CLASS 11 CHAPTER-11
LINES

Exercise 10.3

Q5.Find the points on the x-axis, whose distances from the line $\frac{x}{3}+\frac{y}{4}=1$ are 4 units.

Solution: Given line is

$$\frac{x}{3}+\frac{y}{4}=1 \tag{1}$$

which can be written as

$$4x+3y=12 \tag{2}$$

this equation can be expressed as

$$\mathbf{n}^\top\mathbf{x}=c \tag{3}$$

$$\text{where } \mathbf{n}=\begin{pmatrix}4\\3\end{pmatrix}, c=12 \tag{4}$$

Now we know the distance formula is given as

$$d=\frac{\left|\mathbf{n}^\top\mathbf{P}-c\right|}{\|\mathbf{n}\|} \tag{5}$$

Since it is given that point lies on the x-axis so let

$$\mathbf{P}=x\mathbf{e}_1=\begin{pmatrix}x\\0\end{pmatrix} \tag{6}$$

Substituting the values in the given formula we get

$$d=\frac{\left|\mathbf{n}^\top\mathbf{P}-c\right|}{\|\mathbf{n}\|} \tag{7}$$

$$=\frac{\left|x\mathbf{n}^\top\mathbf{e}_1-c\right|}{\|\mathbf{n}\|} \tag{8}$$

So,

$$\left|x\mathbf{n}^\top\mathbf{e}_1 - c\right| = d\left\|\mathbf{n}\right\| \tag{9}$$

So, either

$$x = \frac{d\left\|\mathbf{n}\right\| + c}{\mathbf{n}^\top\mathbf{e}_1} \tag{10}$$

Or,

$$x = \frac{-d\left\|\mathbf{n}\right\| + c}{\mathbf{n}^\top\mathbf{e}_1} \tag{11}$$

where,

$$\mathbf{n} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{12}$$

$$\left\|\mathbf{n}\right\| = \sqrt{4^2 + 3^2} = 5 \tag{13}$$

$$d = 4 \tag{14}$$

$$c = 12 \tag{15}$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{16}$$

So, substituting the values we get either

$$x = 8 \text{ Or } x = -2 \tag{17}$$

Hence, the two points that satisfy the above criteria are $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$