Excercise 10.3

Q10.If $\mathbf{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\mathbf{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\mathbf{c} = 3\hat{i} + \hat{j}$ are such that $\mathbf{a} + \lambda\mathbf{b}$ is perpendicular to $\mathbf{c}$, then find the value of λ.
Solution:

$$\mathbf{a} + \lambda\mathbf{b} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 - \lambda \\ 2 + 2\lambda \\ 3 + \lambda \end{pmatrix} \tag{1}$$

Now we know,

$$(\mathbf{a} + \lambda\mathbf{b})^\top \mathbf{c} = 0 \tag{2}$$

Hence,

$$\begin{pmatrix} 2 - \lambda & 2 + 2\lambda & 3 + \lambda \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = 0 \tag{3}$$

$$(2 - \lambda)3 + 2 + 2\lambda = 0 \tag{4}$$

$$6 - 3\lambda + 2 + 2\lambda = 0 \tag{5}$$

$$\lambda = 8 \tag{6}$$