LNMIIT/B. Tech./IC/2016-17/EVEN/MTH108/ET

LNMIIT
The LNM Institute of Information Technology

THE LNM INSTITUTE OF INFORMATION TECHNOLOGY
DEPARTMENT OF MATHEMATICS
MATHEMATICS-II & MTH108
END TERM

Time: 180 minutes Date: 28/04/2017 Maximum Marks: 100

Instruction: You should attempt all questions. Your writing should be legible and neat. Marks awarded next to the question. Please make an index showing the question number and page number on the front page of your answer sheet in the following format.

Question No.				
Page No.				

1. (a) Let $W = \{(a, b, c) : a - 2b + c = 0\}$. Show that W is a subspace of R^3 (considered with usual vector addition and scalar multiplication). Find the basis and dimension of W. [6]
   (b) Consider a second order linear differential equation $xy'' - (1 + x)y' + y = x^2e^{2x}, x > 0$. If e^x is a solution of corresponding homogeneous equation then find the general solution of the given differential equation. [8]

2. (a) Find the value of k in the matrix $A = \begin{pmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & k & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

   such that the eigenspace for the eigenvalue $\lambda = 5$ is two-dimensional. Justify answer. [6]
   (b) Solve $y'' + 2xy = 0$ using power series method. [6]

3. (a) Solve $(xy \sin xy + \cos xy)y\, dx + (xy \sin xy - \cos xy)x\, dy = 0$ [8]
   (b) If y_1 and y_2 are two linearly independent solutions of $y'' + p(x)y' + q(x)y = 0, x \in I$, then prove that between two consecutive zeros of y_1 there exists unique zero of y_2 in I. (Here I is an open interval) [6]

4. Find all the nontrivial solutions of the boundary value problem $y'' + \lambda y = 0$ for $x \in (0, \frac{\pi}{2})$ with $y(0) = 0, y'\left(\frac{\pi}{2}\right) = 0$. Compute the coefficients c_n for the eigenfunction expansion of the function $f(x) = x$ on $[0, \frac{\pi}{2}]$. [9]

5. (a) Let $P_n(x)$ be the Legendre polynomial of degree n. Then prove that [6]

$$\|P_n(x)\|^2 = \int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$$

   (Hint: Use Rodrigues' formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}[(x^2-1)^n]$)
   (b) Let A be a 6×9 matrix ($A = [a_{ij}]_{6\times9}$). Could A have a two dimensional null space? What is the largest possible dimension of the column space of A? Justify each answer. [6]

6. (a) Using the **Laplace transformation**, solve the differential equation $xy'' + (1-x)y' + ny = 0$ where n is an non-negative integer. [8]
   (b) Show that the following second order linear non-homogeneous IVP has unique solution. [5]

$$y'' + 2x^2y'e^x + y\cos x = \sin x$$
$$y(0) = 2, \qquad y'(0) = 1.$$

7. (a) Show that if A^2 is the zero matrix, then the only eigenvalue of A is 0. [6]
   (b) Find Fourier sine and cosine series of $f(x) = x$ $0 < x < \pi$ and use this to find the sum of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ [8]

8. (a) Sketch the following functions and then find it's Laplace transforms: [6]

$$f(t) = \begin{cases} t[u(t) - u(t-1)], & 0 \le t < 2, \\ f(t-2), & t > 2. \end{cases}$$

   (b) Find sine integral of $f(x) = e^{-x}, x > 0$ and then evaluate the integral $\int_0^\infty \frac{\omega \sin x\omega}{1+\omega^2}d\omega$. [6]

End of paper