CHAPTER-7
COORDINATE GEOMETRY

Excercise 7.1

Q6. Name the type of quadilateral formed,if any, by the following points, and give reasons for your answer:

1. $(-1,-2),(1,0),(-1,2),(-3,0)$
2. $(-3,5),(3,1),(0,3),(-1,-4)$
3. $(4,5),(7,6),(4,3),(1,2)$

Solution:

1. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{1}$$

$$AB = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{2}$$

$$BC = \mathbf{C} - \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{3}$$

$$DC = \mathbf{C} - \mathbf{D} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{4}$$

$$AD = \mathbf{D} - \mathbf{A} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{5}$$

$$AC = \mathbf{C} - \mathbf{A} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{6}$$

$$BD = \mathbf{D} - \mathbf{B} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{7}$$

If rank of $\begin{pmatrix} AB & DC \end{pmatrix} < 2$ then AB is parallel to DC else it is not.
Rank of $\begin{pmatrix} AB & DC \end{pmatrix}$ = Rank of $\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

Rank $= 1$
Hence, AB is parallel to DC
Similarly,If rank of $\begin{pmatrix} BC & AD \end{pmatrix} < 2$ then BC is parallel to AD else it is not.
Rank of $\begin{pmatrix} BC & AD \end{pmatrix}$ = Rank of $\begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix}$
Rank $= 1$
Hence, BC is parallel to AD
From here we can conclude that ABCD is parallelogram.
Now checking if the adjacent sides are orthogonal to each other

$$(AB)^\top (BC) = \begin{pmatrix} 2 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = -4 + 4 = 0 \tag{8}$$

Now if the diagonals are also orthogonal then it is a square else a rectangle

$$(AC)^\top (BD) = \begin{pmatrix} 0 & 4 \end{pmatrix} \begin{pmatrix} -4 \\ 0 \end{pmatrix} = 0 \tag{9}$$

Hence the diagonals are orthogonal to each other. So, we can conclude that ABCD is a square as shown in Figure 1

2. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \tag{10}$$

$$AB = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \tag{11}$$

$$BC = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \tag{12}$$

$$DC = \mathbf{C} - \mathbf{D} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \tag{13}$$

$$AD = \mathbf{D} - \mathbf{A} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -9 \end{pmatrix} \tag{14}$$

$$AC = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{15}$$

$$BD = \mathbf{D} - \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \tag{16}$$

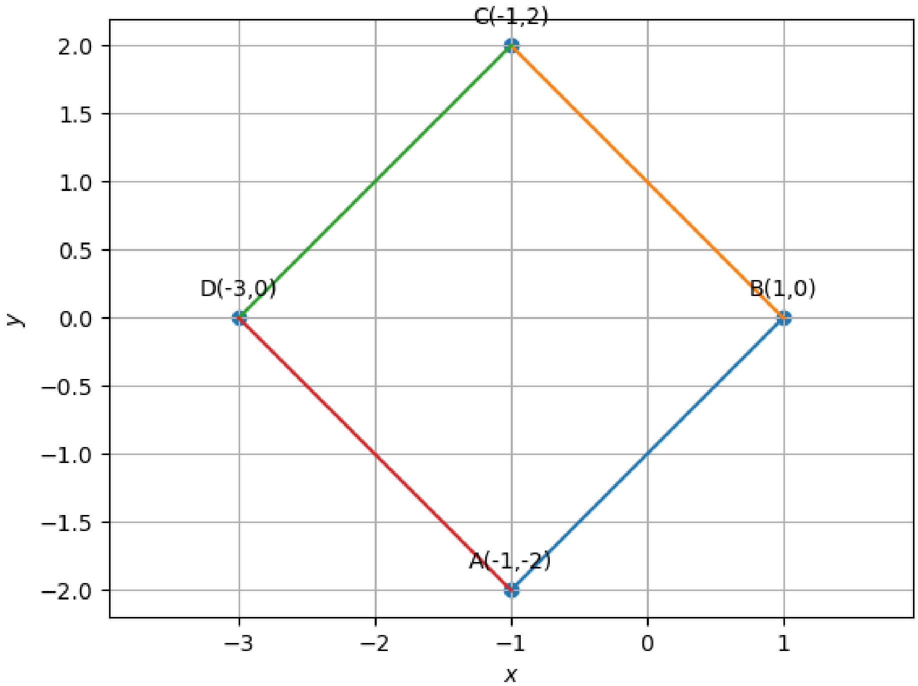

Figure 1:

If rank of $\begin{pmatrix} AB & DC \end{pmatrix} < 2$ then AB is parallel to DC else it is not.

Rank of $\begin{pmatrix} AB & DC \end{pmatrix}$ = Rank of $\begin{pmatrix} 6 & -1 \\ -4 & 7 \end{pmatrix}$

Rank $= 2$

Hence, AB is not parallel to DC

Similarly,If rank of $\begin{pmatrix} BC & AD \end{pmatrix} < 2$ then BC is parallel to AD else it is not.

Rank of $\begin{pmatrix} BC & AD \end{pmatrix}$ = Rank of $\begin{pmatrix} -3 & 2 \\ 2 & -9 \end{pmatrix}$

Rank $= 2$

Hence, BC is not parallel to AD

Now to check if any three points are co-linear

if rank of $\begin{pmatrix} AB & BC \end{pmatrix} < 2$ then points are co-linear

Rank of $\begin{pmatrix} AB & BC \end{pmatrix}$ = Rank of $\begin{pmatrix} 6 & -3 \\ -4 & 2 \end{pmatrix}$
Rank $= 1$
Since none of the opposite sides are parallel to each other and three points are co-linear so these does not form a quadilateral as shown in Figure 2

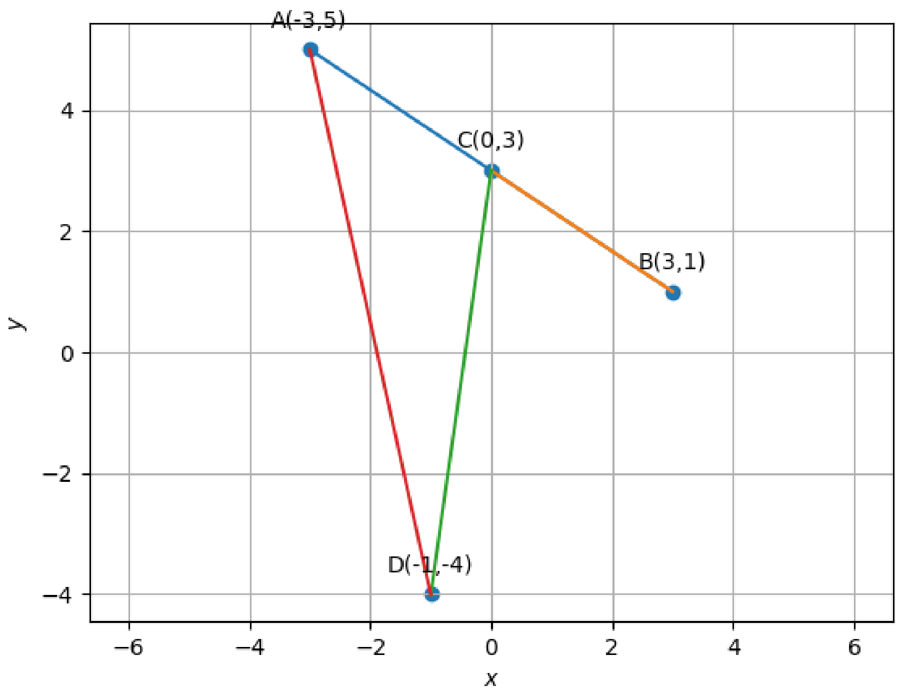

Figure 2:

3. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{17}$$

$$AB = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{18}$$

$$BC = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \tag{19}$$

$$DC = \mathbf{C} - \mathbf{D} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{20}$$

$$AD = \mathbf{D} - \mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \tag{21}$$

$$AC = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{22}$$

$$BD = \mathbf{D} - \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix} \tag{23}$$

If rank of $\begin{pmatrix} AB & DC \end{pmatrix} < 2$ then AB is parallel to DC else it is not.
Rank of $\begin{pmatrix} AB & DC \end{pmatrix}$ = Rank of $\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$
Rank $= 1$
Hence, AB is parallel to DC
Similarly,If rank of $\begin{pmatrix} BC & AD \end{pmatrix} < 2$ then BC is parallel to AD else it is not.
Rank of $\begin{pmatrix} BC & AD \end{pmatrix}$ = Rank of $\begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix}$
Rank $= 1$
Hence, BC is parallel to AD
From here we can conclude that ABCD is parallelogram.
Now checking if the adjacent sides are orthogonal to each other

$$(AB)^\top (BC) = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ -3 \end{pmatrix} = -9 - 3 = -12 \tag{24}$$

Since inner product is not zero so AB and BC are not orthogonal.
Hence, we can say that ABCD is neither a rectangle nor a square.
Now if the diagonals are also orthogonal then it is a Rhombus

$$(AC)^\top (BD) = \begin{pmatrix} 0 & -2 \end{pmatrix} \begin{pmatrix} -6 \\ -4 \end{pmatrix} = 0 + 8 = 8 \tag{25}$$

Hence the diagonals are also not orthogonal so we conclude that ABCD is a parallelogram as shown in Figure 3

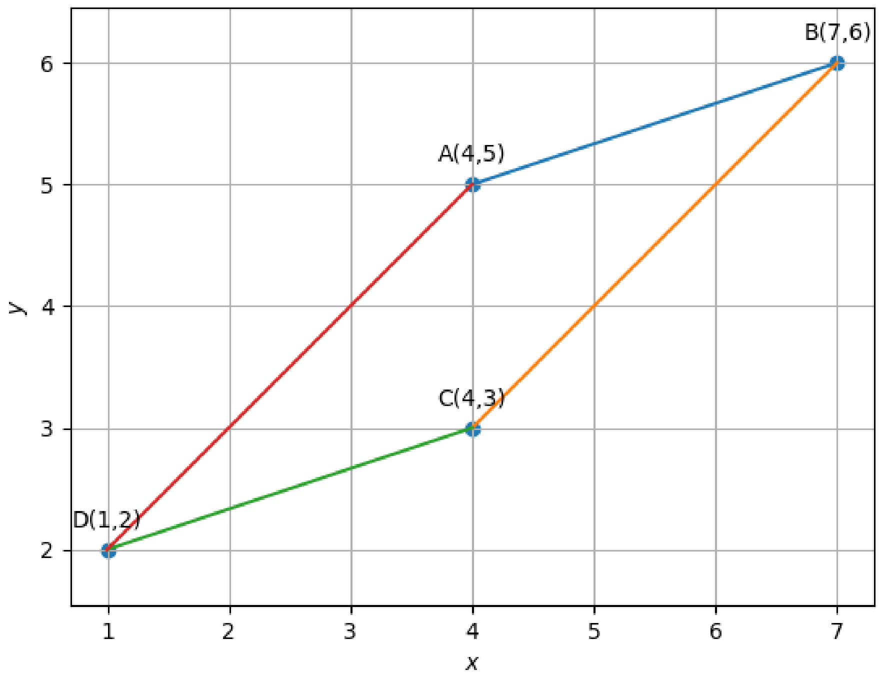

Figure 3: