ELSEVIER Resource and Energy Economics 17 (1995) 327–340

RESOURCE and ENERGY ECONOMICS

# Monopsony power and the existence of natural monopoly in energy utilities

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Received 15 April 1994; accepted 15 June 1995

**Abstract**

The justification for price and entry regulation of firms hinges on whether the firms are natural monopolies, and empirical tests have been used to determine the natural monopoly status of public utilities. However, these tests are biased if the utilities possess monopsony power – a likely case. The bias is against finding natural monopoly status, which can lead to improper policies. An alternative method of testing is proposed which eliminates the bias.

*JEL classification:* L51; L94; L98

*Keywords:* Natural monopoly; Monopsony; Utilities

## 1. Introduction

Natural monopoly refers to a market structure in which one firm serving the entire market is less costly than multiple firms. Electric and natural gas utilities often are cited as examples of natural monopolies. Where natural monopoly exists efficient use of society's resources calls for regulatory policies that promote supply by one firm. These policies may range from regulation of entry by new firms to no regulation and letting the market discipline the natural monopolist. [1]

[1] Berg and Tschirhart (1988) summarize these regulatory policies.


*SSDI* 0928-7655(95)00008-9

The important point, however, is that knowing whether an industry is a natural monopoly is important for the proper design of policies.

More technically, a firm is a natural monopoly if it has a superadditive technology. If the firm is a price taker in all of its input markets, then a superadditive technology is equivalent to the firm's costs being subadditive in the relevant region of production. [2] Although estimating cost functions to test for subadditivity can be very difficult owing to the global nature of the concept, several authors have devised innovative methods for such estimations. [3] However, these estimates have all used standard procedures with respect to the firms' input markets, assuming them to be competitive and input prices exogenous. If input markets are not competitive, or more generally, if the firm is not a price taker but has market power in its input markets, then this should be recognized when estimating the cost function. Failure to do so will yield biased estimates and can lead to incorrect conclusions about the natural monopoly status of the firm. [4] In turn, policies with respect to price and entry regulation of the firm's markets may be misguided if the natural monopoly status is misjudged.

In this paper, the relationship between monopsony power and natural monopoly status is examined. We show that a firm with a superadditive technology, hence a natural monopoly, may not appear to be a natural monopoly if the estimated cost function does not account for monopsony power. In showing this result, an alternative cost function is constructed in which the firm behaves 'as if' the input prices are competitive. This alternative or behavioral cost function can be used to reveal the true natural monopoly status; thus, it is more useful for policy making.

Currently, policies with respect to market structures in the energy and telecommunication industries are under careful and extensive scrutiny. 'In reaction to the shocks of the seventies and eighties, the industry and its regulators are asking fundamental questions about the extent to which the public utility institution should be modified, if not abandoned, in favor of a greater reliance on competition' (Kahn, 1992, p. 177). Although this statement was made with respect to the electric industry, it pertains as well to the telecommunications and natural gas industries. Responses to the fundamental questions Kahn alludes to are already

[2] See Baumol et al. (1982) for the relationship between superadditive technology and subadditive cost.

[3] Evans and Heckman (1983) estimate a multiproduct cost function for AT&T prior to divestiture, and they reject the natural monopoly hypothesis. Using a very different approach (goal programming), Charnes et al. (1988) come to the opposite conclusion of Evans and Heckman which prompted a lively debate on the methodologies. Roeller (1990) modified the Evans and Heckman approach and also came to the opposite conclusion of Evans and Heckman. In a recent paper that discusses these earlier papers in more detail, Shin and Ying (1992) use post divestiture local exchange carrier LEC data and reject the natural monopoly hypothesis for these carriers.

[4] Hughes (1990) and Ofori-Mensa (1990) estimate input demand functions taking into account the rate-of-return constraint that electric utilities operate under. They take the market input prices as exogenous, however, and they do not address whether utilities are natural monopolies.

forthcoming in the these industries. Entrants into the electric industry, non utility generators, accounted for about 3.5% of total U.S. capacity in 1986, and the percent is expected to double by 2000. Local distribution companies of natural gas are being bypassed by their larger customers who are taking advantage of spot market prices. And in telecommunications, local exchange companies are being bypassed by competitive access providers.

Whether these responses are welfare enhancing is an open question. The answer will ultimately have to do with the sustainability of the regulated firms, the contestability of the markets they operate in, the costs of traditional regulation, and, most fundamentally, whether or not the firms are natural monopolies. Natural monopoly status is at the heart of the economic justification for regulation, providing the rationale for regulating electricity, natural gas, telecommunications, cable television, and water. [5] In the electric utility industry, for example, if the vertically integrated structure of generation, transmission and distribution is a natural monopoly, then the advantages of competition are questionable (see Tolley et al. (1992) or Tschirhart (1991)).

In the next Section a simple example is used to illustrate how monopsony power can appear to alter the natural monopoly status of a firm. Specifically, the analysis shows that a firm with a natural monopoly technology may give rise to a cost function that indicates the firm is not a natural monopoly. In Section 3, the relationship between monopsony and natural monopoly is formally developed. With competitive input markets, a superadditive technology is necessary and sufficient for a firm to be a natural monopoly. But with monopsony, a superadditive technology is necessary but not sufficient. In showing this, a behavioral cost function is derived that is more useful for policy making. The last Section covers policy implications and provides a brief discussion of empirical estimation of the behavioral cost function.

## 2. Monopsony in a special case

A special case is provided both to illustrate how monopsony power can alter a firm's natural monopoly status and to introduce notation. Consider a firm employing inputs $x = (x_1, \dots, x_n)$ to produce a single output $y$ according to the homothetic production function $y = F(x)$. Given perfectly competitive input markets, the firm's cost function is defined by

$$C(y,w) = \min_{x}\{wx\colon F(x) = y;\ x > 0\} \qquad (1)$$

[5] See Schmalensee (1979), Sharkey (1982) or Braeutigam (1989) for support of this position.

where $w=(w_1,\ldots,w_n)$ is the vector of input prices. The firm is a natural monopoly if and only if $C(y,w)$ is subadditive in $y$, or

$$C(y,w) < \sum_{j=1}^{m} C(y^j,w) \text{ for } \sum_{j=1}^{m} y^j = y. \tag{2}$$

Because the firm's technology is represented by a homothetic production function, cost can be written as a separable function of output and input prices:

$$C(y,w) = G(y)c(w) \tag{3}$$

where $G(y)=F^{-1}(y)$ and $c(w)$ is sometimes referred to as the unit cost function. Subadditivity of cost in this case is equivalent to

$$G(y) < \sum_{j=1}^{m} G(y^j) \text{ for } \sum_{j=1}^{m} y^j = y. \tag{4}$$

Baumol et al. (1982; Chapter 6) point out that input price changes can move the minimum point of a firm's average cost curve, thus changing its natural monopoly status. As (3) indicates, such a change is not generally true, because natural monopoly status for a firm with a homothetic production function is independent of the competitive input prices. However, if input markets are monopsonistic, then the natural monopoly status is dependent on the input markets even in the case of homothetic production.

To demonstrate, assume the firm acquires monopsony power in all input markets. The cost for input $i$ becomes $w_i(x_i,b_i)x_i$ where $w_i(x_i,b_i)$ is the inverse supply function for input $i$, and $b_i$ represents the parameters of the supply function. Assume too that the supply functions for all inputs are convex and homogeneous of degree $t$ in $x_i$. The monopsony firm's cost function can be derived by rewriting (1) as

$$C(y,b) = \min_x \{ w(x,b)x \colon F(f(x)) = y;\ x > 0 \}. \tag{5}$$

Here, $f(x)$ is homogeneous of degree one, and $F(\cdot)=y$ because a homothetic function can be written as a function of a linearly homogeneous function. Rewrite (5) as

$$\begin{aligned} C(y) &= \min_x \{ (w(x,b)x \colon f(x) = G(y);\ x > 0 \} \\ &= \min_x \{ w(x,b)x \colon [1/G]\ f(x) = 1;\ x > 0 \} \\ &= \min_x \{ w(x,b)x \colon f(x/G) = 1;\ x > 0 \} \end{aligned} \tag{6}$$

where (6) follows because $f(x)$ is homogeneous of degree one. Because supply functions are homogeneous of degree $t$

$$C(y,b) = \min_x \{ G^{t+1} w(x/G,b)x/G \colon f(x/G) = 1;\ x > 0 \}$$

$$= \min_x \{ G^{t+1} w(z,b) z \colon f(z) = 1 \colon x > 0 \}$$
$$= G^{t+1} \min_x \{ w(z,b) z \colon f(z) = 1;\ x > 0 \} = G^{t+1}(y) h(b). \tag{7}$$

Subadditivity for the monopsonistic firm holds it

$$G^{t+1}(y) < \sum_{j=1}^{m} G^{t+i}(y^j) \text{ for } \sum_{j=1}^{m} y^j = y. \tag{8}$$

Comparing (8) to (4), monopsonistic behavior can alter the firms natural monopoly status. For instance, suppose the firm with competitive input markets has an additive cost function so that

$$G(y) = \sum_{j=1}^{m} G(y^j) \text{ for } \sum_{j=1}^{m} y^j = y. \tag{9}$$

Then under monopsony, if $t > 0$, $G^{t+1}$ yields a superadditive cost function and if $t < 0$, $G^{t+1}$ yields a subadditive cost function. Because we would expect $t > 0$, monopsonistic power would make the firm's cost function less subadditive; or acquiring monopsony power can change the firm from a natural monopoly to a monopoly (see Baumol et al. (1982), p. 54). The reason is that the technological advantages of large scale production are offset by the pecuniary disadvantages. To produce at a larger scale requires addition inputs, but for a monopsonist the additional inputs can only be purchased at ever increasing prices.

## 3. Natural monopoly and monopsony

In this section we provide a framework for examining monopsony effects on natural monopoly status. [6] We continue to assume the firm produces a single output using $n$ nonnegative inputs. Let $T$ be the production-possibility set which lists all possible combinations of inputs and outputs. Formally, input and output vectors are in compact sets $X \subset R_+^n$ and $Y \subset R_+$, respectively, and $T$ is a nonempty closed subset of $X \times Y$. In addition, positive output requires positive input use: $(0,y) \in T$ iff $y = 0$; and there is free disposal of inputs: $(x,y) \in T$, $(x',y') \in X \times Y$, $x' \geq x$, $y' \leq y$ imply that $(x',y') \in T$. The input requirement set is defined as

$$A(y) = \{ x \colon (x,y) \in T \}. \tag{10}$$

$A(y)$ is assumed to be closed, convex and bounded and it is the set of inputs that can be used to produce output $y$.

[6] This section relies on duality results between production and cost functions. For a brief but informative survey of duality when output or input markets are imperfect, see Diewert (1982).

If the firm is a price taker in all $n$ input markets, then duality results allow the input requirement set to be written as

$$A(y) = \{x: wx \geq C(y,w), \forall w \in R_{t}^{n}\} \tag{11}$$

where $C(y,w)$ is the firm's cost function which will be referred to as the competitive input market (CIM) cost function. Baumol et al. prove that the CIM cost function is subadditive if and only if the technology is superadditive. Thus

$$C(y,w) + C(y',w) > C(y+y',w) \ \forall w > 0$$

if and only if

$$x \in A(y) \text{ and } x' \in A(y') \Rightarrow (x+x') \in A^0(y+y')$$

where $A^0(y)$ is the technically inefficient portion of $A(y)$ defined as

$$A^0(y) = \{x: (x,y) \in T, \exists x' \leq x, x' \neq x, (x',y) \in T\}. \tag{12}$$

We want to show that under monopsony a superadditive technology is necessary, but not sufficient for a subadditive monopsony input market (MIM) cost function. In doing so we make use of a behavioral input market (BIM) cost function that retains the properties of the CIM cost function, and which is useful for policy making.

Under monopsony the firm is assumed to be a price taker in input markets $(1,\ldots,r)$ and a monopsonist in markets $(r+1,\ldots,n)$. Cost can be written as

$$\sum_{i=1}^{r} w_i x_i + \sum_{i=r+1}^{n} w_i(x_i) x_i \tag{13}$$

where the parameters $b_i$ in the input supply functions are suppressed for convenience. The input supply functions are assumed to be well behaved to ensure a unique solution to the following cost minimization problem: [7]

$$\min_{x} \sum_{i=1}^{r} w_i x_i + \sum_{i=1+r}^{n} w_i(x_i) x_i \text{ subject to } y = f(x) \tag{14}$$

where the production function, $f(x)$ is assumed to be strictly quasiconcave and twice differentiable. This problem gives rise to the MIM cost function

$$c^m(y,w^r) \tag{15}$$

where $w^r = (w_1,\ldots,w_r)$.

[7] Strict convexity of the input supply functions will ensure a unique solution to (14), but it is a stronger condition than necessary.

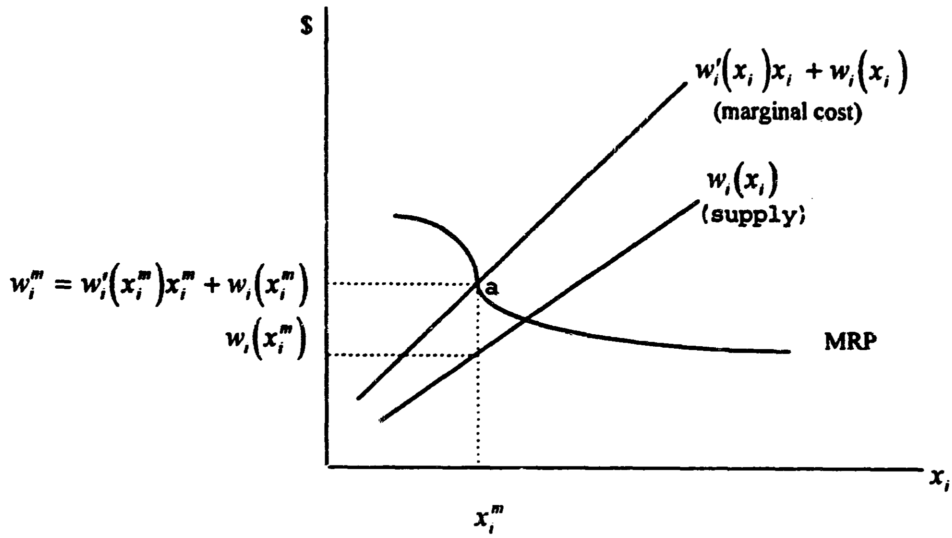


Fig. 1. Monopsony input demand.

Let $x^m$ be the solution to (14) and define a behavioral problem as

$$\min_x \sum_{i=1}^{r} w_i x_i + \sum_{i=1+r}^{n} w_i^m x_i \text{ subject to } y = f(x) \text{ and}$$

$$w_i^m = w_i'(x_i^m)x_i^m + w_i(x_i^m) \tag{16}$$

where the prime on the supply function indicates a derivative. Fig. 1 depicts the situation. The monopsonist equates marginal revenue product with marginal cost of the input at point a and employs $x_i^m$ units at a price of $w_i(x_i^m)$ each. In the behavioral problem, the monopsonist behaves as if it were a price taker. The price $w_i = w_1'\ (x_i^m)x_i^m + w_i(x_i^m)$ is treated as a parameter as if $x_i$ were in perfectly elastic supply. In this case the monopsonist again employs $x_i^m$. Converting the monopsony problem to a competitive problem permits the use of standard duality results below.

Because the solutions to problems (14) and (16) are characterized by identical first-order conditions, the solution to the behavioral problem is also $x^m$. Thus, for any monopsony problem using fixed input prices $w^r$, we can define a behavioral problem using the same fixed input prices and yielding the same solution vector $x^m$. The cost function for the behavioral problem is

$$C^b(y,w^r,w^m) \tag{17}$$

where $w^m = (w_{r+1}^m, \ldots, w_n^m)$. Note that for any output $y$, we have

$$C^m(y,w^r) < C^b(y,w^r,w^m) \tag{18}$$

because $w_i^m > w_i\ (x^m)$ for $i = r+1, \ldots, n$ (see Fig. 1). The $w^m$ can be thought of a vector of pseudo input prices.

Next, define an input requirement set for the monopsony problem as

$$A_m(y)=\left\{x:\sum_{i=1}^{r} w_i x_i+\sum_{i=r+1}^{n} w_i(x_i)x_i \geq C^m(y,w^r),\ \forall w\in R_+^r\right\} \tag{19}$$

and for the behavioral problem as

$$A_b(y)=\left\{x:\sum_{i=1}^{r} w_i x_i+\sum_{i=r+1}^{n} w_i^m x_i \geq C^b(y,w^r,w^m),\ \forall w\in R_+^r\right\}. \tag{20}$$

In the Appendix, we show that $A_m(y)=A_b(y)$. Moreover, because $A_b(y)$ is constructed as if all input markets are competitive, duality results can be used in the usual fashion to show the equivalence of $A_b(y)$ and (10). Therefore,

$$A(y)=A_m(y)=A_b(y) \tag{21}$$

and following Baumol et al. we can write the technically inefficient portion of $A_m(y)$ as

$$A_m^0=\left\{x:\sum_{i=1}^{r} w_i x_i+\sum_{i=r+1}^{n} w_i(x_i)x_i > C(y,w^r),\ \forall w\in R_+^r\right\}. \tag{22}$$

Finally, we can show that under monopsony a superadditive technology is necessary, but not sufficient for a subadditive MIM cost function.

**Theorem 1.** If $C^m(y',w^r)+C^m(y'',w^r)>C^m(y'+y'',w^r)\forall w^r>0$, then $x'\in A_m(y')$ and $x''\in A_m(y'')$ implies that $(x'+x'')\in A_m^0(y'+y'')$.

**Proof.** Let $x'\in A_m(y')$ and $x''\in A_m(y'')$ so that for $w^r>0$ we have

$$\sum_{i=1}^{r} w_i x_i'+\sum_{i=r+1}^{n} w_i(x_i')x_i' \geq C^m(y',w^r)$$

and

$$\sum_{i=1}^{r} w_i x_i''+\sum_{i=r+1}^{n} w_i(x_i'')x_i'' \geq C^m(y'',w^r).$$

Then,

$$B\equiv\sum_{i=1}^{r} w_i[x_i'+x_i'']+\sum_{i=r+1}^{n} w_i(x_i'+x_i'')[x_i'+x_i''] > C^m(y',w^r) + C^m(y'',w^r)$$

because the $w_i$ are strictly convex, and by subadditivity

$$B>C^m(y',w^r)+C^m(y'',w^r)>C^m(y'+y'',w^r).$$

But this implies that $x'+x''\in A^0(y'+y'')$.

That a superadditive technology is not sufficient for a subadditive cost function can be shown by counter example. Suppose a firm employing two inputs exhibits a Cobb–Douglas technology. The problem

$$\min_{x} w_i x_i + w_2(x_2)x_2 \text{ subject to } y = x_1^{\alpha} x_2^{\beta} \text{ and } w_2(x_2) = ax_2$$

generates the cost function

$$C(y,w) = y^{\frac{2}{2\alpha+\beta}} \left[\frac{\beta w_1}{2a\beta}\right]^{\frac{2\alpha}{2\alpha+\beta}} \frac{\alpha}{\beta}[2\alpha+\beta].$$

Clearly, for $\alpha+\beta>1$ the technology is superadditive but the cost function is subadditive only for $2\alpha+\beta>2$. Thus, there is a range of values for $\alpha$ and $\beta$ such that a superadditive technology does not yield subadditive costs.

## 4. Policy implications

The justification for price and entry regulation of firms revolves around their natural monopoly status. To determine their status, cost functions can be estimated and then tested for subadditivity. The standard procedure is to estimate a CIM cost function. But if these firms possess monopsony power, this estimation procedure can yield biased estimates of the CIM cost function and incorrect conclusions with respect to natural monopoly status.

Empirical studies that test for natural monopoly use data from large public utilities, or firms which reasonably can be expected to exhibit monopsony power in their input markets. Although the extent of monopsony power is not conclusive, there is evidence of its existence. A large percent of the telecommunications industry's labor force are members of the American Telecommunications Union, and wage negotiations are common. In the electric industry, Atkinson and Kerkvliet (1986) find evidence that coal-fired utilities exercise monopsony power in their coal input markets.

To avoid the potentially incorrect conclusions associated with the CIM cost function, an alternative approach would be to estimate both the MIM cost function and the BIM cost function. Estimating MIM is straightforward, and in a sense easier than estimating the CIM function. Less data is needed for the input prices, because the monopsony market prices do not appear as functional arguments in the cost function.

Estimating the BIM cost function given by (17) is more problematic, because the pseudo input prices, $w^m$, are not observed. Moreover, the elements of $w^m$ are functions of the chosen inputs so that estimating the BIM cost function requires estimating input supply functions. However, the estimation procedure can be greatly simplified by assuming a specific functional form for the input supply functions. In particular, suppose an input supply function over the relevant input range exhibits constant elasticity. Then, for input $i$, the supply function can be written

$$w_i(x_i) = ux_i^{\upsilon}.$$

From (16), the pseudo price for the $i$th input is

$$w_i^m = \mathrm{d}w_i(x_i)x_i/\mathrm{d}x_i = \mathrm{d}ux_i^{\upsilon+1}/\mathrm{d}x_i = k_i w_i(x_i)$$

where $k_i = \upsilon + 1$. The unobserved, behavioral price can be written as the product of a constant and the observed, actual price. Moreover, the constant can be estimated in the cost function. [8] Because the BIM cost function is derived with the monopsonist behaving 'as if' she was a price taker for her inputs, the cost function contains all the economically relevant information about the production technology.

Once the MIM and BIM cost functions are estimated, they can be tested for subadditivity. From Theorem 1, if the MIM cost function is subadditive, then the firm is a natural monopoly. If the MIM cost function is not subadditive, no conclusion can be drawn on natural monopoly status. With respect to the BIM cost function, if the MIM cost function is subadditive, the BIM function will also be subadditive and no additional information is gained. However, if the BIM function is not subadditive, then we can conclude that the firm is not a natural monopoly.

Information on the firm's natural monopoly status can be combined with information on barriers to entry into the firm's markets and the sustainability of the firm to formulate regulatory policy, including possibilities for deregulation. [9] Briefly, if a firm is not a natural monopoly, then regulation is not justified. If a firm is a natural monopoly, then regulation may be justified depending on whether its markets are contestable and on whether sustainable prices exist. Without contestability, regulation is justified to protect consumers from monopoly output pricing practices. With contestability, regulation is justified only if the firm is not sustainable, for if it is sustainable then the threat of entry will compel the firm to charge break-even prices.

The possibility of monopsony power complicates these broad policy prescriptions. Monopsony implies that inputs can only be purchased at ever increasing prices, and that less than the socially optimum level of input is employed. While

[8] Estimating the $k_i$ was originally suggested by Lau and Yotopoulos (1971) who referred to them as shadow prices perceived by the firm that differ from the market prices. Atkinson and Halvorsen (1984) and Atkinson and Halvorsen (1990), e.g.s have made extensive use of this technique to test for technical and allocative efficiency for regulated public utilities. Their method takes the $k_i$ as functions of exogenous variables instead of being endogenously determined by the firm, and is, therefore, inappropriate for the monopsony case. A somewhat different approach is used by Appelbaum (1979) to test for price taking behavior. None of these authors addressed the natural monopoly and monopsony issue.

[9] Berg and Tschirhart (1988) summarize these regulatory policies.

entry of new firms into the output markets will diminish the monopsony power and increase the level of input employed, this may not be desirable if the incumbent firm is also a natural monopolist. Although the competing entrant firms hire an increased level of input, now that input is too large for the final output produced, because the technological advantages of natural monopoly are lost.

More to the point for this analysis is the possibility that entry is encouraged in natural monopoly markets, because cost studies which do not take into account monopsony power erroneously determine that firms in the markets are not natural monopolies. This scenario could be important in the electric industry which traditionally has been dominated by large, regulated utilities, but which is now experiencing entry by relatively small, unregulated electric generating firms. Proponents of entry appeal to studies that suggest generation is no longer a natural monopoly, yet no account is taken of the monopsony implications derived in this paper.

## Acknowledgements

Scott Atkinson, Sanford Berg, Bruce Forster, Tracy Lewis, Richard Romano, Edward Zabel, an anonymous referee and participants at a University of Florida Seminar provided useful comments and suggestions. Partial funding was provided by NSF/EPSCoR Grant # RII-8610680. Responsibility for any errors lies with the author.

## Appendix

**Theorem 2.** The input requirement set

$$A_m(y) = \left\{ x \colon \sum_{i=1}^{r} w_i x_i + \sum_{i=r+1}^{n} w_i(x_i) x_i \geq C^m(y, w^r), \forall w^r > 0 \right\} \tag{19'}$$

is equivalent to the set

$$A_b(y) = \left\{ x \colon \sum_{i=1}^{r} w_i x_i + \sum_{i=r+1}^{n} w_i^m x_i \geq C^b(y, w^r, w^m), \forall w^r > 0 \right\} \tag{20'}$$

where $w_i^m$ is defined in the behavioral problem.

**Proof.** For a given $y$ and $w^r$ we know that if $x^m$ satisfies

$$\sum_{i=1}^{r} w_i x_i^m + \sum_{i=r+1}^{n} w_i(x_i^m) x_i^m = C^m(y, w^r) \tag{A.1}$$

then it also satisfies

$$\sum_{i=1}^{r} w_i x_i^m + \sum_{i=r+1}^{n} w_i^m x_i^m = C^b(y,w^r,w^m). \tag{A.2}$$

Because the left side of (A.1) is a convex function of $x$ by assumption (recall the $w_i(x_i)$ are strictly convex for all $i$), the set

$$S \equiv \left\{ x | \sum_{i=1}^{r} w_i x_i^m + \sum_{i=r+1}^{n} w_i(x_i^m) x_i^m \leq C^m(y,w^r) \right\}$$

is convex in input space. Moreover, (A.2) defines a supporting hyperplane for $s$ at the boundary point $x^m$. Therefore, by standard separation theorems, for any $x$ such that

$$\sum_{i=1}^{r} w_i x_i + \sum_{i=r+1}^{n} w_i^m x_i \geq C^b(y,w^r,w^m)$$

it must be that

$$\sum_{i=1}^{r} w_i x_i + \sum_{i=r+1}^{n} w_i(x_i) x_i \geq C^m(y,w^r).$$

Thus, if $x \in A_b(y)$ then $x \in A_m(y)$.

Next we need to show that if $x \notin A_b(y)$ then $x \notin A_m(y)$. Suppose $\hat{x} \notin A_b(y)$ so

$$\sum_{i=1}^{r} w_i \hat{x}_i + \sum_{i=r+1}^{n} w_i^m \hat{x}_i < C^b(y,w^r,w^m) \tag{A.3}$$

but

$$\sum_{i=1}^{r} w_i \hat{x}_i + \sum_{i=r+1}^{n} w_i(\hat{x}_i) \hat{x}_i \geq C^m(y,w^r). \tag{A.4}$$

We need to find another $w^r$ such that inequality (A.4) is reversed. By (A.3), we know $\hat{x}$ cannot produce $y$, otherwise $x^m$ would not be a cost minimizing input bundle. By continuity of the production function, we can define an $\tilde{x}$ such that $y = f(\tilde{x})$ and $\tilde{x}_i = \bar{x}_i + \delta$, $i = 1,\ldots,n$, and

$$\lambda f_i(\tilde{x}) = \tilde{w}_i \qquad i = 1,\ldots,r$$
$$\lambda f_j(\tilde{x}) = \tilde{w}_j^m \qquad j = r+1,\ldots,n$$

where $f_i(\tilde{x}) = \partial f/\partial x_i$ and $\lambda$ is a constant. But $\tilde{x}$ is the solution to the problems:

$$\min_x \sum_{i=1}^{r} \tilde{w}_i x_i + \sum_{i=r+1}^{n} \tilde{w}_i^m x_i \text{ subject to } y = f(x)$$

and

$$\min_x \sum_{i=1}^{r} \tilde{w}_i x_i + \sum_{i=r+1}^{n} w_i(\tilde{x}_i) x_i \text{ subject to } y = f(x)$$

where $\tilde{w}_i^m = w_i'(\tilde{x})x_i + w_i(\tilde{x})$. Thus,

$$\sum_{i=1}^{r} \tilde{w}_i \tilde{x}_i + \sum_{i=r+1}^{n} w_i(\tilde{x}_i)\tilde{x}_i = C^m(y, \tilde{w}^r)$$

but

$$\sum_{i=1}^{r} \tilde{w}_i \hat{x}_i + \sum_{i=r+1}^{n} w_i(\hat{x}_i)\hat{x}_i < C^m(y, \tilde{w}^r)$$

because $\bar{x} < \tilde{x}$ and $w_i(x_i)x_i$ is monotonically increasing in $x_i$. Therefore $\bar{x} \notin A(y)$.

## References


Appelbaum, E., 1979, Testing price taking behavior, Journal of Econometrics 9, 283–294.

Atkinson, S.E. and R. Halvorsen, 1984, Parametric efficiency tests, economies of scale, and input demand in U.S. electric power generation, International Economic Review 25, 647–662.

Atkinson and Halvorsen, 1990, Tests of allocative efficiency in regulated multiproduct firms, Resources and Energy (April) 12, 65–77.

Atkinson, S.E. and J. Kerkvliet, 1986, Measuring the multilateral allocation of rents: Wyoming low sulphur coal, RAND Journal of Economics (Autumn) 17, 416–430.

Baumol, W.J., J.C. Panzar and R.D. Willig, 1982, Contestable markets and the theory of industry structure (Harcourt Brace Jovanovich, New York).

Berg, S.V. and J. Tschirhart, 1988, Natural monopoly regulation: Principles and practice (Cambridge University Press, USA).

Braeutigam, R.R., 1989, Optimal policies for natural monopolies, in: R. Schmalensee and R. Willig, eds., Handbook of Industrial Organization, Vol. 2 (North-Holland, Amsterdam), 1289–1346.

Charnes, A., W.W. Cooper and T. Sueyoshi, 1988, A goal programming/constrained regression review of the Bell system breakup, Management Science 34, 1–26.

Diewert, W.E., 1982, Duality approaches to microeconomic theory, in: K.J. Arrow and M.D. Intriligator, eds., Handbook of Mathematical Economics II, 535–599.

Evans, D.S. and J.J. Heckman, 1983, Multiproduct cost function estimates and natural monopoly tests for the Bell system, in: D.S. Evans, ed., Breaking up Bell (North-Holland, Amsterdam), 253–282.

Hughes, J.P., 1990, Profit maximizing input demand under rate-of-return regulation, Resources and Energy 12, 75–95.

Kahn, A.E., 1992, Least-cost planning generally and DSM in particular, Resources and Energy 14, 177–185.

Lau, L.J. and P.A. Yotopoulos, 1971, A test of relative efficiency and an application to Indian agriculture, American Economic Review 63, 214–223.

Ofori-Mensa, C., 1990, Tests of perverse input demand behavior by the rate-of-return regulated firm, Resources and Energy 12, 97–106.

Roeller, L.H., 1990, Proper quadratic cost functions with an application to the Bell system, Review of Economics and Statistics 72, 202–210.

Schmalensee, R., 1979, The control of natural monopolies (D.C. Heath and Co., Lexington, MA).

Sharkey, W.W., 1982, The theory of natural monopoly (Cambridge University Press, New York).

Shin, R.T. and J.S. Ying, 1992, Unnatural monopolies in local telephone, RAND Journal of Economics 23(2), 171–183.

Tolley, G.S., P.H. Griffes, R.S. Chirinko, R.R. Geddes and E. Bodmer, 1992, Emerging issues in the regulation of electric utilities, Resources and Energy 14, 3–35.

Tschirhart, J., 1991, Entry into the electric power industry, Journal of Regulatory Economics 3, 27–43.