

CHAPTER-7
COORDINATE GEOMETRY

Exercise 7.1

Q6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

1. $(-1, -2), (1, 0), (-1, 2), (-3, 0)$
2. $(-3, 5), (3, 1), (0, 3), (-1, -4)$
3. $(4, 5), (7, 6), (4, 3), (1, 2)$

Solution:

1. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (2)$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad (3)$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (4)$$

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad (5)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (6)$$

$$\mathbf{D} - \mathbf{B} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (7)$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \text{ and } \mathbf{C} - \mathbf{B} = \mathbf{D} - \mathbf{A}. \quad (8)$$

Hence, $ABCD$ is a parallelogram.

Now checking if the adjacent sides are orthogonal to each other

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 2 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = -4 + 4 = 0 \quad (9)$$

Now if the diagonals are also orthogonal then it is a square else a rectangle.

$$(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{B}) = \begin{pmatrix} 0 & 4 \end{pmatrix} \begin{pmatrix} -4 \\ 0 \end{pmatrix} = 0 \quad (10)$$

Hence the diagonals are orthogonal to each other.

So, we can conclude that $ABCD$ is a square as shown in Figure 1

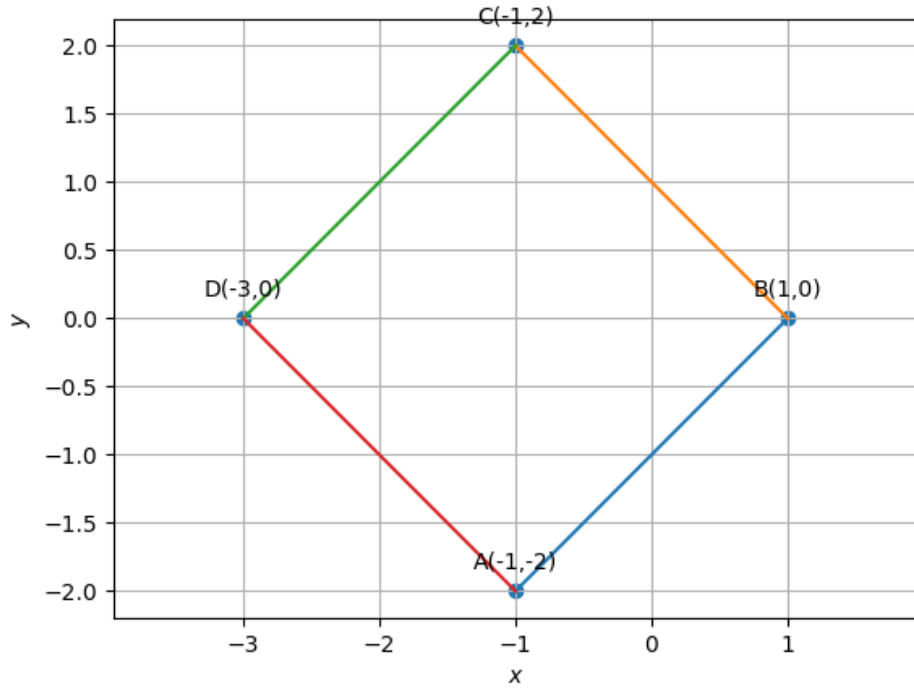


Figure 1:

2. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \quad (11)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \quad (12)$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \quad (13)$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \quad (14)$$

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -9 \end{pmatrix} \quad (15)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (16)$$

$$\mathbf{D} - \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \quad (17)$$

$$\mathbf{B} - \mathbf{A} \neq \mathbf{C} - \mathbf{D} \text{ and } \mathbf{C} - \mathbf{B} \neq \mathbf{D} - \mathbf{A}, \quad (18)$$

Hence, $ABCD$ is not a parallelogram, it can be a irregular quadilateral.

Now to check if any three points are collinear,

if rank of $(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{B}) = 1$ then points are collinear

Forming the collinearity matrix

$$\begin{pmatrix} 6 & -3 \\ -4 & 2 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 + \frac{2}{3}R_1} = \begin{pmatrix} 6 & -3 \\ 0 & 0 \end{pmatrix} \quad (19)$$

Hence, rank = 1

Since none of the opposite sides are parallel to each other and three points are collinear so these does not form a quadilateral as shown in Figure 2

3. The coordinates are given as

$$\mathbf{A} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (20)$$

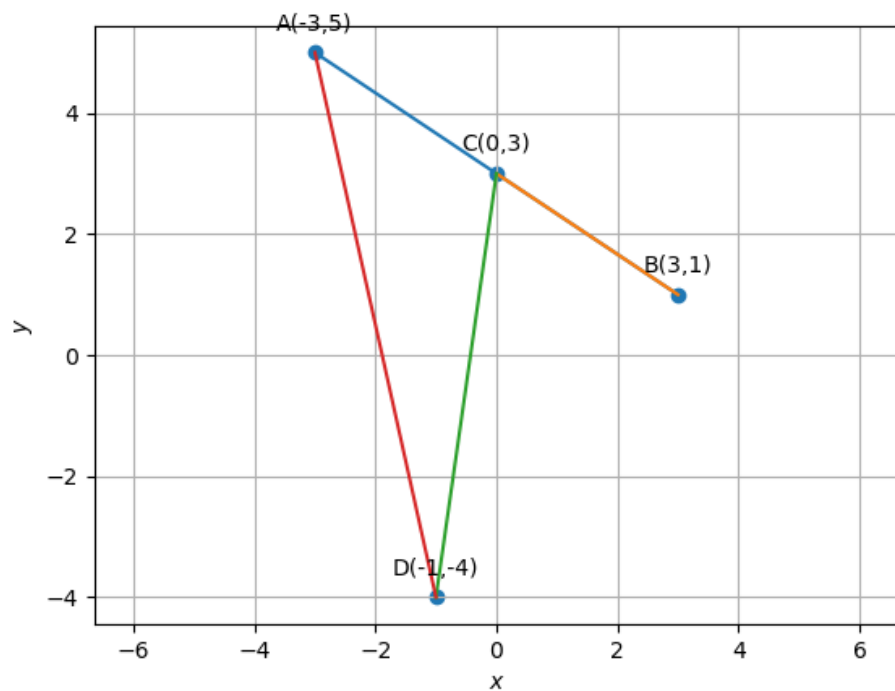


Figure 2:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (21)$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \quad (22)$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (23)$$

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \quad (24)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad (25)$$

$$\mathbf{D} - \mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 7 \\ 6 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix} \quad (26)$$

$$(27)$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \text{ and } \mathbf{C} - \mathbf{B} = \mathbf{D} - \mathbf{A}, \quad (28)$$

Hence, $ABCD$ is a parallelogram.

Now checking if the adjacent sides are orthogonal to each other

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{B}) = (3 \ 1) \begin{pmatrix} -3 \\ -3 \end{pmatrix} = -9 - 3 = -12 \quad (29)$$

Since inner product is not zero so adjacent sides are not orthogonal.

Hence, we can say that $ABCD$ is neither a rectangle nor a square.

Now if the diagonals are orthogonal then it is a Rhombus.

$$(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{B}) = (0 \ -2) \begin{pmatrix} -6 \\ -4 \end{pmatrix} = 0 + 8 = 8 \quad (30)$$

Hence the diagonals are also not orthogonal so we conclude that $ABCD$ is a parallelogram as shown in Figure 3

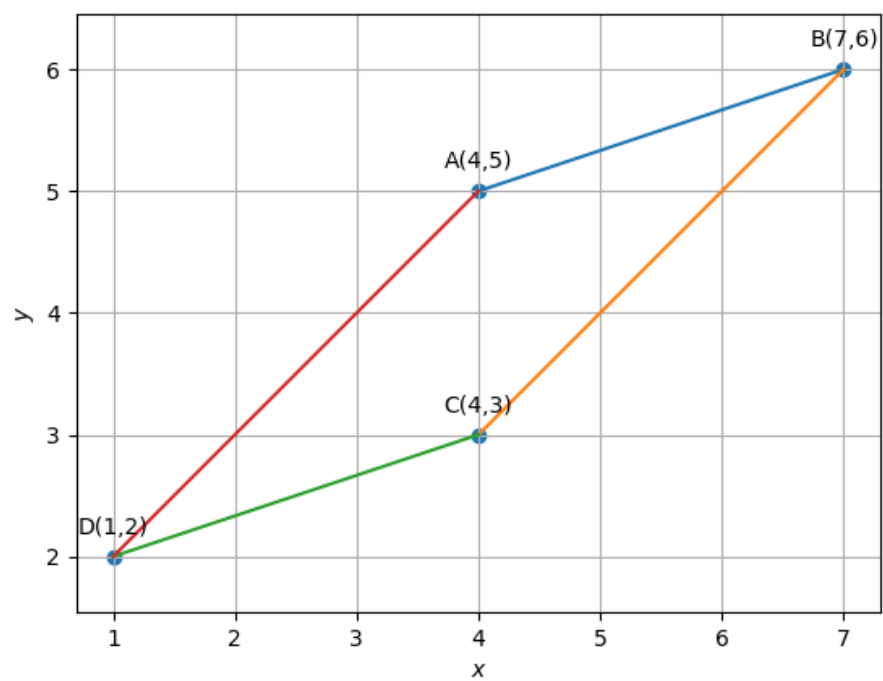


Figure 3: