$$\begin{array}{ccc} \boxed{\text{Newtonian Physics}} & \xrightarrow{\text{fast}} & \boxed{\text{Special relativity.}} \\ \downarrow \text{small} & \searrow^{\text{fast \& small}} & \\ \boxed{\text{Quantum Physics}} & & \boxed{\text{Quantum field theory}} \end{array}$$

# Eg of classical Physics:

① In classical world, harmonic oscillator gives cont. energy.

$x(t) = A \sin\omega t$

$F = -kx$

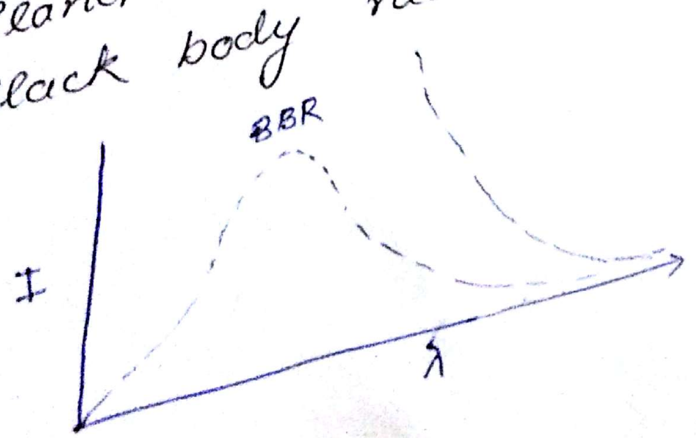
Quantum Harmonic oscill.
diatomic molecule
P.E depends on $x^2$
Energy is quantised

② Pendulum $T = 2\pi\sqrt{\frac{l}{g}}$ $\sqrt{g} = $

$\boxed{\text{Waves}}$ → spread in
→ superposed.

$\boxed{\text{Particles}}$ → Localized
→ cannot pass

# Max Planck - father of Q.T.
# Planck's Quantum Theory.
# Black body radiation (84)




A black body absorb EM radiat
radiates

# Stefan Boltzmann Law (87)

$I = a\sigma T^4$ $\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}$ stefan boltzmann const

$a \leq 1$

$u = \sigma A T^4$ A = surface area.
$\sigma$ = stefan-boltzman const.
$u$ = energy of thermal radiation emitted by BB

$u = \sigma e A T^4$
$0 < e \leq 1$ $e = a$
e = emittivity
a = absorptive power

# Wien's law (86).

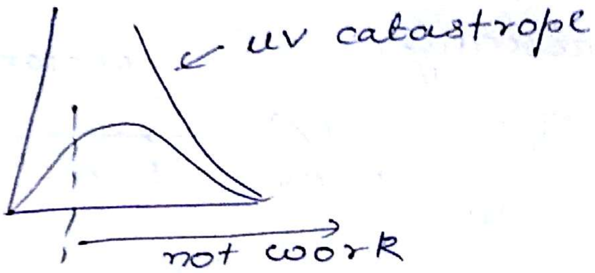
$\lambda_{max} T = const = 2$…
(fails at low $\nu$)

Q- $\lambda_{yellow\ light} = 600\,nm$ T
$\lambda_{blue} = 470\,nm$ $T = \frac{289}{4}$…

Temp of sun = 6000 K.
Temp of human body = 37°C.

# Rayleigh Jeans Law

$I = \frac{2\pi c K T}{\lambda^4}$




Classical average energy of atom = KT

$I(\lambda, T) = \frac{2hc^2}{\lambda^5} \times \frac{1}{e^{\frac{hc}{\lambda KT}} - 1}$ $I(\nu, T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{\frac{h\nu}{KT}} - 1}$

Q- corresponding $\lambda$. $= \frac{2898}{310} \times \cancel{10^3}\, \mu m = 9.34\, \mu m$

$\lambda = \frac{2898\, \mu m}{288} = 10.06\, \mu m$.

# photoelectric effect;

T↑ peak $\lambda$ ↓ energy emitted ↑
(total area under graph ↑)

$I \propto$ max KE
Current $\not\propto \nu/\lambda$

classicaly average energy of standing wave $= KT$.
average energy of oscillator $= \dfrac{h\nu}{(e^{\frac{h\nu}{kT}} - 1)}$

Intensity $\propto$ max KE of photoelectron (classical concept)

# PEE

$K = h\nu - W$

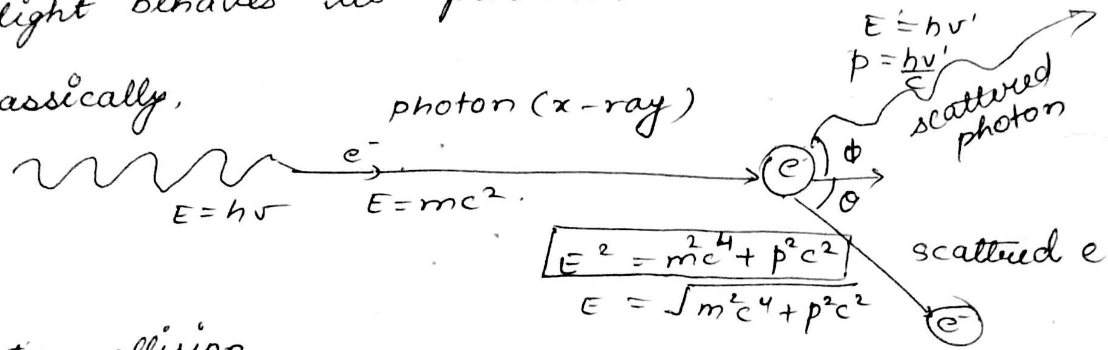
Intensity $\propto$ no. of photoe's emitted

# Compton effect.

Objective:

① light behaves as particle.

classically,




$$\boxed{E^2 = m^2c^4 + p^2c^2}$$

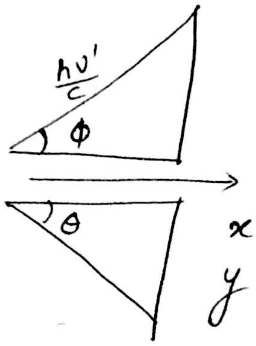
$$E = \sqrt{m^2c^4 + p^2c^2}$$

elastic collision

Loss in photon energy = Gain in $e^-$ energy.

$$\boxed{h\nu - h\nu' = K.E} \text{ — ①}$$

momentum is vector quanlity.




$X = \frac{h\nu'}{c}\cos\phi$

$Y = \frac{h\nu'}{c}\sin\phi$

$x = p\cos\theta$

$y = p\sin\theta$.

momentum is conserved in direction of motion

Initial momentum = Final momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu}{c}\cos\phi + p\cos\theta$$

$$pc\cos\theta = h\nu - h\nu\cos\phi \text{ — ②}$$

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$$0 = \frac{h\nu'}{c} \sin\phi - p \sin\theta$$

$$pc \sin\theta = h\nu' \sin\phi \quad \text{— (3)}$$

square eq (2) & (3) and add it

$$p^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\phi$$

energy is conserved.
↓
(Relativistic)

Before collision = After collision

$$E_{electron} + E_{photon} = E_{\substack{scattered \\ photon}} + E_{\substack{scattered \\ e^-}}$$

$$mc^2 + h\nu = h\nu' + \sqrt{m^2c^4 + p^2c^2}$$

after rearranging and solving it.

$$p^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + 2(h\nu - h\nu')mc^2 \quad \text{— (5)}$$

$$h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + 2(h\nu - h\nu')mc^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\phi$$

$$2mc^2(h\nu - h\nu') = 2h^2\nu\nu'(1 - \cos\phi)$$

$$mc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{h}{\lambda\lambda'}(1 - \cos\phi)$$

$$\Delta\lambda = \lambda_c(1 - \cos\phi)$$

$$\boxed{\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)}$$ compton formula.

$\lambda_c$ = compton $\lambda$ = $2.426 \times 10^{-12}$ m.

So, it behaves as particle as ($\lambda$ changing).

Compton effect (recap)

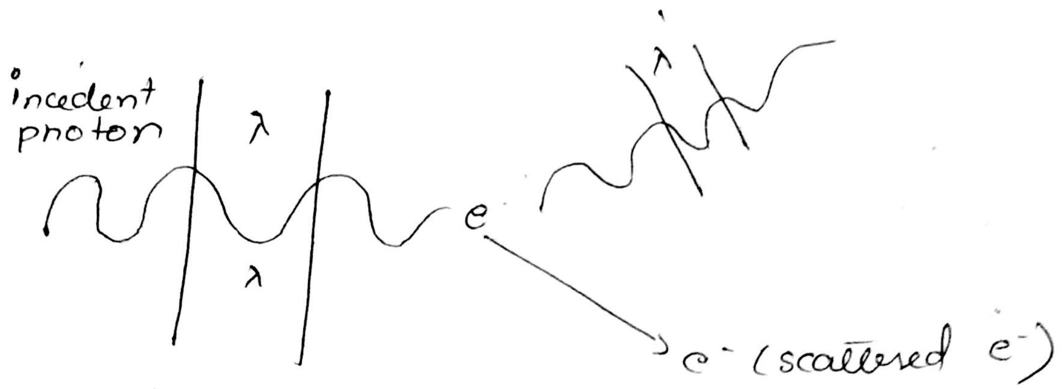
This experiment confirms particle nature of light.

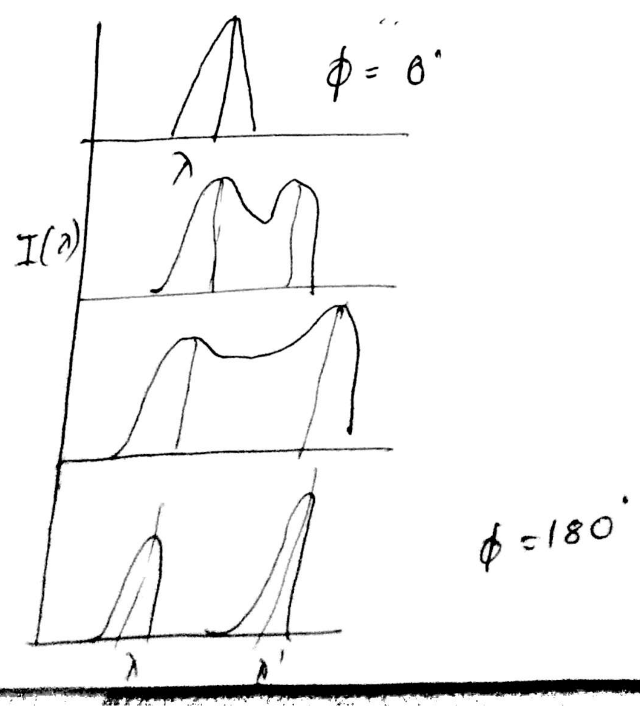
$\Delta\lambda = \lambda_c(1-\cos\phi)$ $\phi \rightarrow$ scattering angle.

$\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$

$\lambda_c = 2.43 \times 10^{-12}$ m

Q Show that mathematically ~~ea~~ every $e^-$ free cannot absorb a photon
(It does not follow energy momentum conservation law)



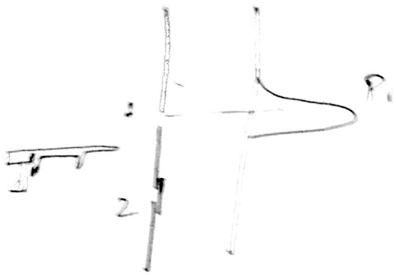



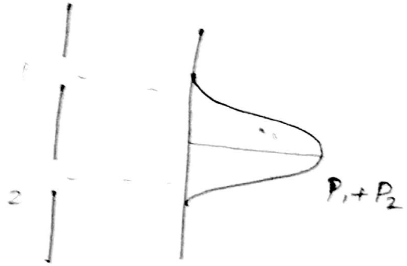
① with bullets

① Bullets are indestructable

② Bullets arrive at screen in lumps.

$P_1$ = prob. of getting bullets on screen 1.
$P_2$ = " 2





when $1^{st}$ slit open

$I = |h_1|^2$

$h_1$ = hight of ripple.

$I_{12} = |h_1 + h_2|^2$

$I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta.$

② with electrons.

$\phi$ = prob. amplitude

$P_1 = |\phi_1|^2$

$\phi$ = complex no. ~~$\phi = a+ib$~~

$P_{12} = |\phi_1 + \phi_2|^2 = \phi_1^2 + \phi_2^2 + 2|\phi_1 \phi_2|\cos\delta$

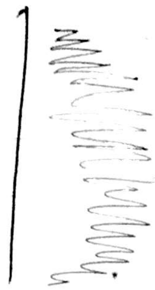
$= |\phi_1|^2 + |\phi_2|^2 + \underbrace{\phi_1 * \phi_2 + \phi_1 * \phi_2}_{\text{Interference.}}$

when light source is placed, electron it behaves as bullet.

col I      col II      col III

N      N      N''

high intensity      low intensity



$P_{12}$

actual (schematic)

$P_{12}$

observed. (bullets)

→ $\phi = \phi_1 + \phi_2$

$P = |\phi_1 + \phi_2|^2$

$P = P_1 + P_2$

$= |\phi_1|^2 + |\phi_2|^2$

+ Uncertainty principle

$$\boxed{\Delta x . \Delta p \geq \frac{h}{4\pi}}$$

$\Delta x$ = uncertainity of posn.

$\Delta p$ = uncertainty of momentum.

$$\lambda = \frac{h}{p} \qquad h = 6.63 \times 10^{-34}$$

+ Young's double slit experiment

$$n\lambda = d\sin\theta$$

+ Bragg's law (due to interference)

$$2d\sin\theta = n\lambda.$$

d = distance between 2 crystal layers

---

$$F_e = \frac{mv^2}{r}$$

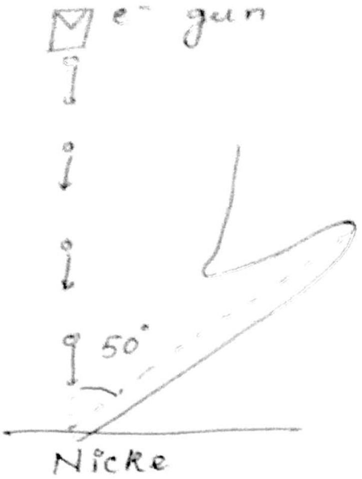
$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^{\cancel{2}}} = \frac{mv^2}{\cancel{r}}$$

$$r = \frac{Ze^2}{4\pi\varepsilon_0 mv^2}$$

Total energy $= \frac{mv^2}{2} - \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$

$= KE + PE$

Davisson Germer expr
(wave nature)




(due to interference)

$d \sin\theta = n\lambda$

$p = mv$

$K.E = eV$

$\frac{1}{2} mv^2 = eV$

$\frac{1}{2} \frac{m^2 v^2}{m} = eV$

$\lambda = \frac{h}{\sqrt{2meV}}$ (A°)

$\lambda = \sqrt{\frac{1.5}{V}}$ (nm)

$h\nu = E_u - E_i$

$L = mvr = \frac{nh}{2\pi}$

$r_n = \frac{n^2 a_0}{Z}$

$E_n = \frac{kze^2}{2r_n} = -E_1 \frac{z^2}{n^2}$

$v_n = \frac{n\hbar}{m r_n} = \frac{e^2}{4n\pi\varepsilon_0 \hbar}$

$\boxed{\begin{matrix} E = U/2 \\ E = -K \end{matrix}}$

$E_1 = -\frac{m_e e^4}{8\varepsilon_0^2 h^2} \cdot \frac{z^2}{n^2}$

Q- determine $\lambda$
(a) for ball. = 46 gram (mass)
(b) for $e^- = 9.1 \times 10^{-31}$ kg.

(a) $\lambda = \frac{h}{mV} = \frac{6.63 \times 10^{-34}}{46 \times 10^{-3} \times 50} = 0.0028 \times 10^{-31}$
$= 2.8 \times 10^{-34}$

(b) $\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 10^{8}} = 3.6 \times 10^{-12}$ $\cancel{10^{-11}}$

+ Pressure waves periodically varies. in water waves
+ Prob. / wave function varies periodically in matter waves.

wave function $\psi = \phi$.

$P = |\psi|^2$

1. $\Psi(x, y, z, t)$

Probability amplitude $= |\Psi|^2$ has large value.

↓ strong possibility to get the object.

↓

Dirac notation

$\langle \quad \rangle$

$| \rangle \longrightarrow$ Ket

$|x = 3\rangle$

$|b\rangle$

$|x = 3, x = 5\rangle \neq$

It represents the initial state.

$\langle | \longrightarrow$ bra

represents final state.

$\langle x | \Psi \rangle = \Psi(x)$

The probability amplitude that a system $|\Psi\rangle$ has posn. ~~+ve~~ $x$.

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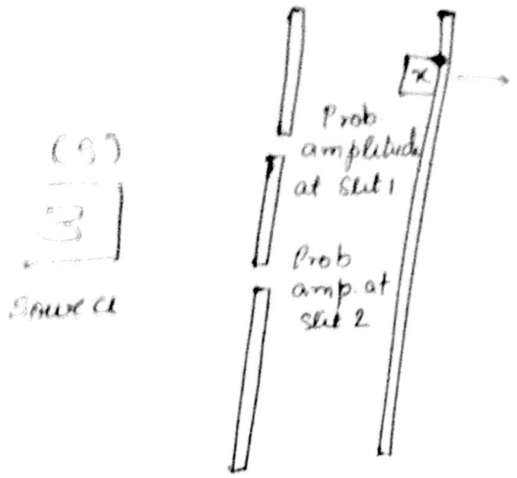



detector (x)

$\langle x | S \rangle$

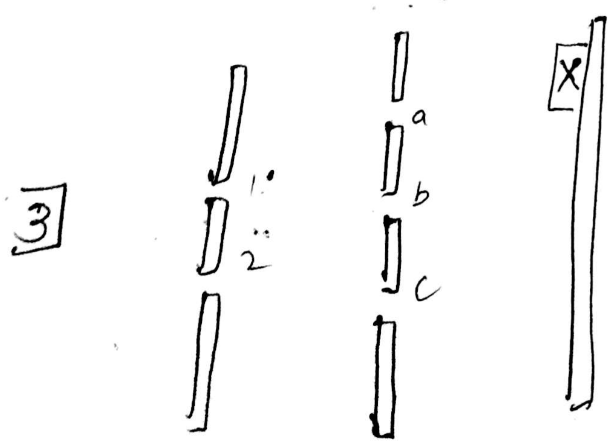
$\psi_s(s) = 1$

$\langle s / s \rangle = 1$

Probability amplitude at $x$ due to waves from S.
= Probability amplitude
$= \psi_s(x)$.

$\varphi$ - write down prob ampl.
= ⟨Final posn / initial posn⟩
= ⟨particle arrives at $x$ / particle leaves S⟩
$= \langle x / S \rangle$ both slits open.

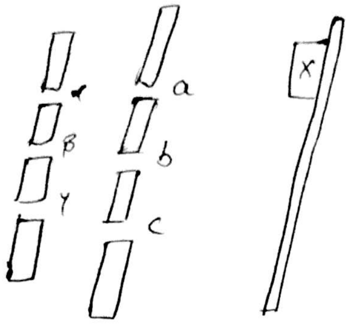
$\langle x / S \rangle_{both}$ = particle through 1 + particle through 2.
$= \langle x/S \rangle$ through 1 + $\langle x/S \rangle$ through 2 $= \psi(x)$
$= \langle x / 1 \rangle \langle 1 / S \rangle + \langle x / 2 \rangle \langle 2 / S \rangle$



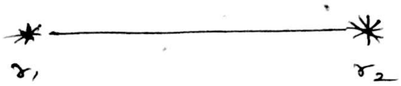

$P(A) = \langle x/a \rangle \langle 1/S \rangle \langle a/1 \rangle$
$+ \langle x/a \rangle \langle 2/S \rangle \langle a/2 \rangle$
$+ \langle x/b \rangle \langle 1/S \rangle \langle b/1 \rangle$
$+ \langle x/b \rangle \langle 2/S \rangle \langle b/2 \rangle$
$+ \langle x/c \rangle \langle 1/S \rangle \langle c/1 \rangle$
$+ \langle x/c \rangle \langle 2/S \rangle \langle c/2 \rangle$

$P(A) = \langle x|a\rangle\langle a|1\rangle\langle 1|s\rangle$

$\boxed{3}$




$$\langle x|s\rangle = \sum_{\substack{i=\alpha,\beta,\gamma \\ \alpha=a,b,c}} \langle x|\alpha\rangle\langle\alpha|i\rangle\langle i|s\rangle$$




momentum $p$

$$\langle r_2|r_1\rangle = \frac{e^{i p \vec{r}_{12}/\hbar}}{r_{12}}$$

$$p^2c^2 = E^2 - m_0^2c^4$$

De Broglie Phase Velocity

$$V_p = \nu\lambda = \frac{c^2}{v}$$

$V_p \not> c$

$V_p$ = phase velocity of particle

$$E = h\nu = \gamma mc^2$$

$$\nu = \frac{\gamma mc^2}{h}$$

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} \qquad V_p = \frac{\gamma mc^2}{h} \times \frac{h}{\gamma mv} = \frac{c^2}{v}$$

$V_p = \dfrac{c^2}{v} \longrightarrow$ mathematical expr.

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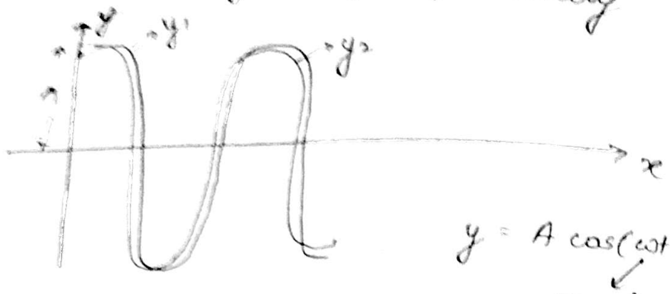
v of object c

$V_p > C$

\# Phase velocity & group velocity




$\omega = 2\pi f$

$k = \frac{2\pi}{\lambda}$

$y = A\cos(\omega t - kx)$

angular frequency

propagation constant or wave velocity

$$\psi = A e^{-i(\omega t - kx)} = A e^{i(kx - \omega t)}$$

**Phase velocity**: velocity of all components of wave.

$$\phi = \omega t - kx = \text{const.}$$

$$kx = \omega t + \text{const.}$$

$$x = \frac{\omega t}{k} + C$$

$$\frac{dx}{dt} = \frac{\omega}{k} + 0$$

$$\therefore V_p = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda$$

**Group velocity**

Let us say $y_1$ & $y_2$ waves are superimposing and amplitude of two waves are same.

Let us assume that

$y_1 \longrightarrow \omega$ & $K$

$y_2 \longrightarrow \omega + \Delta\omega$ & $K + \Delta K$

$$y = y_1 + y_2 \quad \text{—— (1)}$$

$$y_1 = A\cos(\omega t - kx) \quad \text{—— (2)}$$

$$y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x] \quad \text{—— (3)}$$

$$y = y_1 + y_2$$

$$= \boxed{A(\cos\alpha + \cos\beta) = 2A\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}$$

where $\alpha = \omega t - kx$

$\beta = (\omega + \Delta\omega)t - (k + \Delta k)x$

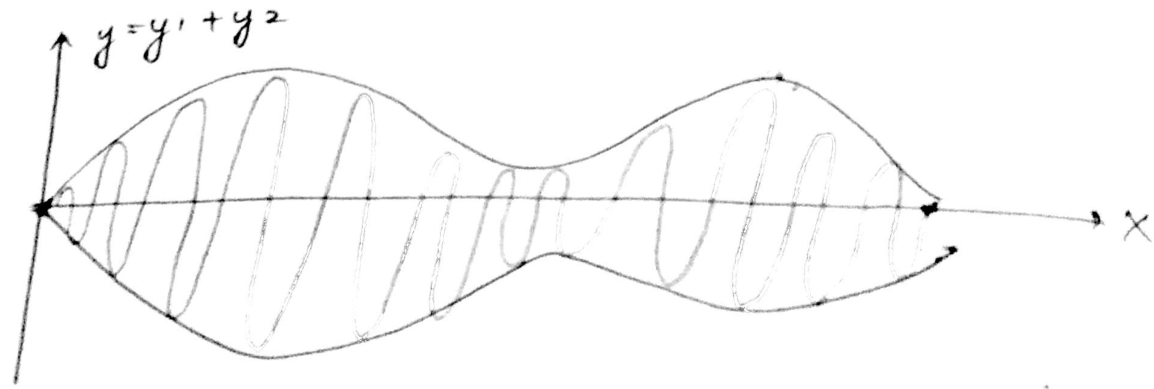
$$y = 2A\cos\left(\frac{(2\omega+\Delta\omega)}{2}t - \frac{(2k+\Delta k)}{2}x\right)\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right)$$

$$2\omega + \Delta\omega \simeq 2\omega \quad \& \quad 2K + \Delta K = 2K$$

$$\Rightarrow \quad y = \underbrace{2A\cos(\omega t - kx)}_{\text{Amplitude}}\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k x}{2}\right)$$

Small scale oscillations

large scale oscillations




$$V_g = \frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta K}$$

# Relationship between $V_p$ & $V_g$

$$V_g = \frac{d\omega}{dK} = \frac{d(V_p K)}{dK}$$

$$V_g = V_p + K\frac{dV_p}{dK}$$

$$V_g = V_p - \lambda\frac{dV_p}{d\lambda} \longrightarrow \text{in dispersive medium.}$$

# Dispersive medium

A medium in which wave v is $\nu/\lambda$ dependent.

For const. $V_p$, $V_g = V_p$

# Group velocity for matter waves:

$$V_g = \frac{d\omega}{dK}$$

$$\omega = 2\pi\nu = 2\pi\frac{\gamma mc^2}{h} = \frac{d\omega}{dv}$$

$$y = A\cos(\omega t - kx)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi\gamma mv}{h} = \frac{dK}{dv}$$

$$\cancel{V_g} = \cancel{\frac{d\omega/dv}{dK/dv}} = \frac{\cancel{2\pi\gamma mc^2}\,\cancel{h}}{\cancel{h}\,\cancel{2\pi\gamma mv}} = \frac{c^2}{v}$$

$$\frac{d\omega}{d\gamma} = d\left(\frac{2\pi\lambda mc^2}{h}\right)$$

$$\frac{dK}{d\gamma} = d\left(\frac{2\pi\gamma mV}{h}\right) = d\left[\frac{2\pi m}{h} v\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= \frac{2\pi mc^2}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$\cancel{= -\frac{v}{2}\left(1-\right.}$$

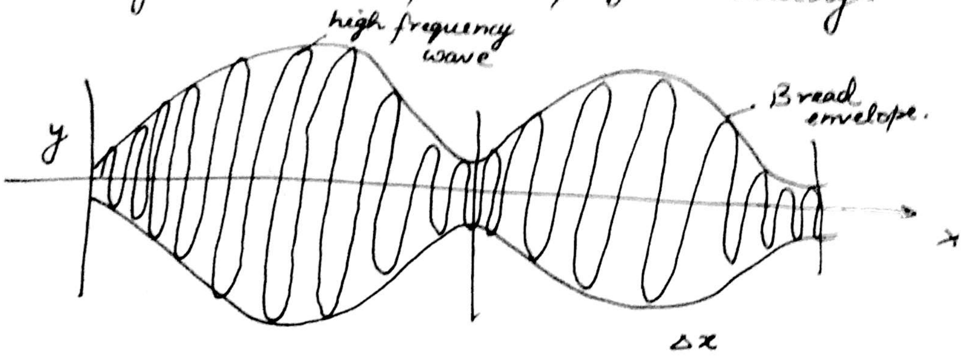
$$d\left[\frac{2\pi mc^2}{h}\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= -\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\frac{2\pi mc^2}{h}\left(\frac{-2v}{\cancel{c^2}}\right)$$

$$\frac{dK}{dv} = d\left(\frac{2\pi\gamma mV}{h}\right) = \frac{2\pi m}{h} v\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$= \frac{2\pi m}{h}\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} + -\frac{v}{2}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)\right.$$

$V_g = v$ particle / object velocity.




$$y = \phi = \underbrace{2A\cos\left(\frac{\Delta k x}{2} - \frac{\Delta\omega t}{2}\right)}_{\text{amplitude modulation}} \cos(kx - \omega t) \simeq \cos(-\theta) = \cos\theta$$

Distance between 2 adjacent minima

$= (X_2)_{node} - (X_1)_{node}$

at fix time $t = t_0$

$$2A\cos\left(\frac{\Delta k}{2}x_1 - \frac{\Delta\omega}{2}t_0\right) = 0 \quad \text{for } x_1$$

$$2A\cos\left(\frac{\Delta k}{2}x_2 - \frac{\Delta\omega}{2}t_0\right) = 0 \quad \text{for } x_2$$

phase difference between $x_1$ & $x_2$

$$\frac{\Delta k}{2}(x_2 - x_1) = \pi$$

$$\Delta k(x_2 - x_1) = 2\pi$$

$$\Delta k \Delta x = 2\pi$$

.. multiply $\hbar$ in above eq.

$$\hbar \Delta k \cdot \Delta x = 2\hbar\pi$$

$$\vec{\Delta p} \cdot \Delta x = \hbar\pi \times 2$$

$$\Delta p \cdot \Delta x = h$$

## Wave function ($\psi$)

$\psi$ has no physical significance but $|\psi|^2$ has

$|\psi|^2 = \psi^* \times \psi$ $\quad$ $\psi = A + iB$

$\psi^*$ = complex conjugate of $\psi$.

$= (A+iB) \times (A-iB)$

$= A^2 - i^2 B^2 = A^2 + B^2 \quad (i^2 = -1)$

# acceptable wave function:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\psi(x,y,z)|^2 dx\, dy\, dz = 1$$

$$\int_{-\infty}^{\infty} |\psi|^2 dv = 1$$ Particle exist in space

$$= 0$$ Particle does not exist

Well behaved wave function:

① $\psi$ must be continuous & single valued.

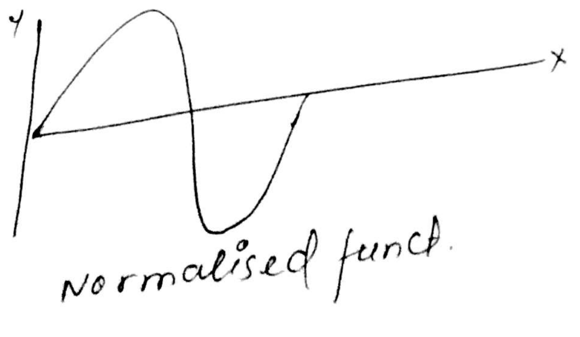
② $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be cont. & single valued

③ $\psi$ must be normalizable. which means that $\psi$ must go to zero as $x \to \pm\infty$, in order that $\int |\psi|^2 dv$ overall space be in finite constant.

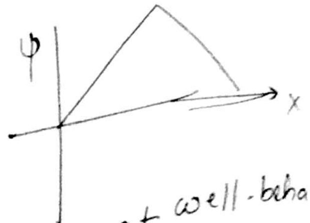
④ One can multiply $\psi(r,t)$ by a constant say $N$ so that $N\psi$ satisfies the condition in ①

$$|N|^2 \int_{-\infty}^{\infty} |\psi(r,t)|^2 dV = 1$$

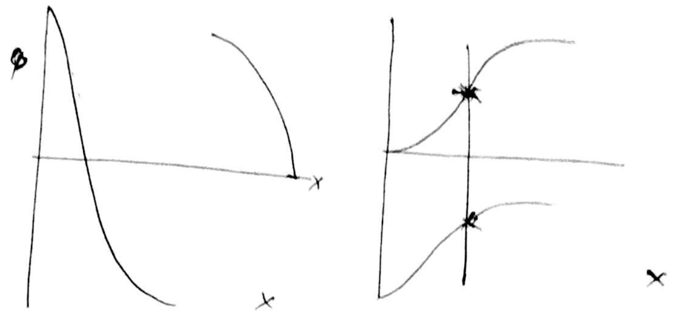
where $N$ is called normalization const.




Normalised funct.




not well-behaved




$\psi = A \sec x$ (discont.)

$\psi = A \tan x$ ×

$\psi = A e^{x^2}$ ×

$\psi = A e^{-x^2}$ ✓

Q- $\psi = A e^{-ax^2}$ determine normalization const. A.

$$|A| \int_{-\infty}^{\infty} \exp(-2ax^2)\, dx = \sqrt{\frac{\pi}{2a}}$$

$$A = \left(\frac{2a}{\pi}\right)^{\frac{1}{4}}$$

$$|A|^2 \sqrt{\frac{\pi}{2a}} = 1$$

Schrodinger eq.

$$\psi = A e^{-i(\omega t - kx)}$$

$$\psi = A e^{-\frac{i}{\hbar}(\hbar \omega t - \hbar k x)}$$

$E = \hbar \omega$

$p = \hbar k$

$$\psi = A e^{-\frac{1}{\hbar}(Et - px)}$$

Total energy of system

$$E = K.E + P.E$$

$$E = \frac{1}{2}\frac{p^2}{m} + U(x) \quad \text{———} \; (2)$$

diff wrt x

$$\frac{d\psi}{dx} = \left(-\frac{1}{\hbar}\right) A (-p)\, e^{-\frac{1}{\hbar}(Et - px)}$$

$$\frac{\partial \psi}{\partial x} = \left(\frac{p}{\hbar}\right)\psi$$

Once again differentiate eq. ① w.r.t x.

$$\boxed{p^2\psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2}} \quad —③$$

When we differentiate eq. ① w.r.t "t".

$$E\psi = \frac{-i}{\hbar}\frac{\partial \psi}{\partial t} \quad —④$$

now multiply $\psi$ in eq ②

$$E\psi = \frac{1}{2}\frac{p^2\psi}{\cancel{8}m} + U_x[\psi] \quad —⑤$$

from ③ & ④

$$-\frac{\hbar}{i}\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U_x[\psi]$$

$(-1 = i^2)$

$$i\hbar\frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x} + U_{(x)}\psi_{(x)}$$

Time dependent Schrodinger eq.

$i\hbar$

# TIME INDEPENDENT Schrodinger eq. (TISE)

$$\psi_{(x,t)} = Ae^{-i(\omega t - kx)} = Ae^{-\frac{i}{\hbar}(Et - px)} \quad —(1)$$

$$\psi_{(x,t)} = \psi_{(x)}\phi(t) = \psi_{(x)}\underbrace{e^{-\frac{i}{\hbar}Et}}_{\phi(t)} \quad —(2)$$

separation of variables technique

We know TDSE,

$$\frac{i\hbar\,\delta\psi(x,t)}{\delta t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}(x,t) + U\phi(x,t) \quad —(3)$$

Put the value of $\psi(x,t)$ from eq (2).

$$\frac{i\hbar\,\partial}{\partial t}\left(\psi_{(x)}e^{-\frac{iEt}{\hbar}}\right) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(\psi_{(x)}e^{-\frac{iEt}{\hbar}}\right) + U\psi_{(x)}e^{-\frac{iEt}{\hbar}}$$

$$i\hbar\,\psi(x)\left(-\frac{i}{\hbar}\right)E\,e^{-\frac{iEt}{\hbar}} = \frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}e^{\frac{iEt}{\hbar}} + U(x)\psi(x)e^{\frac{iEt}{\hbar}}$$

$$E\,\psi(x)\,e^{-\frac{iEt}{\hbar}} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}e^{-\frac{iEt}{\hbar}} + U(x)\,e^{-\frac{iEt}{\hbar}}$$

now we can divide by $e^{-\frac{iEt}{\hbar}}$

$$E\,\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x)$$

~~TD~~ TISE

We know TDSE

$$\frac{i\hbar\,\partial\,\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + U\phi(x,t)$$

The amplitude $\psi(x,y,z,t)$ for object moving in a potential $U(x,y,z)$ obeys TDSE

$$\frac{i\hbar\,\partial\,\psi(r,t)}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi(r,t) + U(r)\,\psi(r,t)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

↓

Laplacian operator

$$\hat{E}\,\psi(r,t) = \hat{H}\,\psi(r,t)$$ TDSE

$\wedge \longrightarrow$ hat / cap.

$$\hat{E} = i\hbar\frac{\partial}{dt}$$

$$\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + U$$

↓

Hamiltonian operator

$E\,\psi(x) = \hat{H}\,\psi(x)$ TISE

$E \rightarrow$ total energy

$\hat{H} = -\dfrac{\hbar^2}{2m}\nabla^2 + U$

# Position of the particle

$\langle x \rangle$ = Expectation value of position

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} x\,|\psi|^2\,dx}{\int_{-\infty}^{\infty} |\psi|^2\,dx = 1}$$

(Normalization condition)
prob. of getting posn of particle = 1

$$= \int_{-\infty}^{\infty} x\,|\psi|^2\,dx$$

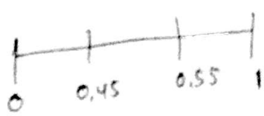
Q- $\psi = \begin{cases} ax & 0 \le x \le 1 \\ 0 & \end{cases}$ $a \in$ const

given wave func.

(a) find prob. that particle can be found b/w $x = 0.45$ and $x = 0.55$.

(b) find expectation value of particle posn.

Ans:




(a) $P = \int_{-\infty}^{\infty} |\psi|^2\,dx$

$= \int_{0.45}^{0.55} a^2 x^2\,dx$

$= \dfrac{a^2}{3}\left[x^3\right]_{0.45}^{0.55} = 0.0251\,a^2$

Scanned by CamScanner

(b) $\langle x \rangle = \int_{-\infty}^{\infty} x|\psi|^2 dx$

$= \int_0^1 x(a^2x^2)dx$

$= \frac{a^2}{4}$

we can't find expectation value of p, E due to uncertainty principle

$\Delta p . \Delta x \geq \frac{h}{4\pi}$

$\Delta E . \Delta t \geq \frac{h}{4\pi}$

# Operator

Eigen value

$\hat{O}\psi = a\psi$

operator

Eigen function

| Function | Operator |
|---|---|
| position | $\hat{x}$ |
| momentum | $\hat{p}$ — $\hat{p_x} = \frac{\hbar}{i}\frac{\partial}{\partial x} = -i\frac{\hbar \partial}{\partial x}$ <br> $\hat{p_y} = \frac{\hbar}{i}\frac{\partial}{\partial y} =$ <br> $\hat{p_z} =$ |
| Energy | $\hat{E} = i\hbar\frac{\partial}{dt}$ |

Total Energy (time independent) $\hat{H} = -\frac{\hbar^2}{2m} \hat{p}^2 + \hat{U}$

Potential E $\hat{U}$

Angular momentum

$\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$

$\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$

$\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$