# Compton Effect

## Objective

1. Scattering of radiation from an electron
2. Confirmation of particle ~~li~~ nature of radiation (Compton effect)

### Compton Effect

At a time (early 1920's) when the particle (photon) nature of light suggested by the (photoelectric effect) was still being debated, the Compton experiment gave clear and independent evidence of particle behaviour. Compton was awarded the Nobel Prize in 1927 for the "discovery of the effect named after him".

"The effect is important because it demonstrates that light can not be explained as wave phenomenon".

### Compton Effect according to classical wave theory of light

The wave picture predicts scattered radiation having the same wavelength though less energetic than the incident radiation.

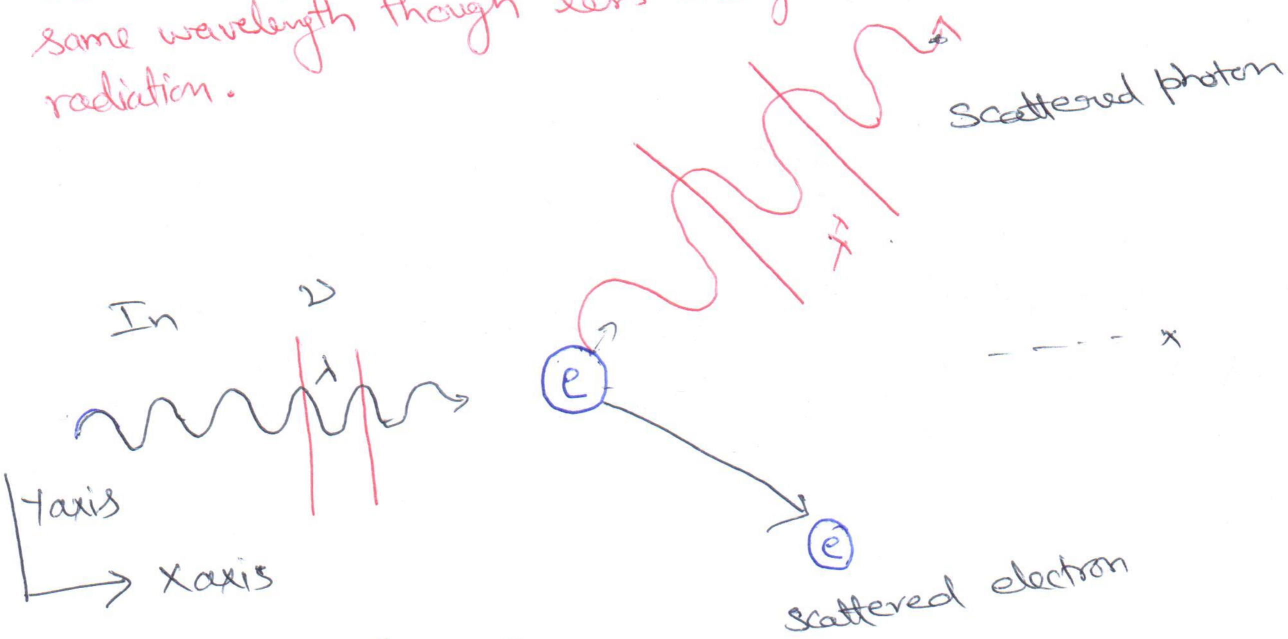



figure I

Figure 2 shows such a collision: an X-ray photon collides an electron (assumed to be initially at rest in the laboratory coordinate system) and is scatted away from its original direction of motion while electron receives an impulse and begins to move.

We can think the photon as losing an amount of energy in the collision that is the same as the K.E. gained by electron.

Loss in photon energy = Gain in electron Energy

$$\boxed{h\nu - h\nu' = K.E.} \quad (1)$$

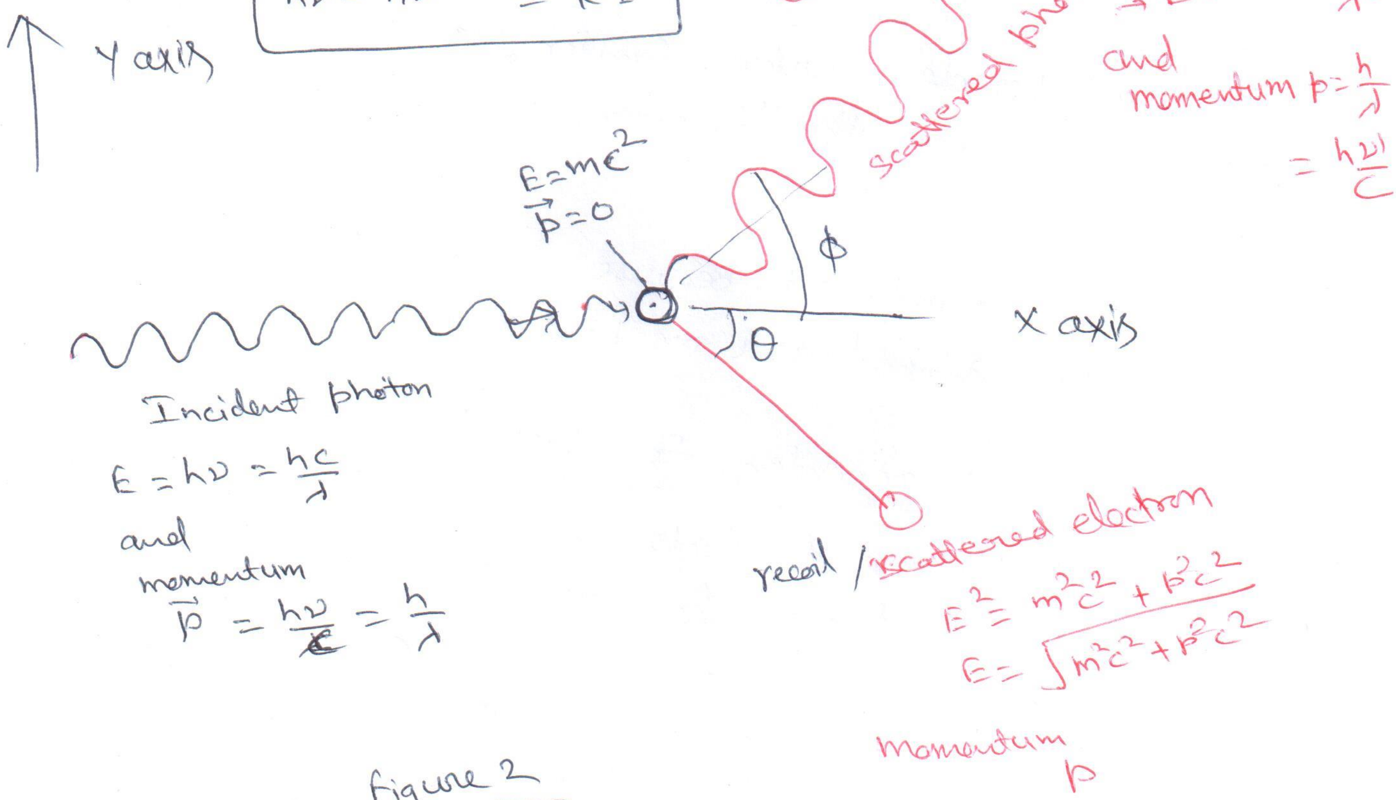
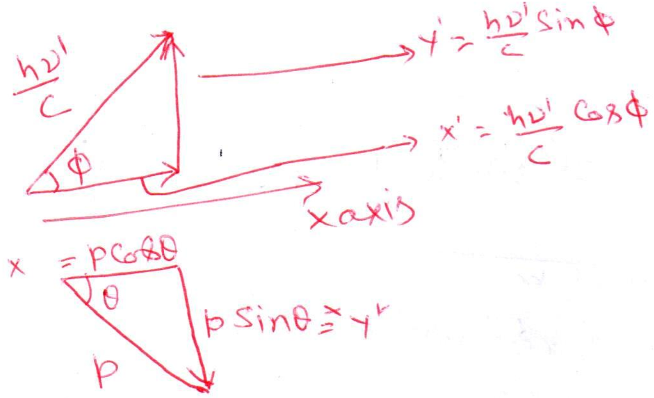



Figure 2

~~Here~~ Compton's observation is consistent with what we expect if photons, considered as particle, collide with ~~elastic in~~ electrons in an elastic collision.

Vector diagram of momentum and their components of the incident and scattered photons and scattered electrons.




Since energy and momentum is conserved in such an event and as a result scattered photon has less energy (longer wavelength than the incident photon).

Momentum is conserved

It's a vector quantity and in collision must be conserved in each of two mutually perpendicular direction

Case I direction of motion

Initial momentum (only photon) = final momentum (s. photon + s. electron)

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta$$

or

$$pc\cos\theta = h\nu - h\nu'\cos\phi \quad \text{--- (2)}$$

Case 2 for $\perp$r direction

Initial momentum = Final momentum

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta$$

or

$$pc\sin\theta = h\nu'\sin\phi \quad \text{--- (3)}$$

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Square equation (2) & (3) and add it

$$p^2c^2(\cos^2\theta + \sin^2\theta) = (h\nu - h\nu'\cos\phi)^2 + (h\nu')^2 \sin^2\phi$$

$$\boxed{p^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\phi} \quad --- \quad (4)$$

↳ Now determine the value of $pc$.

Conservation of energy: (relativistic effect)

Energy before the collision = Energy after collision

$E_{e(rest)} + E_{photon}(h\nu) = E'_{(photon)}(h\nu') + E_{scattered}$

$$mc^2 + h\nu = h\nu' + \sqrt{m^2c^4 + p^2c^2}$$

$$mc^2 + h\nu - h\nu' = \sqrt{m^2c^4 + p^2c^2}$$

from equation (1) $K.E. = h\nu - h\nu'$

$$mc^2 + K.E. = \sqrt{m^2c^4 + p^2c^2}$$

square above equation

$$(mc^2 + K.E.)^2 = m^2c^4 + p^2c^2$$

$$\boxed{p^2c^2 = K.E^2 + 2K.E\, mc^2} \quad --- \quad (5)$$

now ~~from~~ put the value of $p^2c^2$ from equation (4)

$$h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\phi = (h\nu - h\nu')^2 + 2(h\nu - h\nu')mc^2$$

$$\cancel{2mc^2 h(\nu-\nu') =}$$

$$\cancel{h^2\nu^2} + \cancel{h^2\nu'^2} - 2h\nu h\nu'\cos\phi = \cancel{h^2\nu^2} + \cancel{h^2\nu'^2} - 2h^2\nu\nu' + 2(h\nu - h\nu')mc^2$$

$$2mc^2(h\nu - h\nu') = 2h^2\nu\nu'(1-\cos\phi)$$

$$\nu = \frac{c}{\lambda}$$

$$\cancel{2}mc^{\cancel{3}}\cancel{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{\cancel{2}h^{\cancel{2}}\cancel{c^2}}{\lambda\lambda'}(1-\cos\phi)$$

$$mc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{h}{\lambda\lambda'}(1-\cos\phi)$$

or

$$\boxed{\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)} \quad \text{---} (6)$$

Compton Formula

or

$$\boxed{\Delta\lambda = \lambda_c(1-\cos\phi)} \quad \text{---} (6)$$

where $\Delta\lambda = \lambda' - \lambda$ and $\lambda_c = \frac{h}{mc}$ Compton wavelength.

for an electron $\lambda_c = 2.426\times10^{-12}\,m$

$\lambda_c = 2.426\ pm$

$1\ pm = 1\ pico\ meter = 10^{-12}\ meter$

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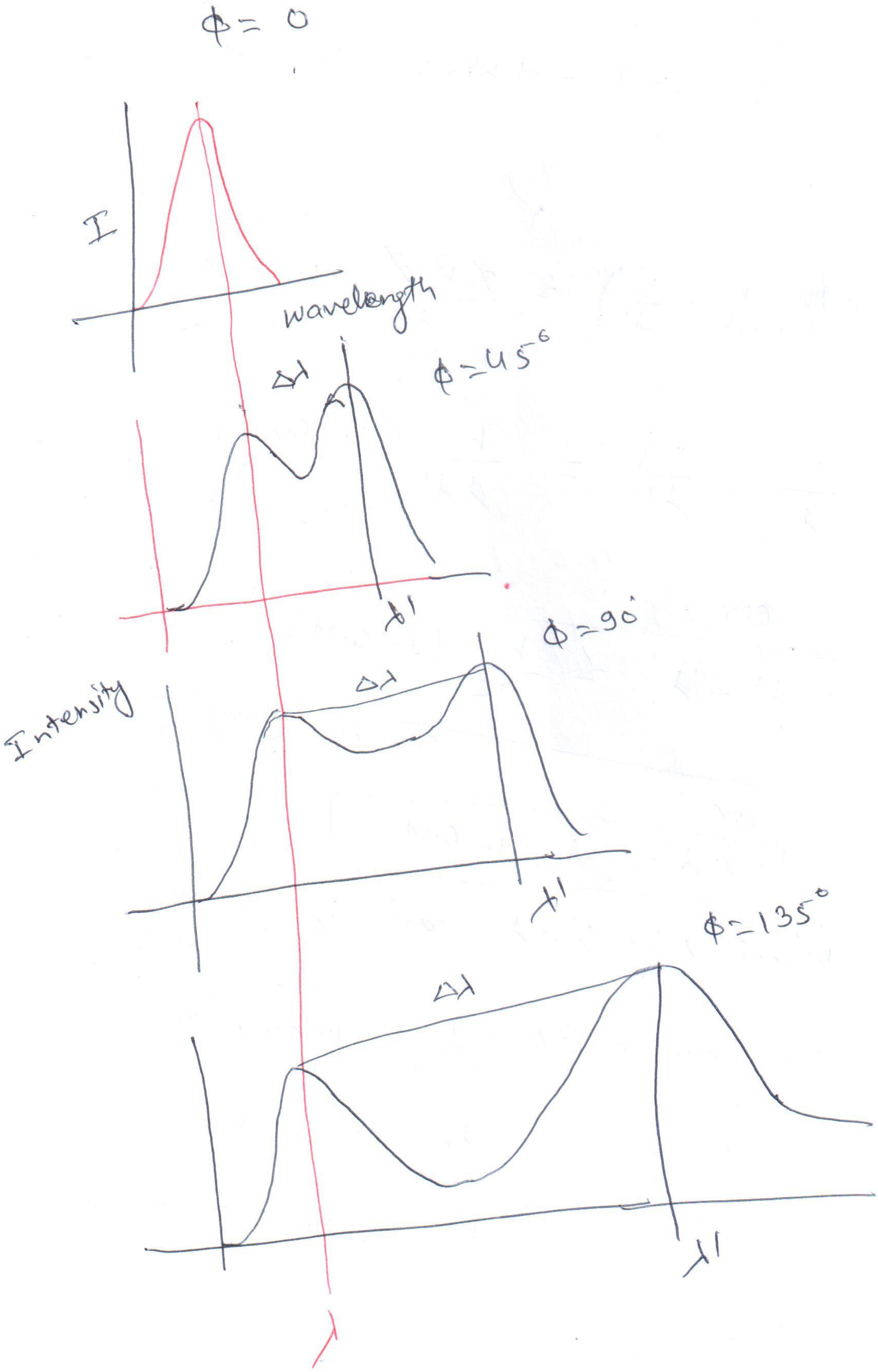
Change in wavelength corresponds to $\phi$.

Greatest wavelength change possible corresponds to $\phi \simeq 180^\circ$.



## Recap

① Compton Effect is a process in which x-rays collide with electrons are scattered.

② Unlike the prediction of classical wave theory, the wavelength of the scattered radiation does not depend on the intensity of incident light & scattering but ~~of~~ depends on wavelength of angle.

③ Compton effect can be explained by particle nature of light.

④ Compton effect is best exhibited with short wavelength of light like x-rays.

⑤ A free electron can not absorb a photon because it is not possible to simultaneously satisfy energy-momentum conservation.

**Question** In compton effect experiment, why a free electron can not absorb a photon? Show that mathematically.

**Ans.**


~~Com~~ Consider ~~a~~ free electron at rest which absorbs a photon energy of $h\nu$.

The final energy of electron would be

$$= h\nu + mc^2$$

According to relativistic principle, if momentum of the electron $p$, the total energy is given by

$$E = \sqrt{p^2c^2 + m^2c^4}.$$

When the electron absorbs the incident photon, the momentum of the photon would be transferred to the electron. Since the electron was initially at rest (i.e. with zero momentum) its final momentum $p$ is $p(= h\nu/c)$. Thus we have

~~E + m.~~ total energy

$$pc + m_0c^2 = \sqrt{p^2c^2 + m_0^2c^4}$$

$$\cancel{p^2c^2} + \cancel{m^2c^4} + 2pc\cdot mc^2 = \cancel{p^2c^2} + \cancel{m^2c^4}$$

$$\boxed{2mpc^3 = 0}$$ which is not possible.

The reason why an electron bound to an atom can absorb a photon (as in compton effect) is that the electron can share some of the resulting momentum with the ion which has a much larger mass.