Excercise 6.6

Q4. Find the equation of normal to the curve $x^2 = 4y$ which passes through the point (4,-2)
Solution: The given equation of the curve can be written as

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{3}$$

$$f = 0 \tag{4}$$

We are given that

$$\mathbf{h} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{5}$$

This can be formulated as optimization problem as follows:

$$\min_{\mathbf{x}} \quad \mathrm{f}(\mathbf{x}) = \|\mathbf{x} - \mathbf{h}\|^2 \tag{6}$$

$$\text{s.t.} \quad g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{7}$$

It is already proved that the optimization problem is non-convex. However, by relaxing the constraint in (7) as

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f \leq 0 \tag{8}$$

the optimization problem can be made convex. Now we use Gradient Descent to find the optimum value. We define

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \alpha \nabla g(\mathbf{x}_n) \tag{9}$$

And the condition is given as

$$(\mathbf{x} - \mathbf{h})^\top \nabla g(\mathbf{x}_n) \neq 0 \tag{10}$$

Now we choose the parameters as

1. $\alpha = 0.001$

2. precision = 0.001
3. n = 10000
4. $\mathbf{x}_0 = 4$

We get the minimum value of $\mathbf{x}$ as

$$\mathbf{x}_{min} = \begin{pmatrix} 1.695 \\ 0.718 \end{pmatrix} \tag{11}$$

See figure 1

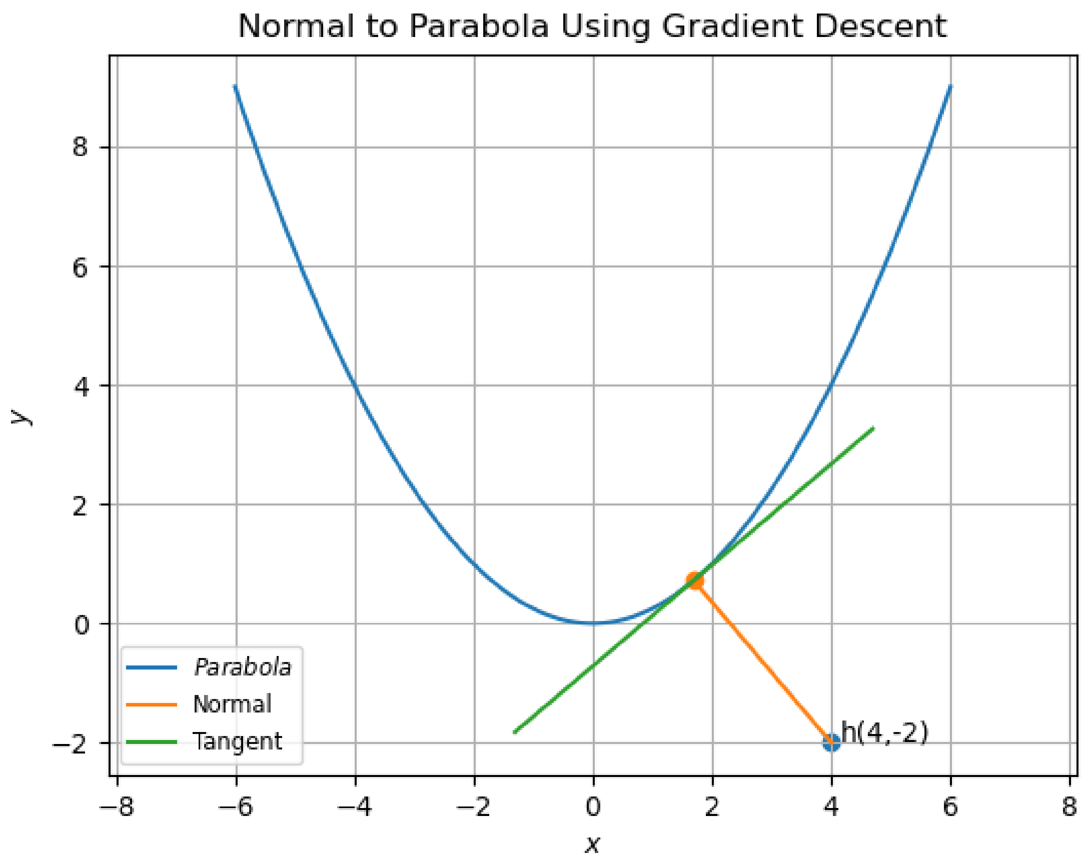

Figure 1: