Assignment 9

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Python code link:`https://github.com/ParvezAlam123/Assignment-9`

1 Prob. Misc. 5.29

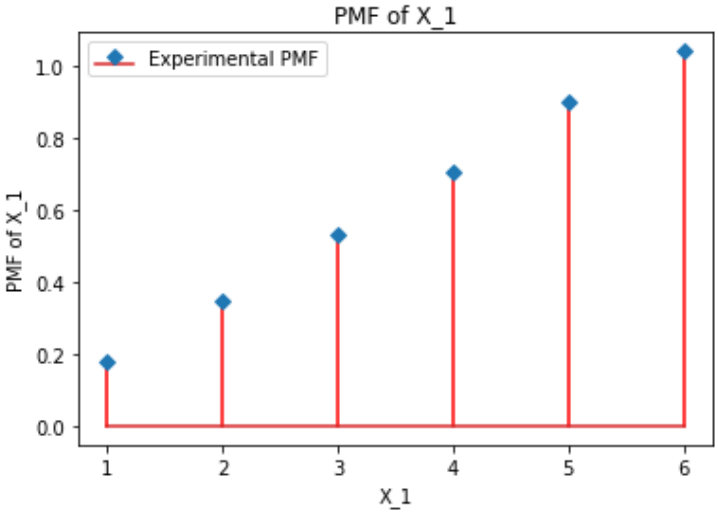
Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

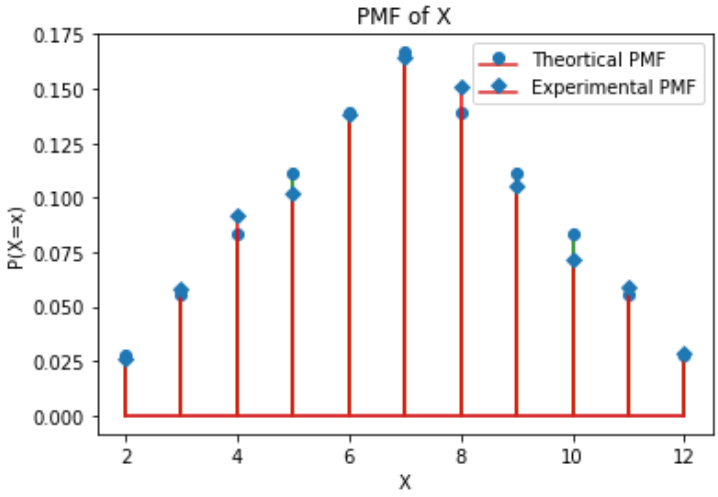
Solution: Let X_1 be random variable for first dice andX_2 be random variable for second dice

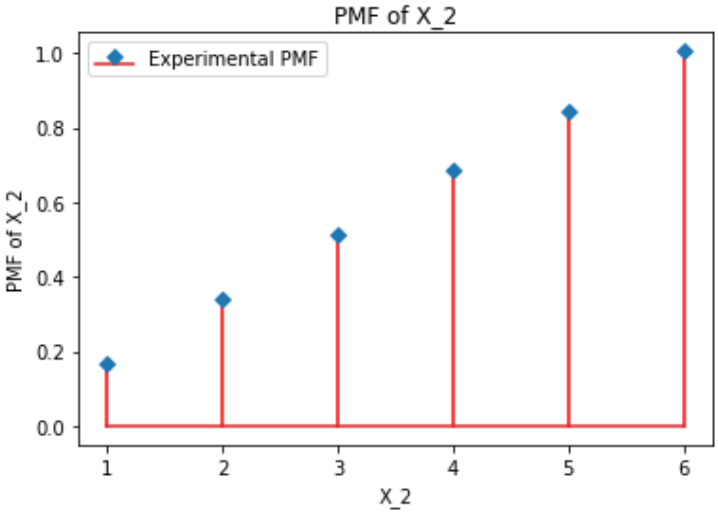
$$
\begin{aligned}
X_1, X_2 &\in \{1,2,3,4,5,6\} \\
X &= X_1 + X_2 \\
X &\in \{1,2,3,4,5,6,7,8,9,10,11,12\} \\
p_{X_i}(n) &= \begin{cases} \frac{1}{6} & 1 \le X_i \le 6 \\ 0 & otherwise \end{cases} \\
p_X(n) &= \Pr(X_1 + X_2 = n) \\
&= \Pr(X_1 = n - X_2) \\
&= \sum_k \Pr(X_1 = n-k | X_2 = k) p_{X_2}(k) \\
&= \sum_k p_{X_1}(n-k) p_{X_2}(k) \\
&= \frac{1}{6} \sum_{k=1}^{6} p_{X_1}(n-k) \\
&= \frac{1}{6} \sum_{k=n-6}^{n-1} p_{X_1}(k)
\end{aligned}
$$

$$
p_X(n) = \begin{cases} 0 & n \le 1 \\ \frac{1}{6}\sum_{k=1}^{n-1} p_{X_1}(k) = \frac{n-1}{36} & 2 \le n \le 7 \\ \frac{1}{6}\sum_{k=n-6}^{6} p_{X_1}(k) = \frac{13-n}{36} & 7 < n \le 12 \\ 0 & n > 12 \end{cases}
$$

$$
\begin{aligned}
E[X] &= \sum_{n=1}^{12} n p_X(n) \\
&= \sum_{n=2}^{7} n \times \frac{n-1}{36} + \sum_{n=8}^{12} n \times \frac{13-n}{36} \\
&= \frac{2}{36} + \sum_{n=3}^{7} n \times \frac{n-1}{36} + \sum_{n=8}^{12} n \times \frac{13-n}{36} \\
&= \frac{2}{36} + \frac{1}{36}\left(\sum_{n=3}^{7} n(n-1) + (n+5)(13-(n+5))\right) \\
&= \frac{2}{36} + \frac{1}{36}\left(\sum_{n=3}^{7} n^2 - n + (n+5)(8-n)\right) \\
&= \frac{2}{36} + \frac{1}{36}\left(\sum_{n=3}^{7} n^2 - n + 8n - n^2 + 40 - 5n\right) \\
&= \frac{2}{36} + \frac{1}{36}\left(\sum_{n=3}^{7} 2n + 40\right) \\
&= \frac{1}{18} + \frac{1}{18}\left(\sum_{n=3}^{7} n + 20\right) \\
&= \frac{1}{18}\left(1 + \sum_{n=3}^{7} n + 20\right) \\
&= \frac{1}{18}(1 + 23 + 24 + 25 + 26 + 27) \\
&= 7
\end{aligned}
$$


PMF of X_1
Experimental PMF
PMF of X_1
X_1
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6


PMF of X
Theortical PMF
Experimental PMF
P(X=x)
X
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
2
4
6
8
10
12


PMF of X_2
Experimental PMF
PMF of X_2
X_2
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6