The LNM Institute of Information Technology
Jaipur, Rajsthan

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Math-II (2016-17), Quiz-2:

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Section: A-2. Tutorial Batch: T-4

Time: 15 Minutes Maximum Marks: 10

1. If y_1 and y_2 are any two solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, then prove that their Wronskian is either identically zero or never zero on $[a, b]$. [4]

2. Work out first three terms of Lagrange polynomial expansion of x^4. What would be the coefficient of x^6 in that expansion, give justification. [6]

   (Note: $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}[(x^2-1)^n]$, is called the Rodrigues' formula for Legendre polynomial P_n of degree n.)

Q1. Wronskian $W(y_1, y_2) = \begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix} = y_1 y_2' - y_2 y_1'$

$I = [a, b]$

Then $y_1'' + P(x_0) y_1' + Q(x_0) y_1 = 0 \quad \times y_2$

$y_2'' + P(x_0) y_2' + Q(x_0) y_2 = 0 \quad \times y_1$

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$y_1 - y_2$ also satisfies thus

$\Rightarrow y_1'' - y_2'' + P(x) y_1' - P(x) y_2' + Q(x)(y_1 - y_2)$

$y_1'' y_2 + P(x_0) y_1' y_2 + Q(x_0) y_1 y_2 = 0$

$y_2'' y_1 + P(x_0) y_2' y_1 + Q(x_0) y_1 y_2 = 0.$

$P(x_0)\left(y_1' y_2 - y_2' y_1\right) = -\left(y_1'' y_2 - y_2'' y_1\right)$

$P(x_0) = -\dfrac{(y_1'' y_2 - y_2'' y_1)}{(y_1' y_2 - y_1' y_1)} = -\dfrac{(y_1'' y_2 - y_2'' y_1)}{W}$

Here $W = c e^{-c_1 x}$ satisfies.

Thus $\boxed{W = c e^{-c_1 x}}$ Here either $c = 0$ or $c \neq 0$

Since $e^{-c_1 x}$ can't be zero

thus W is either $0 \ \forall x \in I$

or not $0 \ \forall x \in I.$

Q2. $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$

First three terms of Legrange polynomial expansion of x^5

Coeff of x^6 in the expansion.

~~$y(x) = a_0 + a_1 x + a_2 x^2 + \dots$~~

$$a_n = \frac{(2n)!}{2^n (n!)^2}$$

$(1-x^2)y'' - 2x y' + p(p+1)y = 0$ — Legendre eq^n

$$a_5 = \frac{(10)!}{2^5 (5!)^2} \rightarrow \text{coeff of } x^6.$$

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