EE200 Report: Frequency and Spatial Domain Analysis

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1 Objective

This experiment explores:

- The 2D Discrete Fourier Transform (DFT) of images and their visual interpretation.
- Synthesizing a hybrid image by combining low-frequency and high-frequency components from two images.
- Understanding how perception changes based on viewing distance and frequency content.

2 Tools Used

- Python 3.x
- OpenCV (cv2)
- NumPy
- Matplotlib

3 Theory and Equations

3.1 2D Discrete Fourier Transform (DFT)

The DFT for a 2D signal $f(x, y)$ of size $M \times N$ is defined as:

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} \tag{1}$$

This transforms spatial domain data into the frequency domain.

3.2 Magnitude Spectrum

To visualize the frequency domain:

$$|F(u,v)| = \sqrt{\mathrm{Re}(F)^2 + \mathrm{Im}(F)^2} \tag{2}$$

In decibel (dB) scale:

$$S(u,v) = 20\log_{10}(|F(u,v)| + 1) \tag{3}$$

3.3 Centering the Spectrum

The zero-frequency component is usually at the corner. Using NumPy:

> `np.fft.fftshift()` shifts the spectrum so that the DC component is at the center.

3.4 Rotation Effect

Rotating an image rotates its Fourier spectrum equivalently. A 90-degree counter-clockwise rotation causes a 90-degree rotation in the spectrum.

4 Experimental Procedure

4.1 Fourier Transform of an Image

1. Load a grayscale image (e.g., cat).
2. Apply `np.fft.fft2()` to compute the 2D FFT.
3. Use `np.fft.fftshift()` to center the spectrum.
4. Plot:
   - Magnitude spectrum (log scale)
   - Magnitude spectrum (dB scale)
   - Rotated image and its FFT

Observation: The center of the shifted spectrum corresponds to low-frequency components. Rotation causes a corresponding rotation in the spectrum.

4.2 Hybrid Image Creation

Goal: Merge low frequencies of one image (cat) and high frequencies of another (dog).

4.2.1 Steps:

1. Low-pass filter the cat image (convolution is denoted by $*$):
$$I_{\text{cat}}^{\text{low}} = G_{\sigma_{\text{low}}} * I_{\text{cat}} \tag{4}$$
2. High-pass the dog image:
$$I_{\text{dog}}^{\text{high}} = I_{\text{dog}} - G_{\sigma_{\text{high}}} * I_{\text{dog}} \tag{5}$$
3. Boost the dog component:
$$I_{\text{dog}}^{\text{boosted}} = \alpha \cdot I_{\text{dog}}^{\text{high}} + \beta \tag{6}$$
4. Fuse images:
$$I_{\text{hybrid}} = a \cdot I_{\text{cat}}^{\text{low}} + b \cdot I_{\text{dog}}^{\text{boosted}} \tag{7}$$

Transfer Functions:

- Low-pass filter (Gaussian):
$$H_{\text{low}}(u, v) = e^{-\frac{D(u,v)^2}{2\sigma^2}} \tag{8}$$
- High-pass filter (Gaussian):
$$H_{\text{high}}(u, v) = 1 - H_{\text{low}}(u, v) \tag{9}$$

5 Results and Observations

- The hybrid image prominently shows the dog.
- When viewed from a distance or by squinting, the cat becomes visible.
- This confirms that:
  - Low frequencies contribute to the general shape.
  - High frequencies contribute to fine details.
- Rotation of an image causes a corresponding rotation in its Fourier spectrum.

6 Conclusion

- Frequency mixing enables perceptual illusions based on distance and focus.
- The 2D Fourier Transform is a powerful tool to analyze and manipulate image content.
- Hybrid images show different information depending on the viewer's distance or visual focus.