

Presentation Outline:

- 1) Introducing the 4-Queen problem
- 2) **Activity**: Solving 4-Queen problem using artifacts
- 3) Solution of 4-Queen problem in Backtracking approach
- 4) Demerits of Backtracking approach
- 5) Introducing 8-Queen problem
- 6) Discussion on Genetic Algorithm
- 7) Solution of 8-Queen problem using GA
- 8) Conclusion



The 4-Queen Problem

Once upon a time, there was a great king in India. However, it was a matter of shame that he had 4 Queens. The Queens were so arrogant and they didn't even want to see one another. Therefore, the King built a castle of 4 x 4 rooms. However, he couldn't find a way to place the 4 Queens in 4 separate rooms, so that they couldn't see each other.

Would, you please help the King to place the Queens? Avoid placing two Queens in a same row, same column and even same diagonal rooms.

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Solution of the 4-Queen Problem Using Backtracking Approach

Therefore , the king called Professor of an University to solve the 4-Queen problem. And Professor solved the 4-Queen problem in backtracking approach.

Complexity: NQueens : $O(N!)$





The 5-Queen Problem



One month later, Professor received a call from the great King to solve his 5-Queen problem. Professor, solved the 5-Queen problem in backtracking approach.



Solution of the 5-Queen Problem Using Backtracking Approach





6-Queen Problem



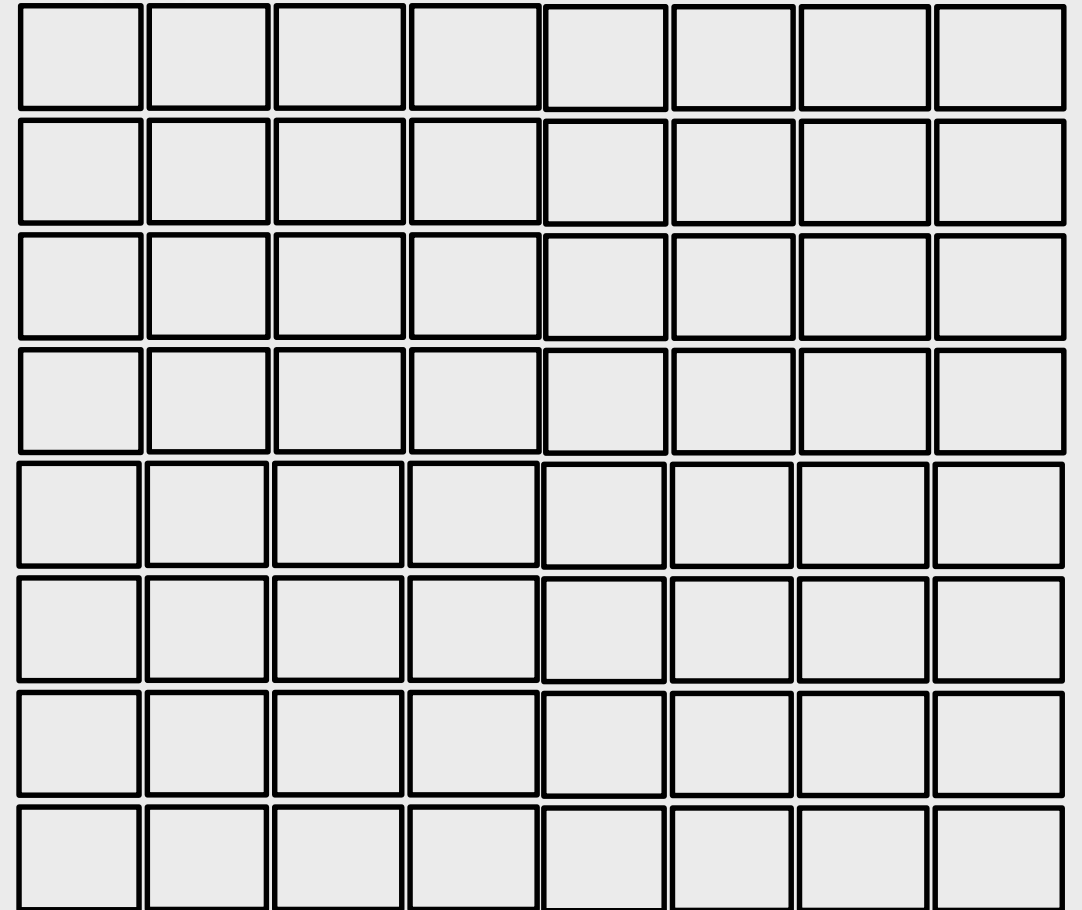
Fortunately, one month later, the King requested the professor to solve 6-Queen problem. The professor thought that the King may request him to solve 16-Queen problem within next 10 months.

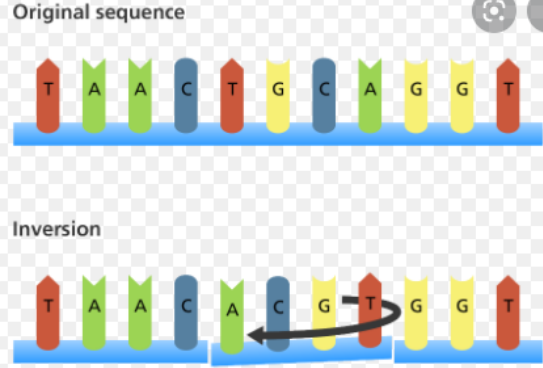
Backtracking approach will not be efficient to solve the 8 or 16-Queen problems.

Therefore, professor invented Genetic Algorithm to solve the n-Queen problem.



8-Queen Problem





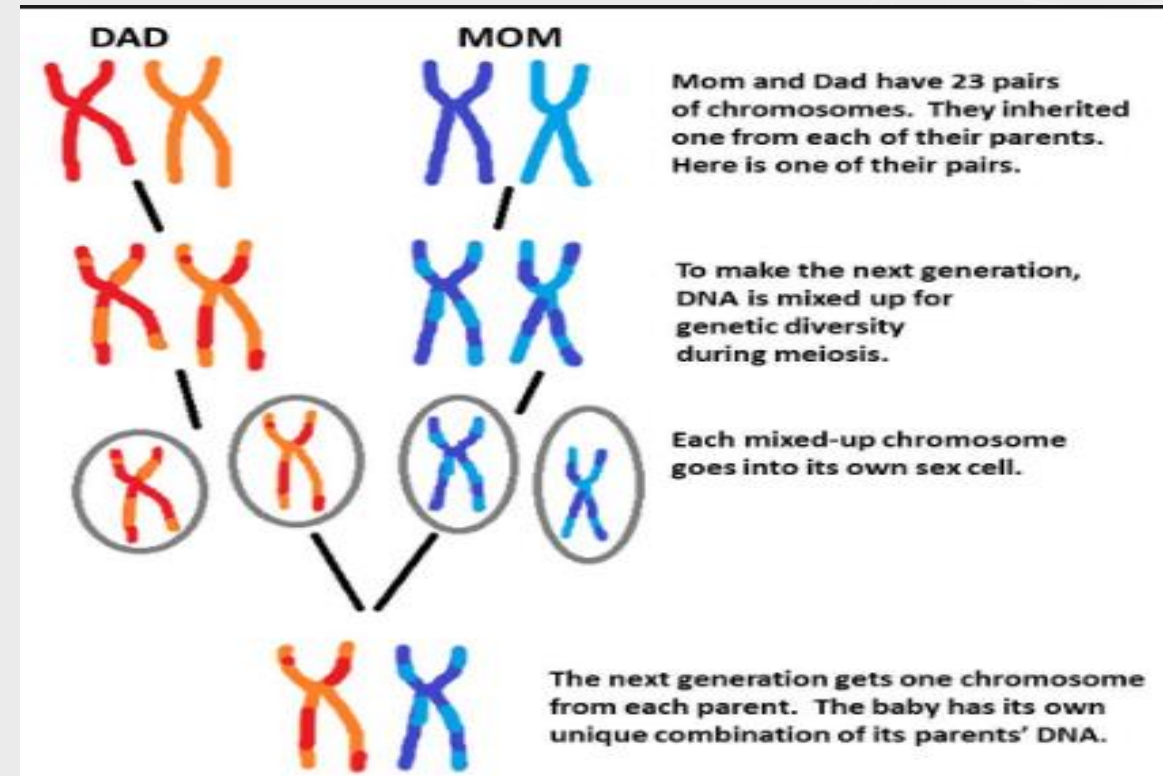
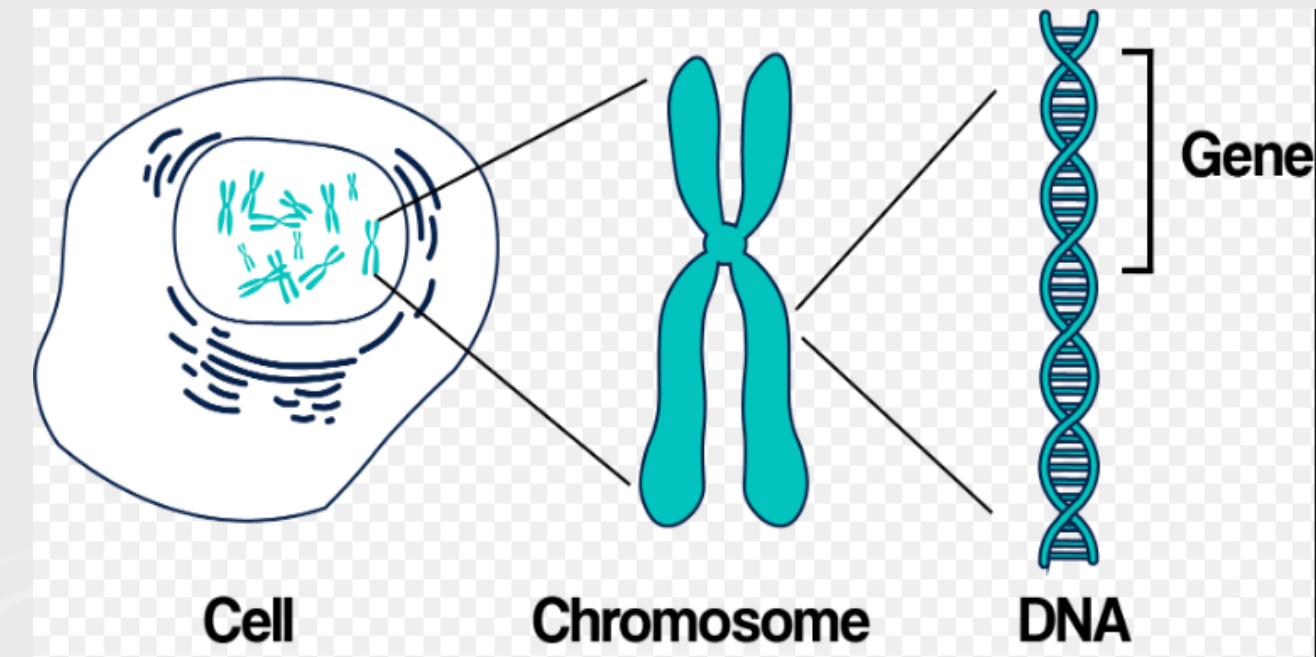
John Holland introduced **Genetic Algorithm (GA)**



Genetic Algorithm

Darwin's theory of evolution

The **genetic algorithm** is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution.



Introduced in the 1970s by John Holland at University of Michigan

- ▶ begin with k randomly generated states (population)
- ▶ each state (individual) is a string over some alphabet (chromosome)
- ▶ fitness function (bigger number is better)
- ▶ crossover
- ▶ mutate (evolve?)

John Holland introduced **Genetic Algorithm (GA)**

Darwin's theory of evolution



GA: PROCEDURE

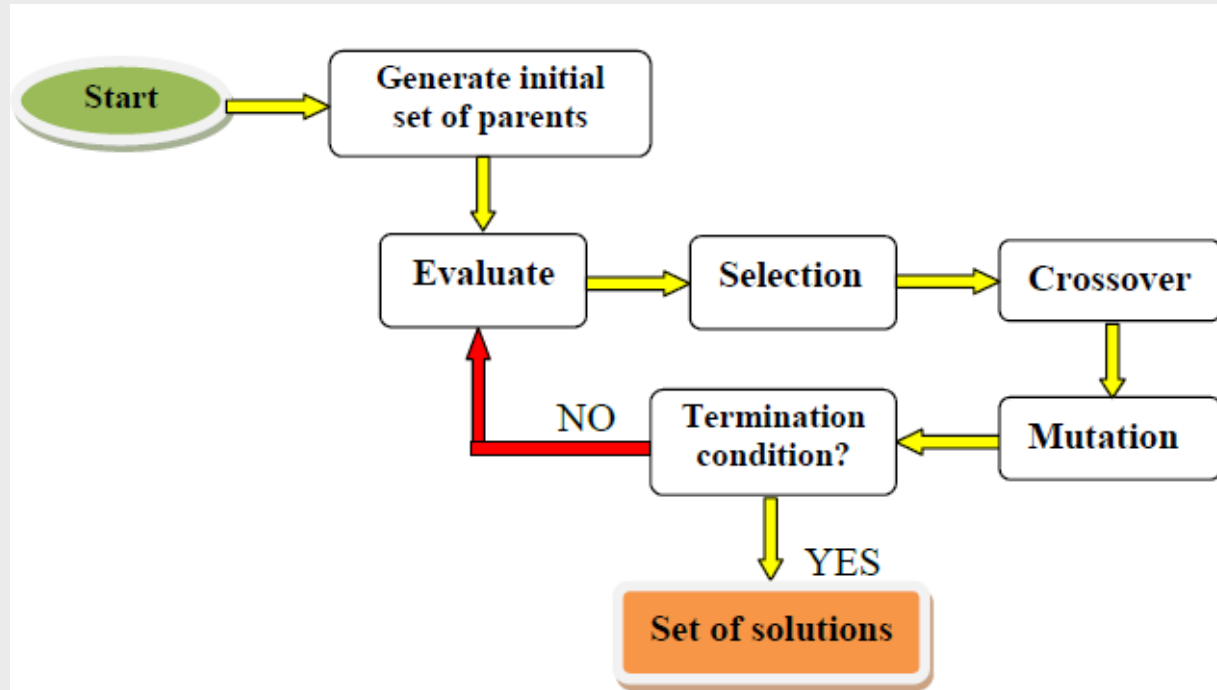


Fig.1 Process of genetic algorithm

Pseudo-code of GA:

START

 Generate the initial population

 Compute fitness

 REPEAT

 Selection

 Crossover

 Mutation

 Compute fitness

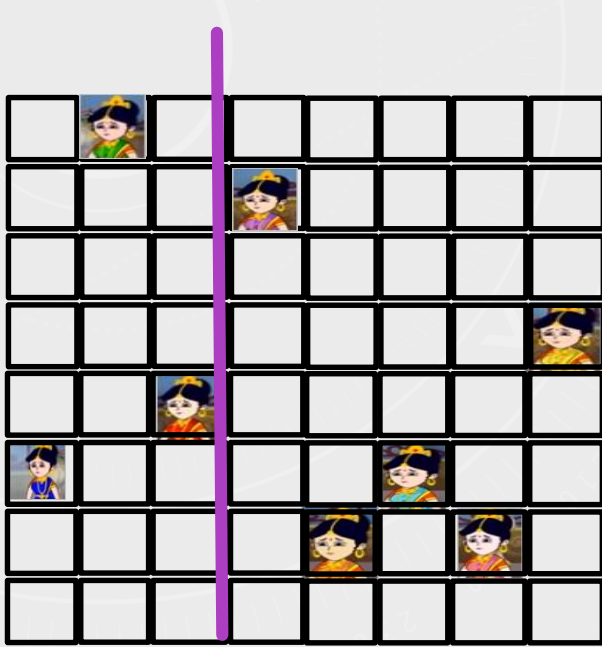
 UNTIL population has converged

STOP



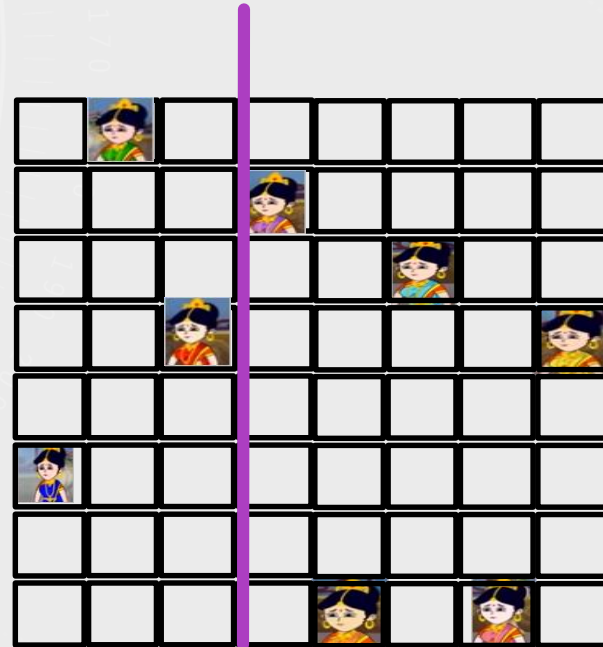
Formulation of Genetic Algorithm

John Holland introduced **Genetic Algorithm (GA)**
Darwin's theory of evolution



3 8 4 7 2 3 2 5

Fitness=28-7=21



3 8 5 7 1 6 1 5

Fitness=28-4=24

Chromosome of Father: 3 8 4 7 2 3 2 5

Chromosome of Mother: 3 8 5 7 1 6 1 5

Fitness function: number of non-attacking pairs of queens

Maximum number of attacking pairs: $8 \times 7/2 = 28$

[Q1 Q2]

[Q1 Q3]

[Q1 Q4]

[Q1 Q5]

[Q1 Q6]

[Q1 Q7]

[Q1 Q8]

.....

[Q8 Q7]

Crossover:

Chromosome from Father:

3 8 4 7 2 3 2 5

Chromosome from Mother:

3 8 5 7 1 6 1 5

Crossover point

3 8 4 7 2 3 2 5

3 8 5 7 1 6 1 5

2 4 4 1 5 1 2 4

3 2 5 4 3 2 1 3

Chromosome of Father:

3 8 4 7 2 3 2 5

Chromosome of Mother:

3 8 5 7 1 6 1 5

Offspring 1:

3 8 4 7 1 6 1 5

Offspring2:

3 8 5 7 2 3 2 5

Mutation:

Before Mutation:

Offspring 1: 3 8 4 7 1 6 1 5









Offspring2: 3 8 5 7 2 3 2 5

After Mutation:

Offspring 1: 3 8 4 7 1 6 2 5

Offspring2: 3 8 6 7 2 3 2 5





Offspring 1:

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|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|--|-------------------------------------------------------------------------------------|
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3 8 4 7 1 6 2 5

Fitness=28-0=28

Offspring2:

| | | | | | | | |
|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
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3 8 6 7 2 3 2 5

Fitness=28-7=21

Genetic Algorithms Example

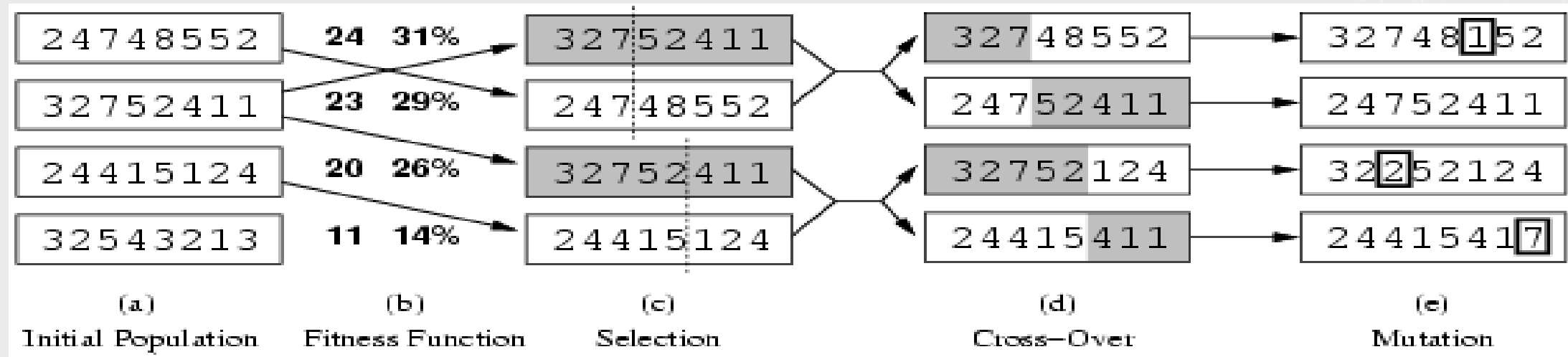
Represent states and compute fitness function.

| | |
|----------|---------------|
| 24748552 | 24 |
| 32752411 | 23 |
| 24415124 | 20 |
| 32543213 | 11 |
| | <u>77</u> |

(a)

Initial Population

GENETIC ALGORITHMS

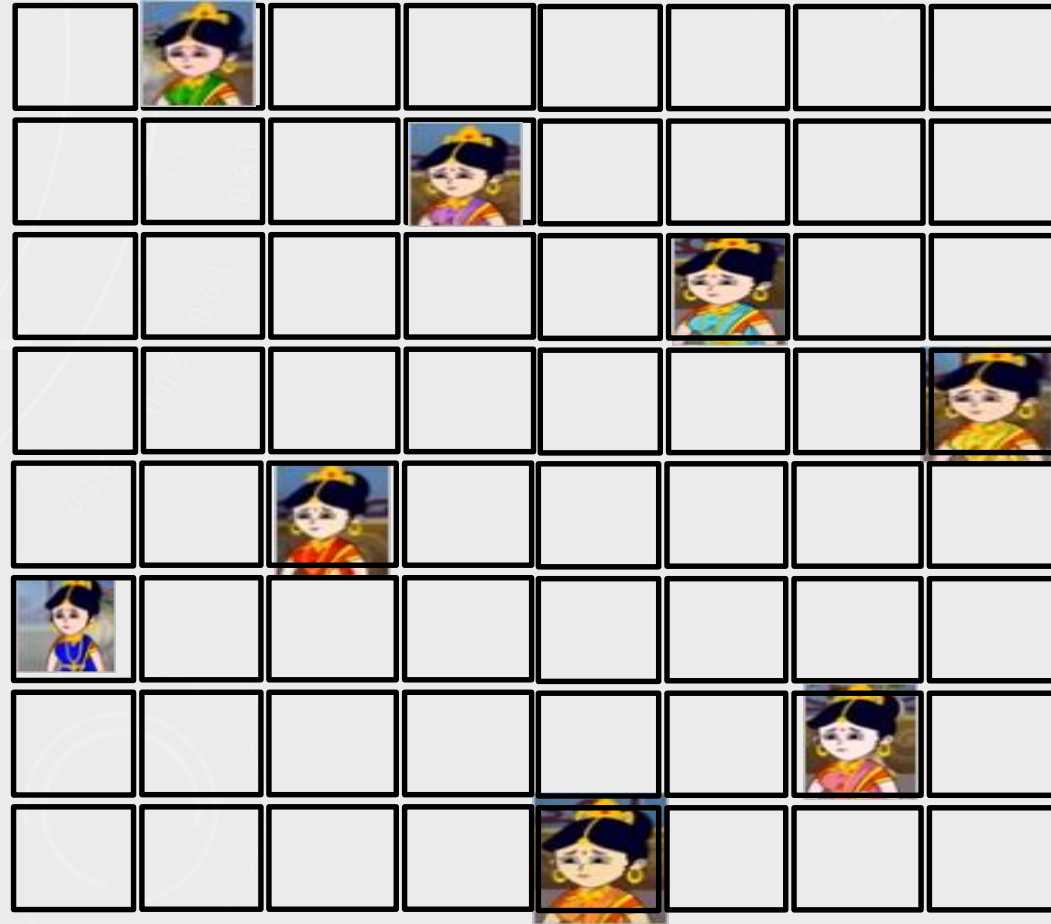


- Fitness function: number of non-attacking pairs of queens (min = 0, max = $8 \times 7/2 = 28$)
- $24/(24+23+20+11) = 31\%$
- $23/(24+23+20+11) = 29\%$ etc



Solution of 8-Queen Problem using Genetic Algorithm

John Holland introduced **Genetic Algorithm (GA)**
Darwin's theory of evolution

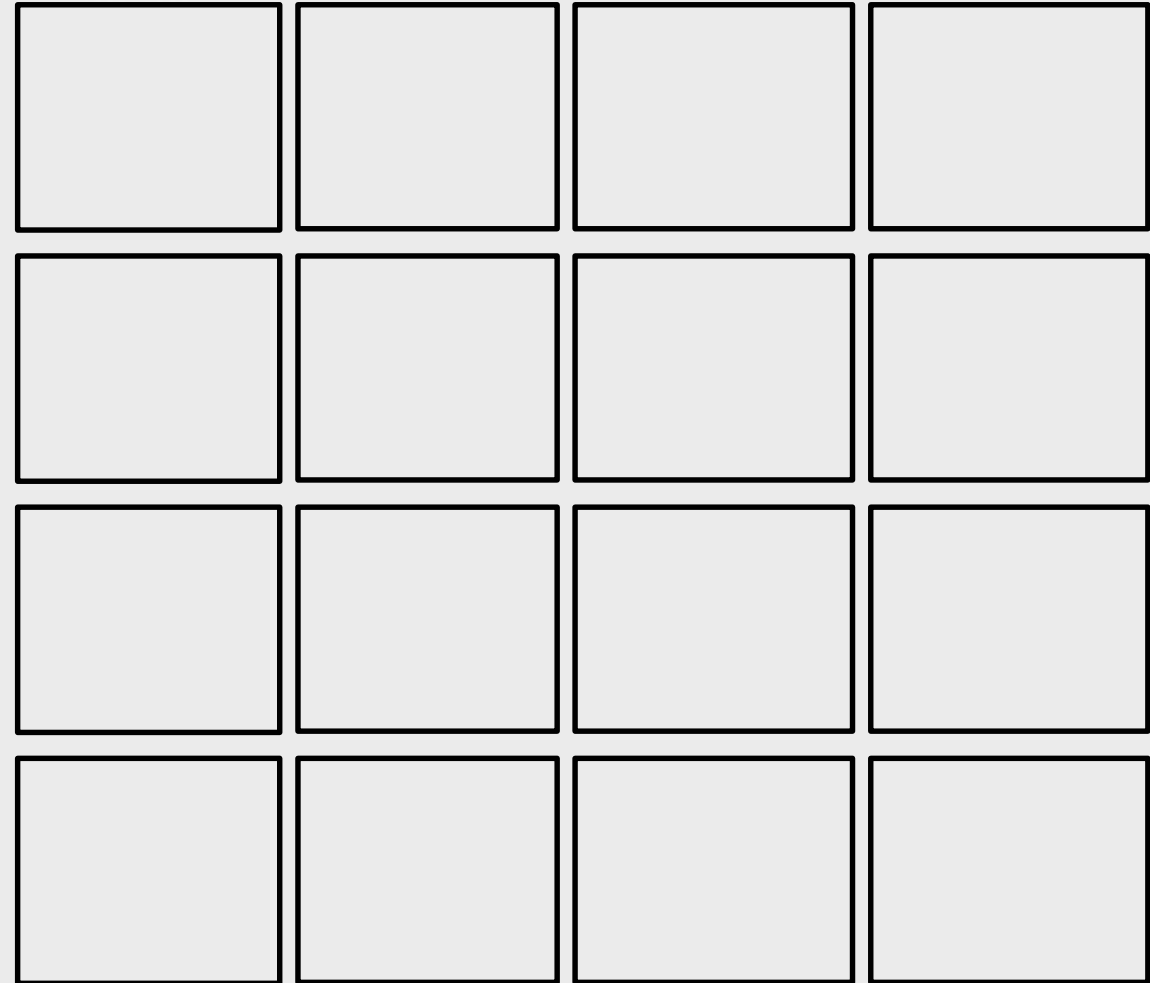




The 4-Queen Problem

Fitness function: number of non-attacking pairs of queens

What is the Maximum fitness value: ????





4-Queen Problem Using Backtracking vs GA Approach

Therefore , the king called Professor John Holland of the University of Michigan to solve the 4-Queen problem. And solved the 4-Queen problem in backtracking approach.

Complexity: NQueens : $O(N!)$

Genetic Algorithms complexity is $O(g(nm + nm + n))$ with **g the number of generations, n the population size and m the size of the individuals**. Therefore the complexity is on the order of $O(gnm)$





Try to Solve the 4-Queen Problem Using GA?



Initial Population

GA: ANALYSIS

| <i>n</i> | Size of solution space (<i>n</i> !) | Number of solutions |
|----------|--------------------------------------|---------------------|
| 1 | 1 | 1 |
| 2 | 2 | 0 |
| 3 | 6 | 0 |
| 4 | 24 | 2 |
| 5 | 120 | 10 |
| 6 | 720 | 4 |
| 7 | 5040 | 40 |
| 8 | 40320 | 92 |
| 9 | 362880 | 352 |
| 10 | 3628800 | 724 |
| 11 | 39916800 | 2680 |
| 12 | 479001600 | 14200 |
| 13 | 6227020800 | 73712 |
| 14 | 87178291200 | 365596 |
| 15 | 1307674368000 | 2279184 |
| 16 | 20922789888000 | 14772512 |
| 17 | 355687428096000 | 95815104 |
| 18 | 6402373705728000 | 666090624 |
| 19 | 121645100408832000 | 4968057848 |
| 20 | 2432902008176640000 | 39029188884 |
| 21 | 51090942171709440000 | 314666222712 |
| 22 | 1124000727777607680000 | 2691008701644 |
| 23 | 25852016738884976640000 | 24233937684440 |
| 24 | 620448401733239439360000 | 227514171973736 |
| 25 | 15511210043330985984000000 | 2207893435808352 |
| 26 | 403291461126605635584000000 | 22317699616364044 |