Sum of Subsets

Problem:

Given positive numbers w_i, $1 \leq i \leq n$, and m, find all subsets of the w_i whose sum is m.

Example:

$(w_1, w_2, w_3, w_4) = (11, 13, 24, 7)$ and $m = 31$.

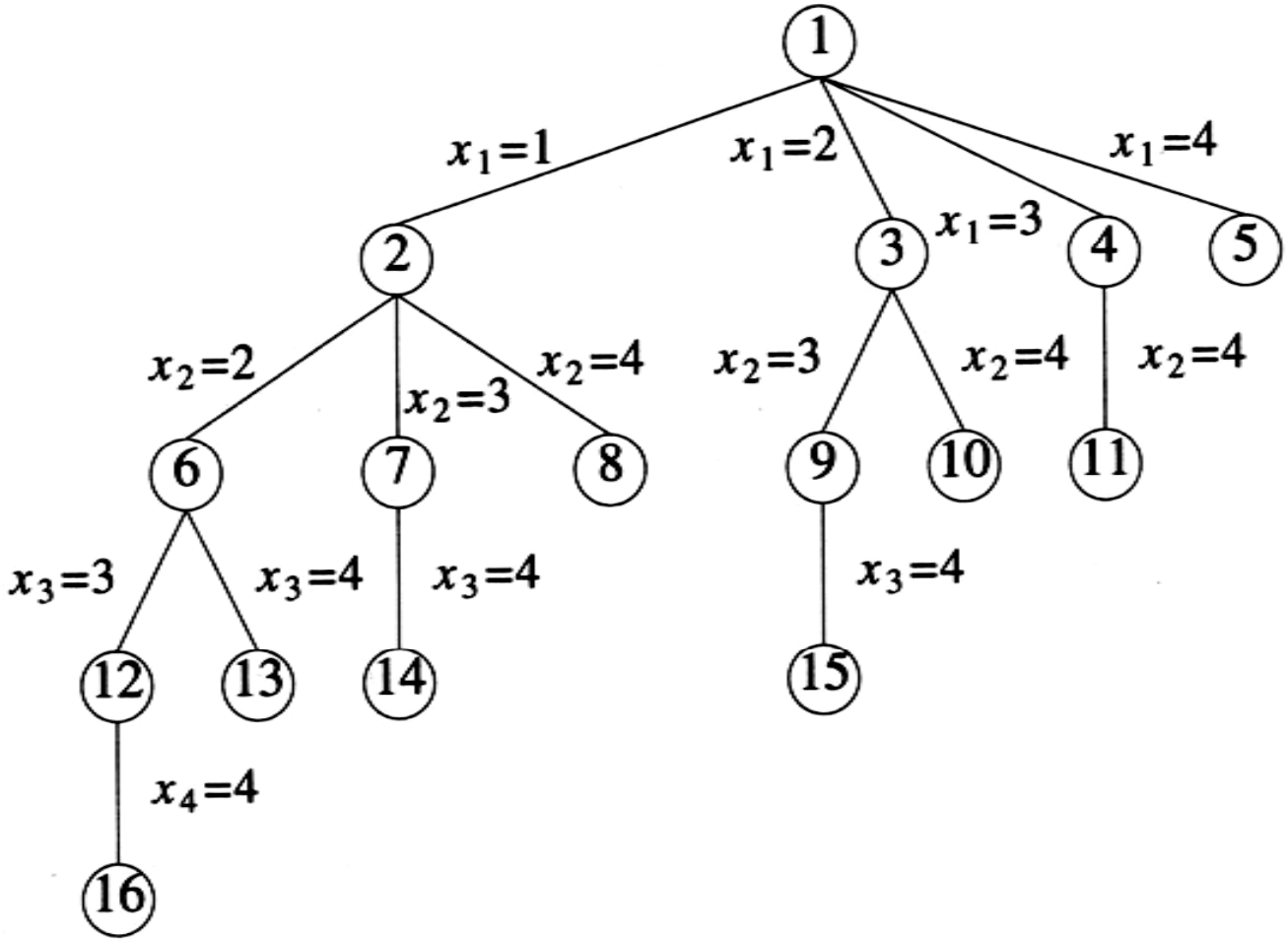
Solutions are (11,13,7) and (24,7).

Representation of Solution Vector

- Variable-sized
  - By giving indices
    - ✓ In the above example, (1,2,4) and (3,4)
- Fixed-sized
  - n-tuple $(x_1,x_2,\ldots,x_n)$ where x_i is in {0,1}, $1 \le i \le n$
    - ✓ In the above example, (1,1,0,1) and (0,0,1,1)

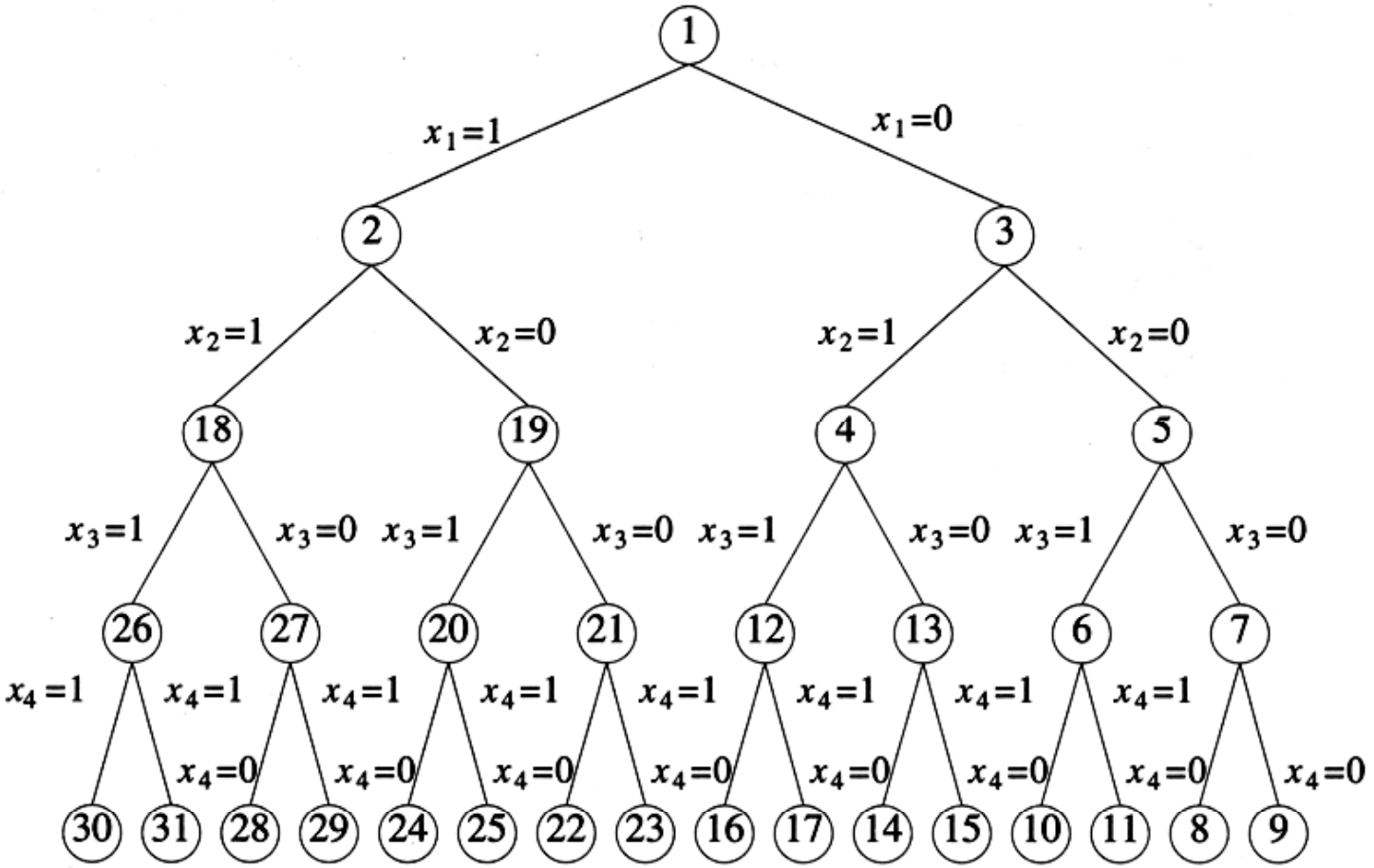
Variable tuple size (Sahni: Figure 7.3)

Solution space defined by all paths from the root to any node



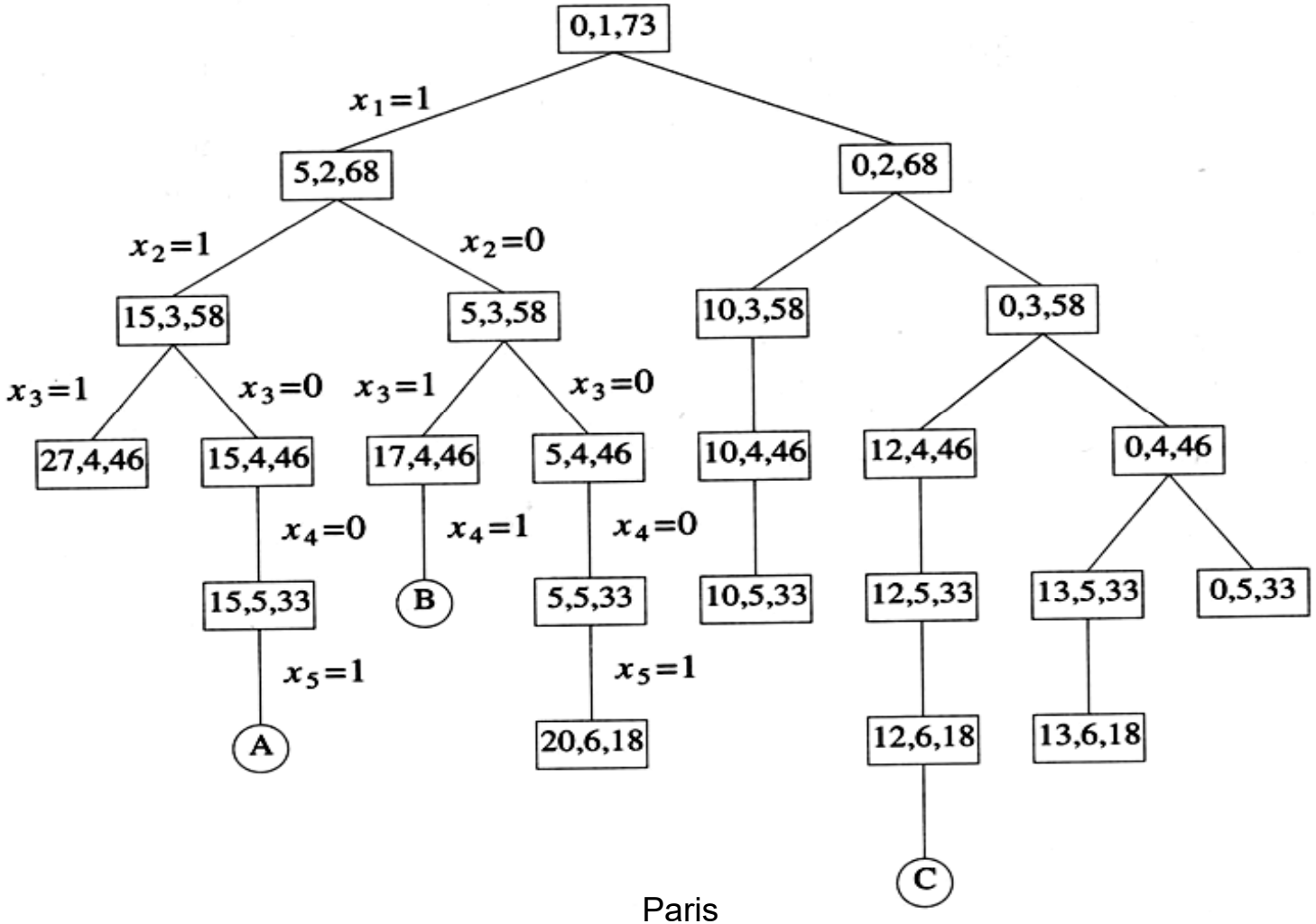
Fixed tuple size (Sahni: Figure 7.4)

Solution space defined by all paths from the root to a leaf node



Example (Sahni: Figure 7.10)

n=6, w[1:6] = {5,10,12,13,15,18}, m=30



Algorithm: Sum of Subsets (Algorithm 7.6)

Invoked by SumOfSub(0,1,$\sum_{i=1,n} w_i$)

s: Sum of elements already chosen r: Sum of elements remaining

k: Index of element which is now active for selection

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1   Algorithm SumOfSub(s, k, r)
2   // Find all subsets of w[1 : n] that sum to m. The values of x[j],
3   // 1 ≤ j < k, have already been determined. s = Σ_{j=1}^{k-1} w[j] * x[j]
4   // and r = Σ_{j=k}^{n} w[j]. The w[j]'s are in nondecreasing order.
5   // It is assumed that w[1] ≤ m and Σ_{i=1}^{n} w[i] ≥ m.
6   {
7       // Generate left child. Note: s + w[k] ≤ m since B_{k-1} is true.
8       x[k] := 1;
9       if (s + w[k] = m) then write (x[1 : k]); // Subset found
10          // There is no recursive call here as w[j] > 0, 1 ≤ j ≤ n.
11      else if (s + w[k] + w[k + 1] ≤ m)
12          then SumOfSub(s + w[k], k + 1, r − w[k]);
13      // Generate right child and evaluate B_k.
14      if ((s + r − w[k] ≥ m) and (s + w[k + 1] ≤ m)) then
15      {
16          x[k] := 0;
17          SumOfSub(s, k + 1, r − w[k]);
18      }
19  }
```

Thank You

Stay Safe