Assignment-4

2) $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}_{3\times 4}$

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 2 & -3 & -1 \\ 0 & 4 & -6 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 2 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$r_1, r_2 \Rightarrow$ rowspace $\dim(R(A)) = 2$
$c_1, c_2 \Rightarrow$ column space $\dim(C(A)) = 2$

$c \cdot D \Rightarrow c_3, c_4$ free variable $x_3 = s$ $x_4 = t$

$x_1 + 2x_2 - x_4 = 0$

$x_1 + 2x_2 = t$

$2x_2 = 3s + t$ $\Rightarrow x_1 = -3s$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3s \\ 3/2\, s + t/2 \\ s \\ t \end{bmatrix} = s\begin{bmatrix} -3 \\ 3/2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$\dim N(A) = 2$

rank + nullity $= \dim R(A) + \dim N(A) = n$

$$\boxed{2 + 2 = 2 + 2 = 4}$$

3) $A = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 7 & 7 & 4 \\ 1 & 2 & 2 & 5 & 5 & 6 \\ 3 & 6 & 6 & 15 & 14 & 15 \end{bmatrix}$

a) $M_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ rank = 1

$M_2 = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ rank = 1

$M_3 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \\ 3 & 6 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ ~~rank~~

$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ rank = 2

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$$M_4 = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 3 & 7 \\ 1 & 2 & 2 & 5 \\ 3 & 6 & 6 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank = 2

$$M_5 = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 3 & 7 & 7 \\ 1 & 2 & 2 & 5 & 5 \\ 3 & 6 & 6 & 15 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 3 & 9 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ rank = 3

$$M_6 = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 7 & 7 & 4 \\ 1 & 2 & 2 & 5 & 5 & 6 \\ 3 & 6 & 6 & 15 & 14 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 3 & 2 & 5 \\ 0 & 0 & 3 & 9 & 5 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rank = 3

b) $(C_1, C_3, C_5) \Rightarrow$ column space

c) $C_4 = aC_1 + bC_3 + cC_5$

$(2,3,0,0) = a(1,0,0,0) + b(1,1,0,0) + c(3,1,1,0)$

$= (a+b+3c, \; b+c, \; c, \; 0)$

$a+b+3c = 2$

$b+c = 3$

$c = 0 \quad \Rightarrow b = 3 \quad a = -1$

d) rank of $A = 3$

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4) largest possible $\dim(R(A)) = 3$
(having all non-zero rows)

$\dim R(A) + \dim N(A) = n = 4$

least $\dim R(A) = 1$

$\Rightarrow 1 + \dim N(A) = 4$

$\underline{\dim N(A) = 3}$

largest $\dim(N(A)) = 3$

5) according to rank-nullity

$\dim R(A) + \dim N(A) = n$

$\dim R(A) + 4 = 6$

$\underline{\dim R(A) = 2}$

7) a) $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda)(1-\lambda) - 4 = 0$

$(1-\lambda)^2 = 4 \qquad 1-\lambda = \pm 2$

$\lambda = -1, \quad \lambda = 3$

eigen vector for $\lambda = -1$

$(A - \lambda I)x = 0 \quad = \quad (A + I)x = 0$

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{array}{l} 2x_1 + x_2 = 0 \\ 4x_1 + 2x_2 = 0 \end{array}\right\} \text{L.D.}$$

$$x_2 = -2x_1$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$t \in R$
$\Rightarrow$ eigen vectors for $\lambda = -1$

for $\lambda = 3$

$(A - 3I)x = 0$

$$\begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{array}{l} -2x_1 + x_2 = 0 \\ 4x_1 - 2x_2 = 0 \end{array}\right\} \text{L.D}$$

$$x_2 = 2x_1$$

$$X_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = s \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$s \in R$
$\Rightarrow$ eigen vector for $\lambda = 3$

b) $\begin{bmatrix} -1 & 1 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{bmatrix} = A$

$|A - \lambda I| = 0$

$$\begin{vmatrix} -1-\lambda & 1 & 2 \\ 2 & 2-\lambda & 2 \\ -3 & -6 & -6-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} -3-\lambda & \lambda-1 & 0 \\ 2 & 2-\lambda & 2 \\ -3 & -6 & -6-\lambda \end{vmatrix} = 0$$

$$-3-\lambda\left((2-\lambda)(-6-\lambda)\right) - (\lambda-1)\left[-12-2\lambda-6\right]$$

$$(-3-\lambda)\left(-12-2\lambda-6\lambda+\lambda^2\right) - (\lambda-1)\left[-2\lambda-18\right]$$

$$(-3-\lambda)(\lambda^2-8\lambda-12) - (\lambda-1)(-2\lambda-18)$$

$$-3\lambda^2+24\lambda+36-\lambda^3+8\lambda^2+12\lambda+2\lambda^2+18\lambda$$
$$-2\lambda-18 = 0$$

$$-\lambda^3+7\lambda^2+52\lambda+18 = 0$$
$$\lambda^3-7\lambda^2-52\lambda-18 = 0$$

a) $(A - 2I)x = 0$

$$\begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 - x_2 + 6x_3 = 0$

$x_1 = s$

$x_2 = t$

$$\begin{bmatrix} s \\ t \\ -\frac{s}{3} + \frac{t}{6} \end{bmatrix} = s \begin{bmatrix} 1 \\ 0 \\ -1/3 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 1/6 \end{bmatrix}$$

eigen vectors and they are L.I. w.r.t

$\lambda = 2$