| = |I| = 1$, ~~since~~ & $|A| = |A^T|$ & $|AA^T| = |A||A^T|$

$|A|^2 = 1 \Rightarrow |A| = \pm 1$.

5. A is nilpotent matrix

$\Rightarrow A^m = 0 \quad (m \geq 1)$

Now, we have to prove that $(A+I)$ is invertable

$\Rightarrow$ Inverse exist & determinant Non zero.

(1) —— $(A+I)(A+I)^{-1} = (A+I) = (I - A + A^2 - \_\_\_ + (-1)^{m-1} A^{m-1} + (-1)^m A^m + \_\_\_)$

$\rightarrow$ Higher terms will be zero, $A^m = 0 \Rightarrow A^{m+1} = 0,\ A^{m+2} = 0 \ldots$ so on.

Expension

$\Rightarrow (1+x)^{-1} = (1 - x + x^2 - x^3 + x^4 - + \_\_\_ \infty)$

from (1) $\quad (A+I)(A+I)^{-1} = I + (-1)^{m-1} A^m \quad$ {on simplifying}

Now, $A^m = 0$ {Given}

$\Rightarrow (A+I)(A+I)^{-1} = I$

$\rightarrow$ taking determinants both sides, we have

$\Rightarrow |A+I|\,|(A+I)^{-1}| = |I| = 1 \neq 0$

$\Rightarrow \quad \Rightarrow |A+I| \neq 0 \Rightarrow A+I$ is invertable.

6. (a) $\begin{bmatrix} 4 & 3 \\ -8 & -6 \\ 16 & 12 \end{bmatrix}$ $\quad R_3 \leftarrow R_3 - 4R_1$

$R_2 \rightarrow R_2 + 2R_1$

$\sim \begin{bmatrix} 4 & 3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\rightarrow$ Row echelon form

(b) $\begin{bmatrix} 1 & 5 & 8 \\ 3 & 2 & 9 \end{bmatrix}$ $\quad R_2 \leftarrow R_2 - 3R_1$

$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & -13 & -13 \end{bmatrix}$ $\rightarrow$ Row echolon form.

(c) $\begin{bmatrix} 0 & 2 & -3 \\ 2 & 0 & 5 \\ -3 & 5 & 0 \end{bmatrix}$ $\quad R_1 \leftrightarrow R_3 \quad R_2 \rightarrow R_2 + \frac{2}{3}R_1$

$\sim \begin{bmatrix} -3 & 5 & 0 \\ 2 & 0 & 5 \\ 0 & 2 & -3 \end{bmatrix} \sim \begin{bmatrix} -3 & 5 & 0 \\ 0 & \frac{10}{3} & 5 \\ 0 & 2 & -3 \end{bmatrix}$

$R_2 \rightarrow 3R_2$

& then $R_3 \rightarrow R_2 - 5R_3$

$\sim \begin{bmatrix} -3 & 5 & 0 \\ 0 & 10 & 15 \\ 0 & 0 & -30 \end{bmatrix}$

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7. (a)

$$\left.\begin{aligned} x-y+z&=0\\ -x+y-z&=0\\ 10y+25z&=90\\ 20x+10y&=80 \end{aligned}\right\}\rightarrow$$

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$$[A|b]=\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0\\ -1 & 1 & -1 & 0\\ 0 & 10 & 25 & 90\\ 20 & 10 & 0 & 80 \end{array}\right]$$

convert in Row echelon form

$$\sim\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0\\ 0 & 10 & 25 & 90\\ 0 & 0 & 95 & -190\\ 0 & 0 & 0 & 0 \end{array}\right]$$

Backward Substitution

$(x, y, z) = (\ ,\ )$

$10y+25z-2z=90$

(c) $z=-2,\quad 10y=140$

$y=14.\qquad x=y+z=14+2=16.$

8.

$$\begin{aligned} x+y+z&=5\\ 2x+3y+5z&=8\\ 4x+0y+5z&=2 \end{aligned}$$

(a) coefficient Matrix : $A=\begin{bmatrix} 1 & 1 & 1\\ 2 & 3 & 5\\ 4 & 0 & 5 \end{bmatrix}$

augmented matrix $[A|b]=\begin{bmatrix} 1 & 1 & 1 & 5\\ 2 & 3 & 5 & 8\\ 4 & 0 & 5 & 2 \end{bmatrix}$

(b) Applying Gauss Jordan elimination method on $[A|b]$

$R_2\leftarrow R_2-2R_1,\quad R_3\leftarrow R_3-4R_1$

$$\sim\begin{bmatrix} 1 & 1 & 1 & 5\\ 0 & 1 & 3 & -2\\ 0 & -4 & 1 & -18 \end{bmatrix}$$

$R_3\leftarrow R_3+4R_2$

$$\sim\begin{bmatrix} 1 & 1 & 1 & 5\\ 0 & 1 & 3 & -2\\ 0 & 0 & 13 & -26 \end{bmatrix}\sim\begin{bmatrix} 1 & 1 & 1 & 5\\ 0 & 1 & 3 & -2\\ 0 & 0 & 1 & -2 \end{bmatrix}$$

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$$\sim \begin{bmatrix} 1 & \cancel{0} & \cancel{0}^{-2} & 7 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$-2 \; \cancel{+3}$

$-\cancel{3}$

-2

$R_2 \leftarrow R_1 + 2R_3$

$-2 \; -6$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \cancel{-8} \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$(x, y, z) = (3, 4, -2)$

unique solution : Exist solution.

10. (d)
$$\left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 1 & 0 & 0 \\ 1 & 5 & -1 & 0 & 1 & 0 \\ 3 & 13 & 6 & 0 & 0 & 1 \end{array}\right]_{3 \times 6}$$

$$\sim \begin{bmatrix} 1 & 3 & -4 & 1 & 0 & 0 \\ 0 & 2 & 3 & -1 & 1 & 0 \\ 0 & 4 & 18 & -3 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -4 & 1 & 0 & 0 \\ 0 & 1 & 3/2 & -1/2 & 1/2 & 0 \\ 0 & 4 & 18 & -3 & 0 & 1. \end{bmatrix}$$

Linear Combination :

A vector α is said to be a linear combination of vector $\alpha_1, \alpha_2, — \alpha_n$ if

$$\alpha = a_1\alpha_1 + a_2\alpha_2 + — + a_n\alpha_n$$

$$a_1, a_2, — a_n \in F.$$

Eg.

$$\alpha = (2, 3, \sqrt{2})$$

$$\{\alpha_1 = (1,0,0), \alpha_2 = (0, \tfrac{1}{2}, 0), \alpha_3 = (0,0,2)\}$$

$$\Rightarrow \alpha = 2\alpha_1 + 6\alpha_2 + \frac{1}{\sqrt{2}}\alpha_3$$

{Here It is Easy}

Q. Let $S = \{(1,2), (1,1)\}$

Can I write $(2,5)$ in L.C. of S.

Solu. Let

$$(2,5) = x(1,2) + y(1,1) \quad — ①$$

$$(2,5) = (x+y, 2x+y)$$

$$\Rightarrow \left.\begin{aligned} x+y &= 2 \\ 2x+y &= 5 \end{aligned}\right\} \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 1 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -1 & 1 \end{array}\right]$$

$$y = -1, \ x = 3.$$

by putting in ①.

$$RHS = (+3)(1,2) + (-1)(1,1) = (2,5)$$

$\Rightarrow$ values of x & y satisfy $\Rightarrow$ Yes we can write $(2,5)$ in L.C.

Q. Let $S = \{(1,-2), (-2,4)\}$.

Can I write $(2,5)$ in L.C. of S.

$$(2,5) = x(1,-2) + y(-2,4)$$

$$(2,5) = (x-2y, -2x+4y)$$

$$\left.\begin{aligned} x-2y &= 2 \\ -2x+4y &= 5 \end{aligned}\right\} \left[\begin{array}{cc|c} 1 & -2 & 2 \\ -2 & 4 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 9 \end{array}\right]$$

$$R_2 \leftarrow R_2 + 2R_1$$

Linear span of a Vector Space.

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} ; a,b,c,d \in R \right\}$$ V is the

$$R^2 = \{(a,b) : a,b \in R\}.$$
$$S = \{(1,0), (0,1).\}$$ $(a,b) = a(1,0) + b(0,1)$

Subspace: ① V(F)
② $W \subseteq V$. ③ W is it self a vector space under the given operations.

Theorem: V is a vector space & W is non empty subset of V. Then W is a subspace of V if and only if
$$a\alpha + \beta \in W \quad \forall \alpha, \beta \in W$$
$a \in F$.

Eg ⑧ Let V be a set of all real valued continuous functions over R and W be a set of all solution of ~~real~~ valued continuous functions y such that
$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = 0.$$
F → Real No.

Solu. ① T/show
$$W_1 = \left\{ y : 2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = 0 \right\} \subseteq V.$$

Let $y_1, y_2 \in W$ $\Rightarrow 2\frac{d^2y_1}{dx^2} + 3\frac{dy_1}{dx} + 5y_1 = 0$ — ①

$\Rightarrow 2\frac{d^2y_2}{dx^2} + 3\frac{dy_2}{dx} + 5y_2 = 0$ — ②

Let $y = ay_1 + y_2$, T/P: $ay_1 + y_2 \in W$.

$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = 2\frac{d^2(ay_1+y_2)}{dx^2} + 3\frac{d(ay_1+y_2)}{dx} + 5(ay_1+y_2)$$

$$= 2\frac{d^2ay_1}{dx^2} + 2\frac{d^2y_2}{dx^2} + 3\frac{day_1}{dx} + 3\frac{dy_2}{dx} + 5ay_1 + 5y_2$$

$$= a\left(2\frac{d^2y_1}{dx^2} + 3\frac{dy_1}{dx} + 5y_1\right) + \left(2\frac{d^2y_2}{dx^2} + 3\frac{dy_2}{dx} + 5y_2\right)$$

Using ① & ②

$$= a \times 0 + 0 = 0.$$

So, we can conclude that $ay_1 + y_2$ also satisfy this condition
$\Rightarrow ay_1 + y_2 \in W \Rightarrow W$ is a subspace of V.

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Ex. $V = \{(a,b,c) : a,b,c \in R\}$

$W_1 = \{(a,b,c) : a+b+c = 1,\ a,b,c \in R\}$: definitely $W \subseteq V$.

$W_2 = \{(a,b,c) : 2a+3b-c = 0;\ a,b,c \in R\}$

$W_3 = \{(a,b,c) : 2a = b,\ a,b,c \in R^2\}$

impressions eg → $W_4 = \{y : 2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = 7\} \subseteq V$.

Linear Span :→

Let V(F) is a vector space and S is a non empty set of V. Then the linear span of S is the set of all linear combination of finite sets of elements of S such that

$L(S) = \{a_1\alpha_1 + a_2\alpha_2 + \ldots + a_n\alpha_n : \alpha_1, \alpha_2, \ldots \alpha_n \in S,\ a_1, a_2, \ldots a_n\}$

↳ vector subspace

Q. Which of the following are vector space

$\cancel{R(C)}$ $C(R)$ ✓ $R(R)$ ✓ $R(Q)$ ✓ $\cancel{Q(R)}$

$2 \times \sqrt{2} = 2\sqrt{2} \notin Q$
$\downarrow_Q \quad \downarrow_R$

eg → $S = \{(1,1), (1,2)\} \subseteq R^2$.

Let $(x,y) \in R^2$

$a(1,1) + b(1,2) = (x,y)$

$(a+b,\ a+2b) = (x,y)$

$a+b = x$

$a+2b = y$

$$\left[\begin{array}{cc|c} 1 & 1 & x \\ 1 & 2 & y \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 1 & x \\ 0 & 1 & y-x \end{array}\right]$$

No. of Non Zero rows = Rank, in A & [A|B].

Applying back substitution

$b = y - x$

$a = 2x - y$.

$(x,y) = (2x-y)(1,1) + (y-x)(1,2)$

Linearly dependent:

Let V(F) be a vectorspace over F and $S = \{\alpha_1, \alpha_2, \ldots$

$\alpha_n\} \subseteq V$. Then the elements of V are said to be linearly dependent if there exist some scalars $a_1, a_2, \dots a_n \in F$ which all are not zero such that

$$a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n = 0.$$

Eg, $S = \{ \overset{\alpha_1}{(1,1)}, \overset{\alpha_2}{(1,2)} \} \subseteq R^2$. checking whether they are linearly dependent or NOT.

$a_1\alpha_1 + a_2\alpha_2 = 0$

$a_1(1,1) + a_2(1,2) = 0.$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right].$$

By back substitutent.
~~$(a_1, a_2) = (0,0)$~~

$\Rightarrow a_1 \& a_2 = 0$ It shows that $\alpha_1 \& \alpha_2$ are linearly independent.

Basis → Let V be a vector space over F and S is a non empty ~~space~~ subset of V. S is said to be a basis of V if

① Elements of S are linearly - independent.

② S span V, ie, $L(S) = V$.

← Homogeneous

← NonHomogeneous {unknown Nos.}

Dimension = No. of elements in basis

Eg → $S = \{(1,1), (1,2)\}$ is a basis of R^2.

$S_1 = \{(1,0), (0,1)\}$; $S_2 = \{(1,1)\}$; $S_3 = \{(1,0), (0,1), (1,1)\}$,

$S_4 = \{(0,0)\}$.

$S_1 : \{(1,0),(0,1)\}$ ① $a_1(1,0) + a_2(0,1) = (0,0)$

$a_1 = 0 \quad a_2 = 0 \Rightarrow$ Linearly Independent $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \rightarrow$

② $a_1(1,0) + a_2(0,1) = (x,y)$ $\Rightarrow S_1$ is basis

$a_1 \cdot (x,y) = (x)(1,0) + (y)(0,1)$ $\Rightarrow S_1$ is span ✓

$S_2 = \{(1,1)\}$, ① $a_1(1,1) = (0,0)$ $a_1 = 0$ $\Rightarrow$ linearly independent

② $a_1(1,1) = (x,y) \Rightarrow a_1 = x = y \Rightarrow L(S) \neq V \rightarrow$ we are unable to write all elements in linear combination of S_2

$S_3 = \{(1,0),(0,1),(1,1)\}$ ① $a_1(1,0) + a_2(0,1) + a_3(1,1) = (0,0)$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$a_1 + a_3 = 0$

$a_2 + a_3 = 0$ if $a_3 = K$

$a_1 = -K = a_2$ $\Rightarrow$ linearly dependent $\Rightarrow$ Not a Basis

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$S_4 : \{(0,0)\}$ $a_1(0,0) = (0,0)$ $a_1 \in R$, ⇒ linearly dependent
→ Not a Basis

Summary:

Vector Space : $V(F)$

$V = \{(a,b,c) : a,b,c \in R\}$.

$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1+a_2, b_1+b_2, c_1+c_2)$

$k(a_1, b_1, c_1) = (ka_1, kb_1, kc_1)$

Subspace : $W \subseteq V(F)$

& $a\alpha + \beta \in W$, $\alpha, \beta \in W$ & $a \in F$

$\alpha = (a,a,a)$, $\beta = (b,b,b)$, $k \in R$

$k(a,a,a) + (b,b,b) = (ka+b, ka+b, ka+b)$

Linear Dependence & ~~Impro~~ Independence
LD LI

$S_4 = \{(1,1,1), (0,1,1), (0,0,1), (0,0,0)\}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$ already in echlon form

One zero row ⇒ Linearly ~~independent~~ dependent.

For ~~Any~~ Linearly independent ⇒ No Zero row should be there in Matrix form.

$S_2 = \{\overset{e_1}{(1,0,0)}, \overset{e_2}{(0,1,0)}, \overset{e_3}{(0,0,1)}\}$

Since, we have No Zero Row this collection is LI.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Linear Span (Linear Combination)

$L(S) = \{a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n ; \alpha_1, \alpha_2, \dots \alpha_n \in S$
$a_1, a_2, \dots a_n \in F\}$

$\alpha = a_1e_1 + a_2e_2 + a_3e_3 = a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1)$

coordinates.

$\int_{-\infty}^{\infty} e^{-ax^2}$ · $\int_0^{\infty} e^{-ax^2}$

Coordinates :-

$a_1, \quad a_2, \quad a_3$
$\Downarrow \quad \Downarrow \quad \Downarrow$
$x \quad y \quad z$ in case of S_2

(x, y, z). $\Leftarrow$ Here, we get simple one, we can other complex things too

$S_1 : \{(1,1,1)\}$ $\quad \alpha = a_1 e_1$

$(x, y, z) = a_1(1,1,1)$

S_1 = Span W, but not V. $\rightarrow$ contradiction. $(1,0,0) \neq (a_1, a_1, a_1)$
$x=1, \; y=z=0$.

Basis : Linearly Independent + S span V.

S_2 is basis for V
S_1 is basis for W
S_2 is NOT basis for W $\leftarrow$ Not subset {Reason}

Dimension :
No. of vector in V.
~~dim~~ $\dim W = 1, \quad \dim V = 3$

Eg $\rightarrow$ $S_3 = \{\overset{e_1}{(1,0,0)}, \overset{e_2}{(0,1,0)}, \overset{e_3}{(0,0,1)}, \overset{e_4}{(1,1,1)}\}$ $\quad e_4 = e_1 + e_2 + e_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \boxed{1} & \boxed{1} & \boxed{1} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_4 \leftarrow R_4 - (R_1 + R_2 + R_3)$$

atleast 1 Rows = 0 $\Downarrow$ (LD)

$\boxed{\text{Rank} = 3}$.

if we can write any one element in the L Combination of others $\Rightarrow$ LD $\Rightarrow$ NOT Basis.

Eg $\rightarrow$ Rank of the Matrix :-

$$\text{eg}\quad \begin{bmatrix} 1 & 2 & 3 & 1 \\ \boxed{2} & 4 & -1 & 2 \\ \boxed{1} & +2 & 10 & 1 \end{bmatrix}$$

(pivot: 1)

$R_2 \leftarrow R_2 - 2R_1$
$R_3 \leftarrow R_3 - R_1$

$$\sim \begin{bmatrix} \boxed{1} & 2 & 3 & 1 \\ 0 & 0 & \boxed{-7} & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & -7 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \leftarrow R_3 + R_2$$

every Row is considered as Vector.

Here, $R_3 = 3R_1 - R_2$

$S_4 = \{ (1,2,3), (2,0,1), (3,1,2) \}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$R_3 \leftarrow R_3 - R_1 - R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -5 \\ 0 & -1 & 1 \end{bmatrix}$$

$R_3 \leftarrow R_3 - \frac{1}{4} R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -5 \\ 0 & 0 & -\frac{3}{4} \end{bmatrix}$$

→ Zero → Zero Rows ⇒ LI.

★ for checking LI, we can check Determinant too.

$\boxed{\Delta \neq 0 \text{ for LI}}$

$\boxed{\Delta = 0 \text{ for LD}}$ → as atleast one Zero Rows

~~for~~ for Linear Span:

$(x,y,z) = a(1,2,3) + b(2,0,1) + c(3,1,2)$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & x \\ 2 & 0 & 1 & y \\ 3 & 1 & 2 & z \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & x \\ 0 & -4 & -5 & y-2x \\ 0 & -5 & -7 & z-3x \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & x \\ 0 & -4 & -5 & y-2x \\ 0 & 0 & -\frac{3}{4} & \frac{4z-8x-5y}{4} \end{array}\right]$$

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Coordinates $\Rightarrow (a, b, c)$

$$= \Big(\underbrace{?, ?}, -\frac{1}{3}(4z - 2x - 5y)\Big)$$

can be ~~find out~~ got by backward Substitution.

$\boxed{*}$ Dimension is fixed but Basis need not to be Unique.

Eg Vector Space : $V(F)$

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in R \right\}$$

Subspace : $W \subset V(F)$

$$W_1 = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in R \right\}$$

collection

Let $S_5 = \left\{ \begin{array}{c} R_1 \\ R_2 \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

$e_1 = [\overbrace{1\ 0}^{R_1}\ \overbrace{0\ 0}^{R_2}]$, $e_2 = [\overbrace{0\ 1}^{R_1}\ \overbrace{0\ 0}^{R_2}]$, $e_3 = [0, 0\ 1\ 0]$

$e_4 = [0\ 0\ 0\ 1]$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow LI \text{ as } |A| \neq 0.$$

Linear Span :

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

coordinates $(x, y, z, w) = (a, b, c, d) \Rightarrow$ dim $= 4$ as $n(S_5) = 4$

Inner Product Space : $\boxed{V \times V \to F}$

Let F be the field of real numbers or the field of complex numbers, and V a vector space over F. An inner product on V is a function, which assigns to each order pair of vectors α, β in V a scalar $\langle \alpha, \beta \rangle$ in F such a way that for all $\alpha, \beta, \gamma \in V$ and all scalars $c \in F$.

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$\boxed{1}$ $\langle \alpha+\beta, \gamma\rangle = \langle \alpha,\gamma\rangle + \langle \beta,\gamma\rangle$, $\langle a\alpha+\beta,\gamma\rangle = a\langle\alpha,\gamma\rangle+\langle\beta,\gamma\rangle$

$\boxed{2}$ $\langle c\alpha, \beta\rangle = c\langle\alpha,\beta\rangle$ — Linearity property

$\boxed{3}$ $\langle\alpha,\beta\rangle = \overline{\langle\beta,\alpha\rangle}$ conjugate Symmetry. Real: $\langle\alpha,\beta\rangle = \langle\beta,\alpha\rangle$. Symmetry

$\boxed{4}$ $\langle\alpha,\alpha\rangle \geq 0$ and $\langle\alpha,\alpha\rangle = 0$ if & only if $\alpha = 0$ — Non-Negativity property / Positive definite

Verify that it is Inner Product Space.

Standard IPS on complex Nos.

Ex → On $V_n(C)$ there is an inner product, which we call the standard inner product if $\alpha = (a_1, a_2, \_\_ a_n)$, $\beta = (b_1, b_2, \_\_ b_n) \in V_n$, $\gamma = (c_1, c_2, \_\_ c_n)$ then we define

$$\langle\alpha,\beta\rangle = a_1\bar{b}_1 + a_2\bar{b}_2 + \_\_ + a_n\bar{b}_n \quad \text{—(1)}$$

Soln. To show Inner product space (1) should satisfy Linearity, conjugate & non-negative property.

Linearity: $\alpha,\beta,\gamma \in V_n(C)$

then $a\alpha+\beta = (aa_1+b_1, aa_2+b_2, \_\_ aa_n+b_n)$

$\langle a\alpha+\beta,\gamma\rangle = \langle(aa_1+b_1, aa_2+b_2, \_\_, aa_n+b_n), (c_1, c_2, c_3 \_\_ c_n)\rangle$

$= (aa_1+b_1)\bar{c}_1 + (aa_2+b_2)\bar{c}_2 + \_\_ + (aa_n+b_n)\bar{c}_n$

$= (aa_1\bar{c}_1 + aa_2\bar{c}_2 + \_\_ + aa_n\bar{c}_n) + (b_1\bar{c}_1 + b_2\bar{c}_2 + \_\_ + b_n\bar{c}_n)$

$= a(a_1\bar{c}_1 + \_\_ + a_n\bar{c}_n) + (b_1\bar{c}_1 + \_\_ + b_n\bar{c}_n)$

$= a\langle\alpha,\gamma\rangle + \langle\beta,\gamma\rangle$

Conjugate Symmetry:

$\langle\beta,\gamma\rangle = b_1\bar{a}_1 + b_2\bar{a}_2 + \_\_ + b_n\bar{a}_n$

$\overline{\langle\beta,\alpha\rangle} = \overline{(b_1\bar{a}_1)}, \overline{(b_2\bar{a}_2)} + \_\_ + \overline{(b_n\bar{a}_n)}$

$= \bar{b}_1 a_1 + \bar{b}_2 a_2 + \_\_ + \bar{b}_n a_n$

$= a_1\bar{b}_1 + a_2\bar{b}_2 + \_\_ + a_n\bar{b}_n$

$= \langle\alpha,\beta\rangle$.

Since, $|z|^2 = z\bar{z}$

$\overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2$

$\overline{(\bar{a}_1)} = a_1$, $\overline{z_1 z_2} = \bar{z}_1\bar{z}_2$

$z_1 z_2 = z_2 z_1$

Property used

Non Negativity property:

$\langle\alpha,\alpha\rangle = a_1\bar{a}_1 + a_2\bar{a}_2 + \_ + a_n\bar{a}_n$

$= |a_1|^2 + |a_2|^2 + \_\_ + |a_n|^2 > 0.$

if $\langle \alpha, \alpha \rangle = 0$

$|a_1|^2 + |a_2|^2 + \_\_ + |a_n|^2 = 0$

$\Rightarrow a_1 = a_2 = a_3 = \_\_ = a_n = 0.$

$\alpha = (0, 0, \_\_ 0).$

$\langle \alpha, \beta \rangle = a_1\bar{b}_1 + a_2\bar{b}_2 + \_\_ + a_n\bar{b}_n.$

Here, $V_n(C)$ is unitary space,

$V_n(R)$ is Euclidean space.

$\langle \alpha, \beta \rangle = a_1b_1 + a_2b_2 + \_\_ + a_nb_n.$

Ex. $V(R)$; $\alpha = (a_1, a_2) \in V$, $\beta = (b_1, b_2) \in V$, $\gamma = (c_1, c_2) \in V$.

$\langle \alpha, \beta \rangle = a_1b_1 - a_2b_2 - a_1b_2 + 4a_2b_1$

Check whether $V(R)$ is IPS or NOT.

Soln. $a\alpha + \beta = (aa_1 + b_1, aa_2 + b_2).$

$\langle a\alpha+\beta, \gamma \rangle = (aa_1+b_1)c_1 - (aa_2+b_2)c_2 - (aa_1+b_1)c_2 + 4(aa_2+b_2)c_1$

$\langle \beta, \alpha \rangle = b_1a_1 - b_2a_2 - b_1a_2 + 4b_2a_1 \neq \langle \alpha, \beta \rangle.$

$\Rightarrow$ Not having Symmetry property

Not Inner Product Space (IPS) $\Leftarrow$

Eg. Let $V(R)$ be a vector space of all continuous real valued function on the interval $[0,1]$ if $f, g \in V$ defined

$\langle f, g \rangle = \int_0^1 f(t)g(t)\,dt.$

Show that it is an inner product on V.

Soln. ① Linear

$f(t) \in V$, $g(t) \in V$, $h(t) \in V$ $\}$ $a \in F.$

$\langle af(t)+g(t), h(t) \rangle = \int_0^1 (af(t) + g(t))h(t)\,dt.$

$= \int_0^1 [af(t)h(t) + g(t)h(t)]\,dt$

$= \int_0^1 af(t)h(t)\,dt + \int_0^1 g(t)h(t)\,dt$

$= a\langle f(t), h(t) \rangle + \langle g(t), h(t) \rangle = a\langle f,h \rangle + \langle g, h \rangle.$

② Symmetry.

$$\langle f,g\rangle = \int_0^1 f(t)\,g(t)\,dt$$

$$= \int_0^1 g(t)\,f(t)\,dt$$

$f(t)\cdot g(t) = g(t)\cdot f(t)$ in case of Real valued functions we will ... same ...

$$= \langle g,f\rangle.$$

③ $\langle f,f\rangle = \int_0^1 f(t)\,f(t)\,dt$

$$= \int_0^1 f^2(t)\,dt.$$

since, $f^2(t) \geq 0$.

$\Rightarrow \langle f,f\rangle \geq 0$

for $\langle f,f\rangle = 0$.

$$\Rightarrow \int_0^1 f^2(t)\,dt = 0.$$

$$\Rightarrow f^2(t) = 0. \Rightarrow f(t) = 0$$

or $f = 0$.

Eg $\rightarrow$ $f(t) = t-1$ $\{$ under previous definition
$g(t) = t+2$

$$\langle f,g\rangle = \int_0^1 (t-1)(t+2)\,dt$$

$$= \int_0^1 (t^2+t-2)\,dt$$

$$= \left(\frac{1}{3}+\frac{1}{2}-2\right)-(0) = -\frac{7}{6}$$

⊛ Norm of a Vector :→ / Length of Vector

Let V be an inner product space. If $\alpha \in V$, then norm (or length of vector) is defined as

$$\|\alpha\| = \sqrt{\langle \alpha,\alpha\rangle}$$

$\| f(t) \| = \langle f, f \rangle = \int_0^1 (t-1)^2 dt = \frac{1}{3}$ {previous example extended}

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Normalization of vector.

$$\hat{\alpha} = \frac{1}{\|\alpha\|} \alpha$$

eg → $\hat{f} = \frac{1}{\|f\|} f = \cancel{\frac{1}{\sqrt{3}} f = \frac{1}{\sqrt{3}}} \sqrt{3}(t-1)$

$\|\hat{f}\| = \sqrt{(\hat{f}, \hat{f})}$ if this comes out 1. then → Normalization of vector ✓

eg → $V_2(R) = R^2 = \{ (a_1, a_2) ; a_1, a_2 \in R \}$

$\alpha = (a_1, a_2)$, $\beta = (b_1, b_2)$

(I) $\langle \alpha, \beta \rangle = a_1^2 + a_1 b_1 + a_2 b_2 + b_1^2$

(II) $\langle \alpha, \beta \rangle = a_1 a_2 + 2 b_1 b_2$

(III) $\langle \alpha, \beta \rangle = a_1 b_1 + 2 a_2 b_2 + 5$ ← 1 property DNH

(IV) $\langle \alpha, \beta \rangle = 2 a_1 b_1 + 3 a_2 b_2$ ✓

(V) $\langle \alpha, \beta \rangle = 2 a_1 b_1 - 3 a_2 b_2$ PDP

OPERATIONS. check which one of them IPS.

$\gamma = (c_1, c_2)$

Soln. ~~$\langle \alpha + \beta, \gamma \rangle = a$~~

(I) linearity property

$\alpha + \beta = (a_1 + b_1, a_2 + b_2)$

$(\alpha + \beta, \gamma) = \{ (a_1 + b_1)^2 + (a_1 + b_1) c_1 + (a_2 + b_2) c_2 + c_1^2$

&

$(\alpha, \gamma) + (\beta, \gamma) = a_1^2 + a_1 c_1 + a_2 c_2 + c_1^2 + b_1^2 + b_1 c_1 + b_2 c_2 + c_1^2$

$[(\alpha + \beta, \gamma)] - [(\alpha, \gamma) + (\beta, \gamma)] = \cancel{a_1^2} + 2 a_1 b_1 + \cancel{b_1^2} + \cancel{a_1 c_1} + \cancel{b_1 c_1} + \cancel{a_2 c_2} + \cancel{b_2 c_2} + \cancel{c_1^2}$
$- \cancel{a_1^2} - \cancel{a_1 c_1} - \cancel{a_2 c_2} - c_1^2 - \cancel{b_1^2} - \cancel{b_1 c_1} - \cancel{b_2 c_2} - \cancel{c_1^2}$

$= 2 a_1 b_1 - c_1^2 \neq 0$ ⇒ Not IPS under this operation

(II) $\langle \alpha, \beta \rangle = a_1 a_2 + 2 b_1 b_2$, $\langle \beta, \alpha \rangle = b_1 b_2 + 2 a_1 a_2$

since, $\langle \alpha, \beta \rangle \neq \langle \beta, \alpha \rangle$ ⇒ Not IPS.

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(III) $\langle \alpha, \beta \rangle = a_1 b_1 + 2a_2 b_2 + 5$.

Also Linear property DNH

~~$\langle \alpha, \alpha \rangle = a_1^2 + 2a_2^2 + 5$~~

$\langle \alpha, \alpha \rangle = a_1^2 + 2a_2^2 + 5$.

~~$\langle \alpha, \alpha \rangle = a_1^2 + \geq 5$.~~

Since $\langle \alpha, \alpha \rangle \geq 5$.

$\Rightarrow$ there is no α such that $\langle \alpha, \alpha \rangle = 0$.

Orthogonal vectors

eg, $\overset{\alpha}{(1,0)}, \overset{\beta}{(0,1)} \in \mathbb{R}^2$.

$\langle \alpha, \beta \rangle = \langle (1,0), (0,1) \rangle = 0$.

Let $V(F)$ be an inner product space and $\alpha, \beta \in V$, then α and β are said to be Orthogonal, if Inner product ~~of α & β under usual~~ using α & β $= 0$.

$\langle \alpha, \beta \rangle = 0$.

if not mentioned any operation $\Longrightarrow$ under usual standard operations on IP.

eg, $(0,0)$ is orthogonal to every element $\Rightarrow$ Self orthogonal.

$\langle (0,0), (a,b) \rangle = 0.a + 0.b = 0$.

Orthogonal set:

eg, $S_1 : \{(0,1), (1,0), (0,0)\} \Leftarrow$ Orthogonal set.

$S_2 : \{(0,1), (1,0), (1,1)\} \Leftarrow$ it is Not " — ".

as, $\langle (0,1), (1,1) \rangle \neq 0$. also $\langle (1,0), (1,1) \rangle \neq 0$.