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ANALYTICAL EXPRESSIONS FOR THE DETERMINATION OF THE MAXIMUM POWER POINT AND THE FILL FACTOR OF A SOLAR CELL

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Summary

Simple approximate analytical expressions for calculating the values of current and voltage at the maximum power point and the fill factor of a solar cell are proposed. The ratios I_m/I_L and V_m/V_{oc}, and hence the fill factor, are shown to depend on the two normalized parameters

$$v_{oc} = \frac{V_{oc}}{mkT/e}$$

and

$$v_R = \frac{R_S I_L}{mkT/e}$$

which are closely related to the diode quality factor and the series resistance. The accuracy of the approach proposed here is fairly good for $v_R < 3$ and $v_{oc} > 15$, for relative errors of less than 1%. These ranges for v_R and v_{oc} cover the real situations for most silicon and GaAs solar cells. The usefulness of the analytical expressions as indicators of the behaviour of the cell is discussed, and the influence of the series resistance and the diode quality factor is emphasized. The application of these expressions to the determination of the series resistance and the diode quality factor is also discussed.

1. Introduction

An implicit equation has to be solved to calculate the current and voltage values at the maximum power point in a solar cell. A numerical solution to such an equation can be obtained easily and enables the maximum power point values, I_m and V_m, to be determined exactly. However, the derivation of analytical expressions for current and voltage with an acceptable degree of accuracy is of great interest. Analytical expressions allow a much simpler quicker calculation and, more importantly, they show clearly the effect of different parameters on the values of I_m and V_m, and

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hence on the fill factor FF. This information is very useful in determining directions for research, development and application work.

In this paper we present simple analytical expressions which enable I_m and V_m to be determined with great accuracy over a relatively wide range of $R_s I_L$ values.

2. Derivation of analytical expressions for I_m and V_m

A solar cell with a significant series resistance R_s can be treated analytically by considering R_s separately from the intrinsic device. With this assumption and in those cases in which the intrinsic device satisfies the shift approximation [1], the solar cell can be modelled by

$$I = I_d(V_j) - I_L \tag{1}$$

$$V_j = V - IR_s \tag{2}$$

where V_j is the voltage applied to the intrinsic device, $I_d(V_j)$ represents any equation suitable for modelling the dark current–voltage (I–V) characteristic of the intrinsic device and the other parameters have their usual meaning.

The shift approximation applies to a large number of practical solar cells under different operating conditions. For low injection conditions it applies to p–n junction solar cells with light or moderate doping concentrations and to single-crystal silicon cells with highly doped base regions or polycrystalline silicon cells. In all these cases, $I_L \approx I_{sc}$. The shift approximation also holds for high injection conditions; in this case, I_L is essentially the maximum current that can be drawn from the solar cell and in general is greater than I_{sc} [1].

The maximum power condition for a device modelled by eqns. (1) and (2) is

$$\mathrm{d}(VI) = V_m\,\mathrm{d}I + I_m\,\mathrm{d}V = 0 \tag{3}$$

This equation can be rewritten in terms of V_{jm}. By differentiating eqn. (2) and substituting $\mathrm{d}V$ in eqn. (3) we obtain

$$(V_{jm} + 2R_s I_m)\,\mathrm{d}I + I_m\,\mathrm{d}V_j = 0 \tag{4}$$

and hence

$$V_{jm} = -\left(2R_s + \frac{1}{G_{jm}}\right) I_m \tag{5}$$

where

$$G_{jm} = \left[\frac{\mathrm{d}I}{\mathrm{d}V_j}\right]_{V_j = V_{jm}} \tag{6}$$

Equation (5) is implicit in V_j and can be solved numerically. It should be noted that eqn. (5) is a general equation that does not depend on the equation used to model the intrinsic device.

Now, let us consider the usual single-exponential model for a solar cell:

$$I = I_0 \left\{ \exp\left(\frac{V_j}{V_t}\right) - 1 \right\} - I_L \tag{7}$$

where $V_t = mkT/e$. Equation (7) becomes

$$I = -I_L \left\{ 1 - \exp\left(\frac{V_j - V_{oc}}{V_t}\right) \right\} \tag{8}$$

where V_{oc} is the open-circuit voltage for the condition $\exp(V_{oc}/V_t) \gg 1$, which is obviously true for all practical cells.

It should be noted that using the single-exponential model and neglecting shunt resistance effects is a reasonable approximation when the calculations are restricted to the high voltage range near the open-circuit and maximum power point conditions, as is the case here. This model is even more accurate when the cell is under concentrated illumination.

When eqns. (8) and (6) are taken into account, eqn. (5) becomes

$$V_{jm} = \psi$$
$$= 2R_s I_L \left\{ 1 - \exp\left(\frac{V_{jm} - V_{oc}}{V_t}\right) \right\} + V_t \left\{ \exp\left(\frac{V_{oc} - V_{jm}}{V_t}\right) - 1 \right\} \tag{9}$$

A graphical solution of eqn. (9) is shown in Fig. 1 for typical practical values of a solar cell; the solution is found at the intersection of the straight line and the curve defined by the two equalities of eqn. (9).

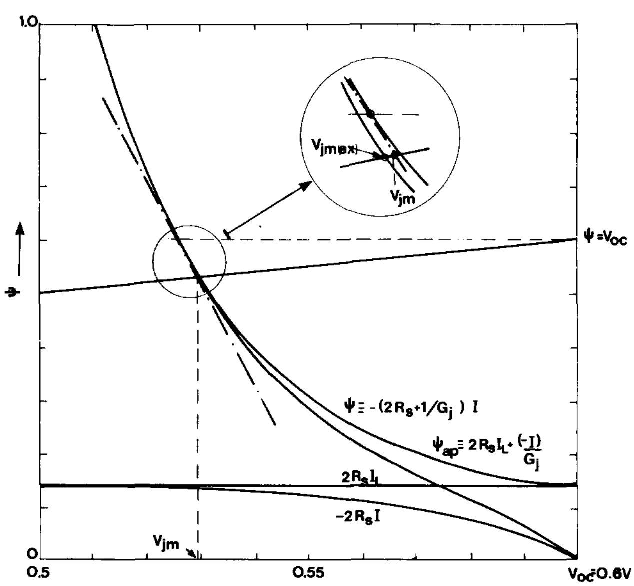

Fig. 1. Graphical illustration of the proposed approach to the calculation of V_{jm}.

The approximate analytical solution of eqn. (9) that we propose here is obtained as the intersection of the straight line $\psi = V_{jm}$ with the straight line resulting from a linear approximation of the function

$$\psi = 2R_s I_L + V_t \left\{ \exp\left(\frac{V_{oc} - V_{jm}}{V_t} \right) - 1 \right\} \tag{10}$$

around the point where $\psi = V_{oc}$. Substitution of the first term on the right-hand side of eqn. (9) by $2R_s I_L$ significantly simplifies the mathematical treatment and is a good approximation because $|I_m|$ is only slightly less than I_L in most practical cases. Furthermore, for $R_s I_L$ values in the range found in real cells, this approximation improves the resulting accuracy because of the slight upward shift of the line given by eqn. (10) with respect to the true line. This shift partially compensates for the downward shift due to the linear approximation of eqn. (10).

The linear expansion of eqn. (10) around the point where $\psi = V_{oc}$ leads to

$$\psi = V_{oc} - a(V_{jm} - V_{oc} + V_t \ln a) \tag{11}$$

where

$$a = v_{oc} + 1 - 2v_R \tag{12}$$

and v_{oc} and v_R are two normalized voltages defined by

$$v_{oc} = V_{oc}/V_t \qquad v_R = R_s I_L / V_t \tag{13}$$

The intersection of eqn. (11) with $\psi = V_{jm}$ leads to the following expressions for the current and voltage values at the maximum power point:

$$\frac{V_{jm}}{V_{oc}} = 1 - \frac{b}{v_{oc}} \ln a \tag{14}$$

$$-\frac{I_m}{I_L} = 1 - a^{-b} \tag{15}$$

$$\frac{V_m}{V_{oc}} = 1 - \frac{b}{v_{oc}} \ln a - \frac{v_R}{v_{oc}}(1 - a^{-b}) \tag{16}$$

where $b = a/(a + 1)$.

3. Results and discussion

The exact values of $-I_m/I_L$, V_m/V_{oc} and the fill factor, the numerical values calculated by solving eqn. (5) for the one-exponential characteristic given by eqn. (8), the approximate values given by eqns. (15) and (16) of this work and the approximate values given by eqns. (25) and (26) of ref. 2 are summarized in Table 1. The exact values of $-I_m/I_L$ and V_m/V_{oc} together with our approximate values are also plotted in Figs. 2 and 3 as functions of the two key parameters v_{oc} and v_R.

TABLE 1

Brief summary of the results in the ranges $v_R < 3$ and $15 < v_{oc} < 30$

v_{oc}	v_R	$-I_m/I_L$			V_m/V_{oc}			FF		
		AP1	*EXT*	*AP2*	*AP1*	*EXT*	*AP2*	*AP1*	*EXT*	*AP2*
15	0.0	0.9264	0.9254	0.9248	0.8260	0.8270	0.8275	0.7653	0.7653	0.7653
15	1.5	0.9076	0.9082	0.9091	0.7505	0.7500	0.7493	0.6811	0.6811	0.6811
15	3.0	0.8767	0.8831	0.8934	0.6851	0.6803	0.6721	0.6006	0.6008	0.6004
20	0.0	0.9453	0.9448	0.9445	0.8547	0.8552	0.8555	0.8080	0.8080	0.8080
20	1.5	0.9353	0.9353	0.9357	0.7929	0.7930	0.7926	0.7416	0.7416	0.7416
20	3.0	0.9210	0.9225	0.9270	0.7349	0.7337	0.7301	0.6769	0.6769	0.6768
30	0.0	0.9641	0.9639	0.9638	0.8891	0.8893	0.8894	0.8572	0.8572	0.8572
30	1.5	0.9599	0.9599	0.9600	0.8448	0.8448	0.8447	0.8109	0.8109	0.8109
30	3.0	0.9547	0.9549	0.9562	0.8014	0.8012	0.8001	0.7651	0.7651	0.7651

EXT, exact values; AP1, this work; AP2, values obtained using ref. 2, eqns. (25) and (26).

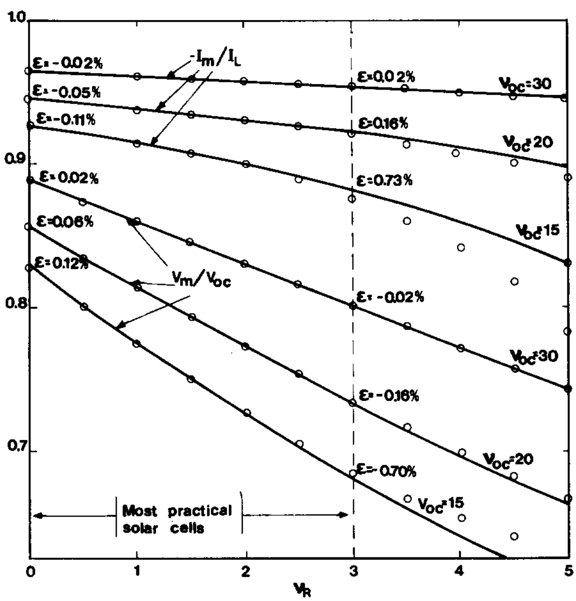

Fig. 2. Exact (——) and approximate (○) values of $-I_m/I_L$ and V_m/V_{oc} as functions of v_R for various values of v_{oc}.

As can be seen, the agreement between the exact (numerical) and the approximate (analytical) solutions proposed here is fairly good for $v_R < 3$ and for $15 < v_{oc}$. These ranges of v_R and v_{oc} cover the real situation for most practical silicon and GaAs solar cells, including concentration cells. As can also be seen, the approximation obtained using eqns. (15) and (16) is

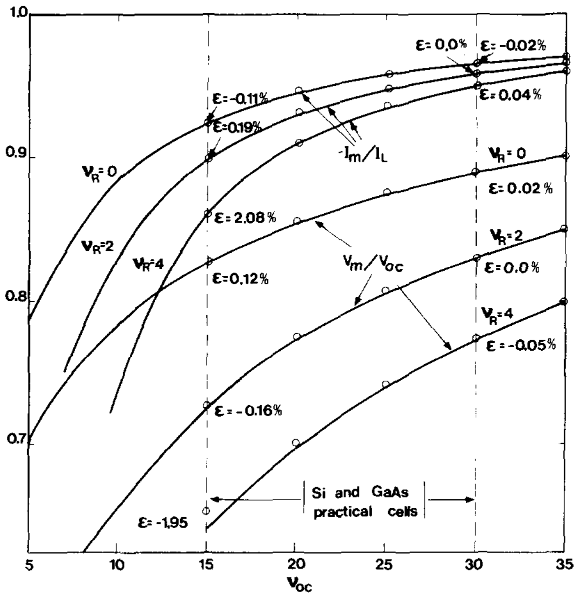

Fig. 3. Exact (——) and approximate (○) values of $-I_m/I_L$ and V_m/V_{oc} as functions of v_{oc} for various values of v_R.

similar to that obtained with ref. 2, eqns. (25) and (26), but our expressions are much simpler.

As shown in Figs. 2 and 3, the error between the exact and the approximate solutions changes its sign as v_R increases from zero. This is due to the shift involved in the use of eqn. (10), as discussed above, and means that for some v_R values greater than zero the approximate and the exact solutions coincide. This is the case, for example, for $v_{oc} = 30$ and $v_R = 2$. Thus the greatest accuracy is obtained for values of v_R around 2, which is the case for an appreciable number of practical cells. In any event, in the ranges of v_{oc} and v_R given above, the relative error is always lower than 1%, and the errors for standard silicon cells ($v_{oc} \approx 24$ for a cell with $V_{oc} = 0.600$ V at room temperature) are less than 0.2%. In addition, it should be noted that, because the relative errors for $-I_m/I_L$ and V_m/V_{oc} are of similar magnitude and opposite sign, the fill factor of the solar cell is given by

$$\mathrm{FF} \approx -\frac{I_m}{I_L}\frac{V_m}{V_{oc}} \tag{17}$$

when $I_{sc} \approx I_L$, and this has a resulting accuracy that is much greater than those of eqns. (15) and (16). The relative error for the fill factor in the range $v_R < 3$ and $v_{oc} > 15$ is less than 0.03%, as illustrated in Fig. 4. For $v_R < 3$ a further approximation for $-I_m/I_L$ is still acceptable in many cases. As the parameter b approaches unity, eqn. (15) can be rewritten as

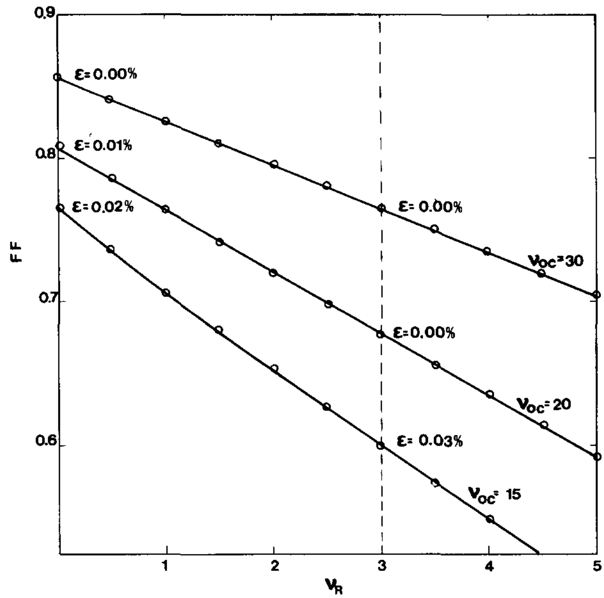

Fig. 4. Exact (——) and approximate (○) values of FF as functions of v_R for various values of v_{oc}.

$$
-\frac{I_m}{I_L} \approx 1 - \frac{1}{a}
$$

$$
= 1 - \frac{V_t}{V_{oc}}\left(1 + \frac{2R_sI_L - V_t}{V_{oc}}\right) \tag{18}
$$

This expression is similar to ref. 3, eqn. (9). The relative error corresponding to this approximation is less than 2% for $v_R < 3$ and $v_{oc} > 15$. For standard silicon solar cells the error is about 1% or less.

4. Application to the determination of series resistance

The following expression can be obtained from eqns. (2), (5), (6) and (8):

$$
R_s = -\frac{V_m}{I_m} - \frac{V_t}{I_L + I_m} \tag{19}
$$

This formula allows the series resistance to be determined experimentally from the illuminated I–V characteristic. The method is similar to that described in ref. 4, but does not require any dynamic measurements.

A new expression for R_s can also be obtained by combining eqns. (15) and (16) and using the definitions given by eqn. (13):

$$R_s = \frac{V_m}{I_m} - \frac{V_{oc}}{I_m} - \frac{V_t}{I_m} \ln\left(1 + \frac{I_m}{I_L}\right) \tag{20}$$

When the approximation used to obtain eqn. (18) is valid, *i.e.* when $a^{-b} \approx 1/a$, eqn. (19) becomes

$$R_s = \frac{V_{oc}}{I_L} + \frac{V_m}{I_m} \tag{21}$$

Equation (21) can also be derived from eqn. (20) on the assumption that the third term of the right-hand side is negligible and that the second term $-V_{oc}/I_m$ can be approximated by V_{oc}/I_L. This expression has previously been proposed [5] as a way of measuring the series resistance. It has the advantage that V_t need not be known and that temperature variations during the measurement have little influence on the results. However, we found that eqn. (21) can give R_s values far from the true values. We believe that this is because the error involved in changing from eqn. (19) or eqn. (20) to eqn. (21) can produce a greater relative error in the calculation of R_s using eqn. (21).

An alternative to eqns. (19) and (20), in which V_t must be known, and to eqn. (21), which can be inaccurate, is to calculate v_{oc} and v_R from experimental values of $-I_m/I_L$ and V_m/V_{oc} by solving the system formed by eqns. (15) and (16). Once the two parameters are known, R_s and V_t can be calculated from

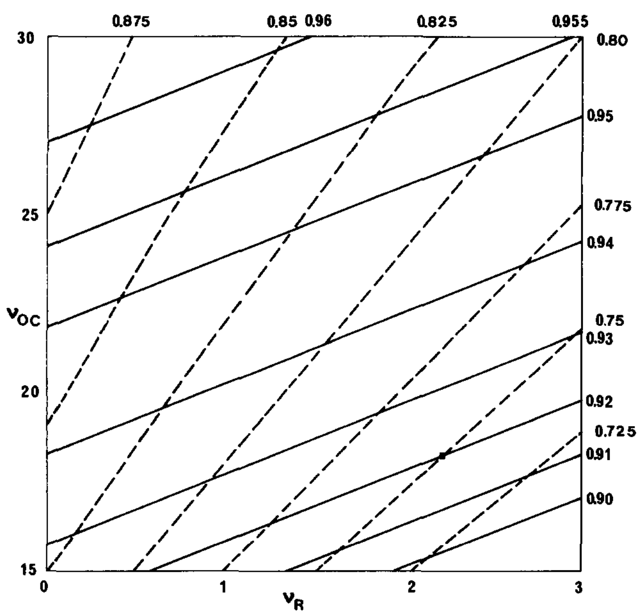

Fig. 5. Graphical method of obtaining the parameters v_{oc} and v_R for given values of $-I_m/I_L$ and V_m/V_{oc}: ——, constant values of $-I_m/I_L$; – – –, constant values of V_m/V_{oc}.

TABLE 2

Examples of the determination of series resistance

Case number[a]	V_{oc} (V)	I_L (A)	V_m (V)	$-I_m$ (A)	V_m/V_{oc}	$-I_m/I_L$	R_s *from eqn. (22)* (mΩ)	R_s *from eqn. (21)* (mΩ)	V_t *from eqn. (23)* (V)
1	0.600	1	0.440	0.93	0.9311	0.7333	100	127	0.025
2	0.761	20	0.607	19.09	0.7977	0.9547	4	6.3	0.025
3	0.761	20	0.601	19.28	0.7898	0.9640	4.7	6.9	0.022
4	0.600	0.1	0.450	0.092	0.75	0.920	0.700	1.109	0.034
5	0.713	12.35	0.584	11.87	0.9611	0.8191	5.1	8.5	0.021

[a]See text for explanation.

$$R_s = \frac{V_{oc}}{I_L}\frac{v_R}{v_{oc}} \tag{22}$$

and

$$V_t = V_{oc}/v_{oc} \tag{23}$$

A graphical solution can be obtained from Fig. 5 where the relation between the parameters v_{oc} and v_R given by eqns. (15) and (16) is illustrated for constant values of $-I_m/I_L$ and V_m/V_{oc}. This figure shows that the determination of R_s is rather sensitive to errors in the measurements of currents and voltages. Particular difficulties can arise from inadequate accuracy in the experimental determination of the maximum power point.

Some illustrative examples are given in Table 2. Cases 1 and 2 are representative of non-concentrator and concentrator solar cells with series resistances of 100 mΩ and 4 mΩ respectively. Case 3 is the same as case 2, but errors of +2% in the measurement of $-I_m$ and of -1% in the measurement of V_m are assumed. Case 4 is the same as that proposed in ref. 2, Section 5.2. This example is shown in Fig. 5 by the full square at $v_R = 2.21$ and $v_{oc} = 18.3$. As can be seen, our solution is slightly more accurate. Case 5 is a real 2 in low resistivity concentrator solar cell measured in our laboratory.

5. Conclusions

Simple approximate analytical expressions have been obtained for the current and voltage values of the maximum power point of the illuminated I–V characteristic of a solar cell. The expressions show that the behaviour depends on the two key normalized parameters v_{oc} and v_R.

The expressions are accurate enough to be used in the simulation of practical cells. However, it is more important that, because of the variation in the diode quality factor along the characteristics when a single-exponential model is used, the uncertainty included in the calculations (if a constant, and to some extent arbitrary, value of m is assumed) can be much higher than that due to the approximation involved in eqns. (15) and (16). The accuracy of the calculations of the fill factor of the cells using the proposed approximations is even better because the relative errors for $-I_m/I_L$ and V_m/V_{oc} are of similar magnitude and opposite sign.

A new method of determining the series resistance and the diode quality factor at the maximum power point, based on V_{oc}, I_{sc}, V_m and I_m measurements, has been proposed.

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Nomenclature

a	$v_{oc} + 1 - 2v_R$
b	$a/(a+1)$
G_j	dI/dV_j, intrinsic dynamic conductance
I	illumination current
I_d	dark current
I_L	photogenerated current
I_m	maximum power point current
I_0	saturation dark current
m	diode ideality factor
R_s	series resistance
v_{oc}	V_{oc}/V_t, normalized open-circuit voltage
v_R	R_sI_L/V_t, normalized ohmic drop
V	external voltage
V_j	$V - IR_s$, junction voltage
V_m	maximum power point voltage
V_{oc}	open-circuit voltage
V_t	mkT/e, thermal voltage
ϵ	relative error between the exact and the approximate values

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