The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 8

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1. Verify that $y = x^2 \sin x$ and $y = 0$ both are solutions of the initial value problem:
$$x^2y'' - 4xy' + (x^2 + 6)y = 0, \quad y(0) = y'(0) = 0.$$
Does it contradict the uniqueness?

2. Find the curve $y = y(x)$ passing through origin for which $y'' = y'$ and the line $y = x$ is tangent at the origin.

3. Find the differential equation satisfied by each of the following two-parameter families of plane curves:
$$(i)\ y = \cos(ax + b) \qquad (ii)\ y = ax + \frac{b}{x} \qquad (iii)\ y = ae^x + bxe^x$$

4. (a) Find the values of m such that $y = e^{mx}$ is a solution of
$$(i)\ y'' + 3y' + 2y = 0 \qquad (ii)\ y'' - 4y' + 4y = 0 \qquad (iii)\ y''' - 2y'' - y' + 2y = 0.$$
(b) Find the values of m such that $y = x^m$ $(x > 0)$ is a solution of
$$(i)\ x^2y'' - 4xy' + 4y = 0 \qquad (ii)\ x^2y'' - 3xy' - 5y = 0.$$

5. Let $p(x)$, $q(x)$, $r(x)$ are continuous functions on the interval I. Further, suppose $y_1(x)$, $y_2(x)$ are any two solutions of the linear non-homogeneous equation
$$y'' + p(x)y' + q(x)y = r(x), \qquad x \in I. \tag{1}$$
Obtain conditions on the constants a and b such that $ay_1 + by_2$ is also its solution.

6. If $p(x)$, $q(x)$ are continuous functions on the interval I, then Show that $y = x$ and $y = \sin x$ are not solutions of the linear homogeneous equation
$$y'' + p(x)y' + q(x)y = 0, \qquad x \in I. \tag{2}$$

7. (a) Let $y_1(x)$, $y_2(x)$ be two linearly independent C^2 functions on the interval I, such that the wronskian $W(y_1, y_2)$ is not zero at any point on I. Show that there exists unique $p(x)$, $q(x)$ on I such that (2) has y_1, y_2 as fundamental solutions.
(b) Construct equations of the form (2), from the pairs of linearly independent solutions:
$$(i)\ e^{-x},\ xe^{-x} \qquad (ii)\ e^{-x}\sin 2x,\ e^{-x}\cos 2x$$

8. Show that a solution to (2) with x-axis as tangent at any point in I must be identically zero on I.

9. Let $y_1(x)$, $y_2(x)$ are two linearly independent solutions of (2). Show that
   (i) between consecutive zeros of y_1, there exists a unique zero of y_2.
   (ii) $\phi(x) = \alpha y_1(x) + \beta y_2(x)$ and $\psi(x) = \gamma y_1(x) + \delta y_2(x)$ are two linearly independent solutions <u>iff</u> $\alpha\delta \neq \beta\gamma$.

10. Let $y_1(x)$, $y_2(x)$ are two solutions of (2) with a common zero at any point in I. Show that y_1, y_2 are linearly dependent on I.