Assignment 1

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1

Given that

$$P_{X_1,X_2}(x_1,x_2) = x_1^2 + x_2^2$$

Since

$$P_{V,\theta}(v,\theta) = |J| P_{X_1,X_2}(x_1,x_2)$$

let

$$x_1 = \sqrt{v}cos\theta$$

$$x_2 = \sqrt{v}sin\theta$$

$$J = \begin{vmatrix} \frac{1}{2v}cos\theta & -vsin\theta \\ \frac{1}{2v}sin\theta & vcos\theta \end{vmatrix} = \frac{1}{2}$$

Joint distribution of X1 and X2 is

$$P_{X_1,X_2}(x_1,x_2) = x_1^2 + x_2^2$$

then joint distribution of V and θ is

$$P_{V,\theta}(v,\theta) = \frac{1}{2}(x_1^2 + x_2^2)$$

then

$$P_{V,\theta}(v,\theta) = \frac{1}{2}(vcos^2\theta + vsin^2\theta)$$

$$= \frac{1}{2}v$$

2 Problem 6.2.4

Find $P_V(v)$
Solution: Say θ varies from 0 to 2π
, then
$P_V(v) = \int_0^{2\pi} P_{V,\theta}(v,\theta)\, d\theta$
$= \int_0^{2\pi} \frac{v}{2}\, d\theta$
$= \frac{v}{2}\left[\theta\right]_0^{2\pi}$
$= v\pi$

3 Problem 6.2.5

Find $P_\theta(\theta)$
Solution : Say v varies from 0 to constant R, then
$P_\theta(\theta) = \int_0^{R} P_{V,\theta}(v,\theta)\, dv$
$= \int_0^{R} \frac{v}{2}\, dv$
$= \frac{1}{2}\left[\frac{v^2}{2}\right]_0^R$
$= \frac{1}{4}(R^2 - 0)$
$= \frac{R^2}{4}$

4 Problem 6.2.6

Since

$$P_{v,\theta}(v,\theta) \neq P_V(v) * P_\theta(\theta)$$

hence V and θ are not independent.