CHAPTER-7
COORDINATE GEOMETRY

# Excercise 7.4

Q4.The two opposite vertices of a square are $(-1, 2)$ and $(3, 2)$. Find the coordinates of the other two vertices.

**Solution :**

Given points

$$\mathbf{A} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

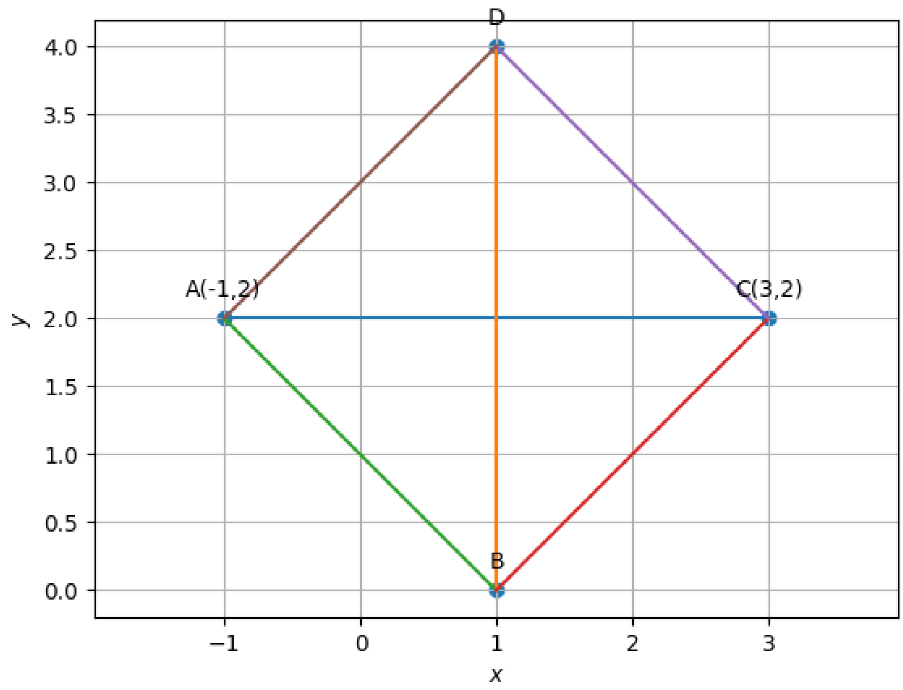


Figure 1:

Shifting point A to origin with refrence to Figure 2

$$\mathbf{A}' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C}' = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

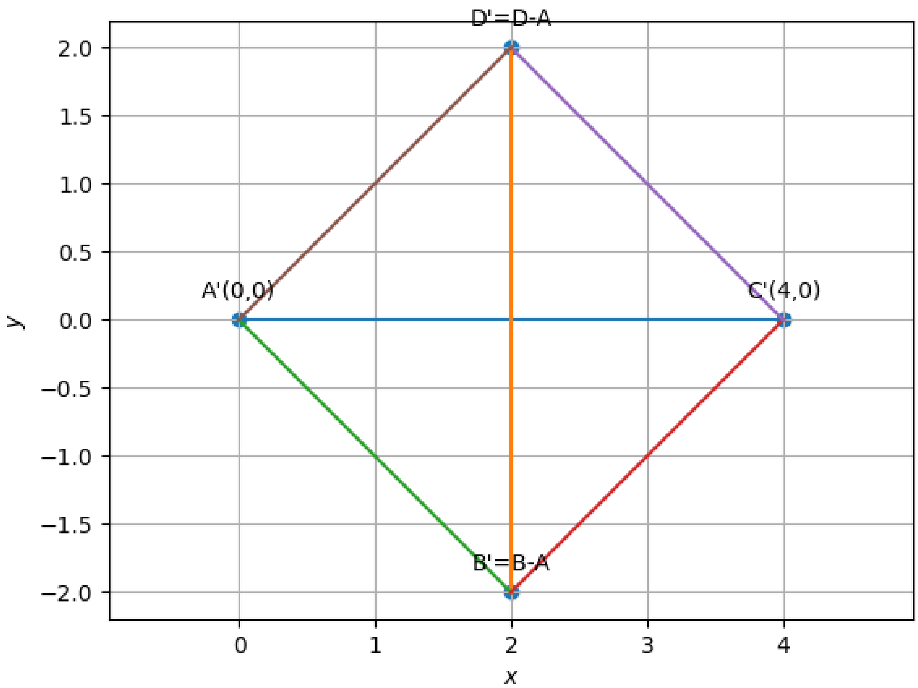


Figure 2:

For a generalised case, the angle made by **AC** on the x-axis will be $\theta+45°$, where $\theta$ is the angle made by **AB** on the x-axis. Hence, we can calculate the angle made by **AB** on the x-axis as.

$$tan(\theta + 45) = \text{slope of } \mathbf{AC}$$
$$\beta = \theta + 45 = tan^{-1}(\text{slope of } \mathbf{AC})$$

However in this particular case, angle made by **AC** on the x-axis

$$tan\beta = \frac{0-0}{4-0}$$
$$\beta = 0°$$

Therefore, here $\theta = -45°$

We know the rotation matrix for CW rotation is given as

$$P^{\top} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$

Since the angle here is negative so ultimately the coordinates will be rotated in ACW direction. Now the transformed(rotated) coordinates are with refrence to Figure 3.

$$\mathbf{C}\text{''} = P(\mathbf{C} - \mathbf{A}) = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{4}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} \end{pmatrix}$$

$$\mathbf{B}\text{''} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{C}\text{''} = \begin{pmatrix} \frac{4}{\sqrt{2}} \\ 0 \end{pmatrix}, \mathbf{D}\text{''} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{C}\text{''} = \begin{pmatrix} 0 \\ \frac{4}{\sqrt{2}} \end{pmatrix} \text{ and } \mathbf{A}\text{''} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

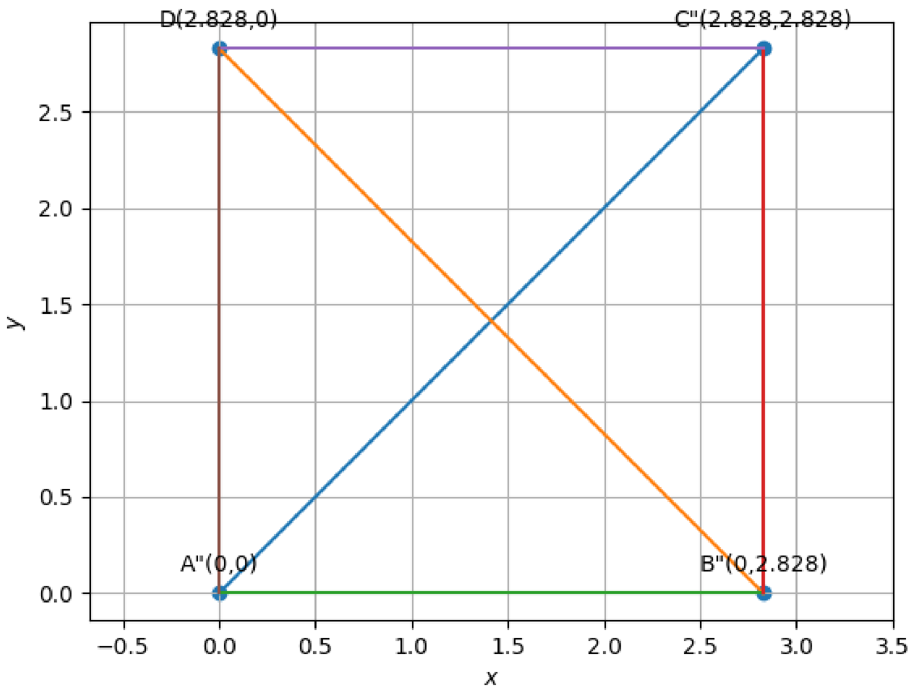


Figure 3:

Again tranforming(rotating) the coordinates back to the original axis. We know for anti-clockwise direction the rotation matrix is given as

$$P = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

Again we know that the angle is negative so the rotation will be in clockwise direction. So now the transformed(rotated) coordinates B and D are with refrence to Figure 4

$$\mathbf{B}' = P\mathbf{B}\text{''} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{4}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\mathbf{D}' = P\mathbf{D}\text{''} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ \frac{4}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

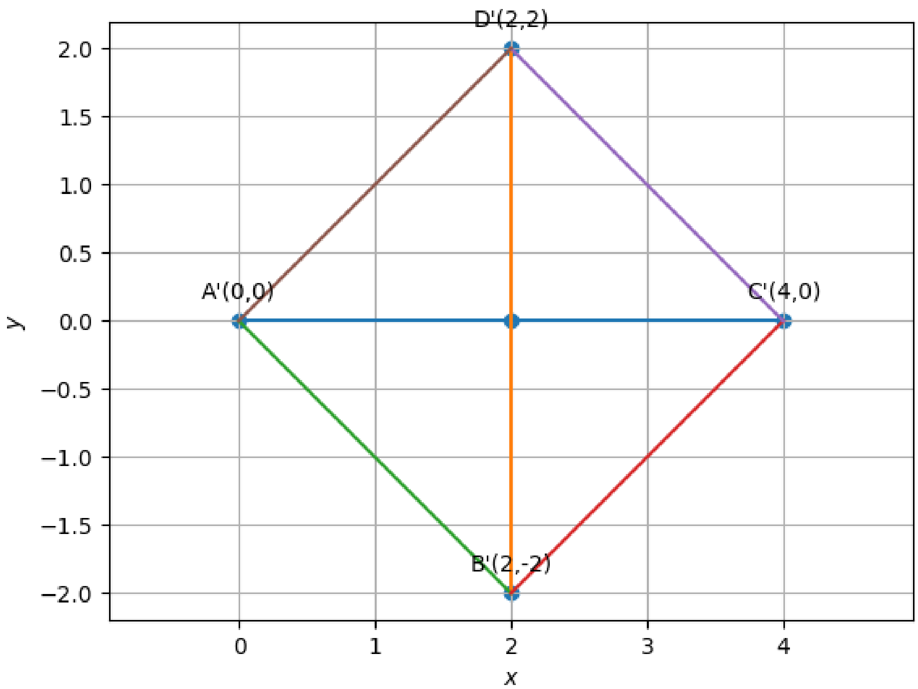


Figure 4:

Again transforming(shifting) the axis back to the original with refrence to Figure 5

$$\mathbf{B} = \mathbf{B}' + \mathbf{A} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{D} = \mathbf{D}' + \mathbf{A} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

Hence, the other two vertices are $\mathbf{B}(1,0)$ and $\mathbf{D}(1,4)$

The direct formula for calculation of the vertices is:

$$\mathbf{B} = \mathbf{A} + P[\mathbf{e_1}\ \mathbf{0}]P^{\top}(\mathbf{C} - \mathbf{A})$$

$$\mathbf{D} = \mathbf{A} + P[\mathbf{0}\ \mathbf{e_2}]P^{\top}(\mathbf{C} - \mathbf{A})$$

where $P$ is the rotation matrix and $\mathbf{A}$ and $\mathbf{C}$ are the position vectors of opposite vertices.

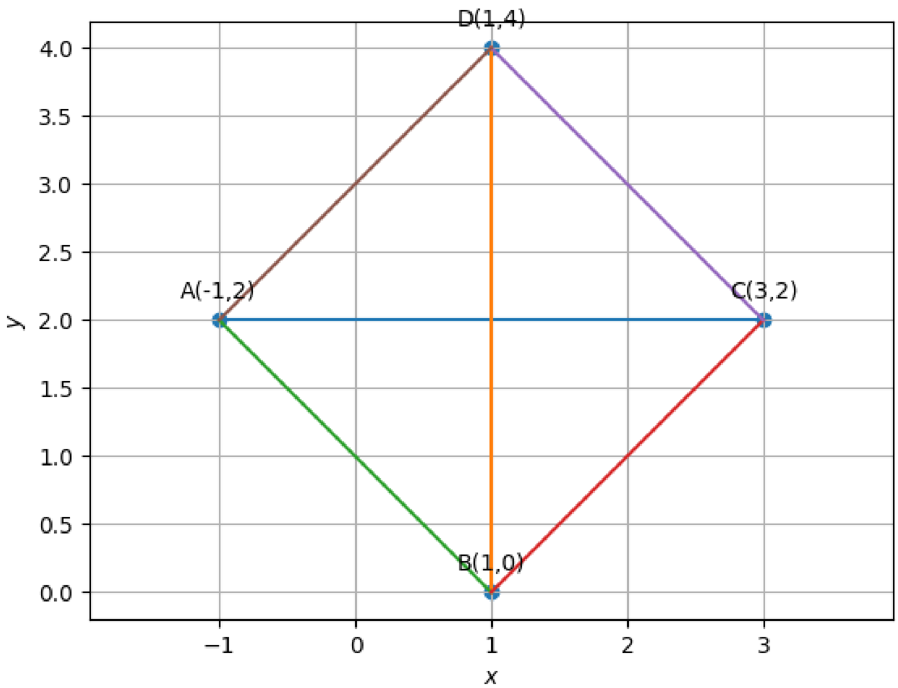


Figure 5: