Department of Mathematics, LNMIIT,Jaipur

MATH-3 Linear Algebra And Complex Analysis
Additional info., Sem. I 2014-15,

Consider the system of 2 equations:

$$x_1 + 4x_3 - x_4 = 7$$
$$x_2 - 2x_3 - 3x_4 = 8$$

which also can be written as

$$x_1 + 0X_2 + 4x_3 - x_4 = 7$$
$$0X_1 + x_2 - 2x_3 - 3x_4 = 8$$

A quick look at the equations tells us if we know x_2, x_3, x_4 then x_1 is determined by the first equation. We may treat x_3, x_4 as free variables and determine x_2 from the second equation and x_1 from first equation by back substitution. Let $x_3 = a$ and $x_4 = b$.Then

$$x_2 = 8 + (2a + 3b) \text{ and } x_1 = 7 + ((b - 4a)$$

Let S denote the set of all solutions of the given system of 2 equations. Then

$$S = \{(7, 8, 0, 0) + a(-4, 2, 1, 0) + b(1, 3, 01), a, b, \in \mathbb{R}\}.$$

We also note that $(7, 8, 0, 0)$ is a solution of the given system while $(-4, 2, 1, 0), (1, 3, 0, 1)$ are the solutions of the corresponding linear equation (of the given system) viz

$$x_1 + 4x_3 - x_4 = 0$$
$$x_2 - 2x_3 - 3x_4 = 0$$

Comments: !. By assigning various vales for a, b we indeed generate the solutions of the given system.
2. The linear combination of the 2 solutions $(-4, 2, 1, 0), (1, 3, 0, 1)$ of the homogeneous equation is yet another solution of the homogeneous equation and also any solution of the homogeneous equation is a linear combination of $(-4, 2, 1, 0), (1, 3, 0, 1)$.
3.

$$a(-4, 2, 1, 0) + b(1, 3, 01), a, b, \in \mathbb{R}\}.$$

is called the general solution of the homo.equn. while

$$(7, 8, 0, 0) + a(-4, 2, 1, 0) + b(1, 3, 01), a, b, \in \mathbb{R}\Gamma$$

is called the general solution of the non-homo. equn.