

**The LNM Institute of Information Technology, Jaipur**  
**Mid-Term Examination (September 2016)**  
**Sub: Mathematics-III**

Duration : 90 minutes

Max.Marks: 14+16

Name: Shubham Sethi

Roll No.: 15UCS139 Date: 12-09-2016

Note: **Submit Part-I after 45 minutes of commencement of the exam.** Encircle/Tick the most appropriate answer for objective type questions. Overwriting will be treated as a wrong answer. Use only the last page of main answer sheet for rough work and calculation. Use only pen to write answers. Answers written by pencils will not be evaluated.

### Part-I

(To be returned within 45 minutes)

1. Which of the following function is bounded on complex plane  $\mathbb{C}$  [1 mark]  
 (a)  $\sin z$  (b)  $z^2 - i$  (c)  $\cos^2 z$  ☒ (d)  $e^{i\operatorname{Re}(z)}$
2. The function  $f(z) = \operatorname{Log}(z) + |z|^2$  is [1 mark]  
 (a) Differentiable in  $\mathbb{C}$  except 0 ☒ (b) Differentiable nowhere  
 (c) Differentiable at the origin (d) Analytic at the origin.
3. Which of the following statement is true [1 mark]  
☒ (a) Contour integral of an analytic function defined on a simply connected domain is independent of path.  
 (b) If  $f(z)$  is analytic in a simply connected domain, then for any curve  $C$ ,  $\int_C f(z) dz = 0$ .  
 (c) If  $f(z)$  is analytic and non-constant function on the entire complex plane  $\mathbb{C}$ , then  $f(z)$  can be a bounded function on  $\mathbb{C}$ .  
 (d) If  $f(z)$  is any function defined on a domain  $D$  such that  $\int_C f(z) dz = 0$ , then  $f(z)$  is analytic.
4. Which of the following statement is always true for two complex numbers  $z_1$  and  $z_2$  [1 mark]  
 (a)  $\operatorname{Arg}(z_1 z_2) = \operatorname{Arg}(z_1) + \operatorname{Arg}(z_2)$  ☒ (b)  $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$   
 (c)  $\operatorname{Log}(z_1 z_2) = \operatorname{Log}(z_1) + \operatorname{Log}(z_2)$  (d)  $|z_1 + z_2| = |z_1| + |z_2|$
5. Which of the following statement is false for the complex numbers  $z$  [1 mark]  
 (a)  $\sin iz = i \sinh z$  (b)  $\cos iz = \cosh z$   
☒ (c)  $\sin \bar{iz} = i \sinh \bar{z}$  (d)  $\cos \bar{iz} = \cosh \bar{z}$
6. The value of contour integral  $\oint_C \frac{e^z}{z^2+1} dz$  is, where  $C : |z + \pi| = 2$  [1 mark]  
 (a)  $\frac{-ie^{-i}}{2}$  (b)  $\frac{ie^{-i}}{2}$  (c)  $\frac{-ie^i}{2}$  ☒ (d) 0.
7. The principle value of  $(-1 - i)^{3i}$  is  $e^{9\pi/4} \left[ \cos\left(\frac{3\log 2}{2}\right) + i \sin\left(\frac{3\log 2}{2}\right) \right]$  [2 mark]
8. The zeroes of the function  $\sin(iz - 1)$  are  $\left[ (n\pi - 1)i \right]^{\pm 2} = \frac{(n\pi - 1)i^{\pm 2}}{2}$  [2 mark]
9. The value of  $\oint_C \frac{\sin z}{z(z-5)} dz$  where  $C : |z - 1| = 2$  is 0 [2 mark]
10. The sufficient conditions for the function  $f(z) = u(x, y) + iv(x, y)$  to be differentiable at the point  $z_0$  are/is [2 mark]  
 $u_x, u_y, v_x, v_y$  are continuous  
C-R eq: ?