

4th Assignment  
 Subject : Physics II (Electrodynamics)  
 Date: 9<sup>th</sup> April 2014

**Problem 4.15** A thick spherical shell (inner radius  $a$ , outer radius  $b$ ) is made of dielectric material with a “frozen-in” polarization

$$\mathbf{P}(\mathbf{r}) = \frac{k}{r} \hat{\mathbf{r}},$$

where  $k$  is a constant and  $r$  is the distance from the center (Fig. 4.18). (There is no *free* charge in the problem.) Find the electric field in all three regions by two different methods:

- (a) Locate all the bound charge, and use Gauss’s law to calculate the field it produces.  
 (b) Use Eq. 4.23 to find  $\mathbf{D}$ , and then get  $\mathbf{E}$  from Eq. 4.21. [Notice that the second method is much faster, and avoids any explicit reference to the bound charges.]
- $$\oint \mathbf{D} \cdot d\mathbf{a} = Q_{\text{enc}} \quad (4.23)$$

**Problem 4.20** A sphere of linear dielectric material has embedded in it a uniform free charge density  $\rho$ . Find the potential at the center of the sphere (relative to infinity), if its radius is  $R$  and its dielectric constant is  $\epsilon_r$ .

**Problem 4.21** A certain coaxial cable consists of a copper wire, radius  $a$ , surrounded by a concentric copper tube of inner radius  $c$  (Fig. 4.26). The space between is partially filled (from  $b$  out to  $c$ ) with material of dielectric constant  $\epsilon_r$ , as shown. Find the capacitance per unit length of this cable.

**Problem 1** For the spherical conducting shells filled with two linear dielectric materials, shown in Fig.1, the potential of inner and outer shell is 0 and  $V_0$ , respectively. Determine the potential distribution in the region  $c < r < a$ .

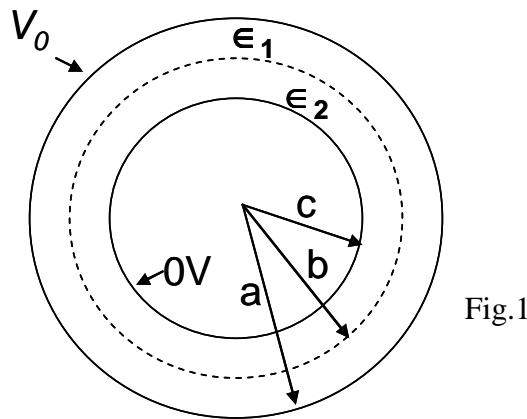
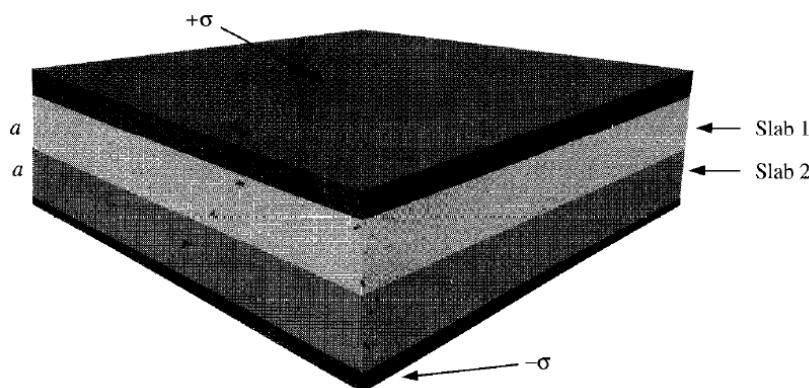


Fig.1

**Problem 4.18** The space between the plates of a parallel-plate capacitor (Fig. 4.24) is filled with two slabs of linear dielectric material. Each slab has thickness  $a$ , so the total distance between the plates is  $2a$ . Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is  $\sigma$  and on the bottom plate  $-\sigma$ .

- Find the electric displacement  $\mathbf{D}$  in each slab.
- Find the electric field  $\mathbf{E}$  in each slab.
- Find the polarization  $\mathbf{P}$  in each slab.
- Find the potential difference between the plates.
- Find the location and amount of all bound charge.
- Now that you know all the charge (free and bound), recalculate the field in each slab, and confirm your answer to (b).



**Problem 4.26** A spherical conductor, of radius  $a$ , carries a charge  $Q$  (Fig. 4.29). It is surrounded by linear dielectric material of susceptibility  $\chi_e$ , out to radius  $b$ . Find the energy of this configuration (Eq. 4.58).

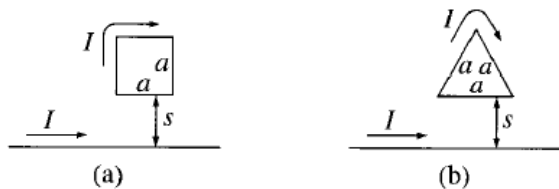


Figure 5.24

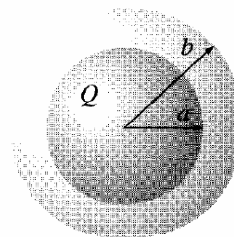


Figure 4.29

**2. Helmholtz coil:** Two identical coaxial circular loops are placed exactly parallel to each other. The current carrying loops are separated by a distance  $2d$ . If there are  $N$  turns in each of these loops, find out the expression of magnetic field at any axial point between the two loops. Derive the condition of achieving uniform magnetic field in the region between the loops.

**Problem 5.10**

- (a) Find the force on a square loop placed as shown in Fig. 5.24(a), near an infinite straight wire. Both the loop and the wire carry a steady current  $I$ .
- (b) Find the force on the triangular loop in Fig. 5.24(b).

**Problem 5.12** Suppose you have two infinite straight line charges  $\lambda$ , a distance  $d$  apart, moving along at a constant speed  $v$  (Fig. 5.26). How great would  $v$  have to be in order for the magnetic attraction to balance the electrical repulsion? Work out the actual number. . . Is this a reasonable sort of speed?<sup>8</sup>

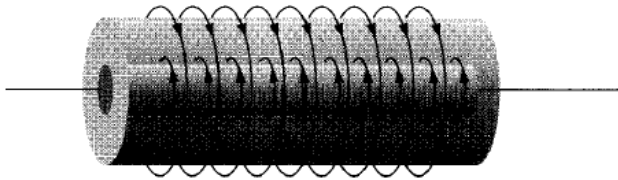


Figure 5.42

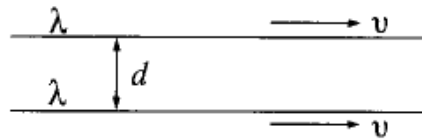


Figure 5.26

**Problem 5.15** Two long coaxial solenoids each carry current  $I$ , but in opposite directions, as shown in Fig. 5.42. The inner solenoid (radius  $a$ ) has  $n_1$  turns per unit length, and the outer one (radius  $b$ ) has  $n_2$ . Find  $\mathbf{B}$  in each of the three regions: (i) inside the inner solenoid, (ii) between them, and (iii) outside both.

**Problem 6.8** A long circular cylinder of radius  $R$  carries a magnetization  $\mathbf{M} = ks^2 \hat{\phi}$ , where  $k$  is a constant,  $s$  is the distance from the axis, and  $\hat{\phi}$  is the usual azimuthal unit vector (Fig. 6.13). Find the magnetic field due to  $\mathbf{M}$ , for points inside and outside the cylinder.

**Problem 6.16** A coaxial cable consists of two very long cylindrical tubes, separated by linear insulating material of magnetic susceptibility  $\chi_m$ . A current  $I$  flows down the inner conductor and returns along the outer one; in each case the current distributes itself uniformly over the surface (Fig. 6.24). Find the magnetic field in the region between the tubes. As a check, calculate the magnetization and the bound currents, and confirm that (together, of course, with the free currents) they generate the correct field.

**Problem 6.17** A current  $I$  flows down a long straight wire of radius  $a$ . If the wire is made of linear material (copper, say, or aluminum) with susceptibility  $\chi_m$ , and the current is distributed uniformly, what is the magnetic field a distance  $s$  from the axis? Find all the bound currents. What is the *net* bound current flowing down the wire?

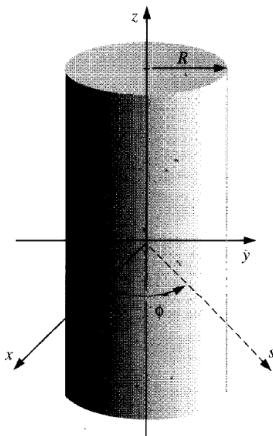


Figure 6.13

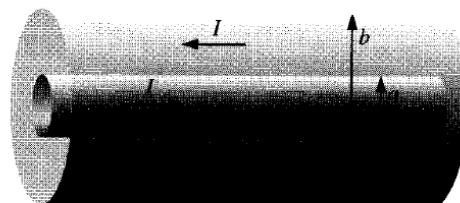


Figure 6.24