

The LNM Institute of Information Technology
Jaipur, Rajasthan
MATH-II
Assignment 6

1. Show that the following equations are exact and hence find their general solution:

$$(i) (\cos x \cos y - \cot x) = (\sin x \sin y + 1)y' \quad (ii) y' = 2x(ye^{-x^2} - y - 3x)/(x^2 + 3y^2 + e^{-x^2}).$$

2. Verify that

$$\frac{1}{2}(Mx + Ny)d(\ln(xy)) + \frac{1}{2}(Mx - Ny)d(\ln(x/y)) = Mdx + Ndy.$$

Hence show that (i) if the differential equation $M(x, y)dx + N(x, y)dy = 0$ is homogeneous, then $1/(Mx + Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$, (ii) if the differential equation $M(x, y)dx + N(x, y)dy = 0$ is not exact but is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$, then $1/(Mx - Ny)$ is an integrating factor unless $Mx - Ny \equiv 0$.

3. Assuming that the differential equation $M(x, y)dx + N(x, y)dy = 0$ has an integrating factor which is a function of $x + y^2$, find the relation to be satisfied by M and N . Show that the equation $(3y^2 - x) + 2y(y^2 - 3x)y' = 0$ admits an integrating factor which is a function of $(x + y^2)$. Hence solve this differential equation.
4. Show that $2\sin(y^2) + xy\cos(y^2)y' = 0$ admits an integrating factor which is a function of x only. Hence solve the differential equation.
5. Reduce the following differential equations into linear form and solve:

$$(i) y^2 y' + y^3/x = \sin x \quad (ii) y' \sin y + x \cos y = x \quad (iii) y' = y(xy^3 - 1).$$

6. Find the orthogonal trajectories of the following families of curves:

$$(i) e^x \sin y = c \quad (ii) y^2 = cx^3.$$

7. Find the family of oblique trajectories which intersect the family of straight lines $y = cx$ at an angle of 45° .

Note: An oblique trajectory is a curve that intersect each member of a given family of curve at a constant angle $\alpha \neq 90^\circ$.

8. Show that the following families of curves are self-orthogonal:

$$(i) y^2 = 4c(x + c) \quad (ii) x^2/c^2 + y^2/(c^2 - 1) = 1.$$

Supplementary problems from “Advanced Engg. Maths. by E. Kreyszig (8th Edn.)

- (i) Page 32, Q.10,12,17,26,29,35
(ii) Page 39, Q.13,18,20,28,29,33,34
(iii) Page 51, Q.7,9,10,15,17