9.1 Letter Combination of a Phone Number (Backtracking)

```
import java.util.*;
class LetterCombinations {
  private static final Map<Character, String> phoneMap = new HashMap<>();
  static {
     phoneMap.put('2', "abc");
     phoneMap.put('3', "def");
     phoneMap.put('4', "ghi");
     phoneMap.put('5', "jkl");
     phoneMap.put('6', "mno");
     phoneMap.put('7', "pqrs");
     phoneMap.put('8', "tuv");
     phoneMap.put('9', "wxyz");
  public List<String> letterCombinations(String digits) {
     List<String> result = new ArrayList<>();
     if (digits == null || digits.length() == 0) return result;
     backtrack(result, new StringBuilder(), digits, 0);
     return result;
  }
  private void backtrack(List<String> result, StringBuilder current, String digits, int
index) {
    if (index == digits.length()) {
       result.add(current.toString());
       return;
     String letters = phoneMap.get(digits.charAt(index));
     for (char letter : letters.toCharArray()) {
       current.append(letter);
       backtrack(result, current, digits, index + 1);
       current.deleteCharAt(current.length() - 1); // Backtrack
     }
  public static void main(String[] args) {
     LetterCombinations obj = new LetterCombinations();
     String digits = "23";
     List<String> combinations = obj.letterCombinations(digits);
     System.out.println("Letter combinations for " + digits + ": " + combinations);
  }
}
```

9.2 Regular Expression Matching using Backtracking

Coding:

```
class RegexMatching {
  public boolean isMatch(String s, String p) {
     return matchHelper(s, p, 0, 0);
  }
  private boolean matchHelper(String s, String p, int i, int j) {
     if (j == p.length()) return i == s.length();
     boolean firstMatch = (i < s.length() &&
                  (s.charAt(i) == p.charAt(j) || p.charAt(j) == '.'));
     if (j + 1 < p.length() && p.charAt(j + 1) == '*')  {
       return matchHelper(s, p, i, j + 2) || (firstMatch && matchHelper(s, p, i + 1, j));
     } else {
       return firstMatch && matchHelper(s, p, i + 1, j + 1);
     }
  }
  public static void main(String[] args) {
     RegexMatching regex = new RegexMatching();
     System.out.println(regex.isMatch("aa", "a*")); // Output: true
     System.out.println(regex.isMatch("mississippi", "mis*is*p*.")); // Output: false
     System.out.println(regex.isMatch("ab", ".*")); // Output: true
     System.out.println(regex.isMatch("aab", "c*a*b")); // Output: true
  }
}
```

9.3 Sequential Digits Using Backtracking

Coding:

```
import java.util.*;
class SequentialDigits {
  public List<Integer> sequentialDigits(int low, int high) {
     List<Integer> result = new ArrayList<>();
     for (int start = 1; start \leq 9; start++) {
       generateNumbers(start, 0, low, high, result);
     }
     Collections.sort(result);
     return result;
  }
  private void generateNumbers(int digit, int num, int low, int high, List<Integer> result)
     if (num >= low && num <= high) {
       result.add(num);
     }
     if (num > high || digit > 9) {
       return;
     }
     generateNumbers(digit + 1, num * 10 + digit, low, high, result);
  }
  public static void main(String[] args) {
     SequentialDigits obj = new SequentialDigits();
     int low = 100, high = 300;
     System.out.println("Sequential digits in range: " + obj.sequentialDigits(low, high));
}
```