CHAPTER-10
VECTOR ALGEBRA

Excercise 10.4

Q5.Find λ and μ if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\mathbf{0}$
Solution:

$$\text{Let } \mathbf{A} = \begin{pmatrix}3\\2\\2\end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix}1\\\lambda\\\mu\end{pmatrix} \tag{1}$$

$$\tag{2}$$

The cross product or vector product of $\mathbf{A},\mathbf{B}$ is defined as

$$\mathbf{A}\times\mathbf{B} = \begin{pmatrix}\begin{vmatrix}\mathbf{A}_{23} & \mathbf{B}_{23}\\\mathbf{A}_{31} & \mathbf{B}_{31}\\\mathbf{A}_{12} & \mathbf{B}_{12}\end{vmatrix}\end{pmatrix} \tag{3}$$

Now we know

$$\mathbf{A}\times\mathbf{B} = \mathbf{0} \tag{4}$$

Hence

$$\begin{vmatrix}\mathbf{A}_{23} & \mathbf{B}_{23}\end{vmatrix} = 0, \begin{vmatrix}\mathbf{A}_{31} & \mathbf{B}_{31}\end{vmatrix} = 0 \text{ and } \begin{vmatrix}\mathbf{A}_{12} & \mathbf{B}_{12}\end{vmatrix} = 0 \tag{5}$$

Substituting the values

$$\begin{vmatrix}\mathbf{A}_{23} & \mathbf{B}_{23}\end{vmatrix} = 0 \tag{6}$$

$$\begin{vmatrix}6 & \lambda\\27 & \mu\end{vmatrix} = 0 \tag{7}$$

$$6\mu - 27\lambda = 0 \tag{8}$$

$$\begin{vmatrix}\mathbf{A}_{31} & \mathbf{B}_{31}\end{vmatrix} = 0 \tag{9}$$

$$\begin{vmatrix}27 & \mu\\2 & 1\end{vmatrix} = 0 \tag{10}$$

$$27 - 2\mu = 0 \tag{11}$$

$$\mu = 13.5 \tag{12}$$

$$\begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} = 0 \tag{13}$$

$$\begin{vmatrix} 2 & 1 \\ 6 & \lambda \end{vmatrix} = 0 \tag{14}$$

$$2\lambda - 6 = 0 \tag{15}$$

$$\lambda = 3 \tag{16}$$

Hence, the values are $\lambda = 3$ and $\mu = 13.5$