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Assignment-5.

Q1.

a) $u_t + x u_x = 0$ Quasi ✓

~~Lin~~ Semi ✓

$\boxed{\text{Linear}}$

b) $u_t + a u_x = u^2$

$\boxed{\text{Semilinear}}$

c). $x u_t + u_x = u$.

$\boxed{\text{Linear}}$

d) $u_t + u u_x = 0$

$\boxed{\text{Quasilinear}}$

e)

f)

Q2. a) $z(x,y) = xy + f(x^2+y^2)$

$\cancel{u = x^2 + y^2}$ $\cancel{du = 2x\,dx + 2y\,dy}$

~~dz~~

$$z_x = p = y + 2x f'(x^2+y^2)$$
$$z_y = q = x + 2y f'(x^2+y^2)$$

$$\frac{p-y}{2x} = \frac{q-x}{2y}$$

$$py - y^2 = qx - x^2$$

b) $z(x, y) = f(x/y)$

$x/y = u$

$\frac{1}{y} = u_x$

$\frac{-x}{y^2} = u_y$

$p = \frac{1}{y} f'(u)$

$q = \frac{-x}{y^2} f'(u)$

$py = \frac{-qy^2}{x}$

c) $f(x-z, y-z) = 0$

$x - z = u$

$y - z = v$

$u_y = 0 - q$

$u_x = 1 - p$

$v_y = 1 - q$ $\quad v_x = -p$

$u_z = -1$

$\frac{\partial f(x-z, y-z)}{\partial u} + \frac{\partial f(x-z, y-z)}{\partial v}$

$-p f'(u,v) + (-q) f'(u,v).$

$\frac{\partial f(u,v)}{\partial (x,y)}$

$0 = \frac{\partial f(u,v)}{\partial x} = \frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial y}{\partial x}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial x}\right)$

$f'(u,v)\left(-p + (-p)\frac{\partial z}{\partial x}\right) + f'(u,v)(-p + \ldots)$

$f'(u,v)$

~~$P = \dfrac{\partial(u,v)}{\partial(x,y)}$~~

$$P = \frac{\partial(u,v)}{\partial(y,z)}$$

$u = x - z$

$v = y - z$

$$Q = \frac{\partial(u,v)}{\partial(z,x)}$$

$u_y = -q$

$u_x = 1-p \quad u_z = -1$

$$R = \frac{\partial(u,v)}{\partial(x,y)}$$

$v_x = -p$

$v_y = 1-q$

$v_z = -1$

$$\frac{\partial(u,v)}{\partial(y,z)} = \begin{vmatrix} u_y & u_z \\ v_y & v_z \end{vmatrix}$$

$$= \begin{bmatrix} -q & -1 \\ 1-q & -1 \end{bmatrix} = 1 = P$$

$$Q = \begin{bmatrix} u_z & u_x \\ v_z & v_x \end{bmatrix} = \begin{bmatrix} -1 & 1-p \\ -1 & -p \end{bmatrix} = p + 1 - p = 1 = Q$$

$$R = \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} = \begin{bmatrix} 1-p & -q \\ -p & 1-q \end{bmatrix} = \cancel{1-p-pq}$$

$(1-p)(1-q) - pq$

~~$(q+y)p + qQ = 1-p-pq$~~

~~$2pq + 2p + q = 1 - Q$~~

$1-p-q$

$\boxed{p+q = 1/2}$

$1-p-q = p+q$

03.

a) $z(x,y) = \cancel{xyz}\ (x+a)(y+b)$

$\cancel{z}\ p = y+b$

$q = x+a$

$$Z = pq$$

b) $Z = ax+by$

$p = a$

$q = b \qquad Z = px+qy$

c) $Z^2(1+a^3) = 8(x+ay+b)^3$

$2ZZ_x(1+a^3) = 24(x+ay+b)^2$

$2ZZ_y(1+a^3) = 24a(x+ay+b)^2$

$$p = \frac{(x+ay+b)^2}{2Z(1+a^3)}$$

$\cancel{\frac{q}{a}} = p \qquad a = p/q$

$$Z^2\left(1+\frac{p^3}{q^3}\right) = 8\left(x+\frac{p}{q}y+b\right)^3$$

$$\sqrt[3]{\frac{Z^3\left(1+p^3/q^3\right)}{8}} - x - \frac{p}{q}y = b$$

04. a) $x^2+y^2+(z-c)^2-a^2=0$

$2x+2(z-c)Z_x = 0$

$2y+2(z-c)Z_y = 0$

$$\frac{+2x}{p} = \frac{+2y}{q}$$

$$qx-py=0.$$

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$$(x^2+y^2)\cos^2\theta - (z-c)^2\sin^2\theta = 0$$

$$(x^2+y^2)\cos^2\theta = (z-c)^2\sin^2\theta$$

$$2x\cos^2\theta = 2(z-c)z_x\sin^2\theta.$$
$$2y\cos^2\theta = 2(z-c)z_y\sin^2\theta.$$

$$\frac{x}{y} = \frac{p}{q} \Rightarrow qx - py = 0;$$

$$z(x,y) = f(x^2+y^2)$$

$$u = x^2+y^2$$
$$u_x = 2x$$
$$u_y = 2y.$$

$$p = 2xf'(u)$$
$$q = 2yf'(u).$$

$$\frac{p}{q} = \frac{x}{y} \Rightarrow qx - py = 0;$$

$$u(x,y) = f(v)$$

$$u(x,y) = f(v(x,y))$$

$$u_x = v_x f'(v)$$
$$u_y = v_y f'(v) \qquad \frac{u_x}{u_y} = \frac{v_x}{v_y}.$$

b) $u(x,y) = f(x-ay) + g(x+ay)$

$u_x = f'(x-ay) + g'(x+ay)$
$u_y = -af'(x-ay) + ag'(x+ay)$.

$u_{xx} = f''(x-ay) + g''(x+ay)$
$u_{xy} = -af''(x-ay) + ag''(x+ay)$
$u_{yy} = a^2 f''(x-ay) + a^2 g''(x+ay)$
$u_{yx} = -af''(x-ay) + ag''(x+ay)$

$$\frac{u_{xx}}{u_{yy}} = a^2$$

$$u_x + \frac{u_y}{a} = 2g'(x+ay)$$

$$u_{yx} = u_{xy}$$

$$u_x - \frac{u_y}{a} = 2f'(x+ay)$$

Q6. a) $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$

$\ln x = \ln y + c_1 \quad \ln y = \ln z + c_2$

$\ln \frac{x}{y} = c_1 \quad \ln y/z = c_2$

$F\left(\frac{x}{y}, \frac{y}{z}\right) = 0$

$$x^2 p + y^2 q = (x+y)z$$

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{x+y}$$

$$\frac{dx - dy}{x^2 - y^2} = \frac{dz}{x+y}$$

$$\frac{dx - dy}{x-y} = dz$$

$$\ln x - y = z + c_1$$

$$\frac{-1}{x} + \frac{1}{y} \qquad F\left(\frac{x-y}{z}, \frac{1}{y} - \frac{1}{x}\right) = 0$$

$$yzp + xzq = xy$$

$$\frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{xy}.$$

$$\frac{x^2}{2} = \frac{y^2}{2} + c_1 \qquad \frac{y^2}{2} = \frac{z^2}{2} + c_2$$

$$F(x^2 - y^2, y^2 - z^2) = 0.$$

$$(z^2 - 2yz - y^2)p + x(y+z)q = x(y-z)$$

$$\frac{dx}{z^2 - 2yz - y^2} = \frac{dy}{x(y+z)} = \frac{dz}{x(y-z)}$$

$$\cancel{\frac{dy - dz}{2z} = \frac{dy + dz}{2y}}.$$

$$y\,dy - z\,dy - y\,dz - z\,dz = 0$$

$$\boxed{-zy + \frac{y^2}{2} + \frac{z^2}{2} = c_1}$$

$$\frac{dx}{z^2-2yz-y^2} = \frac{dy}{x(y+z)} = \frac{dz}{x(y-z)}$$

$$\frac{z\,dy}{x(yz+z^2)} = \frac{y\,dz}{x(y^2-yz)}$$

$\cancel{(z-(y))^2}$

$\cancel{z^2-y^2-2yz}$

$$\cancel{z\,dy + y\,dz}$$

$$\frac{x\,dx + y\,dy + z\,dz}{x}$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = C_1$$

$$F\left(y^2+z^2-2zy,\ x^2+y^2+z^2\right) = 0$$

Q7. $2p + 3q = -8z$

a) $z = 1-3x \qquad y = 0$

$$\frac{dx}{2} = \frac{dy}{3} = \frac{dz}{-8z}$$

$$\cancel{\frac{y}{3} + \frac{z}{8} = C = \frac{x}{2} + \frac{y}{2}}$$

$-8\frac{y}{3} = \ln(8z)$

$\cancel{\frac{z}{8} = C = \frac{x}{2}}$

$\frac{e^{-8y/3}}{3} \cancel{\in B} = C$

$\cancel{1-3x = 4x}$

$\cancel{x = \frac{1}{7}}$

$3e^{8y/3} = C_1$

$3e^{8y/3} = \frac{x}{2} - \frac{y}{3}$

$\frac{x}{2} - \frac{y}{3} = C_2$

$1-3x = \frac{x}{2}$

$x = 2/7$

$\frac{x}{2} - \frac{y}{3} = c_1$ $\qquad$ $-\frac{8x}{2} = \ln z + \ln C$

$z e^{-4x} = C_2$

$z = 1 - 38x$

$\cancel{x} \quad s \quad x$

$y = 0.$

$(1-3s) e^{4s} = C_2.$

$\frac{s}{2} = C_1$

$\frac{x}{2} - \frac{y}{3} = c_1$ $\qquad$ $\cancel{c_2 = (1-3s)(e^{4s})}$

$c_2 = (1-3x)(e^{4x})$

$\frac{x}{2} = c_1$ $\qquad$ $\boxed{C_2 = (1-(3\times 2c_1)(e^{(4\times 2c_1)})}$

$x = 2c_1$

$$z e^{4x} = \left(1 - (6)\left(\frac{x}{2} - \frac{y}{3}\right) e^{\left(8\left(\frac{x}{2} - \frac{y}{3}\right)\right)}\right)$$

b) $z = x^2$ $\qquad$ $2y = 1 + 3x$

$\frac{x}{2} - \frac{y}{3} = c_1$ $\qquad$ $z e^{4x} = C_2$

$\frac{x}{2} - \left(\frac{1+3x}{6}\right) = c_1$ $\qquad$ $x^2 e^{4x} = C_2.$

$\frac{3\cancel{x} - 1 - 3\cancel{x}}{6} = c_1$ $\qquad$ $x^2 e^{4x} = C_2$

No Solution.

c) $z = e^{-4x}$ $\quad 2y - 3x = 0$

$$ze^{4x} = C_2$$
$$3x - 2y = C_1$$

$x = 1$
$y = s$
$z = s(1+s)$

$$s(s+1)e^{4} = C_2$$
$$3 - 2s = C_1$$

$$\frac{3 - C_1}{2} = s$$

$$\left(\frac{3-C_1}{2}\right)\left(\frac{5-C_1}{2}\right)e^{4} = C_2$$

$$\left(\frac{3 - 3x + 2y}{2}\right)\left(\frac{5 - 3x + 2y}{2}\right)e^{4} = ze^{4x}$$

Q8. d) $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$

$x = 1 \quad y = 0 \quad z = s$

$$\frac{dx}{2xy - 1} = \frac{dy}{(z - 2x^2)} = \frac{dz}{2(x - yz)}$$

$x \times \quad y \quad \times \frac{1}{2}$

$$\frac{z\,dx}{2xyz - z} = \frac{dy}{z - 2x^2} = \frac{x\,dz}{2x^2 - 2xyz}$$

$$z\,dx + dy + dz = 0$$

$$\cancel{\int z\,dx + \int dy + \int dz = 0}$$

$$z\,dx + x\,dz + dy = 0$$

$$d(zx) + dy = 0$$

$$zx + y = C_1$$

(xyz)

$(x(y\frac{dz}{dt} + z\frac{dy}{dt}) + (yz)x$

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$\frac{x\,dx}{} + y\,dy + \frac{1}{2}dz = 0$

$\cancel{d(xy) + \frac{1}{2}dz = 0}$

$\frac{x^2}{2} + \frac{y^2}{2} + \frac{1}{2}z = 0$

$\frac{1}{2} + \frac{x}{2} = c_1$

$x = c_2$

$1 + c_2 = 2c_1$

$x7a_3$

$\cancel{1 + x = 1 + x}$

$1 + x = 2(3x + y)$

Q8. b) $x^3 p + y(3x^2 + y)q = z(2x^2 + y)$

$x = 1 \quad y = s \quad z = (1+s)s$

$\frac{dx}{x^3 y} = \frac{dy}{y(3x^2+y)} = \frac{dz}{z(2x^2+y)}$

xxz

xzy^2

dy

$\frac{dx}{2z^{x3}} = \frac{dy}{(y)(3x^2+y)} = \frac{dz}{(2x^2+y)(z)(x)}$

(xy^3)

$(x\,y\frac{dz}{dx} + z\frac{dy}{dx})$

$+(yz)(\frac{dx}{dx})$

$(x^3)(yz) \qquad 3x^3yz + (xz)(xy^2) \qquad (2x^3z)(y) + (xy^2)(z)$

$d(\frac{x}{y})$

$yz\,dx - \cancel{3x^2y}$

$yz\,dx - xz\,dy + xy\,dz = 0$

$\frac{(z)(y\,dx - x\,dy)}{xy^2} + \frac{x}{y}dz = 0$

Q8. b) $x^3 p + y(3x^2+y)q = z(2x^2+y)$

$$yz - x\,\frac{dx}{x^3} = \frac{dy\,(-xz)}{y(3x^2+y)} = \frac{dz\,(xy)}{z(2x^2+y)}$$

$$\frac{z\,dx}{zx^3} = \frac{dy}{3x^2y+y^2} = \frac{dz}{2x^2z+yz}.$$

$$\frac{z\,dx}{zx^3} \qquad \frac{x\,dz}{2x^3z+xyz}.$$

$$\frac{yz\,dx}{x^3yz} = \frac{-xz\,dy}{-3x^3yz-y^2xz} = \frac{xy\,dz}{2x^3yz+y^2xz}$$

$$yz\,dx - xz\,dy + xy\,dz = 0.$$

$$xy\,dz + yz\,dx = xz\,dy.$$

$$\ln xz = \ln yc$$

$$\frac{xz}{y} = c_1.$$

$$\frac{dx}{x^3} = \frac{dy}{y(3x^2+y)}$$

$\cancel{yd}$ $y3x^2dx + y^2dx = x^3dy$

$$dx = \frac{x^3dy - 3x^2y\,dx}{y^2}$$

$$\cancel{x} = \frac{x^3}{y}c_2$$

$$\frac{y}{x^2} = c_2$$

$s = c_2$

$\frac{\cancel{s}(s+1)}{\cancel{s}} = c_1$ $\qquad c_2+1 = c_1$

$$\frac{y}{x^2} + 1 = \frac{xz}{y}.$$