CHAPTER-7
COORDINATE GEOMETRY

Excercise 7.4

Q4. The two opposite vertices of a square are $(-1, 2)$ and $(3, 2)$. Find the coordinates of the other two vertices.

Solution :

Given points $A = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, C = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Let $B = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $D = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

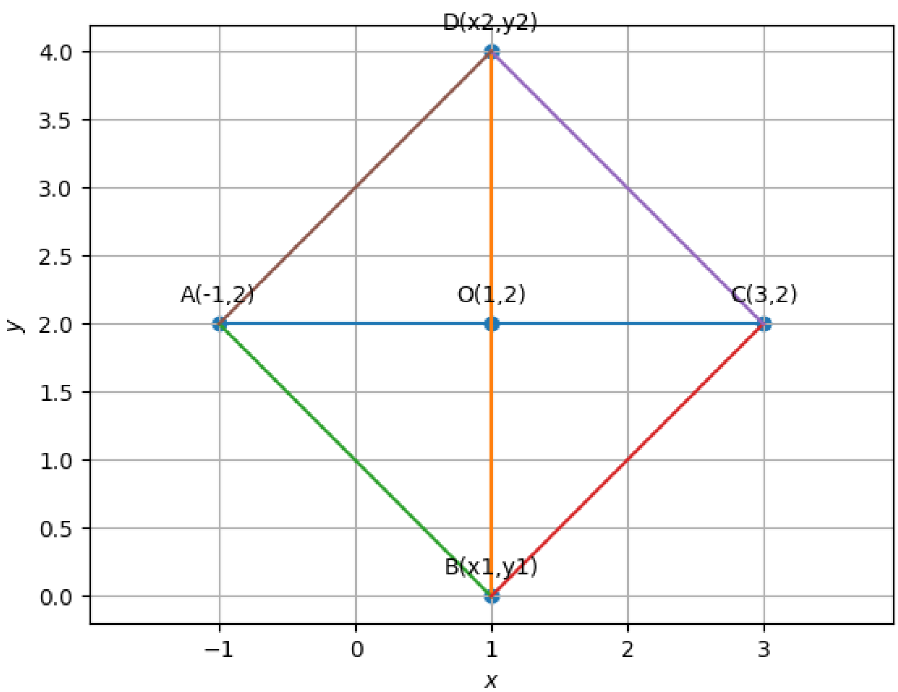

Figure 1:

Shifting point A to origin $A' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $B' = B - A = \begin{pmatrix} x_1 + 1 \\ y_1 - 2 \end{pmatrix}$
$C' = C - A = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $D' = D - A = \begin{pmatrix} x_2 + 1 \\ y2 - 2 \end{pmatrix}$

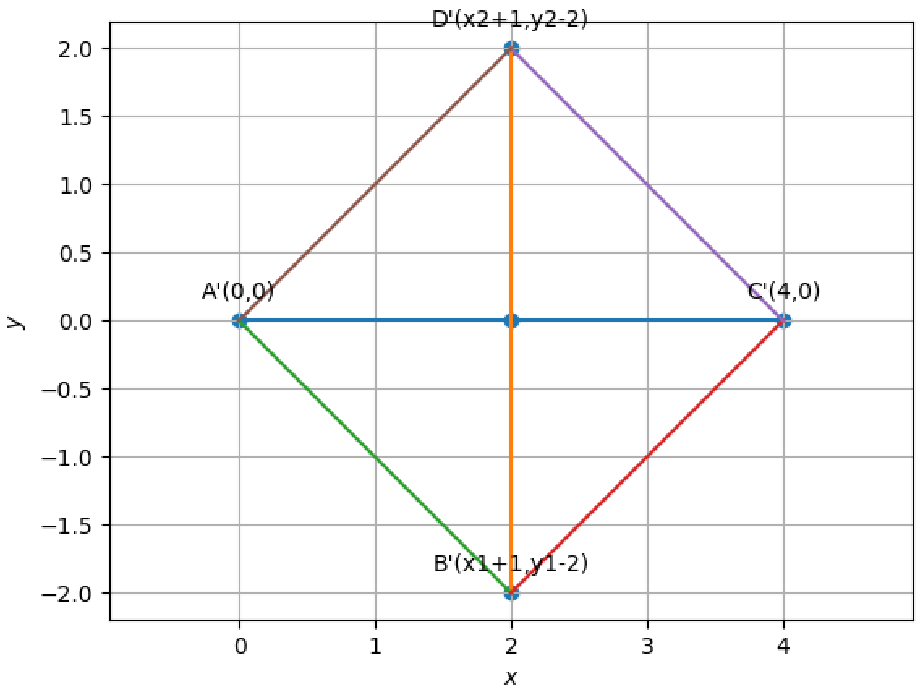

Figure 2:

Now caculating the angle the **AB** makes with the x-axis.
Let **AB** makes an angle θ with the x-axis and we know $\angle C'A'B' = 45°$
Angle made by $\mathbf{AC} = \theta + 45$ on the x-axis

$$tan(\theta + 45) = \frac{0 - 0}{4 - 0}$$
$$\theta = -45°$$

We know the rotation matrix is given as $P = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$

Now the transformed coordinates are

$$C" = P(C - A) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{4}{\sqrt{2}} \\ -\frac{4}{\sqrt{2}} \end{pmatrix}$$

$$B" = \begin{pmatrix} 0 \\ -\frac{4}{\sqrt{2}} \end{pmatrix}, D" = \begin{pmatrix} \frac{4}{\sqrt{2}} \\ 0 \end{pmatrix} \text{ and } A" = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

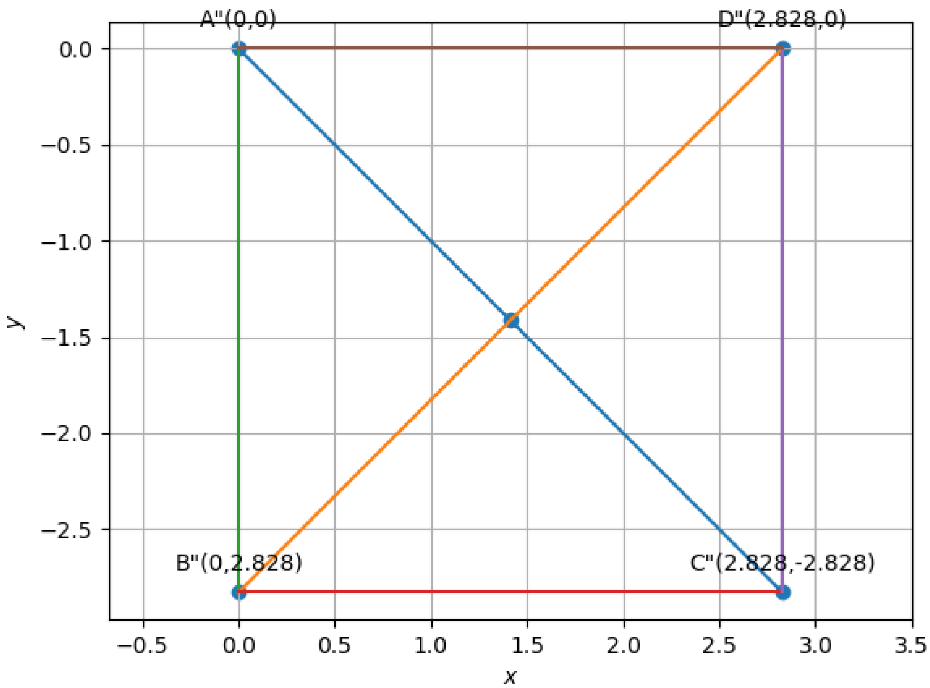

Figure 3:

Again tranforming the coordinates back to the original axis.
We know for anti-clockwise direction the rotation matrix is given as

$$P = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$

So now the transformed coordinates B and D will be

$$B' = PB" = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{4}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$D' = PD" = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{4}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

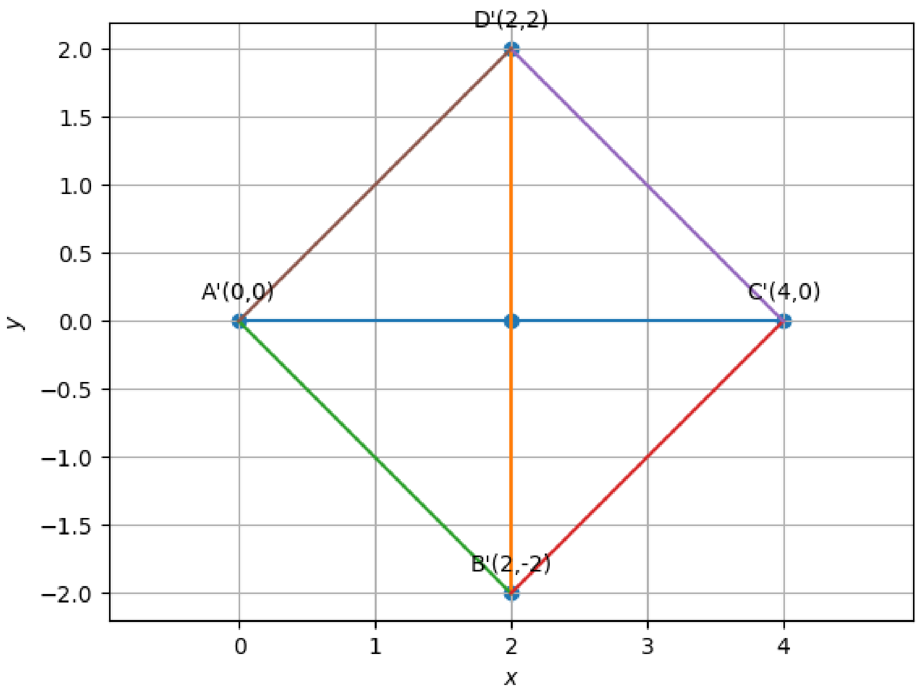

Figure 4:

Again transforming the axis back to the original

$$B = B' + A = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$D = D' + A = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

Hence, the other two vertices are $B(1,0)$ and $D(1,4)$

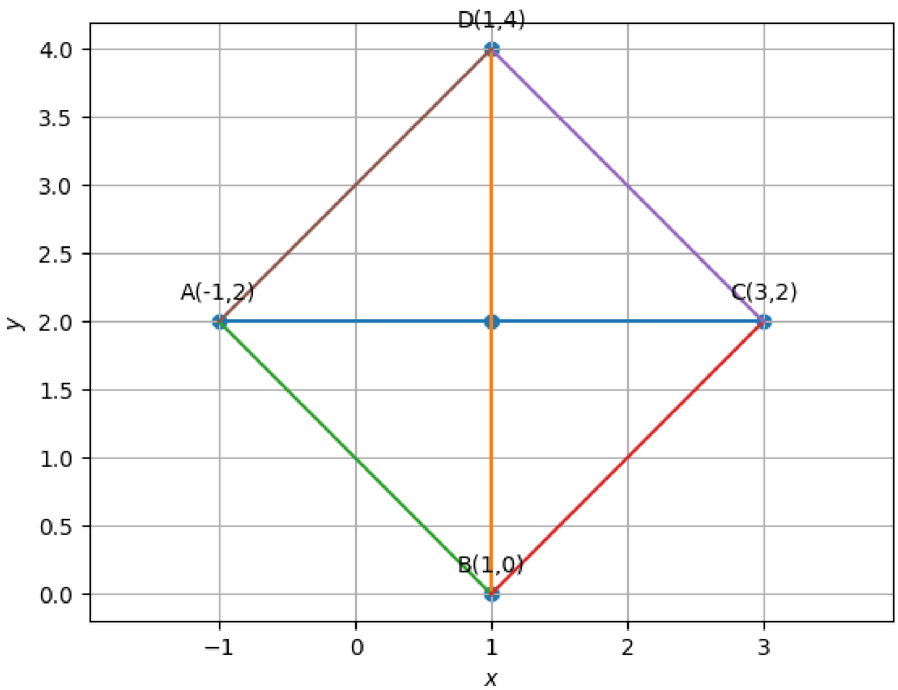

Figure 5: