

CLASS 11 CHAPTER-11
LINES

Exercise 10.2

Q18. $P(a, b)$ is the mid-point of the line segment between axes. Show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$

Solution: Let

$$\mathbf{A} = x\mathbf{e}_1, \mathbf{B} = y\mathbf{e}_2 \text{ and } \mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (1)$$

where

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2)$$

as shown in Figure 1

Now we know

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{x\mathbf{e}_1 + y\mathbf{e}_2}{2} \quad (3)$$

$$2\mathbf{P} = x\mathbf{e}_1 + y\mathbf{e}_2 \quad (4)$$

$$\mathbf{e}_1^\top (2\mathbf{P}) = \mathbf{e}_1^\top (x\mathbf{e}_1 + y\mathbf{e}_2) \quad (5)$$

$$= x \quad (6)$$

$$\mathbf{e}_2^\top (2\mathbf{P}) = \mathbf{e}_2^\top (x\mathbf{e}_1 + y\mathbf{e}_2) \quad (7)$$

$$= y \quad (8)$$

So, from here we can say that

$$x = 2\mathbf{e}_1^\top \mathbf{P} = 2a \quad (9)$$

$$y = 2\mathbf{e}_2^\top \mathbf{P} = 2b \quad (10)$$

$$\mathbf{A} = \begin{pmatrix} 2a \\ 0 \end{pmatrix} \quad (11)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 2b \end{pmatrix} \quad (12)$$

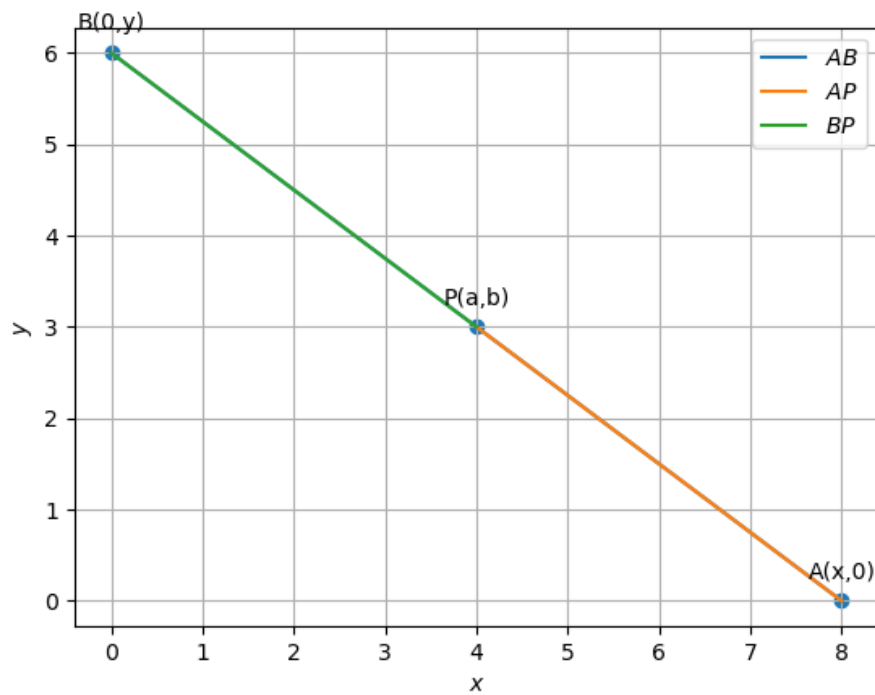


Figure 1:

Now direction vector is

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \quad (13)$$

$$= \begin{pmatrix} 2a \\ -2b \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-b}{a} \end{pmatrix} \quad (14)$$

so normal vector is

$$\mathbf{n} = \begin{pmatrix} 1 \\ \frac{a}{b} \end{pmatrix} \quad (15)$$

So, the equation of line passing through \mathbf{P}

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{P}) = 0 \quad (16)$$

$$\left(1 \quad \frac{a}{b}\right) \left(\mathbf{x} - \begin{pmatrix} a \\ b \end{pmatrix}\right) = 0 \quad (17)$$

$$\left(1 \quad \frac{a}{b}\right) \mathbf{x} = 2a \quad (18)$$

$$\left(1 \quad \frac{a}{b}\right) \begin{pmatrix} x \\ y \end{pmatrix} = 2a \quad (19)$$

$$x + \frac{ay}{b} = 2a \quad (20)$$

$$bx + ay = 2ab \quad (21)$$

$$\frac{x}{a} + \frac{y}{b} = 2 \quad (22)$$

Hence proved.