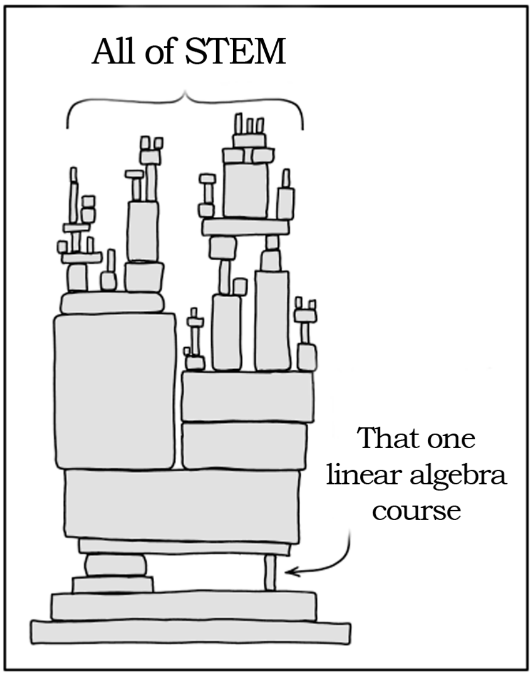
Applied Linear Algebra in Data Analysis



Source: https://twitter.com/Quasilocal/status/1664701458200121351

The above figure is not that far away from the truth. Linear algebra today is as important as calculus, if not more.

Linear algebra serves as the first step towards high-dimensional thinking, a prerequisite skill for modern data analysis. Paired with probability theory and optimization, it comprises a potent trio of tools driving the current machine learning revolution.

In this course, our mission is to lay a solid foundation in applied linear algebra, along with basic probability theory and optimization. But why these three topics?

- **Linear Algebra:** Think in high dimensions & visualize complex structures.
- **Probability Theory:** Navigate uncertainty with precision. Probability theory provides a principled approach to handling the unpredictable nature of real-world data.
- **Optimization:** Seek the best solutions. Optimization allows us to navigate the vast solution space and pinpoint the most efficient paths to solving problems.

Throughout this course, we will:

a) Introduce fundamental concepts in linear algebra, matrix methods, probability theory, and optimization.
b) Demonstrate the practical application of these concepts in real-world scenarios.
c) Provide hands-on experience for applying these concepts to solve real-world problems.

Course GitHub Page

All course-related documents will be available on the GitHub page: https://github.com/siva82kb/ALADA-Course

Course Content

(Modules in orange cover concepts and Modules in blue are applications)

Vectors and Vector Spaces; Matrices; Solving Linear Equations; Orthogonality; Matrix Inverses; Signal processing: Fourier and Wavelet Transforms; Eigenvectors and Eigenvalues; Systems theory: Linear Dynamical Systems; Positive Definiteness & Semi-definiteness; Matrix Norms; Singular Value Decomposition; Dimensionality Reduction: Principal Component Analysis; Image compression; Optimization: A brief introduction; Linear Least Squares and its Variants; Linear programing; Signal processing: Filtering; Machine learning: Cross-validation; Linear Regression Models; Linear Models for Classification; Neuromechanics: Linear Programming applied to neuromuscular control, and radiation planning. Probability and Statistics: Gaussian distribution.

Assignments

- This is a hands-on course heavy on pen-and-paper and programming assignments. Students are encouraged to use Python or Julia for their programming assignments.
- All assignments will need to be submitted using the submission link on the course GitHub page.
- All assignments will be released one after the other as we progress through the course

- You have a cumulative 3-day late submission grace period for your assignments for the entire course.
- If you use up your 3-day grace period, all subsequent late assignments will receive 0 marks.

Resources

The topics covered in this course are quite mature, so it's no surprise that there are numerous wonderful resources. Here are some resources that cover the topics discussed in this course. There are several wonderful resources:

1. G Strang, Introduction to Linear Algebra. Wellesley, *MA: Wellesley-Cambridge Press*, 1993.
2. CD Meyer, Matrix Analysis and Applied Linear Algebra. *Siam*; 2000 Jun 1.
3. S Boyd and L Vandenberghe, Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares. Online Book.
4. FJ Valero-Cuevas. Fundamentals of Neuromechanics. *Berlin: Springer,* 2016.
5. CM Bishop, and MN Nasrabadi. Pattern recognition and machine learning. *New York: Springer*, 2006.
6. Online lectures: Linear Algebra by Prof. Gilbert Strang.
7. Selected online lectures: Matrix Methods in Data Analysis, Signal Processing, and Machine Learning, by Prof. Gilbert Strang.
8. Selected online lectures: Linear Dynamical Systems by Prof. Stephen Boyd.
9. Selected online lectures: Convex Optimization by Prof. Stephen Boyd.

Grading

The final grade for the course will consider the performance on the assignments, quizzes, mid-term, and the final exam.

Assignments	15
Quizzes	10
Mid-term	20
Final	55
Total	100

An absolute grading scheme will be followed for the course:

S: >=90; **A**: >=80; **B**: >= 70; **C**: >= 60; **D**: >=50; **E**: >=40; **F**: <40

Lectures Schedule (August – December 2024)

Lecture timings:
Thursday, Friday: 7:30 – 9:00 AM
Saturday: 7:30 – 8:30 AM

Week	Day	Date	Details	Details
1	1	2/8/24	Concepts in Vector Spaces - 1	Vector spaces, Subspace, Span, Linear Independence
	2	3/8/24	Concepts in Vector Spaces - 2	Norm, Inner Products, Basis, Orthonormal Basis, Linear Functions
2	3	8/8/24	Matrices – 1	Matrices, Matrix Multiplication
	4	9/8/24	Matrices – 2	Rank of. Matrix, Matrix Inverse, Complex Vectors & Matrices
	5	10/8/24	Tutorial 1	
3	6	15/8/24	Holiday	
	7	16/8/24	Solving linear equations 1	Linear Equations, Understanding Solutions to Linear Equations
	8	17/8/24	Tutorial 2	
4	9	22/8/24	Solving linear equations 2	General Solution to Linear Equations, Four Fundamental Subspaces of a Matrix
	10	23/8/24	Orthogonality	Orthogonality, Orthogonal Projections, Gram-Schmidt Procedure, QR Decomposition
	11	24/8/24	Tutorial 3	
5	12	29/8/24	Matrix Inverses	Meaning of matrix inverses, Left and Right Inverses, Generalized Inverses
	13	30/8/24	**Application**: Signal Processing – Fourier & Wavelet Transform	
	14	31/8/24	Tutorial 4	
6	15	3/10/24	Eigenvectors and Eigenvalues	Similarity Transformation, Eigenvectors and Eigenvalues, Diagonalization of a Matrix, Jordan Forms
	16	4/10/24	**Application**: Linear Dynamical Systems	
	17	5/10/24	Tutorial 5	
7	18	10/10/24	Positive Definiteness and Matrix Norms	Positive Definite Matrices, Matrix Norms
	19	11/10/24	**Application**: Dimensionality reduction, PCA and Image compression.	Introduction to PCA
	20	12/10/24	Tutorial 6	
8	21	17/10/24	Optimization - 1	Continuity, Derivatives, Taylor series, Univariate minimization (line search)
	22	18/10/24	Optimization - 2	Multivariate minimization
	23	19/10/24	Tutorial 7	
9	24	24/10/24	Univariate optimization	Continuity, Derivatives, Taylor series, Univariate minimization (line search)
	25	25/10/24	Multivariate, Unconstrained Optimization	Multivariate optimization and its different methods
	26	26/10/24	Tutorial 8	
10	27	31/10/24	Constrained Optimization	Constrained optimization with equality constraints
	28	1/11/24	Constrained Optimization	Constrained optimization with equality constraints
	29	2/11/24	Tutorial 9	
11	30	7/11/24	Constrained Optimization	Constrained optimization with inequality constraints
	31	8/11/24	Additional topics in Optimization	Multiobjective optimization, Introduction to Convex Optimization
	32	9/11/24	Tutorial 10	
12	33	14/11/24	Linear Least Squares and its Variants	Linear Least Squares, Multiobjective Least Squares, Constrained Least Squares

	34	15/11/24	**Application**: Signal Processing – Filtering, CT Image Reconstruction	
	35	16/11/24	Tutorial 11	
13	36	21/11/24	**Application**: Machine learning – Cross-validation; Linear regression models	
	37	22/11/24	Linear Programs	Definition, Basic Concepts
	38	23/11/24	Tutorial 12	
14	39	28/11/24	Linear Programs	Simplex Algorithm
	40	29/11/24	Linear Programs	Simplex Algorithm
	41	30/11/24	Tutorial 13	
15	42	5/12/24	**Application**: Neuromechanics - Neuromuscular control, Radiation planning	
	43	6/12/24	Introduction to Probability	
	44	7/12/24	Tutorial 14	
16	45	12/12/24	Introduction to Probability	
	46	13/12/24	Introduction to Statistics	
	47	14/12/24	Tutorial 15	

Midsem Exam: 7:30 – 8:30 AM, 15 October 2024
Final Exam: 7:30 – 10:30 AM, 21 December 2024

Assignment Schedule

Refer to the GitHub page.