Book → Sydsæter & Hammond

Revision M1

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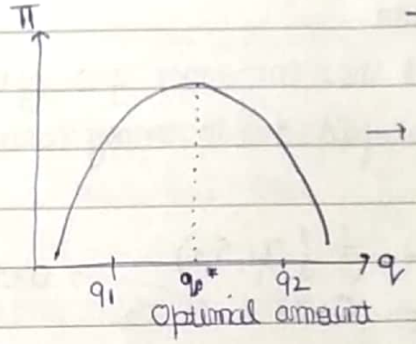
Zwaefgz

Applied Economics

1 Necessary & Sufficient Condition for Profit Max
2 Implicit function
3 Constrained Maximization → Interpret Lagrangian multiplier
4 Homogeneous function & Eulers theorem (Pg 76)

1) $\pi = \pi(q)$ → quantity

Profit function



→ at q_1, $\frac{d\pi}{dq} > 0$

→ at $\boxed{q^*, \frac{d\pi}{dq} = 0}$

at q_2, $\frac{d\pi}{dq} < 0$

Necessary but not sufficient Condition

Sufficient condition → $\frac{d^2\pi}{dq^2} < 0$

2) Implicit → $Q = f(K, L)$ as labour is function capital

$y = f(x_1, x_2)$ → explicit

→ $y = f(x_1, x_2(x))$

$dy = \frac{\partial f}{\partial x_1} \cancel{dx_1} + \frac{\partial f}{\partial x_2} \cdot \frac{dx_2}{dx_1}$

If x_1 changes by 1 unit by how much x_2 changes

If $dy = 0 \Rightarrow \frac{dx_2}{dx_1} = \frac{-\partial f / \partial x_1}{\partial f / \partial x_2}$ gives degree of substitution b/w products

3) $y = f(x_1, x_2)$

$dy = f_1 dx_1 + f_2 dx_2$ $\quad f_1 = \frac{\partial f}{\partial x_1} \quad f_2 = \frac{\partial f}{\partial x_2}$

$d^2y = \frac{\partial}{\partial x_1}(f_1 dx_1 + f_2 dx_2) + \frac{\partial}{\partial x_2}(f_1 dx_1 + f_2 dx_2)$

$d^2y = f_{11} dx_1^2 + f_{22} dx_2^2 + 2 f_{12} dx_1 dx_2$

For maximization → $d^2y < 0$

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unconstrained

→ Hessian Determinant

if f11 concave

$$\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} \rightarrow \begin{matrix} \text{If } > 0 \Rightarrow \text{concave function} \\ \text{If } < 0 \Rightarrow \text{convex function} \end{matrix}$$

$$f_{11} = \frac{\partial^2 f}{\partial x_1^2}, \quad f_{12} = \frac{\partial^2 f}{\partial x_2 \partial x_1}, \quad f_{21} = \frac{\partial^2 f}{\partial x_1 \partial x_2}, \quad f_{22} = \frac{\partial^2 f}{\partial x_2^2}$$

$f_{21} = f_{12} \rightarrow$ Young's Theorem

Constrained maximization

Z is satisfaction level that consumer gets after consuming x_1 & x_2

cannot be always satisfied due to money constrained

s.t (subject to)

$$Z = f(x_1, x_2)$$

budget $b = c_1x_1 + c_2x_2$ ← cost

$$M = P_1x_1 + P_2x_2$$

money income cannot exceed beyond this

Producer

$$Q = f(K, L)$$

$$C = wL + rK$$

s.t a cost ~~constant~~ constrained

Lagrangian Method → To find optimal solution to constrained Problem

$$L = f(x_1, x_2) + \lambda [b - c_1x_1 - c_2x_2], \quad \lambda > 0$$

FOC
First order condition

$$\frac{\partial f}{\partial x_1} = 0 \Rightarrow \frac{\partial f}{\partial x_1} - \lambda c_1 = 0 \rightarrow (1)$$

$$\frac{\partial f}{\partial x_2} = 0 \Rightarrow \frac{\partial f}{\partial x_2} - \lambda c_2 = 0 \rightarrow (2)$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow b - c_1x_1 - c_2x_2 = 0 \rightarrow (3)$$

$$\Rightarrow \boxed{b = c_1x_1 + c_2x_2}$$

$$\rightarrow \frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}} = \frac{c_1}{c_2}$$

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$-f_1(f_1 f_{22} - f_2 f_{12}) + f_2(f_1 f_{21} - f_2 f_{11})$

$-f_1^2 f_{22} + 2 f_1 f_2 f_{12} - f_2^2 f_{11}$

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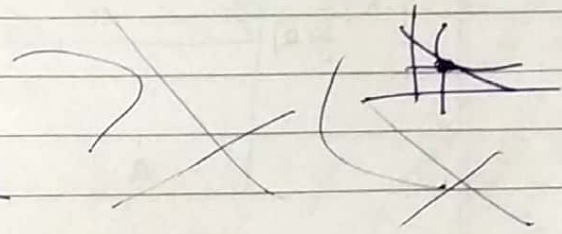
$$c_1 = c_2 \frac{\partial f / \partial x_1}{\partial f / \partial x_2} \longrightarrow \text{Put this value of } c_1 \text{ in } b = c_1 x_1 + c_2 x_2$$



$$\Rightarrow \quad b = c_2 \frac{\partial f / \partial x_1}{\partial f / \partial x_2} x_1 + c_2 x_2$$

Ex → If labour increased by 1 amount then by how much quantity increases

Shadow Prices

$$\lambda = \frac{\partial f / \partial x_1}{c} = \frac{\text{marginal Benefit}}{\text{marginal cost}}$$

$$\lambda = \frac{MU_1}{P_1} \text{ or } \frac{MP_2}{w}$$

$$\lambda = \frac{MU_i}{P_i} \text{ or } \frac{MP_i}{c_i} \quad \forall i = 1(1)n$$

Each good Purchased → Extra P | Extra Cost

Extra Utility

→ Now to check if maximization holds or not

Now let $\frac{\partial f}{\partial x_1} = f_1$ & $\frac{\partial f}{\partial x_2} = f_2$

$$\frac{f_1}{f_2} = \frac{c_1}{c_2}, \quad y = f(x_1, x_2)$$

$$\hookrightarrow \boxed{d^2y = f_{11} dx_1^2 + f_{22} dx_2^2 + 2 f_{12} dx_1 dx_2}$$

$$\boxed{b = c_1 x_1 + c_2 x_2} \quad \text{Now this is included}$$

$$0 = c_1 dx_1 + c_2 dx_2$$

$$\boxed{dx_1 = \frac{-c_2 dx_2}{c_1}}$$

$$d^2y = f_{11} dx_1^2 - 2 f_{12} \frac{f_1}{f_2} dx_1^2 + f_{22} \frac{f_1^2}{f_2^2} dx_1^2$$

$$\frac{f_{11} f_2^2 - 2 f_{12} f_1 f_2 + f_{22} f_1^2}{f_2^2}$$

Bordered Hessian

$$\begin{bmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{21} & f_{22} \end{bmatrix} > 0$$

+ve means Quasi Concave

$d^2y < 0$

utility function or Production f^n

Objective f^n is Quasi Concave

Why do we study quasi concavity

$$f[(1-\lambda)a + \lambda b] \geq \min [f(a), f(b)]$$

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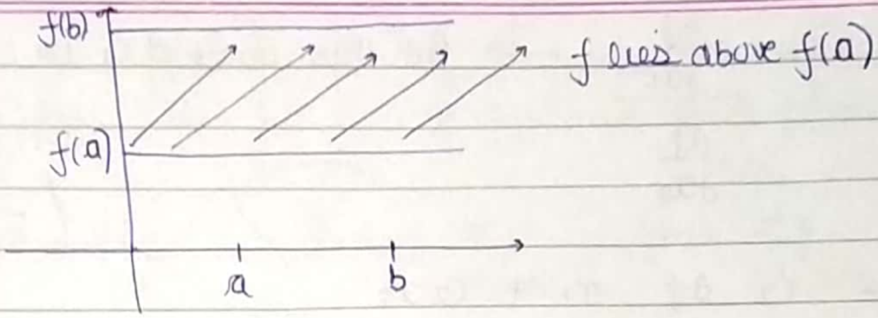
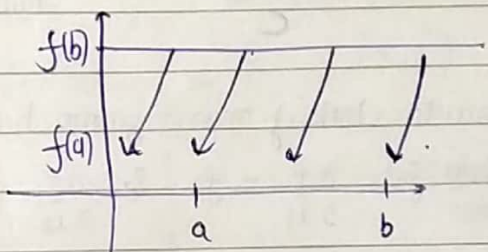


Quasi Convex

$$\begin{bmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{21} & f_{22} \end{bmatrix} > 0 \quad , \quad f\left[(1-\lambda)a + \lambda b\right] \leq \max\left[f(a), f(b)\right]$$



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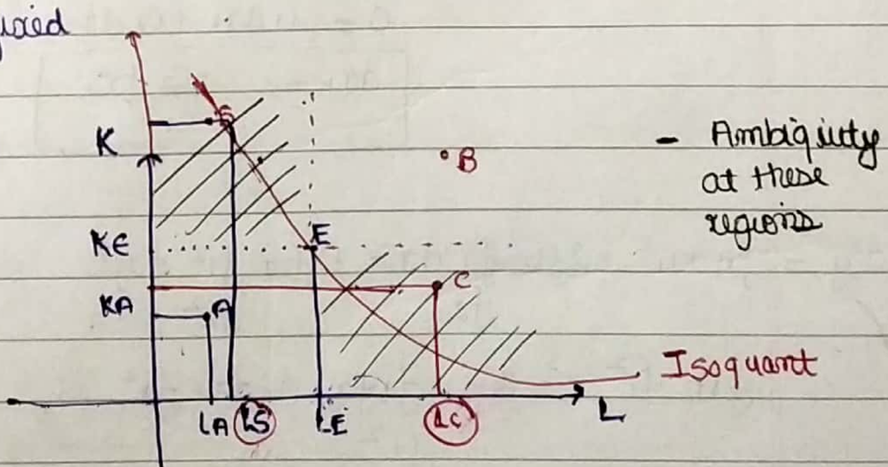
Unit 1 Production

Isoquant

Locus of combination b/w two or many inputs such that the production level remains fixed

$Q = f(K, L)$

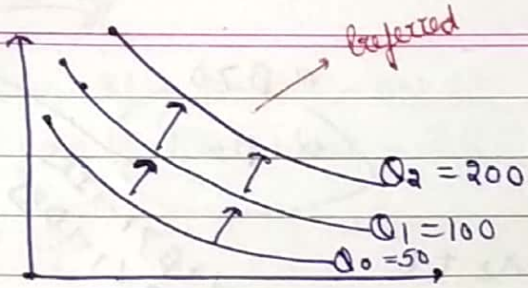


In all circumstances E is preferred over A

$E >_p A$

$E <_p B$

Countour lines which show different levels of output



Along isoquant $dQ = 0$

$Q = f(K, L)$

MRTSK,L → amount of capital that producer will sacrifice to gain a marginal unit of labour.

MRTL,K

$$dQ = 0$$

$$\frac{\partial f}{\partial K} dK + \frac{\partial f}{\partial L} dL = 0$$

$$\boxed{\frac{dK}{dL} = \frac{-f_L}{f_K} < 0} \Rightarrow \text{downward sloping}$$

Second order derivative

$$\frac{d^2K}{dL^2} = \frac{d}{dL}\left(\frac{-f_L}{f_K}\right) = \frac{f_K\left(f_{LL} + f_{LK}\frac{dK}{dL}\right) - f_L\left(f_{KL} + f_{KK}\frac{dK}{dL}\right)}{f_K^2}$$

$$\frac{f_K\left(f_{LL} + f_{LK}\left(\frac{-f_L}{f_K}\right)\right) - f_L\left(f_{KL} + f_{KK}\left(\frac{-f_L}{f_K}\right)\right)}{f_K^2}$$

$$\frac{f_K f_{LL} - \frac{f_K f_{LK} f_L}{f_K} - f_L f_{KL} + \frac{f_L^2 f_{KK}}{f_K}}{f_K^2}$$

$$\frac{f_K f_{LL} - f_{LK} f_L - f_L f_{KL} + \frac{f_L^2 f_{KK}}{f_K}}{f_K^2}$$

$$= \boxed{\frac{f_K^2 f_{LL} - 2 f_{LK} f_L f_K + f_L^2 f_{KK}}{f_K^3} < 0}$$

by Cobb - Douglas $(\sigma \neq 1)$

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$$Q = AK^{\alpha}L^{\beta}$$

$\alpha, \beta > 0$

$\alpha + \beta = 1$ → CRS, DRS, IRS

$\alpha + \beta > 1 \to IRS$

$\alpha + \beta < 1 \to DRS$

$$y = \alpha + \beta_1 X_1 + \beta_2 X_2 + t$$

$$Q = \alpha + \beta_1 K + \beta_2 L + t$$

1^{st} derivative $\frac{dK}{dL} = -\frac{f_L}{f_K} = \frac{-AK^{\alpha}\beta L^{\beta-1}}{A\alpha K^{\alpha-1}L^{\beta}} = \frac{-\beta}{\alpha}\frac{K}{L}$

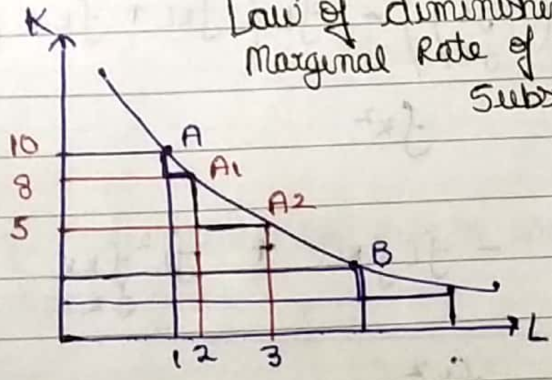
2^{nd} derivative

$$\frac{\overset{(+)(-)}{f_K^2 f_{LL}} \; \overset{-}{-} \; \overset{(+)}{2 f_L f_K f_{KL}} \; \overset{+}{+} \; \overset{(-)}{f_L^2 f_{KK}}}{f_K^3} \Rightarrow \text{convex to } 0$$

or quasi-concave

$f_K^2 > 0$ $\qquad$ $f_{LL} = A\beta(\beta-1)K^{\alpha}L^{\beta-2} < 0$

Law of diminishing Marginal Rate of Technical Substitution



Slope is called MRTS

Slope A > Slope B

from A → B getting more labourers by giving more capital
giving more capital to hire more labours

After point B, amount of capital given is lesser to hire one extra labour

Slope of Isoquant → MRTS

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Ex $Q = KL - 0.8K^2 - 0.2L^2$

if $K = 10$ $\quad APL_{max} = ?$

$APL = \frac{Q}{L}$

at $K=10$, $Q = 10L - 80 - 0.2L^2$

$APL = \frac{Q}{L} = 10 - \frac{80}{L} - 0.2L$

$\frac{80}{L^2} = 0.2$ (1^{st} derivative $= 0$) for maximum

$L^2 = \frac{800}{2} \Rightarrow L = 20$

Ques at what L, $MPL = 0$

$MPL = \frac{\partial Q}{\partial L} = K - 0.4L = 0 \Rightarrow L = \frac{100}{4} = 25$

Elasticity of substitution

$y = f(x_1, x_2)$
$= f(x_1, g(x_1))$
a kind of substitution

$\sigma = \frac{\%\text{ change in } (K/L)}{\%\text{ change in MRTS}}$

$\sigma = \frac{d(K/L)\,/\,K/L}{d(MRTS)\,/\,MRTS} = \frac{d(K/L)}{d(MRTS)} \frac{MRTS}{K/L}$

Ex $Q = AK^{\alpha}L^{\beta}$, σ ??

$MRTS = \frac{\beta}{\alpha}\left(\frac{K}{L}\right)$

$\boxed{\sigma = 1}$

$\sigma = \frac{d(K/L)}{K/L} \frac{\left(\frac{\beta}{\alpha}\frac{K}{L}\right)}{d(MRTS)}$

$= \frac{d(K/L)}{(K/L)} \frac{\left(\frac{\beta}{\alpha}\frac{K}{L}\right)}{\left(\frac{\beta}{\alpha}\,d(K/L)\right)}$

$= 1 \Rightarrow$ one unit of labour can be exchanged for one unit of capital

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Perfect Substitutes

Ex $Q = aK + bL$

↳ K & L are $\boxed{\text{perfectly substitutes}}$

$w \to L$ | if wage rate increases then substitute labour with Capital

$r \to K$ |

$$\frac{dK}{dL} = \frac{-f_L}{f_K} = \frac{-b}{a}$$

$$\boxed{MRTS = \left|\frac{dK}{dL}\right|}$$

MRTS = slope / mod of slope

$$\sigma = \frac{d(K/L)}{K/L} \cdot \frac{\cancel{\left(\frac{\beta}{\alpha}\frac{K}{L}\right)}\ (MRTS)}{d(MRTS)}$$

$MRTS = -b/a$

$d(MRTS) = 0$

$\sigma \Rightarrow \infty$ $\Rightarrow$ Capital & Labour can be infinitely substituted

Perfect Compliments

$Q = \min\{\alpha K, \beta L\}$

$\frac{K}{L}$ is constant

inputs are used in fixed proportion

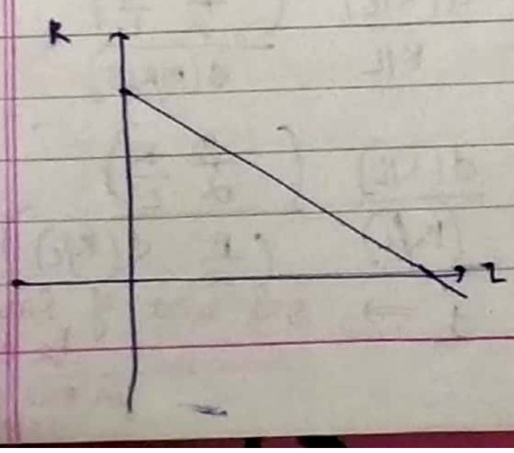
$\sigma = 0$

have to use both capital and labour in some proportion.

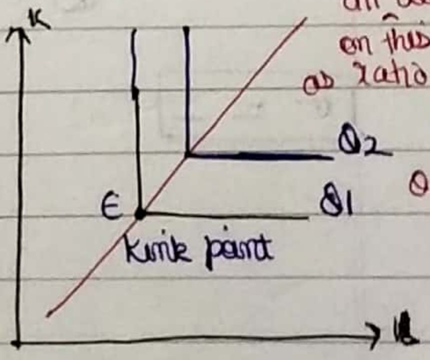
ex one can't buy only one shoe for one leg. One has to buy for both left & right

Perfect Substitutes → Shape of isoquant

$Q = aK + bL$



Perfect Compliments



prod all decision on this line as ratio fixed

$Q_2 > Q_1$

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Euler's Theorem

$Z = f(x, y)$

$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = nZ$ n is degree of homogeneity

$Z = x^2 + y^2 + xy$

$x(2x+y) + y(2y+x)$

$= 2(x^2+y^2+xy)$

$= 2Z$

$\Rightarrow n = 2$

* Product Exhaustion Theorem

If a production f^n is of homogenous of degree 1 then the factors of production are ~~divided~~ equal to their marginal product ~~regarded~~ / rewarded

$Q = f(K, L)$

$\hookrightarrow$ HOD $\rightarrow 1$

K, L

$K \frac{\partial f}{\partial K} + L \frac{\partial f}{\partial L} = 1 Q$

wage rate + rent rate

whatever is the earning it is distributed equally b/w labour & capital

If >1 $\rightarrow$ Total Payments $>$ Total O/P

If <1 $\rightarrow$ " $<$ Total O/P

18/1/19 Convexity & Concavity through the hessian determinant function

$$H = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix}$$

Concave f^n

$\rightarrow f_{11} < 0$ $f_{22} < 0$

$f_{11} f_{22} - (f_{12})^2 \geq 0$

Convex f^n

$\rightarrow f_{11} \geq 0$, $f_{22} \geq 0$

$\rightarrow f_{11} f_{22} - (f_{12})^2 \geq 0$

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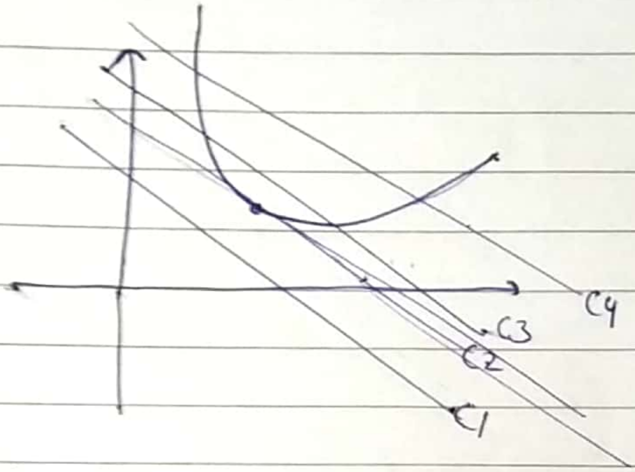
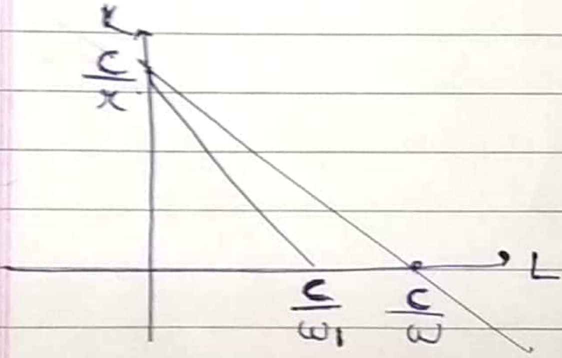
$C = wL + rK$



$C^* = w'L + rK$ (wage change)

$w_1 > w$



<u>Dual Problem</u>

→ Q maximize → cost constraint
→ Cost minimize → Q constraint

} constraint optimization

$$\mathcal{L} = wL + rK + \lambda\left(\bar{Q} - f(K,L)\right)$$

$$\frac{d\mathcal{L}}{dL} = w - \lambda f_L$$

$$\frac{d\mathcal{L}}{dK} = r - \lambda f_K$$

$$\boxed{\frac{f_K}{f_L} = \frac{r}{w}}$$

Write either L in terms of K or K in terms of L

Put L (in terms of K) in Q to get K^*

Put K (" " " L) " " " " L^*

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all quasiconcave are concave function but not wise vera

~~so quant a concave~~ 2nd order condition

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$$Q = K^{\alpha} L^{\beta}$$

$$C = WL + rK$$

$$Z = WL + rK + \lambda\left[\bar{Q} - K^{\alpha} L^{\beta}\right]$$

$$K^* = \left(\frac{\alpha W}{\beta r}\right)^{\frac{\beta}{\alpha+\beta}} (q_0)^{1/\alpha+\beta}$$

$$L^* = \left(\frac{\beta r}{\alpha W}\right)^{\frac{\alpha}{\alpha+\beta}} (q_0)^{\frac{1}{\alpha+\beta}}$$

$$C^* = WL^* + rK^*$$

$$C^* = W^{\frac{\beta}{\alpha+\beta}} r^{\frac{\alpha}{\alpha+\beta}} \left[\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{\alpha+\beta}}\right] (q_0)^{\frac{1}{\alpha+\beta}}$$

Shepard's Lemma

$$\frac{\partial C^*}{\partial W} = L^* \quad , \quad \frac{\partial C^*}{\partial r} = K^*$$

Significant

Ques $Q = K^{1/2} L^{1/2}$ minimise cost subject to o/p constraint

$W = 12,\ r = 3$

$q_0 = 40$ Find K^*, L^*

$$K^* = \left[\frac{\alpha}{\beta}\left(\frac{12}{3}\right)\right]^{\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}}} (40)^{\frac{1}{\frac{1}{2}+\frac{1}{2}}}$$

$$= 1(4)^{1/2} (40)$$

$$= 80$$

$$L^* = \left(\frac{1}{4}\right)^{1/2} (40)^{\frac{1}{1}} = 20$$

$$C^* = 20W + 80r$$

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$$Z = 12L + 3K + \lambda\left[40 - K^{1/2}L^{1/2}\right]$$

$$\frac{\partial Z}{\partial L} = 0 \qquad \frac{\partial Z}{\partial \lambda} = 0 \qquad \frac{\partial Z}{\partial K} = 0$$

$\rightarrow$ $12 - \frac{\lambda}{2} K^{1/2} L^{-1/2} = 0 \rightarrow (1) \Rightarrow 24 = \lambda K^{1/2} L^{-1/2}$

$\rightarrow$ $40 - K^{1/2} L^{1/2} = 0 \rightarrow (3)$

$\rightarrow$ $3 - \frac{\lambda}{2} K^{-1/2} L^{1/2} = 0 \rightarrow (2) \Rightarrow 6 = \lambda K^{-1/2} L^{1/2}$

$$\Rightarrow 4 = \frac{K}{L}$$

$$40 = K^{1/2} L^{1/2}$$
$$40 = \sqrt{4L}\,\sqrt{L}$$
$$40 = 2L$$
$$\boxed{\begin{matrix} L = 20 \\ K = 80 \end{matrix}}$$

$\left(-(f_1)^2\right) > 0$

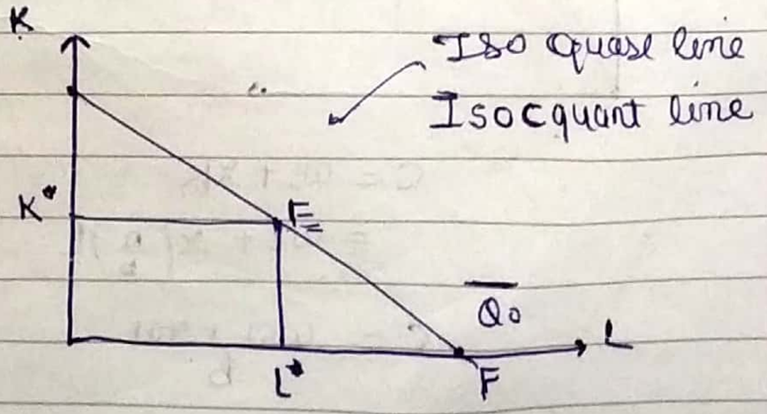
$f_1^2 > 0$

If $\begin{vmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{21} & f_{22} \end{vmatrix} < 0 \Rightarrow$ optimum

Perfect Substitutes

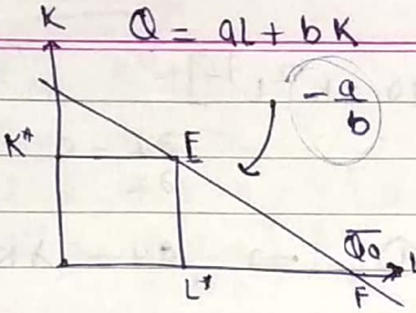
eq at corner

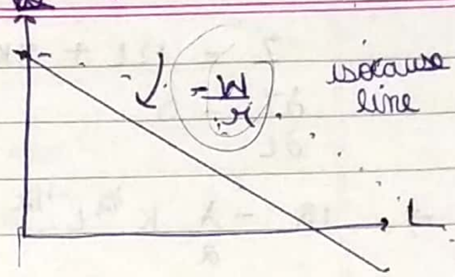
$$Q = aL + bK$$



$y = mx + c$

$C = wL + rK$





If $\frac{a}{b} = \frac{w}{r}$ (infinitely many sol^n) $\quad \frac{f_L}{f_K} = \frac{w}{r}$

If $\frac{a}{b} < \frac{w}{r}$, $L^* = 0$, $K^* = \frac{C}{r}$

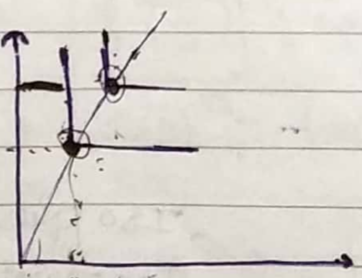
If $\frac{a}{b} > \frac{w}{r}$, $K^* = 0$, $L^* = \frac{C}{w}$

Slope of isoquant

$MP = \frac{MP_L}{MP_K}$

Perfect Complements

$$Q = \min\left\{\frac{K}{a}, \frac{L}{b}\right\}$$



eq'b at kink

$\frac{K}{a} = \frac{L}{b}$

$\Rightarrow bK = aL$

$K = \frac{a}{b}L$

$C = wL + rK$

$= wL + r\left(\frac{a}{b}\right)L$

$C = \frac{wbL + raL}{b}$

$L^* = \frac{bC}{wb + ra}$ $\quad K^* = \frac{aC}{wb + ra}$

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Ques. $q = \min\{5K, 10L\}$ $\quad a = \frac{1}{5}, \; b = \frac{1}{10}$

$C = wL + rK$ $\quad K = 2L$

K^*, L^*

$L =$

$C = wL + 2rL$

$C = L(w + 2r)$

$\Rightarrow L = \frac{C}{w + 2r}$

$q = 5K = 10L$

$TC = q\left(\frac{w}{10} + \frac{r}{5}\right)$ $\quad$ Cost in Terms of Q

$AC = \frac{TC}{q} = \frac{w}{10} + \frac{r}{5}$

$MC = \frac{dTC}{dq} = \frac{w}{10} + \frac{r}{5}$

$\Rightarrow AC = MC$

Ques. $x = A\left(q_1^{\alpha} + q_2^{\beta}\right)$

$P_1 \,\&\, P_2$ $\quad \alpha, \beta > 1$

$\pi =$ Revenue $-$ Cost

$\pi = P_1 q_1 + P_2 q_2 - Cx$

$\pi = P_1 q_1 + P_2 q_2 - CA\left(q_1^{\alpha} + q_2^{\beta}\right)$

$\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow P_1 - C\alpha A q_1^{\alpha-1} = 0 \Rightarrow q_1 = \left(\frac{P_1}{C\alpha A}\right)^{\frac{1}{\alpha-1}}$

$\frac{\partial \pi}{\partial q_2} = 0 \Rightarrow P_2 - C\beta A q_2^{\beta-1}$

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$$\Pi = Pq - C = Pq - wL - rK = P\left(AK^{\alpha}L^{1-\alpha}\right) - wL - rK$$

L, K

$$\frac{d\Pi}{dL} = 0 \Rightarrow PAK^{\alpha}(1-\alpha)L^{-\alpha} - w = 0$$

$$\frac{d\Pi}{dK} = 0 \rightarrow PA\alpha K^{\alpha-1}L^{1-\alpha} - r = 0$$

$$PAK^{\alpha}(1-\alpha)L^{-\alpha} = w$$

$$PAK^{\alpha-1}\alpha L^{1-\alpha} = r$$

$$\frac{w}{r} = \frac{(1-\alpha)/\alpha}{K^{-1}L}$$

Let $1-\alpha = \beta$

$$\boxed{\frac{w}{r} = \frac{\beta K}{\alpha L}}$$

$$L^{*} = \left(\frac{\alpha}{W}\right)^{\frac{1-\beta}{\gamma}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{\gamma}} (AP)^{\frac{1}{\gamma}}$$

$$K^{*} = \left(\frac{\alpha}{W}\right)^{\frac{\alpha}{\gamma}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{\gamma}} (AP)^{\frac{1}{\gamma}}$$

$\gamma = 1 - \alpha - \beta$ Unconditional demand

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micro economics
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$$\pi = PQ - C$$

$$Q = L^{\alpha} K^{\beta}$$

$$\pi = P(L^{\alpha} K^{\beta}) - wL - rK$$

$$\frac{\partial \pi}{\partial L} = 0 \Rightarrow \alpha P L^{\alpha-1} K^{\beta} = w \longrightarrow (1)$$

$$\frac{\partial \pi}{\partial K} = 0 \Rightarrow \beta P L^{\alpha} K^{\beta-1} = r \longrightarrow (2)$$

$$\frac{1}{2} = \frac{w}{r} = \frac{\alpha}{\beta}\frac{K}{L}$$

$$\Rightarrow \alpha K r = \beta w L$$

$$K = \frac{\beta w L}{\alpha r} \longrightarrow (3)$$

Substitute eq (3) in eq (1)

$$\alpha P L^{\alpha-1}\left[\frac{\beta}{\alpha}\frac{w}{r}L\right]^{\beta} = w$$

$$L = f(w, r, P)$$

$$L^{\alpha+\beta-1} = \frac{w}{P\alpha\left(\frac{\beta w}{\alpha r}\right)^{\beta}}$$

Ques Explain why this data proof that α Corporation is done inefficiently

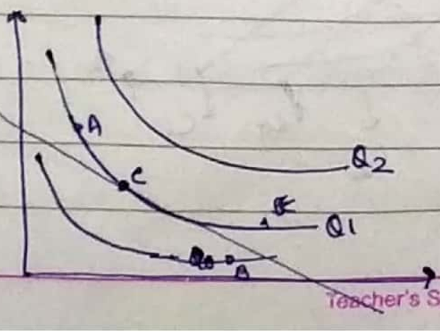
$MP_L = 10$ $w = 4$ $MP_K = 15$ $r = 3$

Slope of isocost & slope of isoquant equal at point of efficiency

Slope of isoquant $= \frac{MP_L}{MP_K} = \frac{10}{15} = \frac{2}{3}$

Isocost $= \frac{w}{r} = \frac{4}{3}$

we are checking for tangency



$Q_0 < Q_1 < Q_2$

$\frac{w}{r} \neq \frac{MP_L}{MP_K}$

not efficiently working

Teacher's Sign

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Now to make it efficiently work, $\frac{MP_L}{MP_K} = \frac{w}{r}$

why not do this

either increase Productivity by ↑ MP_L

Ques $Q = L^{0.5} K^{0.5}$, $K = 2$

Find TC, FC

Short run production function

Sol Square both sides

$Q^2 = L\bar{K}$

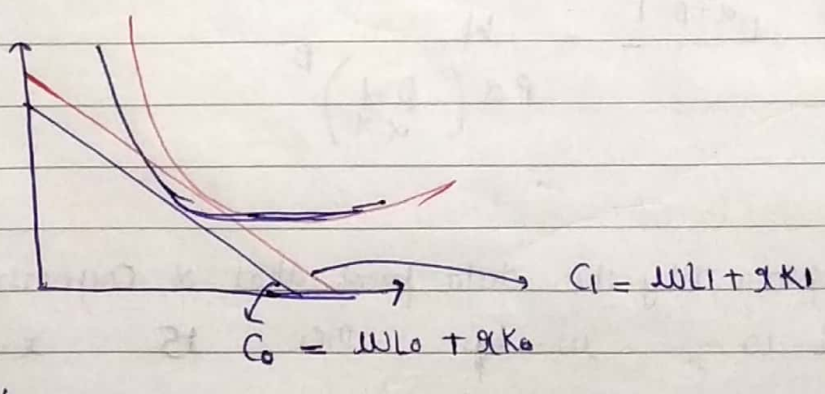
$L = \frac{Q^2}{K}$

$C = wL + rK$

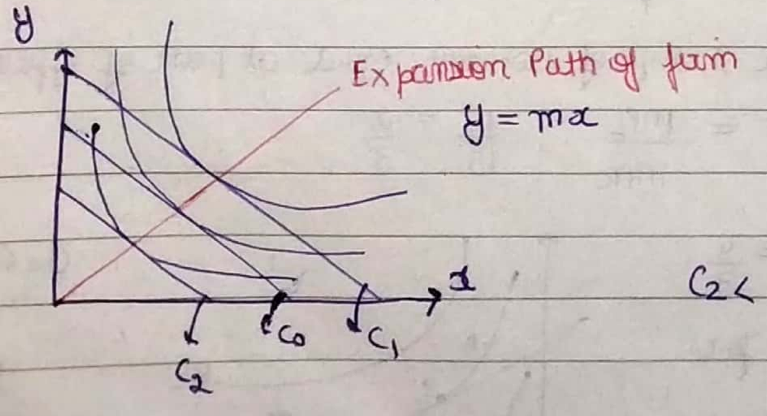
$C = \frac{wQ^2}{K} + rK = \underbrace{\frac{wQ^2}{2}}_{\text{variable cost}} + \underbrace{2r}_{\text{Fixed cost}}$

$TC = TVC + TFC$

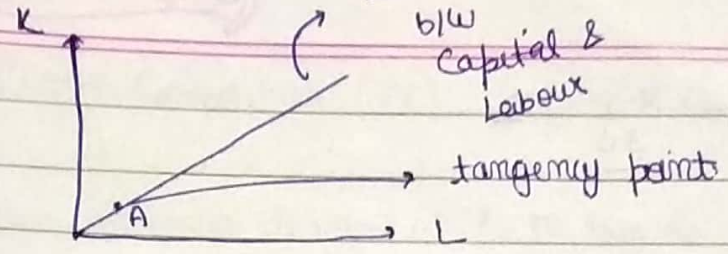
Expansion Path



$C_1 > C_0$



$C_2 < C_0$



$y = mx$

$K = mL$

$m = \frac{fL}{fK} = \frac{w}{r}$

$\frac{eq\ 1}{eq\ 2} = \frac{fL}{fK} = \frac{w}{r}$

$K^{0.5} L^{0.5}$

$\frac{K}{L} = \frac{w}{r}$

$L = \frac{Kr}{w}$

$\frac{\partial Z}{\partial L} = 0 \rightarrow (1)$ $\quad \frac{\partial Z}{\partial \lambda} = 0 \rightarrow (3)$

$\frac{\partial Z}{\partial K} = 0 \rightarrow (2)$

Ques Suppose that a firm is employing 20 workers & it is a only variable input, wage = 60, Avg Product of Labour = 30. The last worker added 12 units to total output & the total FC is 3600. what is MC, AVC, ATC

$L = 20$ $\quad C = wL + rK$

$w = 60$

$AP_L = 30$

$TFC = 3600$

$\cancel{AC} = \frac{\cancel{TC}}{L} = \frac{3600}{20} = \cancel{180}$

Sol $AP_L = \frac{Q}{L}$ $\quad AC = \frac{TC}{Q}$ $\quad MC = \frac{\partial TC}{dQ}$

$MP_L = \frac{dQ}{dL}$

$MP_L \rightarrow$ 1 unit L ↑ → Q Chang

$MC = \frac{W}{MP_L}$ $\quad AC = \frac{W}{AP_L}$

$\frac{1}{MP_L} \rightarrow$ amount of Labour required to get 1 unit of o/p

$\frac{w}{MP_L} \rightarrow$ amount of cost to bear to get 1 unit of o/p

Teacher's Sign

$$AVC = \frac{w}{APL} = \frac{60}{30} = 2$$

$$AVC = \frac{w}{APL} = \frac{TVC}{Q} \Rightarrow 2 = \frac{20 \times 60}{Q} \Rightarrow Q = 600$$

$$AFC = \frac{3600}{600}$$

$$AC = AVC + AFC$$
$$= 2 + 6$$
$$= 8$$

$$MP = \frac{60}{12}$$

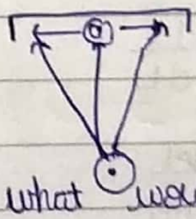
Ques $C = q\, w^{\frac{2}{3}} r^{\frac{1}{3}}$

$$L^* = \frac{\partial C^*}{\partial w} = \frac{2}{3} q\, w^{-1/3} r^{1/3}$$ Shepard's Lemma

$$K^* = \frac{\partial C^*}{\partial r} = \frac{1}{3} q w^{2/3} r^{-2/3}$$

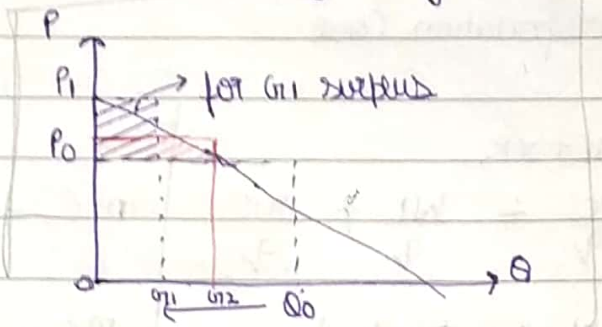
Finding optimal labour & capital given cost function.

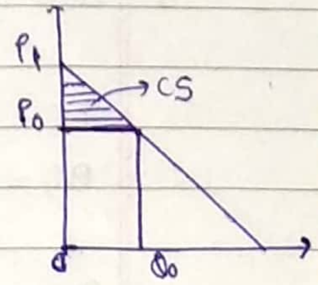
26/8/19
- Perfect Competition (ideal)
- monopoly (Price ~~Distribute~~ discrimination)
- monopolistic (Similar quality of diff variety eg Pepsi, coca cola)
- Oligopolistic (Game Theory)
  - ✓ If I would cut down price then what would rival do
  - → When some firms ↑ Quantity, what would others do
  - → when some firms ↑ price



In Perfect Competition (PC), Consumer Surplus [CS] is max

How consumer decides at P_0 to buy Q_0 goods





→ One would continue purchasing till CS = 0

$\frac{1}{2} \times OQ_0 \times (OP_1 - OP_0)$

→ eg pair of jeans (G_1) at 4500 in march
" " " (G_1) at 1500 in June
→ 4500 - 1500 = 3000 CS

Assumptions

1. Large No of buyers & sellers
N → Sellers selling T shirts, each seller has insignificant share
Similarly buyer's share is negligible

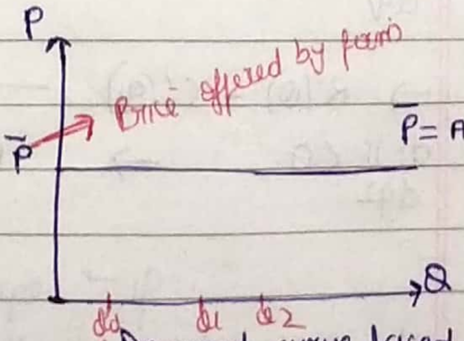
2. Homogenous → Quality of products is identical, no varieties.

3. Firm is a Price-Taker
↳ when any firms have same quantity & quality then they are price taker.

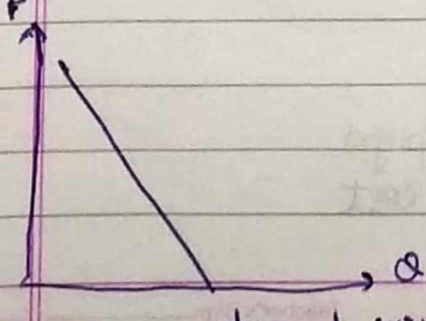
$TR = \bar{P}Q$

$AR = \frac{TR}{Q} = \bar{P}$

$MR = \bar{P}$



Demand curve faced by the firm
→ Quantity demanded by consumers



demand curve for consumers

Eq → MC = MR → q

Seller's Price ⇒ Pat that q

4. Free entry & exit of firms

A firm who is willing to enter market in PC, need not pay any licence amount to government

5 Zero Transportation Cost

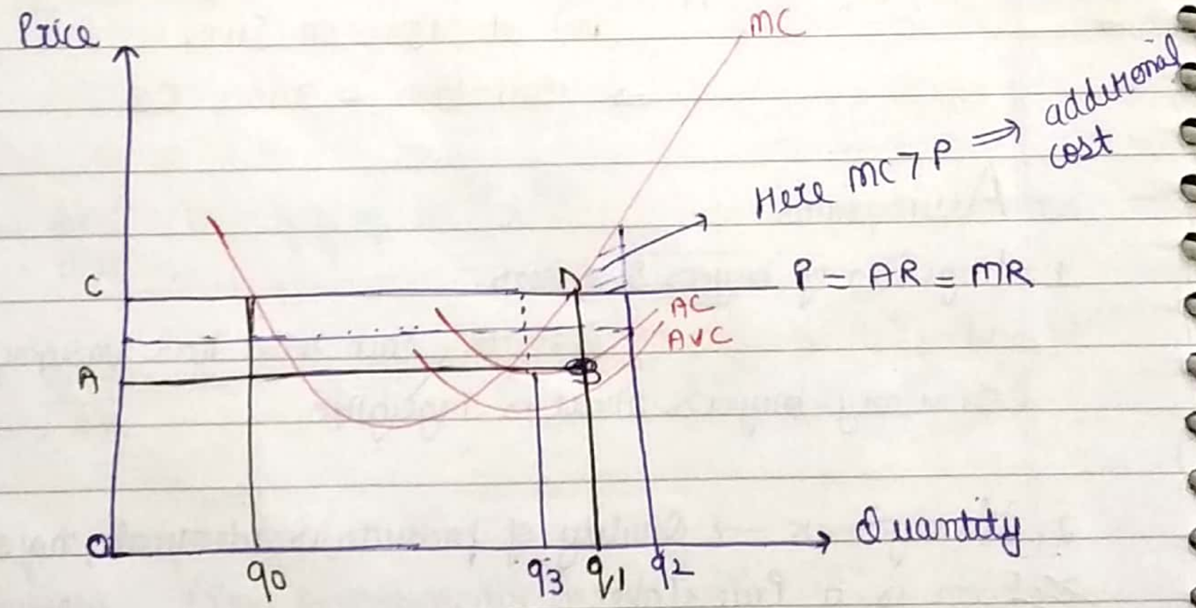
$TC = WL + rK$

$AC = \frac{TC}{q} = \frac{WL}{q} + \frac{rK}{q}$

$AC = W.\frac{1}{AP_L} + r\frac{1}{AP_K}$

$MC = \frac{dTC}{dQ}$

$MC = W.\frac{1}{MP_L} + r\frac{1}{MPK}$



$\pi(q) = R(Q) - C(Q)$ → Producer will try to maximise $\pi(q)$

$\frac{d\pi}{dq} = 0$

$\Rightarrow R'(Q) = C'(q)$ → 1 ⇒ MR = MC

$\frac{d^2\pi}{dq^2} < 0$ ⇒ $R''(Q) < C''(q)$ → 2

q_1 → equilibrium point

if move $q_3 \to q_1$ then earn more profit

" " $q_1 \to q_2$ then additional cost

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$$\Pi = R(q) - C(q)$$

$$\Pi = \bar{P}Q - C(q)$$

$$\frac{d\Pi}{dQ} = \bar{P} - C'(q) = 0 \Rightarrow P = MC$$

$$\frac{d^2\Pi}{dQ^2} < 0 \Rightarrow -C''(q) < 0$$

$P > MC \rightarrow (\uparrow)$ Production
$P < MC \rightarrow (\downarrow)$ Production

If $P > MC \rightarrow CS = 0$
$P = MC \rightarrow CS$ max

28/08/19

why would firm close its business

$TC = wL + rK$

Short Run (SR) $TC = TFC + TVC$ → dependent on q
↙ independent on Q

$$ATC = \frac{TC}{q}$$

SR: $AFC = TFC/q$ $AVC = \frac{TVC}{q}$

$TC = WL + rK$

$SR \rightarrow TC = WL + r\bar{K}$

$$\frac{TC}{q} = w\left(\frac{L}{q}\right) = w\,APL \rightarrow AVC$$

Ques When does firm decides to shut down

→ Supply curve of firm

Long Run → Concept of $AFC = 0$

Production function of Iso-quant

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2/9/19

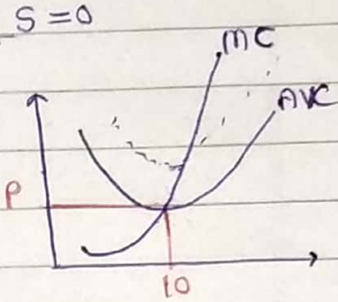
Supply

Shut down Condition of a firm in Perfect Competition Short Run

$P \geq AVC_{min} \rightarrow S > 0$

$P < AVC_{min} \rightarrow S = 0$

Supply function for the firm



$TC = C(q_i) = 0.1q_i^3 - 2q_i^2 + 15q_i + 10$

Step1 Calculate AVC

Step2 Find minima

Step3 In PC, $\boxed{P = MC}$

↓

Producer will Produce here

Step4 Find Price level P

(a) $AVC = \frac{C(q_i)}{q_i} = \boxed{0.1q_i^2 - 2q_i + 15}$

(b) $TVC = 0.1q_i^3 - 2q_i^2 + 15q_i$

(c) $\frac{dAVC}{dq} = 0 \Rightarrow 0.2q_i - 2 = 0$

$\boxed{q_i = 10}$ minima

→ Check second order condition

(d) $MC = \frac{dTC}{dq} = 0.3q_i^2 - 4q_i + 15$

$\boxed{P = MC = 0.3q_i^2 - 4q_i + 15}$

$\Rightarrow 0.3q_i^2 - 4q_i + (15 - P) = 0$

$ax^2 + bx + c$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$q_i = \frac{4 \pm \sqrt{16 - 1.2(15-P)}}{0.6}$

Supply eqn for i^{th} firm

$q_i = \frac{4 + \sqrt{1.2P - 2}}{0.6}$ → for i^{th} firm at level P this will be quantity supplied

$AVC|_{q_i=10} = 0.1(10)^2 - 2(10) + 15 = 10 + 15 - 20 = 5$

$\boxed{\text{Firm operate if } P \geq 5, \ S > 0}$

if $P < 5$, $S = 0$

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For n firms in previous ones total quantity supplied for a price level P is.

$$q = \left(\frac{4+\sqrt{1.2P-2}}{0.6}\right)n$$

II When firms do not have identical cost function

Example take 2 types of firms

(a) $C_{1i} = 0.04 q_{1i}^3 - 0.8 q_{1i}^2 + 10 q_{1i}$

$AVC_{1i} = 0.04 q_{1i}^2 - 0.8 q_{1i} + 10$

$MC = 0.12 q_{1i}^2 - 1.6 q_{1i} + 10$

$P = MC \Rightarrow 0.12 q_{1}^2 - 1.6 q_{1i} + 10 - P$

$$q_{1i} = \frac{1.6 + \sqrt{2.56 - 4(0.12)(10-P)}}{2(0.12)}$$

$$q_{1i} = \frac{1.6+\sqrt{2.56-4.8+0.48P}}{0.24}$$

$\frac{dAVC}{dq} = 0 \Rightarrow 0.08 q_{1i} - 0.8 = 0$

$\boxed{q_{1i} = 10}$

$\boxed{P = AVC_1|_{10} = 0.04(100) - 0.8(10) + 10 = 6}$

(b) $C_{2i} = 0.04 q_{2i}^3 - 0.8 q_{2i}^2 + 20 q_{2i}$

$AVC_2 = 0.04 q_2^2 - 0.8 q_2 + 20$

$\frac{dAVC}{dq} = 0.08 q_2 - 0.8 = 0 \Rightarrow \boxed{q_{2i} = 10}$

$MC = 0.12 q_2^2 - 1.6 q_2 + 20$

$P = MC$

$0.12 q_2^2 - 1.6 q_2 + (20-P) = 0$

$$q_2 = \frac{1.6 + \sqrt{2.56 - (0.12)(4)(20-P)}}{2(0.12)}$$

$\boxed{P = 4 - 8 + 20 = 16}$

$$q_{1i} = \frac{1.6+\sqrt{0.48P-2.56}}{0.24}$$

$$q_{2i} = \frac{1.6+\sqrt{0.48P-7.04}}{0.24}$$

Teacher's Sign

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for $P \geq 6$, Type 1 firm will supply
for $P \geq 16$, Type 2 firm will supply

$P < 6$, $SF1 = 0$ $SF2 = 0$
$6 < P < 16$, $SF1 > 0$ $SF2 = 0$
$P > 16$, $SF1 > 0$ $SF2 > 0$

Ques $C = q^3 - 4q^2 + 8q$
The firm will enter if profit is positive and leave if profit is -ve
1 Find supply function ⟶ Total market demand function
2 Given demand f^n $D = 2000 - 100P$
Determine the eq^n price, quantity & no of firms that are operating in the industry

$AC = q^2 - 4q + 8$

$\frac{dAC}{dq} = 2q - 4 = 0 \Rightarrow q = 2$ → Corresponds to min AC

$\frac{d^2AC}{dq^2} = 2 > 0$ (minima)

$MC = 3q^2 - 8q + 8$
For perfect competition $MC = P$
$3q^2 - 8q + (8 - P) = 0$

$P = AC|_{q=2} = \cancel{(16) - 8(4) + 8} = 4 - 4(2) + 8 = 4$
$P \geq 4$, supply > 0
(1) $q_i = \frac{8 + \sqrt{64 - 12(8-P)}}{6}$ Supply function

(2) $D = 2000 - 100(4) = 1600$ At eq^n level market demand = 1600

each firm at eq^n produces 2 units

Total firms at eq $= \frac{1600}{2} = 800$

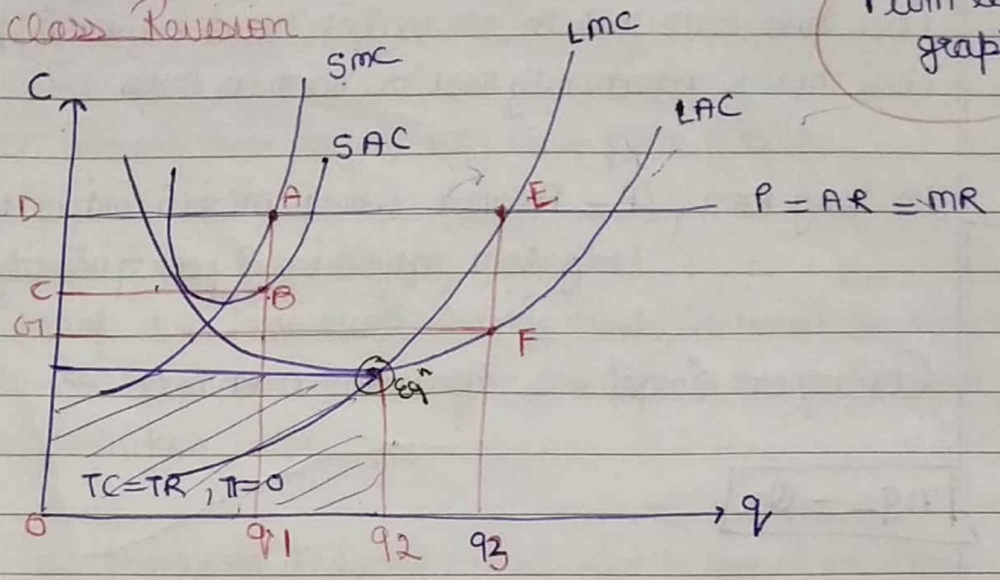
$\boxed{D_{eq} = n \times q_{equb}}$

Teacher's Sign ______

$P \rightarrow$ Selling Price

Last class Revision

Firm level graph



$\pi =$ ABCD

TC = CBOq1

TR = DAq1O

Profit margin has Increased

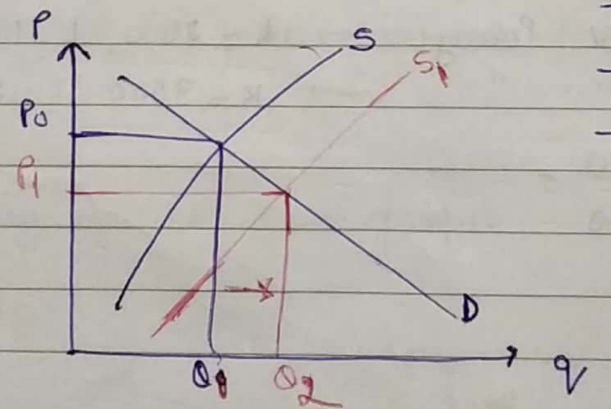
For long Run

TC = GFq3O

TR = DEq3O

π = DEFG

Industry level graph



→ more firms enter market

→ Supply ↑

→ Supply curve shift Right

Existing firms produce at q3 and enjoy profit so more firms join here which lead to ↓ in price level ⟹ now producer profit = 0

π (economic profit) = 0

Accounting profit > 0

Price level will fall in such a way that there will be no opportunity cost

Due to competition q3 moves to q2. Its a loss of q3

This is the price of competition that firm has to pay

Price level will fall to an extent where economic profit $(\pi) = 0$
This means opportunity cost is no more there

In long Run, $P = AC_{min}$ where MC_{min} intersects.
Long run equilibrium ~~per~~ quantity

Consumer Surplus is maximum over here

$$\boxed{nq_2 = Q_2}$$

4/9/19 Case Study on Effluent fees on Steel plant
→ Isoquant
→ isocost
→ Elasticity of Substitution

Pre Gov Policy → $K = 2000$, $L = 10{,}000$ Point A
Post " " → $K = 3500$, $L = 5000$ Point B

} Quantity of o/p same

$x = 40$
$w = 10$

A → B

higher o/c   (+) Eco π

when o/c becomes zero No firm will move from A to B

Industrial Long Run Supply Curve

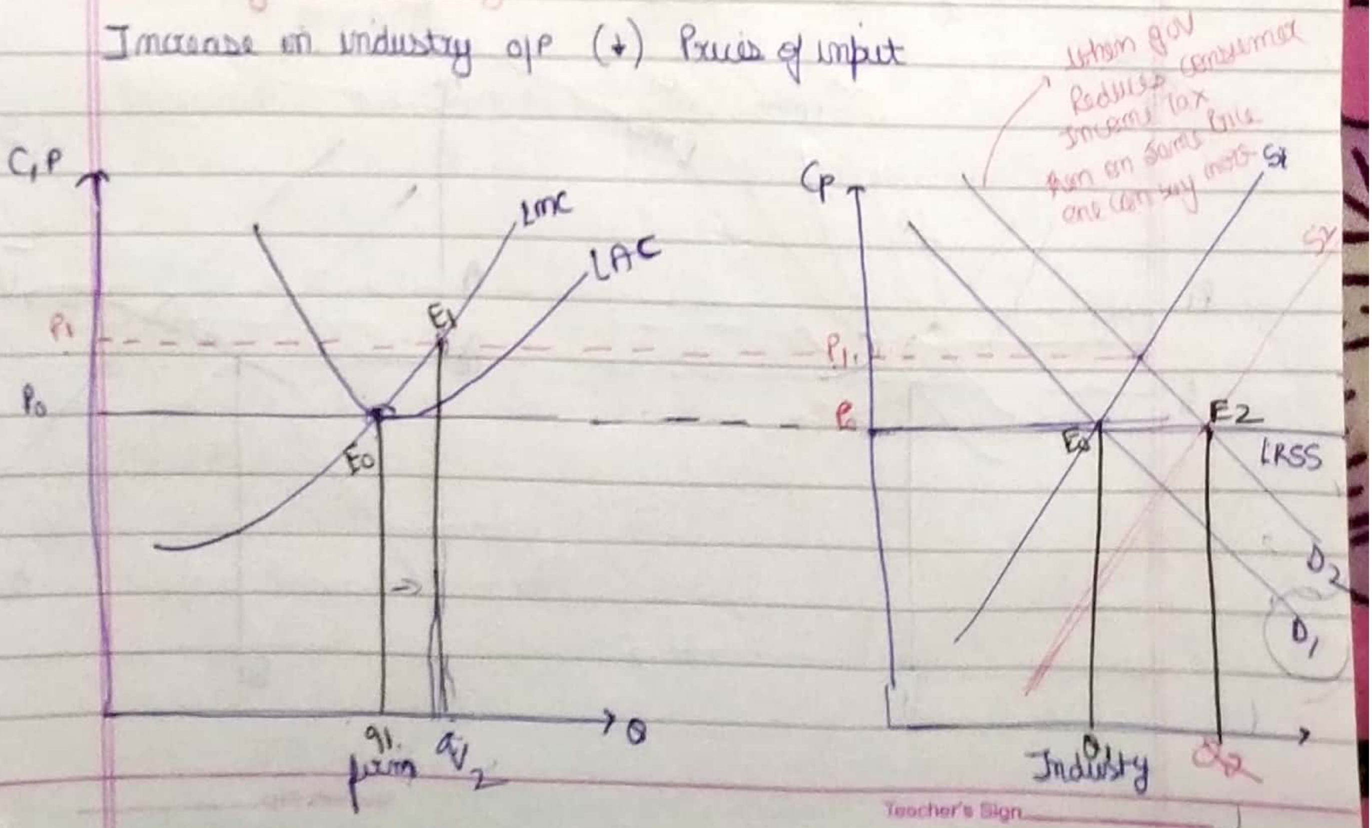
The shape of LR Industrial supply curve depends on an extent to which changes in industry o/p affect the prices the firm must pay for input to production curve.

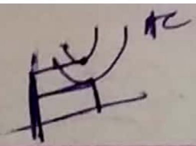
How changes in Industry output affects cost of inputs

1. Constant Cost Industry (unskilled labour)
changes in industry o/p does not change the Prices of inputs

2. Increasing Cost Industry (skilled labour)
Increase in industry o/p (↑) prices of inputs

3. Decreasing Cost Industry
Increase in industry o/p (↓) Prices of input





moving $Q_1 \rightarrow Q_2$ per unit cost of production remains same
Overall cost changes

Ques $TC = q^3 - 8q^2 + 30q + 5$

At what level of q MC cuts AVC

Sol $MC = 3q^2 - 16q + 30$

$AVC = q^2 - 8q + 30$

$3q^2 - 16q = q^2 - 8q$

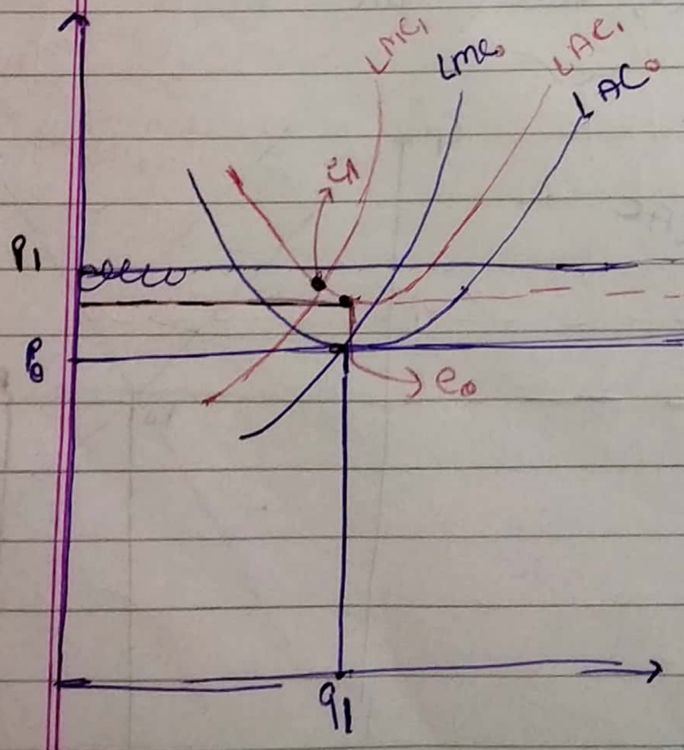
$2q^2 = 8q$

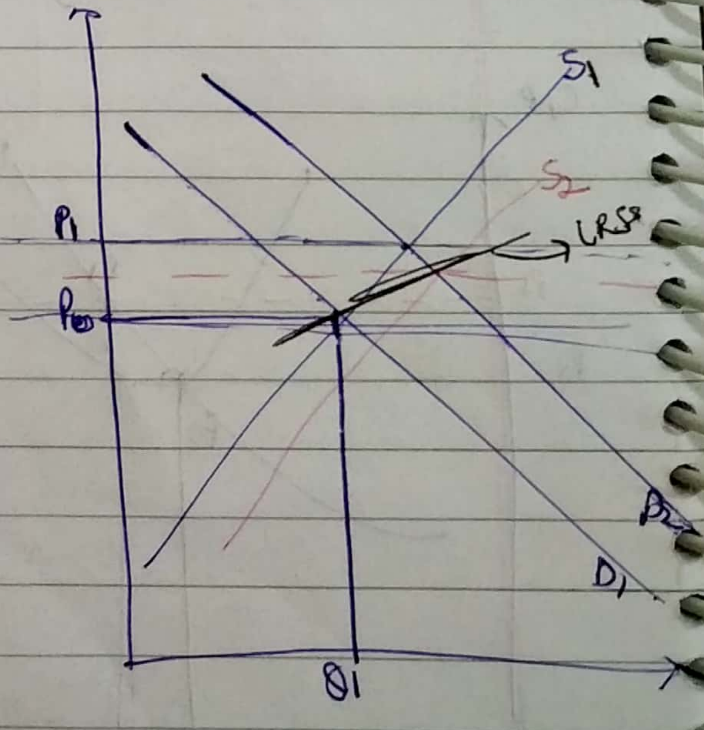
$\boxed{q = 4}$ At $q = 4$ MC cuts AVC

→ In constant cost firm / industry the additional input(s) necessary to produce a higher o/p can be purchased without increasing per unit Price.

Increasing Cost Industry

Gov reduces Income tax





Teacher's Sign

By default per unit cost (↑) as o/p (↑). AC & MC curve shifts up. Long Run equilibrium reached at that point where zero Economic π prevails (LMC = LAC).

Firms will enter as long as ECO π > 0

Ques Market DD : $Q^D = 6500 - 100P$

Market SS : $Q^S = 1200P$

TC : $C(q) = 722 + \frac{q^2}{200}$

Question of Short Run as Fixed cost Present

(a) Eqⁿ Price & Quantity

(b) output supply by firm

(c) No of firms

Eqⁿ Price → $6500 - 100P = 1200P$

$6500 = 1300P$

$P = 5$

~~Quantity~~ demand → 6000

Demand = n × Quantity

(c) $6000 = n \times Q \implies n = \frac{6000}{500} = 12$

(b) $P = MC$

$P = \frac{q}{100}$

$100P - q = 0$

$q = 100P$

$q = 500$ as $P = 5$

Quantity supplied by each firm

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What will happen in Long Run?

→ Currently in Short Run 12 firms are operating
→ Check Economic profit
→ Check LR equl^b condition
→ If (+) Economic profit
→ if > 0 then firms enter

Long Run Eq^b condition

$P = LMC = LAC\,min$

$TC_{LR} = \frac{q^2}{200}$

$LMC = \frac{q}{100}$ $LAC = \frac{q}{200}$

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Monopoly

Mono → one

What makes a Firm monopoly → lobbying

1 Control over critical inputs

→ De-Beers controls Diamond Supply over world [movie Blood Diamond]

→ Alu company of America

→ ONGC controls petroleum needs of India, Jio for 4G services

They had Test Server

Had no cost of operation

2 Technical Barriers

$Q(\uparrow) \rightarrow AC / MC(\downarrow)$ experience economies of scale or economies of scope

3 Legal Restrictions → Some legal instructions by gov can lead to monopoly

eg Defence, Electricity

In monopoly also one cannot charge as much high as they want

$\boxed{\text{Revenue}}$

$P = P(Q)$

$TR = P(Q).Q$

$AR = P(Q)$

$MR = P(Q) + Q P'(Q)$

Elasticity of demand

$P(Q)$ $P(\uparrow), Q(\downarrow)$

$P(\downarrow), Q(\uparrow)$

$$e = -\left[\frac{\text{\% change in Quantity demanded}}{\text{\% change in Price}}\right] = -\frac{dQ/Q}{dP/P}$$

$e = 1 \Rightarrow$ unit elastic

$e \rightarrow \infty \Rightarrow$ Perfectly elastic

$e = 0 \rightarrow$ Perfectly inelastic

$$MR = P + \frac{Q\,dP}{dQ} = P\left(1 - \frac{1}{e}\right) = P\left(1 + \frac{1}{|e|}\right)$$

$\pi = TR(Q) - TC(Q)$

In perfect competition $MR = MC$ equilibrium Condition

$$P\left(1 + \frac{1}{|e|}\right) = MC$$

$$1 + \frac{1}{|e|} > 0 \Rightarrow |e| < 1 \Rightarrow \cancel{\quad}$$

$$1 - \frac{1}{e} > 0 \Rightarrow e > 1$$

A monopolist will always operate in region $e > 1$

$$P - \frac{P}{e} = MC \Rightarrow \boxed{\frac{P - MC}{P}} = \frac{1}{e} \rightarrow$$

Mark over marginal cost as a % of Price

Lerner's index (tells how much Power monopoly has)

$$L = \frac{P - MC}{P}$$

① Monopolist will operate in elastic market ($e > 1$)

② $\frac{P - MC}{P} = \frac{1}{e}$

Lerners index
$L \in [0, 1]$

If $L = 0 \Rightarrow P = MC$ (Perfectly Competitive

(No monopoly Point)

Here monopoly can not implement it's Power control & Price.

③ If $L \approx 1$ (High monopoly Power)

- Inverse Elasticity Rule of Pricing

Ques $D = 2000 - 20P$

$C = 0.05\, q^2 + 10000$

Find equilibrium Quantity & Price for the monopolist if Possible make AR & MR curve.

$\cancel{AR = Q}$

Sol $Q = 2000 - 20P$ | $MC = 0.10q$

$TR = Q\,P(Q) = (2000 - 20P)\,P(Q)$

$P = \dfrac{2000 - Q}{20}$

$\cancel{TR = (2000 - 20P)\left[\dfrac{2000 - Q}{20}\right]}$

$\cancel{= 100(2000 - Q) - P(2000 - Q)}$

$=$

$\cancel{TR = Q\,P(Q)}$

$\cancel{= Q\left[\dfrac{2000 - Q}{20}\right]}$

$\cancel{= Q100 - \dfrac{Q^2}{20}}$

$\cancel{MR = \dfrac{-Q}{10} + 100}$

$MC = \cancel{0.01q}\ 0.1q$

$MR = MC$

$\Rightarrow \dfrac{-Q}{10} + 100 = 0.1Q$

$= 0.2Q = 100$

$$\boxed{\begin{array}{c} Q = 500 \\ P = 75 \end{array}}$$

$TR = Q\,P(Q)$

$= Q\left[\dfrac{2000 - Q}{20}\right]$

$= \dfrac{Q^2}{20} - 1000$

$MR = \dfrac{Q}{10} - 100$

$MC = 0.10q$

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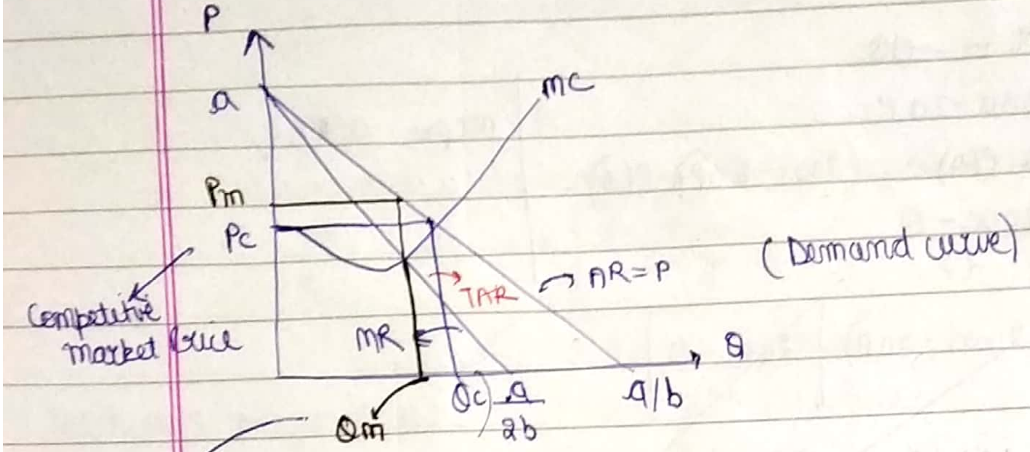
$P(Q) = \alpha - bQ$

$TR = aQ - bQ^2$

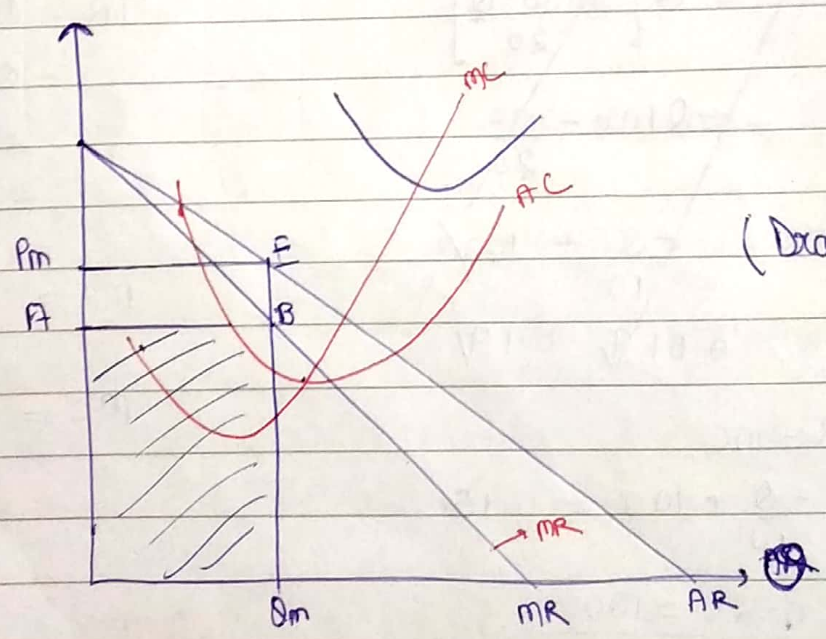
$AR = a - bQ = P$

$MR = a - 2bC$



Monopolist output

Demand curve faced by monopolist ~~for its curve~~ products



(Draw again)

$TC = OABQm$

$TR = OPmEQm$

$\pi = APmEB$

A monopolist will always earn a +ve economic profit

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Multi Plant Monopoly

→ Assume 2 factories / Plants

→ 1 market

→ F_1 , F_2 → In city

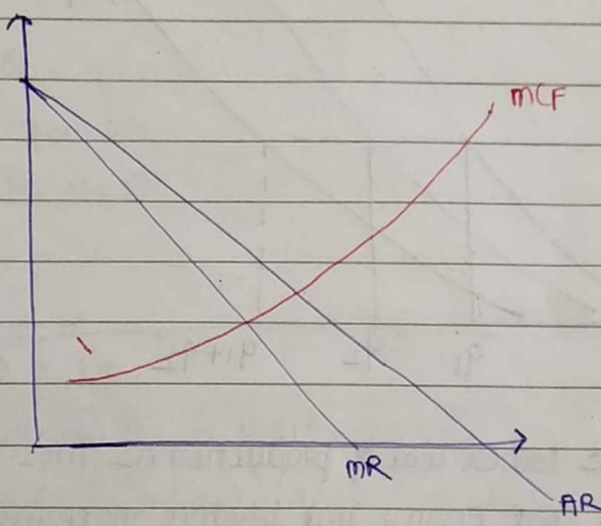
In Haridwar

There cost of production won't be same

If cost of Production in Haridwar is less then will F_2 shift its plant to Haridwar ??

→ MR is same for both

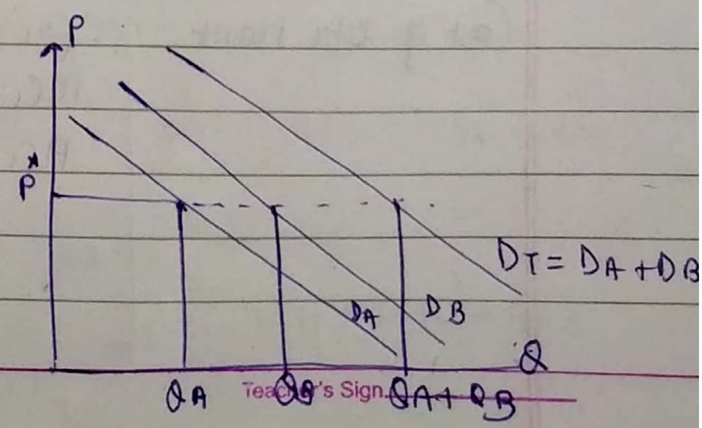
$MR = MCF_1 = MCF_2$ (This should satisfy) for profit



16/9/19 OPEC (Petroleum firm) is Cartel → 1 market
Different Plants

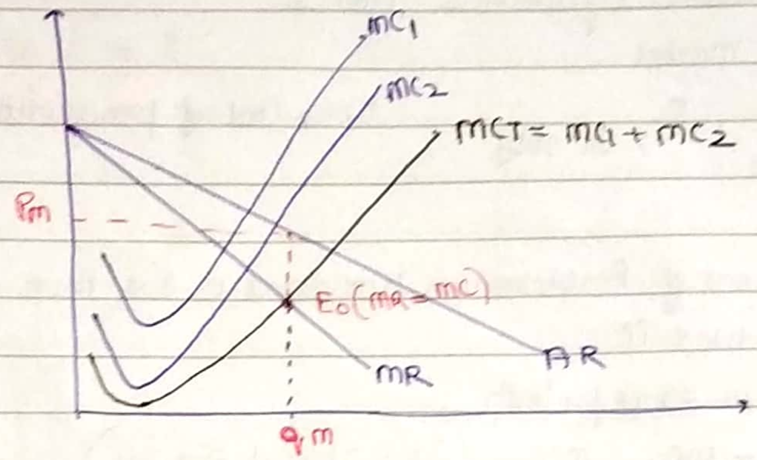
There was a Law MRTP act (1950)

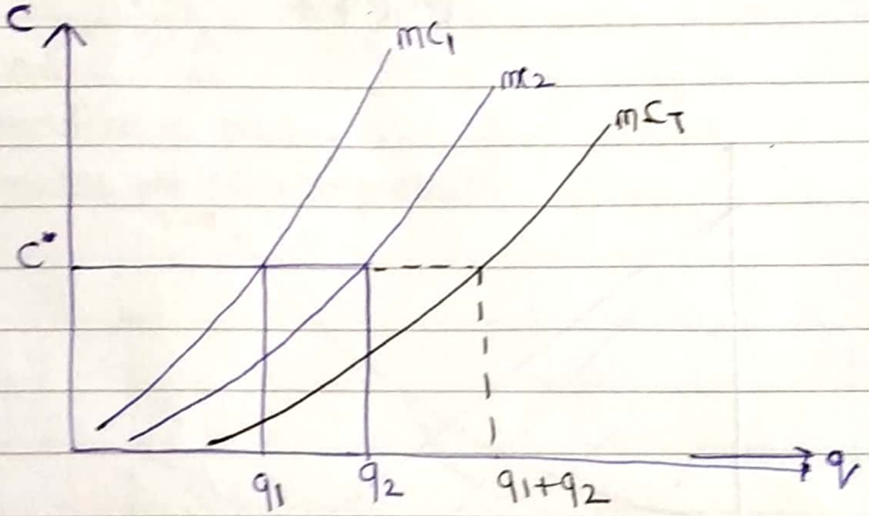
Horizontal Summation



Teacher's Sign.

Eqlb'm Condition for a Multiplant Monopolist / cartel





Firm 2 has lower cost of production as MC_2 lies below MC_1 So at a given Cost Firm 2 will produce more quantity than firm 1

Market DD function $P = a - Q$

$$Q = \sum_{i=1}^{n} q_i$$

Cost of i^{th} Plant $C(q_i) = F + cq_i^2$

$$MC_i = 2cq_i$$

$$AC_i = \frac{F}{q_i} + cq_i$$

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$$\pi = TR - TC$$

$$= PQ - \sum_{i=1}^{n} (F + Cq_i^2)$$

$$\pi = [a - (q_1 + q_2 + \dots + q_n)]Q - (nF + c(q_1^2 + q_2^2 + \dots + q_n^2))$$

$$\frac{d\pi}{dq_i} = 0$$

$$\Rightarrow \quad a - 2\sum_{i=1}^{n} q_i - 2cq_i = 0$$

$$a - 2nq_i - 2cq_i = 0$$

$$P = a - \sum q_i \qquad P = \frac{an + 2ac}{2(n+c)}$$

Ques DD: $Q = q_1 + q_2$

$$P = 700 - 5Q$$

$$C_1(q_1) = 10q_1^2$$

$$C_2(q_2) = 20q_2^2$$

$$\pi = PQ - TC$$

$$= (700 - 5Q)Q - 10q_1^2 - 20q_2^2$$

$$= (700 - 5(q_1 + q_2))(q_1 + q_2) - 10q_1^2 - 20q_2^2$$

$$\frac{d\pi}{dq_1} = 0 \rightarrow$$

$$\pi = 700(q_1 + q_2) - 5(q_1^2 + q_2^2 + 2q_1q_2) - 10q_1^2 - 20q_2^2$$

$$\frac{d\pi}{dq_1} = 700 - 10q_1 - 10q_2 - 20q_1$$

$$700 - 30q_1 - 10q_2 = 0 \Rightarrow 30q_1 + 10q_2 = 700$$

$$\frac{d\pi}{dq_2} = 700 - 10q_2 - 10q_1 - 40q_2$$

$$700 - 50q_2 - 10q_1 = 0 \Rightarrow 10q_1 + 50q_2 = 700$$

Teacher's Sign

$30q_1 + 10q_2 = 10q_1 + 50q_2$

$20q_1 = 40q_2$

$\boxed{q_1 = 2q_2}$

$700 = 30q_1 + 10q_2$

$700 = 30(2q_2) + 10q_2$

$700 = 60q_2 + 10q_2$

$\boxed{q_2 = 10}$, $\boxed{q_1 = 20}$

$p^* = 700 - 5q_1^* - 5q_2^*$

$p^* = 700 - 5(2\times10) - 5(10)$

$p^* = 700 - 100 - 50$

$\boxed{p^* = 550}$

Price - Discrimination

First Degree

Andrew odlyzko

If a company can do price discrimination then how much profit it can have.

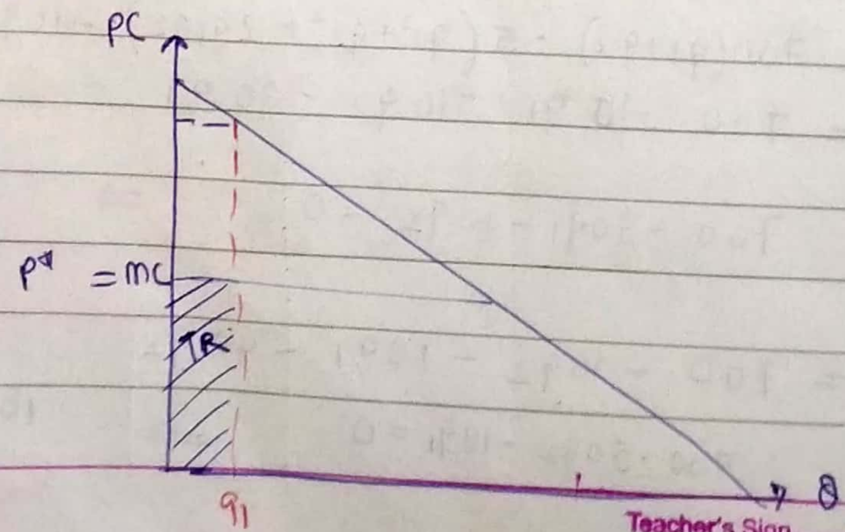
First Degree → In this monopolist is able to extract consumer surplus

For eg a Tea cost = Rs 7

& Consumer is ready to Pay Rs 15

But actual Price is also Rs 7

So when seller sells at Rs 15 → Surplus = Rs 8 taken by seller.



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$$P = 100 - \frac{Q}{20}$$

$$C(Q) = 0.05\,Q^2 + 10{,}000$$

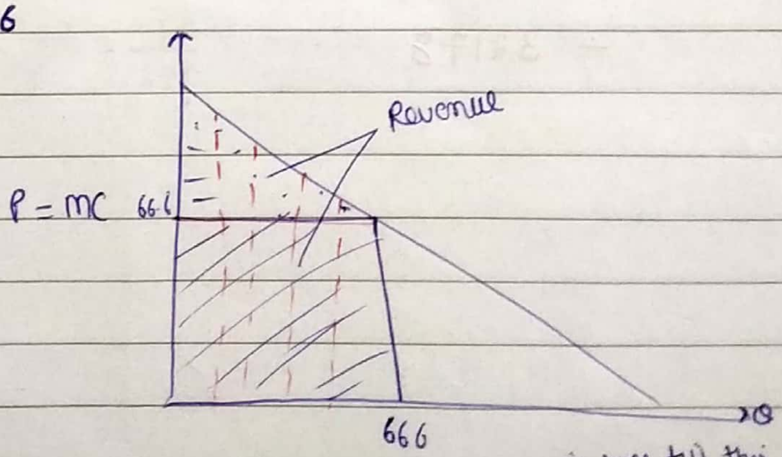
$$\pi = PQ - C(Q)$$
$$= \left(100 - \frac{Q}{20}\right)Q - 0.05\,Q^2 - 10000$$

$$\frac{d\pi}{dQ} = 100 - \frac{Q}{10} - 0.1Q = 0$$

$$1000 - Q - Q = 0$$
$$1000 = 2Q$$
$$\boxed{\begin{array}{l} Q^* = 500 \\ \pi = 15000 \end{array}}$$

Last price till which producer will sell is $P = MC$ (Below this producer won't sell)

$$P = MC$$
$$100 - \frac{Q}{20} = 0.1Q$$
$$2000 - Q = 2Q$$
$$Q^* = \frac{2000}{3} = 666.6$$
$$P^* = 66.6$$



(Producer will sell till this quantity at $P = 66.6$)

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$$\int_0^{666} R(Q)\,dQ = \int_0^{666} P(Q)\,Q\,dQ = \int_0^{666}\left(100Q - \frac{Q^2}{20}\right)dQ$$

$$= \frac{100Q^2}{2} - \frac{Q^3}{60}\Big|_0^{666}$$

$$= \frac{100}{2}(666)^2 - \frac{(666)^3}{60}$$

$$= (666)^2\left(50 - \frac{666}{60}\right)$$

$$\int_0^{666} P(Q)\,dQ = \int_0^{666} 100 - \frac{Q}{20}\,dQ = 100Q - \frac{Q^2}{40}\Big|_0^{666}$$

$$= 100(666) - \frac{(666)^2}{40}$$

$$= 66600 - 11088.9$$

Revenue $= 55511.1$

$$\pi = \text{Revenue} - 32178$$

$$C(Q) = 0.05Q^2 + 10000$$

$$= (0.05)(666)^2 + 1000$$

$$= 32178$$

18/9/19

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Learner's Index

Revision

$$TR = P(Q)\,Q$$

$$MR = \frac{dTR}{dQ} = P(Q) + Q\,\frac{dP(Q)}{dQ}$$

$$= P\left[1 + \frac{Q}{P}\frac{dP}{dQ}\right]$$

$$= P\left[1 + \frac{1}{-e}\right]$$

$$= P\left[1 - \frac{1}{e}\right]$$

$$|e| = \frac{P}{Q}\frac{dQ}{dP}$$

$$e = \frac{-P}{Q}\frac{dQ}{dP}$$

$$MR = P\left[1 + \frac{1}{|e|}\right]$$

$$MC = MR$$

$$MC = P\left(1 - \frac{1}{e}\right)$$

$$\Rightarrow \frac{P - MC}{P} = \frac{1}{e}$$

or $P\left[1 + \frac{1}{|e|}\right] = MC$

$$\frac{P - MC}{P} = \frac{1}{|e|}$$

3rd Degree Price Discrimination

HPD (Horizontal Price differentiation) → Different varieties but same ~~quantities~~ qualities of a Product (eg Shirts of different colours in range of 9K to 10K)

VPD → Different varieties but different qualitites of a Product

Srilankan Tea & Assam Tea
↓ ~~lower~~ higher quality; ↓ ~~high~~ low quality

→ Read Intro of Paper Send

Not always the seller has all information about customers
There can be two group of people for a seller at Inox for eg.
One group that comes on weekend & other that come when Price low on Tuesday. so if seller keeps same price whole week then seller is going to loss one group of people.

money to run movie on Tue is same as that on weekends.
so marginal Cost will remain same. There won't be extra added cost.
But Price discrimination is happening.

eg In Indigo Senior citizen ~~have~~ get discounts → Price discrimination

$$X \begin{cases} X_H \rightarrow P_H \\ X_L \rightarrow P_L \end{cases}$$

$$\pi = P_H X_H + P_L X_L - C(x)$$

how much quantity

$$\frac{d\Pi}{\partial X_H} = 0 \qquad \frac{\partial \Pi}{\partial X_L} = 0$$

$MR_H = MC \rightarrow (1)$ $\qquad$ $MR_L = MC \rightarrow (2)$

$MR_H = MR_L = MC$ (Marginal Revenue from People on weekends same as that on Tuesday)

Ques $P_1 = 80 - 5Q_1$

$P_2 = 170 - 20Q_2$

$MC = 10$

$P_1^d, P_2^d, Q_1^d, Q_2^d = ?$

$TR_1 = P_1Q_1$

$= (80 - 5Q_1)Q_1$

$= 80Q_1 - 5Q_1^2$

$MR_1 = 80 - 10Q_1$

$MR_1 = MC$

$80 - 10Q_1 = 10$

$Q_1^d = 7$

$P_1^d = 45$

$TR_2 = P_2Q_2$

$MR_2 = 170 - 40Q_2$

$MR_2 = MC$

$q_2^d = 4$

$P_2^d = 90$

$$MR_i = P_i\left[1 - \frac{1}{e_i}\right] \quad \forall i$$

$MR_1 = MR_2$

$$P_1\left[1 - \frac{1}{e_1}\right] = P_2\left[1 - \frac{1}{e_2}\right]$$

$$\frac{P_1}{P_2} = \frac{\left[1 - \frac{1}{e_2}\right]}{\left[1 - \frac{1}{e_1}\right]} = \frac{(e_2 - 1)e_1}{(e_1 - 1)e_2}$$

If $e_1 < e_2$

$P_1 > P_2$ {Customer 1 does not get affected much by change in Price)

if $e_1 = e_2 \rightarrow P_1 = P_2$

e_1 & e_2 are not 1

Teacher's Sign.

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Higher Price is charged to that group of Customer that has inelasticity i.e inelastic demand

value of |e| is less

23/9/19 Two Part - Tariff

Walter Oi (1971)

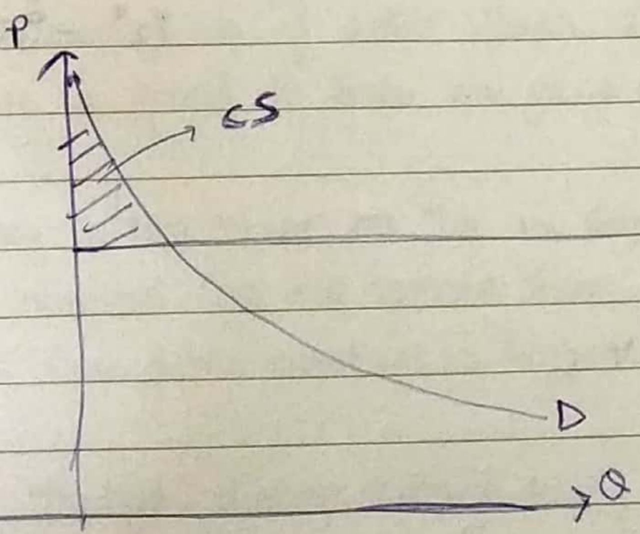
→ A disney land dilemma :

Two part tariff for a mickey mouse monopoly

→ Entry Fee

→ Usage Fee

1.	Single consumer → Single consumer or group of people having same demand.	
2.		consumer
3.	n	consumers



Single Consumer

(per unit cost of producing that product is constant)

Usage fee → P = MC

entree fee → The consumer surplus

Now when Consumer comes

→ entry fee → P = MC

usage fee → The CS

If n rides in disney land with Price P_i

$\boxed{nP_i = CS}$

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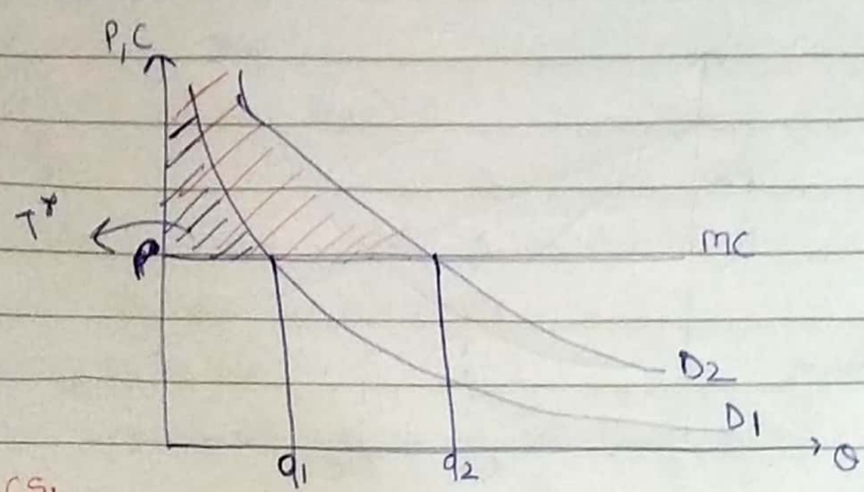
Two Consumers



$CS_2 > CS_1$

Entry fee $\longrightarrow 2T^*$

$\pi \rightarrow 2T^* + \underbrace{(P^* - MC)(Q_1+Q_2)}_{\text{usage fee}}$

For n users

→ If entry fee is high → no of people will drop

→ If usage fee is high all those who are entering wouldn't go for ride

$\pi = T \cdot n(T) + (P - MC)\, Q(n)$ → Total Quantity available

Number of People entering market is function of entry fee.
From this set of people entering, a subset of people go for usage.

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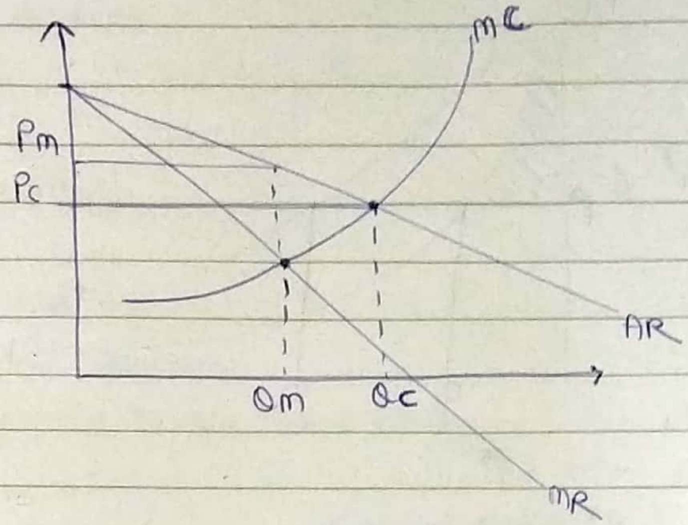
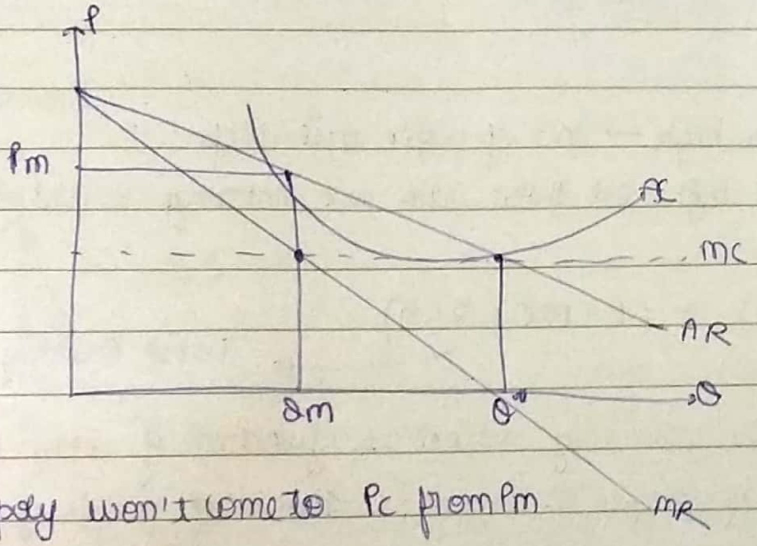
Regulation



Pc → Perfect Competition
Pm → monopolist

For monopoly



AC cannot be greater than AR

Monopoly won't come to Pc from Pm as it would be a loss
So gov says keep $P = AC = P^*$

For eg Price of toll is fixed before construction monopoly will charge Price equal to their avg cost
= ~~gov will provide this extra revenue~~
$P_m - P^*$ to monopoly

Teacher's Sign ______

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Ques $LRC = 20Q - Q^2/8$

$P = 50 - Q$

MCP (marginal cost Price)?

Marginal cost

$20 - \frac{Q}{4} = 50 - Q$

$\frac{3Q}{4} = 30$

$Q = 40$

$P = 10$

Avg cost

$20 - \frac{Q}{8} = 50 - Q$

$\frac{7Q}{8} = 30$

$Q = \frac{240}{7}$

$P = \frac{350 - 240}{7}$

$P = \frac{110}{7}$

Production Function

→ Isoquant are not quasi concave

It is convex to origin

To prove downward sloping → $\frac{dK}{dL} < 0$ | $\frac{dL}{dK} < 0$

Law of diminishing MRTS

$\frac{d^2L}{dK^2} < 0$

$$\begin{vmatrix} 0 & f_L & f_K \\ f_L & f_{LL} & f_{LK} \\ f_K & f_{KL} & f_{KK} \end{vmatrix} < 0$$

$A = -f_L(f_L f_{KK} - f_K f_{LK}) + f_K(f_L f_{KL} - f_K f_{LL})$

$A > 0$

$-A < 0$

Concept of duality

Max $Q \to f(K, L)$

st $C = wL + rK$

or min $C = wL + rK$

st $Q = f(K, L)$

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Marginal cost

$20 - \frac{Q}{4} = 50 - Q$

$\frac{3Q}{4} = 30$

$Q = 40$

$P = 10$

Avg cost

$20 - \frac{Q}{8} = 50 - Q$

$\frac{7Q}{8} = 30$

$Q = \frac{240}{7}$

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$P = \frac{110}{7}$

Ques $LRC = 20Q - Q^2/8$

$P = 50 - Q$

MCP (marginal cost Price)?

Production Function

→ Isoquant are not quasi concave

It is convex to origin

To prove downward sloping → $\frac{dK}{dL} < 0$ | $\frac{dL}{dK} < 0$

Law of diminishing MRTS

$\frac{d^2L}{dK^2} < 0$

$$\begin{vmatrix} 0 & f_L & f_K \\ f_L & f_{LL} & f_{LK} \\ f_K & f_{KL} & f_{KK} \end{vmatrix} < 0$$

$$A = -f_L(f_L f_{KK} - f_K f_{LK}) + f_K(f_L f_{KL} - f_K f_{LL})$$

$A > 0$

$-A < 0$

Concept of duality

Max $Q \rightarrow f(K,L)$

st $C = wL + rK$

or min $C = wL + rK$

st $Q = f(K,L)$

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I $\quad Z = f(K,L) + \lambda_0 \left[C - \omega L - r K \right]$

$$\frac{\partial Z}{\partial K} = \frac{\partial f}{\partial K} - \lambda_0 r = 0 \longrightarrow (1)$$

$$\frac{\partial Z}{\partial L} = \frac{\partial f}{\partial L} - \lambda_0 \omega = 0 \longrightarrow (2)$$

$$\lambda_0 = \frac{MP_K}{r} = \frac{MP_L}{\omega}$$

$$\frac{\partial Z}{\partial \lambda_0} \Rightarrow C - \omega L - rK = 0 \longrightarrow (3)$$

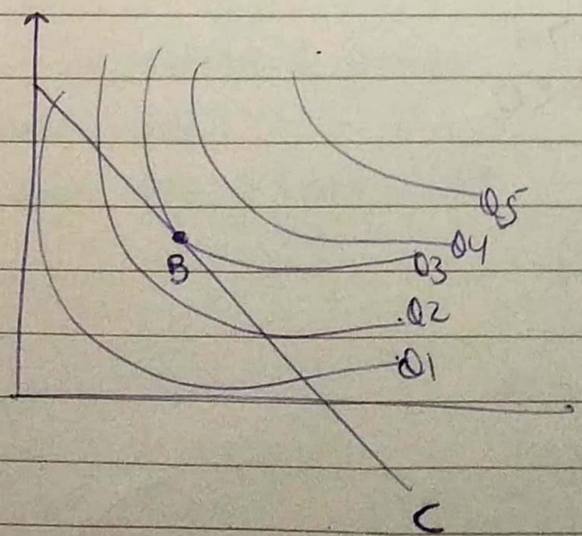
II $\quad L = \omega C + rK + \lambda \left(\bar{Q} - f(L,K) \right)$

$$\lambda_1 = \frac{r}{MP_K} = \frac{\omega}{MP_L}$$

$$\boxed{\lambda_0 = \frac{1}{\lambda_1}} \Longrightarrow \text{Duality}$$

Tangency condition s.t at same cost, more quantity (higher isoquant)

↳ Max Q s.t cost constraint give point of tangency



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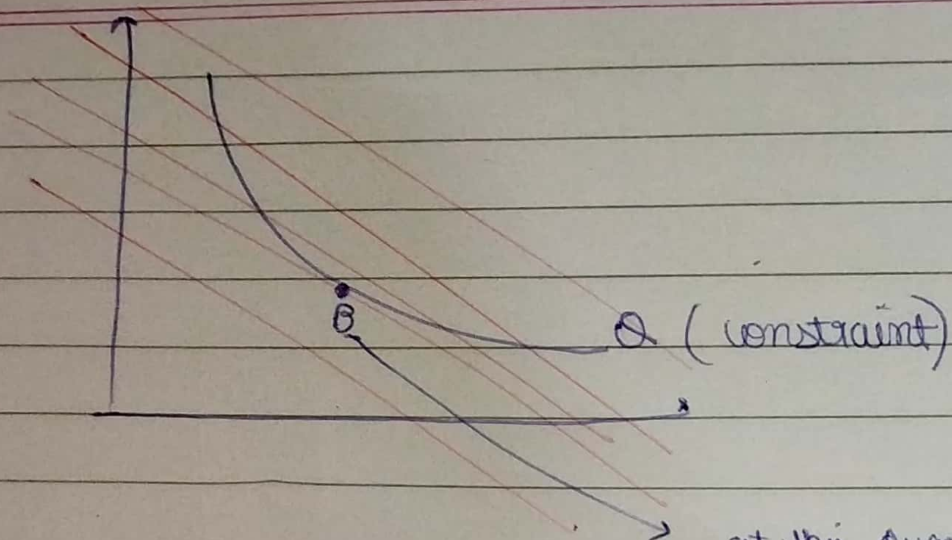

at this quantity cost is minimum