MATH 2600–Problem Set 3

Daniel Park

January 31, 2024

1. All Possible Products

A is of shape 3×2, B is of shape 2×4, C is of shape 4×3, and D is of shape 3×3. Thus, the possible products are: AB, BC, CD, DA, CA.

$$AB = \begin{pmatrix} 2(1)+5(9) & 2(7)+5(2) & 2(2)+5(7) & 2(9)+5(1) \\ 1(1)+4(9) & 1(7)+4(2) & 1(2)+4(7) & 1(9)+4(1) \\ 2(1)+1(9) & 2(7)+1(2) & 2(2)+1(7) & 2(9)+1(1) \end{pmatrix}$$

$$= \begin{pmatrix} 47 & 24 & 39 & 23 \\ 37 & 15 & 30 & 13 \\ 11 & 16 & 11 & 19 \end{pmatrix}$$

$$BC = \begin{pmatrix} [1(1)+7(2)+2(1)+9(3)] & [1(0)+7(1)+2(1)+9(2)] & [1(4)+7(3)+2(5)+9(1)] \\ [9(1)+2(2)+7(1)+1(3)] & [9(0)+2(1)+7(1)+1(2)] & [9(4)+2(3)+7(5)+1(1)] \end{pmatrix}$$

$$= \begin{pmatrix} 44 & 27 & 44 \\ 23 & 11 & 78 \end{pmatrix}$$

$$CD = \begin{pmatrix} [1(1)+0(2)+4(1)] & [1(0)+0(1)+4(3)] & [1(7)+0(2)+4(0)] \\ [2(1)+1(2)+3(1)] & [2(0)+1(1)+3(3)] & [2(7)+1(2)+3(0)] \\ [1(1)+1(2)+5(1)] & [1(0)+1(1)+5(3)] & [1(7)+1(2)+5(0)] \\ [3(1)+2(2)+1(1)] & [3(0)+2(1)+1(3)] & [3(7)+2(2)+1(0)] \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 12 & 7 \\ 7 & 10 & 16 \\ 8 & 16 & 9 \\ 8 & 5 & 25 \end{pmatrix}$$

$$DA = \begin{pmatrix} [1(2)+0(1)+7(2)] & [1(5)+0(4)+7(1)] \\ [2(2)+1(1)+2(2)] & [2(5)+1(4)+2(1)] \\ [1(2)+3(1)+0(2)] & [1(5)+3(4)+0(1)] \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 12 \\ 9 & 16 \\ 5 & 17 \end{pmatrix}$$

$$CA = \begin{pmatrix} [1(2)+0(1)+4(2)] & [1(5)+0(4)+4(1)] \\ [2(2)+1(1)+3(2)] & [2(5)+1(4)+3(1)] \\ [1(2)+1(1)+5(2)] & [1(5)+1(4)+5(1)] \\ [3(2)+2(1)+1(2)] & [3(5)+2(4)+1(1)] \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 9 \\ 11 & 17 \\ 13 & 14 \\ 10 & 24 \end{pmatrix}$$

2. AB vs. BA

2b. $\mathbf{A} = \begin{pmatrix} 1 & -1 & 5 \\ 3 & 0 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 3 & 6 \\ 1 & 5 \end{pmatrix}$

A is of shape 2×3 and B is of shape 3×2. Both AB and BA are thus defined. AB is of shape 2×2 and BA is of shape 3×3, so they do not have the same number of rows and columns.

2d. $\mathbf{A} = \begin{pmatrix} 3 & 1 & -4 \\ -2 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

A is of shape 3×3 and B is of shape 3×3. Both AB and BA are thus defined. AB is of shape 3×3 and BA is of shape 3×3, so they *do* have the same number of rows and columns.

$$AB = \begin{pmatrix} [3(2)+1(0)+-4(0)] & [3(0)+1(5)+-4(0)] & [3(0)+1(0)+-4(-1)] \\ [-2(2)+0(0)+5(0)] & [-2(0)+0(5)+5(0)] & [-2(0)+0(0)+5(-1)] \\ [1(2)+-2(0)+3(0)] & [1(0)+-2(5)+3(0)] & [1(0)+-2(0)+3(-1)] \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 5 & 4 \\ -4 & 0 & -5 \\ 2 & -10 & -3 \end{pmatrix}$$

Since both A and B are square matrices and $AB \neq I_3$, $B \neq A^{-1}$, and thus BA is not going to equal to AB. To check...

$$BA = \begin{pmatrix} [2(3)+0(-2)+0(1)] & [2(1)+0(0)+0(-2)] & [2(-4)+0(5)+0(3)] \\ [0(3)+5(-2)+0(1)] & [0(1)+5(0)+0(-2)] & [0(-4)+5(5)+0(3)] \\ [0(3)+0(-2)+-1(1)] & [0(1)+0(0)+-1(-2)] & [0(-4)+0(5)+-1(3)] \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 2 & -8 \\ -10 & 0 & 25 \\ -1 & 2 & -3 \end{pmatrix}$$

Indeed, $AB \neq BA$.

3. $A = \begin{pmatrix} 2 & -5 \\ 3 & 1 \end{pmatrix}$

3a. Evaluate A^2

$$A^2 = \begin{pmatrix} [2(2) + -5(3)] & [2(-5) + -5(1)] \\ [3(2) + 1(3)] & [3(-5) + 1(1)] \end{pmatrix}$$
$$= \begin{pmatrix} -11 & -15 \\ 9 & -14 \end{pmatrix}$$

3b. α, β, γ

Find α, β, $\gamma \in \mathbb{R}$, such that $\alpha I + \beta A + \gamma A^2 = \mathbf{0}$ and α, β, γ not all 0.

$$\alpha I_2 + \beta A + \gamma A^2 = \mathbf{0}$$

$$\begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} + \begin{pmatrix} 2\beta & -5\beta \\ 3\beta & \beta \end{pmatrix} + \begin{pmatrix} -11\gamma & 15\gamma \\ 9\gamma & -14\gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha + 2\beta - 11\gamma & -5\beta + 15\gamma \\ 3\beta + 9\gamma & \alpha + \beta - 14\gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\alpha + 2\beta - 11\gamma = 0$$

$$-5\beta + 15\gamma = 0$$

$$3\beta + 9\gamma = 0$$

$$\alpha + \beta - 14\gamma = 0$$

From the 2nd and 3rd equations, solving for γ...

$$\beta = -3\gamma$$

Plugging that into the 4th equation...

$$\alpha - 17\gamma = 0$$

$$\alpha = 17\gamma$$

$$\gamma = \gamma$$

There are infinitely many solutions. If not all zero, one solution is $\alpha = 17$, $\beta = -3$, $\gamma = 1$.

5. Proof

If A and B are matrices s.t. $I - AB$ is invertible, then the inverse of $I - BA$ is given by the formula $(I - BA)^{-1} = I + B(I - AB)^{-1}A$.
Let's denote C as $(I - AB)^{-1}$. Then,

$$(I - BA)^{-1} = I + BCA$$

$$(I - BA)(I - BA)^{-1} = (I - BA)(I + BCA)$$

We will prove that $(I - BA)(I + BCA) = I$, since $(I - BA)(I - BA)^{-1} = I$.

$$(I - BA)(I + BCA) = I$$

$$I^2 + IBCA - BAI + BABCA = I$$

$$I + BCA - BA + BABCA = I$$

$$I + B(C - I - ABC)A = I$$

$$I + B(C - ABC - I)A = I$$

$$I + B[(I - AB)C - I]A = I$$

Since $C = (I - AB)^{-1}$, $C^{-1} = (I - AB)$, because $I - AB$ is assumed to be invertible.

$$I + B(C^{-1}C - I)A = I$$

$$I + B(I - I)A = I$$

$$I + B\mathbf{0}A = I$$

$$I + \mathbf{0} = I$$

$$I = I$$

Thus, the inverse of $I - BA$ is given by $I + B(I - AB)^{-1}A$. ■

6. Finding inverses

6a. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 + (-R_1) = \left[\begin{array}{ccc|ccc} 0 & -2 & 0 & -1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\frac{R_2}{-2} = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & \frac{-1}{-2} & \frac{1}{-2} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_1 + (-R_2) = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_1 + (-R_3) = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -1 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6b. $\mathbf{B} = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 5 & 3 \\ 2 & 6 & -1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 1 & 5 & 3 & 0 & 1 & 0 \\ 2 & 6 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 + (-R_1) = \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & -1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 3 & 5 & -1 & 1 & 0 \\ 2 & 6 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 + (-2R_1) = \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 3 & 5 & -1 & 1 & 0 \\ 0 & 2 & 3 & -2 & 0 & 1 \end{array}\right]$$

$$\frac{R_2}{3} = \left[\begin{array}{ccc|ccc} 0 & 1 & \frac{5}{3} & \frac{-1}{3} & \frac{1}{3} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 2 & 3 & -2 & 0 & 1 \end{array}\right]$$

$$R_3 + (-2R_2) = \left[\begin{array}{ccc|ccc} 0 & 0 & \frac{-1}{3} & \frac{-4}{3} & \frac{-2}{3} & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} & -\frac{4}{3} & -\frac{2}{3} & 1 \end{array}\right]$$

$$R_1 + (-2R_2) = \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{-16}{3} & \frac{5}{3} & \frac{-2}{3} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{3} & \frac{5}{3} & -\frac{2}{3} & 0 \\ 0 & 1 & \frac{5}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} & -\frac{4}{3} & -\frac{2}{3} & 1 \end{array}\right]$$

$$-3R_3 = \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 4 & 2 & -3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{3} & \frac{5}{3} & -\frac{2}{3} & 0 \\ 0 & 1 & \frac{5}{3} & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 4 & 2 & -3 \end{array}\right]$$

$$R_2 + (-\frac{5}{3}R_3) = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & -\frac{21}{3} & -\frac{9}{3} & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{3} & \frac{5}{3} & -\frac{2}{3} & 0 \\ 0 & 1 & 0 & -7 & -3 & 5 \\ 0 & 0 & 1 & 4 & 2 & -3 \end{array}\right]$$

$$R_1 + (\frac{16}{3}R_3) = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{69}{3} & \frac{30}{3} & \frac{-48}{3} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 23 & 10 & -16 \\ 0 & 1 & 0 & -7 & -3 & 5 \\ 0 & 0 & 1 & 4 & 2 & -3 \end{array}\right]$$

$$B^{-1} = \begin{bmatrix} 23 & 10 & -16 \\ -7 & -3 & 5 \\ 4 & 2 & -3 \end{bmatrix}$$

6e. $\mathbf{C} = \begin{bmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & a & b+c & 1 & 0 & 0 \\ 1 & b & a+c & 0 & 1 & 0 \\ 1 & c & a+b & 0 & 0 & 1 \end{array}\right]$$

$$R_2 + (-R_1) = \left[\begin{array}{ccc|ccc} 0 & b-a & a-b & -1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & a & b+c & 1 & 0 & 0 \\ 0 & b-a & a-b & -1 & 1 & 0 \\ 1 & c & a+b & 0 & 0 & 1 \end{array}\right]$$

$$R_3 + (-R_1) = \left[\begin{array}{ccc|ccc} 0 & c-a & a-c & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & a & b+c & 1 & 0 & 0 \\ 0 & b-a & a-b & -1 & 1 & 0 \\ 0 & c-a & a-c & -1 & 0 & 1 \end{array}\right]$$

$$\frac{R_2}{b-a} = \left[\begin{array}{ccc|ccc} 0 & 1 & \frac{a-b}{b-a} & \frac{-1}{b-a} & \frac{1}{b-a} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & a & b+c & 1 & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{a-b} & -\frac{1}{a-b} & 0 \\ 0 & c-a & a-c & -1 & 0 & 1 \end{array}\right]$$

$$R_3 + -((c-a)R_2) = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & \frac{b-c}{a-b} & \frac{c-a}{a-b} & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & a & b+c & 1 & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{a-b} & -\frac{1}{a-b} & 0 \\ 0 & 0 & 0 & \frac{b-c}{a-b} & \frac{c-a}{a-b} & 1 \end{array}\right]$$

Since R_3 has no pivot variable, we cannot turn the variables above it (namely $b + c$ and -1) to 0 to reduce it to RREF. Thus, the inverse does not exist.

7. Using inverses to solve systems

7a. Inverse of $\begin{pmatrix} 2 & 5 & 8 & 5 \\ 1 & 2 & 3 & 1 \\ 2 & 4 & 7 & 2 \\ 1 & 3 & 5 & 3 \end{pmatrix}$

$$\left[\begin{array}{cccc|cccc} 2 & 5 & 8 & 5 & 1 & 0 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 & 1 & 0 & 0 \\ 2 & 4 & 7 & 2 & 0 & 0 & 1 & 0 \\ 1 & 3 & 5 & 3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\frac{R_1}{2} = \left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 4 & \frac{5}{2} & \frac{1}{2} & 0 & 0 & 0 \end{array}\right]$$

$$R_2 + (-R_1) = \left[\begin{array}{cccc|cccc} 0 & -\frac{1}{2} & -1 & -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 0 \end{array}\right]$$

$$R_3 + (-2R_1) = \left[\begin{array}{cccc|cccc} 0 & -1 & -1 & -3 & -1 & 0 & 1 & 0 \end{array}\right]$$

$$R_4 + (-R_1) = \left[\begin{array}{cccc|cccc} 0 & \frac{1}{2} & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 4 & \frac{5}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & -1 & -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -1 & -1 & -3 & -1 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 1 \end{array}\right]$$

$$-2R_2 = \left[\begin{array}{cccc|cccc} 0 & 1 & 2 & 3 & 1 & -2 & 0 & 0 \end{array}\right]$$

$$R_3 + R_2 = \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \end{array}\right]$$

$$R_4 + (-\frac{R_2}{2}) = \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & -1 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 4 & \frac{5}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$-R_4 = \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 4 & \frac{5}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$R_2 + (-3R_4) = \left[\begin{array}{cccc|cccc} 0 & 1 & 2 & 0 & -2 & 1 & 0 & 3 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 4 & \frac{5}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & -2 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$R_1 + (-\frac{5}{2}R_4) = \left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 4 & 0 & -2 & \frac{5}{2} & 0 & \frac{5}{2} \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 4 & 0 & -2 & \frac{5}{2} & 0 & \frac{5}{2} \\ 0 & 1 & 2 & 0 & -2 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$R_2 + (-2R_3) = \left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & -2 & 5 & -2 & 3 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 4 & 0 & -2 & \frac{5}{2} & 0 & \frac{5}{2} \\ 0 & 1 & 0 & 0 & -2 & 5 & -2 & 3 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$R_1 + (-4R_3) = \left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 0 & 0 & -2 & \frac{21}{2} & -4 & \frac{5}{2} \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & \frac{5}{2} & 0 & 0 & -2 & \frac{21}{2} & -4 & \frac{5}{2} \\ 0 & 1 & 0 & 0 & -2 & 5 & -2 & 3 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right]$$

$$R_1 + (-\frac{5}{2}R_2) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 3 & -2 & 1 & -5 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 3 & -2 & 1 & -5 \\ 0 & 1 & 0 & 0 & -2 & 5 & -2 & 3 \\ 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 & 0 & -1 \end{array}\right]$$

The inverse is $\begin{bmatrix} 3 & -2 & 1 & -5 \\ -2 & 5 & -2 & 3 \\ 0 & -2 & 1 & 0 \\ 1 & -1 & 0 & -1 \end{bmatrix}$.

7b. Solve system of equations

$Ax = b$, where $A = \begin{pmatrix} 2 & 5 & 8 & 5 \\ 1 & 2 & 3 & 1 \\ 2 & 4 & 7 & 2 \\ 1 & 3 & 5 & 3 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$.

We can solve for x by solving for $A^{-1}b$.

$$A^{-1} = \begin{pmatrix} 3 & -2 & 1 & -5 \\ -2 & 5 & -2 & 3 \\ 0 & -2 & 1 & 0 \\ 1 & -1 & 0 & -1 \end{pmatrix}$$

$$A^{-1}b = \begin{pmatrix} 3(0) + -2(1) + 1(0) + -5(1) \\ -2(0) + 5(1) + -2(0) + 3(1) \\ 0(0) + -2(1) + 1(0) + 0(1) \\ 1(0) + -1(1) + 0(0) + -1(1) \end{pmatrix}$$

$$x = A^{-1}b = \begin{pmatrix} -7 \\ 8 \\ -2 \\ -2 \end{pmatrix}$$