Q2. $a_n = \dfrac{(-1)^n}{\sqrt{n}} + i\dfrac{1}{n^2}$

$$\frac{a_{n+1}}{a_n} = \frac{\sqrt{\frac{(-1)^{2n+2}}{n+1} + \frac{1}{(n+1)^4}}}{\sqrt{\frac{(-1)^{2n}}{n} + \frac{1}{n^4}}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\sqrt{\frac{((n+1)^3+1)\,n^4}{(n+1)^4\,(n^3+1)}}$$

$$\cancel{1 + \frac{1}{(n+1)^3}}$$

$$L = \lim_{n\to\infty}\sqrt{\frac{\left(1+\frac{1}{n+1}\right)}{\left(\frac{n+1}{n}\right)\left(1+\frac{1}{n^3}\right)}} = 1$$

Test Fails.

For $a_n = \dfrac{(-1)^n}{n} + i\dfrac{1}{n^2}$ ~~to be con~~

$\Sigma a_n \to$ convergent Conditionally.

⇓ Need to proof.

need to proof $\dfrac{(-1)^n}{\sqrt{n}} \to$ convergent

and $\dfrac{1}{n^2} \to$ convergent

For $\Sigma a_n \to$ Absolute convergence

⇓ Need to proof

$\dfrac{1}{\sqrt{n}} \longrightarrow$ converges $\qquad \dfrac{1}{n^2} \to$ converges

$\dfrac{1}{n^p}$ $\quad p>1$ convergent

$\quad p\le 1$ divergent thus $\dfrac{1}{\sqrt{n}}$ diverges $\to$ X Absolute

$\dfrac{1}{n} + \dfrac{1}{n^{2}}$

$$\frac{(n+1)^3 + (n+1)}{(n+1)^4}$$

$$\frac{n^3+1}{n^4}$$

$$\frac{n^3\left(1+\frac{1}{n}\right)^3 + \left(\frac{1}{n^2}+\frac{1}{n^3}\right)}{n^4\left(1+\frac{1}{n^4}\right)}$$

$n^3\,($

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For $\frac{(-1)^n}{a_n}$

By alternating series test

$\frac{1}{n}$ decreases $\lim_{n\to\infty} \frac{1}{n} = 0$

Thus converges

$\frac{1}{n^2} \rightarrow$ converges thus

converges but not absolutely.

Q3. $\left|\frac{a_n}{a_{n+1}}\right| = R$

a) $\lim_{n\to\infty} \left(\frac{\ln(n)}{\ln(n+1)}\right)^2 = \left(\frac{\frac{1}{n}}{\frac{1}{n+1}}\right)^2 = 1 = R$

b) $\lim_{n\to\infty} \left|\frac{n!}{(n+1)!}\right| = \frac{1}{n+1} = 0 = R.$

c) $\lim_{n\to\infty} \frac{n^2}{(n+1)^2} \frac{4^n + 3n}{4^{n+1} + 3(n+1)} = \frac{1}{(1+\frac{1}{n})^2} \frac{1 + \frac{3n}{4^n}}{4 + \frac{3(n+1)}{4^n}} = \frac{1}{4} = R$

d) $\lim_{n\to\infty} \frac{(n!)^3}{3n!} \times \frac{3(n+1)!}{((n+1)!)^3} = \frac{(3n+3)(3n+2)(3n+1)}{(n+1)^3} \quad 27 = R.$

e) $\lim_{n\to\infty} \left(\frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n^2+n}}\right) \frac{\sqrt{(n+1)^2 + (n+1)}}{\sqrt{n+2} - \sqrt{n+1}}$

$\lim_{n\to\infty} \left(\frac{\sqrt{n+2} + \sqrt{n+1}}{\sqrt{n+1} + \sqrt{n}}\right)\left(\sqrt{\frac{(n+1)(n+2)}{n(n+1)}}\right) = 1 = R$

04. $$\sum_{n=0}^{\infty}(n+1)^2 z^n = \frac{1+z}{(1-z)^3}$$

$$\cancel{\frac{1}{(1-z)^3}} \quad \cancel{\&} \quad \cancel{\frac{z}{(1-z)^3}}$$

~~N At~~ $\cancel{\sum_{n=1}^{\infty}((n+1)^2 z^n)}$

$$\cancel{\sum_{n=0}^{\infty}(n^2+2n+1) z^n}$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$$

$$\frac{1}{(1-z)^2} = \sum_{n=0}^{\infty} n z^{(n-1)}$$

$$\frac{2}{(1-z)^3} = \sum_{n=0}^{\infty} n(n-1) z^{n-2} = \sum_{n=0}^{\infty}(n+2)(n+1) z^n$$

$$\frac{2z}{(1-z)^3} = \sum_{n=0}^{\infty}(n+1)(n) z^n$$

$$\frac{2+2z}{1-z^3} = 2\sum_{n=0}^{\infty}(n+1)^2 z^n = \frac{1+z}{1-z^3}$$

05. a) $f(z) = \frac{1}{z^2}$

$$\frac{1}{a+z-a} = \frac{1}{a} \quad \frac{1}{1+\left(\frac{z-a}{a}\right)} \qquad \left|\frac{z-a}{a}\right| < 1$$

$$\frac{1}{z} = \frac{1}{a}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z-a}{a}\right)^n$$

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$$\frac{-1}{z^2} = \frac{1}{a}\sum_{n=0}^{\infty} \frac{(-1)^{n+1} n (z-a)^{n-1}}{a^n}$$

$$\frac{1}{z^2} = \sum_{n=0} \frac{(-1)^{n+1} n (z-a)^{n-1}}{a^n}$$

b) $f(z) = \dfrac{6z+8}{(2z+3)(4z+5)}$ at $z=1$.

$$= \frac{1}{2z+3} + \frac{1}{4z+5}$$

$$= \frac{1}{2z-2+5} + \frac{1}{4z-4+9}$$

$$= \frac{1}{5}\frac{1}{\frac{2}{5}(z-1)+1} + \frac{1}{9}\frac{1}{\frac{4}{9}(z-1)+1}$$

$$= \frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n(-1)^n(z-1)^n + \frac{1}{9}\sum_{n=0}^{\infty}\left(\frac{4}{9}\right)^n(-1)^n(z-1)^n$$

c) $f(z) = \dfrac{e^{z-1}\,e}{z+1}$ $\quad |z-1|<1$

$$= \cancel{\frac{1+1}{z}\left(1+z+\frac{z^2}{2!}+\frac{z^3}{3!}+\cdots\right)}\ \frac{1}{2+(z-1)}$$

$$= \left(\frac{1}{2}\frac{1}{1+(z-1)}\right)\left(1+z-1+\frac{(z-1)^2}{2!}+\frac{(z-1)^3}{3!}+\cdots\right)$$

$$= \frac{1}{2}\left(1+(z-1)+(z-1)^2+\cdots\right)\left(1+(z-1)+\frac{(z-1)^2}{2!}+\cdots\right)$$

Q6.

a) $f(z) = \dfrac{1}{z-3}$ $\qquad |z| > 3$

$\dfrac{1}{|z|} < \dfrac{1}{3} < 1$

$= \left(\dfrac{1}{z}\right)\left(\dfrac{1}{1-\left(\frac{3}{z}\right)}\right)$ $\qquad \left|\dfrac{3}{z}\right| < 1$

$= \dfrac{1}{z}\left(1 + \dfrac{3}{z} + \left(\dfrac{3}{z}\right)^2 + \text{---}\right)$

$= \dfrac{1}{z} + \dfrac{3}{z^2} + \dfrac{9}{z^3} + \text{---}$

b) $f(z) = \dfrac{1}{z(z-1)}$ $\qquad 0 < |z| < 1$

$= \dfrac{-1}{z}\left(1 + z + z^2 + \text{---}\right) = \dfrac{-1}{z} - 1 - z - z^2 + \text{---}$

c) $f(z) = z^3 e^{\frac{1}{z}}$ for $|z| > 0$

case 1. $\qquad 0 < |z| < 1$

$f(z) = z^3\left(1 + \dfrac{1}{z} + \dfrac{1}{z^2} + \text{---}\right)$

$= z^3 + z^2 + z + 1 + \dfrac{1}{z} + \text{---}$

Case 2 $\qquad |z| > 1 \qquad \left|\dfrac{1}{z}\right| < 1$

$f(z) = z^3 + z^2 + z + 1 + \dfrac{1}{z} + \text{---}$

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Q7. $f(z) = \dfrac{1}{z(1+z^2)}$ $\qquad |z| < 1$

$$\frac{1}{z}\sum_{n=0}^{\infty}(-1)^n z^{2n} = \sum_{n=0}^{\infty}(-1)^n z^{2n-1}$$

$|z| > 1$

$$\frac{1}{|z|^3}\left(\frac{1}{1+\frac{1}{|z|^2}}\right) \qquad \left|\frac{1}{z}\right| < 1$$

$$\frac{1}{|z|^3}\left(1+\frac{(-1)}{|z|^2}+\frac{1}{|z|^4}+\frac{(-1)}{|z|^6}+\cdots\right)$$

$$\frac{1}{|z|^3}-\frac{1}{|z|^5}+\frac{1}{|z|^7}-\frac{1}{|z|^9}+\cdots$$

Q8. $|f(z)| - |g(z)| < 0$

$|g(z)| - |f(z)| > 0$

$\cancel{|g(z)|}$ $\quad \cancel{|g(z) - f(z)| > |g(z)| - |f(z)| \geq 0}$

$\cancel{|f(z) - g(z)| > 0}$

$\left|\dfrac{f(z)}{g(z)}\right| < 1$ $\quad$ by Liouville Theorem

$|f'(z)| < |f(z)|$ $\qquad \left|\dfrac{f(z)}{g(z)}\right|$ is constant say λ

$f(z) = \lambda g(z)$

$f'(z) = k f(z)$ $\qquad \ln f(z) = k + C$

$f(z) = e^{(k+c)z} = e^{pz}$.

Q9.

a) $ze^{1/z}$ $\quad z=0$ Essential Singular Point.

b) $\frac{\sin z}{z}$ $\quad z=0$ Removable Singular Point.

c) $\frac{1-\cos z}{z^2}$ $\quad z=0$ Removable Singular Point.

d) $\frac{\pi \cot \pi z}{z^2}$

$$\pi \frac{1}{z^2}\left(\frac{1}{z} - \frac{z}{3} + \frac{z^3}{245} + \cdots\right)$$

Pole Order 2 ~~Principal part has finite no. of terms~~ ~~Thus removable singularity.~~

If finite terms (b_n) then pole of order = no of terms.

e) $\frac{z-\sin(z-1)}{z-1}$

$$= \frac{z-\left((z-1) - \frac{(z-1)^3}{3!} + \cdots\right)}{z-1}$$

$$= \frac{1 + \frac{(z-1)^3}{3!} - \frac{(z-1)^5}{5!} + \cdots}{(z-1)}$$

Non Removable

~~Essential Singularity.~~ Pole of Order 1.

f) $\frac{z^2+\sin z}{\cos(z) - 1} = \frac{z^2 + z - \frac{z^3}{3!} + \frac{z^5}{5!} + \cdots}{\cancel{1} - \frac{z^2}{2!} + \frac{z^4}{4!} + \cdots - \cancel{1}}$

Essential singularity.

Q10. a) $\frac{\sin z}{z^2-\pi^2}$ $\quad z = \pm\pi$ Removable Singularity.

$$\frac{\sin(\pi - z)}{-(z+\pi)(z-\pi)} \qquad \frac{\sin(z+\pi)}{(z-\pi)(z+\pi)}$$

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b) $z=\pi$ is pole of order 1.

c) $\frac{z\cos z}{1-\sin z}$ $z=\pi/2$

$\frac{z^2\sin(\pi/2-z)}{(1-\cos z)\,z}$ Removable Singularity

Q11.

a) $\frac{1}{z(z+1)} = \frac{1}{z}\left(1-z+z^2-z^3+ \ldots\right)$

$b_1 = 1$

b) $z\cos\frac{1}{z} = z\left(1-\left(\frac{1}{z}\right)^2\frac{1}{2!}+\left(\frac{1}{z}\right)^4\frac{1}{4!}+ \ldots\right)$

$= z - \frac{1}{2z} + \frac{1}{4!z^3} + \ldots$

$b_1 = \frac{-1}{2}$

c) $\frac{z-\sin z}{z} = \frac{z-\left(z-\frac{z^3}{3!}+\frac{z^5}{5!}+ \ldots\right)}{z}$

$= \frac{\frac{z^3}{3!}-\frac{z^5}{5!}+ \ldots}{z} = \frac{z^2}{3!}-\frac{z^4}{5!}+ \ldots$

$b_1 = 0$

d) $\frac{\cot z}{z^4} = \frac{1}{z} - \frac{z^3}{3} + \frac{z^3}{245} + \frac{z^5}{(\quad)} + \ldots$

$b_1 = \frac{+1}{245}$

Q2.

a) $\dfrac{e^{-z}}{z^2}$ $z=0$

$\frac{1}{z^2}\left(1 - z + \frac{z^2}{2!} + \text{---}\right)$

$b_1 = \frac{-1}{2}$

$2\pi i\left(\frac{1}{2}\right) = \pi i$

b) $\dfrac{e^{-z+1}}{(z-1)^2}$ $\frac{1}{e}$

$\frac{1}{e}\frac{1}{(z-1)^2}\left(1 - (z-1) + \frac{(z-1)^2}{2!} + \text{---}\right)$

πi

c) $z^2 e^{\frac{1}{z}}$

$z^2\left(1 + \frac{1}{z} + \frac{1}{2z^2} + \text{---}\right)$

$2\pi i/6$

e) $\dfrac{e^z}{(z^2-1)}$ ~~similarly order 1 for z+1 thus~~

$\dfrac{e^z}{(z-1)(z+1)}$

order 1. $\dfrac{e^{z-1}}{(z-1)(z+1)}$ e $\frac{e}{z+1}\left(\frac{1}{z-1} + \frac{(z-1)}{z-1} + \frac{(z-1)^2}{2!} + \text{---}\right)$

$Res f(z) =$ $\cancel{\frac{1}{(m-1)!} \lim_{z\to z_0} \frac{d^{m-1}}{dz^{m-1}}\left((z-z_0)^m f(z)\right)}$

$\frac{e}{2}$ $\frac{-e}{2}$

$\boxed{0}$

$\frac{1}{2} = e^{-i\theta} =$

d) $\dfrac{z+1}{z^2-2z}$ $z=0$ $z=2$

$\underset{z=0}{\text{Res}}\, f(z) = \cancel{-1/2}\ \dfrac{-1}{2}\ \cancel{z+1}$ $\underset{z=1}{\text{Res}}\, f(z) = \cancel{1/4}\ \dfrac{3}{2}$

$\dfrac{r}{z}$ $\underset{z=0}{\dfrac{d}{dz}} \cancel{\dfrac{z+1}{z-2}}$ $\cancel{\dfrac{(z-2)-(z+1)}{(z-2)^2}}$ $= \cancel{\dfrac{-3}{(z-2)^2}}$

$\dfrac{z+1}{2z-2}$ $z=0$ $\cancel{= \dfrac{-3}{4}}$

$\underset{z=2}{\dfrac{d}{dz}}$ $\cancel{\dfrac{z+1}{z}}$ $\cancel{=}$ $\cancel{\dfrac{1}{4}}$ $\cancel{\dfrac{-1}{z^2}} = \dfrac{-1}{4}$

$+2\pi i$

e) $\dfrac{\pi \cot \pi z}{\left(z+\frac{1}{2}\right)^2}$

$\cancel{\dfrac{d^2\, \pi \cot \pi z}{dz^2}}$ $\cancel{\dfrac{d}{dz} = \pi^2 \operatorname{cosec}^2 \pi z}$ $m=2$

$-\pi^2 \operatorname{cosec}^2 \pi z$ $-\pi^3\, 2\operatorname{cosec}^2 \pi z \cot \pi z$

$-\pi^2$

$+ \cancel{4}\pi i \left(\cancel{\operatorname{cosec}^2 2\pi} \cot \cancel{2\pi}\right)$

$2\pi i(-\pi^2) = -2\pi^3 i$

Q13.

a) $\int_0^\infty \frac{2x^2-1}{x^4+5x^2+4}\,dx$

Roots should be imaginary.

Application of Cauchy Residue theorem

$\cancel{\frac{(2x^2-1)\,dx}{x^4+4x^2+x^2+4}}$

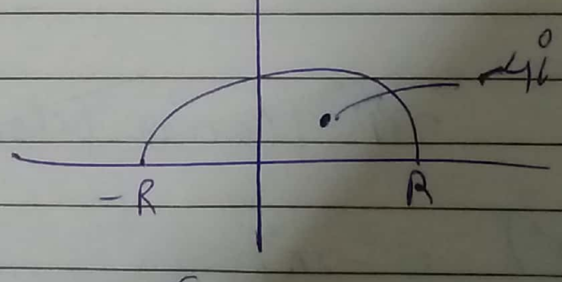
$\cancel{\frac{(2x^2-1)\,dx}{x^2(x^2+4)+1(x^2+4)}} = \cancel{\frac{(2x^2-1)\,dx}{(x^2+4)^2}}$

$\cancel{\frac{d(2z^2-1)\,dz}{dz}} = \cancel{4z} = \cancel{16i}$

$\frac{2x^2-1}{x^4+5x^2+4} \Rightarrow \frac{2-\frac{1}{x^2}}{x^2+5+\frac{4}{x^2}}$

$= \frac{2x}{x^2}\;\frac{(2x^2-1)}{(x^2+4)^2}$

$f(z) < \frac{K}{|z|^2}$ Numerator 2 power less than denominator



$\lim_{R\to\infty}\int_{-R}^{\infty} f(z)\,dz + \lim_{R\to\infty}\int_{C_R} f(z)\,dx = 2\pi i\sum \text{Res } f(z)$

$\lim_{R\to\infty}\int_0^\infty f(z)\,dz = \pi i \operatorname{Res}_{z=2i} f(z) + \pi i \operatorname{Res}_{z=i} f(z)$

$$\frac{d}{dx^2}\frac{(2x^2-1)(x^2+4)^2}{(x^2+x)^2} = 4$$

$$\boxed{4\pi i}$$

$$\frac{2x^2-1}{(x^2+1)(x^2+4)}$$

$$\frac{d}{dx}\frac{2x^2-1}{x^2+1} = \frac{4x(x^2+1)-(2x^2-1)2x}{(x^2+1)^2}$$

$x = 2i$

$$= \frac{8i(-3)+18i\times 2}{9}$$

$$= \frac{(-24+36)i}{9} = \frac{6i}{9} = -\frac{4i}{3}$$

$$\frac{2x^2-1}{x^2+4} = \frac{4x(x^2+4)-(2x^2-1)(2x)}{(x^2+4)^2}$$

$$= \cancel{4i}\ \frac{4i(3)+6i}{9} = \frac{18i}{9} = 2i$$

$$\frac{2x^2-1}{4x^3+10x}$$

$$\frac{-9}{-32i+20i} = \frac{9}{4i} = \frac{-3i}{4}$$

$$\frac{-3}{-4i+10i} = \frac{-3}{6i} = +\frac{i}{2}$$

$$\frac{1}{4}i\times\pi i = \boxed{-\pi/4}$$

Q13 b)

$$\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$$

x^4

$x^4+1 \longrightarrow \text{Roots} \longrightarrow \sqrt{i}, -\sqrt{i}, \sqrt{-i}$

4π

$(-1)^{1/4} = (i)^{1/2}$

2π

$= e$

$(e^{-\pi i})^{1/4} = e^{((-\pi+2k\pi)i)\frac{1}{4}}$

$e^{-i\pi/4} \quad e^{i\pi/4} \quad e^{i3\pi/4} \quad e^{i5\pi/4}$

$\cos(-\pi/4) + i\sin(-\pi/4) = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ ✓

$2\pi - \pi/4$

$\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ ✓

$\frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$ $\quad 2\pi + \pi/4$

$\frac{1}{4x^3} = \cancel{\frac{1}{\sqrt{2}\,4(-i)}}$ $\cancel{2\sqrt{2}}$

$\frac{-i}{2\sqrt{2}} \quad \pi i$

$\boxed{\frac{\pi}{2\sqrt{2}}}$

$\cancel{\frac{1}{\sqrt{2}\,4(1+i)^3}}$

$\cancel{\frac{2\sqrt{2}}{\sqrt{2}\,4(-1+i)^3}}$

$\frac{\frac{\sqrt{2}\,2i}{\sqrt{2}}}{4\left(\frac{i}{2} - \frac{1}{2}\right)}$

$\frac{1}{4e^{3i\pi/4}} \quad \frac{1}{4e^{i\pi/4}}$

$\frac{1}{4\left(\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)} \quad \frac{1}{4\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)}$

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Q14

a) $\displaystyle\int_0^\infty \frac{\cos ax}{x^2+1}\,dx$

Let us consider $x \to z$

$\cancel{\lim_{R\to\infty} \int_{-R}^{R} \frac{e^{iaz}}{z^2+1}\,dz}$

$\cancel{z = \pm i \to z = i}$

$\cancel{\frac{d}{dz}\,\frac{e^{iaz}}{z+i}}$

$\cancel{= \frac{iae^{iaz}(z+i) - e^{iaz}}{(z+i)^2}}$

$\cancel{\frac{+2a}{e^a 4} + \frac{1}{4ae^a} = \frac{a}{2e^a} + \frac{1}{4e^a}}$

~~Any (w) ...~~

$\frac{p(x)}{q'(x)} = \frac{e^{-a}}{2z} \qquad z = i$

$\frac{-ie^{-a}}{2} \times \pi i = \boxed{\frac{\pi e^{-a}}{2}}$

b) $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)}$

$x = bi \qquad \frac{e^{ix}}{\cancel{2x(x^2+b^2)} + 2x(x^2+a^2)} = \frac{e^{-b}}{2bi(a^2-b^2)}$

$= \frac{e^{-a}}{2ai(b^2-a^2)} \Rightarrow \frac{\pi}{a^2-b^2}\left(\frac{e^{-b}}{b} \quad \frac{e^{-a}}{a}\right)$

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Q15.

a) $$\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$$

Answer should be 0

$a > |b|$

$\frac{a}{b} > 1$

$$\sin\theta = \frac{z - \frac{1}{z}}{2i} \qquad z = e^{i\theta}$$

$dz = i e^{i\theta} d\theta$

$$\int_0^{2\pi} \frac{dz}{iz\left(a + \frac{b}{2i}\left(z + \frac{1}{z}\right)\right)}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2}$$

$$\int_0^{2\pi} \frac{dz}{iz\left(a + \frac{b}{2i}\left(z - \frac{1}{z}\right)\right)} = \int_0^{2\pi} \frac{2dz}{z\left(2a + bz - \frac{b}{z}\right)}$$

$$= \int_0^{2\pi} \frac{2dz}{(2az + bz^2 - b)}$$

$bz^2 + 2azi - b$

$bz^2 + azi + azi - b$

$$\frac{-2a \pm \sqrt{4a^2 - 4b^2}}{2b} \qquad \frac{\sqrt{4a^2 - 4b^2} - \sqrt{4a^2}}{4b}$$

$2bz + 2a$

$$\frac{\pm\sqrt{a^2 - b^2} - \sqrt{a^2}}{2b}$$

$2\sqrt{a^2+b^2} \quad -2a + 2a$

$\sqrt{a^2 - b^2} + a$

$$\boxed{\frac{2\pi}{\sqrt{a^2-b^2}}}$$

Q15. b)

$$\int_0^{2\pi} \frac{dz}{zi\left(3-\frac{2}{2}\left(z+\frac{1}{z}\right)+\frac{1}{2i}\left(z-\frac{1}{z}\right)\right)}$$

$$\frac{dz}{3zi - zi\left(z+\frac{1}{z}\right)+\frac{z}{2}\left(z-\frac{1}{z}\right)}$$

$$3zi - z^2 i - i + \frac{z^2}{2} - \frac{1}{2}$$

$$\left(\frac{1}{2}-i\right)z^2 + 3zi - \left(\frac{1}{2}+i\right)$$

$$\frac{-3i \pm \sqrt{9(-1)+4\left(\frac{1}{4}+1\right)}}{2\left(\frac{1}{2}-i\right)} = z.$$

$-9 \cdot 3$

$$\frac{-3\pm 2i}{2-2i} \qquad \cancel{\frac{-3\pm i\sqrt{2}}{1-2i}} = \cancel{\frac{-3+2\sqrt{3}i}{1-2i}}$$

$$(1-2i)z + 3i \qquad -\cancel{3i}+2i+\cancel{3i}$$

$$\cancel{+3+2\sqrt{3}i+3i} \qquad \frac{2\pi i}{2i} = \boxed{\pi}$$