CHAPTER-7
COORDINATE GEOMETRY

Excercise 7.1

Q6.Name the type of quadilateral formed,if any, by the following points, and give reasons for your answer:

1. $(-1,-2),(1,0),(-1,2),(-3,0)$
2. $(-3,5),(3,1),(0,3),(-1,-4)$
3. $(4,5),(7,6),(4,3),(1,2)$

Solution:

1. The coordinates are given as

$$\mathbf{A}=\begin{pmatrix}-1\\-2\end{pmatrix},\mathbf{B}=\begin{pmatrix}1\\0\end{pmatrix},\mathbf{C}=\begin{pmatrix}-1\\2\end{pmatrix} \text{ and } \mathbf{D}=\begin{pmatrix}-3\\0\end{pmatrix} \tag{1}$$

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix} \tag{2}$$

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}-1\\2\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-2\\2\end{pmatrix} \tag{3}$$

$$\mathbf{C}-\mathbf{D}=\begin{pmatrix}-1\\2\end{pmatrix}-\begin{pmatrix}-3\\0\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix} \tag{4}$$

$$\mathbf{D}-\mathbf{A}=\begin{pmatrix}-3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\end{pmatrix}=\begin{pmatrix}-2\\2\end{pmatrix} \tag{5}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}-1\\2\end{pmatrix}-\begin{pmatrix}-1\\-2\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix} \tag{6}$$

$$\mathbf{D}-\mathbf{B}=\begin{pmatrix}-3\\0\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-4\\0\end{pmatrix} \tag{7}$$

Since $\mathbf{B}-\mathbf{A}=\mathbf{C}-\mathbf{D}$ and $\mathbf{C}-\mathbf{B}=\mathbf{D}-\mathbf{A}$, $ABCD$ is a parallelogram. Now checking if the adjacent sides are orthogonal to each other

$$(\mathbf{B}-\mathbf{A})^{\top}(\mathbf{C}-\mathbf{B})=\begin{pmatrix}2 & 2\end{pmatrix}\begin{pmatrix}-2\\2\end{pmatrix}=-4+4=0 \tag{8}$$

Now if the diagonals are also orthogonal then it is a square else a rectangle

$$(\mathbf{C}-\mathbf{A})^{\top}(\mathbf{D}-\mathbf{B}) = \begin{pmatrix}0 & 4\end{pmatrix}\begin{pmatrix}-4\\0\end{pmatrix} = 0 \tag{9}$$

Hence the diagonals are orthogonal to each other. So, we can conclude that $ABCD$ is a square as shown in Figure 1

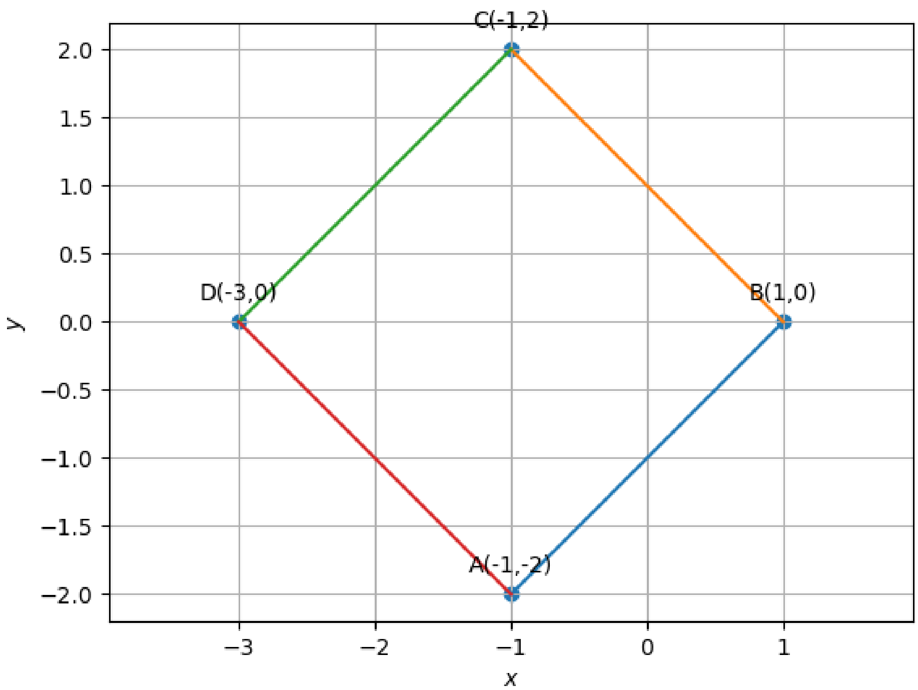

Figure 1:

2. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix}-3\\5\end{pmatrix}, \mathbf{B} = \begin{pmatrix}3\\1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}0\\3\end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix}-1\\-4\end{pmatrix} \tag{10}$$

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}3\\1\end{pmatrix}-\begin{pmatrix}-3\\5\end{pmatrix}=\begin{pmatrix}6\\-4\end{pmatrix} \tag{11}$$

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}0\\3\end{pmatrix}-\begin{pmatrix}3\\1\end{pmatrix}=\begin{pmatrix}-3\\2\end{pmatrix} \tag{12}$$

$$\mathbf{C}-\mathbf{D}=\begin{pmatrix}0\\3\end{pmatrix}-\begin{pmatrix}-1\\-4\end{pmatrix}=\begin{pmatrix}1\\7\end{pmatrix} \tag{13}$$

$$\mathbf{D}-\mathbf{A}=\begin{pmatrix}-1\\-4\end{pmatrix}-\begin{pmatrix}-3\\5\end{pmatrix}=\begin{pmatrix}2\\-9\end{pmatrix} \tag{14}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}0\\3\end{pmatrix}-\begin{pmatrix}-3\\5\end{pmatrix}=\begin{pmatrix}3\\-2\end{pmatrix} \tag{15}$$

$$\mathbf{D}-\mathbf{B}=\begin{pmatrix}-1\\-4\end{pmatrix}-\begin{pmatrix}3\\1\end{pmatrix}=\begin{pmatrix}-4\\-5\end{pmatrix} \tag{16}$$

Since, $\mathbf{B}-\mathbf{A}\neq\mathbf{C}-\mathbf{D}$ and $\mathbf{C}-\mathbf{B}\neq\mathbf{D}-\mathbf{A}$, $ABCD$ is not a parallelogram, it can be an irregular quadilateral.
Now to check if any three points are collinear
if rank of $\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{B}\end{pmatrix}=1$ then points are collinear
Forming the collinearity matrix

$$\begin{pmatrix}6 & -3\\-4 & 2\end{pmatrix}\xleftrightarrow{R_2\to R_2+\frac{2}{3}R_1}=\begin{pmatrix}6 & -3\\0 & 0\end{pmatrix} \tag{17}$$

Hence, rank =1
Since none of the opposite sides are parallel to each other and three points are collinear so these does not form a quadilateral as shown in Figure 2

3. The coordinates are given as

$$\mathbf{A}=\begin{pmatrix}4\\5\end{pmatrix},\mathbf{B}=\begin{pmatrix}7\\6\end{pmatrix},\mathbf{C}=\begin{pmatrix}4\\3\end{pmatrix}\text{ and }\mathbf{D}=\begin{pmatrix}1\\2\end{pmatrix} \tag{18}$$

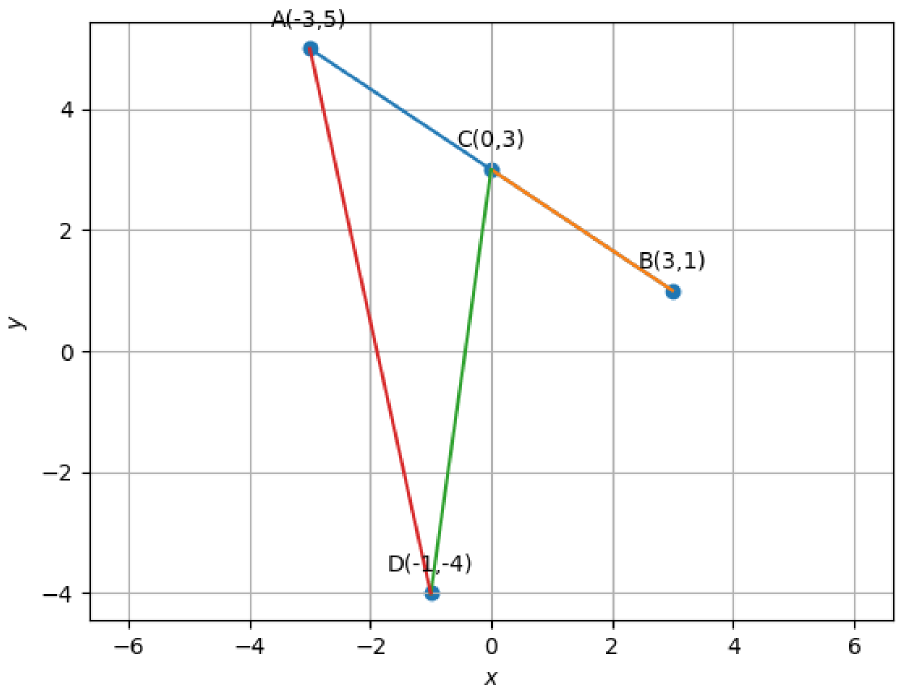

Figure 2:

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}7\\6\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix} \tag{19}$$

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}7\\6\end{pmatrix}=\begin{pmatrix}-3\\-3\end{pmatrix} \tag{20}$$

$$\mathbf{C}-\mathbf{D}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix} \tag{21}$$

$$\mathbf{D}-\mathbf{A}=\begin{pmatrix}1\\2\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}-3\\-3\end{pmatrix} \tag{22}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}0\\-2\end{pmatrix} \tag{23}$$

$$\mathbf{D}-\mathbf{B}=\begin{pmatrix}1\\2\end{pmatrix}-\begin{pmatrix}7\\6\end{pmatrix}=\begin{pmatrix}-6\\-4\end{pmatrix} \tag{24}$$

Since $\mathbf{B}-\mathbf{A}=\mathbf{C}-\mathbf{D}$ and $\mathbf{C}-\mathbf{B}=\mathbf{D}-\mathbf{A}$, $ABCD$ is a parallelogram. Now checking if the adjacent sides are orthogonal to each other

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{B}) = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ -3 \end{pmatrix} = -9-3=-12 \tag{25}$$

Since inner product is not zero so adjacent sides are not orthogonal. Hence, we can say that $ABCD$ is neither a rectangle nor a square. Now if the diagonals are orthogonal then it is a Rhombus.

$$(\mathbf{C}-\mathbf{A})^\top(\mathbf{D}-\mathbf{B}) = \begin{pmatrix} 0 & -2 \end{pmatrix} \begin{pmatrix} -6 \\ -4 \end{pmatrix} = 0+8=8 \tag{26}$$

Hence the diagonals are also not orthogonal so we conclude that $ABCD$ is a parallelogram as shown in Figure 3

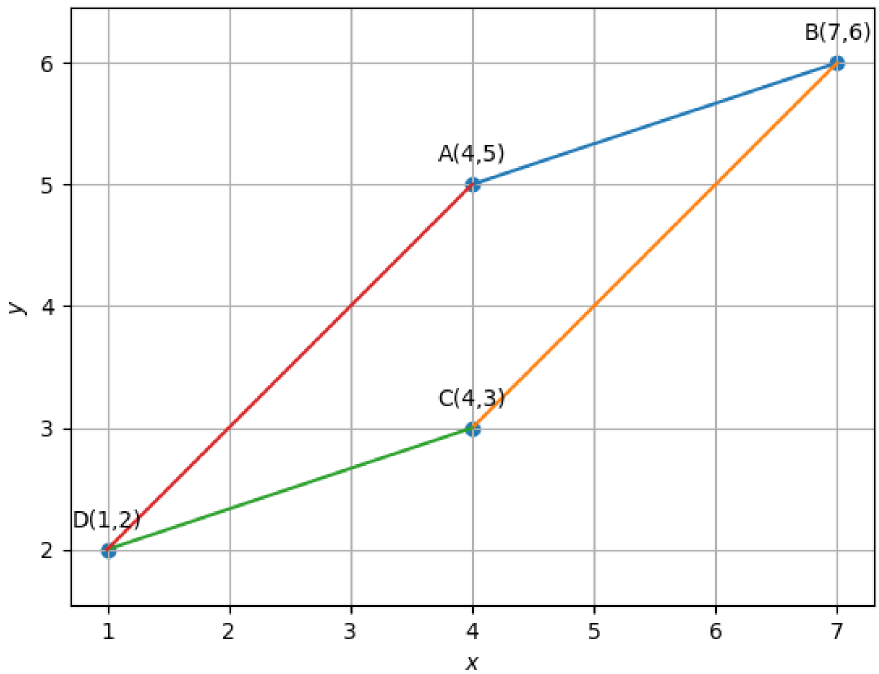

Figure 3: