

CLASS 11 CHAPTER-11
LINES

Exercise 10.3

Q5. Find the points on the x-axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Solution: Given line is

$$\frac{x}{3} + \frac{y}{4} = 1 \quad (1)$$

which can be written as

$$4x + 3y = 12 \quad (2)$$

this equation can be expressed as

$$\mathbf{n}^\top \mathbf{x} = c \quad (3)$$

$$\text{where } \mathbf{n} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, c = 12 \quad (4)$$

Now we know the distance formula is given as

$$d = \frac{|\mathbf{n}^\top \mathbf{P} - c|}{\|\mathbf{n}\|} \quad (5)$$

Since it is given that point lies on the x-axis so let

$$\mathbf{P} = x\mathbf{e}_1 = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (6)$$

Substituting the values in the given formula we get

$$d = \frac{|\mathbf{n}^\top \mathbf{P} - c|}{\|\mathbf{n}\|} \quad (7)$$

$$d = \frac{|x\mathbf{n}^\top \mathbf{e}_1 - c|}{\|\mathbf{n}\|} \quad (8)$$

So,

$$|x\mathbf{n}^\top \mathbf{e}_1 - c| = d \|\mathbf{n}\| \quad (9)$$

So, either

$$x = \frac{d \|\mathbf{n}\| + c}{\mathbf{n}^\top \mathbf{e}_1} \quad (10)$$

Or,

$$x = \frac{-d \|\mathbf{n}\| + c}{\mathbf{n}^\top \mathbf{e}_1} \quad (11)$$

where,

$$\mathbf{n} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad (12)$$

$$\|\mathbf{n}\| = \sqrt{4^2 + 3^2} = 5 \quad (13)$$

$$d = 4 \quad (14)$$

$$c = 12 \quad (15)$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (16)$$

So, substituting the values we get either

$$x = 8 \text{ Or } x = -2 \quad (17)$$

Hence, the two points that satisfy the above criteria are $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$