

## CONIC SECTIONS

### Excercise 10.2

Q2.In fig 1, if TP and TQ are two tangents to a circle with centre O so that  $\angle POQ = 110^\circ$  then  $\angle PTQ$  is equal to.

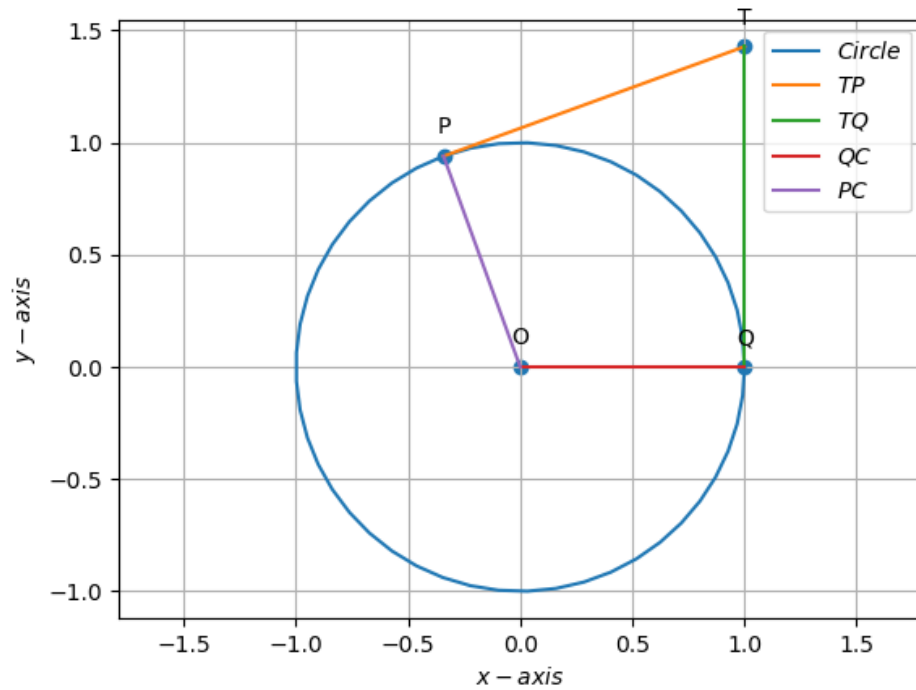


Figure 1:

**Solution:** Let

$$\text{radius} = r \text{ and } \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1)$$

Now we know any point on circle is given as

$$\mathbf{X} = \mathbf{c} + r \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2)$$

So,

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + r \begin{pmatrix} \cos 0^\circ \\ \sin 0^\circ \end{pmatrix} = \begin{pmatrix} r \\ 0 \end{pmatrix} \quad (3)$$

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + r \begin{pmatrix} \cos 110^\circ \\ \sin 110^\circ \end{pmatrix} = \begin{pmatrix} r \cos 110^\circ \\ r \sin 110^\circ \end{pmatrix} \quad (4)$$

For tangent TP

$$\mathbf{n}_1 = \mathbf{P} - \mathbf{c} = \begin{pmatrix} r \cos 110^\circ \\ r \sin 110^\circ \end{pmatrix} = \begin{pmatrix} 1 \\ \tan 110^\circ \end{pmatrix} \quad (5)$$

$$\mathbf{m}_1 = \begin{pmatrix} 1 \\ -\cot 110^\circ \end{pmatrix} \quad (6)$$

For tangent TQ

$$\mathbf{n}_2 = \mathbf{Q} - \mathbf{c} = \begin{pmatrix} r \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (7)$$

$$\mathbf{m}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (8)$$

Now the angle between two lines with slope  $\mathbf{m}_1$  and  $\mathbf{m}_2$  is given as

$$\cos \theta = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (9)$$

Now,

$$\|\mathbf{m}_1\| = \sqrt{1^2 + (-\cot 110^\circ)^2} = \csc 110^\circ \quad (10)$$

$$\|\mathbf{m}_2\| = 1 \quad (11)$$

Substituting the values in (9)

$$\cos \theta = \frac{\begin{pmatrix} 1 & -\cot 110^\circ \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\csc 110^\circ} \quad (12)$$

$$= -\cos 110^\circ \quad (13)$$

$$= \cos (180 - 110) \quad (14)$$

$$\implies \theta = 70^\circ \quad (15)$$

Hence,  $\angle PTQ = 70^\circ$