# Excercise 10.2

Q2.In fig 1, if TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110°$ then $\angle PTQ$ is equal to.
**Solution:** Let the output angle be $\phi$. The input parameters are given as

| Input Parameters | Value | Description |
|---|---|---|
| $\mathbf{O}$ | $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ | Centre of the circle |
| r | 1cm | radius of the circle |
| $\theta$ | 110° | $\angle POQ$ |

Table 1:

Any point $\mathbf{X}$ on the circle is given as

$$\mathbf{X} = \mathbf{O} + r\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{1}$$

So points $\mathbf{P}$ and $\mathbf{Q}$ can be calculated as

$$\mathbf{P} = \mathbf{O} + \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2}$$

$$\mathbf{Q} = \mathbf{e}_1 \tag{3}$$

For tangent $TP$

$$\mathbf{n}_1 = \mathbf{P} - \mathbf{O} \tag{4}$$

$$= \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} 1 \\ \tan\theta \end{pmatrix} \tag{5}$$

$$\mathbf{m}_1 = \begin{pmatrix} 1 \\ -\cot\theta \end{pmatrix} \tag{6}$$

For tangent $TQ$

$$\mathbf{n}_2 = \mathbf{e}_1 - \mathbf{O} \tag{7}$$

$$= \mathbf{e}_1 \tag{8}$$

$$\mathbf{m}_2 = \mathbf{e}_2 \tag{9}$$

The equation of $TP$ is given as

$$\mathbf{n}_1^\top \left(\mathbf{x} - \mathbf{P}\right) = 0 \tag{10}$$

$$\mathbf{n}_1^\top \left(\mathbf{x} - \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}\right) = 0 \tag{11}$$

$$\begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix} \mathbf{x} = 1 \tag{12}$$

The equation of $TQ$ is given as

$$\mathbf{n}_2^\top \left(\mathbf{x} - \mathbf{e}_1\right) = 0 \tag{13}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 1 \tag{14}$$

The tangent point can be calculated by solving (12) and (15)

$$\begin{pmatrix} \cos\theta & \sin\theta \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{15}$$

$$\implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ \tan\frac{\theta}{2} \end{pmatrix} \tag{16}$$

The angle between two lines with slope $\mathbf{m}_1$ and $\mathbf{m}_2$ s given as

$$\cos\phi = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\left\|\mathbf{m}_1\right\| \left\|\mathbf{m}_2\right\|} \tag{17}$$

$$= \frac{\begin{pmatrix} 1 & -\cot\theta \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left(\csc\theta\right)\left(1\right)} \tag{18}$$

$$= -\cos\theta \tag{19}$$

$$\implies \cos\phi = -\cos\theta \tag{20}$$

Hence,

$$\phi = \cos^{-1}\left(\cos\left(180^\circ - \theta\right)\right) \tag{21}$$

$$= 180^\circ - \theta = 70^\circ \tag{22}$$

Hence, $\angle PTQ = 70^\circ$. See Fig 1

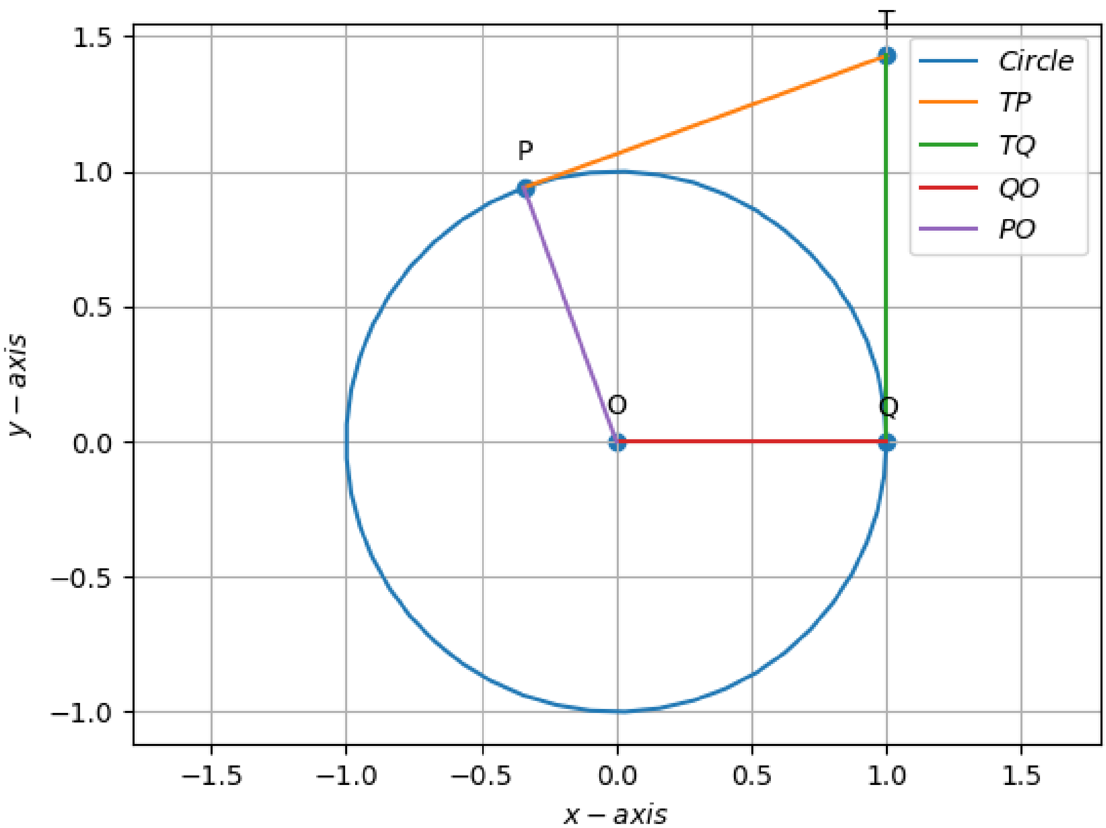
T
P
O
Q
Circle
TP
TQ
QO
PO
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
y − axis
x − axis

Figure 1: