## CLASS 11 CHAPTER-11 LINES

## Exercise 10.3

Q5. Find the points on the x-axis, whose distances from the line  $\frac{x}{3}+\frac{y}{4}=1$  are 4 units.

Solution: Given line is

$$\frac{x}{3} + \frac{y}{4} = 1 \tag{1}$$

which can be written as

$$4x + 3y = 12 \tag{2}$$

this equation can be expressed as

$$\mathbf{n}^{\mathsf{T}}\mathbf{x} = c \tag{3}$$

where 
$$\mathbf{n} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, c = 12$$
 (4)

Now we know the distance formula is given as

$$d = \frac{\left| \mathbf{n}^{\mathsf{T}} \mathbf{P} - c \right|}{\|\mathbf{n}\|} \tag{5}$$

Since it is given that point lies on the x-axis so let

$$\mathbf{P} = x\mathbf{e}_1 = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{6}$$

Substituting the values in the given formula we get

$$d = \frac{\left| \mathbf{n}^{\top} \mathbf{P} - c \right|}{\|\mathbf{n}\|} \tag{7}$$

$$=\frac{\left|x\mathbf{n}^{\top}\mathbf{e}_{1}-c\right|}{\|\mathbf{n}\|}\tag{8}$$

So,

$$|x\mathbf{n}^{\mathsf{T}}\mathbf{e}_{1} - c| = d \|\mathbf{n}\| \tag{9}$$

So, either

$$x = \frac{d \|\mathbf{n}\| + c}{\mathbf{n}^{\mathsf{T}} \mathbf{e}_1} \tag{10}$$

Or,

$$x = \frac{-d \|\mathbf{n}\| + c}{\mathbf{n}^{\top} \mathbf{e}_1} \tag{11}$$

where,

$$\mathbf{n} = \begin{pmatrix} 4\\3 \end{pmatrix} \tag{12}$$

$$\|\mathbf{n}\| = \sqrt{4^2 + 3^2} = 5$$
 (13)  
  $d = 4$  (14)

$$d = 4 \tag{14}$$

$$c = 12 \tag{15}$$

$$\mathbf{e}_1 = \begin{pmatrix} 1\\0 \end{pmatrix} \tag{16}$$

So, substituting the values we get either

$$x = 8 \text{ Or } x = -2 \tag{17}$$

Hence, the two points that satisfy the above criteria are  $\begin{pmatrix} -2\\0 \end{pmatrix}$  and  $\begin{pmatrix} 8\\0 \end{pmatrix}$  as shown in Figure 1

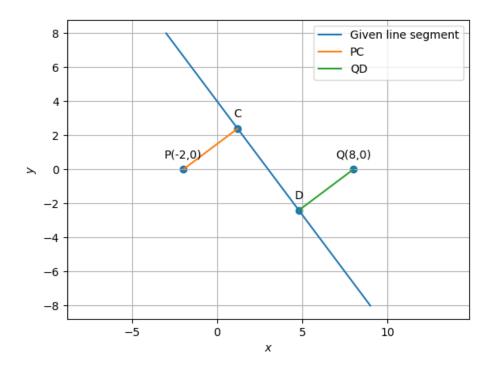


Figure 1: