Assignment 1

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Given that

$$V = x_1^2 + x_2^2$$

The joint pdf is given as :

$$P_{X_1,X_2}(x_1,x_2) = \frac{1}{2\pi}\exp\left(\frac{-(x_1^2+x_2^2)}{2}\right)$$

Since

$$P_{V,\theta}(v,\theta) = |J| P_{X_1,X_2}(x_1,x_2)$$

let

$$X_1 = \sqrt{V}cos\theta$$

$$X_2 = \sqrt{V}sin\theta$$

$$J = \begin{vmatrix} \frac{1}{2v}cos\theta & -vsin\theta \\ \frac{1}{2v}sin\theta & vcos\theta \end{vmatrix} = \frac{1}{2}$$

then joint distribution of V and θ is

$$P_{V,\theta}(v,\theta) = \frac{1}{2\pi}\exp\left(\frac{-(x_1^2+x_2^2)}{2}\right)$$

then

$$P_{V,\theta}(v,\theta) = \frac{1}{4\pi}\exp\left(\frac{-(x_1^2+x_2^2)}{2}\right)$$

$$= \frac{1}{4\pi}\exp\left(\frac{-(vcos^2\theta + vsin^{22})}{2}\right)$$

$$= \frac{1}{4\pi}\exp\left(\frac{-v}{2}\right)$$

$$CDF = \int_0^{2\pi}\int_0^{R^2}\frac{1}{4\pi}e^{\left(\frac{-v}{2}\right)}\,dv\,d\theta$$

$$= 1 - e^{\left(\frac{-R^2}{2}\right)}$$

2 Problem 6.2.4

Find $P_V(v)$
Solution: Say θ varies from 0 to 2π , then
$P_V(v) = \int_0^{2\pi} P_{V,\theta}(v,\theta)\,d\theta$

$= \int_0^{2\pi} \frac{1}{4\pi}\exp\left(\frac{-v}{2}\right)d\theta$

$= \frac{1}{4\pi}\exp\left(\frac{-v}{2}\right)[\theta]_0^{2\pi}$

$= \frac{1}{2}\exp\left(\frac{-v}{2}\right)$

3 Problem 6.2.5

Find $P_\theta(\theta)$
Solution : Say $\sqrt{v}$ varies from 0 to constant R, then v would vary from 0 to R^2
$P_\theta(\theta) = \int_0^R P_{V,\theta}(v,\theta)\,dv$

$= \int_0^R \frac{1}{4\pi}\exp\left(\frac{-v}{2}\right)dv$

$= \frac{-1}{2\pi}\left[e^{\frac{-v}{2}}\right]_0^{R^2}$

$$= \frac{1}{2\pi}\left[1 - e^{\frac{-R^2}{2}}\right]$$

4 Problem 6.2.6

Since

$$P_{v,\theta}(v, \theta) \neq P_V(v) * P_\theta(\theta)$$

hence V and θ are not independent.