The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 9

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1. Verify that $y = x^2 \sin x$ and $y = 0$ both are solutions of the initial value problem:

$$x^2 y'' - 4xy' + (x^2 + 6)y = 0, \quad y(0) = y'(0) = 0.$$

   Does it contradict the uniqueness?

2. Find general solution of the following differential equations given a known solution y_1:

   (i) $x(1-x)y'' + 2(1-2x)y' - 2y = 0 \qquad y_1 = \frac{1}{x}$
   (ii) $(1-x^2)y'' - 2xy' + 2y = 0 \qquad y_1 = x.$

3. Verify that $\sin x/\sqrt{x}$ is a solution of $x^2 y'' + xy' + (x^2 - \frac{1}{4})y = 0$ over any interval on the positive x-axis and hence find its general solution.
   Note: This ODE is the special case of Bessels equation corresponding to $p = 1/2$.

4. Solve the following differential equations:

$$(i)\ y'' - 4y' + 3y = 0 \qquad (ii)\ y'' + 2y' + (\omega^2 + 1)y = 0,\ \omega \text{ is real.}$$

5. Solve the following initial value problems:

   (i) $y'' + 4y' + 4y = 0 \qquad y(0) = 1, y'(0) = -1$
   (ii) $y'' - 2y' - 3y = 0 \qquad y(0) = 1, y'(0) = 3.$

6. The equation

$$x^2 \frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0,$$

   where a, b are constants, is called the Euler-Cauchy equation. Show that under the transformation $x = e^t$ for the independent variable, the above reduces to

$$\frac{d^2y}{dt^2} + (a-1)\frac{dy}{dt} + by = 0,$$

   which is an equation with constant coefficients. Hence solve:

$$(i)\ x^2y'' + 2xy' - 12y = 0 \quad (ii)\ x^2y'' + xy' + y = 0 \quad (iii)\ x^2y'' - xy' + y = 0.$$

7. By using the method of variation of parameters, find the general solution of:

   (i) $y'' + 4y = 2\cos^2 x + 10e^x \qquad (ii)$ $y'' + y = x \sin x$
   (iii) $y'' + y = 1 + \sin x \qquad (iv)$ $xy'' - y' = x^2(3+x)e^x.$

Supplementary problems from "Advanced Engg. Maths. by E. Kreyszig (8^{th} Edn.)
(a) Page $75 - 76$, Q.10,17,27,28 $\qquad (b)$ Page 80, Q.13,17,19,20
(c) Page 96, Q.2,6,7 $\qquad (d)$ Page 100, Q.12,16
(e) Page 104, Q.16 $\qquad (f)$ Page $131 - 132$, Q.4,11,18,20
(g) Page 137, Q.14,16,20