

CHAPTER-7
COORDINATE GEOMETRY

Exercise 7.4

Q4. The two opposite vertices of a square are $(-1, 2)$ and $(3, 2)$. Find the coordinates of the other two vertices.

Solution: $A = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, C = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Let $B = \begin{pmatrix} x \\ y \end{pmatrix}$

From the properties of square we know that $\|\mathbf{AB}\| = \|\mathbf{BC}\|$

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} x + 1 \\ y - 2 \end{pmatrix} \text{ and } \mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 3 - x \\ 2 - y \end{pmatrix}$$

Now using $\|\mathbf{AB}\| = \|\mathbf{BC}\|$

$$\sqrt{(x + 1)^2 + (y - 2)^2} = \sqrt{(3 - x)^2 + (2 - y)^2} \quad (1)$$

$$(x + 1)^2 = (3 - x)^2 \quad (2)$$

$$x^2 + 1 + 2x = 9 + x^2 - 6x \quad (3)$$

$$8x = 8 \quad (4)$$

$$x = 1 \quad (5)$$

Now we also know that two sides of a square are perpendicular to each other.

$$\mathbf{AB} \perp \mathbf{BC}$$

$$(\mathbf{AB})^T(\mathbf{BC}) = 0 \quad (6)$$

$$(x+1 \quad y-2) \begin{pmatrix} 3-x \\ 2-y \end{pmatrix} = 0 \text{ (substituting value of x)} \quad (7)$$

$$(2 \quad y-2) \begin{pmatrix} 2 \\ 2-y \end{pmatrix} = 0 \quad (8)$$

$$4 + (y-2)(2-y) = 0 \quad (9)$$

$$4 + 2y - y^2 - 4 + 2y = 0 \quad (10)$$

$$y^2 - 4y = 0 \quad (11)$$

$$y(y-4) = 0 \quad (12)$$

$$y = 0, 4 \quad (13)$$

Therefore the other two vertices are $(1, 0)$ and $(1, 4)$

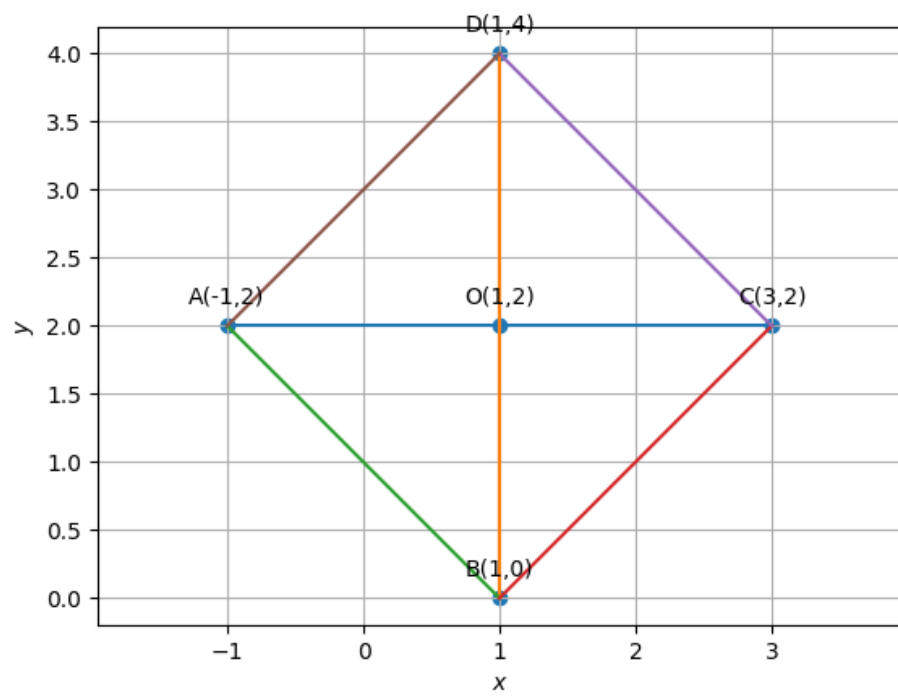


Figure 1: