Intertemporal Cost Allocation and Investment Decisions

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This paper considers the profit-maximization problem of a firm that must make sunk investments in long-lived assets to produce output. It is shown that if per-period accounting income is calculated using a simple and natural allocation rule for investment, called the relative replacement cost (RRC) rule, under a broad range of plausible circumstances, the firm can choose the fully optimal sequence of investments over time simply by choosing a level of investment each period in order to maximize the next period's accounting income. Furthermore, in a model in which shareholders delegate the investment decision to a better-informed manager, it is shown that if accounting income based on the RRC allocation rule is used as a performance measure for the manager, robust incentives are created for the manager to choose the profit-maximizing sequence of investments, regardless of the manager's own personal discount rate or other aspects of the manager's personal preferences.

I.   Introduction

In a variety of industries, firms must make sunk investments in long-lived assets to produce output. Calculation of profit-maximizing investment levels and evaluation of the firm's performance in such a situation are inherently complicated because of the need to consider implications for cash flows over multiple future periods. One technique that firms routinely use to create simplified single-period "snapshots" of their performance is to calculate per-period accounting income using accounting

measures of cost that allocate the costs of purchasing long-lived assets over the periods that the assets will be used. Firms use these single-period snapshots of performance both to directly guide their investment decisions and to evaluate the performance of managers who make investment decisions. Given their widespread use to both directly and indirectly guide investment decisions, it is perhaps surprising that there has been almost no formal analysis in the economics, finance, or accounting literature that attempts to investigate whether there is any basis for these accounting practices and, if so, how the choice of an allocation rule ought to be affected by factors such as the pattern of depreciation of the underlying asset, the firm's discount rate, the rate at which asset prices are changing over time, and the manager's own rate of time preference. This paper provides a theory that addresses these questions. It shows that, under a broad range of plausible circumstances, a natural and simple allocation rule, which will be called the relative replacement cost (RRC) rule, can be used both to simplify calculation of the optimal level of investment and to create robust incentives for managers to choose this level of investment when the decision is delegated to them.

In particular, two major results are proven. First, it is shown that, when accounting income is calculated using the RRC allocation rule, the firm can choose the fully optimal sequence of investments simply by choosing a level of investment each period in order to maximize the next period's accounting income. Second, in a model in which shareholders delegate the investment decision to a better-informed manager, it is shown that if shareholders base the manager's wage each period on current and past periods' accounting income calculated using the RRC rule, the manager will have the incentive to choose the fully optimal sequence of investments as long as each period's wage is weakly increasing in current and past periods' accounting income. Furthermore, this result holds regardless of the manager's own personal discount rate or other aspects of the manager's personal preferences. Therefore, the investment incentive problem is solved in a robust way, and the firm is left with considerable degrees of freedom to address any other incentive problems that may exist, such as providing incentives for the manager to exert effort each period, by choosing the precise functional form of the wage function each period.

In the formal model of this paper, it is assumed that assets have a known but arbitrary depreciation pattern and that the purchase price of new assets changes at a known constant rate over time. The RRC allocation rule is defined to be the unique allocation rule that satisfies the following two properties: (i) the cost of purchasing an asset is allocated across periods of its lifetime in proportion to the relative cost of replacing the surviving amount of the asset with new assets, and (ii)

the present discounted value of the cost allocations using the firm's discount rate is equal to the initial purchase price of the asset.

Property i can be interpreted as a version of the "matching principle" from accrual accounting that states that investment costs should be allocated across periods so as to match costs with benefits, where the "benefit" that an asset contributes to any period is interpreted to be the avoided cost of purchasing new capacity in that period. Property ii can be viewed as stating that the investment should be fully allocated, taking the time value of money into account. Most traditional accounting systems ignore the time value of money when allocating investment costs over time. The term "residual income" is generally used in the accounting literature to describe income measures that are calculated using an allocation rule for investment that takes the time value of money into account (Horngren and Foster 1987, 873–74). Recently there has been an explosion of applied interest in using residual income both to directly guide capital budgeting decisions and as a performance measure for managers who make capital budgeting decisions. Management consulting companies have renamed this income measure "economic value added" (EVA) and very successfully marketed it as an important new technique for maximizing firm value. *Fortune*, for example, has run a cover story on EVA, extolling its virtues and listing a long string of major companies that have adopted it (Tully 1993).[1] This paper provides an explicit formal model that justifies the use of residual income in the capital budgeting process and also specifically identifies the particular allocation rule that should be used to calculate residual income and how it depends on the depreciation pattern of the underlying assets.

Most of the literature on the optimal investment problem under certainty restricts itself to considering the case of exponential depreciation, in which a constant share of the capital stock is assumed to depreciate each year regardless of the age profile of the capital stock.[2] The assumption of exponential depreciation dramatically simplifies the analysis because the age profile of the existing capital stock can be ignored. However, for the purposes of this paper's study of cost allocation rules, it is important to allow for general patterns of depreciation because one of the most interesting questions to investigate regarding cost allocation rules is how the nature of the appropriate cost allocation rule should change as the depreciation pattern of the underlying assets changes. Obviously the pattern of depreciation must be a factor that can be exogenously varied in order to investigate this question. Furthermore, the case of exponential depreciation is not a particularly natural case

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[1] See the roundtable discussion in the *Continental Journal of Applied Corporate Finance* (Stern and Stewart 1994) and the associated articles (Sheehan 1994; Stewart 1994).

[2] See Jorgensen (1963) for an early analysis, and see Abel (1990) for a more extensive discussion of the optimal investment literature and further references.

to consider for most real applications. In most real applications, a much more natural case to consider is the so-called case of "one-hoss shay" depreciation, in which assets are assumed to have finite lifetimes and to remain equally productive over their lifetimes. This paper's analysis of the general case will, in particular, apply to the case of one-hoss shay depreciation.

This paper's results are based on Arrow's (1964) analysis of the optimal investment problem under certainty for general patterns of depreciation. Arrow shows that for any given depreciation pattern of assets, a vector of "user costs" can be calculated with the property that, under a broad range of plausible circumstances, the seemingly complex optimal investment problem collapses into a series of additively separable single-period problems by which the firm can be viewed as choosing the amount of capital to "rent" each period with rental rates given by the vector of user costs. This paper's basic insight is that a simple cost allocation rule (namely, the RRC rule) can be defined with the property that the cost it allocates to any period of an asset's lifetime is equal to the surviving amount of the asset multiplied by that period's user cost. The desirable properties of the RRC rule then follow from this. As part of the proof that the RRC allocation rule has this relationship to user cost, this paper derives a different and much simpler formula for calculating user cost than the formula derived by Arrow. In particular, Arrow's formula for calculating user cost depends on the vector of marginal investment rates, which describe the series of changes in investments sufficient to increase the stock of capital by one unit in a given period while holding the stock of capital constant in all other periods. For the case of general depreciation patterns, the formula for the vector of marginal investment rates is complicated and difficult to calculate and is defined by an infinite series of recursively defined functions. This paper shows that it is possible to derive an alternate and much simpler formula for user cost that does not depend on marginal investment rates.[3] In particular, it shows that a very simple formula exists to calculate hypothetical, perfectly competitive rental prices for assets and then proves that these hypothetical, perfectly competitive rental prices must be equal to user costs. The fact that user cost can be calculated by a very simple formula that does not depend on marginal investment rates is an interesting result independent of its application to cost allocation rules. Furthermore, the fact that user costs can be interpreted as hypo-

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[3] For the special case of exponential depreciation, the formula that determines marginal investment rates is very simple, and Arrow observes that his formula for user cost collapses into the simple formula directly derived by Jorgensen (1963) for this case. The incremental contribution of this paper is to show that a similarly simple formula to calculate user costs exists for general depreciation patterns even when there is no simple formula to calculate the vector of marginal investment rates.

thetical, perfectly competitive rental prices provides some extra economic intuition to explain the user cost result.

Two recent groups of papers have considered the role of accounting measures of income in the capital budgeting process. Anctil (1996) and Anctil, Jordan, and Mukherji (1998) consider a model in which depreciation is exponential, there are adjustment costs to changing the size of the capital stock, and the environment is stationary. They show that the time path of capital stock when the firm chooses each period's capital stock to maximize that period's residual income converges to the fully optimal time path. Rogerson (1997) considers a model in which the firm only invests once at the beginning of the first period.[4] An allocation rule called the relative benefits rule is shown to have the same sorts of desirable properties that the RRC allocation rule is shown to have in the model of this paper. While the allocation rules identified by both papers can be interpreted as allocating investment costs across periods in proportion to the relative benefit that the investment creates across periods, the relevant notion of "benefit" turns out to be very different in each case. In particular, in the one-time investment model of Rogerson (1997), the optimal allocation rule is determined solely by the demand-side factor of how the level of demand varies across periods. In contrast, the optimal allocation rule in the model of this paper is determined solely by the supply-side factor of how investments made in different periods can substitute for one another in creating capital stock to be used in a given period. In particular, the optimal allocation rule does not depend on how demand varies over time. Therefore the economic factors that determine the optimal allocation rule are quite different depending on whether or not investments in different periods substitute for one another.

In a companion paper to this one (Rogerson 2008), it is shown that the approach of this paper can also be applied to the issue of calculating welfare-maximizing prices for a regulated firm. In particular, it is shown that when there are constant returns to scale within each period (i.e., when output in each period is proportional to the capital stock in each period), the accounting cost of output calculated using the RRC allocation rule is equal to long-run marginal cost. Therefore, prices set equal to accounting cost calculated using the RRC rule are first-best in the sense that they both induce efficient consumption decisions and allow the firm to break even.

The paper is organized as follows. Section II presents the model and Arrow's user cost result. Section III proves that the vector of user costs

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[4] See Rogerson (1992) for an earlier, related result. Papers that have generalized Rogerson's (1997) result and applied it in a number of different settings include Reichelstein (1997, 2000), Dutta and Reichelstein (1999, 2002), Baldenius and Ziv (2003), and Baldenius and Reichelstein (2005).

can be interpreted as a vector of hypothetical, perfectly competitive
rental prices and uses this to construct a simple formula for user cost.
Section IV defines cost allocation rules and shows that the RRC allo-
cation rule has the property that it sets the accounting cost of using a
unit of capital in any period of its lifetime equal to user cost. Sections
V and VI describe the properties of the RRC allocation rule that follow
from this. Section VII briefly explains how the results generalize to the
case where future asset prices do not change at a constant rate. More
technical proofs are contained in the appendix.

II.   The Model and Arrow's User Cost Result

Let $I_t \in [0, \infty)$ denote the number of assets that the firm purchases in
period $t \in \{0, 1, \ldots\}$. Let $\mathbf{I} = (I_0, I_1, \ldots)$ denote the entire vector of in-
vestments. Assume that assets become available for use one period after
they are purchased and then gradually wear out or depreciate over time.
It will be convenient to use notation that directly defines the share of
the asset that survives and is thus available for use in each period, rather
than the share that depreciates. Let s_t denote the share of an asset that
survives until at least the tth period of the asset's lifetime and let $\mathbf{s} =
(s_1, s_2, \ldots)$ denote the entire vector of survival shares. Assume that $s_t \in
[0, 1]$ for every t, $s_1 = 1$, and that s_t is weakly decreasing in t. Two natural
and simple examples of depreciation patterns are the cases of expo-
nential depreciation given by

$$s_t = \beta^{t-1} \tag{1}$$

for some $\beta \in (0, 1)$ and one-hoss shay depreciation given by

$$s_t = \begin{cases} 1, & t \in \{1, 2, \ldots, T\} \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where T is a positive integer.
     For simplicity, assume that the firm begins period 0 with no existing
capital.[5] Let K_t denote the number of assets the firm has available for
use in period $t \in \{1, 2, \ldots\}$ and let $\mathbf{K} = (K_1, K_2, \ldots)$ denote the entire

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[5] All of the analysis in this paper actually applies to the general case where the firm
enters period 0 with existing assets if K_t is interpreted as the firm's incremental capital
stock, that is, its capital stock created by assets purchased in period 0 or later.

vector of capital stocks. The vector of capital stocks generated by any vector of investments is then determined by the linear mapping[6]

$$K = [S]I, \tag{3}$$

where $[S]$ is the lower triangular matrix

$$[S] = \begin{array}{ccc} s_1 & 0 & 0\dots \\ s_2 & s_1 & 0\dots \\ s_3 & s_2 & s_1\dots \\ \vdots & \vdots & \vdots \end{array} \ . \tag{4}$$

It is evident by inspection that $[S]$ is invertible and that its inverse is the matrix $[M]$ defined by

$$[S]^{-1} = [M] = \begin{array}{ccc} m_0 & 0 & 0\dots \\ m_1 & m_0 & 0\dots \\ m_2 & m_1 & m_0\dots \\ \vdots & \vdots & \vdots \end{array} \ , \tag{5}$$

where the values of m_i are determined sequentially by

$$m_0 = 1, \tag{6}$$

$$\sum_{j=0}^{i} m_j s_{i+1-j} = 0 \quad \text{for } i \in \{1, 2, \dots\}. \tag{7}$$

Therefore, the unique vector of investments that generates any given vector of capital stocks is determined by the linear mapping

$$\mathbf{I} = [M]\mathbf{K}. \tag{8}$$

The m_i parameters have a very natural interpretation. Suppose that the firm wishes to increase its stock of capital in period t by one unit while leaving the capital stock in all other periods fixed. Then m_i is the marginal adjustment to investment that the firm must make in period $t - 1 + i$. The vector $\mathbf{m} = (m_0, m_1, \dots)$ will be called the vector of marginal investment rates.

Let $\delta \in (0, 1)$ denote the firm's discount rate. Let $p_t \in [0, \infty)$ denote the price of purchasing a new unit of the asset in period t, and let $\mathbf{p} = (p_0, p_1, \dots)$ denote the vector of all asset prices. It will always be assumed that $p_t\delta^t$ is decreasing in t, so that it is not profitable to stockpile assets ahead of time. Many of the results in this paper will require the

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[6] I would like to thank Nancy Stokey for suggesting that I present the main arguments using matrix notation, which dramatically simplifies and clarifies the analysis. When matrix notation is used, vectors will be interpreted to be column vectors, and row vectors will be denoted by the superscript T.

additional assumption that asset prices change at a constant rate over time, that is, that

$$p_t = p_0 \alpha^{t-1} \quad \text{for some } \alpha \in \left(0, \frac{1}{\delta}\right). \tag{9}$$

However, it will not always be assumed that the vector of asset prices satisfies equation (9). Rather, when results depend on this assumption, this will be explicitly noted.

The present discounted cost of undertaking a vector of investments **I** is given by

$$\sum_{t=0}^{\infty} p_t I_t \delta^t. \tag{10}$$

Define $C(\mathbf{K})$ to be the present discounted cost of undertaking the vector of investments that generates **K**. This is created by substituting equation (8) into (10). Note that, since both equations (8) and (10) are linear, $C(\mathbf{K})$ must be linear in **K**. Therefore, $C(\mathbf{K})$ can always be written in the form

$$C(\mathbf{K}) = \sum_{t=1}^{\infty} c_t^* K_t \delta^t \tag{11}$$

for some vector of constants $\mathbf{c}^* = (c_1^*, c_2^*, \ldots)$. That is, the present discounted cost of providing any vector of capital stocks **K** can actually be calculated as though the firm can rent assets on a period-by-period basis at rental rates given by the vector of constants, $\mathbf{c}^*$. Following Arrow (1964), this vector of constants will be called the vector of user costs. Straightforward matrix multiplication shows that the formula for period t user cost is given by[7]

$$c_t^* = \sum_{i=0}^{\infty} m_i p_{t-1+i} \delta^{i-1}. \tag{12}$$

Equation (12) is the analog in this paper's model of Arrow's original formula for user cost. The formula is very intuitive. The right-hand side of equation (12) is simply the present discounted value, calculated in period t dollars, of the series of marginal changes to investments that will increase the stock of capital by one unit in period t while leaving the stock of capital in all other periods unchanged. Substitution of equation (9) into (12) yields the formula for user cost for the special case where asset prices change at a constant rate,

$$c_t^* = k^* p_t, \tag{13}$$

[7] See the Appendix.

where

$$k^* = \sum_{i=0}^{\infty} m_i(\alpha\delta)^{i-1}. \tag{14}$$

For this special case, note that user cost is proportional to the purchase price of assets so that user cost changes at the same constant rate at which asset prices change.

Let $B(K, t)$ be the function determining the firm's operating profit or "benefit" in period t, given the capital stock K. Let $B_K(K, t)$ denote the marginal benefit function. Assume that for every t, $B_K(K, t)$ exists, is continuous, is strictly decreasing when it is strictly positive, and is equal to 0 for large enough values of K.

The firm's optimization problem can now be stated as follows:

$$\max_{\mathbf{K}} \quad \sum_{t=1}^{\infty} [B(K_t, t) - c_t^* K_t]\delta_t \tag{15}$$

$$\text{subject to} \quad [M]\mathbf{K} \geq 0, \tag{16}$$

$$\mathbf{K} \geq \mathbf{0}. \tag{17}$$

Note that as long as the nonnegativity of investment (NNI) constraint given by equation (16) can be ignored, the problem collapses into a series of additively separable single-period problems by which the firm can be viewed as choosing the level of capital to rent each period at rental rates given by the vector of user costs. This observation is Arrow's user cost result.

More formally, define the relaxed optimization problem to be the problem of maximizing equation (15) subject only to (17) and let $\mathbf{K}^* = (K_1^*, K_2^*, \ldots)$ denote the unique vector of capital stocks that solves this problem, which is defined by

$$B_K(K_t^*, t) = c_t^* \quad \text{and} \quad K_t^* \geq 0 \quad \text{or}$$

$$B_K(K_t^*, t) < c_t^* \quad \text{and} \quad K_t^* = 0. \tag{18}$$

Then Arrow's user cost result can be stated as follows.

PROPOSITION 1 (Arrow 1964).   Suppose that $\mathbf{K}^*$ satisfies the NNI constraint given by equation (16). Then $\mathbf{K}^*$ is the unique solution to the firm's optimization problem given by equations (15)–(17).

Proof.   As above. QED

Note that a sufficient condition for a vector of capital stocks to satisfy equation (16) is that K_t be weakly increasing in t. A sufficient condition for K_t^* to be weakly increasing in t is of course that the marginal product

of capital be increasing at least as quickly as the user cost of capital, that is, that $B_K(K, t) - c_t^*$ be weakly increasing in t for every $K \in (0, \infty)$. For the remainder of this paper it will simply be assumed that this condition is satisfied so that $\mathbf{K}^*$ is the unique solution to the firm's optimization problem.[8]

III.   Hypothetical, Perfectly Competitive Rental Prices and a Simpler Formula for User Cost

Consider a hypothetical situation in which there is a rental market for assets, and a supplier of rental services can enter the market in any period by purchasing one unit of the asset and then renting out the available capital stock over the asset's life. Let c_t denote the price of renting one unit of capital stock in period t and let $\mathbf{c} = (c_1, c_2, \ldots)$ denote the entire vector of rental prices. Assuming that suppliers incur no extra costs besides the cost of purchasing the asset, that they can rent the full remaining amount of the asset every period, and that their discount rate is equal to δ, the zero-profit condition that must be satisfied by a perfectly competitive equilibrium is

$$p_t = \sum_{i=1}^{\infty} c_{t+i} s_i \delta^i \quad \text{for every } t \in \{0, 1, 2, \ldots\}. \tag{19}$$

Proposition 2 below states the intuitively reasonable result that the vector of user costs is the unique vector of rental prices that satisfies the zero-profit constraints in equation (19).

   PROPOSITION 2.   The vector of user costs, $\mathbf{c}^*$, is the unique vector of rental prices satisfying equation (19).

   Proof.   See the appendix. QED

   Recall that Arrow's formula for user cost for the special case where asset prices change at a constant rate is given by equations (13)–(14). While the formula is somewhat simpler than the formula for the general case, it still depends on the entire vector of marginal investment rates that must be calculated by the infinite sequence of the recursively defined equations (6)–(7). However, it is straightforward to directly calculate a very simple formula for a vector of rental prices that satisfies equation (19), where the formula only depends on the vector of survival

[8] For the case of general vectors of asset prices, this sufficient condition is somewhat unsatisfactory, in the sense that it is stated in terms of the behavior of the vector of user costs that is itself determined by a relatively complex formula. However, for the special case where asset prices change at a constant rate, which is the primary focus of this paper, it is sufficient to assume that $B_K(K, t)/\alpha^t$ is weakly increasing in t (since it has already been observed that user costs change at the same rate as asset prices for this case). That is, it is sufficient to assume that the marginal product of capital is growing at least as fast as the price of new assets.

shares and does not depend on the vector of marginal investment rates. By proposition 2, this must therefore also be a formula for the vector of user costs.

PROPOSITION 3. An alternate formula for calculating the constant k^* in equation (13) is given by

$$k^* = \frac{1}{\sum_{i=1}^{\infty} s_i(\alpha\delta)^i}. \tag{20}$$

Proof. It is straightforward to verify that the rental rates defined by equations (13) and (20) satisfy (19).[9] QED

IV. Cost Allocation Rules

The remainder of this paper investigates how cost allocation rules can be used to help guide investment decisions. Since the main positive results are for the special case where asset prices are assumed to change at a constant rate, it will be useful to simplify the formal presentation by restricting attention to this case. In sections IV, V, and VI it will be assumed that the vector of asset prices satisfies equation (9). Then the extent to which the results generalize is discussed in section VII.

A. *Allocation and Depreciation Rules*

Define a depreciation rule to be a vector $\mathbf{d} = (d_1, d_1, \ldots)$ such that $d_i \geq 0$ for every i and

$$\sum_{i=1}^{\infty} d_i = 1, \tag{21}$$

where d_i is interpreted as the share of depreciation allocated to the ith period of the asset's life. Let D denote the set of all depreciation rules. Define an allocation rule to be a vector $\mathbf{a} = (a_1, a_2, \ldots)$ that satisfies $a_i \geq 0$ for every i and

$$\sum_{i=1}^{\infty} a_i \gamma^i = 1 \tag{22}$$

for some discount rate $\gamma \in (0, 1)$. Let A denote the set of all allocation rules and let $\hat{a}(\mathbf{a})$ denote the value of γ such that equation (22) is

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[9] Another way of proving proposition 3 is to directly prove the statement that

$$\sum_{i=0}^{\infty} m_i \gamma^{i-1} = \frac{1}{\sum_{i=1}^{\infty} s_i \gamma^i} \quad \text{for every } \gamma \in (0, 1).$$

This proof is presented in an earlier version of this paper, available from the author.

satisfied. The allocation rule **a** will be said to be complete with respect to the discount rate $\hat{a}(\mathbf{a})$.

Firms generally think of themselves as directly choosing a depreciation rule and a discount rate instead of as directly choosing an allocation rule. The cost allocated to each period is then calculated as the sum of the depreciation allocated to that period plus imputed interest on the remaining (nondepreciated) book value of the asset. Formally, for any depreciation rule **d** and discount rate γ, the corresponding allocation rule is given by

$$a_i = d_i + \left(\frac{1-\gamma}{\gamma}\right) \sum_{j=i}^{\infty} d_j. \tag{23}$$

It is straightforward to verify that the resulting allocation rule determined by equation (23) is complete with respect to γ. It is also straightforward to verify that for any $\mathbf{a} \in A$, there is a unique $(\mathbf{d}, \gamma)$ such that equation (23) maps $(\mathbf{d}, \gamma)$ into **a**. It is defined by $\gamma = \hat{a}(\mathbf{a})$ and

$$d_i = \sum_{j=1}^{\infty} \gamma^j a_{i-1+j} - \sum_{j=1}^{\infty} \gamma^j a_{i+j}. \tag{24}$$

Therefore, one can equivalently think of the firm either as choosing a depreciation rule and discount rate or as choosing an allocation rule. For the purposes of this paper, it is more convenient to view the firm as directly choosing an allocation rule.[10]

B.   Accounting Cost and Accounting Income

Let $A_t(K_1, \ldots, K_t, \mathbf{a})$ denote the accounting cost of capital in period t, conditional on the firm's choice of capital stocks up until that point and the allocation rule it uses. It is defined by

$$A_t(K_1, \ldots, K_t, \mathbf{a}) = \sum_{i=1}^{t} \varphi_{t-i}(K_1, \ldots, K_{t+1-i}) p_{t-i} a_i, \tag{25}$$

where $\varphi_t(K_1, \ldots, K_{t+1})$ denotes the level of investment in period t necessary to produce the vector of capital stocks $(K_1, \ldots, K_t)$ as given by equation (8). Let $Y_t(K_1, \ldots, K_t, \mathbf{a})$ denote the accounting income in period t, defined by

$$Y_t(K_1, \ldots, K_t, \mathbf{a}) = B(K_t, t) - A_t(K_1, \ldots, K_t, \mathbf{a}). \tag{26}$$

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[10] See Rogerson (1992) for a fuller discussion of the relationship between depreciation and allocation rules and their properties.

C. The RRC Allocation Rule

An allocation rule $\mathbf{a} = (a_1, a_2, \ldots)$ can be said to allocate costs in proportion to the cost of replacing the surviving amount of the asset with new assets if it satisfies

$$a_i = k s_i \alpha^i \tag{27}$$

for some positive real number k. It is easy to verify that an allocation rule of the form in equation (27) is complete with respect to δ if and only if the constant k is equal to the value k^* defined by equation (20). Let $\mathbf{a}^*$ denote the allocation rule determined by setting k equal to k^*. This is the RRC allocation rule. It is the unique allocation rule that satisfies the following two properties: (i) it allocates costs in proportion to replacing the surviving amount of the asset with new assets, and (ii) it is complete with respect to δ.

For applied purposes, note that the RRC allocation rule takes a particularly simple form for the case where assets follow the one-hoss shay depreciation pattern defined by equation (3):

$$a_i^* = \begin{cases} k^* \alpha^i, & i \in \{1, \ldots, T\} \\ 0, & i \in \{T+1, \ldots\}. \end{cases} \tag{28}$$

D. The Relationship between Accounting Cost under the RRC Rule and User Cost

For an asset purchased in period t, the allocation rule $\mathbf{a}$ satisfies the property that the total cost allocated to the ith period of the asset's lifetime is equal to that period's user cost multiplied by the surviving amount of the asset if and only if the following statement is true:

$$p_t a_i = c_{t+i}^* s_i. \tag{29}$$

There is obviously a unique value of a_i that satisfies equation (29). Substitution of equations (13) and (9) into (29) and reorganization yields

$$a_i = k^* s_i \alpha^i. \tag{30}$$

Therefore, the RRC allocation rule satisfies equation (29), and it is the unique allocation rule that does so. It follows from this that the RRC rule is the unique allocation rule such that the accounting cost in every period is equal to that period's user cost multiplied by that period's capital stock; that is,

$$A_t(K_1, \ldots, K_t, \mathbf{a}^*) = c_t^* K_t. \tag{31}$$

These results are summarized in proposition 4.

PROPOSITION 4.   The RRC allocation rule is the unique allocation rule that satisfies equation (29) and is also the unique allocation rule that satisfies equation (31).

Proof.   As above. QED

V.   A Simple Rule for Calculating the Optimal Investment Path

Proposition 5 now states that, when the RRC rule is used to calculate accounting income, the firm can choose the fully optimal vector of capital stocks simply by choosing a level of investment each period to maximize next period's accounting income.

PROPOSITION 5.   Suppose that the firm calculates accounting income using the allocation rule $\mathbf{a} = (a_1, \ldots, a_n)$ and chooses a level of investment every period to maximize the next period's accounting income. Then a sufficient condition for the firm to choose $\mathbf{K}^*$ is that a_1 be equal to a_1^*.

Proof.   Suppose that the firm is in period t and that it will therefore choose K_{t+1} by its current-period investment decision. By the user cost result, K_{t+1}^* is chosen to maximize

$$B(K_{t+1}, t+1) - c_{t+1}^* K_{t+1}. \tag{32}$$

Now suppose that the firm chooses K_{t+1} to maximize period $t+1$ accounting income. Then, as long as the nonnegativity constraint on investment does not bind, K_{t+1} is chosen to maximize

$$B(K_{t+1}, t+1) - p_t a_1 K_{t+1}. \tag{33}$$

By comparing equations (32) and (33), a sufficient condition for the firm to choose the fully optimal vector of investments is that $p_t a_1 = c_{t+1}^*$. By proposition 4, a necessary and sufficient condition for this is that $a_1 = a_1^*$. QED

Of course, the sufficient condition in proposition 5 only specifies the first-period allocation share of the allocation rule used by the firm. Therefore, while the RRC allocation rule satisfies this sufficient condition, there are obviously many other allocation rules that also satisfy it. However, the RRC allocation rule is a particularly simple and natural allocation rule, and it is not clear that it would be possible to identify some other equally simple and natural allocation rule that sets a_1 equal to a_1^* but sets a_i unequal to a_i^* for other values of i. Furthermore, the next section considers a more complex model in which shareholders delegate the investment decision to management, accounting income is used as a managerial performance measure, and a sufficient condition

for an allocation rule to create good investment incentives is that the allocation share in every period be set according to the RRC allocation rule.

VI. Managerial Investment Incentives

This section considers an extension of the basic model in which shareholders delegate the investment decision to a better-informed manager and shows that there is a sense in which shareholders can create robust incentives for management to choose the fully efficient investment path by using accounting income calculated using the RRC allocation rule as a performance measure for management.

Suppose that the production/demand environment is as described in the previous sections. Assume that shareholders know $\mathbf{s}$ and α and can therefore calculate the RRC allocation rule but that they do not know the benefit function $B(K, t)$ and therefore do not have sufficient information to calculate the optimal vector of investments. Assume that the manager knows all of the functions and parameters in the model and is therefore able to calculate the optimal vector of investments. Suppose that shareholders delegate the investment decision to the manager and that they create a compensation scheme for the manager by choosing an allocation rule and wage function. The allocation rule is used to calculate each period's accounting income. The wage function determines the wage the manager receives each period as a function of current and past periods' accounting incomes. Assume that the manager has preferences over vectors of wage payments (with the property that the manager weakly prefers a higher wage in any period, holding the wages in all other periods constant) and chooses the sequence of capital stocks to maximize his or her own utility.

For any given allocation rule, it will generally be the case that the vector of capital stocks that the manager finds it optimal to choose will depend in complex ways on the particular wage function being used and the manager's own preferences over vectors of wage payments, including his or her personal discount rate. This is because it will generally be the case that an allocation rule will create trade-offs between increasing accounting income in different periods, and the manner in which the manager weighs these trade-offs will depend on both the wage function and the manager's own preferences. However, suppose that there was an allocation rule that had the property that there was a vector of capital stocks that simultaneously maximized the accounting income in every period. Then as long as each period's wage was increasing in the current and past periods' accounting income, it would obviously be optimal for the manager to choose this vector of capital stocks. More formally, an allocation rule $\mathbf{a}'$ will be said to create robust

incentives for the manager to choose the vector of capital stocks $\mathbf{K}'$, if $\mathbf{a}'$ satisfies

$$(K_1', \ldots, K_t') \in \arg\max_{(K_1, \ldots, K_t)} \ Y_t(K_1, \ldots, K_t, \mathbf{a}') \quad \text{for every } t \in \{1, 2, \ldots\}.$$

(34)

Proposition 6 now states the main result of this section, which is that the RRC allocation rule creates robust incentives for the manager to choose the fully optimal vector of capital stocks.

PROPOSITION 6. The RRC allocation rule, $\mathbf{a}^*$, creates robust incentives for the manager to choose the fully optimal vector of capital stocks, $\mathbf{K}^*$.

Proof. Substitution of equation (31) into (26) shows that accounting income under the RRC rule is given by

$$Y_t(K_1, \ldots, K_t, \mathbf{a}^*) = B(K_t, t) - c_t^* K_t.$$

(35)

Note that accounting income in period t only depends on K_t and that it is maximized at K_t^*. This implies that the vector of capital stocks $\mathbf{K}^* = (K_1^*, K_2^*, \ldots)$ simultaneously maximizes accounting income for every time period. QED

The above result requires some interpretation. In particular, it does not formally show that a contract using the RRC allocation rule is the optimal solution to a completely specified principal agent problem. It is clear that such a result would be straightforward to prove in a model in which it was assumed that the only incentive/information problem was that the manager is better informed than shareholders about some information necessary to calculate the fully optimal investment plan. However, there would be no need in such a model to base the manager's wage on any measure of the firm's performance. This is because one fully optimal contract would be for shareholders to simply pay the manager a constant wage each period that is sufficient to induce the manager to accept the job. Then the manager would be (weakly) willing to choose the profit-maximizing investment plan.

Therefore, in reality, the result of this paper will only be useful in situations where there is some additional incentive problem that requires shareholders to base the manager's wage on some measure of the firm's performance. A natural candidate would be to assume that there is a moral hazard problem within each period, that is, that each period the manager can exert unobservable effort that affects the firm's cash flow that period. This would create a multiperiod moral hazard problem with asymmetric information. The modeling problem this creates is that solutions to such problems are extremely complex, and the nature of the solution generally depends on particular aspects of the

environment (such as the agent's preferences) that the principal is un-likely to have reliable information about. Thus, it is not clear that such contracts would be suitable for use in the real world, where robustness to small changes in the environment is likely to be important.

In light of these difficulties, the result of this paper can be interpreted as offering a useful alternative approach. In particular, this paper shows that, by restricting themselves to choosing a compensation scheme in which accounting income is calculated using the RRC allocation rule and in which each period's wage is a weakly increasing function of current and past periods' accounting income, shareholders can guar-antee in a robust way that the investment incentive problem will be completely solved and still leave themselves considerable degrees of freedom to address remaining incentive issues. For example, by using accounting income based on the RRC allocation rule as a performance measure, shareholders could thereby guarantee that the investment in-centive problem was completely solved and then use a "trial and error" process over time to identify a wage function that appeared to create the appropriate level of effort incentives.

Note that in cases where it is possible to calculate a fully optimal contract, it may well be that the fully optimal contract does not induce the agent to choose the profit-maximizing level of investment. However, it is precisely these sorts of calculations that are exceedingly complex and that are unlikely to be robust to small changes in the contracting environment.

Of course, the question of whether the results of this section can be used to more formally show that a contract using the RRC allocation rule is the optimal solution to a completely specified principal agent problem is an interesting question for future research. One observation that may prove helpful in this regard is that proposition 6 can be stated in a somewhat more general form. Namely, it is clear that for any given discount rate $\gamma \in (0, 1)$, the principal can provide the agent with robust incentives to choose the sequence of investments that would be first-best for the discount rate γ by using the discount rate γ to calculate accounting income under the RRC rule. Therefore, the RRC allocation rule can actually be used to robustly implement the entire continuum of investment strategies, consisting of the set of investment strategies that would be first-best for any discount rate $\gamma \in (0, 1)$.

VII. General Patterns of Future Asset Prices

This section reports the extent to which the results of this paper gen-eralize to the case where asset prices do not necessarily change at a constant rate. In brief, it is still possible to define cost allocation rules in terms of the vector of user costs so that the resulting cost allocation

rules have the same sorts of desirable properties as were shown to hold in previous sections. The main difference is that there is no longer necessarily any simple or natural way to describe these allocation rules in terms of the underlying parameters of the model. Therefore, while the generalization is of analytic interest because it helps clarify precisely why the cost allocation result is true and what it depends on, it may be of more limited practical interest. However, a firm's information about future prices is likely to be somewhat imprecise in any event, so that it may be very natural and reasonable in many applied cases to project future prices by simply specifying a likely average future growth rate.

The remainder of this section provides a very brief sketch of the manner in which the results generalize. For the general case it will be necessary to potentially allow the firm to choose a different allocation rule to allocate each period's investment. Let $\mathbf{a}_t = (a_{t1}, a_{t2}, \ldots)$ denote the allocation rule used to allocate investments made in period t for $t \in \{0, 1, 2, \ldots\}$. Define the user cost allocation rule for period t, denoted by $\mathbf{a}_t^U = (a_{t1}^U, a_{t2}^U, \ldots)$, to be the allocation rule such that the cost allocated to the ith period of the asset's lifetime is equal to that period's user cost multiplied by the surviving amount of the asset,

$$a_{ti}^U = \frac{c_{t+i}^* s_i}{p_t}. \tag{36}$$

Therefore, the vector of user cost allocation rules is constructed to have the property that the cost of purchasing an asset allocated to any period of its lifetime is equal to that period's user cost multiplied by the surviving amount of the assets. Propositions 5 and 6 continue to hold true in the generalized model because of equation (36). Furthermore, proposition 2 implies that each of the allocation rules is complete with respect to δ. When asset prices change at a constant rate, the formula in equation (36) collapses to the formula for calculating the RRC allocation rule. However, in the general case, the formula in equation (36) does not appear to collapse into any simple or natural form.

Appendix

Derivation of Equation (12)

Let $[\Delta]$ denote the matrix with δ^{i-1} in the ith diagonal position and zeroes elsewhere. Note for future reference that the matrix $[\Delta][M][\Delta]^{-1}$ is of the form

$$[\Delta][M][\Delta]^{-1} = \begin{matrix} \delta^0 m_0 & 0 & 0 & 0\ldots \\ \delta^1 m_1 & \delta^0 m_0 & 0 & 0\ldots \\ \delta^2 m_2 & \delta^1 m_1 & \delta^0 m_0 & 0\ldots \\ \vdots & \vdots & \vdots & \vdots \end{matrix}. \tag{A1}$$

Substitute equation (8) into (10) to create the following formula for $C(\mathbf{K})$,[11]

$$C(\mathbf{K}) = \mathbf{p}^T[\Delta][M]\mathbf{K} = \mathbf{p}^T[\Delta][M](\delta[\Delta])^{-1}(\delta[\Delta])\mathbf{K}. \tag{A2}$$

Rewrite equation (11) in matrix notation as

$$C(\mathbf{K}) = \mathbf{c}^{*T}\delta[\Delta]\mathbf{K}. \tag{A3}$$

A comparison of equations (A2) and (A3) shows that

$$\mathbf{c}^{*T} = \mathbf{p}^T[\Delta][M](\delta[\Delta])^{-1} = \frac{\mathbf{p}^T[\Delta][M][\Delta]^{-1}}{\delta}. \tag{A4}$$

Equation (12) follows from (A1) and (A4).

Proof of Proposition 2

In matrix notation, equation (19) can be written as

$$\mathbf{p}^T = \mathbf{c}^T\delta[\Delta][S][\Delta]^{-1}. \tag{A5}$$

Multiply both sides of equation (A5) by $[\Delta][M][\Delta]^{-1}$ and reorganize using the fact that $[M] = [S]^{-1}$ to yield

$$\mathbf{c}^T = \frac{\mathbf{p}^T[\Delta][M][\Delta]^{-1}}{\delta}. \tag{A6}$$

A comparison of equations (A4) and (A6) shows that (A6) is the definition of the vector of user costs.

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[11] Recall that vectors are interpreted to be column vectors and that the superscript T is used to denote row vectors.

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