Asymptotic Notations

Sahni : Page: 40, 41, 48 & 49

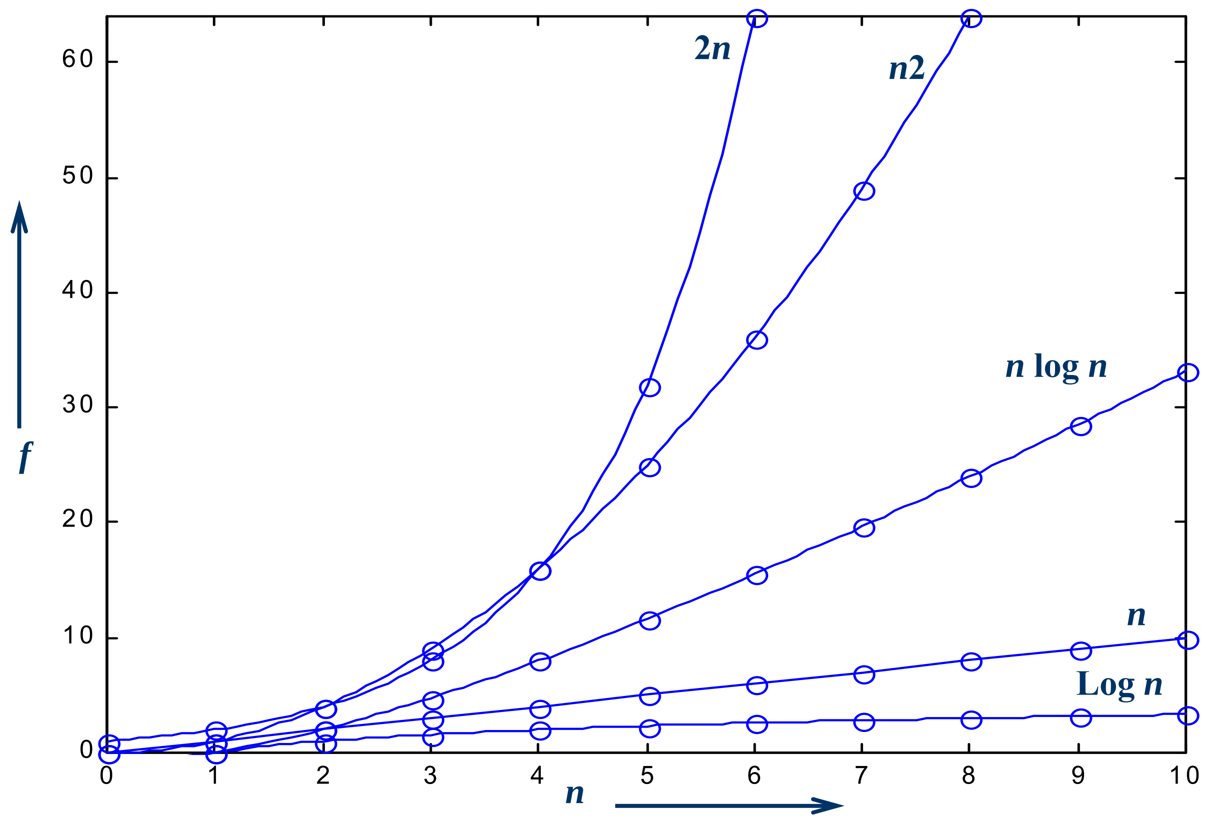
Cormen: Page: 45

Types of Time Functions

		Input size n					
Time	Name	1	2	4	8	16	32
1	constant	1	1	1	1	1	1
$\log n$	logarithmic	0	1	2	3	4	5
n	linear	1	2	4	8	16	32
$n \log n$	log linear	0	2	8	24	64	160
n^2	quadratic	1	4	16	64	256	1024
n^3	cubic	1	8	64	512	4096	32768
2^n	exponential	2	4	16	256	65536	4294967296
$n!$	factorial	1	2	24	40326	2092278988000	26313×10^{33}

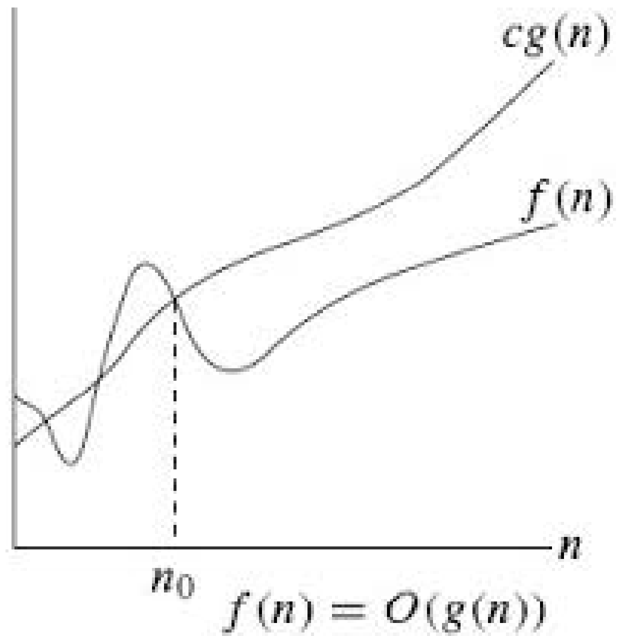
$$1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 < \ldots < 2^n < 3^n < \ldots < n^n$$

Plot of Function Values



Asymptotic Notations: "O"

- **O [Big "oh"] :** The function $f(n) = O(g(n))$
  iff there exist positive constants c and n_0
  such that $f(n) \le c \cdot g(n)$ for all n, $n \ge n_0$.



- Example: $3n + 2 = O(n)$
  as $3n + 2 \le 4n$ for all $n \ge 2$
- Represents : Upper bound

"O" [Upper bound]: Practice

- The function $f(n) = O(g(n))$ iff there exist positive constants c and n_0 such that $f(n) \le c \cdot g(n)$ for all n, $n \ge n_0$.

  $1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 < \ldots < 2^n < 3^n < \ldots < n^n$

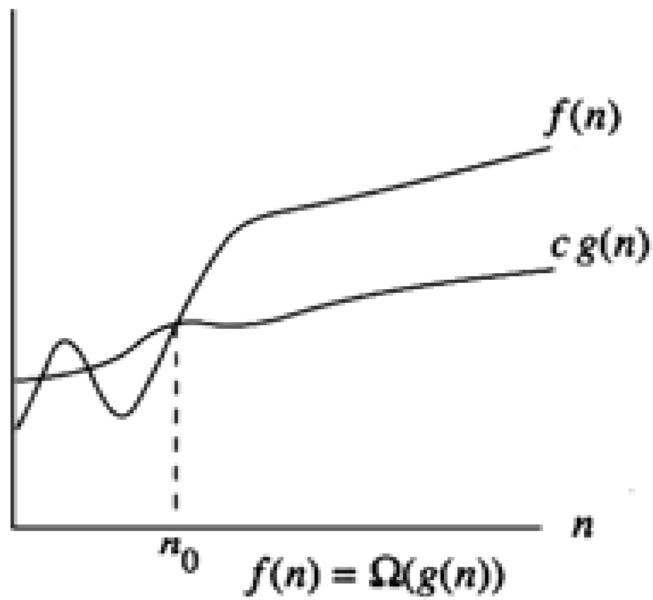
$$f(n) = 10n^2 + 4n + 2$$

Asymptotic Notations: "Ω"

- **Ω [Omega]:** The function $f(n) = \Omega(g(n))$
  iff there exist positive constants c and n_0
  such that $f(n) \geq c \cdot g(n)$ for all n, $n \geq n_0$.

> Example: $3n + 2 = \Omega(n)$
> as $3n + 2 \geq 3n$ for all $n \geq 1$

> Represents : Lower bound



"Ω" [Lower bound]: Practice

- The function $f(n) = \Omega(g(n))$ iff there exist positive constants c and n_0 such that $f(n) \geq c \cdot g(n)$ for all n, $n \geq n_0$.

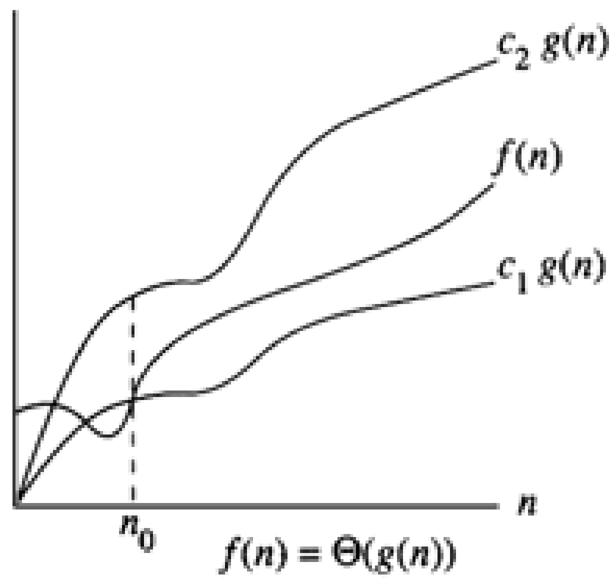
$$1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 < \ldots < 2^n < 3^n < \ldots < n^n$$

$$f(n) = 10n^2 + 4n + 2$$

Asymptotic Notations: "Θ"

- **Θ [Theta]:** The function $f(n) = \Theta(g(n))$
  iff there exist positive constants c_1, c_2 and n_0
  such that $c_1\, g(n) \le f(n) \le c_2\, g(n)$ for all n, $n \ge n_0$.

  - Example: $3n + 2 = \Theta(n)$
    as $3n + 2 \le 4n$ and $3n + 2 \ge 3n$
    for all $n \ge 2$
  - Represents : Tight/Average bound



"Θ"[Average bound]: Practice

- The function $f(n) = \Theta(g(n))$ iff there exist positive constants c_1, c_2 and n_0 such that $c_1\, g(n) \le f(n) \le c_2\, g(n)$ for all n, $n \ge n_0$.

  $1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 < \ldots < 2^n < 3^n < \ldots < n^n$

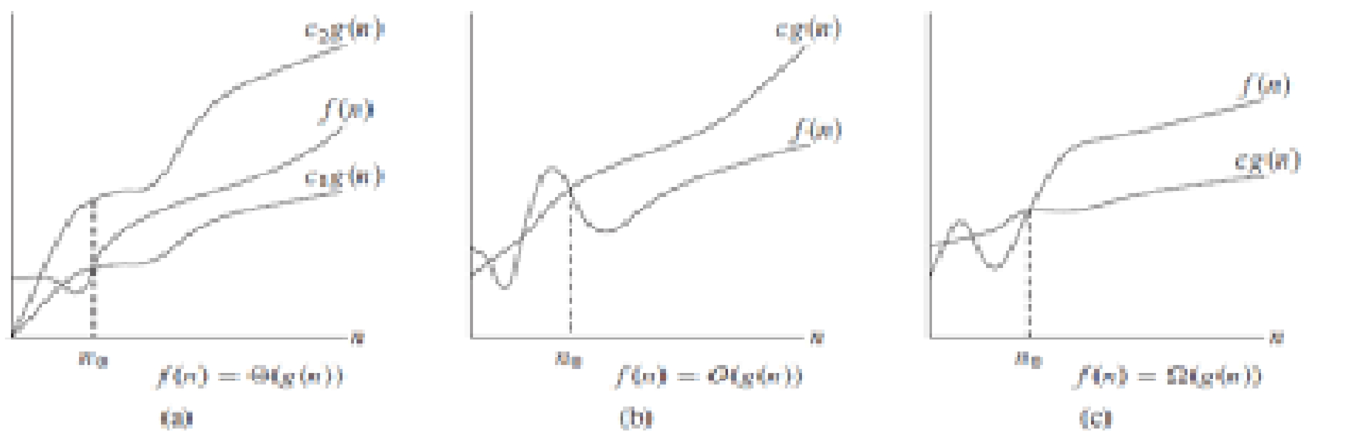
$$f(n) = 10n^2 + 4n + 2$$

Home Work

$$f(n) = n!$$

Notations: At a glance

- O [Upper] : The function $f(n) = O(g(n))$ iff there exist positive constants c and n_0 such that $f(n) \le c \cdot g(n)$ for all n, $n \ge n_0$.
- Ω [Lower]: The function $f(n) = \Omega(g(n))$ iff there exist positive constants c and n_0 such that $f(n) \ge c \cdot g(n)$ for all n, $n \ge n_0$.
- Θ [Tight / Average]: The function $f(n) = \Theta(g(n))$ iff there exist positive constants c_1, c_2 and n_0 such that $c_1\, g(n) \le f(n) \le c_2\, g(n)$ for all n, $n \ge n_0$.



Thank You

Stay Safe