Excercise 10.2

Q2.In fig 1, if TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110^\circ$ then $\angle PTQ$ is equal to.

Solution: Let us assume the centre $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Any point $\mathbf{X}$ on the circle is given as

$$\mathbf{X} = \mathbf{O} + \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{1}$$

The input parameters are given as

Input Parameters	**Value**
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
radius	1cm
$\angle POQ$	110°
P	$\begin{pmatrix} \cos 110^\circ \\ \sin 110^\circ \end{pmatrix}$
Q	$\begin{pmatrix} \cos 0^\circ \\ \sin 0^\circ \end{pmatrix}$

Table 1:

For tangent TP

$$\mathbf{n}_1 = \mathbf{P} - \mathbf{O} \tag{2}$$

$$= \begin{pmatrix} \cos 110^\circ \\ \sin 110^\circ \end{pmatrix} = \begin{pmatrix} 1 \\ \tan 110^\circ \end{pmatrix} \tag{3}$$

$$\mathbf{m}_1 = \begin{pmatrix} 1 \\ -\cot 110^\circ \end{pmatrix} \tag{4}$$

For tangent TQ

$$\mathbf{n}_2 = \mathbf{Q} - \mathbf{O} \tag{5}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{6}$$

$$\mathbf{m}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{7}$$

The equation of TP is given as

$$\mathbf{n}_1^\top \left(\mathbf{x} - \begin{pmatrix} \cos 110^\circ \\ \sin 110^\circ \end{pmatrix} \right) = 0 \tag{8}$$

$$\begin{pmatrix} -0.342 & 0.939 \end{pmatrix} \mathbf{x} = 1 \tag{9}$$

The equation of TQ is given as

$$\mathbf{n}_2^\top \left(\mathbf{x} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) = 0 \tag{10}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 1 \tag{11}$$

The tangent point can be calculated by solving (9) and (11)

$$\begin{pmatrix} -0.342 & 0.939 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{12}$$

$$\implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1.428 \end{pmatrix} \tag{13}$$

The angle between two lines with slope $\mathbf{m}_1$ and $\mathbf{m}_2$ s given as

$$\cos\theta = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \, \|\mathbf{m}_2\|} \tag{14}$$

$$= \frac{\begin{pmatrix} 1 & -\cot 110^\circ \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{(\csc 110^\circ)\,(1)} \tag{15}$$

$$= -\cos 110^\circ \tag{16}$$

$$\implies \theta = 70^\circ \tag{17}$$

Hence, $\angle PTQ = 70^\circ$. See Fig 1

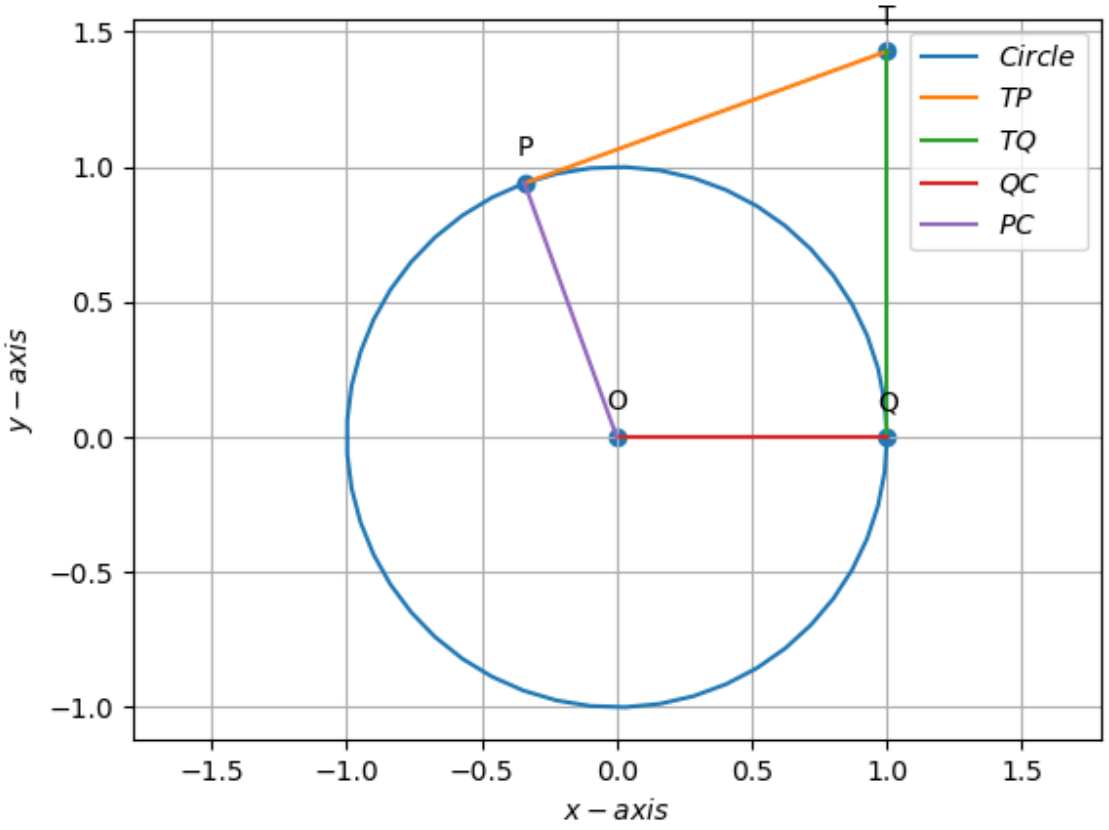
T
P
O
Q
Circle
TP
TQ
QC
PC
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
y − axis
x − axis

Figure 1: