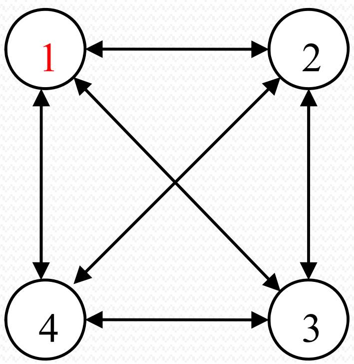
Traveling Salesperson Problem (TSP) using Dynamic Programming (DP)

Book: Sahni, 2nd Edition

Section: 5.9; Page: 318 – 320

Traveling Salesperson Problem (TSP)

Find the shortest path from a city (say 1), visit all other cities exactly once, and return to the city where it started (city 1).



Node	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

Notations

- c_{ij}: Cost from *i* to *j*.

- ***g(i,S)*** : The length of a shortest path
  starting at vertex *i*,
  going through all vertices in *S*,
  and terminating at vertex *1*
  (Source / Destination Vertex).

TSP using DP

Node	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

- $g(2, \Phi) = c_{21} = 5$;
- $g(3, \Phi) = c_{31} = 6$;
- $g(4, \Phi) = c_{41} = 8$.

- g (2, {3}) = c23 + g (3, Φ) = 9 + 6 = 15;
- g (2, {4}) = c24 + g (4, Φ) = 10 + 8 = 18.

- g (3, {2}) = c32 + g (2, Φ) = 13 + 5 = 18;
- g (3, {4}) = c34 + g (4, Φ) = 12 + 8 = 20.

- g (4, {2}) = c42 + g (2, Φ) = 8 + 5 = 13;
- g (4, {3}) = c43 + g (3, Φ) = 9 + 6 = 15.

TSP using DP (2)

Node	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

- $g(2, \{3, 4\}) = \min\{ c23 + g(3, \{4\}),\ c24 + g(4, \{3\})\}$
  $= \min\{ 9 + 20,\ 10 + 15\}$
  $= \min\{ 29,\ 25\}$
  $=25$
- $g(3, \{2, 4\}) = \min\{ c32 + g(2, \{4\}),\ c34 + g(4, \{2\})\}$
  $= \min\{ 13 + 18,\ 12 + 13\}$
  $= \min\{ 31,\ 25\}$
  $=25$
- $g(4, \{2, 3\}) = \min\{ c42 + g(2, \{3\}),\ c43 + g(3, \{2\})\}$
  $= \min\{ 8 + 15,\ 9 + 18\}$
  $= \min\{ 23,\ 27\}$
  $=23$

- $g(2, \Phi) = c_{21} = 5;$
- $g(3, \Phi) = c_{31} = 6;$
- $g(4, \Phi) = c_{41} = 8.$

- $g(2, \{3\}) = 15;$
- $g(2, \{4\}) = 18.$
- $g(3, \{2\}) = 18;$
- $g(3, \{4\}) = 20.$
- $g(4, \{2\}) = 13;$
- $g(4, \{3\}) = 15.$

TSP using DP (3)

Node	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

- $g(1, \{2, 3, 4\}) = \min\{ c_{12} + g(2, \{3, 4\}),$
  $c_{13} + g(3, \{2, 4\}),$
  $c_{14} + g(4, \{2, 3\}) \}$
  $= \min\{ 10 + 25, \quad 15 + 25, \quad 20 + 23\}$
  $= \min\{ 35, \quad 40, \quad 43\}$
  $=35$

- $g(2, \Phi) = c_{21} = 5;$
- $g(3, \Phi) = c_{31} = 6;$
- $g(4, \Phi) = c_{41} = 8.$

- $g(2, \{3\}) = 15;$
- $g(2, \{4\}) = 18.$
- $g(3, \{2\}) = 18;$
- $g(3, \{4\}) = 20.$
- $g(4, \{2\}) = 13;$
- $g(4, \{3\}) = 15.$

- $g(2, \{3, 4\}) = 25$
- $g(3, \{2, 4\}) = 25$
- $g(4, \{2, 3\}) = 23$

- Optimal tour length : 35.
- Optimal tour : $1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1.$

Thank You

Stay Safe