CHAPTER-10
VECTOR ALGEBRA

Excercise 10.4

Q5.Find λ and μ if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \mathbf{0}$
Solution:

$$\text{Let } \mathbf{A} = \begin{pmatrix} 2 \\ 6 \\ 27 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 \\ \lambda \\ \mu \end{pmatrix} \tag{1}$$

$$\tag{2}$$

The cross product or vector product of $\mathbf{A}, \mathbf{B}$ is defined as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \\ \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix} \tag{3}$$

Hence

$$\begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{vmatrix} = \begin{vmatrix} 6 & \lambda \\ 27 & \mu \end{vmatrix} = 6\mu - 27\lambda \tag{4}$$

$$\begin{vmatrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} = \begin{vmatrix} 27 & \mu \\ 2 & 1 \end{vmatrix} = 27 - 2\mu \tag{5}$$

$$\begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 6 & \lambda \end{vmatrix} = 2\lambda - 6 \tag{6}$$

Substituting the values

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 6\mu - 27\lambda \\ 27 - 2\mu \\ 2\lambda - 6 \end{pmatrix} \tag{7}$$

Now we know

$$\mathbf{A} \times \mathbf{B} = \mathbf{0} \tag{8}$$

So,

$$\begin{pmatrix} 6\mu - 27\lambda \\ 27 - 2\mu \\ 2\lambda - 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

Since the number of equations are more than the number of variables so we require only two equations.

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 27 \\ 6 \end{pmatrix} \tag{10}$$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 27 \\ 6 \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 27 \\ 6 \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 13.5 \\ 3 \end{pmatrix} \tag{13}$$

Hence, the values are $\lambda = 3$ and $\mu = 13.5$