2^{nd} Assignment
Subject: Physics II (Electrodynamics)
Date: 23^{th} Jan 2014

1. **Compute the divergence and curl of the following vector fields.**

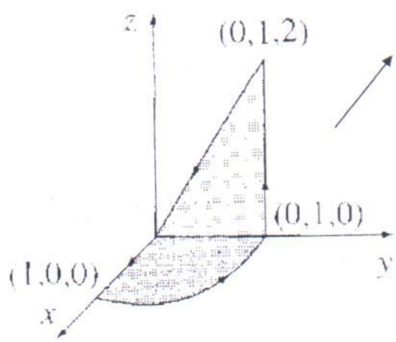
   (i) $\vec{F} = \rho(2+\sin^2\phi)\hat{\rho} + \rho\sin\phi\cos\phi\,\hat{\phi} + 3z\hat{z}$

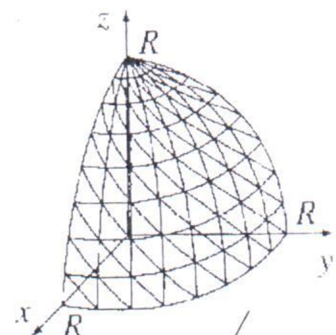
   (ii) $\vec{F} = (r\cos\theta)\hat{r} + (r\sin\theta)\,\hat{\theta} + (r\sin\theta\cos\phi)\,\hat{\phi}$

2. Compute the line integral of

   $$\mathbf{v} = (r\cos^2\theta)\,\hat{\mathbf{r}} - (r\cos\theta\sin\theta)\,\hat{\boldsymbol{\theta}} + 3r\,\hat{\boldsymbol{\phi}}$$ around the path shown in Fig.

   Check your answer, using Stokes' theorem





3. Check the divergence theorem for the function

   $$\mathbf{v} = r^2\cos\theta\,\hat{\mathbf{r}} + r^2\cos\phi\,\hat{\boldsymbol{\theta}} - r^2\cos\theta\sin\phi\,\hat{\boldsymbol{\phi}},$$

   using as your volume one octant of the sphere of radius R Make sure you include the *entire* surface.

$$\vec{\nabla}\cdot\vec{V} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho v_\rho) + \frac{1}{\rho}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla}\times\vec{V} = \left[\frac{1}{\rho}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\rho} + \left[\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial\rho}\right]\hat{\phi} + \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho v_\phi) - \frac{\partial v_\rho}{\partial\phi}\right]\hat{z}$$

$$\vec{\nabla}\cdot\vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{V} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$