Assignment 9

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1 Prob. Misc. 5.29

Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

Solution: Let X_1 be random variable for first dice and X_2 be random variable for second dice

$$X_1, X_2 \in \{1, 2, 3, 4, 5, 6\}$$
$$X = X_1 + X_2$$
$$X \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$
$$p_{X_i}(n) = \begin{cases} \dfrac{1}{6} & 1 \le X_i \le 6 \\ 0 & otherwise \end{cases}$$
$$\begin{aligned} p_X(n) &= \Pr(X_1 + X_2 = n) \\ &= \Pr(X_1 = n - X_2) \\ &= \sum_k \Pr(X_1 = n - k | X_2 = k) p_{X_2}(k) \\ &= \sum_k p_{X_1}(n - k) p_{X_2}(k) \\ &= \frac{1}{6} \sum_{k=1}^{6} p_{X_1}(n - k) \\ &= \frac{1}{6} \sum_{k=n-6}^{n-1} p_{X_1}(k) \end{aligned}$$

$$p_X(n) = \begin{cases} 0 & n \le 1 \\ \dfrac{1}{6} \displaystyle\sum_{k=1}^{n-1} p_{X_1}(k) = \dfrac{n-1}{36} & 2 \le n \le 7 \\ \dfrac{1}{6} \displaystyle\sum_{k=n-6}^{6} p_{X_1}(k) = \dfrac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases}$$

$$\begin{aligned} E[X] &= \sum_{n=1}^{12} n p_X(n) \\ &= \sum_{n=2}^{7} n \times \frac{n-1}{36} + \sum_{n=8}^{12} n \times \frac{13-n}{36} \\ &= \frac{1}{36}(2 + 6 + 12 + 20 + 30 + 42) + \frac{1}{36}(40 + 36 + \\ & \quad 30 + 22 + 12) \\ &= 3.111 + 3.889 \\ &= 7 \end{aligned}$$

PMF of X