Equilibrium Unemployment Theory

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I FOUNDATIONS

1 The Labor Market

The purpose of this chapter is to describe a simple version of the labor-market model that captures the salient features of the theory of unemployment developed in this book. The model does not yet claim to be realistic or empirically implementable. At this stage many of the variables that are likely to be important in an empirical analysis of unemployment are left out. Its purpose is to point out the nature of unemployment in the steady state and to show how wages and unemployment are jointly determined in an otherwise standard equilibrium model.

The model of this chapter is suitable for macroeconomic analysis, and it is with this objective in mind that it is being developed. Chapter 2 extends it in an important direction, by introducing endogenous job destruction. Chapter 3 embeds it into the standard neoclassical model of growth and considers the properties of the balanced growth path. The model is shown to add a rich theory of the natural rate of unemployment to the full-employment models that form the basis of much of modern macroeconomics.

1.1 Trade in the Labor Market

The central idea of the model is that trade in the labor market is a decentralized economic activity. It is uncoordinated, time-consuming, and costly for both firms and workers. Firms and workers have to spend resources before job creation and production can take place, and existing jobs command rents in equilibrium, a property that does not characterize Walrasian labor markets.

We use a simple modeling device to capture the implications of trade for market equilibrium, which has its parallel in the neoclassical assumption of the existence of an aggregate production function. We assume that there is a well-behaved *matching function* that gives the number of jobs formed at any moment in time as a function of the number of workers looking for jobs, the number of firms looking for workers, and possibly some other variables (which are introduced in later chapters).

Trade in the labor market is a nontrivial economic activity because of the existence of heterogeneities, frictions, and information imperfections. If all workers were identical to each other and if all jobs were also identical to each other, and if there was perfect information about their location, trade would be trivial. But without homogeneity on either side of the market and with costly acquisition of information, firms and workers find it necessary to spend resources to find productive job matches. The heterogeneities may be in the skills possessed by workers, on the one hand, and those required by firms, on the other. They may be in the information possessed about the job. Or, they may be in the location of jobs and workers and in the timing of job creation in different locations. In this environment there is uncertainty about the arrival of good jobs to job-seekers and good workers to hiring firms, and firms and workers have to decide whether to accept what is available, wait for a better alternative, or influence the arrival process itself by spending resources on the acquisition of information, retraining employees, or changing location.

The matching function gives the outcome of the investment of resources by firms and workers in the trading process as a function of the inputs. It is a modeling device that captures the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit. In this sense it occupies the same place in the macroeconomist's tool kit as other aggregate functions, such as the production function and the demand for money function. The production function summarizes a relationship that depends on a physical technology that is not made explicit in macroeconomic modeling. The demand for money function summarizes a transaction technology and a portfolio choice that is also rarely made explicit. Similarly the matching function summarizes a trading technology between heterogeneous agents that is also not made explicit.

Like the other aggregate functions in the macroeconomist's tool kit, the usefulness of the matching function depends on its empirical viability and on how successful it is in capturing the salient features of exchange in the labor market. Of course, if an empirically successful microfoundation for the matching function were known, that would make it more convincing, but it is not uncommon to find aggregate functions in the macroeconomist's tool kit without explicit microfoundations. Empirical success and modeling effectiveness are usually sufficient. One of the aims of this book is to show that the matching function is a useful modeling device and that many new and plausible results can be derived from it.

Matching functions have been estimated for a number of countries with good and largely uniform results. Some of these studies are discussed in the Notes on the Literature at the end of this chapter. Matching functions have also been derived from explicit trading processes, but there is as yet no microfoundation for it that dominates all others. In this book we will assume the existence of a general matching function of a few variables and impose some regularity restrictions on it that have been shown to be empirically valid in a large number of studies.

Trade and production are completely separate activities. In order to emphasize this, we assume that there is full specialization in either trade or production. A firm with many jobs may have some of them filled and some vacant, but only vacant jobs can engage in trade. Thus, although firms do not specialize, jobs do. Similarly, in the model of this chapter and in most of the applications in this book, a worker may be employed or unemployed, but only unemployed workers search for jobs. In view of the empirical importance of search on the job and job-to-job moves, however, we will also characterize market equilibrium with search on the job (chapter 4). We will argue that the justification for the assumption that only the unemployed search is naturally not its descriptive accuracy. It is the claim that the theory of unemployment obtained under the assumption of no on-the-job search is not significantly different from the one obtained when on-the-job search is introduced. It will be shown in chapter 4 that although some new results are derived, there are no significant modifications to the theory of unemployment in this and the next chapter, and it is not necessary for equilibrium-matching models to abandon the assumption of a simple matching function with only unemployed workers searching. The assumption of full specialization in either production or trade is a useful modeling device for both jobs and workers.

Vacant jobs and unemployed workers become matched to each other and move from trading to production activities gradually, according to the prevailing matching technology. Unemployment persists in the steady state because during the matching process and before all unmatched job-worker pairs meet, some of the existing jobs break up, providing a flow into unemployment. The separations result from firm-specific shocks, which summarize mainly changes in technology or demand, due to shifts in production or utility functions.

Firms and workers decide what to do with full knowledge of the job-matching and job-separation processes but without any attempt to coordinate their actions. There are many firms and many workers, and each operates as an atomistic competitor. The equilibrium that we describe is a full rational expectations equilibrium. The aggregate equilibrium state is one where firms and workers maximize their respective objective functions, subject to the matching and separation technologies, and where the flow of workers into unemployment is equal to the flow of workers out of unemployment. Our assumptions ensure that there is a unique unemployment rate at which these two flows are equal.

We begin with a formalization of the equilibrium condition for unemployment. Suppose there are L workers in the labor force. We let u denote the unemployment rate—the fraction of unmatched workers—and v the number of vacant jobs as a fraction of the labor force. We refer to v as the vacancy rate, and we assume that only the uL unemployed workers and the vL job vacancies engage in matching. The model is specified in continuous time. The number of job matches taking place per unit time is given by

$$mL = m(uL, vL). \tag{1.1}$$

Equation (1.1) is the matching function. It is assumed increasing in both its arguments, concave, and homogeneous of degree 1. Homogeneity, or constant returns to scale, is an important property, and our reasons for assuming it are similar to the reasons that aggregate production functions are assumed to be of constant returns: It is empirically supported and plausible, since in a growing economy constant returns ensures a constant unemployment rate along the balanced-growth path. Of course one does not need constant returns everywhere, both at the micro and macro levels, to derive a balanced growth path. There are by now, however, convincing empirical reasons for assuming constant returns in matching functions, which are discussed at the end of this chapter and later (chapter 3). The empirical literature has further found that a log-linear (Cobb-Douglas) approximation to the matching function fits the data well. We do not need to impose this additional restriction on the matching function for the results derived in this book.

The job vacancies and unemployed workers that are matched at any point in time are randomly selected from the sets vL and uL. Hence the process that changes the state of vacant jobs is Poisson with rate

$m(uL,vL)/vL$. By the homogeneity of the matching function, this rate is a function of the ratio of vacancies to unemployment only. It is convenient to introduce the v/u ratio as a separate variable, denoted by θ, and write the rate at which vacant jobs become filled as

$$q(\theta) \equiv m\left(\frac{u}{v}, 1\right). \tag{1.2}$$

During a small time interval δt, a vacant job is matched to an unemployed worker with probability $q(\theta)\delta t$, so the mean duration of a vacant job is $1/q(\theta)$. By the properties of the matching technology, $q'(\theta) \leq 0$ and the elasticity of $q(\theta)$ is a number between 0 and -1. Its absolute value is denoted by $\eta(\theta)$.

Unemployed workers move into employment according to a related Poisson process with rate $m(uL, vL)/uL$. Making use of the θ notation, this rate is equal to $\theta q(\theta)$ and has elasticity $1 - \eta(\theta) \geq 0$. The mean duration of unemployment is $1/\theta q(\theta)$. Thus unemployed workers find jobs more easily when there are more jobs relative to the available workers, and firms with vacancies find workers more easily when there are more workers relative to the available jobs. The process that describes the transition out of unemployment is related to the process that describes the filling of jobs by the fact that jobs and workers meet in pairs. By the structure of the model, θ is an appropriate measure of the *tightness* of the labor market. In much of what follows, labor market tightness, θ, is a more convenient variable to work with than the vacancy rate, v.

The dependence of the functions $q(\theta)$ and $\theta q(\theta)$ on the relative number of traders (tightness) is an example of a trading externality that will play a central role in our analysis. The trading externality arises because during trade, price is not the only allocative mechanism. During a short interval of time δt, there is a positive probability $1 - q(\theta)\delta t$ that a hiring firm will not find a worker and another positive probability $1 - \theta q(\theta)\delta t$ that an unemployed worker will not find a job, whatever the set of prices. There is stochastic rationing, which cannot be eliminated by price adjustments. But it can be made better or worse for the representative trader by adjustments in the relative number of traders in the market. If the ratio of hiring firms to searching workers increases, the probability of rationing is higher for the average firm and lower for the average worker, and conversely. We refer to these trade externalities as *search* or *congestion* externalities because they are caused by the congestion that searching firms and workers cause for each other during trade. Their existence is important for most of the properties of equilibrium unemployment that we derive. It also has important implications for the efficiency of equilibrium, which we consider in chapter 8.

The flow into unemployment results from job-specific (idiosyncratic) shocks that arrive to occupied jobs at the Poisson rate λ. The job-specific shocks may be caused by structural shifts in demand that change the relative price of the good produced by a job, or by productivity shocks that change the unit costs of production. In either case they are real shocks associated with a shift in either tastes or technology. Once a shock arrives, the firm has no choice but either to continue production at the new value or to close the job down. In the full specification of the model, we will assume that when an idiosyncratic shock arrives, the net product of the job changes to some new value that is a drawing from a general probability distribution. In the simpler model in this chapter, however, we assume that the probability distribution of idiosyncratic productivity values is a very special one. The relative price of output of an occupied job is either high enough (and constant) to make production profitable or low enough (and hence arbitrarily small) to lead to a separation. Idiosyncratic shocks move the value of output from the high level to the low one at rate λ.

Job creation takes place when a firm and a searching worker meet and agree to form a match at a negotiated wage. When the decision to create a job is made, the firm has a choice of which product value to choose, so it always chooses the high value. Goods or technologies are differentiated but irreversible. Before job creation, there is full choice of technology and product type; once job creation has taken place, the firm has no choice of either. Thus, once the firm and worker meet and a job is created, production continues until a negative idiosyncratic shock arrives, at which point the productivity of the job moves to the low value. *Job destruction* then takes place, which in the framework of this model is equal to job separations: The worker moves from employment to unemployment, and the firm can either withdraw from the market or reopen a job as a new vacancy (in equilibrium firms are indifferent between these two options).

We assume that the job-worker pairs that experience the adverse shocks are randomly selected. During a small time interval δt a worker moves from employment to unemployment with exogenous probability $\lambda\delta t$, and an occupied job separates with the same probability. Thus job separations follow a Poisson process with rate λ which is independent of the processes that describe the filling of jobs, and which in this version of the model is exogenous.

Without growth or turnover in the labor force, the mean number of workers who enter unemployment during a small time interval is $\lambda(1-u)L\delta t$, and the mean number who leave unemployment is $mL\delta t$. We rewrite the latter as $u\theta q(\theta)L\delta t$, where $\theta q(\theta)\delta t$ is the transition probability of the unemployed. The evolution of mean unemployment is given by the difference between the two flows,

$$\dot{u} = \lambda(1-u) - \theta q(\theta)u. \tag{1.3}$$

In the steady state the mean rate of unemployment is constant, so

$$\lambda(1-u) = \theta q(\theta)u. \tag{1.4}$$ *

We assume that the market is large enough, so deviations from the mean can be ignored. We can rewrite (1.4) as an equation determining unemployment in terms of the two transition rates:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}. \tag{1.5}$$

Equation (1.5) is the first key equation of the model. It implies that for given λ and θ, there is a unique equilibrium unemployment rate. λ is a parameter of the model; θ is an unknown. We show in the next section that θ is determined by an equation derived from the assumption of profit maximization and that it is unique and independent of u. Hence the solution for u is also unique. By the properties of the matching function, (1.5) can be represented in tightness-unemployment space or in vacancy-unemployment space, by a downward-sloping and convex to the origin curve. This curve is known as the *Beveridge curve*.

We stated the steady-state condition (1.4) in terms of the flows in and out of unemployment. Alternatively, it can be stated in terms of job flows, which we do here for future reference. The number of jobs created at any moment in time is $m(v, u)$. The empirical literature on job flows defines the job creation rate as the ratio of the number of jobs created to employment, namely as $m(v, u)/(1-u)$. The job destruction rate is similarly defined as the ratio of the total number of jobs destroyed, $\lambda(1-u)$, to employment, $1-u$. Equating the constant job destruction rate, λ, to the job creation rate gives (1.4). Restating the steady-state condition in terms of the job flows makes it clear that the key driving force in this version of the model is job creation. It also makes it easier to compare some of the model's results with the empirical findings, which we do in later sections.

1.2 Job Creation

Job creation takes place when a firm and a worker meet and agree to an employment contract. Before this can take place, however, the firm has to open a job vacancy and search, and unemployed workers also have to search. The employment contract specifies only a wage rule that gives the wage rate at any moment in time as a function of some commonly observed variables. Hours of work are fixed (and normalized to unity), and either side can break the contract at any time.

For convenience, we assume that firms are small. Each has one job that is vacant when it first enters the market, but the job is occupied by a worker after an employment contract has been signed. When the job is occupied, the firm rents capital and produces output, which is sold in competitive markets. The capital decision is not important for our main results, so we will suppress it for the time being and introduce it later in this chapter. We assume instead that the value of a job's output is some constant $p > 0$. When the job is vacant, the firm is actively engaged in hiring at a fixed cost $pc > 0$ per unit time. During hiring, workers arrive to vacant jobs at the rate $q(\theta)$, which for now is independent of what the firm does.

The hiring cost is made proportional to productivity on the ground that it is more costly to hire more productive workers. In a long-run equilibrium it is a natural assumption to make, since the costs of the firm have to rise along with productivity to ensure the existence of a steady state. The assumption may, however, be less easy to justify over the business cycle, when it might be more natural to assume that hiring costs depend on wages, on the ground that hiring is a labor-intensive activity. But even if wages are not proportional to productivity in the short run, assuming proportionality of the hiring cost to productivity is still a good assumption to make. The derivation and explanation of the results that follow are easier with this assumption, and there are no new results of interest that can be derived by making the alternative assumption that hiring costs depend on wages.

The number of jobs is endogenous and determined by profit maximization. Any firm is free to open a job vacancy and engage in hiring. Hence profit maximization requires that the profit from one more vacancy should be zero. In the environment of the simple model of this chapter, with each firm having one job only, profit maximization is equivalent to a zero-profit condition for firm entry. In chapter 3 we show that the same condition can be derived from a standard model of a competitive firm with costs of adjustment for employment, when the firm maximizes the present-discounted value of profits. We follow the single-job approach in this chapter to motivate the wage equation, which plays a key role in the analysis. Some of our later results, however, especially those derived for endogenous job destruction depend critically on this assumption.

Let J be the present-discounted value of expected profit from an occupied job and V the present-discounted value of expected profit from a vacant job. With a perfect capital market, an infinite horizon and when no dynamic changes in parameters are expected, V satisfies the Bellman equation

$$rV = -pc + q(\theta)(J - V). \tag{1.6}$$

A job is an asset owned by the firm. In a perfect capital market the valuation of the asset is such that the capital cost, rV, is exactly equal to the rate of return on the asset: The vacant job costs pc per unit time and changes state according to a Poisson process with rate $q(\theta)$. The change of state yields net return $J - V$. Since we are in a steady state, there are no capital gains or losses from expected changes in the valuation of jobs. Both V and J are constant.

In equilibrium all profit opportunities from new jobs are exploited, driving rents from vacant jobs to zero. Therefore the equilibrium condition for the supply of vacant jobs is $V = 0$, implying that

$$J = \frac{pc}{q(\theta)}. \tag{1.7}$$

This is the second key equation of the equilibrium model. For an individual firm, $1/q(\theta)$ is the expected duration of a vacancy. Condition (1.7) states that in equilibrium, market tightness is such that the expected profit from a new job is equal to the expected cost of hiring a worker. Since in the environment of this model a firm cannot enter the market with a filled job, there are rents in equilibrium associated with filled jobs. Competition for vacant jobs drives those rents down to the expected cost of finding a worker.

The asset value of an occupied job, J, satisfies a value equation similar to the one for vacant jobs. The flow capital cost of the job is rJ. In the labor market, the job yields net return $p - w$, where p is real output and w is the cost of labor. The job also runs a risk λ of an adverse shock, which leads to the loss of J. Hence J satisfies the condition,

$$rJ = p - w - \lambda J. \tag{1.8}$$

The firm takes the interest rate and product value as given, but the wage rate is determined by a bargain between the meeting firm and worker. Making use of equation (1.8) to substitute J out of the equilibrium condition (1.7), we derive the equation,

$$p - w - \frac{(r + \lambda)pc}{q(\theta)} = 0. \tag{1.9}$$

Equation (1.9), as we will see in chapter 3, corresponds to a marginal condition for the demand for labor. p is the marginal product of labor and $(r + \lambda)pc/q(\theta)$ is the expected capitalized value of the firm's hiring cost. If the firm had no hiring cost, c would be 0 and (1.9) would reduce to the standard marginal productivity condition for employment in the steady state. Because of the properties of the arrival rate $q(\theta)$, equation (1.9) can be represented by a downward-sloping curve in θ, w space. In this model the properties of the matching technology ensure that the demand for labor is downward-sloping, even for a constant marginal product of labor. We will refer to it (and to the equation representing it) as the *job creation* condition.

The Beveridge curve and job creation condition, (1.6) and (1.9), contain four unknowns—unemployment, the number of jobs, the real wage rate, and the real rate of interest. Between them they yield solutions for the two quantities—unemployment and jobs—in terms of the two prices—real wages and rate of interest—which are still to be determined. To determine them, we have to close the model by considering the behavior of workers and the demand side of the model.

1.3 Workers

Workers normally influence the equilibrium outcome through their job search and their influence on wage determination. In the simple version of the model in this chapter, the size of the labor force and each worker's intensity of search are fixed, whereas the assumption of common productivity in all jobs makes the job-acceptance decision trivial. So the only influence that workers have on the equilibrium outcome is through wages. In this section we derive the typical worker's returns when employed and when unemployed, which we use in the next section to derive the wage equation and in later chapters in other applications.

A typical worker earns w when employed and searches for a job when unemployed. During search the worker enjoys some real return z, which we measure in the same units as real wages. z may include a number of things, distinguished by the fact that they have to be given up when the worker becomes employed. We ignore any income that the worker receives regardless of whether he is employed or unemployed, since with risk neutrality and perfect capital markets such incomes do not play a role in the determination of unemployment.

The most obvious component of z is unemployment insurance benefits. Another component of z is the income that the unemployed worker may be able to earn by doing odd and irregular jobs in a secondary sector of the economy, if such a sector exists. z includes also the imputed real return from any unpaid leisure activities, such as home production or recreation. We show later in this and subsequent chapters that the way in which we specify z is important for many of our results. To begin with, we assume that z is constant and independent of market returns.

Let U and W denote the present-discounted value of the expected income stream of, respectively, an unemployed and an employed worker, including the imputed return from nonmarket activities. The unemployed worker enjoys (expected) real return z while unemployed, and in unit time he expects to move into employment with probability $\theta q(\theta)$. Hence U satisfies

$$rU = z + \theta q(\theta)(W - U). \tag{1.10}$$

Equation (1.10) has the same interpretation as the firm's asset equations (1.6) and (1.8). The asset that is valued is the unemployed worker's human capital and the valuation placed on it by the market is U, which is made up of the yield z and the expected capital gain from change of state $q(\theta)(W - U)$. rU can be given two useful interpretations. Since it is the average expected return on the worker's human capital during search, it is the minimum compensation that an unemployed worker requires to give up search. This makes it the unemployed worker's reservation wage (see also chapter 6). rU is also the maximum amount that the unemployed worker can spend without running down his (human) capital. Hence it is also the unemployed worker's normal or permanent income, with permanent income defined broadly to include imputed non-market returns.

Employed workers earn a wage w; they lose their jobs and become unemployed at the exogenous rate λ. Hence the valuation placed on them by the market, W, satisfies

$$rW = w + \lambda(U - W). \tag{1.11}$$

The permanent income of employed workers, rW, is different from the constant wage w because of the risk of unemployment. If there were job quitting in the model, rW would be the minimum net compensation that the worker would require to give up his job. Without on-the-job search, workers stay in their jobs for as long as $W \geq U$. The necessary and sufficient condition for this to hold is $w \geq z$. Although w is still an unknown, this inequality is assumed to hold. We show later that a sufficient condition for this to hold is $p \geq z$, which is imposed.

Equations (1.10) and (1.11) can be solved for the permanent incomes of unemployed and employed workers, in terms of the returns z and w and the discount and transition rates:

$$rU = \frac{(r + \lambda)z + \theta q(\theta)w}{r + \lambda + \theta q(\theta)}, \tag{1.12}$$

$$rW = \frac{\lambda z + [r + \theta q(\theta)]w}{r + \lambda + \theta q(\theta)}. \tag{1.13}$$

Since $w \geq z$, it follows from (1.12) and (1.13) that with discounting, employed workers have higher permanent incomes than unemployed workers: Unemployment is more costly to those currently experiencing it than to those who expect to experience it some time in the future. But without discounting, unemployed workers are not worse off than employed workers in permanent-income terms. The reason is that with infinite horizons all workers eventually participate equally in employment and unemployment, and the timing of their experience does not affect their ex ante returns.

1.4 Wage Determination

In equilibrium, occupied jobs yield a total return that is strictly greater than the sum of the expected returns of a searching firm and a searching worker. If the firm and worker who are together separate, each will have to go through an expensive process of search before meeting another partner. Since all job-worker pairs are equally productive, the expected joint return of the firm and the worker after they form new matches must be the same as the joint return from their current match. Hence a realized job match yields some pure economic rent, which is equal to the sum of the expected search costs of the firm and the worker (including forgone wages and profits). Wages need to share this economic (local-monopoly) rent, in addition to compensating each side for its costs from forming the job. We assume that the monopoly rent is shared according to the Nash solution to a bargaining problem.

The wage rate for a job is fixed by the firm and the worker after they meet. Because all jobs are equally productive and all workers place the same value on leisure, the wage fixed for each job is the same everywhere. But an individual firm and worker are too small to influence the market. When they meet, they fix the wage rate by taking behavior in the rest of the market as given.

Given our assumptions on productivity and the arrival process of idiosyncratic shocks, a firm and worker who are brought together by the matching process will always form a productive job. An employment contract between the meeting firm and worker is a wage w_i for each period of time that they are together and a separation rule that is contingent on the arrival of an idiosyncratic shock. We assume both here and throughout the analysis that the wage contract is renegotiated whenever new information arrives. This amounts to assuming that the wage rate continually satisfies the Nash sharing rule for the life of the job.

For a wage rate w_i the firm's expected return from the job, J_i, satisfies,

$$rJ_i = p - w_i - \lambda J_i. \tag{1.14}$$

Recall that if the firm is unoccupied the job is worth $V = 0$. The job is worth to the worker W_i, where

$$rW_i = w_i - \lambda(W_i - U). \tag{1.15}$$

The expected return from search, U, is independent of w_i and satisfies, as before, equation (1.12), with w denoting wages in the rest of the market.

The wage derived from the (generalized) Nash bargaining solution is the w_i that maximizes the weighted product of the worker's and the firm's net return from the job match. In order to form the job match, the worker gives up U for W_i and the firm gives up V for J_i. Therefore the wage rate for this job satisfies

$$w_i = \arg\max(W_i - U)^{\beta}(J_i - V)^{1-\beta}, \tag{1.16}$$

where $0 \leq \beta \leq 1$. In symmetric situations β is equal to $\frac{1}{2}$. More generally, there may be plausible bargaining situations that imply a different β, for example, when firms and workers have different rates of impatience. In those more general situations, β may be interpreted as a relative measure of labor's bargaining strength, other than the one implied by the "threat points" U and V. We will treat β as a constant parameter strictly between 0 and 1 throughout the analysis and think of $\frac{1}{2}$ as the most plausible value, since we are modeling symmetric situations.

The first-order maximization condition derived from (1.16) satisfies

$$W_i - U = \beta(J_i + W_i - V - U). \tag{1.17}$$

So in the model of this chapter, β is labor's share of the total surplus that an occupied job creates.

Condition (1.17) can be converted into a wage equation in a number of ways; we show here two. First, by substituting W_i and J_i from (1.15) and (1.14) into (1.17), and by imposing the equilibrium condition $V = 0$, we derive the flow version of (1.17),

$$w_i = rU + \beta(p - rU). \tag{1.18}$$

Workers receive their reservation wage rU and a fraction β of the net surplus that they create by accepting the job: product value net of what they give up, rU. rU is not a particularly interesting variable in the equilibrium solution of the model. A simpler and more appealing version of the wage equation may be derived by noting that (1.18) implies that all jobs will offer the same wage, and by making use of (1.17) and the equilibrium condition for jobs, (1.7), to substitute $W - U$ out of (1.10). This gives the following expression for rU:

$$rU = z + \frac{\beta}{1-\beta} pc\theta, \tag{1.19}$$

which, when substituted into (1.18) yields the aggregate wage equation that holds in equilibrium,

$$w = (1-\beta)z + \beta p(1 + c\theta). \tag{1.20}$$

Equation (1.20) is the most convenient form of the wage equation for the applications that we will study. It is intuitive for a market equilibrium if we note that $pc\theta$ is the average hiring cost for each unemployed worker (since $pc\theta = pcv/u$ and pcv is total hiring cost in the economy). Workers are rewarded for the saving of hiring costs that the representative firm enjoys when a job is formed. The way that market tightness enters the wage equation in our model is through the bargaining power that each party has. A higher θ indicates that jobs arrive to workers at higher rate than workers do to vacant jobs, relative to an equilibrium with lower θ. The worker's bargaining strength is then higher and the firm's lower, and this leads to a higher wage rate.

Equation (1.20) replaces the labor supply curve of Walrasian models. Labor supply in our model is fixed: The labor force size is constant, workers search with constant intensity, and they work a fixed number of hours when in a job. The Walrasian labor supply curve in our model is a vertical line at the fixed labor force size. But the existence of local monopoly power in this model and the sharing rule used to solve for wages imply that even with fixed labor product and labor supply, there is an upward-sloping relation in θ, w space (or alternatively, for given vacancies, a downward-sloping relation between wages and unemployment). We refer to this curve as the *wage curve* for brevity, though it can also be referred to as the wage-setting function or wage-determination curve.

We finally note one more property of the sharing rule (1.17). A job in equilibrium creates a positive surplus for both the firm and the worker, which is equal to the sum of the expected cost of search and the expected cost of hiring. The firm's net return from the job is J which, by (1.7), must be positive and equal to the expected hiring cost. From (1.14) and (1.20) it follows that the product p must be strictly greater than the worker's nonmarket return, z, and labor's share β must be strictly less than 1; otherwise, no firm will have an incentive to open a job. The worker's return from a job, $W - U$, may then be strictly positive, if $\beta > 0$, but it is also feasible to have $\beta = 0$, where the firm enjoys the entire surplus from the job. We will assume that p is sufficiently high to ensure the existence of a nontrivial equilibrium and that β is strictly less than 1.

1.5 Steady-State Equilibrium

Equilibrium is a triple (u, θ, w) that satisfies the flow equilibrium condition (1.5), the job creation condition (1.9), and the wage equation (1.20). The real interest rate is left out of this analysis and for the time being assumed to be exogenous and constant. For given interest rate, equations (1.9) and (1.20) determine the wage rate and the ratio of vacancies to unemployment; given the ratio of vacancies to unemployment, equation (1.5) determines unemployment. With knowledge of θ, the evolution of employment is obtained from the assumption of a constant labor force and the evolution of output from the assumption of a constant output per job.

Because of the structure of the model, it is convenient to refer to θ as the unknown, in place of the number of jobs or job vacancies. Recall that the number of jobs is equal to employment, $(1 - u)L$, plus job vacancies, θuL; therefore, if we know θ and u, we also know the number of jobs. As before, we refer to θ as labor-market tightness (or as the v/u ratio). The three equations determining steady-state equilibrium are reproduced here for convenience:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}, \tag{1.21}$$

$$p - w - \frac{(r + \lambda)pc}{q(\theta)} = 0, \tag{1.22}$$

$$w = (1 - \beta)z + \beta p(1 + c\theta). \tag{1.23}$$

Equilibrium is easily shown to be unique with the help of two diagrams, one that replaces the conventional demand and supply diagram for labor and a new diagram with the Beveridge curve as its centerpiece.

Figure 1.1 shows equilibrium for tightness and wages. Recall that (1.22) is the job creation curve, and in tightness-wage space it slopes down: Higher wage rate makes job creation less profitable and so leads to a lower equilibrium ratio of jobs to workers. It replaces the demand curve of Walrasian economics. Equation (1.23) is the wage curve and slopes up: At higher market tightness the relative bargaining strength of market participants shifts in favor of workers. It replaces the supply curve. Equilibrium (θ, w) is at the intersection of the two curves and it is unique.

Consider now figure 1.2, the Beveridge diagram. Figure 1.1 shows that the equilibrium θ is independent of unemployment. The equation for this θ can be explicitly derived by substituting wages from (1.23) into (1.22), to get

$$(1-\beta)(p-z)-\frac{r+\lambda+\beta\theta q(\theta)}{q(\theta)}pc=0. \tag{1.24}$$

In the vacancy-unemployment space of figure 1.2, this is shown as a line through the origin, with slope θ. The steady-state condition for

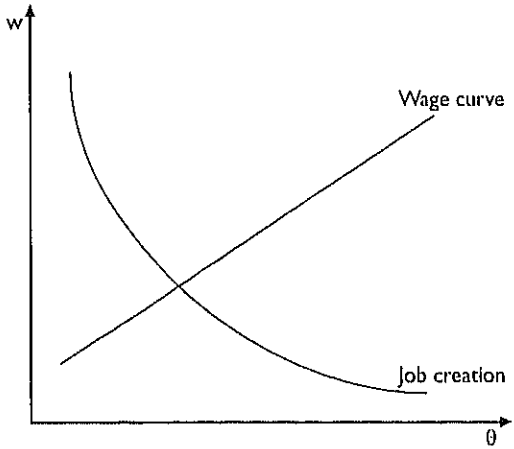

Figure 1.1
Equilibrium wages and market tightness

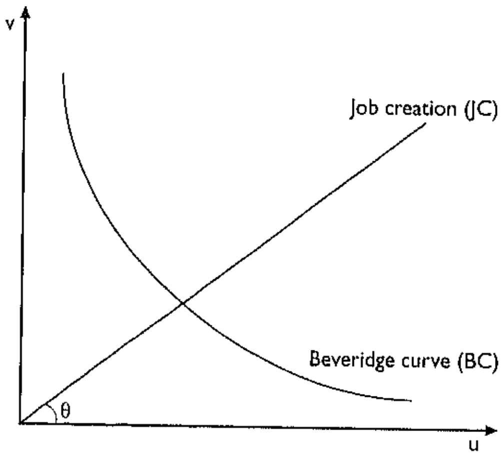

Figure 1.2
Equilibrium vacancies and unemployment

unemployment, (1.21), is the Beveridge curve, and it is convex to the origin by the properties of the matching technology. When there are more vacancies, unemployment is lower because the unemployed find jobs more easily. Diminishing returns to individual inputs in matching imply the convex shape. Equilibrium vacancies and unemployment are at the unique intersection of the job creation line and the Beveridge curve.

Some of the more interesting properties of equilibrium can be shown in the diagrams. Higher labor productivity increases p and shifts the job creation line in figure 1.1 to the right and the wage curve up. Since $\beta < 1$, the job creation curve shifts by more, so both wages and market tightness increase (see also equation 1.24). In figure 1.2 this rotates the job creation line anticlockwise, increasing vacancies and reducing unemployment.

Examination of the wage equation and (1.24) shows that the productivity effect on market tightness and unemployment takes place because of the fixed income (imputed or actual) of unemployed workers, z. The fixed z stops wages from fully absorbing productivity changes, so at higher productivity the profit from job creation is higher, leading to more job creation and lower unemployment. Although this is intuitive within the framework of the model of this chapter, it is not a desirable property of a model in long-run equilibrium. In the long run, wages should fully absorb productivity changes, at least when they are labor-augmenting, and a balanced-growth equilibrium with constant unemployment should exist.

Extensions that imply that wages fully absorb productivity changes are easy to find. The easiest, though not most general, is when z is primarily unemployment insurance income which is fixed in terms of the average wage rate. If we let $z = \rho w$, where ρ is the replacement rate (a policy parameter), then the wage equation becomes

$$w = \frac{\beta(1+c\theta)}{1-(1-\beta)\rho}p. \tag{1.25}$$

With this wage equation the job creation condition (1.22) becomes

$$1 - \frac{\beta(1+c\theta)}{1-(1-\beta)\rho} - \frac{(r+\lambda)c}{q(\theta)} = 0, \tag{1.26}$$

making the equilibrium θ independent of productivity. Equilibrium unemployment is then also independent of the average level of productivity.

Other more general assumptions about the value of nonmarket time also have such neutrality implications, though not necessarily with respect to the rate of growth of productivity. We return to this question in chapter 3, where we argue that some inflexibility in z may be a reasonable feature of short- to medium-run equilibrium but not of long-run equilibrium, and where we look at the effects of productivity growth.

A higher nonmarket return for workers, shown by a higher z (or higher ρ if the wage equation is (1.25)), shifts the wage curve in figure 1.1 up and therefore increases wages but reduces market tightness. Workers claim a higher wage when z is higher because the cost of unemployment is lower, and with higher wages firms create fewer jobs. A higher β has similar effects, for similar reasons. The job creation line in figure 1.2 rotates clockwise, reducing vacancies and increasing unemployment. Note that these effects of unemployment income are obtained by ignoring the disincentive effects that this income has on job search and job acceptance, a question that we address in chapters 5 and 6.

A higher real interest rate or arrival rate of negative idiosyncratic shocks (and so higher job destruction rate at given unemployment rate) shifts the job creation curve in figure 1.1 to the left. The reason is that in the case of the higher interest rate, the future revenues from a job are discounted more heavily, and in the case of the higher destruction rate, the life of the job is on average shorter. They both lead to lower tightness and wage rate because the costs of job creation have to be paid up front. In the Beveridge diagram both changes rotate the job creation line down, but the increase in the arrival rate of idiosyncratic shocks also shifts the Beveridge curve out. The higher interest rate increases unemployment and reduces vacancies. The higher job destruction rate increases unemployment but has uncertain effect on vacancies.

The higher λ shifts the Beveridge curve out because at given unemployment rate a higher λ implies a bigger flow into unemployment than out of it. Unemployment needs to increase to bring the flow out of unemployment into equality with the higher inflow. Another change that can shift the Beveridge curve out is an exogenous fall in the rate of job matching, namely a downward shift in the matching function for given vacancies and unemployment. We have not yet discussed reasons for such a shift in this chapter, but we show in chapters 5 and 6 that exogenous changes in job matching can be caused by changes in a small number of parameters that affect, in equilibrium, the searching behavior of firms and workers. There is, however, another cause of exogenous changes in the rate of job matching, which is related to the usefulness of the concept of the aggregate matching function, and which we discuss here.

This cause is the degree of mismatch in the economy. As we have already argued, job matching in a real economy does not take place instantaneously because of heterogeneities in the qualities of jobs and workers, differences in their location, and imperfect information about these and other relevant parameters. The matching function in our model is a convenient modeling device that summarizes the effects of these factors on the speed of job formation. Mismatch can be thought of as an empirical concept that measures the degree of heterogeneity in the labor market. If mismatch in an economy were identically zero, the matching function would not exist and jobs and workers would meet instantaneously. It is because of the existence of some mismatch that meetings take place only after a search and application process. Therefore, if there is an exogenous rise in mismatch, the rate of job matching at given labor-market tightness must fall, and so the Beveridge curve must shift to the right, away from the origin. Since a fall in the rate of job matching also reduces the arrival rate of workers to jobs at given market tightness, it also shifts the job creation curve in figure 1.1 down and to the left, reducing wages and equilibrium tightness. In figure 1.2 this is shown by a downward rotation of the equilibrium job creation line, giving rise to more unemployment but to no apparent change in vacancies.

Of course, if empirically mismatch changes are frequent, the usefulness of the concept of the matching function is reduced. But the requirement that changes in mismatch are not frequent is not different from the one of other aggregate functions, such as the aggregate production function and the money demand function. If empirically there are many shifts in the aggregate production function because of problems associated with the aggregation of capital and labor, its usefulness in macroeconomic modeling is reduced. Our use of an aggregate matching function relies as much on the absence of serious aggregation problems as do the other aggregate functions used in macroeconomics. Whether in practice the matching function is a useful device or not is an empirical question. The available empirical evidence supports it sufficiently well to justify its use in the macroeconomic modeling of unemployment (see the notes on the literature at the end of this chapter).

In the empirical literature, mismatch bears some relationship to the frequently discussed sectoral shifts hypothesis, and to the older view of structural unemployment, which was thought to be unemployment arising from fast structural change in the economy as a whole. For example, it has been argued that the oil, technology, and other supply shocks of the 1970s and 1980s increased the speed with which unemployed workers needed to adapt to the changing requirements of employers. This led to increased mismatch, which increased unemployment at given vacancies. However, neither the sectoral shifts hypothesis in the United States, nor the mismatch hypothesis in Europe, has had much success in accounting for a large fraction of fluctuations in employment, which is another indication of the stability of the aggregate matching function.

1.6 Capital

We now introduce the capital decision in the model of the preceding sections and show that under the assumption that there is a perfect second-hand market for capital goods, on the one hand, the essential features of the unemployment model remain unaltered and on the other hand the capital decision is unaffected by the existence of matching frictions. We continue assuming that the interest rate is exogenous, an assumption that we relax in chapter 3. The assumptions that follow will justify our use of the model without capital in later chapters.

Suppose that the firm can buy and sell capital at the price of output in a perfect market. No time lapses between the decision to trade in capital and the execution of the decision. Since capital is costly, the firm will buy it only when there is a worker occupying the job; that is, vacancies do not own capital.

The productivity p of the model of preceding sections is reinterpreted as a labor-augmenting productivity parameter that measures the efficiency units of labor. Letting K and N, respectively, denote aggregate capital and employment, there is an aggregate production function $F(K, pN)$, with positive but diminishing marginal products and constant returns to scale. Defining k as the ratio K/pN (i.e., the capital stock per efficiency unit of labor), we define $f(k)$ as output per efficiency unit of labor, $F(K/pN, 1)$. The per-unit production function $f(k)$ satisfies $f'(k) > 0$ and $f''(k) < 0$.

When the job is vacant, its asset value is given by V and satisfies the same value equation as before, (1.6). But when a worker arrives and a wage rate agreed, the firm hires capital k for each efficiency unit of labor. The capital stock owned (or rented, it makes no difference with a perfect second-hand market for capital goods) by the firm becomes part of the value of the job, so the asset value of an occupied job is now given by $J + pk$, the sum of the present-discounted value of profits and the value of the rented capital stock. The real capital cost of the job is $r(J + pk)$. In the labor market the job yields net return $pf(k) - \delta pk - w$, where $pf(k)$ is real output, δpk is capital depreciation, and w is the cost of labor. The job also runs a risk λ of an adverse shock, which leads to the loss of J but not of k, which can be sold in the second-hand market. Hence J is determined by the asset-valuation condition

$$r(J + pk) = pf(k) - \delta pk - w - \lambda J, \tag{1.27}$$

which generalizes (1.8).

The firm takes the interest rate and the wage rate as given and rents as much capital as is necessary to maximize the value of the job. The maximization of J with respect to k gives the familiar equilibrium condition for the firm's capital stock,

$$f'(k) = r + \delta. \tag{1.28}$$

The marginal product of capital is equal to the marginal cost of capital, the rental plus the depreciation rate.

A rearrangement of equation (1.27) shows that it is equivalent to (1.8) with the generalization that job product p is replaced by $p[f(k) - (r + \delta)k]$. Of course, with (1.28) holding, the term in the square brackets is the marginal product of an efficiency unit of labor, $f(k) - kf'(k)$. None of the other asset value equations is affected by the introduction of the capital stock. Also, because the firm can buy and sell capital in a free market, the wage bargain is not affected by the introduction of capital—the worker cannot hold up the firm that has committed itself to a capital investment because capital investments are liquid (and reversible, although technology is still irreversible). Therefore the model can be solved as before but with the generalization that product p is multiplied by $[f(k) - (r + \delta)k]$ in all expressions. We restate here the equilibrium conditions with this generalization:

$$f'(k) = r + \delta, \tag{1.29}$$

$$p[f(k) - (r + \delta)k] - w - \frac{(r + \lambda)pc}{q(\theta)} = 0, \tag{1.30}$$

$$w = (1 - \beta)z + \beta p[f(k) - (r + \delta)k + c\theta], \tag{1.31}$$

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}. \tag{1.32}$$

For given interest rate, the equilibrium system in (1.29) to (1.32) is recursive. With knowledge of r, equation (1.29) gives the capital-labor ratio. With knowledge of r and k, the block (1.30) to (1.31) gives wages and market tightness, very much along the lines illustrated in figure 1.1. Finally, for given tightness, equation (1.32) determines unemployment, along the lines of figure 1.2. The parametric effects described with the aid of figures 1.1 and 1.2 are not altered by the introduction of capital.

The equilibrium aggregate capital stock in this economy is $L(1 - u)pk$ and equilibrium employment $L(1 - u)$, so aggregate output is $L(1 - u)pf(k)$. Since we assumed a constant interest rate, the supply of capital needs to be infinitely elastic to satisfy the equilibrium conditions (1.29) to (1.32). We will return to this issue in chapter 3, when we consider the determination of the interest rate for an endogenous supply of capital.

1.7 Out-of-Steady-State Dynamics

The discussion of the preceding sections was entirely about steady states. The theory of unemployment in it, however, was explicitly derived from (1) a dynamic equation for the evolution of the unemployment stock, which was derived from models of the job creation and job destruction flows, and (2) forward-looking rational expectations behavior by firms and workers in job creation and wage determination. We now examine the implications of these assumptions for the dynamic behavior of the economy out of steady state.

Although the analysis of out-of-steady-state dynamics suggests various directions for the development of a stochastic model of fluctuations, we do not pursue the development of a full model of the business cycle here. Our focus is on the dynamic behavior of unemployment, vacancies, and wages when there is a matching function. A full model of the cycle would require a more general treatment of capital than the one offered in the preceding section and would also require the introduction of consumption and savings (see the Notes on the Literature for some recent work in this area of research). Instead, here we ignore capital and focus on the dynamics of the triple (θ, w, u) with the help of figures 1.1 and 1.2, ignoring all markets other than the one for labor. In this context the questions of interest are whether (1) there is a unique stable path to the steady state of figures 1.1 and 1.2, and (2) the out-of-steady-state dynamics of unemployment and vacancies are consistent with the cyclical stylized fact, that when the economy is off the Beveridge curve of figure 1.2 it traces anticlockwise loops around the curve.

In the steady-state model we derived the equilibrium value of tightness from the assumption that the expected profit from the creation of a new job vacancy was zero. We now assume that this property holds out of the steady state as well. To ensure that this assumption can hold, job vacancies, and so market tightness, have to be "jump" variables; that is, firms have to be able to open up or close vacancies instantaneously so as to ensure that the value of a new vacancy is always zero.

Similar assumptions are made about wage determination. Wages in the steady state were derived from the assumption that the firm and worker shared the rents from a job according to (1.17). We now assume that the same sharing rule holds out of steady state, consistent with our assumption that the firm and worker can renegotiate any time new information arrives. This assumption also requires that wages are a jump variable and change without delay and as necessary during adjustment, which is needed if (1.17) is to be always satisfied.

The dynamic behavior of unemployment contrasts with that of vacancies and wages. Although firms and workers can decide in isolation and without delay whether to form or break a match once they meet, at the aggregate level match formation is governed by the matching technology. The matching technology does not allow jumps in job formation; it describes a slow, stable and backward-looking process that is governed by the difference between the job creation and job destruction flows, as shown in equation (1.3). This makes unemployment a predetermined variable at any moment in time.

It is natural to expect the out-of-steady-state dynamics of a model that combines a backward-looking stable process with forward-looking jump variables to be characterized by a saddle path. Under the assumption of rational expectations we can show that the saddle path is the unique stable path that takes the economy to its steady-state equilibrium.

In order to derive the dynamic equations for wages and market tightness, we need to specify the expected returns of firms and workers out of steady state. The net worth of jobs and workers is now an explicit function of time. The arbitrage equations determining their value are similar to the ones that hold in the steady state, except that now they recognize the fact that there may be capital gains or losses from changes in the valuation placed by the market on jobs and workers.

Let again V denote the asset value of a vacant job. With a perfect capital market and perfect foresight it satisfies the arbitrage equation

$$rV = -pc + \dot{V} + q(\theta)(J - V). \tag{1.33}$$

As in the steady state, the left-hand side of (1.33) is the capital-market cost of the asset. The right-hand side is the labor-market return: a yield $-pc$, expected capital gains from changes in the valuation of the asset $\dot{V}$, and expected capital gains from the chance of finding a worker to take the vacancy. Comparison of (1.33) with (1.6) shows that the only thing that has changed in the valuation of a vacant job is that now account has to be taken of the changes in the value of the job during adjustment.

The value of a filled job, J, satisfies a similar arbitrage condition. In the absence of capital we get

$$rJ = p - w + \dot{J} - \lambda J. \tag{1.34}$$

$\dot{J}$ is the expected capital gain from changes in job value during adjustment.

Our assumption that firms exploit all profit opportunities from new jobs, regardless of whether they are in the steady state or out of it, implies that $V = \dot{V} = 0$. Therefore J is determined by the two equations

$$J = \frac{pc}{q(\theta)}, \tag{1.35}$$

$$\dot{J} = (r + \lambda)J - (p - w). \tag{1.36}$$

The net worth of employed and unemployed workers are given by two arbitrage equations that are similar to (1.33) and (1.34). As before, U denotes the net worth of an unemployed worker and W the net worth of an employed worker. The arbitrage equations when changes in valuations take place because of out-of-steady-state dynamics are

$$rU = z + \dot{U} + \theta q(\theta)(W - U) \tag{1.37}$$

and

$$rW = w + \dot{W} + \lambda(U - W). \tag{1.38}$$

All differential equations for asset values are unstable because of arbitrage and perfect foresight.

Wages are determined by the Nash solution to the bargaining problem, which as before implies the sharing rule (1.17). Because we allow wages to be renegotiated continually, (1.17) holds also in rates of change. Going through the same substitutions as for the steady-state wage equation, we derive the same equation as before, (1.20), which now holds both in and out of the steady state. Thus, for given productivity and income during unemployment, the out-of-steady-state dynamics of wages are driven entirely by the dynamics of labor-market tightness.

We are now ready to describe the out-of-steady-state dynamics of wages and tightness (figure 1.1). By (1.35), the job value J is a monotonically increasing function of tightness. Because wages are also a monotonically increasing function of tightness without sticky dynamics, the dynamic system in (1.35) and (1.36) can have only one rational expectations solution, $\dot{J} = \dot{\theta} = 0$. (To see this more explicitly, substitute J from (1.35) and wages from (1.17) into (1.36).) Therefore the out-of-steady-state equilibrium of tightness and wages is still given by the intersection of the two curves in figure 1.1. Whatever the initial conditions in this economy, vacancies and wages instantaneously jump to the intersection of the two curves.

This contrasts with the dynamics associated with the Beveridge curve. To demonstrate this, we combine the dynamics of unemployment with those of tightness into a two-equation system with u and θ as the unknowns. The equation for the evolution of unemployment, (1.3), is stable with driving force θ. Substitution of wages and job value J from (1.17) and (1.35) into (1.36) gives an unstable equation in θ, with no other unknown in it. The critical point (equilibrium) of the two-equation system is a saddle point. The sign pattern of a first-order linear approximation to the two differential equations is

$$\begin{pmatrix} \dot{u} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} - & - \\ 0 & + \end{pmatrix} \begin{pmatrix} u \\ \theta \end{pmatrix}. \tag{1.39}$$

With negative determinant in (1.39), the necessary and sufficient conditions for a saddle-point equilibrium are satisfied.

The saddle point arises because one of the variables, unemployment, is sticky and stable, whereas the other, vacancies, is forward-looking and unstable. Firms in this model treat vacancies as an asset; it is the price that has to be paid now in order to attract employees in the future. The expected arrival of employees is the rate of return on the asset held by the firm, and as with other assets, there is an instability inherent in the supply of vacancies. If the arrival rate of employees is expected to fall, the firm wants to have fewer vacancies when the fall takes place. It therefore endeavors to reduce its vacancies by hiring its employees before the expected fall. But to hire more employees sooner, the firm needs to open up more vacancies. Thus, an expected fall in the arrival rate of employees leads to more vacancies coming into the market and to an immediate fall in the arrival rate of employees to each vacancy.

The expected changes in the arrival rate of employees play the role of expected capital gains or losses on the firm's outstanding vacancies. The unique feature of vacancies as an asset is that to get to a situation where the firm needs fewer of them, it needs to open more of them initially. This implies that vacancies overshoot their equilibrium value when an adjustment is expected to take place. This can also be shown more formally.

The perfect foresight path in the neighborhood of equilibrium is unique: The number of stable roots in (1.39) is equal to the number of predetermined variables. The initial condition on the predetermined variable, and the requirement that the perfect foresight path should converge (a terminal condition), uniquely define an initial point in (θ, u) space, from which adjustment to equilibrium takes place. In the absence of anticipated changes in the exogenous variables, the initial point is always on the saddle path, since this is the unique convergent path.

In system (1.39) the saddle path is easily found because of the independence of the second equation from unemployment. Since θ is the unstable variable, if θ is not in equilibrium, it will diverge. So the saddle path is the θ-stationary (figure 1.3). If at any point in time unemployment is, say, equal to u_0, in the absence of anticipated future changes in the equilibrium position, the system must be at point A on the saddle path. Adjustment then takes place along the saddle path, with θ constant and unemployment falling until there is convergence to equilibrium.

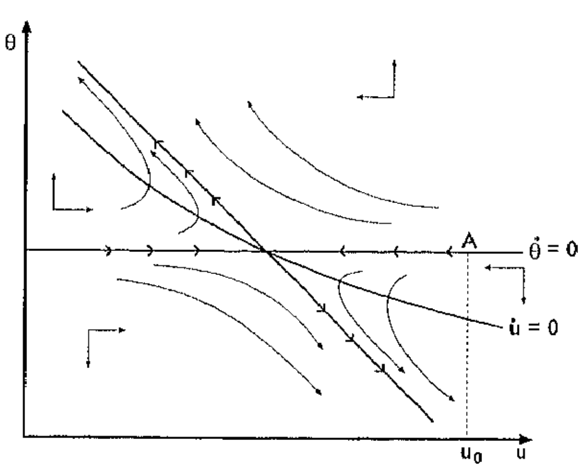

Figure 1.3
Adjustment paths in labor-market tightness and unemployment space

The translation of the dynamics of figure 1.3 to vacancy-unemployment space is straightforward (figure 1.4). Since the θ-stationary is a line through the origin in vacancy-unemployment space, the saddle path is this line. So in the absence of anticipated changes, the perfect foresight adjustment path implies that both unemployment and vacancies change in the same direction during adjustment, even though their equilibrium locus (the Beveridge curve) is downward-sloping. This is the overshooting feature of vacancies. If, say, unemployment is expected to fall from some initial value u_0 toward the intersection of the two curves in figure 1.4, the return from vacancies when unemployment is at u_0 is higher than the anticipated return during adjustment. This is because at higher unemployment, the rate at which workers arrive to vacancies is higher. So firms open up more vacancies at the beginning of adjustment than the number they expect to have in equilibrium. During adjustment the number of vacancies falls through the matching process. But generally, the ratio of vacancies to unemployment does not stay the same when vacancies and workers are matched in pairs, unless in equilibrium vacancies and unemployment happen to be equal (i.e., unless by chance, $\theta = 1$). If vacancies and unemployment are not equal to each other ($\theta \neq 1$), vacancies enter or exit during adjustment so as to maintain the ratio θ constant.

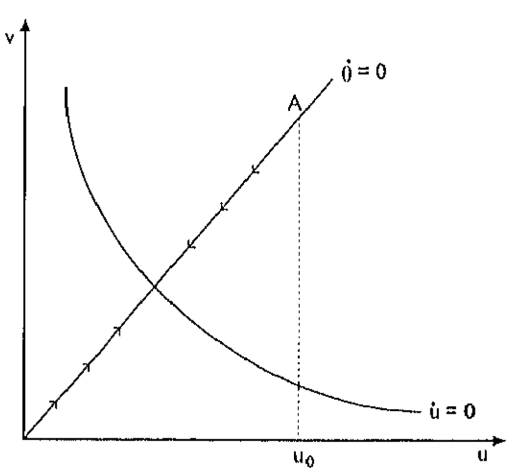

Figure 1.4
Adjustments in vacancy-unemployment space

How do wages, vacancies, and unemployment respond to changes in productivity p? A rise in p shifts the wage curve up and the job creation curve to the right, causing an immediate rise in both market tightness and wages (figure 1.1). The two variables jump to their new equilibrium; there are no adjustment dynamics. In the Beveridge curve diagram, figure 1.5, the impact effect is an anticlockwise rotation of the job creation line. If the initial equilibrium point is A, initially equilibrium jumps to B, as firms open more vacancies to take advantage of the higher productivity. This sets in motion unemployment dynamics, which move the economy down the new job creation line, toward the new steady-state equilibrium point C. In the case of a fall in p, the adjustment dynamics move the economy in the opposite direction, from C to D and then up to A.

Thus wages and tightness move in jumps in response to news about productivity changes, whereas vacancies and unemployment trace anticlockwise loops around the Beveridge curve. Although the loops have spikes due to the overshooting of vacancies, which is not a feature of the data, the out-of-steady-state dynamics derived here are broadly consistent with the stylized observation that over the business cycle vacancies and unemployment trace anticlockwise loops in vacancy-unemployment space. Whether the model is consistent with other business cycle facts of

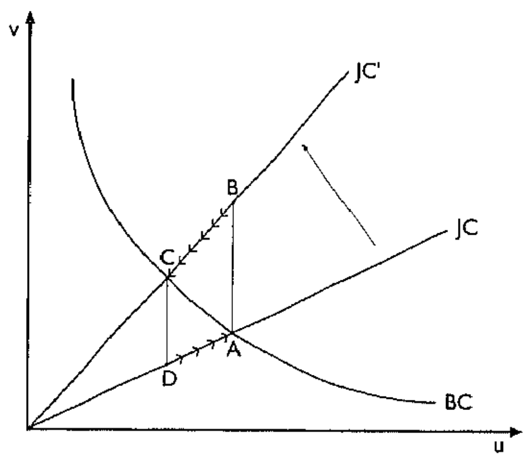

Figure 1.5
Adjustments in response to productivity changes in vacancy-unemployment space

labor markets is a difficult question that needs more explicit modeling of the business cycle and the capital market.

1.8 Notes on the Literature

The idea that a theory of unemployment can fruitfully be built on the assumption that trade in the labor market is an economic activity was first explored by a number of authors in the late 1960s, in what became known as search theory. The most influential papers in this tradition were Alchian (1969), Phelps (1968), and Mortensen (1970); they were collected with other contributions in the same spirit in the Phelps volume (Phelps et al. 1970). The impetus to this research came from Phelps's (1967) and Friedman's (1968) reappraisal of the Phillips curve and the natural rate approach to which this led.

Early search theory assumed the existence of a distribution of wage offers for identical jobs; unemployment arose in equilibrium because workers rejected low-wage jobs. This aspect of the theory was criticized both on logical grounds (Rothschild 1973) and on empirical grounds (Tobin 1972; Barron 1975). An equilibrium model that met Rothschild's criticisms, but with a trivial role for workers looking for alternative jobs, was first presented in Lucas and Prescott (1974).

Lucas and Prescott's model did not consider matching problems. Several early contributions (most notably Phelps 1968) had a matching function relating unemployment and vacancies to hirings, but this device was not generally used to bypass the need to model reservation wages as the main economic mechanism driving unemployment. Early applications of the concept of the matching function that downplay the role of reservation wages include Hall (1979), Pissarides (1979), and Bowden (1980). Diamond and Maskin (1979) used the similar concept of "search technology" in a related context.

The application of zero-profit conditions for new jobs, leading to a closed model with endogenous demand for labor, was first discussed in Pissarides (1979, 1984b).

Early search theory (e.g., Mortensen 1970) had a theory of monopolistic wage setting by firms. The Nash solution was first applied in this context with fixed numbers of traders by Diamond (1982b), though earlier papers by Mortensen (1978) and by Diamond and Maskin (1979) discussed similar sharing rules for the division of the surplus from a job match. Pissarides (1984a, 1985a, b) also applied the Nash rule to derive a wage equation. The use of the Nash solution to the bargaining problem in models of this kind is justified by Binmore, Rubinstein, and Wolinski (1986) by an application of sequential bargaining theory. For alternative assumptions about wage setting, see Burdett and Mortensen (1998), Moen (1997), and the survey of Mortensen and Pissarides (1999b).

The unemployment-vacancy curve has a long history in the British literature on structural change in the labor market. Its existence was noted by Beveridge (1944) and now bears his name. Pioneering work on the Beveridge curve was done by Dow and Dicks-Mireaux (1958), Holt and David (1966), and Hansen (1970). Hansen derived the curve from a model of distinct labor markets interacting at different levels of disequilibrium. His approach does not rely on the existence of a stable matching function but, as he points out, is consistent with it. Much of the early work on vacancy-unemployment interactions was motivated by the desire to find a suitable measure of the excess demand for labor to use in wage-inflation (Phillips curves) studies. In this context, see Dicks-Mireaux and Dow (1959), Lipsey (1960, 1974), and Phelps (1968).

The Phillips curve motivation has been absent from the recent literature (but see the recent paper by Cooley and Quadrini 1998). Instead, authors have been preoccupied with (1) understanding the dynamics of labor markets within explicit dynamic models and (2) building real macroeconomic models with frictions that can better match the business cycle facts. The matching function has played a key role in both strands of the literature.

Despite its importance there are very few attempts to derive the matching function from primitive assumptions about trade. Hall (1979), Pissarides (1979), and Blanchard and Diamond (1994) have borrowed Butters's (1976) urn-ball game to derive an exponential function. Their derivation is, however, mechanical and assumes the absence of all information about potential trading partners. Julien, Kennes, and King (1998) derive a similar function by assuming that job candidates auction their labor services to potential buyers. Ioannides (1997), Lagos (1997), and Lagos and Violante (1998) have studied micro models of agent interaction and derived some properties of the matching technology from more primitive assumptions about market exchange.

In most empirical applications, matching functions are usually assumed to be of the Cobb-Douglas form with constant returns to scale. Coles and Smith (1998) proposed an alternative form where the existing stock of unemployed workers can match only with the inflow of vacancies and the existing stock of vacancies can match only with the inflow of unemployed workers. A similar model was also estimated by Gregg and Petrongolo (1997).

There is now a large number of empirical estimates of aggregate matching functions and the implied Beveridge curves. They generally establish the existence and stability of an aggregate matching function with constant returns to scale. The empirical literature, both micro and macro, is surveyed by Divine and Kiefer (1991). For studies directly relevant to the approach taken in this chapter and using aggregate data, see Pissarides (1986) and Layard, Nickell, and Jackman (1991) for the United Kingdom; Abraham (1987), Blanchard and Diamond (1989) and Berman (1997) for the United States; Gross (1997) and Entorf (1998) for Germany; Schager (1987) for Sweden; Feve and Langot (1996) for France; and van Ours (1991) and Broersma and van Ours (1998) for the Netherlands. These studies generally accept the assumption of a log-linear function with constant returns to scale. An exception is Warren (1996), who estimates a translog matching function by making use of monthly U.S. manufacturing data and finds increasing returns to scale.

For estimates using disaggregated regional data, see Anderson and Burgess (1995), Burda and Profit (1996), Linderboom, van Ours, and Renes (1994), Gorter and van Ours (1994), Boeri and Burda (1996), Coles and Smith (1996), and Burgess and Profit (1998).

Structural estimation of the Beveridge curve and the other equilibrium conditions of the model was undertaken by Yashiv (1997b). The vacancy (job creation) curve was also estimated with Dutch data by van Ours and Ridder (1991, 1992). Evidence for a wage curve of the kind drawn in figure 1.1 was accumulated by Blanchflower and Oswald (1994).

The out-of-steady-state analysis of unemployment and vacancies was first discussed in Pissarides (1985a, 1987). In Pissarides (1985a), imputed unemployment income was assumed fixed, but the model contained more features than the models in this chapter. In Pissarides (1987) unemployment income was allowed to depend on wealth (see chapter 3).

Real business cycle extensions of the model of this chapter have been calibrated by Merz (1995), Andolfatto (1996), and Yashiv (1997a). The existence of frictions in the labor market and the noncompetitive elements that it implies provide a richer framework for the analysis of fluctuations in employment than usually found in other models. The search explanation of unemployment is consistent with the assumptions of Hansen (1985) and Rogerson (1988) of indivisible labor because the assumption that workers either search or work follows naturally from the existence of indivisibilities. In calibrations, matching models are usually compared with Hansen's calibrated model and shown to perform at least as well. Shi and Wen (1997) integrated the search equilibrium model with capital accumulation derived from an intertemporal utility maximization framework and derived various analytical results, including a hump-shaped response of output to a productivity shock. For an alternative model of the cycle where job matching drives the fluctuations in employment, see Howitt (1988).

The anticlockwise loops around the Beveridge curve were present in the data examined by the pioneering works on unemployment-vacancy dynamics, such as Dow and Dicks-Mireaux (1958) and Holt and David (1966), as in more recent data. Various explanations have been put forward, for example, by Phelps (1968), Hansen (1970), and Bowden (1980), that rely on the idea that labor demand is more flexible than employment. The phenomenon is regular enough to have the status of a stylized fact of business cycles.

The framework of the model of the next chapter is more suitable for the study of sectoral shifts, so discussion of the relevant literature is postponed.

2 Endogenous Job Destruction

The model of the preceding chapter highlighted an important property of equilibrium: In the steady state the rate of job creation is equal to the rate of job destruction. Differences in the two rates induce out-of-steady-state employment dynamics. In the simple model of chapter 1, however, the rate of job destruction was a constant λ. So, with the exception of exogenous changes in the rate of job destruction, the influence of shocks and parameter changes on the natural rate of unemployment operated through the rate of job creation.

But empirical evidence shows that both job creation and job destruction respond to exogenous shocks. In some instances, as in the case of business cycle shocks, there is evidence that the rate of job destruction is even more responsive than the rate of job creation. This chapter generalizes the simple model of chapter 1 by making job destruction an unknown of the model that depends on the optimizing actions of firms and workers. Parameter or policy influences on unemployment now work through both job creation and job destruction. The assumptions of the model are consistent with the evidence of Davis, Haltiwanger, and Schuh (1996) and others on the nature of the job destruction and job creation processes in industrial countries.

2.1 Productivity Shocks and Reservation Rules

We endogenize the job destruction decision by considering a more general distribution of idiosyncratic productivity shocks than assumed in chapter 1. At some of the idiosyncratic productivities that firms now face production is profitable, but at some others it is not. The firm chooses a *reservation productivity* and destroys jobs whose productivity falls below it. The productivity of a job falls below the reservation value either because of the arrival of an idiosyncratic shock or because of the arrival of a general shock that hits many firms. In the steady state we consider only idiosyncratic shocks, which is consistent with the evidence that the main reason for job destruction is the arrival of idiosyncratic shocks. But when we consider out-of-steady-state dynamics, we show that general productivity shocks can also cause changes in the rate of job destruction.

When job destruction takes place, the firm and worker separate. This generates an endogenous flow of workers into unemployment, which in the steady state is equal to the matching rate. We continue assuming that the only reason for job separations is job closure, an assumption relaxed in chapter 4.

In the model of the preceding chapter, newly created jobs had constant productivity p until a negative shock arrived, which led to the destruction of the job. Without loss of generality, we can assume that the negative shock reduced the productivity of the job to 0. We now generalize this assumption by writing px for the productivity of the job, where p denotes, as before, a general productivity parameter and x an idiosyncratic one. The model of the preceding chapter can be reinterpreted as one that allowed only two values for the idiosyncratic parameter, 1 and 0. In this chapter we assume that when an idiosyncratic shock arrives, the productivity of the job moves from its initial value x to some new value x', which is a drawing from a general distribution $G(x)$ with support in the range $0 \leq x \leq 1$. No other assumptions are needed to derive the results of this chapter, though for convenience we will also assume that the distribution is free of holes. As with the model of job creation, the capital decision does not play an important role in the determination of the rate of job destruction provided that there is a perfect market in second-hand capital, so we will first develop the model without explicit reference to capital.

As before, idiosyncratic shocks arrive to jobs at Poisson rate λ. The idiosyncratic productivity that is drawn after the arrival of the shock is independent of initial productivity and is irreversible. The firm has the choice either to continue production at the new productivity or to close the job down and separate from the worker. The idiosyncratic shock process has persistence because $\lambda < \infty$, it is memoryless because the new x' after the shock is independent of initial x, and it is irreversible because the only choice that the firm has is either to produce at the new x' or shut down. But as previously, at job creation the firm has complete choice of job productivity. Profit maximization trivially requires that all new jobs are created at maximum productivity, p.

The intuition behind our new assumptions is similar to the one that was briefly discussed in preceding sections. Jobs are differentiated by product and technology. Changes in tastes and technologies move the relative value of products up or down a scale, here from 0 to p. The firm cannot adapt its technology or influence tastes for its product once production has started, but it can choose its product and technology before the job is created.

The assumption of complete irreversibility is obviously extreme and is made for convenience. Firms might be able, in some situations, either to update their technologies or destroy one job and immediately create another one, switching production to another product that commands higher value, without dismissing the worker. These are complications that do not influence the main results that we discuss here, provided that there is at least *some* irreversibility that necessitates job destruction and separation of firm and worker when a negative shock arrives. The empirical literature, mainly for data reasons, defines as job destruction the joint event of job destruction and separation of firm and worker, as we do here.

Because now jobs are distinguished by productivity, we have to make productivity explicit in the expression for the value of a filled job. Let $J(x)$ be the value of a filled job with idiosyncratic productivity x, and let $w(x)$ be the wage associated with it. When an idiosyncratic shock arrives, the firm has the choice of either destroying the job, with return 0, or continuing at the new productivity. Since free disposal is always a choice, the optimal decision is that production should continue if $J(x) \geq 0$ but stop if $J(x) < 0$. We show below that $J(x)$ is a continuous function of x, so the job destruction rule $J(x) < 0$ satisfies the reservation property with respect to the *reservation productivity* R, defined by

$$J(R) = 0. \tag{2.1}$$

By the reservation property, firms destroy all jobs with idiosyncratic productivity $x < R$ and continue producing in all jobs with productivity $x \geq R$. This, and the property that all jobs are created at maximum productivity, makes the productivity of a filled job a stochastic Poisson process with initial value p and terminal value pR.

Equilibrium unemployment is obtained as before from the equality of the flow into unemployment with the flow out of it. The flow into unemployment in this model is equal to the fraction of jobs that get hit by a productivity shock below reservation value. In a large market this is given by the product of the fraction of firms that get hit by a shock, λ, and the probability that the shock is below reservation, $G(R)$. Therefore the flow into unemployment (job destruction) is given by $\lambda G(R)(1 - u)$. As before, the flow out of unemployment is equal to job creation, $m(v, u) = \theta q(\theta)u$. The evolution of unemployment is therefore given by

$$\dot{u} = \lambda G(R)(1-u) - \theta q(\theta)u, \tag{2.2}$$

and its steady-state value by

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}. \tag{2.3}$$

Equation (2.3) is the Beveridge curve for the economy with endogenous job destruction. But because the Beveridge curve now depends on both R and θ, which are both unknowns, it is less useful as a tool of diagrammatic analysis than it was in the model with exogenous job destruction rate. Most of the variables that shift the job creation line in vacancy-unemployment space in this model also shift the Beveridge curve through their influence on the reservation productivity. We will return to this diagram when we derive the equilibrium conditions for R and θ.

2.2 Steady-State Equilibrium

To derive the conditions that characterize equilibrium, we first need to derive the asset values of jobs and workers with the more general distribution of productivity shocks. Equilibrium is fully characterized by the employment contract, which is now a wage rate $w(x)$ for each productivity x and a reservation value R, a market equilibrium condition for tightness θ that is to be derived from the job creation decision and the condition for the evolution of unemployment, (2.2).

The asset value of a job with productivity in the range $1 \geq x \geq R$ satisfies

$$rJ(x) = px - w(x) + \lambda \int_R^1 J(s)dG(s) - \lambda J(x). \tag{2.4}$$

For the worker the returns from working at a job with idiosyncratic productivity x satisfy

$$rW(x) = w(x) + \lambda \int_R^1 W(s)dG(s) + \lambda G(R)U - \lambda W(x). \tag{2.5}$$

In (2.4), whenever an idiosyncratic shock arrives, the firm has to give up the value $J(x)$ for a new value $J(s)$ if the new idiosyncratic productivity is in the range $1 \geq s \geq R$, or destroy the job for a zero return otherwise. In (2.5) the worker in a job with productivity x enjoys expected returns $W(x)$, which he has to give up when a shock arrives. If the new productivity is in the range $1 \geq s \geq R$, the worker remains employed, but if it is below that range, he becomes unemployed for an expected return U.

As before, we assume that the wage rate divides the job surplus in fixed proportions at all x, so the sharing rule that generalizes (1.17) is

$$W(x) - U = \beta[J(x) + W(x) - V - U] \tag{2.6}$$

for all $1 \geq x \geq R$. Implicit in this sharing rule is the assumption that the wage rate is renegotiated every time a productivity shock arrives. V is the firm's expected return from a vacancy. Since U and V are both independent of x, it follows immediately from (2.4), (2.5), and (2.6) that $J'(x) \geq 0$, which is a sufficient condition for the optimality of the reservation rule in (2.1).

Also as before, job creation follows the same rules as when idiosyncratic productivity took only two values. Noting that all jobs are created at maximum idiosyncratic productivity, $x = 1$, the expected profit from a new job vacancy satisfies

$$rV = -pc + q(\theta)[J(1) - V]. \tag{2.7}$$

$q(\theta)$ is the rate at which workers arrive to job vacancies. Firms open vacancies until all rents from vacant jobs are exhausted. Therefore job creation satisfies a condition similar to (1.7),

$$J(1) = \frac{pc}{q(\theta)}. \tag{2.8}$$

The four equations that uniquely solve for the four unknowns in the steady state—unemployment, the reservation productivity, wages, and market tightness—are (2.3), (2.1), (2.6), and (2.8). We solve the model by deriving first the wage equation at all productivities, use it to substitute wages out of the job creation and job destruction conditions, and then use the latter two conditions to solve for R and θ. With knowledge of R and θ, unemployment can then be obtained from the Beveridge curve. Unlike the model of chapter 1, the key two-equation block now is not the wage equation and job creation condition but the reduced form job creation and job destruction conditions.

To derive the wage equation, we proceed as in chapter 1 and use the job creation condition (2.8) and the sharing rule (2.6) to write the unemployed worker's expected returns as

$$\begin{aligned} rU &= z + \theta q(\theta)[W(1) - U] \\ &= z + \frac{\beta}{1-\beta} pc\theta. \end{aligned} \tag{2.9}$$

Multiplying the asset equation for the firm, (2.4), by β and the one for the worker, (2.5), by $1 - \beta$, then subtracting one from the other and making use of the sharing rule (2.6), the zero-profit condition $V = 0$, and equation (2.9), gives a wage equation that is the natural generalization of (1.20):

$$w(x) = (1-\beta)z + \beta p(x + c\theta). \tag{2.10}$$

Wages depend on job productivity and the worker's unemployment income but not on other jobs' productivities. Market conditions influence wages only through market tightness, θ. As before, the reason for this influence is that θ influences the firm's and the worker's bargaining strength. A higher θ makes it easier for workers to find a job elsewhere, and more difficult for the firm to recruit a worker, so wages are higher at all productivities.

Following the derivation of the wage equation, we derive two new expressions, one for the job creation condition and one for the job destruction condition. The two equations will form a self-contained block that will give a unique solution for R and θ, which can then be used in (2.10) and (2.3) to solve for wages and unemployment.

We note first that by the nature of the sharing rule (2.6), the firm and worker agree about the jobs that should be destroyed. The firm wants to destroy all jobs with productivity below the R that satisfies $J(R) = 0$. By the zero-profit condition for new vacancies, $V = 0$, and so (2.6) implies that R also satisfies $W(R) = U$. But this is precisely the point where the worker will want to quit a job and join unemployment, since at all productivities $x < R$, $W(R) < U$ and the worker is better off in the unemployment pool than in the job. Thus there are no voluntary job separations for one side and involuntary for the other; all job separations are privately efficient. (They are not socially efficient because of the search externalities discussed in chapter 1, a question that we address in chapter 8.) Private efficiency implies that once we make use of the wage equation, we can derive the expression for the reservation productivity by analyzing either the firm's decision or the worker's. We concentrate on the firm's decisions.

Substitution of the wage equation into (2.4) gives

$$(r+\lambda)J(x) = (1-\beta)(px - z) - \beta pc\theta + \lambda \int_R^1 J(s)dG(s). \tag{2.11}$$

Evaluating now (2.11) at $x = R$ and subtracting the resulting equation from (2.11) after noting that $J(R) = 0$ by the definition of the reservation productivity in (2.1), we get

$$(r+\lambda)J(x) = (1-\beta)p(x-R). \tag{2.12}$$

Substitution now of $J(x)$ from (2.12) into the integral expression of (2.11), gives

$$(r+\lambda)J(x) = (1-\beta)(px - z) - \beta pc\theta + \frac{\lambda(1-\beta)p}{r+\lambda}\int_R^1 (s-R)dG(s). \tag{2.13}$$

The conditions for job creation and job destruction can be obtained by making use of (2.12) and (2.13). To derive first the condition for job creation, we make use of (2.12) for $x = 1$ and the zero-profit condition (2.8) to arrive at the key equation

$$(1-\beta)\frac{1-R}{r+\lambda} = \frac{c}{q(\theta)}. \tag{2.14}$$

Equation (2.14) says that the expected gain from a new job to the firm must be equal to the expected hiring cost that the firm has to pay. This is drawn in figure 2.1 as a downward-sloping curve and labeled the job creation curve. It slopes down because at higher R the expected life of a job is shorter, because in any short interval of time δt the job is destroyed with probability $\lambda G(R)\delta t$. Firms create fewer jobs as a result, leading to a fall in market tightness, θ.

The parameters that shift the job creation curve for given R are few. General productivity p does not enter this expression because both the firm's expected revenues and costs are proportional to p. Higher β leads to less job creation because it decreases expected profits by giving labor more of the surplus from new jobs. Higher r or λ also decrease job

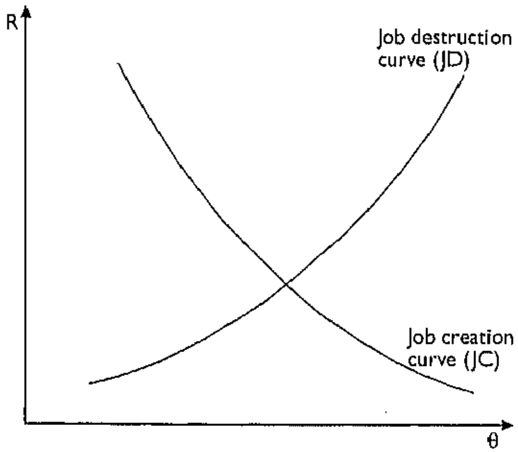

Figure 2.1
Equilibrium reservation productivity and market tightness

creation because the future returns from new jobs are discounted at higher rates. Finally, higher mismatch, in the sense of lower arrival rate of workers at given tightness, also reduces job creation because it increases the expected duration of a vacancy and, by implication, the expected hiring cost facing the firm. All these changes shift the job creation curve to the left.

The job destruction condition is derived from (2.13) by evaluating it at $x = R$ and substituting the result into the zero-profit condition for the reservation job, (2.1):

$$R - \frac{z}{p} - \frac{\beta c}{1-\beta}\theta + \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s) = 0. \tag{2.15}$$

Equation (2.15) is the second of the two key equations that we use to solve for R and θ. It is shown in figure 2.1 as an upward-sloping curve and labeled the job destruction curve. It slopes up because at higher θ, the worker's outside opportunities are better (and wages are higher) and so more marginal jobs are destroyed.

Before discussing the properties of the equilibrium R and θ, we note an important property implied by the job destruction condition. By equation (2.9), the reservation productivity is less than the reservation wage of unemployed workers, rU. The reason for this is that occupied jobs have a positive option value, which implies that there is some labor hoarding. This option value is shown by the integral expression in (2.15). Because of the possibility that a job productivity might change, the firm keeps some currently unprofitable jobs occupied. By doing this, it is able to start production at the new productivity immediately after arrival, without having to pay a recruitment cost and forgo production during search. As intuition would justify, if productivities change more frequently (higher λ), the option value of keeping a worker is higher. The option value is also higher if the discount rate is lower, because the returns from a productivity change accrue in the future, and if the expected gain that can be obtained from new productivities, shown by the integral expression, is higher.

Now, at given θ, the reservation productivity is higher when unemployment income is higher and also when labor's share of profits, β, is higher. As with θ, both these effects work through the worker's reservation wage. From (2.9) labor's reservation wage is higher when z, β, θ, and c are higher. The reservation productivity is also higher when the rate of arrival of idiosyncratic shocks is lower and when the interest rate is higher, in each case because the option values of the job is lower.

Finally the reservation productivity is lower when productivities in all jobs increase by the same proportion, represented by an increase p. The reason for this is that with higher general productivities, the worker's opportunity cost becomes relatively less attractive because z is independent of p. As we emphasized in chapter 1, this is a plausible effect of higher p in a short- to medium-run equilibrium but not in a long-run growth equilibrium. If we follow the suggestion made in chapter 1 and write z as a proportion of the mean wage, the effect of p on the reservation productivity disappears. (See also chapter 3 for more discussion of this point.)

To see this, suppose that z is a fixed proportion of the mean wage rate observed in the market,

$$z = \rho E[w(x) \mid x \geq R], \tag{2.16}$$

where $0 \leq \rho \leq 1$ is the replacement rate. Then from (2.10),

$$E[w(x) \mid x \geq R] = (1-\beta)z + \beta p[E(x \mid x \geq R) + c\theta]. \tag{2.17}$$

Therefore

$$z = \frac{\rho\beta}{1-\rho(1-\beta)} p[E(x \mid x \geq R) + c\theta]. \tag{2.18}$$

As expected, unemployment income is higher when the replacement rate, labor's wage share, expected productivities, and market tightness are higher. Crucially, unemployment income is now proportional to the general productivity parameter, p.

Substitution of z from (2.18) into (2.15) gives the new job destruction condition:

$$R - \frac{\rho\beta}{1-\rho(1-\beta)} E(x \mid x \geq R) - \frac{\beta}{1-\beta}\frac{c\theta}{1-\rho(1-\beta)} + \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s) = 0. \tag{2.19}$$

The reservation productivity is now independent of p. It depends positively on the replacement rate ρ and on all the other parameters as before. The positive slope of the job destruction curve in figure 2.1 is not affected by these substitutions. In the remainder of this chapter, we will use the simpler expression (2.15) for the reservation productivity, though we will return to something like (2.19) when we discuss growth.

Figure 2.1 shows that the equilibrium θ and R are unique and so the equilibrium wage rate for each productivity and equilibrium unemployment are also unique. The equilibrium wage rate at each productivity x is obtained by substituting the solution for θ into (2.10), and equilibrium unemployment is obtained by substituting both θ and R into (2.3). We examine the properties of equilibrium in the next section.

2.3 Unemployment, Job Creation, and Job Destruction

The properties of equilibrium unemployment in the extended model with endogenous job destruction are obtained from the simultaneous solution of three of the equations of the model, the job creation condition (2.14), the job destruction condition (2.15), and the Beveridge curve, (2.3). The first two are illustrated in figure 2.1. Unemployment (and vacancies) are obtained from the Beveridge diagram. As we argued in section 2.1, this diagram is now less useful for the analysis of unemployment than it was in the simpler model of chapter 1 because the

Beveridge curve now depends on the endogenous R, but we still use it to illustrate some of the properties of the model.

The Beveridge curve that is obtained from equation (2.3) is drawn, as before, as a downward-sloping curve in vacancy-unemployment space. But the restrictions on the matching function are not now sufficient for the negative slope because higher θ (and so, for given unemployment, more vacancies), on the one hand, imply more job matchings and, on the other hand, imply more job destruction. The former implies a negative slope but the latter a positive one. We assume that the direct effect of θ through the matching function dominates the indirect effect through the reservation productivity, an assumption that we will make every time a similar conflict arises. The justification for it is empirical: Estimated Beveridge curves slope down, and a set of sufficient restrictions on the probability distribution of productivities that ensures that the effect through θ dominates is weak. This conflict was first noted in partial models of search, where the probability of leaving unemployment is the product of a contact probability and an acceptance probability. The relevant literature is cited in the Notes on the Literature at the end of chapter 6.

Under the assumption that the job matching effect of vacancies on unemployment dominates the job destruction effect, the Beveridge diagram has the same shape as before. The solution for θ is still unique and independent of unemployment. Equilibrium vacancies and unemployment are at the intersection of the Beveridge curve with the job creation line, drawn through the origin at an angle θ, obtained from figure 2.1 The Beveridge diagram is not drawn because it is the same as the one in chapter 1, figure 1.2.

The empirical job destruction rate, defined as the ratio of total job destruction to employment, is $\lambda G(R)$, and the job creation rate is, as before, $m(v, u)/(1 - u) = \theta q(\theta)u/(1 - u)$. Although in steady state the two rates are equal, their impact response to productivity and other changes are interesting in their own right (see also section 2.5). R is a sufficient statistic for the behavior of the job destruction rate and θ for the behavior of the job creation rate at given unemployment. The difference between the impact and steady-state effects of a parameter change on the job creation and job destruction rates is due to the behavior of unemployment (or employment). Because the two-equation system (2.14) and (2.15) that determines equilibrium R and θ is independent of unemployment, when a parameter changes, R and θ jump to their new equilibrium values instantaneously. Unemployment starts moving according to (2.2) only if the new job creation and job destruction rates implied by the change in R and θ are not equal. But the job destruction rate, $\lambda G(R)$, is independent of unemployment, so it does not change further in response to any change in unemployment. The job creation rate depends on unemployment; therefore its response is different on impact, for given unemployment, and in steady state, when by definition it is equal to the job destruction rate.

Consider now the influence of productivity on the job creation rate, job destruction rate, and unemployment. Higher general productivities, shown by higher p, shift the job destruction curve in figure 2.1 (for given z) down and to the right. This increases market tightness and reduces the reservation productivity. At given unemployment the job destruction rate decreases, and the job creation rate increases. Unemployment has to decrease until the job creation rate falls down to the level of the lower job destruction rate. So the steady-state effect of higher general productivity is to reduce the job creation and job destruction rates and unemployment.

The effect of productivity on unemployment can also be shown in the Beveridge diagram, where the rise in θ rotates the job creation line anticlockwise and the fall in the reservation productivity shifts the Beveridge curve toward the origin (figure 2.2). Equilibrium moves from point A to point B, where unemployment is lower. The effect on vacancies is ambiguous, though, if we assume that the effect through θ dominates the effect through R, vacancies increase. The fact that empirically vacancies and unemployment move in opposite directions over the business cycle is another reason for making the assumption that the effect through θ always dominates the effect through R, when the two effects conflict with each other.

As in the model of chapter 1, the productivity effects are due to the fact that higher productivity increases the returns from work but has no influence on the return from nonmarket activities, z. It becomes jointly optimal for the firm and the worker to devote more time to work. In decentralized equilibrium this is signaled to firms by a smaller increase in wages than in productivity, because of the fixed z which enters the wage equation. The smaller change in the wage rate increases profits from both job creation and ongoing jobs, increasing job creation and reducing job destruction at given unemployment. As we argued in the

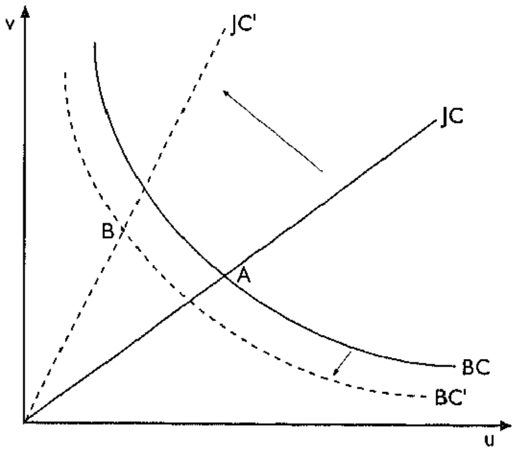

Figure 2.2
Effect of higher productivity on equilibrium vacancies and unemployment

preceding section, if z was proportional to wages, these effects would not materialize.

Because of the property of the model that the level of labor productivity influences job creation and job destruction only to the extent that it influences the ratio z/p, changes in z have the same effect on equilibrium as changes in p but with opposite sign. Thus higher nonmarket income increases wages, reduces job creation at given unemployment, and increases job destruction, leading to higher unemployment in the steady state. These effects still operate if unemployment income is proportional to wages provided that changes in unemployment income are caused by exogenous changes in the replacement rate ρ. This can easily be derived from the pair of equations (2.19) and (2.14) which determine the job creation and job destruction flows in this case.

Now, since all active jobs have productivities in the range pR to p, higher p is equivalent to proportionally higher productivities in all jobs. Two other productivity shifts that are of interest are a translation of the idiosyncratic productivity distribution to the right and a mean-preserving shift in the distribution. The former is another way of analyzing an increase in the productivity of all jobs, but instead of an equal proportional increase it represents an equal absolute increase in all productivities. The mean-preserving shift is a way of representing an increase in the variance of productivity shocks. We will follow a parametric approach to the analysis of these shifts, along the lines suggested by Arrow (1965).

For the analysis of additive shifts, we suppose that all idiosyncratic productivities x depend on an additive shift parameter h, such that

$$x(h) = x + h. \tag{2.20}$$

The influence of h on equilibrium is evaluated by considering the effects of a small displacement of h at the point $h = 0$.

Higher variance in job productivities is similarly represented by a parameter h, which is now multiplicative around the mean of the productivity distribution, $\bar{x}$. It is debatable whether the multiplicative shift should be around the mean of the whole distribution or the mean of the conditional distribution, $E(x \mid x \geq R)$. The latter is the observed mean productivity, since productivities below the reservation R are not taken up. We will assume that the multiplicative shift is around the unconditional mean $\bar{x}$ because it is free of the other parameters of the model, but we will impose the restriction that $z \leq p\bar{x}$, namely that the income of unemployed workers is below mean productivity. This restriction is always satisfied by the conditional productivity mean, $z \leq pE(x \mid x \geq R)$. Imposing this restriction ensures that some active jobs suffer a fall in productivity when there is a multiplicative shock to the distribution. Thus, in examining the effects of a change in the variability of the productivity distribution, we write

$$x(h) = x + h(x - \bar{x}) \tag{2.21}$$

for all x, and evaluate the effects of a small displacement of h at $h = 0$ under the assumption that $z \leq p\bar{x}$.

Consider first the implications of a uniform absolute increase in productivities, shown by the additive shift parameter in (2.20). Reworking the job creation and job destruction conditions (2.14) and (2.15) with $x + h$ replacing x is straightforward. We find that the effect of the additive parameter is to shift the job destruction curve in figure 2.1 down but not move the job creation curve. Therefore the additive shift parameter in the idiosyncratic productivity distribution has the same effect on job creation and job destruction as higher p: It raises market tightness and reduces the reservation productivity. At given unemployment rate the job creation rate rises and the job destruction rate falls, and in equilibrium unemployment also falls.

A multiplicative shift parameter has similarly unambiguous effects on market tightness and the reservation productivity, though their derivation is now less straightforward. Reworking the job creation and job destruction conditions for $x(h)$ as defined in (2.21) gives

$$(1-\beta)(1+h)\frac{1-R}{r+\lambda}=\frac{c}{q(\theta)} \tag{2.22}$$

and

$$(1+h)R-h\bar{x}+\frac{(1+h)\lambda}{r+\lambda}\int_R^1(s-R)dG(s)=\frac{z}{p}+\frac{\beta}{1-\beta}c\theta. \tag{2.23}$$

It is straightforward to see that higher h shifts to the right the job creation curve in figure 2.1, implying more job creation at given R. But the shift in the job destruction curve is ambiguous under our restriction $\bar{x}\geq z/p$ alone (though it would shift it up if the restriction was strengthened to $\bar{x}\geq rU$, the reservation wage of the unemployed). Thus figure 2.1 is not helpful in the analysis of multiplicative shifts.

Differentiation of (2.22) and (2.23) with respect to h, however, shows that at $h=0$ both market tightness and the reservation productivity rise unambiguously. Differentiation of (2.23) gives

$$\left[1-\frac{\lambda}{r+\lambda}[1-G(R)]\right]\frac{\partial R}{\partial h}=\bar{x}-R-\frac{\lambda}{r+\lambda}\int_R^1(s-R)dG(s)+\frac{\beta}{1-\beta}c\frac{\partial\theta}{\partial h}. \tag{2.24}$$

Differentiation also of (2.22) with respect to h gives

$$\frac{c\eta(\theta)}{\theta q(\theta)}\frac{\partial\theta}{\partial h}=\frac{1-\beta}{r+\lambda}\left[1-R-\frac{\partial R}{\partial h}\right], \tag{2.25}$$

where we have used the elasticity notation

$$\eta(\theta)=-\frac{\partial q(\theta)}{\partial\theta}\frac{\theta}{q(\theta)}. \tag{2.26}$$

As we noted in section 1.1, this elasticity need not be a constant, but it is always a number between 0 and 1. Substitution of $\partial R/\partial h$ from (2.24) into (2.25) reveals that the sign of $\partial\theta/\partial h$ is the same as the sign of

$$\left[1-\frac{\lambda}{r+\lambda}[1-G(R)]\right](1-R)-\bar{x}+R+\frac{\lambda}{r+\lambda}\int_R^1(s-R)dG(s). \tag{2.27}$$

Collecting terms, we find that the sign of (2.27) is the same as the sign of

$$1-\bar{x}-\frac{\lambda}{r+\lambda}\int_R^1(1-s)dG(s) \tag{2.28}$$

which is unambiguously positive because

$$1-\bar{x}=\int_0^1(1-s)dG(s). \tag{2.29}$$

Hence the effect of higher h on θ is positive. To show that it is also positive on R, we substitute $\partial\theta/\partial h$ from (2.25) into (2.24) and find that the sign of $\partial R/\partial h$ is the same as the sign of

$$\bar{x}-R-\frac{\lambda}{r+\lambda}\int_R^1(s-R)dG(s)+\frac{\beta\theta q(\theta)}{\eta(\theta)}\frac{1-R}{r+\lambda}. \tag{2.30}$$

Making use of the job creation condition (2.14) to get rid of $1-R$ from the last term of this expression, we find that the sign of (2.30) is the same as the sign of

$$\bar{x}-R-\frac{\lambda}{r+\lambda}\int_R^1(s-R)dG(s)+\frac{\beta}{(1-\beta)\eta(\theta)}c\theta, \tag{2.31}$$

which is unambiguously positive under the restriction $\bar{x}\geq z/p$, given the job destruction condition (2.15) and the fact that $0\leq\eta(\theta)\leq 1$.

Our analysis of the effects of increased variance of the productivity distribution shows that at given unemployment both the job destruction and job creation rates are higher. Because the job destruction rate is higher, the job reallocation rate, the sum of the job creation and the job destruction rates, is also higher in steady state. But whether unemployment is higher or lower depends on the relative magnitude of the impact effect on each rate. If the parameters are such that the initial increase in the job destruction rate is higher than the initial increase in the job creation rate, unemployment goes up to bring the job creation rate into equality with the job destruction rate, and conversely if the initial increase in the job destruction rate is lower. In terms of the Beveridge diagram, a multiplicative shock shifts the Beveridge curve out and rotates the job creation line up, in contrast to the additive shock, which has the same effect on the job creation line but shifts the Beveridge curve in. This difference has been used by a number of authors to distinguish between aggregative shocks, interpreted as a general increase or decrease in job productivities, and reallocation shocks, interpreted as shocks that increase or decrease the variance of productivities, as causes of the cyclical change in unemployment. Although our analysis has been entirely in terms of steady states, the principles underlying the cyclical analysis and our own are similar.

The reasons that a mean-preserving shift in productivities increases job creation (at given unemployment) are easy to see. The mean-preserving shift makes productivities above the mean better and productivities below the mean worse. The firm and worker, however, do not take up productivities below the reservation value; the distribution of productivities that enters their calculus is truncated at R. Therefore the benefits from the higher productivity of jobs above the mean outweigh the costs from the lower productivity of jobs below the mean.

The multiplicative shift has three effects on the reservation productivity, which are shown by the terms on the right-hand side of (2.24). The first operates directly and pushes the reservation productivity up if $R \leq \bar{x}$, as is likely the case under our restriction that $\bar{x} \geq z/p$, or down otherwise. If $R \leq \bar{x}$, the reservation job yields negative profit after the shift. The second effect always increases the reservation productivity because of the increase in market tightness, which improves the workers' outside options. The final effect increases the option value of the job and so it reduces the reservation productivity. The reason for the increase in option value is the same as the reason for the increase in job creation, the truncation of the productivity distribution at R. The restriction that we have imposed, $z \leq p\bar{x}$, is sufficient to ensure that the negative effects on the profit from the reservation job outweigh the positive, and so get an increase in job destruction after a multiplicative shift. The restriction ensures that the reservation productivity is sufficiently low, when compared with the mean of the distribution, to avoid large increases in the productivity of the reservation job after the shift.

We now consider the influence of the other parameters of the model on the equilibrium job creation and job destruction rates, beginning with the rate of interest. Higher real rate of interest shifts the job creation curve in figure 2.1 to the left and the job destruction curve up. For given reservation productivity, there is less job creation because future profits from new jobs are discounted more heavily. Similarly, for given market tightness, the higher interest rate reduces the option value of the job, and so the reservation productivity is higher. The effect of these shifts on market tightness is unambiguously negative, but it is ambiguous on the reservation productivity. To see this, differentiate (2.14) and (2.15) with respect to r to get

$$\frac{c\eta(\theta)}{\theta q(\theta)}\frac{\partial\theta}{\partial r}=-\frac{1-\beta}{r+\lambda}\left(\frac{1-R}{r+\lambda}+\frac{\partial R}{\partial r}\right), \tag{2.32}$$

$$\left[1-\frac{\lambda}{r+\lambda}[1-G(R)]\right]\frac{\partial R}{\partial r}=\frac{\lambda}{(r+\lambda)^2}\int_R^1(s-R)dG(s)+\frac{\beta c}{1-\beta}\frac{\partial\theta}{\partial r}. \tag{2.33}$$

Getting rid of the unambiguous $\partial\theta/\partial r$ from these expressions and making use of the original equations (2.14) and (2.15) to simplify the resulting expression, we find that the sign of $\partial R/\partial r$ is the same as the sign of

$$\frac{z}{p}+\frac{\beta c\theta}{1-\beta}-R-\frac{\beta c\theta}{(1-\beta)\eta(\theta)}. \tag{2.34}$$

This expression is generally of ambiguous sign. To see this, suppose that equilibrium is such that θ is close to zero. Then the job destruction condition (2.15) implies that (2.34) is approximately equal to the option value of the job, and so it is positive. In slack markets the higher rate of interest increases job destruction. If market tightness is away from zero, then it is easy to see that sufficiently small elasticity $\eta(\theta)$ can turn (2.34) negative. But even if $\eta(\theta)$ is close to its upper value of 1, (2.34) approximates $z/p - R$, which is not necessarily positive if θ is large.

In contrast, the rate of arrival of idiosyncratic shocks, which in the job creation expression plays the role of a discount rate, has unambiguous effects on job creation and job destruction. Higher rate of arrival of idiosyncratic shocks reduces the expected life of a job, so it reduces job creation, shifting the job creation curve in figure 2.1 to the left. The shorter expected life of a job, however, reduces the reservation productivity because now the option value of the job is higher. The firm is more willing to hold on to labor if it expects a quick arrival of better conditions. The two effects unambiguously imply a lower reservation productivity but as yet undetermined effects on market tightness. Differentiation of the job creation and job destruction conditions, however, yields

$$\frac{c\eta(\theta)}{\theta q(\theta)}\frac{\partial\theta}{\partial\lambda} = -\frac{1-\beta}{(r+\lambda)^2}(1-R) - \frac{1-\beta}{r+\lambda}\frac{\partial R}{\partial\lambda}, \tag{2.35}$$

$$\left[1-\frac{\lambda}{r+\lambda}[1-G(R)]\right]\frac{\partial R}{\partial\lambda} = -\frac{r}{(r+\lambda)^2}\int_R^1 (s-R)dG(s) + \frac{\beta c}{1-\beta}\frac{\partial\theta}{\partial\lambda}. \tag{2.36}$$

It follows by substitution that

$$\begin{aligned}&\left[\left[1-\frac{\lambda}{r+\lambda}[1-G(R)]\right]\frac{(r+\lambda)c\eta(\theta)}{(1-\beta)\theta q(\theta)} + \frac{\beta c}{1-\beta}\right]\frac{\partial\theta}{\partial\lambda}\\ &\quad = \frac{r}{(r+\lambda)^2}\int_R^1 (s-R)dG(s) - \left[1-\frac{\lambda}{r+\lambda}[1-G(R)]\right]\frac{1-R}{r+\lambda}.\end{aligned} \tag{2.37}$$

The right-hand side of this equation is negative because it is equal to

$$-\frac{1-R}{r+\lambda}\left[1-\frac{\lambda}{r+\lambda}[1-G(R)] - \frac{r}{r+\lambda}\int_R^1 \frac{s-R}{1-R}dG(s)\right], \tag{2.38}$$

and the last two terms in the square brackets are the weighted average of two numbers less than 1.

Thus market tightness falls with λ. At given unemployment, faster arrival of idiosyncratic shocks reduces job creation. The job destruction rate, $\lambda G(R)$, is subject to two opposing influences. On the one hand, it increases because there are now more shocks on average, but on the other hand, it decreases because the firm holds on to jobs longer. If the direct effect dominates, which is the assumption normally (and frequently implicitly) adopted in the literature, the impact effect of faster arrival of idiosyncratic shocks is to increase job destruction and reduce job creation. Unemployment therefore unambiguously increases. In the Beveridge space of figure 1.2, higher λ shifts the Beveridge curve out and rotates the job creation line down.

A related parametric influence to the one just studied is the degree of mismatch. A higher mismatch, as we argued in chapter 1, is represented by a negative shift in the aggregate matching function which reduces the rate of arrival of workers to firms at given market tightness. In figure 2.1 this shifts the job creation curve to the left, reducing both market tightness and the reservation productivity. Job destruction falls and job creation at given unemployment also falls for two reasons, the higher mismatch and the lower degree of market tightness. In the Beveridge diagram this rotates the job creation line clockwise. The Beveridge curve is subject to two influences. The fall in the arrival rate of workers shifts it out, but the fall in the job destruction rate shifts it in. In general, it is not possible to say which effect dominates, but empirically the direct effect of the fall in the matching rate at given tightness is always assumed to be the dominant one. Under this assumption the Beveridge curve shifts out, implying higher equilibrium unemployment.

We consider finally the implications of a higher labor share in the wage bargain, β. Differentiation of (2.15) with respect to β gives,

$$\left[1-\frac{\lambda}{r+\lambda}[1-G(R)]\right]\frac{\partial R}{\partial \beta}=\frac{1}{1-\beta}\left[\frac{c\theta}{1-\beta}+\beta c\frac{\partial \theta}{\partial \beta}\right]. \tag{2.39}$$

Differentiation of (2.14) gives

$$\frac{c\eta(\theta)}{\theta q(\theta)}\frac{\partial \theta}{\partial \beta}=-\frac{1-R}{r+\lambda}-\frac{1-\beta}{r+\lambda}\frac{\partial R}{\partial \beta}. \tag{2.40}$$

The effect of higher labor share is to shift the job destruction curve in figure 2.1 up and the job creation line to the left. Job destruction rises at given market tightness because wages increase and job creation falls for a similar reason. The net effect is to reduce market tightness, and therefore job creation at given unemployment, but it has ambiguous effects on the reservation productivity.

Interestingly substitution of $\partial\theta/\partial\beta$ from (2.40) into (2.39) shows that the sign of $\partial R/\partial\beta$ is the same as the sign of $\eta(\theta)-\beta$. So R reaches a unique maximum at $\beta=\eta(\theta)$. Job destruction increases with labor share at low β and falls at high β because of the nonlinear response of market tightness and the reservation wage to β. We will argue later (chapter 8) that although there is no reason why the two parameters should be equal, even when $\eta(\theta)$ is a constant, the restriction of a constant η equal to β is a natural benchmark to adopt. Under this restriction the reservation productivity is independent of labor's share, and the net effect of labor's share on market tightness becomes

$$\frac{\partial \theta}{\partial \beta}=-\frac{\theta}{(1-\beta)\eta}. \tag{2.41}$$

In the Beveridge diagram higher labor share rotates the job creation line down and does not shift the Beveridge curve, implying higher equilibrium unemployment and lower vacancies.

2.4 Capital

The introduction of capital into the analysis can take place along the lines discussed in chapter 1, without important changes to the results. Moreover the assumption that there is a perfect second-hand market for capital goods, ensures that the rules governing investment are the same as before and as in other neoclassical models.

In the model of this chapter a job productivity can be anywhere in the range pR to p. We interpret px as the efficiency units of the job, and as before, we define k as the units of capital per efficiency unit of labor and $f(k)$ as the per unit production function. The firm with productivity px then buys capital pxk at the price of output and produces $pxf(k)$. If a shock arrives that changes the productivity of the job to px', the firm sells $p(x - x')k$ of its capital stock if $x' \geq R$ and stays in production; however, if $x' < R$, the firm sells its entire capital stock and closes down. Therefore the value of a job with idiosyncratic productivity parameter x now satisfies

$$r[J(x) + pxk] = px[f(k) - \delta k] - w(x) + \lambda \int_R^1 J(s)dG(s) - \lambda J(x). \tag{2.42}$$

Maximization of job value with respect to capital gives the familiar condition

$$f'(k) = r + \delta. \tag{2.43}$$

The other value expressions do not change by the introduction of the capital stock. The value of a vacancy, employment, and unemployment still satisfy (2.7), (2.5), and (2.9), respectively. The wage-sharing rule is still (2.6), and the job creation and job destruction conditions are still given by the two zero-profit requirements, $V = 0$ and $J(R) = 0$.

To derive the job creation and job destruction conditions, we note that the wage-sharing rule gives the wage equation

$$w(x) = (1 - \beta)z + \beta px[f(k) - (r + \delta)k] + \beta pc\theta. \tag{2.44}$$

When this is substituted into (2.42) and use is made of the job destruction condition, we get

$$(r + \lambda)J(x) = (1 - \beta)p(x - R)[f(k) - (r + \delta)k]. \tag{2.45}$$

Therefore (2.42) becomes

$$(r+\lambda)J(x) = (1-\beta)p[f(k)-(r+\delta)k]\left[x + \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s)\right] \\ -(1-\beta)z - \beta pc\theta. \tag{2.46}$$

With (2.45) and (2.46) in hand, the job creation and job destruction conditions can easily be derived. The job creation condition, which as before satisfies (2.8), is derived from (2.45), and it is

$$(1-\beta)\frac{1-R}{r+\lambda}[f(k)-(r+\delta)k] = \frac{c}{q(\theta)}. \tag{2.47}$$

Comparison with the condition without capital, (2.14), shows an obvious generalization.

The job destruction condition is derived from (2.46):

$$[f(k)-(r+\delta)k]\left[R + \frac{\lambda}{r+\lambda}\int_R^1 (s-R)dG(s)\right] = \frac{z}{p} + \frac{\beta}{1-\beta}c\theta. \tag{2.48}$$

Once again, comparison with (2.15) shows that the only generalization is that productivity is multiplied by $f(k) - (r+\delta)k$.

Equilibrium is now defined by the capital condition (2.43), the job creation and job destruction conditions (2.47) and (2.48), the wage equation (2.44), and the evolution for unemployment (2.2). The equilibrium model is recursive for a given interest rate, with the capital stock for each efficiency unit of labor determined first, the pair θ, R determined next, and wages and unemployment last. Aggregate capital in this economy is

$$K = L(1-u)pk\int_R^1 x dG(x) \tag{2.49}$$

and aggregate output $F(L(1-u), K)$, or in per unit terms,

$$Y = L(1-u)pf(k)\int_R^1 x dG(x). \tag{2.50}$$

Clearly, the properties of the key block of equations, for job creation and job destruction, are still the same as in figure 2.1. The introduction of the capital stock does not alter any of the properties previously described provided that we impose the condition the there is a perfect rental market for capital goods. This justifies our continued use of the model without capital goods to study the properties of job creation and job destruction and those of wages and unemployment.

2.5 Out-of-Steady-State Dynamics

The dynamics of job creation, job destruction and unemployment out of steady state are similar to those studied in chapter 1 but with one important difference. We highlight in this section that difference, discussing the other properties only briefly. We ignore capital and the issues associated with its slow adjustment to equilibrium, for the reasons that we gave in chapter 1.

Recall that in chapter 1 the wage equation and job creation condition gave a unique solution for w and θ, which did not involve any sticky variables. Therefore both w and θ were always at their steady-state values, jumping from any arbitrary initial values to the steady state following unanticipated parametric changes. (Of course, anticipated or preannounced parametric changes can produce nontrivial dynamics in both w and θ, along the lines explored in the rational expectations literature, a question that we do not address in this book.)

We make the same assumptions about job creation and wage determination in the model of this chapter as we did in the model of chapter 1, namely that firms can open and close vacancies without delay and that the wage bargain can be renegotiated at any time. As before, these assumptions imply that the zero-profit condition for new vacancies, $V = 0$, or its equivalent (2.8) hold both in and out of steady state, as does the sharing rule (2.6). In the model of chapter 1, however, the job destruction rate was a constant λ, whereas in the model of this chapter it depends on the reservation productivity R and is given by $\lambda G(R)$. A natural assumption to make is that firms can shut down unprofitable jobs without delay, and so the zero-profit condition satisfied by R, (2.1), holds both in and out of steady state.

Under this assumption, R becomes a jump variable. Because the job destruction condition (2.15) does not depend on sticky variables (and as before, neither the job creation condition (2.14) nor the set of wage equations (2.10) do), in the absence of expected parameter changes all three variables, R, θ and $w(x)$ must be on their steady state at all times. This follows for the same reasons that θ and w were always on their steady state in the model of chapter 1: Jobs are treated as assets with values determined by forward-looking arbitrage equations. Any deviation from the steady state leads to divergence and violates the transversality conditions of the maximization problem underlying the arbitrage equations.

This "instability" can be derived more explicitly from dynamic asset-valuation equations of the kind introduced in section 1.7, an exercise not pursued here.

The out-of-steady-state dynamics of unemployment are given by equation (2.2). As before, unemployment is a sticky variable and is driven by jumps in the two forward-looking variables, R and θ. The equilibrium of the triple, u, R, θ is a saddle, with one stable root (for unemployment) and two unstable ones (one each for R and θ).

The important difference between the dynamics of this version of the model and the model of chapter 1 can now be stated. It is due to the endogeneity of the job destruction rate and the assumption that the firm can shut down jobs without delay. Suppose that a parametric change increases the equilibrium reservation productivity from R to some other value R'. Then the rate of job destruction increases to $\lambda G(R')$, but also by the zero-profit condition satisfied by the reservation productivity, the jobs with idiosyncratic productivity x in the range $R \leq x < R'$ become unprofitable and close down. If initially the economy was in a steady state, the set of jobs that close down has nontrivial mass $[G(R') - G(R)](1 - u)$. The job destruction flow and the unemployment rate immediately increase by this number. Slower dynamic adjustments in unemployment follow this jump, however, according to equation (2.2). The job destruction flow drops to $\lambda G(R')$, where it remains in the absence of further changes in parameters.

The jump in unemployment is asymmetric, in the sense that it does not take place if the reservation productivity falls. If R' falls to R, following a parameter change, firms are now willing to keep in the productivity range (R', R) jobs that were previously destroyed. But they cannot instantly increase employment by $[G(R') - G(R)](1 - u)$ because the hiring of employees is slow, by the restrictions on the matching function, and the arrival of idiosyncratic shocks that pushes productivities in the range (R', R) is also slow. Therefore there are no jumps in unemployment if the reservation productivity falls, the initial value of unemployment remains the same as before the change, and the dynamic adjustment that it follows is smooth throughout the path.

We are now in a position to discuss the dynamic behavior of the job creation and job destruction rates, following parametric changes of the kind discussed in section 2.3. In this discussion we do not make explicit reference to the dynamics of wages, which are driven by the dynamics of

θ; we rather conduct it in terms of the two equations that give θ and R, (2.14) and (2.15).

Consider the two most interesting productivity shocks, a change in common productivity p and a multiplicative shift in the distribution of idiosyncratic productivities, $G(x)$. We saw in section 2.3 that a rise in p decreases the job destruction rate $\lambda G(R)$ and for given unemployment increases the job creation rate $\theta q(\theta)u/(1-u)$, through a downward jump in R and an upward jump in θ. Following the initial changes, unemployment falls smoothly, decreasing the job creation rate until it converges to the new and lower level of the job destruction rate. These adjustments are shown in figure 2.3, panel a. In the figure, initially the job destruction and job creation rates are equal. When the change takes place, the job creation rate jumps from its initial value A to a new value B, and the job destruction rate jumps down from A to C. Eventually unemployment falls until the job creation rate falls down to the level of the job destruction rate when the economy reaches a new steady state.

If common aggregate productivity decreases, R jumps to a higher value and θ to a lower one. The job destruction rate now goes up to $\lambda G(R')$, where R' is the higher value, but also on impact a number of jobs $[G(R') - G(R)](1-u)$ are destroyed. Hence, as shown on panel b of figure 2.3, the job destruction rate on impact jumps from its initial value A to a new higher value D, but then returns to the value C, which mirrors the one in panel a. The difference between D and C represents the jobs destroyed on impact as a ratio of employment, $G(R') - G(R)$. The job creation rate now falls, but because unemployment rises on impact by $[G(R') - G(R)](1-u)$, it does not fall by the full amount that would have fallen at given u. After the initial impact of the change, the job creation rate rises to match eventually the higher job destruction rate, at which point a new steady state is reached with higher unemployment.

The analysis of adjustment after a mean-preserving spread in the distribution of productivities follows similar lines. After the spread, both the reservation productivity and market tightness increase. On impact, a mass of jobs is destroyed, causing a jump in the job destruction rate. Following this jump, the job destruction rate falls to a level that is still higher than the initial level, completing its adjustment to the new steady state. The adjustment of the job destruction rate is similar to the one shown in figure 2.3, panel b, for a fall in aggregate productivity. Unlike the path shown in the same diagram, however, the job creation rate first rises, on

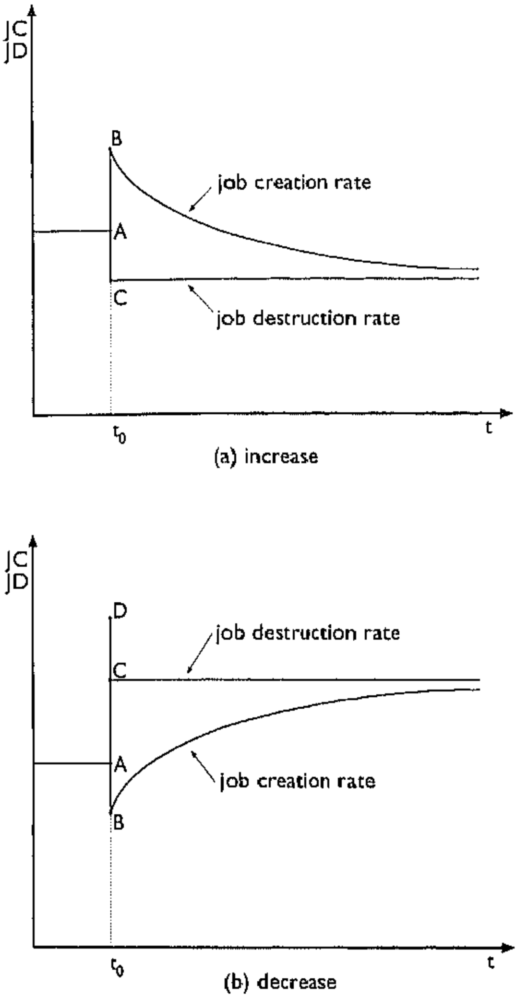

Figure 2.3
Adjustments in the job creation and job destruction rates following a change in aggregate productivity

impact, as shown in panel a of figure 2.3. After the initial rise the job creation rate adjusts gradually to the new level of the job destruction rate, in response to changes in the unemployment rate. It may increase further or decrease depending on whether the initial increase in the job destruction rate is bigger or smaller than the initial rise in the job creation rate. The new steady-state level of unemployment may be higher or lower than in the initial steady state depending on the extent of the change in R and θ. But on impact unemployment rises because the job destruction rate responds faster to the change than the job creation rate does.

If the distribution of idiosyncratic productivities takes a negative mean-preserving spread both the job destruction and job creation rates fall. The job destruction rate falls to its new equilibrium level once for all, along similar lines to the change shown in panel a of figure 2.3. The job creation rate also falls on impact, but whether it rises or falls after the initial change depends again on the relative fall in the two rates.

It follows that the initial change in the job creation rate is negatively correlated with that in the job destruction rate in the case of a uniform change in productivities but positively correlated in the case of a change in the spread of the productivity distribution. In both cases there is an asymmetry in the behavior of the job destruction rate, with a bigger initial jump when the change is positive (i.e., when it increases). Following the initial change, again in both cases, the job creation rate adjusts to the level of the job destruction rate through induced changes in unemployment.

2.6 Notes on the Literature

The model in this chapter is based on Mortensen and Pissarides (1994). The empirical motivation for the model was provided by Davis's and Haltiwanger's (1990, 1992) data for U.S. manufacturing industry. See Davis, Haltiwanger, and Schuh (1996) for more documentation and discussion. Other empirical studies of job flows include Leonard (1987) and Dunne, Roberts, and Samuelson (1989) for the United States; Konings (1995) and Blanchflower and Burgess (1993) for the United Kingdom; Boeri and Cramer (1992) for Germany; Broersma and den Butter (1994) and Gautier (1997) for the Netherlands; Lagarde, Maurin, and Torelli (1994) for France; Albaek and Sorensen (1995) for Denmark; and

Contini et al. (1995) for countries of the European Union. See also the papers collected in OECD (1996) for a discussion of models and data availability for member countries and the OECD *Employment Outlook* for 1994 for comparable data across the OECD. Outside the OECD, job creation and job destruction data for Poland during the transition was reported by Konings, Lehmann, and Schaffer (1996) and for other transition economies by Bilsen and Konings (1998).

The empirical observations of Davis and Haltiwanger and others generated a lot of theoretical interest in job turnover. A variant of the model of Mortensen and Pissarides (1994) was calibrated by Cole and Rogerson (1996) who found that the model accounts for the Davis-Haltiwanger observations provided the pool of nonemployed job seekers is about twice as large as the unemployment rate. This is consistent with evidence presented by Blanchard and Diamond (1990), who found that the flow from out of the labor force to employment justifies the assumption that there are about as many job seekers outside the labor force as there are unemployed. Den Haan, Ramey, and Watson (1997) embedded the model into a real business cycle model and calibrated it. They showed that the model can account for the job creation and job destruction flows and that in addition the labor market frictions magnify and make more persistent the effects of the business cycle shocks.

Other approaches to the modeling of the job creation and job destruction flows include Caballero and Hammour (1994, 1996), Bertola and Caballero (1994), Ramey and Watson (1997), Greenwood, MacDonald, and Zhang (1995), Hopenhayn and Rogerson (1993), and Bertola and Rogerson (1997). In contrast to the approach in Mortensen and Pissarides (1994) and to that in this chapter, which take the match as the unit of analysis, these papers place more emphasis on the firm as the unit of analysis.

The asymmetries in the job creation and job destruction flows, which were a topic of analysis in Mortensen and Pissarides (1993, 1994), have also been analyzed in an alternative framework by Campbell and Fisher (1998). Theoretical explanations of the asymmetry in job turnover take the Davis-Haltiwanger observation of a more volatile job destruction than job creation rate as a fact in need of explanation. Boeri (1996), however, showed that this U.S. fact is not a feature of job reallocation in European countries, with the exception of the United Kingdom. In continental European countries job creation typically shows more cyclical variability. Garibaldi (1998) explained this difference by arguing that in Europe policy restrictions imply that the firm cannot close jobs down without delay, an assumption that is needed to get the asymmetries discussed in this chapter. But even in the United States, Foote (1988) noted that nonmanufacturing exhibits a different asymmetry than manufacturing, with job creation more volatile. He explained it by arguing that declining sectors of the economy exhibit more volatility in job destruction and expanding sectors more in job creation.

The cyclical variability of unemployment was also studied by Gomes, Greenwood and Rebelo (1997), in a model with incomplete markets driven by the decisions of workers who decide each period whether to work or search, given stochastic shocks to the value of their jobs.

The analysis of multiplicative productivity shocks in this chapter (or more generally that of increases in variance) was motivated by the claim, originally made by Lilien (1982), that sectoral shifts in production are a driving force of the cycle. His claim, however, has been disputed. Abraham and Katz (1986) and Blanchard and Diamond (1989) have used the fact that the Beveridge curve responds differently to aggregate and reallocation shocks to differentiate between them, with results favoring aggregate shocks as the predominant driving force. Davis and Haltiwanger and others have attempted to differentiate between the two driving forces by making use of the related prediction that if the driving force is a multiplicative shock, job creation and job destruction should be positively correlated over the cycle. The data presented by Davis, Haltiwanger, and Schuh (1996) showed a strong negative correlation between the job creation and job destruction rates.

In terms of the stylized facts that we discussed in chapter 1, an explanation of the cycle based on multiplicative shifts would produce loops that are too "flat" around the Beveridge curve (i.e., vacancies would not fluctuate enough). It is more likely that structural change takes place in recessions more frequently than it does in booms, which are caused by aggregate shocks, since in recessions the opportunity cost of relocation and production restructuring is less. See Davis (1987) and Caballero and Hammour (1994) for more discussion of this point.