Assignment-6.

Q.

a) $yu_{xx} - xu_{yy} = 0.$ $\quad y^2 + x^2 \neq 0$

$-4(-x)(y) = 4xy$ $\quad x > 0$, $y > 0$.

Hyperbola

both can't be 0 simultaneously. Parabola. $\quad x = 0$ or $y = 0$

Ellipse. $\quad x < 0, y > 0$ or $x > 0, y < 0$

b) $u_{yy} - xu_{xy} + yu_x + xu_y = 0$

$A = 1 \quad B = -x \quad C = y.$

$1 + x^2 + y^2 \neq 0.$

$x^2 - 4y$

$x^2 > 4y \rightarrow$ Hyperbola

$x^2 = 4y \rightarrow$ parabola

$x^2 < 4y \rightarrow$ Ellipse.

c) $y^2 u_{xx} + 2xyu_{xy} + x^2 u_{yy} = 0.$

$A = y^2 \quad B = x^2 \quad C = 2xy.$

$(x+y)^2 \neq 0 \quad x \neq y.$

$x^4 - 8xy^3$

$x^4 \cdot x^3 > 8y^3$

Hyperbola $\quad x > 2y$

Parabola. $\quad x = 2y$

Ellipse. $\quad x < 2y.$

d) $A = 1 \quad B = 2x \quad C = 1 - y^2$ $\quad 1 + 4x^2 + (1-y^2)^2 \neq 0$

$4x^2 - 4(1-y^2)$

Hyperbola $\quad x^2 = 1 + y^2 > 0$

Parabola $\quad x^2 + y^2 = 1$

Ellipse $\quad x^2 + y^2 < 1$

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Q2.

a) $u_{xx} - x^2 y\, u_{yy} = 0$

$A = 1$

$B = 0$ $\qquad \lambda^2 - x^2 y = 0 \Rightarrow \lambda = \pm x\sqrt{y}$

$C = -x^2 y$ $\qquad \frac{dy}{dx} + x\sqrt{y} = 0 \qquad \frac{dy}{dx} - x\sqrt{y} = 0$

$\frac{dy}{\sqrt{y}} = -x\,dx \Rightarrow 2\sqrt{y} - \frac{x^2}{2} = \xi \qquad 2\sqrt{y} + \frac{x^2}{2} = \eta$

$u_x = u_\eta \eta_x + u_\xi \xi_x \qquad \eta_x = x \quad \eta_y = \frac{1}{\sqrt{y}}$

$\xi_x = -x \quad \xi_{xx} = -1$

$\xi_y = \frac{1}{\sqrt{y}} \quad \xi_{yy} = \frac{-1}{2\sqrt{y^3}}$

$\eta_{yy} = \frac{-1}{2\sqrt{y^3}} \quad \eta_{xx} = 1$

$u_{xx} = \eta_x\left(u_{\eta\eta}\eta_x + u_{\eta\xi}\xi_x\right) + u_\eta \eta_{xx} + \xi_x\left(u_{\xi\xi}\xi_x + u_{\xi\eta}\eta_x\right) + u_\xi \xi_{xx}$

$= x\left(x u_{\eta\eta} + u_{\eta\xi}(-x)\right) + u_\eta - x\left(u_{\xi\xi}(-x) + u_{\xi\eta} x\right) - u_\xi$

$u_y = u_\eta \eta_y + u_\xi \xi_y$

$u_{yy} = \eta_y\left(u_{\eta\eta}\eta_y + u_{\eta\xi}\xi_y\right) + u_\eta \eta_{yy} + \xi_y\left(u_{\xi\xi}\xi_y + u_{\xi\eta}\eta_y\right) + \xi_{yy} u_\xi$

$= \frac{1}{\sqrt{y}}\left(\frac{u_{\eta\eta}}{\sqrt{y}} + \frac{u_{\eta\xi}}{\sqrt{y}}\right) - \frac{u_\eta}{2\sqrt{y^3}} + \frac{1}{\sqrt{y}}\left(\frac{u_{\xi\xi}}{\sqrt{y}} + \frac{u_{\xi\eta}}{\sqrt{y}}\right) - \frac{u_\xi}{2\sqrt{y^3}}$

$x^2 y u_{yy} = \frac{x^2 y u_{\eta\eta}}{y} + \frac{x^2 y u_{\eta\xi}}{y} - \frac{x^2 y u_\eta}{2\sqrt{y}\,y} + \frac{x^2 y u_{\xi\xi}}{y} + \frac{x^2 y u_{\xi\eta}}{y} - \frac{x^2 y u_\xi}{2y\sqrt{y}}$

$= \cancel{x^2 u_{\eta\eta}} + x^2 u_{\eta\xi} - \frac{x^2 u_\eta}{2\sqrt{y}} + \cancel{x^2 u_{\xi\xi}} + x^2 u_{\xi\eta} - \frac{x^2 u_\xi}{2\sqrt{y}}$

$\cancel{x^2 u_{\eta\eta}} - x^2 u_{\eta\xi} + u_\eta + \cancel{x^2 u_{\xi\xi}} - x^2 u_{\xi\eta} - u_\xi$

$-2x^2 u_{\eta\xi} + u_\eta + \frac{x^2 u_\eta}{2\sqrt{y}} - 2x^2 u_{\xi\eta} - u_\xi + \frac{x^2 u_\xi}{2\sqrt{y}} = 0$

11a) $4x^2 u_{\eta\xi} = u_\eta + \frac{x^2 u_\eta}{2\sqrt{y}} - u_\xi + \frac{x^2 u_\xi}{2\sqrt{y}}$

$\xi + \eta = 4\sqrt{y}$

$\eta - \xi = x^2$

$4(\eta-\xi) u_{\eta\xi} = u_\eta + \frac{(\eta-\xi) u_\eta}{2\frac{(\eta+\xi)}{4}} - u_\xi + \frac{(\eta-\xi) u_\xi}{2\frac{(\eta+\xi)}{4}}$

$u_{\eta\xi} = \frac{u_\eta}{4(\eta-\xi)} + \frac{u_\eta}{2(\eta+\xi)} - \frac{u_\xi}{4(\eta-\xi)} + \frac{u_\xi}{2(\eta+\xi)}$

$\frac{(\eta+\xi+2\eta-2\xi) u_\eta}{4(\eta-\xi)(\eta+\xi)} + u_\xi \left(\frac{-\eta-\xi+2\eta-2\xi}{4(\eta-\xi)(\eta+\xi)} \right)$

$\frac{(3\eta-\xi) u_\eta}{4(\eta-\xi)(\eta+\xi)} - \frac{(\eta-3\xi) u_\xi}{4(\eta-\xi)(\eta+\xi)}$

b) $e^{2x} u_{xx} + 2e^{x+y} u_{xy} + e^{2y} u_{yy} = 0$

$A = e^{2x} \quad B = 2e^{x+y} \quad C = e^{2y}$

$4e^{2x+2y} - 4e^{2x+2y} = 0.$

$e^{2x}\lambda^2 + 2e^{x+y}\lambda + e^{2y} = 0.$

$(e^x\lambda + e^y)^2 = 0$

$\lambda = -e^{y-x} \qquad \frac{dy}{dx} + e^{y-x} = 0$

$\frac{dy}{e^y} = \frac{-dx}{e^x} \Rightarrow \frac{e^{-y}}{-1} = e^{-x} + C_1$

$C_1 = e^{-x} + e^{-y} \qquad C_2 = e^{-x} - e^{-y}$

$$\xi = e^{-x} + e^{-y} \qquad \eta = e^{-x} - e^{-y}$$
$$\xi_x = -e^{-x} \qquad \eta_x = -e^{-x}$$
$$\xi_{xx} = e^{-x} \qquad \eta_{xx} = e^{-x}$$
$$\xi_y = -e^{-y} \qquad \eta_y = e^{-y}$$
$$\xi_{yy} = e^{-y} \qquad \eta_{yy} = -e^{-y}$$
$$\xi_{xy} = 0 \qquad \eta_{xy} = 0$$

$$u_x = u_\eta \eta_x + u_\xi \xi_x$$
$$u_{xx} = \eta_x\left(u_{\eta\eta}\eta_x + u_{\eta\xi}\xi_x\right) + u_\eta \eta_{xx} + \xi_x\left(u_{\xi\eta}\eta_x + u_{\xi\xi}\xi_x\right) + u_\xi \xi_{xx}$$
$$= -e^{-x}\left(u_{\eta\eta}(-e^{-x}) + u_{\eta\xi}(-e^{-x})\right) + u_\eta e^{-x} + (-e^{-x})\left(u_{\xi\eta}(-e^{-x}) + u_{\xi\xi}(-e^{-x})\right)$$
$$+ u_\xi e^{-x}$$

$$u_{xx} = e^{-2x}u_{\eta\eta} + e^{-2x}u_{\eta\xi} + e^{-x}u_\eta + e^{-2x}u_{\xi\eta} + e^{-2x}u_{\xi\xi} + e^{-x}u_\xi$$

$$u_y = u_\eta \eta_y + u_\xi \xi_y$$

$$u_{yy} = \eta_y\left(u_{\eta\eta}\eta_y + u_{\eta\xi}\xi_y\right) + u_\eta \eta_{yy} + \xi_y\left(u_{\xi\xi}\xi_y + u_{\xi\eta}\eta_y\right)$$
$$+ \xi_{yy}u_\xi$$
$$= e^{-y}\left(u_{\eta\eta}e^{-y} + (-e^{-y})u_{\eta\xi}\right) + (-e^{-y})u_\eta + (-e^{-y})\left((-e^{-y})u_{\xi\xi} + (e^{-y})u_{\xi\eta}\right)$$
$$+ e^{-y}u_\xi$$
$$= e^{-2y}u_{\eta\eta} - e^{-2y}u_{\eta\xi} - e^{-y}u_\eta + e^{-2y}u_{\xi\xi} - e^{-2y}u_{\xi\eta} + e^{-y}u_\xi$$
$$e^{2y}u_{yy} = u_{\eta\eta} - u_{\eta\xi} - e^{y}u_\eta + u_{\xi\xi} - u_{\xi\eta} + e^{y}u_\xi$$

$$u_x = u_\eta \eta_x + u_\xi \xi_x$$
$$u_{xy} = \eta_x\left(u_{\eta\eta}\eta_y + u_{\eta\xi}\xi_y\right) + \cancel{u_\eta \eta_{xy}}^{\,0} + \xi_x\left(u_{\xi\eta}\eta_y + u_{\xi\xi}\xi_y\right) + \cancel{u_\xi \xi_{xy}}$$
$$= (-e^{-x})\left(u_{\eta\eta}e^{-y} + (-e^{-y})u_{\eta\xi}\right) - e^{-x}\left(e^{-y}u_{\xi\eta} - e^{-y}u_{\xi\xi}\right)$$
$$u_{xy} = -e^{-x-y}u_{\eta\eta} + e^{-x-y}u_{\eta\xi} - e^{-x-y}u_{\xi\eta} + e^{-x-y}u_{\xi\xi}$$

$$e^{2x}u_{xx} = u_{\eta\eta} + u_{\eta\xi} + e^{x}u_\eta + u_{\xi\eta} + u_{\xi\xi} + e^{x}u_\xi$$
$$2e^{x+y}u_{xy} = -2u_{\eta\eta} + 2u_{\eta\xi} - 2u_{\xi\eta} + 2u_{\xi\xi}$$

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~~$u_{\eta\xi} + e^{x}u_\eta + u_{\xi\eta} + u_{\xi\xi} + e^{x}u_\xi + 2e^{x+y}u_{xy} - 2u_{\eta\eta} + 2u_{\eta\xi}$~~

~~$-2u_{\xi\eta} + 2u_{\xi\xi} + u_{\eta\eta} - u_{\eta\xi} - e^{y}u_\eta + u_{\xi\xi} - u_{\xi\eta} + e^{y}u_\xi = 0$~~

$u_{\eta\xi}(1+1+2-2-1-1)$

~~$e^{x}u_\eta + 4u_{\xi\xi} + e^{x}u_\xi$~~

$$e^{x}u_\eta + 4u_{\xi\xi} + e^{x}u_\xi - e^{y}u_\eta + e^{y}u_\xi = 0.$$

$$4u_{\xi\xi} = \cancel{-2e^{y}u_\xi} - e^{x}u_\xi - e^{y}u_\xi + e^{y}u_\eta - e^{x}u_\eta$$

$$u_{\xi\xi} = u_\xi\frac{(-e^{x}-e^{y})}{4} + u_\eta\frac{(e^{y}-e^{x})}{4}$$

c) ~~$u_{xx} - e^{x} y^2 u_{xx} + 2xy u_{xy} + x^2 u_{yy} = 0$~~

$x^2 u_{xx} + y^2 u_{yy} = 0$

$A = x^2$; $B = 0$; $C = y^2$.

$x^2 + y^2 \neq 0$.

$-4x^2y^2 < 0$ Elliptic PDE

$\lambda^2 x^2 + y^2 = 0$ $\lambda = \pm \frac{iy}{x}$

$\frac{dy}{dx} + \frac{iy}{x} = 0$ $\frac{dy}{dx} - \frac{iy}{x} = 0$

$\frac{-dy}{iy} = \frac{dx}{x}$ $\frac{dy}{iy} = \frac{dx}{x}$

$i\frac{\ln y}{1} - \frac{\ln x}{1} = C_1$ $-i\frac{\ln y}{1} - \ln x = C_2$

$\beta = C_1 + C_2 = -2\ln x$ $\frac{C_1 - C_2}{i} = 2\ln y. = \gamma$

$\beta = -2\ln x \qquad \gamma = 2\ln y.$

$$u_x = u_\beta \beta_x + u_\gamma \gamma_x$$

$$u_{xx} = \beta_x(u_{\beta\beta}\beta_x + u_{\beta\gamma}\gamma_x) + u_\beta \beta_{xx} + \gamma_x(u_{\gamma\gamma}\gamma_x + u_{\gamma\beta}\beta_x) + u_\gamma \gamma_{xx}$$

$\beta_x = \frac{-2}{x} \quad \gamma_x = 0 \quad \gamma_y = \frac{2}{y} \quad \beta_y = 0.$

$\beta_{xx} = \frac{2}{x^2} \quad \gamma_{xx} = 0 \quad \gamma_{yy} = \frac{-2}{y^2} \quad \beta_y = 0$

$$u_{xx} = \frac{-2}{x}\left(u_{\beta\beta}\left(\frac{-2}{x}\right) + \cancel{u_{\beta\gamma}}\right) + \frac{2u_\beta}{x^2}$$

$$\cancel{u_{yy}} \quad u_y = u_\beta \beta_y + u_\gamma \gamma_y$$

$$u_{yy} = \beta_y(u_{\beta\beta}\beta_y + u_{\beta\gamma}\gamma_y) + u_\beta\beta_{yy} + \gamma_y(u_{\gamma\gamma}\gamma_y + u_{\gamma\beta}\beta_y) + u_\gamma\gamma_{yy}$$

$$u_{yy} = \frac{2}{y}\left(\frac{2u_{\gamma\gamma}}{y}\right) + \cancel{u_{\gamma\beta}} - \frac{2u_\gamma}{y^2}$$

$$x^2 u_{xx} = 4u_{\beta\beta} - 2u_\beta$$

$$y^2 u_{yy} = 4u_{\gamma\gamma} - 2u_\gamma$$

$$4u_{\beta\beta} + 4u_{\gamma\gamma} - 2u_\beta - 2u_\gamma = 0$$

$$2u_{\beta\beta} + 2u_{\gamma\gamma} = u_\gamma + u_\beta$$

Q2.

d). $u_{xx} + 2u_{xy} + 5u_{yy} = xu_x$

$A = 1$

$B = 2 \qquad 4 - 20 < 0 \qquad$ Eliptical PDE

$C = 5 \qquad 1^2 + 2^2 + 5^2 \neq 0.$

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