**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

As the manager intends to begin the work after 10 min,

The average time now taken to complete the work would be µ = 45 + 10 = 55

The probability that the work would not be completed within 1 hour

P(x > 60), Given σ = 8

import scipy.stats

from scipy.stats import norm

scipy.stats.norm.cdf(60,55,8)

0.7340

#probabiltiy that work extends above 1 hour

1-scipy.stats.norm.cdf(60,55,8)

0.265

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans – False. The Probability of employees being older than 44 is less Than that of employees being between the age of 38 and 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans- True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

The difference between  and  is .

**Step-by-step explanation:**

According to the **Central Limit Theorem**, any **large sum** of **independent**, **identically distributed(iid)** random variables is approximately **Normal**.

The **Normal distribution** is defined by two parameters, the **mean**, , and the **variance**,  and written as .

Given   are two independent identically distributed random variables.

From the properties of **normal random variables**,

if  and  are two independent identically distributed random variables then

* the **sum** of normal random variables is given by

,

* and the **difference** of normal random variables is given by



* When  , the **product** of X is given by



* When  , the **linear combination** of X and Y is given by



Given to find, 

Thus, following the property of multiplication, we get



and following the property of addition,



And the difference between the two is given by



The mean of  and  is same but the var() of   is 2 times more than the variance of .

The difference between the two says that the two given variables are **identically** and **independently** distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

ANS - µ = 100

σ = 20

P(x) = 0.99

α = 0.005

Z1- α = Z.995

scipy.stats.norm.ppf(.995) =2.575

Z = (x - µ)/ σ

C.I = 100 +- 2.575\*20

48.5, 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans- Profit1 ~ N(5, 32) + Profit2 ~ N(7, 42) = Profittotal ~ N(12, 52)

µ = 12

σ = 5

P(x) = 0.95

α = 0.025

Z1- α = Z.975

scipy.stats.norm.ppf(.975) =1.96

Z = (x - µ)/ σ

C.I = 12 +- 2.575\*5

2.2M, 22.8M or 9.9 Crore Rupees, 98.1 Crore Rupees,

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans- scipy.stats.norm.ppf(.05)

-1.644

12 – 1.644\*5 =3.78 M or 17.01 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans- scipy.stats.norm.cdf(0,5,3)

0.0477

scipy.stats.norm.cdf(0,7,4)

0.0400

Division 2 has a larger probability of making a loss in a given year