MATHEMATICS

CBSE class X

***Abstract*—General instructions :**

1) ***All questions are compulsory.***
2) ***The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.***
3) ***All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.***
4) ***There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.***
5) ***Use of calculators is not permitted. You may ask for logarithmic tables, if required.***

Section A

Question numbers 1 to 4 carry 1 mark each.

1) Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}\sqrt{-3}$
2) If the matrix A = $\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.
3) Find the magnitude of each of the two vectors $\mathbf{a}$ and $\mathbf{b}$ having the same magnitude such that the angle between them is 60° and there scalar product is $\frac{9}{2}$
4) If $a \circledast b$ denotes the larger od 'a' and 'b' and if $a \circ b = (a \circledast b) + 3$ then the value of $(5 \circ 10)$, where $\circledast$ and $\circ$ are binary operations.

Section B

Question numbers 5 to 12 carry 2 mark each.

5) Prove that
$$3\sin^{-1}(x) = \sin^{-1}(3x - 4x^3), \quad x\epsilon \ [\frac{-1}{2}, \frac{1}{2}]$$
6) Given A = $\begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix}$, ,compute A^{-1} and show that $2A^{-1} = 9I - A$
7) Differentiate $\tan^{-1}(\frac{1+\cos x}{\sin x})$
8) The total cost C(x) associated with the production of x units of an item is given by C(x) = $0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.
9) Evaluate :
$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} \, dx$$
10) Find the differential equation representing the family of curves $y = ae^{bx+5}$, where a and b are arbitrary constants.
11) If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.
12) A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Section C

Question numbers 13 to 23 carry 4 marks each.

13) Using properties of determinant prove that
$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$
.
14) If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$

OR

If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.
15) If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$
16) Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point $(x_1, y_1$ where $x_1 = 2$ and $y_1 > 0$.
17) An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be

borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question ?

18) Find :

$$\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}\,dx$$

19) Find the particular solution of the differential equation $e^x \tan y dx + (2-e^x)\sec^2 y dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$

20) Let $\mathbf{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\mathbf{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\mathbf{c} = 3\hat{i} + \hat{k} - \hat{k}$. Find a vector **d** which is perpendicular to both **c** and **b** and $\mathbf{d}.\ \mathbf{a} = 21$

21) Find the shortest distance between the lines

$$\mathbf{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \ \ and$$
$$\mathbf{r} = \hat{i} - \hat{j} + 2\hat{k} + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$$

22) Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ?

23) Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X.

Section D

Question numbers 24 to 29 carry 6 marks each.

24) Let A = { xϵ Z : 0≤ x≤12}. Show that R = {(a,b):a,b ϵ A,|a-b| is divisible by 4} is an equivalance class[2]

OR

Show that the function f : R → R defined by $f(x) = \frac{x}{x^2+1}$, ∀ x ϵ R is neither one-one nor onto. Also, if g : R → R is defined as g(x) = 2x - 1, find fog(x).

25) If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use it to solve the system of equations

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y – 2z = –3.$$

OR

Using elementary row transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

26) Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line y = x and the circle $x^2 + y^2 = 32$.

27) Evaluate :

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9\sin 2x}\,dx$$

OR

Evaluate

$$\int_1^2 (x^2 + 3x + e^x)\,dx$$

as the limit of the sum.

28) Find the distance of the point (– 1, – 5, – 10) from the point of ntersection of the line $\mathbf{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\mathbf{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$

29) A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of ¡ 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit ? Formulate the above LPP and solve it graphically and find the maximum profit.