Energy xxx (2011) 1–9

Contents lists available at SciVerse ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

Energy and the state of nations

Dietmar Lindenberger [a], Reiner Kümmel [b,*]

[a] Energiewirtschaftliches Institut an der Universität Köln, D-50923 Köln, Germany
[b] Institut für Theoretische Physik und Astrophysik, Universität Würzburg, D-97074 Würzburg, Germany

ARTICLE INFO

Article history:
Received 16 February 2011
Received in revised form
2 July 2011
Accepted 8 August 2011
Available online xxx

Keywords:
Energy
Economic growth
Macroeconomic production functions
Output elasticities

ABSTRACT

The mathematical conditions for the existence of macroeconomic production functions that are state functions of the economic system are pointed out. The output elasticities and the elasticities of substitution of energy-dependent Cobb-Douglas, CES and LinEx production functions are calculated. The output elasticities, which measure the productive powers of production factors and whose numerical values have been obtained for Germany, Japan, and the USA, are for energy much larger and for labor much smaller than the cost shares of these factors. Energy and its conversion into physical work accounts for most of the growth that mainstream economics attributes to "technological progress" and related concepts. It decisively determines the economic state of nations. Consequences for automation and globalization and perspectives on growth are discussed.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

The gross domestic product (GDP) is a nation's output of goods and services. It measures the value added created by economic activities within the country and does not include imported intermediate goods and services. The GDP has been traditionally identified with the wealth of nations, and its growth has been the aim of economic policy everywhere. Critics object that the GDP does not say enough about the quality of life. On the one hand, it does not include activities like unpaid housekeeping, the upbringing of children, and honorary social services. And severe inequalities of wealth distribution, which may cause social turmoils, are no issue either. On the other hand, it comprises all services and goods required in order to mitigate the effects of traffic accidents, repair damage from man-made disasters like the explosion of maritime oil-drilling platforms, and cure diseases caused by pollution. But more than two decades of research into the development of a reliable, quantitative indicator for the quality of life have not yet succeeded in producing a generally accepted substitute for GDP. Furthermore, people like to live in countries where a high GDP per capita indicates that the average individual commands a rich consumer basket of material goods and services. The increasing migration from countries of low GDP per capita to countries of high GDP per capita demonstrates this. People agree in general that the latter are better off than the former, because they use technology and natural resources more intensively for the production of material wealth. Since the GDP is an important, although not sufficient quantitative indicator of the socio-economic state of nations, this paper tries to outline the general mathematical framework for its computation. Methods of calculating key quantities of an economic system are presented.

The plan of the paper is the following. Section 2 discusses the physical justificatiton and general mathematical properties of macroeconomic production functions that depend on capital, labor, energy, and time. The explicit time dependence models the specific human contribution to production and growth, which we call creativity. We indicate the fundamental flaw of standard economic theory: the disregard of technological constraints on the combination of production factors in the determination of macroeconomic equilibrium by the maximization of profit or time-integrated utility. Appendix A gives more details. The system of differential equations, from which the economic weights of the production factors, called output elasticities, and the production functions themselves are to be calculated, is presented. Appendix B relates these basic equations to the production functions' twice differentiability. Section 3 presents three special solutions of the partial differential equations for the output elasticities, the corresponding energy-dependent Cobb-Douglas, CES and LinEx functions, and the elasticities of substitution of these production functions. Findings from prior econometric analyses of economic growth in Germany, Japan, and the USA are reported, according to which the output elasticity of energy far outweighs its small share in total factor cost

* Corresponding author.
E-mail address: kuemmel@physik.uni-wuerzburg.de (R. Kümmel).

0360-5442/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.energy.2011.08.014

while for labor just the opposite is true. Section 4 discusses the political and social consequences that result from the fact that energy decisively determines the economic state of nations.

2. Twice differentiable production functions as state functions of economic systems

Standard economics describes the GDP, or the output of subsectors of an economy, by twice-differentiable functions of the factor inputs. These macroeconomic production functions have the important property that they depend only on the actual numerical values of the production factors and *not* on the path in factor "space" along which the economy has arrived at the magnitudes of the inputs. Macroeconomic production functions are state functions of the economic system in the same sense as entropy, internal energy, enthalpy, and (Helmholtz and Gibbs) free energy are state functions of thermodynamic systems.

2.1. Why macroeconomic production function do make sense

The concept of the twice-differentiable macroeconomic production function is widely accepted. Nevertheless, it has been criticized by scholars who have been concerned about the physical aspects of production [1–6]. Their criticism centers on three principal objections. The first is the problem of aggregating the heterogeneous goods and services of output into one monetary quantity, represented in the national accounts by the deflated GDP, or parts thereof. The second is the related problem of aggregating the heterogeneous components of the capital stock (machines, structures, etc.) into one monetary quantity "capital", measured by deflated currency in the national accounts. The third problem is the unclear relationship between the micro theory of production in individual firms, for which the concept of the (micro)production function is not questioned, and the macro theory.

These concerns have led to a scheme of aggregating output and capital in the physical terms of work performance and information processing and relating the physical aggregates to the usual inflation-corrected monetary aggregates via equivalence factors [7,8].[1] Since work performance and information processing are subject to the causal laws of nature, their result, the economic output, should depend as uniquely on the work-performing and information-processing production factors capital, labor, and energy as any state function of physical systems depends on its physical variables.

2.2. Flaws of standard economics

Thus, we think that the concept of the macroeconomic production function is not the weak point of mainstream economics. But by combining this concept with the observance of technological constraints in the derivation of economic equilibrium one finds the true breaking point of standard economic theory: As sketched in Appendix A, the cost share theorem, according to which the economic weight (output elasticity) of a production factor should be equal to the factor's share in total factor cost, is destroyed by shadow prices, which translate technological constraints into monetary terms. The technological constraints are: 1. the degree of capacity utilization cannot exceed 1, and 2. the degree of automation cannot exceed the technologically possible degree of automation, whose maximum value is 1 [8,9]. The often-used duality between factor-dependent production functions and the price functions that result from them by a Legendre transformation is not valid either as a consequence of the technological constraints. This confers new importance to the search for appropriate methods of computing macroeconomic production functions.

Traditionally, capital and labor have been considered as the principal factors of production. In industrialized countries their cost shares have been in the 25–30 percent range and 65–70 percent range, respectively. After the oil price shocks of the 1970s and 80s, energy was occasionally taken into account as a third production factor with the small cost-share weight of roughly 5 percent. But mainstream economics has the problem that it can explain only about half, or less, of the observed economic growth of industrialized countries by the growth of the cost-share weighted production factors. The other half, or more, is attributed to "technological progress". This is just a word for what is not understood. The dominating role of technological progress "has lead to a criticism of the neoclassical model: it is a theory of growth that leaves the main factor in economic growth unexplained" [10], as the founder of neoclassical growth theory, Robert A. Solow, stated himself. On the other hand, analyses of economic growth that forgo cost-share weighting find that energy and its conversion into physical work accounts for most of the growth that mainstream economics attributes to "technological progress" and related concepts [9,11,12]. This is in line with the observation that "universal history can be subdivided in three parts. Each part is characterized by a certain energy system. This energy system establishes the general framework, within which the structures of society, economy and culture form. Thus, energy is not just one acting factor among many. Rather it is possible, in principle, to determine the formal basic structures of a society from the pertaining energetic system conditions." [13].

2.3. Structure and factors of modern economies

We consider an industrialized economy. It consists, roughly speaking, of a physical basis that produces goods and services, and a market superstructure, where economic actors trade the products of the physical basis. Price signals from supply and demand provide the feedback between the productive physical basis and the market superstructure, see Fig. 1. Three production factors are busy in the physical basis and produce the output Y, which is the GDP, if the system is the national economy. 1) Energy-converting and information-processing machines together with all buildings and installations necessary for their protection and operation represent the production factor capital K. Any machine that is activated by energies that do *not* flow out of living bodies has at least one information processor: the valve or switch that opens up and shuts down the energy flow into the machine. (The difference between Newcomen's steam pump and James Watt's first pumping steam engine was the separate condensing chamber, which increased the number of valves from three to four.) 2) The capital stock K is manipulated and supervised by people, who constitute the production factor labor L. 3) The machines of the capital stock are activated by energy

[1] In brief, the relation between physical and monetary aggregates is the following. The *physical* capital unit is defined as 1 ATON $\equiv$ 1 kW $\times$ κkB/s (kW = kilowatt, kB/s = kilobits per second). The equivalence factor κ is given by $\kappa = (1/N)\sum_{i=1}^{N} S_i T_i$, where the definitions of N, S_i, and T_i imply the measurement description of the ATON: The capital stock K is partitioned in $N \gg 1$ pieces K_i that all have the same *monetary* value, say ν euros. Then S_i = number of kilowatts performed and T_i = number of kilobits per second processed by the fully employed i th capital good K_i. Consequently, the physical value of capital, which is A_k, grows proportionally to the monetary value of capital, which is M_k and shown in the national accounts in constant currency, as long as κ (and thus the ATON) does not change in time: $A_\kappa = N$ ATON $= \sum_{i=1}^{N} S_i T_i$ kW $\times$ kB/s; $M_K \equiv N_\nu$ euro; thus, $A_K = (M_K/\nu)$ ATON/euro. Changes of κ occur, when the monetary valuation of the capabilities of work performance and information processing changes. An equivalence factor ζ, similar to κ, appears in the technical definition of output in terms of the physical work performed and the number of information units processed in its generation.

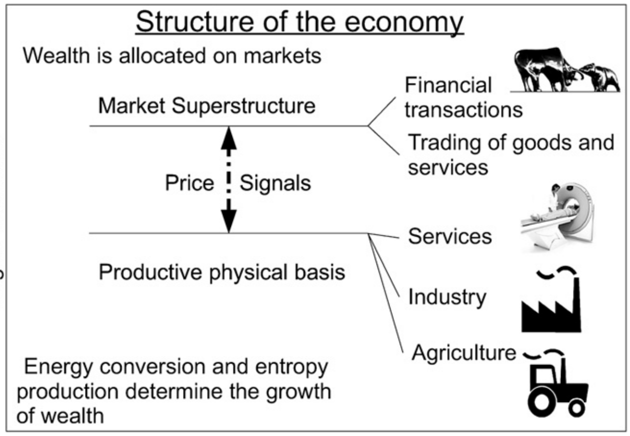

Fig. 1. The productive physical basis of an industrialized economy consists of the sectors agriculture, industry, and services. Their energy intensities differ, but none of them could do without energy conversion. Entropy production is coupled to energy conversion, with repercussions on growth discussed in Section 4. The wealth produced in the basis is allocated on the market, whose legal framework determines the outcome [8].

(more precisely exergy [14]), which is the production factor E. (As a rule, the professional qualification of labor has to increase with the energy flows through the capital stock it controls.) The measuring units are: deflated monetary units for output Y and instrumental capital K as listed by the national accounts, manhours worked per year for routine labor L as shown by the labor statistics, and energy quantities like petajoules "consumed" per year for the factor E.[2] The working of human creativity via ideas, inventions and value decisions is modeled by an explicit time dependence of the production function, which manifests itself, e.g., in a time change of the energy-demand parameter of the capital stock when capital's energy efficiency increases.

It is convenient to introduce new, dimensionless variables, for which we use lower case letters, by writing inputs and output as multiples of their quantities K_0, L_0, E_0, and Y_0 in a base year t_0. The transformation to the dimensionless time series of capital, $k(t)$, labor, $l(t)$, and energy, $e(t)$, is given by

$$k(t) \equiv \frac{K(t)}{K_0},\ l(t) \equiv \frac{L(t)}{L_0},\ e(t) \equiv \frac{E(t)}{E_0}, \tag{1}$$

and the dimensionless production function at time t is

$$y[k,l,e;t] \equiv \frac{Y(kK_0, lL_0, eE_0; t)}{Y_0}. \tag{2}$$

2.4. Growth equation and output elasticities

The production function $y[k,l,e;t]$ is supposed to be twice differentiable. Therefore its infinitesimal change is the total differential dy. Dividing dy by y one obtains the growth equation

$$\frac{dy}{y} = \alpha\frac{dk}{k} + \beta\frac{dl}{l} + \gamma\frac{de}{e} + \delta\frac{dt}{t - t_0}, \tag{3}$$

where the output elasticities

$$\alpha \equiv \frac{k}{y}\frac{\partial y}{\partial k},\ \beta \equiv \frac{l}{y}\frac{\partial y}{\partial l},\ \gamma \equiv \frac{e}{y}\frac{\partial y}{\partial e}, \tag{4}$$

represent the weights with which the growth rates of capital, labor and energy contribute to the growth of output. The output elasticities indicate the productive powers of the production factors. Creativity, in the above-mentioned sense, gives rise to

$$\delta \equiv \frac{t - t_0}{y}\frac{\partial y}{\partial t}. \tag{5}$$

We emphasize that the infinitesimal change of a production function that were *not* twice differentiable would not be a total differential. Its integral would depend on the path of integration in (k,l,e) space and contain not more information on the state of an economic system than the integral of a non-conservative frictional force along a line in position space contains information on the state of a dissipative physical system. (A motor car that slows down because it dissipates its kinetic energy by friction with air and road is such a system.) On the other hand, the total differential in the growth Eq. (3) can be integrated along any convenient path in (k,l,e) space. The calculation of the LinEx function in Section 3 indicates such a path. The explicit condition for twice differentiability and path-independent integrability is that the second-order mixed derivatives of $y[k,l,e;t]$ are equal. As it is shown in Appendix B, this condition results in the partial differential equations for the output elasticities

$$l\frac{\partial \alpha}{\partial l} = k\frac{\partial \beta}{\partial k},\ e\frac{\partial \beta}{\partial e} = l\frac{\partial \gamma}{\partial l},\ k\frac{\partial \gamma}{\partial k} = e\frac{\partial \alpha}{\partial e}. \tag{6}$$

Formally, these equations correspond to the Maxwell relations in thermodynamics.

At any *fixed* time t the contributions from the growth rates of all factors to the growth of output must add up to 100 percent, so that the output elasticities must satisfy the the so-called "constant returns to scale" relation

$$\alpha + \beta + \gamma = 1. \tag{7}$$

This relation characterizes linearly homogeneous production functions, whose value increases by a certain factor, say λ, if *all* inputs increase by the same factor[3] λ. We assume that $y[k,l,e;t]$ is linearly homogeneous indeed. Appendix B shows that the combination of Eqs. (6) and (7) yields the set of partial differential equations [7,15].

$$l\frac{\partial \alpha}{\partial l} = k\frac{\partial \beta}{\partial k},\ k\frac{\partial \alpha}{\partial k} + l\frac{\partial \alpha}{\partial l} + e\frac{\partial \alpha}{\partial e} = 0,$$
$$k\frac{\partial \beta}{\partial k} + l\frac{\partial \beta}{\partial l} + e\frac{\partial \beta}{\partial e} = 0. \tag{8}$$

The most general solutions of the second and the third differential equation in (8) are[4]

[2] Fossil and nuclear fuels, kinetic and potential energy, and solar radiation as well, are practically 100 percent exergy. Therefore, it is appropriate to work with the primary energy data of the national energy balances that include these energy forms properly.

[3] An increase of all inputs by λ must increase output by λ, because at the fixed state of technology that exists at the given time t a, say, doubling of the production system doubles output; in other words: two identical factories with identical inputs of capital, labor and energy produce twice as much output as one factory. Thus, the production function must be linearly homogeneous in (k,l,e) so that $y(\lambda k, \lambda l, \lambda e) = \lambda y(k,l,e)$ for all $\lambda > 0$ and all possible factor combinations. Differentiating this equation with respect to λ according to the chain rule and then putting $\lambda = 1$ one obtains the Euler relation $k(\partial y/\partial k) + l(\partial y/\partial l) + e(\partial y/\partial e) = y$. Dividing this by y yields $(k/y)(\partial y/\partial k) + (l/y)(\partial y/\partial l) + (e/y)(\partial y/\partial e) = 1$ With Eq. (4) this becomes Eq. (7).

[4] The derivation according to the theory of partial differential equations is given in [15]. Verification by insertion into (8) is easy.

$$\alpha = A\left(\frac{l}{k}, \frac{e}{k}\right), \; \beta = B\left(\frac{l}{k}, \frac{e}{k}\right), \tag{9}$$

where A and B are any differentiable functions of their arguments. Because of the first equation in (8) they are coupled together by

$$\beta = \int^{k} \frac{l}{k'} \frac{\partial A}{\partial l} \, \mathrm{d}k' + J\left(\frac{l}{e}\right); \tag{10}$$

here $J(l/e)$ is any differentiable function of l/e.

The output elasticities, and thus the combinations of k, l, e, must satisfy the restrictions

$$\alpha \geq 0, \; \beta \geq 0, \; \gamma = 1 - \alpha - \beta \geq 0, \tag{11}$$

which result from the technical-economic requirement that all output elasticities must be non-negative. Otherwise the increase of an input would result in a decrease of output – a situation the economic actors will avoid.

The general form of the twice-differentiable, linearly homogeneous production function with the output elasticities (9) is

$$y = e\mathcal{F}\left(\frac{l}{k}, \frac{e}{k}\right). \tag{12}$$

Of course, one can write this function also as $y = k\mathcal{G}$, with $\mathcal{G} = (e/k)\mathcal{F}$, or as $y = l\mathcal{H}$, with $\mathcal{H} = (e/l)\mathcal{F}$.

According to the theory of partial differential equations the output elasticities, and thus the production function, could be uniquely determined, if β were known on a boundary surface in k, l, e-space and if one knew α on a boundary *curve* in that space [15]. However, it is practically impossible to obtain this technical-economic information on α and β. Without that one cannot compute the output elasticities at a given time t *exactly*. One must make do with approximations. We indicate some of them in the next section, and the pertaining production functions as well. (Factor prices do not play a role in that, because, according to Eqs. (26) and (27) of Appendix A, the cost share theorem is not valid.)

2.5. Elasticities of substitution

Besides output elasticities, the elasticities of substitution between production factors are of interest to economists. The (Hicks, or direct) elasticity of substitution σ_{ij} between two factors x_i and x_j is defined as

$$\sigma_{ij} \equiv -\frac{\mathrm{d}(x_i/x_j)}{(x_i/x_j)} \cdot \left[\frac{\mathrm{d}\left(\dfrac{\partial y/\partial x_i}{\partial y/\partial x_j}\right)}{\left(\dfrac{\partial y/\partial x_i}{\partial y/\partial x_j}\right)}\right]^{-1}. \tag{13}$$

It gives the ratio of the relative change of factor quotients to the relative change of the quotients of the marginal productivities, if only the factors x_i and x_j vary and all other factors stay constant.

In the three-factor model with $(x_1, x_2, x_3) = (k, l, e)$ the elasticities of substitution can be expressed by the output elasticities α, β and γ. After some algebraic manipulations one finds [16]

$$\sigma_{kl} = \frac{-(\alpha+\beta)\alpha\beta}{\beta^2(k\partial\alpha/\partial k - \alpha) + \alpha^2(l\partial\beta/\partial l - \beta) - 2\alpha\beta l\partial\alpha/\partial l}, \tag{14}$$

$$\sigma_{ke} = \frac{-(\alpha+\gamma)\alpha\gamma}{\gamma^2(k\partial\alpha/\partial k - \alpha) + \alpha^2(e\partial\gamma/\partial e - \gamma) - 2\alpha\gamma e\partial\alpha/\partial e}, \tag{15}$$

$$\sigma_{le} = \frac{-(\beta+\gamma)\beta\gamma}{\gamma^2(l\partial\beta/\partial l - \beta) + \beta^2(e\partial\gamma/\partial e - \gamma) - 2\beta\gamma e\partial\beta/\partial e}. \tag{16}$$

3. Cobb-Douglas, CES, and LinEx functions

We present three special solutions of the partial differential Eq. (8). The pertaining production functions have been used for some time in production and growth theory. Historically, the Cobb-Douglas and the CES functions were designed before the partial differential Eq. (8) had been derived. Nevertheless, the output elasticities of their energy-dependent versions do satisfy these equations. LinEx functions, on the other hand, follow from solutions of (8) and the expected behavior of output elasticities, if factor ratios approach certain limiting values. (Energy-dependent translog functions and their relation to Cobb-Douglas and LinEx functions are discussed in [17]).

3.1. Energy-dependent Cobb-Douglas function

The simplest approach to output elasticities and production functions is choosing the trivial solutions of Eq. (8), which are constants: $\alpha = \alpha_0$, $\beta = \beta_0$. If one inserts them into Eq. (3) at fixed t, observes $\gamma_0 = 1 - \alpha_0 - \beta_0$, and integrates y from y^0_{CDE} to y_{CDE} and the factors from $(1,1,1)$ to (k,l,e), one obtains the energy-dependent Cobb-Douglas function (CDE)

$$y_{\mathrm{CDE}} = y^0_{\mathrm{CDE}} k^{\alpha_0} l^{\beta_0} e^{1-\alpha_0-\beta_0}. \tag{17}$$

This function is often used in quantitative analyses of standard economics,[5] where α_0, β_0, and $1 - \alpha_0 - \beta_0$ are set equal to the cost shares of capital, labor, and energy; these shares have happened to be approximately constant until recently. However, the equilibrium conditions (26) in the presence of technological constraints no longer justify their use in y_{CDE}.

The conceptual problem with the Cobb-Douglas function is that it is a production function with complete substitutability of the production factors. Therefore, its use for computing scenarios of the future, for instance in [19], is problematical. Things are different, if Cobb-Douglas functions are employed to describe economic growth of the past, when, of course, the empirical inputs of capital, labor and energy trivially stayed within the range of the physically possible. Whenever this has been done independently from any cost-share considerations, for instance by fitting y_{CDE} to the time series of output [17,20] and by cointegration analysis [21], the output elasticity of energy turns out to be much larger than 0.05. We will see below that this finding is substantiated by production functions that are more sophisticated than Cobb-Douglas.

3.2. Energy-dependent CES function

Constant elasticities of substitution production functions were introduced into econometrics by Arrow et al. [22] and extended to more than two factors by Uzawa [23]. The linearly homogeneous CES function of k, l, e has the form [16]

$$y_{\mathrm{CES}} = y^0_{\mathrm{CES}} \left[a k^{-\rho} + b l^{-\rho} + (1-a-b) e^{-\rho}\right]^{-1/\rho}. \tag{18}$$

The parameters a and b are usually called "distribution parameters". They must be non-negative. The constant $\rho \equiv 1/\sigma - 1$, which is determined by the constant elasticity of substitution σ, must be

---

[5] Analyses of the economic impacts of climate change like the DICE model of Nordhaus assume that "Output is produced by a Cobb-Douglas production function in capital, labor, and energy" [18].

ARTICLE IN PRESS

larger than −1. The CES function can be easily brought into the form of Eq. (12). It satisfies the fundamental set of Eqs. (3)–(8). Its output elasticities are obtained from (4). With the definition $q \equiv y_{CES}/y^0_{CES}$, they become [16]

$$\alpha_{CES} = a(q/k)^\rho,\ \beta_{CES} = b(q/l)^\rho,\ \gamma_{CES} = 1 - \alpha_{CES} - \beta_{CES}. \quad (19)$$

In the limit $\sigma \to 1$, when $\rho \to 0$, the CES function (18) turns into the Cobb-Douglas function (17). This is most easily seen from the CES output elasticities (19), which become constants in this limit.

If there were no technological constraints and shadow prices, so that Eq. (28) and the cost share theorem would hold, one could relate the CES parameters a, b and ρ to factor prices via Eq. (19) and the combination of Eq. (28) in Appendix A with the Euler relation in footnote 1. But as things are, this is no option (any more). As in the case of y_{CDE}, fitting to empirical time series of output would avoid the shadow-price problem.[6]

3.3. LinEx function

As indicated above, the exact boundary conditions on the partial differential Eq. (8) are unknowable. But how about *asymptotic* technological boundary conditions on the output elasticities? Consider two examples.

1. Machines do not run without energy and (still) require people for handling them. Therefore, the additional output due to an additional unit of capital should decrease as the ratio of labor and energy to capital decreases. This is the case, if the output elasticity of capital, α, vanishes, if l/k and e/k go to zero.
2. In principle, with sufficient capital and energy, it should be possible to produce a given quantity of output y in a state of automation such that the addition of another unit of routine labor does not contribute to output any more. We call this the state of maximum automation. More precisely, if $k_m(y)$ is the fully employed capital stock in the state of maximum automation, and $e_m \equiv ck_m(y)$ is its energy (exergy) demand, then the output elasticity of labor, β, should vanish if k approaches $k_m(y)$ and e approaches $e_m \equiv ck_m(y)$. (As pointed out in Appendix A, the achievable state of automation represents one of the technological constraints on factor combinations.)

The simplest output elasticities that satisfy the differential Eq. (8), constant returns to scale and these asymptotic boundary conditions are [7]

$$\alpha = a\frac{l+e}{k}, \beta = a\left(\frac{cl}{e} - \frac{l}{k}\right), \gamma = 1 - \alpha - \beta = 1 - a\frac{e}{k} - ac\frac{l}{e}. \quad (20)$$

The parameter a indicates the effectiveness with which energy activates and labor handles the capital stock. The negative term in β is a direct consequence of the choice of α, as can be seen from the integral in (10). The positive term in β is a special choice of the function $J\left(\frac{l}{e}\right)$ in (10) so that the asymptotic boundary condition for β is fulfilled.

The restrictions (11) are important for understanding the economic meaning of the output elasticities (20) and of the LinEx function (21), which follows from them. They imply $\beta \le 1$ and thus require that l goes to zero as e does. This is consistent with the fact that workers lose their jobs if production ceases because of the lack of energy.

Inserting the output elasticities (20) into Eq. (3) at fixed t, and integrating y from y^0_{L1} to y_{L1} and the production factors from $(1, 1, 1)$ to (k, l, e) [7] one obtains the (first) LinEx production function

$$y_{L1} = y^0_{L1} e \exp\left[a\left(2 - \frac{l+e}{k}\right) + ac\left(\frac{l}{e} - 1\right)\right], \quad (21)$$

which depends linearly on energy and exponentially on quotients of capital, labor and energy [7]. In contrast to the energy-dependent Cobb-Douglas function the restrictions (11) *do* constrain the combinations of factors in the LinEx function.[8]

The LinEx function (21) contains the technology parameters a (capital effectiveness), c (energy demand of the capital stock), and y^0_{L1}, which are determined econometrically subject to the restrictions (11). It is a phenomenological function that describes the output of an economy *approximately*. Its deviations from the exact production function correspond to the deviations of the asymptotic boundary conditions from the exact boundary conditions. One cannot expect that the LinEx approximation maps all details of production. What matters is the overall picture, while details may be blurred or distorted. This is as inevitable as the incomplete description of the physical world by the natural sciences and their model approximations. In the end, comparison of the model results with experience will eliminate those models whose approximations are too crude. Modified asymptotic boundary conditions yield modified LinEx functions [16,17].

The phenomenological LinEx parameters a, c and y^0_{L1} become time dependent, separately or altogether, when creativity acts and the LinEx function acquires an explicit time dependence: $y_{L1} = y_{L1}[k, l, e; t]$. For instance, $c(t)$ decreases when investments in energy conservation improve the energy efficiency of the capital stock. This occurred quite noticeably in response to the oil price shocks in the 1970s and 1980s and is an example for the (thermodynamically limited) substitution of capital for energy. Structural changes by outsourcing energy-intensive industries may also decrease $c(t)$. The LinEx function reproduces economic growth in Germany, Japan and the USA in good agreement with the empirical data. Recent empirical and theoretical growth curves for Germany, Japan and the USA are shown in [26] and [8]. Figs. 2–4 indicate the time-averaged output elasticities of capital, labor, energy, and creativity.

Ayres and Warr [12,20] have replaced primary energy by "useful work" in the LinEx function. "Useful work" is defined as exergy, multiplied by appropriate conversion efficiencies, plus physical work by animals. The "useful work" data [14] include most of the efficiency improvements that have occurred in energy-converting systems during the 20th century. In this case, two constant technology parameters suffice to reproduce well the gross domestic product of the US economy between 1900 and 1998. The time averages of the corresponding k, l, e output elasticities are similar to the ones in Fig. 4.

The statistical quality measures of the analyses that result in Figs. 2–4, and the error bars, are shown in Appendix C. The adjusted coefficients of determination R^2 exceed 0.995, and the Durbin-Watson coefficients d_W are between 1.46 and 1.9. They are much better than in the case of fitting the energy-dependent Cobb-Douglas

[6] If one had somehow obtained appropriate output elasticities and would *postulate* that output is the sum of factor prices times factor quantities, one could introduce factor prices into the CES function via Eq. (19).

[7] A convenient path is one along whose three different segments only one factor changes at a time while the other two stay constant.

[8] Explicitly, these restrictions limit factor substitutability by the relations $k/(l+e) \ge a(\ge 0)$, $e/k \le c(\ge 0)$, $0 \le a(e/k + cl/e) \le 1$. Therefore, the substitutability objection raised in [24] against the LinEx function is not valid. Furthermore, the production function proposed in [24], see also [25], would deserve discussions with respect to twice differentiability and limitationality, among others. But here is not the place for that.

Please cite this article in press as: Lindenberger D, Kümmel R, Energy and the state of nations, Energy (2011), doi:10.1016/j.energy.2011.08.014

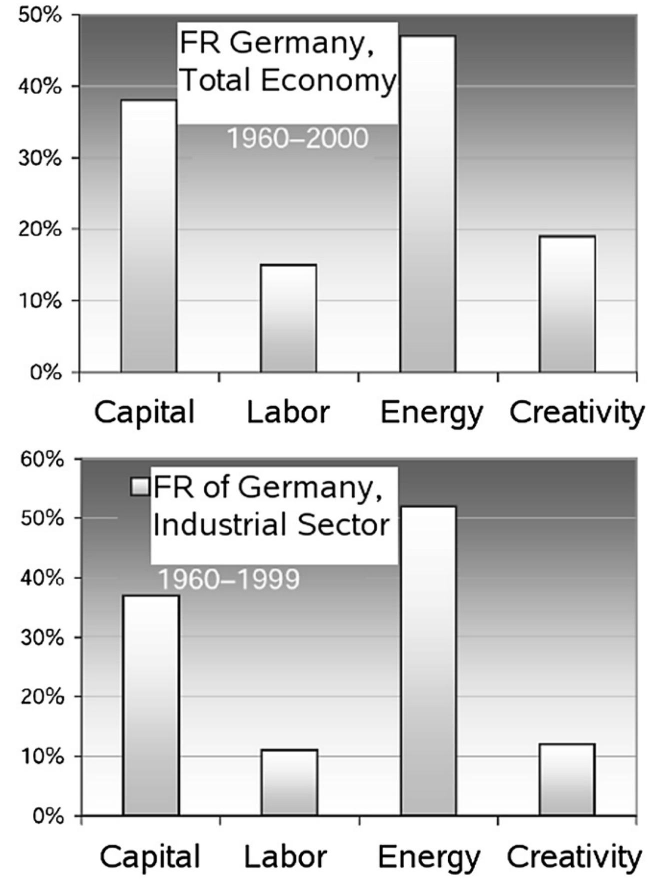

Fig. 2. Time-averaged output elasticities (productive powers) in the total economy of the Federal Republic of Germany (top) and in Germany's industrial sector "Warenproduzierendes Gewerbe" (bottom) [8].

function to the empirical time series of output. As in earlier studies [11,17] the output elasticities are much larger for energy and much smaller for labor than the cost shares of these factors. Social consequences, such as shifting the burden of taxes and levies from the factor labor to the factor energy, are discussed in [26] and Section 4.

Finally, the elasticities of substitution of the LinEx function are obtained by inserting the output elasticities of Eq. (20) into Eqs. (14)–(16). After some algebraic manipulations one can express the

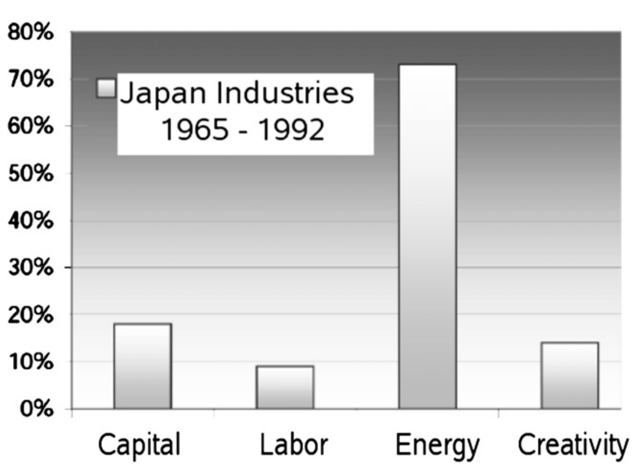

Fig. 3. Time-averaged output elasticities in the Japanese sector "Industries", which produces about 90% of Japanese GDP [8].

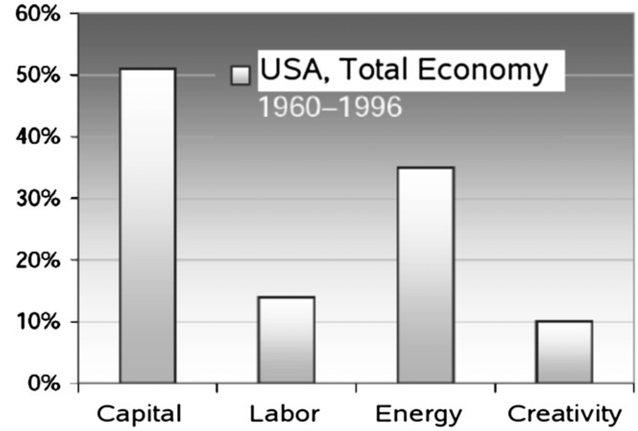

Fig. 4. Time-averaged output elasticities in the total US economy [8].

LinEx elasticities of substitution by the LinEx output elasticities $\alpha = a(l+e)/k$, $\beta = a(cl/e - l/k)$, and the term al/k:

$$\sigma_{kl} = \frac{\alpha + \beta}{2(\beta + al/k)}, \tag{22}$$

$$\sigma_{ke} = \frac{(1-\beta)(1-\alpha-\beta)}{2(1-\beta)(1-\beta-al/k) - \alpha}, \tag{23}$$

$$\sigma_{le} = \frac{-(1-\alpha)(1-\alpha-\beta)}{\beta(1-2\alpha) + 2(1-\alpha)al/k}. \tag{24}$$

Obviously, LinEx is a variable elasticities of substitution (VES) function. Time series of i) the LinEx output elasticities, ii) the technology parameter $a(t)$, and iii) the factor quotient l/k can be computed from German, Japanese and US data contained, e.g., in [8] and [20]. The actual computation of the elasticities of substitution (22)–(24) with the help of these data is left to future work.

4. Summary and outlook

An important, although not sufficient indicator of the socio-economic state of nations is the gross domestic product. Its description by twice-differentiable macroeconomic production functions is physically justified. These functions are integrals of the growth equation. The output elasticities in the growth equation are solutions of a set of partial differential equations. They measure the economic weight and productive power of the production factors capital, labor and energy. Because of technological constraints they are *not* equal to the shares of these factors in total factor cost. Rather, they are for energy much larger and for labor much smaller than the cost shares. Thus, energy and its conversion into physical work accounts for most of the growth that mainstream economics attributes to "technological progress" and related concepts. It decisively determines the economic state of nations.

This finding is fundamentally at variance with mainstream economic thinking. The standard objection to this finding is: "If this were true, money would be lying on the street. One only had to increase the input of cheap energy and decrease that of expensive labor until output elasticities and cost shares are equal." This reasoning of orthodox economists overlooks the insurmountable barriers that block access to that side of the street, where the money lies. These barriers are precisely the technological constraints that determine the shadow prices: One can neither increase energy input beyond design capacity of the machines, and decrease labor's handling of the machines correspondingly, nor

Please cite this article in press as: Lindenberger D, Kümmel R, Energy and the state of nations, Energy (2011), doi:10.1016/j.energy.2011.08.014

substitute energy and capital for labor beyond the limit to automation that exists at a given time.

The large discrepancies between productive powers and cost shares of energy and labor explain the pressure to increase automation as quickly as technology permits, substituting cheap energy/capital combinations for expensive labor. (This way one gets bit by bit to a limited street sector where the money lies, indeed.) Increasing automation is the main part of what is called "increasing productivity". In the past, when labor unions were strong, they demanded and got wage increases according to labor's increased productivity. This way the value added created by the exploitation of the energy sources was given to the general population. As a result, labor's share in GDP was between 70 and 60 percent in the highly industrialized countries. By now, however, more and more formerly well-paid industrial full-time jobs are lost to automation.

The imbalance between economic weights and costs of labor and energy also reinforces the trend toward globalization, because goods and services produced in low-wage countries can be cheaply delivered to high-wage countries thanks to cheap energy and increasingly sophisticated, highly computerized transportation systems. Thus, if the disparities between productive powers and cost shares of labor and energy are too pronounced, there is the danger that newly emerging and expanding business sectors cannot generate enough new jobs that can compensate for the ones lost to progress in automation and globalization. This, then, will result in the net loss of full-time routine jobs in high-wage countries and increasing unemployment, or poorly paid part-time employment, in the less qualified part of the labor force. A slowdown of economic growth, as natural constraints may cause, or economic recessions for whatever reasons, will aggravate the employment problems [9]. Shifting the burden of taxes and levies from the factor labor to the factor energy may be a response to the problems that result from energy's high productive power. This is discussed more in detail in [8,26].

The recent history of economic thought shows that energy receives attention as a factor of production only, when restrictions on energy utilization cause recessions. This happened during the so-called first and the second energy crisis, when oil prices exploded between 1973 and 1975 in the wake of the Yom-Kippur War, and between 1979 and 1981 as a consequence of the Iranian revolution and the Iran-Iraq war. At that time Jorgenson saw a relation between the oil price hikes and the recessions [27,28], whereas Denison [29] objected on the basis of the cost share theorem. With the decline of oil prices between 1981 and 1997 to nearly pre-1973 levels, interest in energy was again not more than interest in a natural resource as one commodity among many. By now, inflation-corrected oil prices have reached the 1981 maximum again. But the present economic instabilities are more attributed to the financial crisis that was triggered by the crash of the US real estate market in 2007/2008 than to a precursor of "Peak Oil", i.e. the decline of conventional oil production after a peak in the near future [30].

In any case, mainstream economic theory *will* realize the importance of the factor energy, when the First and the Second Law of Thermodynamics will hit our economies with full force. These laws represent the constitution of the universe. They say: "Nothing happens in the world without energy conversion and entropy production." Thus, energy conversion in the machines of the capital stock is indispensible for the generation of wealth, on the one hand. And the growth of wealth is endangered, on the other hand, because energy conversion is inevitably coupled to entropy production. Entropy production destroys exergy and manifests itself in emissions of heat and particles. These emissions will eventually restrict energy utilization and growth in the finite system Earth, when the emission-absorbing and life-supporting capacities of the biosphere are being exhausted. (A crude model of the limits to growth within Earth's finite biosphere is given by the growth equation whose output elasticities are multiplied by pollution and recycling functions [8,15].) Reduction of emissions at unchanged energy services can be achieved by improving the energy efficiency of the capital stock within the limits drawn by thermodynamics [31,32]. This changes the output elasticities in the growth Eq. (3); for instance, in the LinEx approximation, where the output elasticities are given by Eq. (20), the energy-demand parameter c decreases and the parameter a, which indicates capital effectiveness, increases. Furthermore, energy conservation and the use of renewable energies require investments in the corresponding energy-converting technologies [33,34]. These technologies become additional components of the capital stock. This also changes the output elasticities in Eq. (20). To get the full picture one would have to combine appropriate energy-dependent production functions – the simplest case would be LinEx – with models of energy, emission, and cost optimization, such as the ones used in [33,34].

Finally, further research should evaluate the relations between elasticities of substitution and output elasticities for CES and LinEx functions.

Acknowledgments

We are grateful for comments of two referees, and we have benefited greatly from suggestions of Barney Foran on how to improve the comprehensibility of the paper. All remaining deficiencies are the sole responsibility of the authors.

Appendix A

Entrepreneurial decisions, aiming at producing a certain quantity of output Y within the technology that exists at a given time t, determine the absolute magnitude of the total capital stock, its degree of capacity utilization, and its degree of automation. The machines of the capital stock are designed and built for specific energy inputs and require a certain amount of labor for handling, supervision, and maintenance. The quantities of labor and energy that are combined with the capital stock of a fixed degree of automation determine the degree of capacity utilization. The degree of automation at time t is represented by the ratio of the actual capital stock K to the capital stock $K_m(Y)$ that would be required in order to produce the actual output Y with the actual technology in the state of maximum automation. This state is characterized by a combination of capital and energy such that adding one more unskilled worker adds virtually nothing to gross economic output so that the output elasticity of routine labor would be vanishingly small. In some manufacturing sectors of industrialized countries this point actually does not seem to be far away.

It is obvious from an engineering point of view and by definition that both the degree of capacity utilization and the degree of automation are i) functions of capital, labor and energy and ii) cannot exceed the number 1. (In fact, even these days, after 40 years during which the density of transistors on a microchip has doubled every 18 months, the achievable state of automation of an economy is well below 1.) In other words, a production system cannot operate above design capacity, and the maximum degree of automation cannot be exceeded. These are the two fundamental technological constraints on the combinations of capital, labor and energy in modern economies. They drastically change the conditions for economic equilibrium that result from the behavioral assumptions of standard economics. One such assumption is profit maximization, according to which the actions of all economic agents are supposed to move the economy into a point of factor space where the difference between output and total factor cost is

ARTICLE IN PRESS

maximum. Alternatively one may follow Samuelson and Solow [35] and assume that: "… society maximizes the (undiscounted) integral of all future utilities of consumption subject to the fact that the sum of current consumption and of current capital formation is limited by what the current capital stock can produce."

The optimization calculus according to these two extremum principles is presented in [9]. The following is the summary of its results. For the sake of notational convenience we write the production factors K, L, E as X_1, X_2, X_3, and the output elasticities α, β, γ defined in Eq. (4) as

$$\varepsilon_i \equiv \frac{X_i}{Y}\frac{\partial Y}{\partial X_i}; i = 1, 2, 3. \tag{25}$$

Consider an economic system that produces its output Y with three factors of production X_1, X_2, X_3, whose combinations are subject to technological constraints, labeled by the indices A and B and expressed by the equations $f_A(X_1, X_2, X_3, t) = 0$, $f_B(X_1, X_2, X_3, t) = 0$ with the help of slack variables. They concern the degree of capacity utilization and the degree of automation. Their explicit forms are given in [9]. Then, profit maximization under constant returns to scale results in the three equilibrium conditions

$$\varepsilon_i = \frac{X_i[p_i + s_i]}{\sum_{i=1}^{3} X_i[p_i + s_i]}; i = 1, 2, 3, \tag{26}$$

which relate the output elasticities ε_i of factors X_i to the market prices p_i per factor unit and the factor shadow prices

$$s_i \equiv -\mu_A \frac{\partial f_A}{\partial X_i} - \mu_B \frac{\partial f_B}{\partial X_i}. \tag{27}$$

Here, μ_A and μ_B are the Lagrange multipliers of the two technological constraint equations in the optimization calculus. Thus, the output elasticities in Eq. (26) are equal to "shadowed" cost shares. Intertemporal optimization of utility U as a function of consumption C yields that the shadow price of capital contains an additional term proportional to the time derivative of dU/dC. This term is small for weakly decreasing $U(C)$.

If there were no technological constraints, the Lagrange multipliers would be zero, the equilibrium conditions would read

$$\frac{\partial Y}{\partial K} = p_K, \quad \frac{\partial Y}{\partial L} = p_L, \quad \frac{\partial Y}{\partial E} = p_E, \tag{28}$$

the shadow prices s_i would vanish, and one would have the usual factor cost shares on the r.h.s of Eq. (26). That's why standard economics assumes that in economic equilibrium output elasticities equal factor cost shares. As shown in [9], this would also justify the duality of production factors and factor prices, which is often used in orthodox growth analyses. The essential information on production would be contained in the price function as the Legendre transform of the production function. In the presence of technological constraints and non-zero shadow prices, however, the Lagrange multipliers are finite and functions of the output elasticities ε_i, so that the cost share theorem and duality are not valid. For an understanding of the economy, prices are not enough.

Appendix B

Twice differentiability of the production function $y[k, l, e; t]$ with respect to the production factors means that the second-order mixed derivatives of y with respect to k, l, e must be equal:

$$\frac{\partial^2 y}{\partial k \partial l} = \frac{\partial^2 y}{\partial l \partial k}, \quad \frac{\partial^2 y}{\partial l \partial e} = \frac{\partial^2 y}{\partial e \partial l}, \quad \frac{\partial^2 y}{\partial k \partial e} = \frac{\partial^2 y}{\partial e \partial k}. \tag{29}$$

According to Eq. (4) the first-order derivatives of y can be expressed by the output elasticities:

$$\frac{\partial y}{\partial k} = \frac{y}{k}\alpha, \quad \frac{\partial y}{\partial l} = \frac{y}{l}\beta, \quad \frac{\partial y}{\partial e} = \frac{y}{e}\gamma. \tag{30}$$

Differentiating the first of these equations with respect to l yields

$$\frac{\partial^2 y}{\partial l \partial k} = \frac{\partial y \alpha / k}{\partial l} = (\alpha/k)\frac{\partial y}{\partial l} + (y/k)\frac{\partial \alpha}{\partial l} = (\alpha/k)\frac{y}{l}\beta + (y/k)\frac{\partial \alpha}{\partial l}, \tag{31}$$

where the last equality results from the second equation in (30). Differentiating the second equation in (30) with respect to k yields

$$\frac{\partial^2 y}{\partial k \partial l} = \frac{\partial y \beta / l}{\partial k} = (\beta/l)\frac{\partial y}{\partial k} + (y/l)\frac{\partial \beta}{\partial k} = (\beta/l)\frac{y}{k}\alpha + (y/l)\frac{\partial \beta}{\partial k}, \tag{32}$$

where the last equality results from the first equation in (30). Because of the first equation in (29) the r.h.s. of (31) and (32) must be equal. This is the case, if

$$l\frac{\partial \alpha}{\partial l} = k\frac{\partial \beta}{\partial k}. \tag{33}$$

This is the first of the equations in (6) and in (8).

The second and the third equation in (6) are derived correspondingly. Inserting the constant-returns-to-scale relation (7), i.e. $\gamma = 1 - \alpha - \beta$, into them one obtains

$$e\frac{\partial \beta}{\partial e} = -l\frac{\partial \alpha}{\partial l} - l\frac{\partial \beta}{\partial l} \tag{34}$$

and

$$e\frac{\partial \alpha}{\partial e} = -k\frac{\partial \alpha}{\partial k} - k\frac{\partial \beta}{\partial k}. \tag{35}$$

The combination of Eq. (33) with (34) and (35) yields the second and the third partial differential equation in (8).

Appendix C

Table 1
Time-averaged LinEx output elasticities for the economic systems in Figs. 2–4 [8] [26]. R^2 is the adjusted coefficient of determination. The best value of the Durbin-Watson coefficient d_W would be 2, indicating no autocorrelation at all.

System	FRG TE 1960–2000	FRG I 1960–99	Japan I 1965–92	USA TE 1960–96
$\overline{\alpha}$	0.38 ± 0.09	0.37 ± 0.09	0.18 ± 0.07	0.51 ± 0.15
$\overline{\beta}$	0.15 ± 0.05	0.11 ± 0.07	0.09 ± 0.09	0.14 ± 0.14
$\overline{\gamma}$	0.47 ± 0.1	0.52 ± 0.09	0.73 ± 0.16	0.35 ± 0.11
$\overline{\delta}$	0.19 ± 0.2	0.12 ± 0.13	0.14 ± 0.19	0.10 ± 0.17
R^2	>0.999	0.996	0.999	0.999
d_W	1.64	1.9	1.71	1.46

References

[1] Fisher FM. Aggregation, aggregate production functions and related topics. Cambridge, Mass: MIT Press; 1993.
[2] Kurz HD, Salvadory N. Theory of Production. Cambridge, Melbourne, New York: Cambridge University Press; 1995.
[3] Kurz HD. Wicksell and the problem of the missing equation. Hist Polit Economy 2000;32:765–88.
[4] Pasinetti L. Critique of the neoclassical theory of growth and distribution. Moneta Credito (Banca Nazionale del Lavoro Quarterly Review) 2000;210: 187–232.
[5] Felipe J, Fisher FM. Aggregation in production functions: what applied economists should know. Metroeconomica 2003;54:208–62.
[6] Silverberg G. Private Communication; 2007.
[7] Kümmel R. The impact of energy on industrial growth. Energy 1982;7: 189–203.

Please cite this article in press as: Lindenberger D, Kümmel R, Energy and the state of nations, Energy (2011), doi:10.1016/j.energy.2011.08.014

ARTICLE IN PRESS

[8] Kümmel R. The Second Law of Economics. Energy, Entropy, and the Origins of Wealth. The Frontiers Collection. New York, Dordrecht, Heidelberg, London: Springer, http://www.springer.com/physics/complexity/book/978-1-4419-9364-9.
[9] Kümmel R, Ayres RU, Lindenberger D. Thermodynamic laws, economic methods and the productive power of energy. J Non-Equilib Thermodyn 2010; 35:145–79. doi:10.15.15/JNETDY.2010.009.
[10] Solow RM. Perspectives on growth theory. J Econ Perspect 1994;8:45–54.
[11] Kümmel R, Henn J, Lindenberger D. Capital, labor, energy and creativity: modeling innovation diffusion. Struct Change Econ Dynam 2002;13:415–33. and references therein.
[12] Ayres RU, Warr B. Accounting for growth: the role of physical work. Struct Change Econ Dynam 2005;16:181–209. and references therein.
[13] Sieferle RP. Das vorindustrielle Solarenergiesystem. In: Brauch HG, editor. Energiepolitik. Berlin: Springer; 1997. p. 27–46 (translation of the quote by R.K.).
[14] Ayres RU, Ayres LW, Warr B. Exergy, power and work in the US economy, 1900–1998. Energy 2003;28:219–73.
[15] Kümmel R. Growth Dynamics of the Energy-Dependent Economy. In: Eichhorn W, Henn R, editors. Mathematical Systems in Economics 54. Cambridge, Mass: Oelgeschlager, Gunn & Hain; 1980.
[16] Lindenberger D. Wachstumsdynamik industrieller Volkswirtschaften: Energieabhängige Produktionsfunktionen und ein faktorpreis-gesteuertes Optimierungsmodell. Dissertation, Fakultät für Wirtschaftswissenschaften der Universität Karlsruhe (1999). Published as Hochschulschriften Bd. 61. Marburg: Metropolis; 2000. ISBN 3-89518-273-7.
[17] Kümmel R, Strassl W, Gossner A, Eichhorn W. Technical progress and energy-dependent production functions. J Econ (Z Nationalökon) 1985;45:285–311.
[18] Nordhaus W. A Question of Balance. Weighting the Options on Global Warming Policies. New Haven & London: Yale University Press; 2008.
[19] Goldman Sachs. The GCC Dream: between the BRICs and the developed world. Glob Econ Paper April 17, 2007;155, https://portal.gs.com.
[20] Ayres RU, Warr B. The Economic Growth Engine. Cheltenham: Edgar Elgar; 2009.
[21] Stresing R, Lindenberger D, Kümmel R. Cointegration of output, capital, labor, and energy. Eur Phys J B 2008;66:279–87. see also, http://www.ewi.uni-koeln.de/fileadmin/user/WPs/ewiwp0804.pdf.
[22] Arrow KJ, Chenery HB, Minhas BS, Solow RM. Capital-labor substitution and economic efficiency. Rev Econ Stat 1961;43:225–50.
[23] Uzawa H. Production functions with constant elasticity of substitution. Rev Econ Stud 1962;29:291–99.
[24] Beaudreau BC, Pokrovski VN. On the energy content of a money unit. Physica A 2010;389:2597–606.
[25] Pokrovski VN. Energy in the theory of production. Energy 2003;28: 769–88.
[26] Kümmel R, Schmid J, Lindenberger D. Why production theory and the Second Law of Thermodynamics support high energy taxes. In: Cabalu H, Marinova D, editors. Second International Association for Energy Economics (IAEE) Asian Conference: Energy Security and Economic Development under Environmental Constraints in the Asia–Pacific Region. Perth: Curtin University of Technology, ISBN 978-0-9775369-2-4; 2008. p. 123–40, http://business.curtin.edu.au/business/research/conferences/2nd-iaee-asian-conference/refereed-conference-papers.
[27] Jorgenson DW. The role of energy in the U.S. economy. Nat Tax J 1978;31: 209–20.
[28] Jorgenson DW. The role of energy in productivity growth. Amer Econ Rev 1984;74/2:26–30.
[29] Denison EF. Explanation of declining productivity growth. Surv Curr Bus 1979; 59(8):1–24. Part II.
[30] Strahan D. The Last Oil Shock. London: John Murray; 2007.
[31] Groscurth HM, Kümmel R, van Gool W. Thermodynamic limits to energy optimization. Energy 1989;14:241–58.
[32] Groscurth HM, Kümmel R. The cost of energy conservation: a Thermoeconomic analysis of national energy systems. Energy 1989;14:685–96.
[33] Bruckner T, Groscurth HM, Kümmel R. Competition and synergy between energy technologies in municipal energy systems. Energy 1997;22: 1005–14.
[34] Lindenberger D, Bruckner T, Groscurth HM, Kümmel R. Optimization of solar distric heating systems: seasonal storage, heat pumps, and cogeneration. Energy 2000;25:591–608.
[35] Samuelson PA, Solow RM. A complete capital model involving heterogeneous capital goods. Quart J Econ 1956;70:537–62.

Please cite this article in press as: Lindenberger D, Kümmel R, Energy and the state of nations, Energy (2011), doi:10.1016/j.energy.2011.08.014