# Excercise 10.3

Q.3.2 Reduce the equation $y - 2 = 0$ into normal form. Find the perpendicular distances from the origin and angle between perpendicular and the positive x-axis.
**Solution:** The given equation can be written as

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2 \tag{1}$$

Let $\mathbf{O}$ be the point from where we have to find the perpendicular distance. The perpendicular distance will be the minimum distance from $\mathbf{O}$ to theline. Let $\mathbf{P}$ be the foot of perpendicular. This can be formulated as an optimization problem as follows:

$$\min_{\mathbf{x}} f\left(\mathbf{x}\right) = \left\|\mathbf{x} - \mathbf{O}\right\|^2 \tag{2}$$

$$\text{s.t. } g\left(\mathbf{x}\right) = \mathbf{n}^\top \mathbf{x} - c = 0 \tag{3}$$

where

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

$$c = 2 \tag{6}$$

Now we define

$$H\left(\mathbf{x}, \lambda\right) = f\left(\mathbf{x}\right) - \lambda g\left(\mathbf{x}\right) \tag{7}$$

and we find that

$$\nabla f\left(\mathbf{x}\right) = 2\left(\mathbf{x} - \mathbf{O}\right) \tag{8}$$

$$\nabla g\left(\mathbf{x}\right) = \mathbf{n} \tag{9}$$

We have to find $\lambda \in \mathbb{R}$ such that

$$\nabla H\left(\mathbf{x}, \lambda\right) = 0 \tag{10}$$

$$\implies 2\left(\mathbf{x} - \mathbf{O}\right) - \lambda \mathbf{n} = 0 \tag{11}$$

$$\implies \mathbf{x} = \frac{\lambda}{2}\mathbf{n} + \mathbf{O} \tag{12}$$

Substituting (12) in (1)

$$\mathbf{n}^\top \left(\frac{\lambda}{2}\mathbf{n} + \mathbf{O}\right) - c = 0 \tag{13}$$

$$\implies \lambda = \frac{2\left(c - \mathbf{n}^\top \mathbf{O}\right)}{\left\|\mathbf{n}\right\|^2} \tag{14}$$

Substituting the value of $\lambda$ in (12),

$$\mathbf{x}_{min} = \mathbf{P} = \mathbf{O} + \frac{\mathbf{n}\left(c - \mathbf{n}^\top \mathbf{O}\right)}{\|\mathbf{n}\|^2} \tag{15}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \frac{\begin{pmatrix} 0 \\ 1 \end{pmatrix}\left(2 - \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)}{1} \tag{16}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{17}$$

$$= \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{18}$$

$$OP = \|\mathbf{P} - \mathbf{O}\|^2 \tag{19}$$

$$= \sqrt{0^2 + 2^2} = 2 \tag{20}$$

The angle $\theta$ made by this perpendicular with x-axis is given by

$$\theta = \tan^{-1}\left(\frac{2}{0}\right) \tag{21}$$

$$= 90^\circ \tag{22}$$

The normal form of equation for straight line is given by

$$\begin{pmatrix} \cos 90^\circ & \sin 90^\circ \end{pmatrix} \mathbf{x} = 0 \tag{23}$$

See figure 1

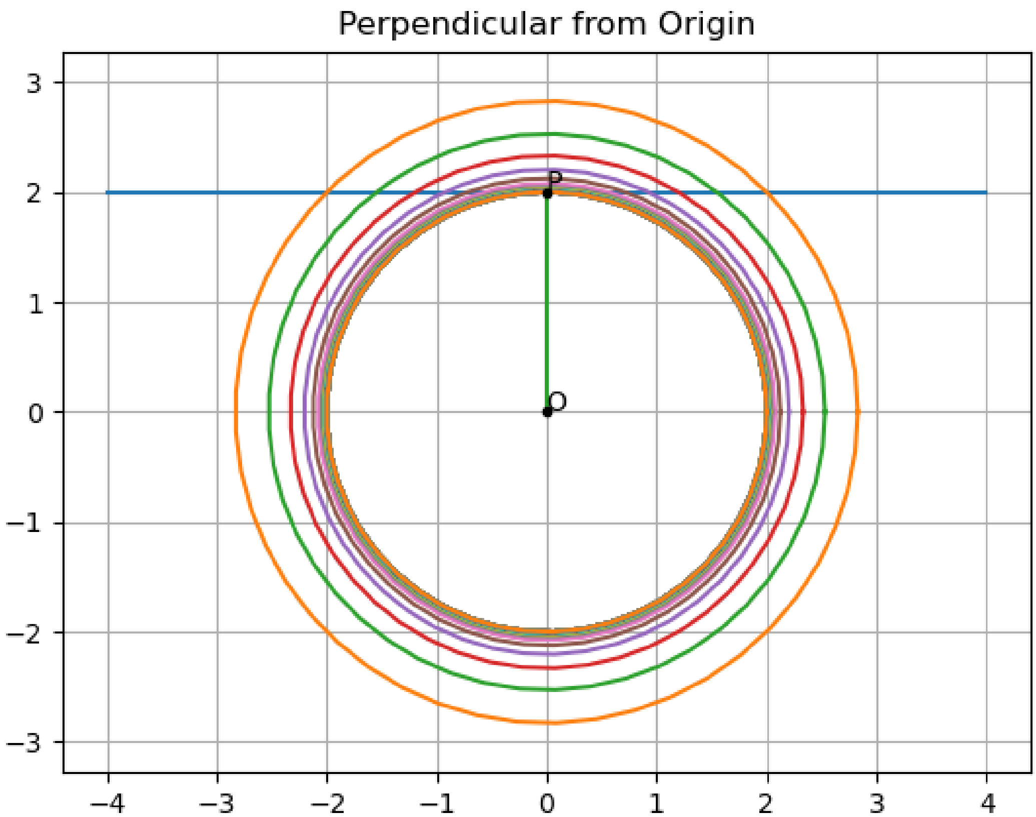


Figure 1: