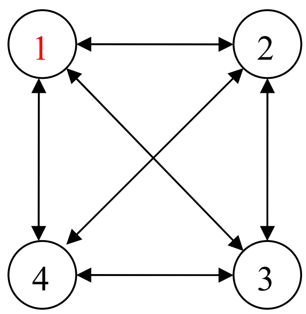
Traveling Salesperson Problem (TSP) Using Dynamic Programming

Problem:

Apply DP approach to solve the travelling salesperson problem starting from node "*1*" for the graph represented as the following cost adjacency matrix.



Node	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

<u>Solution</u>

Let,
c_{ij} be the cost from node ***i*** to node *j* and
g(i,S) be the length of a shortest path starting at vertex ***i***, going through all vertices in ***S***, and terminating at vertex ***1***(Source/Destination Vertex).

Thus,
g (2, Φ) = $\mathbf{c_{21}}$ = 5; g (3, Φ) = $\mathbf{c_{31}}$ = 6; g (4, Φ) = $\mathbf{c_{41}}$ = 8.

g (2, {3}) = $\mathbf{c_{23}}$ + g (3, Φ) = 9 + 6 = 15; g (2, {4}) = $\mathbf{c_{24}}$ + g (4, Φ) = 10 + 8 = 8.
g (3, {2}) = $\mathbf{c_{32}}$ + g (2, Φ) =13 + 5 = 18; g (3, {4}) = $\mathbf{c_{34}}$ + g (4, Φ) = 12 +8 =20.
g (4, {2}) = $\mathbf{c_{42}}$ + g (2, Φ) = 8 + 5 = 13; g (4, {3}) = $\mathbf{c_{43}}$ + g (3, Φ) = 9 + 6 = 15.

g (2, {3, 4}) = min{ $\mathbf{c_{23}}$ + g (3, {4}), $\mathbf{c_{24}}$ + g (4, {3})}
= min{ 9 + 20, 10 + 15}
= min{ 29, 25}
=25

g (3, {2, 4}) = min{ $\mathbf{c_{32}}$ + g (2, {4}), $\mathbf{c_{34}}$ + g (4, {2})}
= min{ 13 + 18, 12 + 13}
= min{ 31, 25}
=25

g (4, {2, 3}) = min{ $\mathbf{c_{42}}$ + g (2, {3}), $\mathbf{c_{43}}$ + g (3, {2})}
= min{ 8 + 15, 9 + 18}
= min{ 23, 27}
=23

g (1, {2, 3, 4}) = min{ $\mathbf{c_{12}}$ + g (2, {3, 4}), $\mathbf{c_{13}}$ + g (3, {2, 4}), $\mathbf{c_{14}}$ + g (4, {2, 3}) }
= min{ 10 + 25, 15 + 25, 20 + 23}
= min{ 35, 40, 43}
=35

So,
An optimal tour of the graph has length 35 and
Optimal tour is: 1 → 2 → 4 → 3 → 1.