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Network Externalities, Competition, and Compatibility

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There are many products for which the utility that a user derives from consumption of the good increases with the number of other agents consuming the good. There are several possible sources of these positive consumption externalities.[1]

1) The consumption externalities may be generated through a direct physical effect of the number of purchasers on the quality of the product. The utility that a consumer derives from purchasing a telephone, for example, clearly depends on the number of other households or businesses that have joined the telephone network. These network externalities are present for other communications technologies as well, including Telex, data networks, and over-the-phone facsimilie equipment.

2) There may be indirect effects that give rise to consumption externalities. For example, an agent purchasing a personal computer will be concerned with the number of other agents purchasing similar hardware because the amount and variety of software that will be supplied for use with a given computer will be an increasing function of the number of hardware units that have been sold. This hardware-software paradigm also applies to video games, video players and recorders, and phonograph equipment.

3) Positive consumption externalities arise for a durable good when the quality and availability of postpurchase service for the good depend on the experience and size of the service network, which may in turn vary with the number of units of the good that have been sold. In the automobile market, for example, foreign manufacturers' sales initially were retarded by consumers' awareness of the less experienced and thinner service networks that existed for new or less popular brands.

In all of these cases, the utility that a given user derives from the good depends upon the number of other users who are in the same "network" as is he or she. The scope of the network that gives rise to the consumption externalities will vary across markets. In some cases, such as the automobile example, the sales of only one firm will constitute the relevant network. In other cases, the relevant network will comprise the outputs of all firms producing the good. For example, the number of stereo phonographs of any one brand is not a determinant of the supply of records that a consumer can play on his or her stereo. In still other markets, the network may comprise the products of a coalition of firms that is a subset of the entire market, as in the case of computers, where some groups of manufacturers adopt common operating systems.

The central feature of the market that determines the scope of the relevant network is whether the products of different firms may be used together. For communications networks, the question is one of whether consumers using one firm's facilities can contact consumers who subscribe to the services of other firms. If two firms' systems are interlinked, or compatible, then the aggregate number of subscribers to the two systems constitutes the appropriate network. If the systems are incompatible, such as Telex and cable, then the size of an individual

[1] In addition to the sources of consumption externalities mentioned in this paper, there are a number of more subtle ones. These include: (*i*) the fact that product information is more easily available for more popular brands; (*ii*) the role of market share as a signal of product quality; and (*iii*) purely psychological, bandwagon effects.

system is the proper network measure for users of that system.

Similarly, for hardware-software markets, the issue is whether software produced for use on one brand of hardware may be run on another brand of hardware. If two brands of hardware can use the same software, then the hardware brands are said to be compatible. The relevant network is the set of users who have compatible brands of hardware. In the personal computer market, the CPM operating system has been designed to allow several brands of computers to use common programs. In the case of quadraphonic audio discs, on the other hand, the records made for one type of player cannot be used on a player that uses a different quadraphonic technology. Here, unlike the case of stereos, the relevant network for a given brand of equipment comprises the set of brands that use the same technology, not the entire market.

For the durables example, the relevant network is the set of brands that require the same parts of servicing skills. If a particular model of automobile has customized parts or requires specialized repair skills, then an owner of the model will find a thinner, and probably more expensive, service system. This smaller network will reduce his or her initial willingness to pay for the model.

Despite the significance of markets in which network externalities are present, relatively little work has been done in this area. The analysis done so far has been set in a monopoly context and has focussed on communications networks. Shmuel Oren and Stephen Smith (1981) is a recent reference to this literature. As the examples above make clear, it is important to extend the study of network externalities to an oligopolistic setting.

In this paper, we develop a simple, static model of oligopoly to analyze markets in which consumption externalities are present. We examine two basic sets of issues. First, we study the effect of consumption externalities on competition and the form of the market equilibrium. When network externalities exist, consumers must form expectations regarding the size of competing networks.

We use a notion of rational, or fulfilled expectations, equilibrium. Our basic findings are that consumption externalities give rise to demand-side economies of scale, which will vary with consumer expectations. As a result, multiple fulfilled expectations equilibria may exist for a given set of cost and utility functions. For some sets of expectations only one firm will produce output, while for other sets of expectations there will be several firms in the market. These equilibria verify the following intuition: if consumers expect a seller to be dominant, then consumers will be willing to pay more for the firm's product, and it will, in fact, be dominant.

The second area that we explore is the compatibility decision. Typically, firms can choose whether to manufacture compatible products, and thus can determine whether individual firm or aggregate market sales are the relevant ones in the evaluation of the consumption externalities. An important question, therefore, is whether firms will have proper incentives to produce compatible goods or services.

Gerald Brock (1975) and Robert Kurdle (1975) have done interesting case studies of compatibility decisions in the U.S. mainframe computer and farm machinery industries, respectively. Neither, however, develops a model of equilibrium in which to analyze firms' compatibility incentives. Using our model, we compare the private and social incentives to produce compatible products. We find that firms with good reputations or large existing networks will tend to be against compatibility, even when welfare is increased by the move to compatibility. In contrast, firms with small networks or weak reputations will tend to favor product compatibility, even in some cases where the social costs of compatibility outweigh the benefits. Viewing firms as a collective decision maker, we find that in our model the firms' joint incentives for product compatibility are lower than the social incentives.

The paper is organized as follows. In Section I, we present the model and define the equilibrium concept. The set of market equilibria is characterized in Section II. We ex-

amine the private and social incentives for compatibility in Section III, primarily under the assumptions that all of the costs of compatibility are fixed. There is a brief summary of our results and a discussion of the relevance of these results for public policy in Section IV. We outline an alternative approach to the formation of consumer expectations in the Appendix. It is shown that the equilibria are qualitatively similar if the firms are able to commit to given network sizes before consumers make their purchase decisions.

I. A Formal Model of Network Competition

A. Consumers

We look at a partial equilibrium oligopoly model in which there are no income effects and consumers act to maximize their surplus. A consumer buys at most one brand and purchases either one or no unit of any given brand.

The surplus that a consumer derives from buying a unit of the good depends on the number of other agents who join the network associated with that product. When the good is durable, an individual's consumption benefits will depend on the future size of the relevant network. Consumers will base their purchase decisions on *expected* network sizes. To capture this important feature of many markets with network externalities in our static, one-period model, we assume that consumers must make their purchase decisions before the actual network sizes are known. The timing is as follows. First, consumers form expectations about the size of the network with which each firm is associated. Second, the firms play an output game, taking consumers expectations as given. This game generates a set of prices. Consumers then make their purchase decisions by comparing their reservation prices (based on expected network sizes) with the prices set by the n firms, $i = 1, \ldots, n$.

We do not explicitly model the process through which consumers' expectations are formed. We will, however, impose the requirement that in equilibrium consumers' expectations are fulfilled. Let x_i^e denote the

number of customers that a consumer expects firm i to have, and let y_i^e be the consumer's prediction of the size of the network with which firm i is associated. All consumers are assumed to have identical expectations of network sizes. When the brands are incompatible, each makes up its own network so $y_i^e = x_i^e$. When m firms' products are compatible, say brands 1 through m, then there is a single network for these brands and

$$y_i^e = \sum_{j=1}^{m} x_j^e \qquad \text{for } i = 1, 2, \ldots, m.$$

Networks are assumed to be homogeneous in the sense that if two networks are of equal size, then all consumers view the two networks as perfect substitutes.

Consumers are assumed to be heterogeneous in their basic willingness to pay for the product, but homogeneous in their valuations of the network externality. Specifically, a consumer of type r has a willingness to pay $r + v(y^e)$ for a product with expected network size y^e. Without further loss of generality we can normalize r and $v(0)$ so that $v(0) = 0$. We can interpret r as the consumer's basic willingness to pay for the good and $v(y)$ as the value he or she attaches to the consumption externality when the number of subscribers is y. The externality function is taken to be twice continuously differentiable, with $v' > 0$, $v'' < 0$, and $\lim v'(y) = 0$ as $y \to \infty$. The basic willingness to pay for the good, r, varies across consumers and is assumed to be uniformly distributed between minus infinity and A with density one.[2] We assume that A is positive.

Each agent purchases the brand that maximizes his or her surplus. Letting p_i denote the price charged for brand i, a consumer of type r chooses the brand for which

$$(1) \qquad r + v(y_i^e) - p_i$$

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[2] The uniform density assumption amounts to assuming a linear demand curve for the product. We assume that the support of r has no finite lower limit in order to avoid having to consider corner solutions, where all consumers enter the market.

is largest. If (1) is negative for all i, then a type r consumer stays out of the market and purchases none of the brands.

B. Firms

Given the homogeneity of the products, two firms i and j will both have positive sales only if

$$(2) \qquad p_i - v(y_i^e) = p_j - v(y_j^e),$$

where $p_i - v(y_i^e)$ is the expected hedonic price of brand i, that is, the price adjusted for the network size. Equation (2) says that the hedonic prices must be equal when multiple firms have positive sales. Let ϕ denote the common value of the hedonic prices given in equation (2).

For a given value of ϕ, only those consumers for whom $r \geq \phi$ enter the market. Given the uniform distribution of r, there are $A - \phi$ such consumers. Thus, if the firms sell a total of $z \equiv \sum_{i=1}^{n} x_i$ units, then prices must be set such that $A - \phi = z$, or

$$(3) \qquad A + v(y_i^e) - p_i = z$$

$$\text{for all } i \text{ such that } x_i > 0.$$

From equation (3) we see that firm i receives a price of

$$(4) \qquad p_i = A + v(y_i^e) - z.$$

The price that firm i receives depends on the expected size of its network, y_i^e, and on the total unit sales of the n firms, z.

There are two types of costs that must be modeled. First, there are costs of production. We assume that production costs are the same for all firms and that these costs take the form of a fixed cost, G, plus a constant per unit variable cost, g. That is, the cost to firm i of producing x units of output is $G + gx$. As long as the fixed costs are smaller than the firm's equilibrium revenues minus variable costs, the fixed costs have no effect on the equilibrium. To simplify the exposition, we assume that the fixed costs of production are equal to zero. Without loss of generality, we also take the variable costs of

production to be equal to zero. Assuming that g is equal to zero is equivalent to redefining r to be the excess of the consumer's basic willingness to pay for the good over the constant per unit cost.[3]

There is a second type of cost that we must consider, the cost of the achieving compatibility. For most of the analysis we will assume that the costs of compatibility are fixed costs, that is, are independent of scale. This amounts to assuming that compatible products have the same marginal production costs as incompatible products. (We discuss the consequences of relaxing this assumption of Section III, Part D). The fixed cost of compatibility that we analyze could include costs of developing and designing a compatible product, the costs of negotiating to select a standard, and the costs of introducing a new, compatible product. Let F_i denote the fixed costs of compatibility incurred by firm i. Note that F_i need not be the same for all firms.

If all of the networks are incompatible, then $y_i^e = x_i^e$, and firm i earns profits equal to

$$(5) \qquad \pi_i = x_i(A - z + v(x_i^e))$$

when it has sales of x_i and total output is z.

When all n products are compatible, $y_i^e = \sum_{j=1}^{n} x_j^e \equiv z^e$, for all i. Therefore, when total output is z and firm i has sales of x_i, the firm's gross profits are

$$(6) \qquad \pi_i = x_i(A - z + v(z^e)),$$

from which we must subtract F_i to get profits net of the fixed costs of compatibility.

C. Fulfilled Expectations Equilibrium

Our equilibrium concept is that of fulfilled expectations Cournot equilibrium, where each firm chooses its output level under the assumptions that: (a) consumers' expecta-

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[3] It is for this reason that negative values of r make sense; the redefined variable r measures the excess of a consumer's basic willingness to pay over the marginal production costs of an additional unit.

tions about the sizes of the networks, $(y_1^e, y_2^e, \ldots, y_n^e)$, are given; and (b) the actual output level of the other firms, $\sum_{j \neq i} x_j \equiv x_{-i}$, is fixed.

Assumption (b) is the standard Cournot assumption. For any fixed set of consumer expectations, the problem is equivalent to the standard linear demand Cournot model with constant marginal costs. Assumption (a) is relaxed in the Appendix, which considers the case in which the y_i^es are formed *after* the firms have selected their output levels. Differentiating equation (5) and rearranging terms, the first-order conditions $d\pi_i/dx_i = 0$ imply that the equilibrium sales levels $(x_1^*, x_2^*, \ldots, x_n^*)$ must satisfy

$$(7) \qquad x_i^* = A + v(y_i^e) - \sum_{j=1}^{n} x_j^*$$

$$\text{for } i = 1, 2, \ldots, n.$$

Note that the right-hand side of equation (7) equals p_i.

For any given set of expectations, we can solve equation (7) simultaneously for the x_i^*s to obtain the unique Cournot equilibrium that corresponds to that set of expectations:

$$(8) \quad x_i^* = \left\{ A + nv(y_i^e) - \sum_{j \neq i} v(y_j^e) \right\}$$

$$\Big/ (n+1) \quad \text{for } i = 1, 2, \ldots, n.$$

The outcome is just the standard linear demand Cournot equilibrium where the differences in $v(y_i^e)$ are analogous to production cost differences. Equation (8) defines a function that maps expectations $(y_1^e, y_2^e, \ldots, y_n^e)$ into Cournot equilibrium network sizes $(y_1^*, y_2^*, \ldots, y_n^*)$ for a given pattern of compatibility. Let $\Gamma(y^e)$ denote this function.

In the absence of rationality constraints on consumer expectations, there is a Cournot equilibrium for any set of expectations. But for most sets of expectations, the expectations will not be fulfilled in the corresponding Cournot equilibrium; the actual network

sizes are not equal to the expected ones. Although it is possible that (in the short run, at least) consumers could be mistaken about network sizes, it is useful to limit the set of possible equilibria by imposing the restriction that expected sales be equal to actual sales in equilibrium. Formally, our equilibrium notion is that of Fulfilled Expectations Cournot Equilibrium (*FECE*), where a *FECE* is an n-vector of network sizes $y^* = (y_1^*, y_2^*, \ldots, y_n^*)$, such that $y^* = \Gamma(y^*)$. If consumers expect the network sizes to be y^*, then in the corresponding Cournot equilibrium the network sizes will indeed equal y^*; consumers' expectations will be fulfilled.

D. *Welfare Formulae*

Given the cost and demand assumptions that we have made, profits and welfare can be written as functions of the firms' individual levels of output. By equation (7), in equilibrium, firm i's output level is equal to the price that firm receives. Thus, the ith firm's profits in equilibrium are $\pi_i = (x_i^*)^2$. We will denote aggregate profits by $\pi \equiv \pi_1 + \ldots + \pi_n$.

The surplus that a consumer derives from joining a network depends on the *actual* size of the network; in equilibrium, the actual size will equal that network's *expected* size. By equations (1) and (3), when market output is z, a type r consumer expects to derive surplus of $r + z - A$ from joining a network. Only those consumers for whom r is greater than $A - z$ join a network; the other consumers stay out of the market and derive no surplus. Integrating over all consumers who do enter the market, we obtain consumers' expected surplus

$$(9) \quad S(z) = \int_{A-z}^{A} (\rho + z - A) \, d\rho = z^2/2.$$

In any fulfilled expectations equilibrium, expected and actual consumers' surplus will be equal, and we can use equation (9) when discussing actual consumers' surplus.

We take the sum of producers' and consumers' surplus as our social welfare measure. Hence, in any fulfilled expectations

Cournot equilibrium, welfare (gross of the fixed costs of compatibility) is given by

$$(10) \quad W(x_1,\ldots,x_n) = \pi(x_1,\ldots,x_n)$$

$$+ S(x_1 + \ldots + x_n) = \sum_{i=1}^{n} x_i^2 + z^2/2.$$

II. The Characterization of Equilibria

In this section, we examine the structure of fulfilled expectations equilibria for compatible and incompatible products, respectively.

A. Complete Compatibility

Suppose that any two products are compatible with one another. Then there is a single network of expected size $z^e = \sum_{i=1}^{n} x_i^e$, and for all i, $y_i^e = z^e$. Equation (8) becomes

$$(11) \qquad x_i^* = (A + v(z^e))/(n+1)$$

$$\text{for } i = 1, 2, \ldots, n.$$

If we impose the fulfilled expectations requirement that $z^e = x_1^* + \ldots + x_n^*$ and sum equation (11) over all i, we obtain

$$(12) \quad z^c = (n/(n+1))(A + v(z^c)),$$

where z^c denotes the fulfilled expectations equilibrium value of total output when the products are compatible. Under our assumptions on $v(\cdot)$, equation (12) has a unique solution, as is clear from Figure 1. This unique compatible-products equilibrium is symmetric: $x_i^c = z_i^c/n$ for all i. We have shown:

PROPOSITION 1: *When all products are mutually compatible, there is a unique FECE. It is symmetric, and the aggregate level of output is given implicitly by equation* (12).

As the number of firms becomes increasingly large, the compatibility equilibrium converges to the perfectly competitive equilibrium; z^c approaches $A + v(z^c)$, and the hedonic price, $A + v(z^c) - z^c$, approaches the marginal cost level of zero.
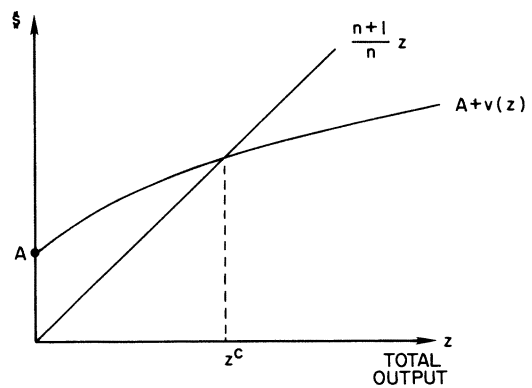

FIGURE 1. EQUILIBRIUM WITH COMPLETE COMPATIBILITY

B. Complete Incompatibility

Now we consider the case where any two brands are incompatible with one another so that $y_i^e = x_i^e$. In equilibrium, each firm i is optimizing given the actions of the other firms, x_j, $j \neq i$, and consumers' expectations, x_i^e. Using equation (7) in conjunction with the fulfilled expectations condition $x_i = x_i^e$, we have $x_i = A + v(x_i) - z$, or

$$(13) \qquad \sum_{j \neq i} x_j = A + v(x_i) - 2x_i$$

$$\text{for } i = 1, 2, \ldots, n.$$

For a given value of x_{-i}, equation (13) can be solved for x_i. The graph of equation (13) is called firm i's *equilibrium reaction correspondence*. One possible shape of this correspondence is illustrated in Figure 2.[4]

The equilibrium reaction correspondence should not be confused with a standard reaction function. The latter merely states firm i's best response to the other firms, given consumer expectations. There will be a different reaction function for each set of expectations. The equilibrium reaction corre-

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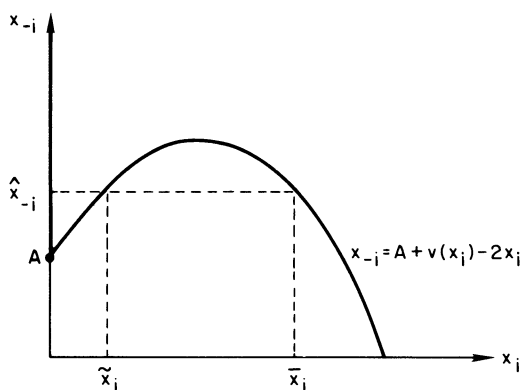[4] It is also possible that firm i's reaction schedule is strictly downward sloping. This will occur if and only if $v'(0) < 2$.

FIGURE 2. FIRM i'S EQUILIBRIUM REACTION
CORRESPONDENCE

FIGURE 3. UNIQUE SYMMETRIC EQUILIBRIUM
WITH COMPLETE INCOMPATIBILITY

spondence gives the set of points such that if the other firms played x_{-i} and consumers expected brand i to have a network size of x_i, then x_i would in fact be firm i's best response. Suppose the other firms set their output at $\hat{x}_{-i}$ in Figure 2. Then firm i's best response to $\hat{x}_{-i}$ would fulfill consumer expectations if these expectations were either $\tilde{x}_i$ or $\bar{x}_i$. Note that firm i treats consumer expectations as exogenous, and thus the firm does not choose between $\tilde{x}_i$ and $\bar{x}_i$.

In Figure 2, firm i's equilibrium reaction correspondence is drawn to include the x_{-i} axis for $x_{-i} > A$. This part of firm i's reaction schedule is not derived from equation (13), which only applies when $x_i > 0$. Instead it is derived from the corner condition, $d\pi_i/dx_i < 0$ at $x_i = 0$. When $x_i^e = x_i = 0$, $d\pi_i/dx_i = A - x_{-i}$, so it is optimal for firm i to set $x_i = 0$ if $x_i^e = 0$ and $x_{-i} > A$.

Having derived the firms' fulfilled expectations reaction schedules, we turn now to the characterization of equilibria. There are three types of equilibria that are possible when the networks of competing firms are incompatible: (i) symmetric oligopoly with n active firms; (ii) symmetric oligopoly with $k < n$ active firms, which we call natural oligopoly; and (iii) asymmetric oligopoly.

1. *Symmetric Oligopoly.*

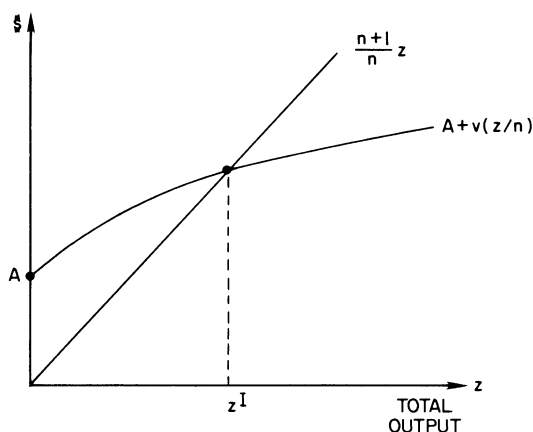PROPOSITION 2: *When each brand is incompatible with all ($n-1$) of the other brands,*

there exists a unique symmetric equilibrium in which $x_i = z^I/n$ and aggregate sales, z^I, are given implicitly by

$$(14) \quad ((n+1)/n)z^I = A + v(z^I/n).$$

PROOF:
Taking $x_i = z/n$ and adding up equation (13) for $i = 1, \ldots, n$, gives $(n-1)z = nA + nv(z/n) - 2z$. Rearranging yields equation (14), which has a unique solution, as Figure 3 illustrates.

2. *Natural Oligopoly (Not all Firms Active).* While a unique symmetric equilibrium always exists, there are asymmetric equilibria that exist for certain parameter values. Given the symmetry in the equilibrium response correspondences, such asymmetric equilibria always come in sets (where the elements of the set differ from each other only by the transposition of the firms' indices). One such type of equilibrium entails some firms exiting the market (i.e., producing no output) and the other firms behaving as oligopolists with a diminished number of competitors.

PROPOSITION 3: *A symmetric equilibrium with k active firms exists if and only if $v(A/k) \geq A/k$.*
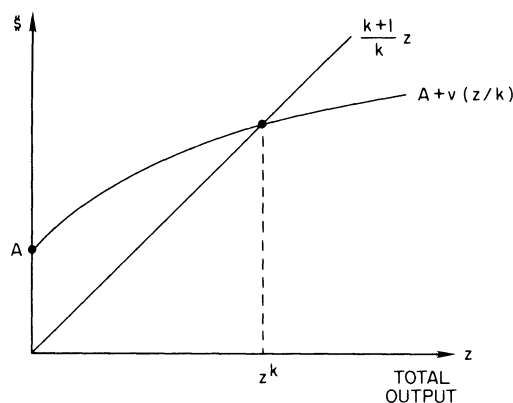
FIGURE 4. NATURAL OLIGOPOLY

FIGURE 5. NATURAL MONOPOLY EQUILIBRIA

PROOF:

Suppose that k firms each produce $x_i = z/k$ units of output, and the remaining $n - k$ firms produce no output. Adding up equation (13) for the k active firms, we obtain $(k-1)z = kA + kv(z/k) - 2z$, or

$$(15) \quad ((k+1)/k)z = A + v(z/k).$$

As Figure 4 illustrates, there will be a unique solution to equation (15). Let z^k denote this solution.

We must check that the remaining $n - k$ firms do not have incentives to produce positive output, that is, that $A + v(0) - z^k = A - z^k < 0$. Again from Figure 4, it is clear that $z^k \geq A$ iff $A + v(A/k) \geq ((k+1)/k)A$, or $v(A/k) \geq A/k$.

COROLLARY 3.1: *For any $k \leq n - 1$, if a k-active-firm symmetric equilibrium exists, then a $(k + 1)$-active-firm symmetric equilibrium exists.*

COROLLARY 3.2: *For any $k \leq n - 1$, if a k-active-firm symmetric equilibrium exists, then $z^k < z^{k+1}$.*

Both corollaries follow from the concavity of $v(\cdot)$. Note that equilibrium with $k = 1$ (the monopoly outcome) or some other low value of k is more likely to obtain when consumers' basic willingness to pay for the good is low (so that A is low) or when the
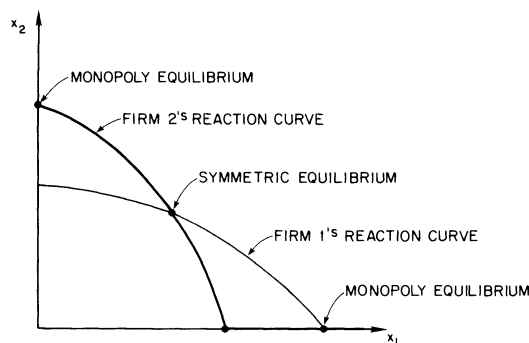
network effects are strong (so that $v(A)$ is large for a given A).

Proposition 3 shows that the network externalities are similar to fixed costs in that they can lead to a limited number of active producers. The analogy between fixed costs and network externalities is not complete, however. This point is demonstrated by Corollary 3.1 and the fact that for the given set of demand conditions, an n-active-firm equilibrium exists for arbitrarily large n. In the case of fixed costs, one cannot squeeze an arbitrarily large number of active producers into the industry.

Figure 5 shows the reaction curves for a case in which a natural monopoly equilibrium exists. It is interesting to note that the monopolist's profits, may be *lower* than the profits of a duopolist in the 2-active-firms symmetric equilibrium. In other words, a monopoly may benefit from entry. This unusual result follows from the fulfilled expectations condition: a monopolist will exploit his position with high prices and consumers know this. Thus, consumers expect a smaller network and are willing to pay less for the good. If the monopolist could commit himself to higher sales he would be better off, but this commitment is not credible so long as he is the sole producer.[5]

3. *Asymmetric Oligopoly.* The third possible equilibrium configuration is one in which

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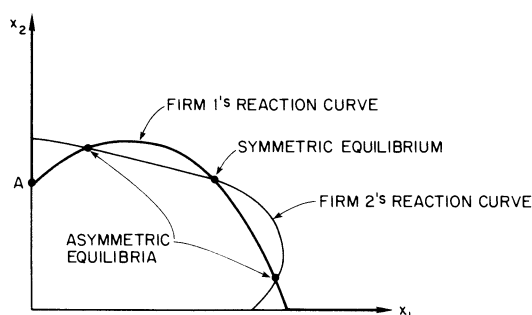[5] We consider the case where commitment is feasible in the Appendix.

FIGURE 6. ASYMMETRIC DUOPOLY

$k \geq 2$ firms produce positive but unequal levels of output. We have constructed examples of such equilibria, although they are difficult to characterize in general. These asymmetric equilibria verify the intuition that a firm may be successful and enjoy a large market share simply because it is expected to by consumers.

One example of an asymmetric duopoly equilibrium is shown qualitatively in Figure 6. Despite a linear demand curve and a concave network valuation function $v(y)$, a variety of possibilities may arise. Figure 6 shows a situation in which asymmetric equilibria exist as well as symmetric and natural monopoly equilibria. In other cases, the only stable equilibria are asymmetric ones.

C. Partial Compatibility

When there are more than two firms, the extent of product compatibility may fall in between complete incompatibility and industrywide compatibility. Assuming that the compatibility relation is symmetric and transitive, the pattern of compatibility can be characterized by the set of compatibility groups, G^j $j = 1, \ldots, J$, where all of the brands within a given group are mutually compatible with each other and are incompatible with any nonmember brands.[6] Thus,

[6] The G^j, $j = 1, \ldots, J$, form a partition of the set $\{1, 2, \ldots, n\}$.

if firm i is in group G^j,

$$y_i = \sum_{k \in G^j} x_k \equiv y^j.$$

For a firm i in group j, the first-order condition is $x_i = A - z + v(y^j)$. Thus, all firms in a given group will choose the same level of output, x^j. Let m^j denote the number of firms in compatibility group j. Then, in equilibrium, for all $x^j > 0$ we must have

$$(16) \qquad x^j = A - z + v(m^j x^j).$$

Equation (16) has the same qualitative properties as our earlier equilibrium conditions, and similar types of equilibria will arise. Here, we will not characterize these equilibria directly. In the next subsection, however, we will compare the equilibria that obtain under different degrees of compatibility.

D. The Output Effects of Compatibility Changes

In analyzing compatibility, it is important to understand the effects of an increase in compatibility on the equilibrium levels of output. What happens to output levels if two compatibility groups "merge" to form a new group where all of the brands in the post-merger group are compatible with one another?

PROPOSITION 4: *The level of total output is greater under industrywide compatibility than in any equilibrium with less than complete compatibility.*

PROOF:
For all firms with positive levels of output, $x_i = A + v(y_i) - z$. Adding up over all firms and rearranging gives us $(n+1)z = nA + \Sigma v(y_i)$, as illustrated in Figure 7. Under complete compatibility, $y_i = z$ for all firms. Absent complete compatibility, $y_i < z$ for at least one firm. Thus, the curve $nA + nv(z)$ lies above $nA + \Sigma v(y_i)$ where the y_i's are
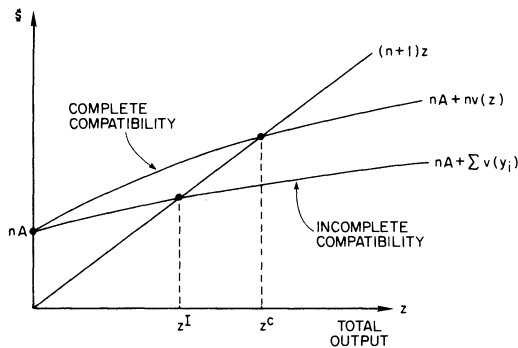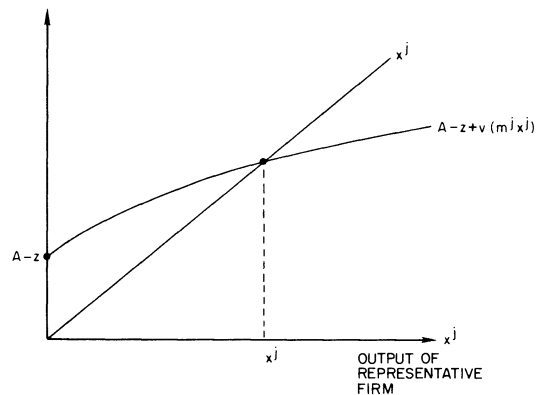
FIGURE 7. COMPLETE VS. INCOMPLETE COMPATIBILITY

FIGURE 8. EQUILIBRIUM OUTPUT OF A FIRM
IN COALITION j

determined under incomplete compatibility. Referring to Figure 7, we see that the equilibrium level of z is greater under industry-wide compatibility.

When the move to increased compatibility does not result in *complete* compatibility, total output need not rise. The following proposition, however, states a sufficient condition for industry output to increase.

PROPOSITION 5: *Suppose that two groups of firms make their products mutually compatible. If premerger total output is less than A, then in any postmerger equilibrium: (a) the average output of the firms in the merging coalitions will rise; (b) the output of any firm not in the merging coalitions will fall; and (c) industry output will rise.*

PROOF:

Let $\hat{x}^j$ denote the premerger output level of a firm in coalition j and $\hat{z}$ denote premerger total output. By equation (16), $\hat{x}^j = A - \hat{z} + v(m^j\hat{x}^j)$. Figure 8 illustrates this condition, where we have made use of the fact that industry output is less than A.

Label the compatibility groups that merge as 1 and 2. Let $\tilde{x}^j$ and $\tilde{z}$ denote the *post*-merger output levels analogous to the $\hat{x}^j$ and $\hat{z}$. If total output falls, then for $j \geq 3$, $A - \tilde{z} + v(m^j x^j)$ will lie everywhere above $A - \hat{z} + v(m^j x^j)$, and from Figure 8 we see that x^j will increase (i.e., $\tilde{x}^j > \hat{x}^j$). By similar argu-

ments, $\tilde{z} = \hat{z}$ implies $\tilde{x}^j = \hat{x}^j$ for $j \geq 3$ and $\tilde{z} > \hat{z}$ implies $\tilde{x}^j < \hat{x}^j$ for $j \geq 3$.

Now, consider the firms in the merged coalitions. For these firms (i.e., $j = 1, 2$), $A - z + v(m^j x^j) < A - z + v(m^1 x^1 + m^2 x^2)$ as long as x^1 and x^2 are positive. The effect of compatibility is to shift up the $A - z + v$ curve when viewed as a function of x^j (as in Figure 8). The effect of z on this curve is the same as for $j \geq 3$. Therefore if $\tilde{z} \leq \hat{z}$, then $\tilde{x}^j > \hat{x}^j$ for $j = 1, 2$.

Suppose that $\tilde{z} \leq \hat{z}$. Then $\tilde{x}^j \geq \hat{x}^j$ for all j, with strict inequality for $j = 1, 2$. But $z = \sum m^j x^j$, so we have a contradiction. Therefore, industry output rises due to the increased compatibility; $\tilde{z} > \hat{z}$. We already have shown that an increase in industry output implies that firms not in the merging coalitions produce less; $\tilde{z} > \hat{z}$ implies $\tilde{x}^j < \hat{x}^j$ for $j \geq 3$. Thus, it must be the case that the firms in the merging coalitions produce more: $m^1\tilde{x}^1 + m^2\tilde{x}^2 > m^1\hat{x}^1 + m^2\hat{x}^2$.

Note that Proposition 5 does *not* state that the new level of per firm output for the enlarged coalition will be larger than both $\hat{x}^1$ and $\hat{x}^2$; it states only that the new level of output will be larger than their mean. Note also that by Proposition 4, if total output under complete compatibility, z^c, is less than A, then $z < A$ in *any* equilibrium. From equation (12), $z^c < A$ if $v(A) < A/n$. So,

$v(A) < A/n$ is sufficient for $\hat{z} < A$, the hypothesis of Proposition 5.

When industry output in the premerger equilibrium equals or exceeds A, an increase in compatibility may be accompanied by a reduction in both industry output and the average level of output of the firms in the merging coalitions. To see this point, consider the following example. There are n firms, $v(A) \geq A$, and initially there is complete incompatibility. By Proposition 2, there exists a unique symmetric equilibrium in which all n firms are active. By Proposition 3, there exist n natural monopoly equilibria, each of which entails the natural monopolist producing A or more units of output. The natural monopolist's output level is less than that of the industry in the n-active-firm symmetric equilibrium (Corollary 3.2).

Suppose that the industry initially is in the n-active-firm symmetric equilibrium, and that some, but not all, of the firms form a compatibility coalition. The natural monopoly equilibria where a nonmerging firm is the natural monopolist will remain equilibria after the merger. Thus, while the premerger equilibrium entailed all firms active, the postmerger equilibrium could be one in which all but one firm shuts down. As we have just stated, industry output falls in this case. Moreover, the firms who formed the compatibility coalition are among those who were active in the premerger equilibrium, but are inactive in the postmerger equilibrium.

III. The Private and Social Incentives for Network Compatibility

To this point, we have treated the compatibility of the products as an exogenous characteristic of the market. In most markets where network externalities are important, the compatibility of the products will be the result of explicit decisions by the firms. When the network externalities are large, the choice of whether to make the products compatible will be one of the most important dimensions of market performance.

There are many cases in which firms will disagree on the desirability of making their products compatible; the move to compatibility may increase the profits of some firms while lowering the profits of others. Thus, we must be careful to specify the mechanism by which compatibility is achieved and whether side payments among firms are feasible.

It is useful to think of there being two basic technologies by which compatibility can be achieved. First, compatibility may arise through the *joint adoption of a product standard*, where a given set of firms must act together to make their products compatible with one another. The CPM operating system for personal computers and the broadcast television standards are two examples. Second, compatibility can be achieved through the *construction of an adapter*, where a single firm can act unilaterally to make its product compatible with those of another firm or group of firms. In the 1960's, for example, Honeywell developed a program that would allow its mainframe computers to run programs initially written for IBM hardware.[7] At present, there is intense competition, both in the market and in the courts, as the video game manufacturers such as Coleco develop adapters that allow their hardware to run the video game programs of competing firms. In other cases there need not be a physical adapter; one firm can adopt another network's specifications for its product design.

When the firms cannot make side payments among one another and when the compatibility mechanism is a product standard, the products of a given set of firms will be made compatible if and only if *all* of these firms would earn greater profits as a result. In contrast, when the compatibility mechanism is an adapter, and side payments are infeasible, the products of two firms will be made compatible if *either* firm would find the move to be profitable.

When side payments are feasible, the firms will make their products compatible if and only if the change in the profits of firms within the set that can make side payments to one another exceeds the joint costs of compatibility.

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[7]See Brock, p. 78.

We will examine several cases that vary in terms of the compatibility technology and feasibility of side payments. In analyzing the private incentives for compatibility, we will look at each firm's change in profits, $\Delta\pi_i = \pi_i^c - \pi_i^I$, and the change in their joint profits $\Delta\pi = \sum_{i=1}^n \Delta\pi_i$, and we compare these with the costs of compatibility. For most of this section we will take these costs to be purely fixed costs. The social incentives for compatibility are given by $\Delta W = W^c - W^I$. We denote the change in consumers' surplus by $\Delta S = S^c - S^I$.

A. The Incentive when Side Payments among All Firms are Feasible

If side payments among all firms are feasible, then a set of side payments could be constructed so that all firms' profits would be increased individually if and only if compatibility would raise joint profits; the private incentives are given by the change in industrywide profits under either compatibility technology.[8] The change in social welfare, $\Delta W = \Delta\pi + \Delta S$, so the social and private incentives will diverge when the move to compatibility changes the level of consumers' surplus. Since $S(z) = z^2/2$, consumers' surplus will go up if and only if output does. By Proposition 4, we know that output and, hence, consumers' surplus will rise with the move to full compatibility. Thus, if $\Delta\pi > 0$, then $\Delta W = \Delta\pi + \Delta S > 0$.

PROPOSITION 6: *When compatibility costs are purely fixed costs, any move to complete compatibility that raises industry profits is socially beneficial.*

Proposition 6 states that firms' incentives for compatibility are not socially excessive. In fact, they may be inadequate. Since $\Delta S > 0$, $\Delta W > \Delta\pi$. If the industrywide costs of

compatibility, F, satisfy $\Delta\pi < F < \Delta W$, then the private firms will fail to adopt a socially desirable system of compatibility. The reason is that the firms cannot appropriate all the benefits of compatibility.[9]

PROPOSITION 7: *Even when arbitrary side payments among all firms are feasible, profit-maximizing firms may fail to achieve complete compatibility in cases where complete compatibility is socially optimal.*

A similar pair of results can be derived from Proposition 5 for markets in which the initial equilibrium entails $z < A$ and compatibility is increased (although not necessarily to industrywide compatibility). This result too relies on the fact that the move to increased compatibility raises total output under the stated conditions.

B. The Adoption of an Industry Standard

When the compatibility mechanism is the adoption of an industry standard, the firms must jointly decide to make their networks compatible. Any firm can veto the move to compatibility. Therefore, if no side payments are feasible, the standard will be adopted if and only if all firms joining the standard benefit from its creation. If we assume that firm i bears a cost F_i to adopt the standard and that side payments are infeasible, then adoption will occur if and only if $\Delta\pi_i > F_i$ for all adopters of the standard.

Suppose that side payments are feasible only when made among firms achieving compatibility. Such side payments might take the form of licensing fees or compensation for the expenses of making the products compatible, for example. In this case, a sufficient condition for achieving compatibility with a standards technology is that the joint profits of the firms achieving compatibility rise.

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[8] Instead of payments made in return for not achieving compatibility, we might see expenditures on legal proceedings aimed at blocking the move to compatibility.

[9] This result is analogous to the fact that a monopolist may be unable to earn a positive profit by providing a socially useful product if there are fixed costs and he or she cannot perfectly price discriminate.

It is clear that, when the compatibility technology is a standard, allowing cost sharing will raise the likelihood of the firms choosing compatibility. If each firm prefers adoption of the standard ($\Delta\pi_i > F_i$ for all i), then the firms in aggregate will ($\sum \Delta\pi_i > \sum F_i$), while the converse is not true. Thus, we can strengthen Proposition 6:

PROPOSITION 8: *The private standardization rule is more stringent when cost sharing is infeasible than when it is feasible. The set of cases in which the firms fail to adopt a socially beneficial standard is therefore larger. It remains true that any privately profitable industrywide standard is desirable.*

To see the effect of cost sharing, suppose that there are only two firms in the industry. The compatibility equilibrium is symmetric, so the firm with the smaller incentive to adopt a standard is the one with initially higher profits, that is, the initially larger producer, say firm 1. If the initial equilibrium is symmetric and $F_1 = F_2 = F$, then $\Delta\pi_1 - F = \Delta\pi_2 - F$ and the presence or absence of cost sharing or other side payments is irrelevant. If the initial equilibrium is asymmetric, however, the condition for standardization, $\Delta\pi_1 > F$ is *strictly* more stringent than the adoption condition with side payments, $\Delta\pi > 2F$. The problem is that the larger firm will lose market share to its smaller rival as a result of standardization. If it can unilaterally block standardization, it may do so, despite the fact that its rival and consumers would benefit. Permitting cost sharing and other side payments will help alleviate the problem of insufficient private adoption incentives for moves to complete compatibility.

When the compatibility increase is to less than complete compatibility, private incentives may be excessive, and such cost sharing may exacerbate the problem. There are two reasons why private incentives may be excessive. First, when the increase is to less-than-complete compatibility, total output and consumers' surplus may fall, so that $\Delta\pi > \Delta W$. Second, there will be some firms that are not members of the groups making their products compatible. As shown in the proof

of Proposition 5, these firms may produce less output and thus have lower profits in the new equilibrium. Absent side payments from these firms, the firms considering compatibility will not take the losses of other firms ($\sum_{j \neq i} \Delta\pi_j < 0$) into account. The social incentives, however, depend on the profits of all firms; $\Delta W = \Delta\pi_i + \sum_{j \neq i} \Delta\pi_j + \Delta S$.

PROPOSITION 9: *When the increase in compatibility leads to less-than-industrywide compatibility, the private incentives to standardize may be excessive.*

C. The Construction of an Adapter

In the adapter case, a firm unilaterally can act to make its product compatible with those of another network. In contrast with the adoption of an industry standard, if side payments to block compatibility are not feasible, then the adapter will be constructed as long as at least one firm earns increased profits from compatibility. When the compatibility mechanism is an adapter, the most reasonable assumption about the costs of compatibility is that the firm that constructs the adapter is the only one to bear the cost, F. Thus, firm i's private incentive to construct an adapter is $\Delta\pi_i - F$, while the social incentive is $\Delta\pi_i + \sum_{j \neq i} \Delta\pi_j + \Delta S - F$. The difference, $\Delta\pi_{-i} + \Delta S$, may in general be either positive or negative, implying that firm i's incentives to construct an adapter may be too low or too high from a social welfare point of view.

To see how the private and social incentives differ, suppose there are only two firms. Since the smaller firm in the initial equilibrium has the most to gain by the move to a symmetric equilibrium with compatibility, we need look only at the incentives of a firm with an initial market share of not more than 50 percent—firm 2, say. Since $\Delta\pi_2 > \Delta\pi_1$, the private decision will be to become compatible if and only if $\Delta\pi_2 > F$.[10] Compatibility is

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[10] We are ignoring the possibility that the firms play a waiting game in which each hopes (in vain) that the

socially optimal if and only if $\Delta W > F$. The divergence between the social and private incentives is given by $\Delta W - \Delta \pi_2 = \Delta \pi_1 + \Delta S$. By Proposition 4, we know that $\Delta S > 0$; the fact that consumers enjoy some of the benefits of compatibility tends to make the private incentives too low. On the other hand, it is not true in general that $\Delta \pi_1 > 0$, so we cannot conclude in general that the private adoption decision is too conservative.

One case in which the private adoption incentives are too low is when the initial equilibrium is symmetric. In that case, $\Delta \pi_1 = \Delta \pi_2$, so that if $\Delta \pi_i > 0$ for one firm, then the change in profits is positive for the other firm as well. As a result, $\Delta W > \Delta \pi_i$ whenever $\Delta \pi_i > 0$, and we have

PROPOSITION 10: *Suppose there are only two firms (or coalitions). If the incompatibility equilibrium is symmetric and there are no side payments, then the private incentives to construct an adapter are too low.*

When there are only two coalitions, permitting side payments to share the costs of the adapter would promote efficiency when the incompatibility equilibrium is symmetric. The adapting coalition confers benefits both on its rival, which free rides on compatibility, and on consumers. Side payments can help solve the free-riding problem, but (by Proposition 6) still leave the firms with insufficient incentives.

When the incompatibility equilibrium is asymmetric, it is possible that the initially larger firm (which does not build the adapter) loses so much market share when the smaller firm builds an adapter that its profits fall, that is, $\Delta \pi_1 < 0$. If this effect dominates the increase in consumers' surplus, that is, if $\Delta \pi_1 + \Delta S < 0$, then $\Delta W < \Delta \pi_2$ and firm 2's compatibility incentives are excessive. The increase in firm 2's market share at the expense of firm 1 is a private gain for which there is no corresponding social benefit.

PROPOSITION 11: *Suppose there are only two coalitions. A coalition with an incompatibility market share of less than 50 percent may have socially excessive incentives to construct an adapter.*

This result is most likely to obtain when the incompatibility equilibrium entails one coalition having a very small market share, as in the monopoly equilibrium.

When the means of achieving compatibility is the construction of an adapter, one firm may attempt to make the networks compatible even though the other firms would prefer that the networks remain incompatible. In such cases, the latter firms may be willing to make expenditures to block compatibility, perhaps through legal channels (currently, there are numerous court cases involving video game and personal computer compatibility). It is not possible to say in general whether such expenditures promote or diminish efficiency. In some cases, $\Delta \pi_1 < 0$, $\Delta \pi_2 > F$, and $\Delta W < F$ as noted in Proposition 11, and firm 1's ability to block the adapter will raise efficiency (if the blocking costs themselves are not too high). In other cases, $\Delta \pi_1 < 0$, $\Delta \pi_2 > F$, and $\Delta W > F$, so blocking the adapter would reduce social welfare, even if the blocking costs are zero. For a given cost and demand structure, one can determine whether ΔW exceeds F or not, but it is not possible to determine this relationship simply on the basis of $\Delta \pi_2$ exceeding F.

D. Extensions and Generalizations

We have made some restrictive assumptions in order to simplify our analysis of the incentives for network compatibility. It is useful to put the results that we have obtained into perspective by discussing the general nature of the divergence between the social and private incentives to achieve compatibility. Essentially, there are two sources of distortion. In making its compatibility decision, each firm ignores the effects that this move will have on: 1) the level of consumers' surplus; and 2) the profits of the other firms.

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other will build an adapter even though each would privately benefit from building the adapter itself.

Consider the first effect. When the move to compatibility raises consumers' surplus, the firms' incentives tend to be too low. Conversely, when the move to compatibility lowers consumers' surplus, the firms are biased towards compatibility. The change in consumers' surplus can itself be decomposed into two components: (a) the change due to the shift in the level of total output; and (b) the change that arises when the marginal consumer values the network externality differently than does the average consumer.

(a) The level of consumers' surplus is an increasing function of the level of total output. When the only costs of compatibility are fixed, we showed that the move to complete compatibility raises output and, hence, consumers' surplus. In this case, $\Delta\pi$ is less than ΔW. Once we relax the assumption that the move to compatibility has no impact on marginal costs, however, output may be lower with complete compatibility than without. The adoption of an industry standard or the construction of an adapter will necessitate the redesign of some or all of the products, which may lead to shifts in the variable costs of production (either upwards or downwards). Whereas the fixed costs of compatibility do not affect the equilibrium output level, changes in marginal costs do. In particular, when the increase in marginal costs is sufficiently large relative to the network effects, total output will be lower under complete compatibility than under incompatibility. In these cases, consumers' surplus will fall as a result of the move to complete compatibility and $\Delta\pi$ is greater than ΔW—the firms' joint incentive are excessive.[11]

(b) The level of consumers' surplus also depends on the relationship between the marginal and inframarginal consumers' valuations of the good. For a given level of output, the firms must set prices low enough to attract the marginal consumer. The lower is the marginal consumer's valuation relative to the average consumer's valuation, the larger will be consumers' surplus.

In our model, all consumers value the network externality equally, and all consumers' valuations of the good rise equally when compatibility is achieved. Thus, for a fixed level of output, the firms can raise prices by just this amount and consumers' surplus is unaffected. More generally, consumers may differ in their valuations of the network externality. If the network externality is stronger for the marginal consumer, then the move to compatibility will raise his or her willingness to pay for the good by more than that of the average consumer. For a given level of output, the firms will be able to raise the price by more than the increase in the average consumer's willingness to pay for the product. Consumers' surplus will fall, and the joint private incentives will tend to be greater than the social incentives. Of course, if the network externality is smaller for the marginal consumer, then the bias will run in the other direction.[12]

The change in consumers' surplus puts a wedge between the change in joint profits and the change in total welfare. When side payments are not feasible, the decision to achieve compatibility depends on the individual profit levels of the firms, and there is a second wedge. The change in profits may be positive for some firms and negative for others. As we have shown, the relationship between the changes in firms' profits will depend on two factors. First, it will depend on their relative changes in market shares

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[11] In fact, when compatibility raises producers' marginal costs, the firms may use the move to compatibility *solely* as a coordinating device to reduce their joint output (i.e., they may have incentives to make their products compatible even if there are no network externalities). This result is an example of the general theory of cost-based facilitating practices (see Steven Salop and David Scheffman, 1983; Katz and Harvey Rosen, 1985; and Jesus Seade, 1983): in an oligopoly, all firms may benefit from jointly increasing their costs because it induces them to reduce their collective output, which may raise their revenues by more than the increase in costs.

[12] It is straightforward, but messy, to extend our model in this direction. There is nothing about this particular problem that is intrinsic to networks. For discussion of the general inability of prices to convey information about the preferences of inframarginal consumers, see A. Michael Spence (1975) and the references cited therein.

and revenues in moving to compatibility. When one group of firms gains market share and profits at the expense of another, the first group will be biased towards compatibility and the latter will be biased away from it. Second, the relationship will depend on the relative costs of compatibility that the producers incur. When the costs of compatibility fall more heavily on some firms than on others, there is a free-rider problem that tends to bias the firms away from compatibility.[13]

IV. Conclusion

We have developed a simple model to capture what we believe is a very significant element of competition in several important markets. Despite this simplicity, some general points emerge. First, the structure of the equilibria in our model confirms the importance of consumers' expectations in markets where network externalities are present. We have subjected expectations to a rationality constraint, but the expectations formation process remains an important element of the market to model explicitly. Given the possibility of multiple equilibria when products are incompatible, firms' reputations may play a major role in determining which equilibrium actually obtains. For example, the existence of a strong reputation for being a market share leader may explain IBM's rapid rise to preeminence in the personal computer market. It would also be useful to consider firms' expenditures to influence consumers' expectations, such as precommitments to a given level of software.

Turning to the compatibility decisions, although we would not want to draw policy conclusions at such an early stage in the analysis, our model does point to areas in which public policy can have an important impact. We have shown that the private decision will depend crucially on the decision locus (whether firms can act unilaterally or if consensus is required) and on the feasibility of side payments. Public policy can influence both of these features. Patent and copyright laws are a significant determinant of whether the compatibility technology is better modeled as the joint adoption of an industry standard (when patents are strictly and broadly enforced), or as the unilateral construction of an adapter (when they are loosely enforced or narrowly applied). From Proposition 1, we know that if the costs of adapting are negligible, and there are no other entry barriers, the market will be perfectly competitive.[14]

Allowing firms to make side payments also may influence the likelihood of compatibility being adopted—upwards when the technology is an industry standard, and either upwards or downwards when the compatibility technology is an adapter. The discussion in Section III, Part D, also points out the need for policymakers to scrutinize the form of the side payments or royalties. Per unit charges may have the effect of implicit cartels by inducing outputs contractions. Finally, public policy can affect the costs of compatibility. Antitrust exemptions that allow industry groups to get together may lower the costs of achieving compatibility and thus make it more likely.

The model here is only a beginning. Explicitly dynamic, multiperiod models are needed to shed additional light on the behavior of markets in which network externalities are important. We hope that this paper will encourage further research in the area of network competition and public policy towards compatibility.

APPENDIX

In the text, we have examined a model where a firm's announcement of its planned level of output has no effect on consumer expectations. This model can be viewed as

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[13] Here we are assuming that when the compatibility technology is an adapter, the costs fall more heavily on the adapting firm.

[14] This outcome may not be the socially optimal one. Absent the ability to earn rents from its network size (through incompatibility), a firm may not have incentives to make the investments necessary to obtain the network. The issues are exactly analogous to those encountered in the analysis of optimal patent policy.

one in which the firms are unable to commit themselves, so that only the output levels of the *FECE* are credible announcements. In this Appendix, we consider the opposite polar case in which firms *can* commit to announced output levels before consumers make their purchase decisions. Firm i commits itself to output level x_i and consumers make their purchase decisions by looking at $v(y_i) - p_i$ across all brands. Firm i chooses its level of output taking the output level of the other firms as given. Thus, we have a standard Cournot equilibrium with demand-side economies of scale.

Given total output z and firm output x_i, firm i has profits of

$$x_i \{ A + v(y_i) - z \}.$$

Differentiating with respect to x_i, the first order conditions are

$$(A1) \quad A + v(y_i) - 2x_i - \sum_{j \neq i} x_j$$

$$+ x_i v'(y_i) = 0 \quad \text{for } i = 1, 2, \ldots, n.$$

The only difference between equation (A1) and our earlier first-order condition is the addition of the term $x_i v'(y_i)$. This term captures the fact that firm i can directly influence consumers' expectations regarding its network size. $x_i v'(y_i)$ is positive, so that firm i's reaction curve will shift upwards in comparison with the earlier equilibrium reaction correspondence.

The analysis is essentially unchanged from that in the text; we simply substitute $v(y_i) + x_i v'(y_i)$ for $v(y_i)$ in firm i's reaction function. Some additional assumptions on v are necessary to ensure that this substitute function is itself concave. In the case of complete incompatibility ($y_i = x_i$ for all i), and a constant elasticity network externality function

$v(x) = \beta x^\alpha$, we simply replace $v(x) = \beta x^\alpha$ by $v(x) + x v'(x) = \gamma x^\alpha$, where $\gamma = (1 + \alpha)\beta$.

Qualitatively, the "commitment" equilibria differ from those analyzed in the text in the following ways. 1) Each firm's reaction correspondence becomes a reaction function, since it can "choose" x_i^e as well as x_i (see the discussion of Figure 2 in Section II, Part B). 2) Equilibrium entails greater output, as each firm accounts for the $x_i v'(y_i)$ term, which shifts its reaction curve outwards. 3) No longer can a firm make greater profits in the $k + 1$-active-firm equilibrium than in the k-firm equilibrium.

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