** AE4872 (2024) – SATELLITE ORBIT DETERMINATION**

A satellite in space above earth

Description automatically generated

**Assignment 3 – Kalman filter**

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Group size: 1 student

Due: Friday, 10 January 2024, 18:00

Page limit: 6 pages

Estimated time: 20 hours

*In your report, explain how you solve the tasks and use figures where appropriate. Describe the intermediate steps and give the intermediate results so we may award partial points (no intermediate results = no full score). Attach your code in the appendix of your report. We will check the code for plagiarism but not grade it.*

In this assignment, you will implement the *extended* Kalman filter (see section 10.3.5 of the lecture notes for the *ordinary* Kalman filter and see the instruction slides for the *extended* Kalman filter) to compute the position and velocity of the GOCE satellite. Use a suitable method (e.g., Matlab’s ode45) when numerically integrating the state vector and state transition matrix.

The following corrections have already been applied to the data:

* Light time correction
* Relativistic correction
* GPS clock correction (transmitter clock correction)
* GOCE receiver clock correction

Do not apply any of these corrections yourself.

Since the precise orbit has superior accuracy, we may interpret the difference between our results and the precise orbits as a good representation of the error in our orbit.

We start counting epochs from 1 or (i.e., not from zero or ).

a) Assume the following standard deviations for the elements of the initial state vector and the observations:

* Initial position: 2 m
* Initial velocity: 0.1 m/s
* Pseudo-range observations: 3 m

Further, assume that the x-component of the initial position is correlated to the x-component of the initial velocity by 70% (correlation coefficient 0.7). Assume the same for the y- and z-components. All other correlations are zero. Assume also that the observations are uncorrelated. Report the covariance matrix of the initial state vector and the covariance matrix of the observations. **(5 points)**

b) The state transition matrix is obtained by integrating the differential equation

together with the state vector. Derive the equation for matrix

for the dynamic model where Earth is considered a point mass (only C00 gravity field coefficient). Integrate the state transition matrix (together with the state vector) from the first to the second epoch, i.e., from to . Use the precise orbit to construct the initial state vector. Report the integrated state vector with mm precision for position and mm/s precision for velocity and the integrated state transition matrix with 10 decimal places (10 digits after the comma or first non-zero digit?).**(15 points)**

*Hint: The size of the state transition matrix elements is approximately (some elements can be smaller):*

c) Choose the precise orbit for the initial state vector and calculate the position and velocity using the extended Kalman filter without process noise. **(20 points)**

* Report the x, y, and z-coordinates of your estimated position and velocity (the updated state vector) for epochs 10, 20, and 30 in a table with millimeter precision (counting epochs from 1).
* Show in a figure the position and velocity errors as the Euclidian distance between:
* Your estimated position and the precise position, .
* Your estimated velocity and the precise velocity, .

*Hint 1: should be well below 10 m for the first 5 epochs.*

*Hint 2: Make sure that all curves are readable. If needed, create two figures.*

*Hint 3: When plotting the errors, use a linear scale (not a logarithmic scale).*

d) For your solution from task c:

* Show standard deviation of the estimated position and velocity: **(5 points)**
* Describe how the standard deviations and evolve and explain this behavior. **(5 points)**
* Describe how the position and velocity errors ( and shown in task c) evolve, compare this to standard deviations and , and explain this behavior. Consider how position and velocity are related. **(5 points)**

e) Suppose you would implement the linear Kalman filter to calculate the position and velocity. Would the position errors be of equal size, smaller, or larger than those in task c, which was based on the extended Kalman filter? Justify for your answer. **(10 points)**

f) Now account for process noise in the Kalman filter. **(10 points)**

* Choose the values for the process noise covariance matrix, note them in your report, and briefly explain your choice.
* Create a figure that shows the effect of accounting for process noise on the estimated state vector and its standard deviation, and explain the effect.

g) For task f, calculate for every epoch the RMS of the observation residuals where in the Kalman filter, compare the size to the position errors, and interpret the comparison. **(10 points)**

h) Assume that we would implement a much more detailed dynamic model, accounting for a complete Earth gravity field model, effects of third bodies (moon, sun, …), atmospheric drag, radiation pressure, etc. **(15 points)**

* Describe what you need to modify in the solution in task f to obtain more accurate positions.
* Explain what would still be limiting the position accuracy.

*Please mention how many hours you spent approximately on the assignment. We will use the information to improve the assignments for next year.*