MATH 2600–Problem Set 2

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1 Reduced Row Echelon Form Uniqueness

Let A denote an arbitrary matrix. Then, the Reduced Row Echelon form (RREF) of A is unique. That is, after performing some elementary row operations on A, there exists no two different matrices that both represent the RREF of A.

Assume two matrices, P and Q, represent the RREF of A. By definition, both P and Q have the same solutions as each other and A if they represent augmented matrices of a system. This implies that by performing elementary row operations on P, we can compute Q.

Call P_i and Q_i the i-th row of P and Q. Then, P_1 and Q_1 must be non-zero, since zero rows must be below non-zero rows. Say that the pivot entry of P_1 is at column j, call this P_{1j}, and the pivot entry of Q_1 is at column k, call this Q_{1k}. If $j = k - 1$, then it would no longer be possible to perform elementary row operations on P to get to Q due to the 0 at P_{1k} or Q_{1j}, implying that P is *not* row-equivalent to Q. In general, $j \not< k$, and similarly, $j \not> k$ for the same reason. Thus, $j = k$; the pivot columns of P_1 and Q_1 must be equal.

We can apply this same logic to the rest of the rows. That is, the pivot column of P_i must be equal to that of Q_i. Earlier, we established that by performing elementary row operations, we can get from P to Q. And since every pivot entry of P and Q are equal and in the same column, the i-th row of P is simply the i-th row of Q multiplied by the constant 1. Therefore, $P = Q$, and no two *different* matrices can represent the RREF of an arbitrary matrix. ■

2 Equal Reduced Row Echelon Forms

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