Binomial Theorem $(x+y)^n = \sum_{k=0}^{n} {}^nC_k\, x^k y^{n-k}$

Kolmogorov's Axiom

1. $0 \le P(E) \le 1$
2. $P(S) = 1$
3. $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

Exponential Random Variable:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Partitioning n obj. into r cells $n_1, n_2 \dots n_r$ $= \dfrac{n!}{n_1! \times n_2! \dots n_r!}$

Matching formula $1 - \frac{1}{2!} + \frac{1}{3!} + \dots (-1)^{N+1}\frac{1}{N!}$

Dearrangement $1 - \left(1 - \frac{1}{2!} + \frac{1}{3!} + \dots\right)$

Baye's Theorem

$$P(B_k|A) = \frac{P(B_k)\,P(A|B_k)}{\sum_{i=1}^{n} P(B_k)\,P(A|B_k)}$$

Probability Mass Function: is a function formula or table which gives the value of probability $P(f=a)$ for each element a in the range set of f.

Cummulative Distribution Function: $F(a) = \sum_{\text{all } x \le a} P(x)$

Expected Value or Expectation:

$$E[x] = \sum_{x: P(x)>0} x P(x)$$

Property: $E[ax+b] = aE[x] + b$

$E[x^n] = \sum_{P(x)>0} x^n P(x)$

$E[g(x)] = \sum_i g(x_i) P(x_i)$

$\Rightarrow Var(x) = E(x^2) - (E(x))^2$

$\Rightarrow Var(ax+b) = a^2 Var(x)$

Binomial Random Variable: $P(i) = {}^nC_i\, p^i (1-p)^{n-i}$

$E[x] = np$; $Var(x) = E[x^2] - (E(x))^2 = np(1-p)$

Poission Random Variable: $P\{x=i\} = \frac{\lambda^i e^{-\lambda}}{i!}$ pmf for Poission Random Variable

$\lambda = np$ $n \to \infty$.

$E[x] = \lambda = np$

Variance $E[x^2] = \lambda(\lambda+1)$

Geometric Random Variable: $P\{x=n\} = (1-p)^{n-1}p$

$E[x] = 1/p$

$Var[x] = \frac{1-p}{p^2}$

Continuous Random Variable:

$P\{x = B\} = \int_B f(x)dx$ $f(x)$: Pdf

Cumulative Distribution Function: $F(a) = P\{x \le a\} = \int_{-\infty}^{a} f(x)dx$

$\boxed{F'(a) = f(a)}$

Uniform Random Variable: $f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$ $E[x] = \frac{\beta+\alpha}{2}$ $Var[x] = \frac{(\beta-\alpha)^2}{12}$