DMS - Assignment - 04

Harshit Khandelwal (16UCS076)

Camlin Page

Date / /

Ans-1 $R = \{(a, b), (b, b), (c, a), (c, c)\}$

$R^2 = \{(a, b), (b, b), (c, b), (c, c), (c, a)\}$

$R^3 = \{(a, b), (b, b), (c, c), (c, b), (c, a)\}$

Ans-2 - $R: A \to B$, $S: B \to A$

$R \circ S = A \to A$

Ans-3 (a) $\{(a, a), (b, b), (c, c)\}$

(b) $\{(a, a), (b, b), (c, c), (b, c), (c, a), (c, b), (a, c)\}$

(c) $\{(a, a), (b, b), (c, c), (a, b)\}$

(d) $\{(a, b), (b, a), (a, a)\}$

(e) $\{(a, a), (b, b), (c, c), (c, a), (a, b)\}$

(f) $\{(a, b), (b, a)\}$

(g) $\{(a, c), (c, b), (a, b)\}$

Ans-4 for antisymmetric $a_{ij} \neq a_{ji}$

& If $a_{ij} = a_{ji} \Rightarrow (i = j)$

So It is antisymmetric.

Scanned by CamScanner

Ans (5) (a) True

(b) False (Not a function)

(c) True

(d) False

Ans-6 by counting manually 25.

Ans-7 (a) 2^{MN}

$A \geq 1 \mid \cancel{B \quad C} \quad 1 \leq B \leq 3 \mid 2 \leq C$

$A+B+C=10$

coeff of x^{10} in $(x^1+x^2+\ldots x^{10})(x^1+x^2+x^3)(x^2+x^3+\ldots x)$

coeff of x^6 in $(1+x+x^2+\ldots x^9)(1+x+x^2)(1+x+\ldots x^1)$

coeff of x^6 in $\frac{x^{10}-1}{x-1} \quad \frac{x^3-1}{x-1} \quad \frac{x^9-1}{x-1}$

Let $A=a+1$, $B=b+1$, $C=c+2$

$0 \leq a \leq 9$, $0 \leq b \leq 2$, $0 \leq c \leq 8$

$a+b+c=6$

Scanned by CamScanner

Ans-8 (a) E E N N E $\Rightarrow \dfrac{5!}{3!\,2!} = \dfrac{5\cdot 4}{2} = 10$

N E E N E

(b). E E N N N N N E E E E N

$$\frac{11!}{6!\,5!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7}{5\cdot 4\cdot 3\cdot 2} = 42\cdot 11 = 462$$

Ans-9 $x_1 + x_2 + x_3 + x_4 = 11$

$x_1 + x_2 + x_3 + x_4 = 7$

$$^{7+4-1}C_4 = {}^{7}C_4 = \frac{7\cdot 6\cdot 5}{3\cdot 2} = 35$$

Ans-10(a) B G

5 4

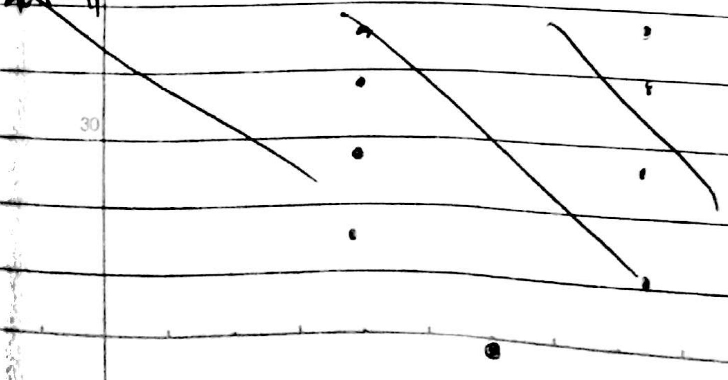
Team member → 5

Total selection = $^{9}C_5$

~~No girl~~ → Answer = $^{9}C_5 - 1$

(b) $\underbrace{1}_{\text{zero girl}} + {}^{4}C_1 \cdot {}^{5}C_4 = 1 + 20 = 21$

Ans-11



(11) 15 29

Ans-12 $\frac{12!}{3!\,4!\,5!}$

(b) $3! = 6$

Ans-13 - $3! = 6$

Ans-14 _ _ _ _ _ ⇒ ~~10 9 8 7 6~~ 36·35·34·33·32

(b) $\frac{0}{1}$ _ _ _ _ ⇒ ~~36·36·36·36~~

(c) 36·36·36·36

Ans-16 $\left[\frac{1976}{2}\right] = 988$

$\left[\frac{1976}{3}\right] = 658$

$\left[\frac{1976}{6}\right] = 329$

⇒ $988 + 658 - 329 = 1317$

(17) { If n == 0 return 1

else

~~return~~ Find (n * fact(n-1))

Scanned by CamScanner

(18) $gcd(x,y) = x$ If $x == y$

$$gcd(x,y) = \begin{cases} y & \text{If } x=0 \\ x & \text{If } y=0 \\ gcd(y, x \bmod y) & \text{else} \end{cases}$$

```
fib(int n)
(17) { If (n==0 || n==1 )
        return n;
   else
     return fib(n-1) + fib(n-1)
   }
```

~~(18)~~

```
(19) search(int x, int start, int end)
  {
    mid= end/2 ;
    If n == a[mid]
        return mid;
    else If n > a[mid]
          start = mid;
       return search(x, start, end)
    else
         end = mid
       return search(x, start, end)
  }
```

Scanned by CamScanner

Camlin Page
Date / /

~~start~~ T (int n)
{
Ans-20 return 1 If n==1

return 2T(n-1)+1 else
}

Ans-21 $a_n = a_{n-1} + 2a_{n-2}$

$a_0 = 3,\ a_1 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1)$

$x = 2, -1$

$A\,2^n + B(-1)^n = a_n$

$a_0 = A + B = 3$ $\Rightarrow A = 1,\ B = 2$

$a_1 = 2A - B = 0$

$a_n = \cancel{2}\ 2^n + 2(-1)^n$

(b). $L_n = L_{n-1} + L_{n-2}$

$x^2 - x - 1 = 0 \quad \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$

$A(\alpha)^n + B(\beta)^n = a_n$

$a_0 = A + B = 1$

$a_1 = A\alpha + B\beta = 3$

Scanned by CamScanner

(i) $n^2 - 6n - 9 = 0$ $\qquad \frac{6 \pm \sqrt{36+36}}{2} = \frac{6 \pm 6\sqrt{2}}{2}$

$\cancel{(n-3)(n+3)}$

$3 \pm 3\sqrt{2}$

$A(3+3\sqrt{2})^n + B(3-3\sqrt{2})^n = a_n$

$2 = A + B \qquad \Rightarrow B = 2 - A$

$3 = A(3+3\sqrt{2}) + B(3-3\sqrt{2})$

$3 = A(3+3\sqrt{2}) + (2-A)(3-3\sqrt{2})$

We can get A & B

(23) $\because$ eqn is $a_n = a a_{n-1} + b a_{n-2}$

$k = 2$

$n^2 - an - b = 0$ (given that roots are complex not real)

~~(25)~~

(26) Proved previously

(28) 123!

Power of 5 in 123! $= \left[\frac{123}{5}\right] + \left[\frac{123}{25}\right] + \left[\frac{123}{125}\right]$

$24 + 4 + 0 = 28$

——" 2 ——" $= \left[\frac{123}{2}\right] + \left[\frac{123}{4}\right] + \cdots \left[\frac{123}{128}\right]$

So number of 0 $= 28$

(30) $\gcd(a,b) = 1 \Rightarrow$ ~~a does not divide b~~ $a = bq_1 + r_1$

$\gcd(b,c) = 1 \Rightarrow$ $b = cq_2 + r_2$

$a = (cq_2 + r_2)q_1 + r_1$

$\gcd(3,8) = 1$

~~$\gcd(8,15) = 1$~~

but $\gcd(3,15) = 3 \neq 1$

(b) let $n = 4$

$n! = 24$

$n! + 1$ is not Prime.

$P(1)$ is true

let $P(k)$ is also true $(k! + 1) \to$ Prime.

$P(k+1) \Rightarrow (k+1)! + 1$

$(k+1)k! + 1$

$k \cdot k! + k! + 1$

Prime Prime

Total can't be prime

So induction is wrong that disproves.

(31) $\phi(10)$ $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\phi(10) = 4$

(32)