Assignment 9

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Python code link:https://github.com/ParvezAlam123/Assignment-9

1 Prob. Misc. 5.29

Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

Solution: Let X_1 be random variable for first dice andX_2 be random variable for second dice

$$X_1, X_2 \in \{1,2,3,4,5,6\}$$
$$X = X_1 + X_2$$
$$X \in \{2,3,4,5,6,7,8,9,10,11,12\}$$

X	outcomes
2	(1,1)
3	(1,2),(2,1)
4	(1,3),(2,2),(3,1)
5	(1,4),(2,3),(3,2),(4,1)
6	(1,5),(2,4),(3,3),(4,2),(5,1)
7	(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)
8	(2,6),(3,5),(4,4),(5,3),(6,2)
9	(3,6),(4,5),(5,4),(6,3)
10	(4,6),(5,5),(6,4)
11	(5,6),(6,5)
12	(6,6)

$$\begin{aligned}
P(X=2) &= P(X_1+X_2=2) \\
&= \frac{1}{36} \\
P(X=3) &= P(X_1+X_2=3) \\
&= \frac{2}{36} \\
P(X=4) &= P(X_1+X_2=4) \\
&= \frac{3}{36} \\
P(X=5) &= P(X_1+X_2) \\
&= \frac{4}{36} \\
P(X=6) &= P(X_1+X_2=6) \\
&= \frac{5}{36} \\
P(X=7) &= P(X_1+X_2=7) \\
&= \frac{6}{36} \\
P(X=8) &= P(X_1+X_2=8) \\
&= \frac{5}{36} \\
P(X=9) &= P(X_1+X_2=9) \\
&= \frac{4}{36} \\
P(X=10) &= P(X_1+X_2=10) \\
&= \frac{3}{36} \\
P(X=11) &= P(X_1+X_2=11) \\
&= \frac{2}{36} \\
P(X=12) &= P(X_1+X_2=12) \\
&= \frac{1}{36}
\end{aligned}$$

Expectation:

$$
\begin{aligned}
E[X] &= \sum_{i=1}^{12} x_i P(X = x_i) \\
&= 2 \times \frac{1}{36} + 3 \times \frac{2}{36} \\
&+ 4 \times \frac{3}{36} + 5 \times \frac{4}{36} \\
&+ 6 \times \frac{5}{36} + 7 \times \frac{6}{36} \\
&+ 8 \times \frac{5}{36} + 9 \times \frac{4}{36} \\
&+ 10 \times \frac{3}{36} + 11 \times \frac{2}{36} \\
&+ 12 \times \frac{1}{36} \\
&= \frac{1}{36}(2 + 6 + 12 + 20 + 30 \\
&+ 42 + 40 + 36 + 30 + 22 + 12) \\
&= 6.444
\end{aligned}
$$