Horner's Rule

Horner's rule for polynomial division is a way to divide a polynomial into monomials to make it easier to evaluate a polynomial f(x) at a certain value x = x0 (polynomials of the 1st degree). Each monomial is made up of no more than one multiplication and one addition. In accumulative addition, the results from one monomial are added to the results from the next monomial, and so on. This way of dividing is also called "synthetic division." To explain the above, let is re-write the polynomial in its expanded form;

$$\mathbf{f(x_0) = a_0 + a_1x_0 + a_2x_0^2 + ... + a_nx_0^n}$$

This can, also, be written as:

$$\mathbf{f(x_0) = a_0 + x_0(a_1 + x_0(a_2 + x_0(a_3 + ... + (a_{n-1} + a_nx_0)....)}$$

The proposed algorithm for this rule is to evaluate the monomials above, starting with the one in the innermost pair of parentheses and moving outwards to the ones in the outermost pair of parentheses. The algorithm is executed following the below steps:

1. Set $k = n$
2. Let $b_k = a_k$
3. Let $b_{k-1} = a_{k-1} + b_kx_0$
4. Set $k = k - 1$
5. If $k \geq 0$ then go to step 3

Else End

This algorithm can, also, be graphically visualized by considering the 5th degree polynomial given by:

$$\mathbf{f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5}$$

which can be evaluated at $x = x_0$ by re-arranging it as:

$$\mathbf{f(x_0) = a_0 + x_0(a_1 + x_0(a_2 + x_0(a_3 + x_0(a_4 + a_5x_0))))}$$

Another way to represent the results using this algorithm is through the tableau given below:

K	5	4	3	2	1	0
	$b_5 = a_5$	$b_4 = a_4 + x_0b_5$	$b_3 = a_3 + x_0b_4$	$b_2 = a_2 + x_0b_3$	$b_1 = a_1 + x_0b_2$	$b_0 = a_0 + x_0b_1$

<u>Example:</u> Evaluate the polynomial $f(x) = x^4 + 3x^3 + 5x^2 + 7x + 9$ at $x = 2$

Solution:

Since the polynomial is of the 4^{th} degree, then n = 4

K	4	3	2	1	0
Step	$b_4 = 1$	$b_3 = 3 + 2 * 1$	$b_2 = 5 + 2 * 5$	$b_1 = 7 + 2 * 15$	$b_0 = 9 + 2 * 37$
Result	1	5	15	37	83

Therefore, f(2) = 83.

Reference:

1. Math10.com, “Horner’s Rule”, https://www.math10.com/en/algebra/horner.html#:~:text=Horner's%20rule%20for%20polynomial%20division,multiplication%20and%20one%20addition%20processes.