CLASS 11 CHAPTER-11
LINES

Excercise 10.2

Q18.$P(a, b)$ is the mid-point of the line segment between axes. Show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$
Solution: Let

$$\mathbf{A} = x\mathbf{e_1}, \mathbf{B} = y\mathbf{e_2} \text{ and } \mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{1}$$

where

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

as shown in Figure 1

Now we know

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{x\mathbf{e_1} + y\mathbf{e_2}}{2} \tag{3}$$

$$2\mathbf{P} = x\mathbf{e_1} + y\mathbf{e_2} \tag{4}$$

$$\mathbf{e_1}^\top (2\mathbf{P}) = \mathbf{e_1}^\top (x\mathbf{e_1} + y\mathbf{e_2}) \tag{5}$$

$$= x \tag{6}$$

$$\mathbf{e_2}^\top (2\mathbf{P}) = \mathbf{e_2}^\top (x\mathbf{e_1} + y\mathbf{e_2}) \tag{7}$$

$$= y \tag{8}$$

$$\tag{9}$$

So, from here we can say that

$$x = 2\mathbf{e_1}^\top \mathbf{P} = 2a \tag{10}$$

$$y = 2\mathbf{e_2}^\top \mathbf{P} = 2b \tag{11}$$

$$\mathbf{A} = \begin{pmatrix} 2a \\ 0 \end{pmatrix} \tag{12}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 2b \end{pmatrix} \tag{13}$$

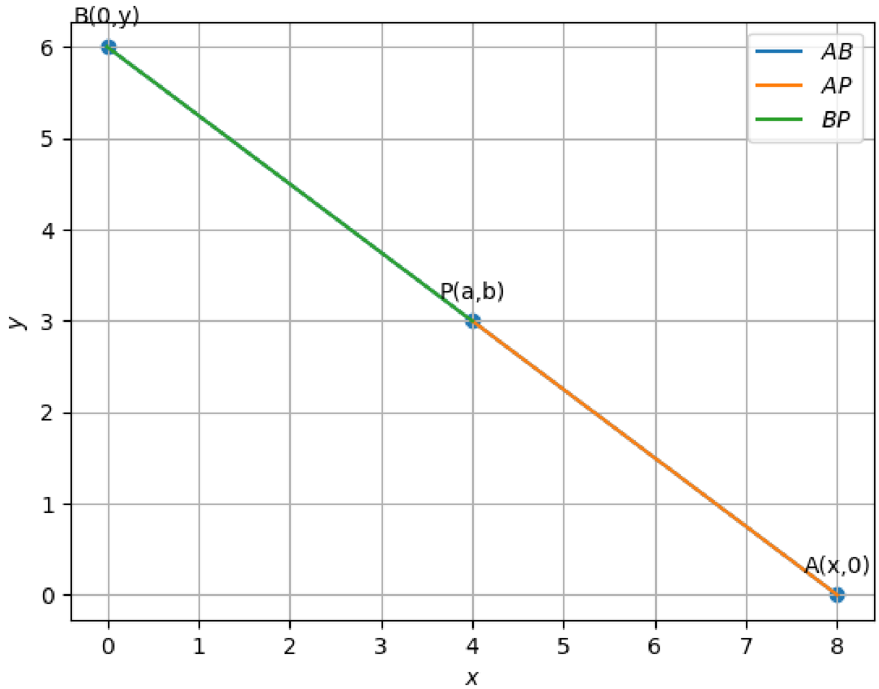

Figure 1:

Now direction vector is

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \tag{14}$$

$$= \begin{pmatrix} 2a \\ -2b \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-b}{a} \end{pmatrix} \tag{15}$$

so normal vector is

$$\mathbf{n} = \begin{pmatrix} 1 \\ \frac{a}{b} \end{pmatrix} \tag{16}$$

So, the equation of line passing through $\mathbf{P}$

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{P}\right) = 0 \tag{17}$$

$$\begin{pmatrix} 1 & \frac{a}{b} \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} a \\ b \end{pmatrix}\right) = 0 \tag{18}$$

$$\begin{pmatrix} 1 & \frac{a}{b} \end{pmatrix} \mathbf{x} = 2a \tag{19}$$

$$\begin{pmatrix} 1 & \frac{a}{b} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2a \tag{20}$$

$$x + \frac{ay}{b} = 2a \tag{21}$$

$$bx + ay = 2ab \tag{22}$$

$$\frac{x}{a} + \frac{y}{b} = 2 \tag{23}$$

Hence proved.