

$$t := 1000 \text{ kg}$$

$$M := \begin{bmatrix} 3t & 0 & 0 \\ 0 & 2t & 0 \\ 0 & 0 & 8t \end{bmatrix} = \begin{bmatrix} 3000 & 0 & 0 \\ 0 & 2000 & 0 \\ 0 & 0 & 8000 \end{bmatrix} \text{ kg}$$

$$k_3 := 2 \cdot 12 \cdot \frac{1.5 \cdot 10^6 \text{ N} \cdot \text{m}^2}{(3.75 \text{ m})^3} = 682666.667 \frac{\text{N}}{\text{m}}$$

$$k_2 := k_3 = 682666.667 \frac{\text{kg}}{\text{s}^2}$$

$$k_1 := 2 \cdot 12 \cdot \frac{1.5 \cdot 10^6 \text{ N} \cdot \text{m}^2}{(4.5 \text{ m})^3} = 395061.728 \frac{\text{N}}{\text{m}}$$

$$K := \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 1077.728 & -682.667 & 0 \\ -682.667 & 1365.333 & -682.667 \\ 0 & -682.667 & 682.667 \end{bmatrix} \frac{\text{kN}}{\text{m}}$$

$$A := \det(-\lambda \cdot M + K) = 0$$

$$A \xrightarrow{\text{expand}}$$

$$\lambda_1 = 871.384 \frac{1}{\text{s}^2}$$

$$\lambda_1 := \text{Re}(\lambda_1) = 871.384 \frac{1}{\text{s}^2}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} :=$$

$$\lambda_2 = 18.549 \frac{1}{\text{s}^2}$$

$$\lambda_2 := \text{Re}(\lambda_2) = 18.549 \frac{1}{\text{s}^2}$$

$$\lambda := \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

$$\lambda_3 = 237.31 \frac{1}{\text{s}^2}$$

$$\lambda_3 := \text{Re}(\lambda_3) = 237.31 \frac{1}{\text{s}^2}$$

$$\omega_1 := \sqrt{\lambda_2} = 4.307 \frac{1}{\text{s}}$$

$$\lambda = \begin{bmatrix} 871.384 \\ 18.549 \\ 237.31 \end{bmatrix} \frac{1}{\text{s}^2}$$

$$\omega_2 := \sqrt{\lambda_3} = 15.405 \frac{1}{\text{s}}$$

$$\omega_3 := \sqrt{\lambda_1} = 29.519 \frac{1}{\text{s}}$$

$$\omega := \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 4.307 \\ 15.405 \\ 29.519 \end{bmatrix} \frac{1}{\text{s}}$$

$$B := K - \omega_1^2 \cdot M = \begin{bmatrix} 1022082.102 & -682666.667 & 0 \\ -682666.667 & 1328235.804 & -682666.667 \\ 0 & -682666.667 & 534276.551 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi_{11} := 1 \quad \Phi_{21} := -\frac{B(0,0)}{B(0,1)} \cdot \Phi_{11} = 1.497$$

$$\Phi_{31} := -\frac{B(1,2)}{B(2,2)} \cdot \Phi_{21} = 1.913$$

$$\Phi_1 := \begin{bmatrix} \Phi_{11} \\ \Phi_{21} \\ \Phi_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 1.497 \\ 1.913 \end{bmatrix} \quad \max(\Phi_1) = 1.913 \quad \Phi_1 := \frac{\Phi_1}{\max\left(\begin{bmatrix} |\Phi_{11}| \\ |\Phi_{21}| \\ |\Phi_{31}| \end{bmatrix}\right)} = \begin{bmatrix} 0.523 \\ 0.783 \\ 1 \end{bmatrix}$$

$$B \cdot \Phi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

$$B := K - \omega_2^2 \cdot M = \begin{bmatrix} 365797.286 & -682666.667 & 0 \\ -682666.667 & 890712.594 & -682666.667 \\ 0 & -682666.667 & -1215816.29 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi_{12} := 1 \quad \Phi_{22} := -\frac{B(0,0)}{B(0,1)} \cdot \Phi_{12} = 0.536$$

$$\Phi_{32} := -\frac{B(1,2)}{B(2,2)} \cdot \Phi_{22} = -0.301$$

$$\Phi_2 := \begin{bmatrix} \Phi_{12} \\ \Phi_{22} \\ \Phi_{32} \end{bmatrix} = \begin{bmatrix} 1 \\ 0.536 \\ -0.301 \end{bmatrix} \quad \max(\Phi_2) = 1 \quad \Phi_2 := \frac{\Phi_2}{\max\left(\begin{bmatrix} |\Phi_{12}| \\ |\Phi_{22}| \\ |\Phi_{32}| \end{bmatrix}\right)} = \begin{bmatrix} 1 \\ 0.536 \\ -0.301 \end{bmatrix}$$

$$B \cdot \Phi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

$$B := K - \omega_{-3}^2 \cdot M = \begin{bmatrix} -1536422.598 & -682666.667 & 0 \\ -682666.667 & -377433.995 & -682666.667 \\ 0 & -682666.667 & -6288402.648 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi_{-13} := 1 \quad \Phi_{-23} := -\frac{B(0,0)}{B(0,1)} \cdot \Phi_{-13} = -2.251$$

$$\Phi_{-33} := -\frac{B(1,2)}{B(2,2)} \cdot \Phi_{-23} = 0.244$$

$$\Phi_{-3} := \begin{bmatrix} \Phi_{-13} \\ \Phi_{-23} \\ \Phi_{-33} \end{bmatrix} = \begin{bmatrix} 1 \\ -2.251 \\ 0.244 \end{bmatrix} \quad \max(\Phi_{-3}) = 1 \quad \Phi_{-3} := \frac{\Phi_{-3}}{\max\left(\begin{bmatrix} |\Phi_{-13}| \\ |\Phi_{-23}| \\ |\Phi_{-33}| \end{bmatrix}\right)} = \begin{bmatrix} 0.444 \\ -1 \\ 0.109 \end{bmatrix}$$

$$B \cdot \Phi_{-3} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

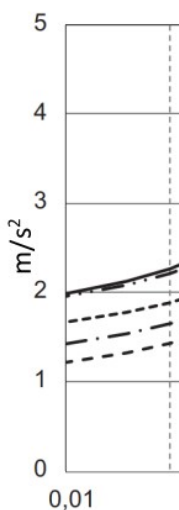
$$\Phi := \text{augment}(\Phi_{-1}, \Phi_{-2}, \Phi_{-3}) = \begin{bmatrix} 0.523 & 1 & 0.444 \\ 0.783 & 0.536 & -1 \\ 1 & -0.301 & 0.109 \end{bmatrix}$$

$$M_s := \Phi^T M \cdot \Phi = \begin{bmatrix} 10044.776 & 0 & 0 \\ 0 & 4298.401 & 0 \\ 0 & 0 & 2686.548 \end{bmatrix} kg$$

$$K_s := \Phi^T K \cdot \Phi = \begin{bmatrix} 186318.177 & 0 & 0 \\ 0 & 1020055.127 & 0 \\ 0 & 0 & 2341014.211 \end{bmatrix} \frac{N}{m}$$

$$T := \frac{2\pi}{\omega} = \begin{bmatrix} 1.459 \\ 0.408 \\ 0.213 \end{bmatrix} s \quad S_1 := 1.4 \frac{m}{s^2} \quad S_2 := 4.225 \frac{m}{s^2} \quad S_3 := 4.225 \frac{m}{s^2}$$

$$\Gamma := M_s^{-1} \Phi^T \cdot M \cdot \text{identity}(3) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.108 \\ 0.387 \\ 0.075 \end{bmatrix} \quad \omega = \begin{bmatrix} 4.307 \\ 15.405 \\ 29.519 \end{bmatrix} \frac{1}{s}$$



$$q_{1\_max} := \Gamma(0) \cdot \frac{1}{\omega_{1}^2} \cdot S_{-1} = 0.084 \text{ m}$$

$$q_{2\_max} := \Gamma(1) \cdot \frac{1}{\omega_{2}^2} \cdot S_{-2} = 0.007 \text{ m}$$

$$q_{3\_max} := \Gamma(2) \cdot \frac{1}{\omega_{3}^2} \cdot S_{-3} = 0 \text{ m}$$

$$\left( (\Phi_{-1} \cdot q_{1\_max})^2 + (\Phi_{-2} \cdot q_{2\_max})^2 + (\Phi_{-2} \cdot q_{2\_max})^2 \right)^{\frac{1}{2}} = \begin{bmatrix} 0.045 \\ 0.066 \\ 0.084 \end{bmatrix} \text{ m}$$

$$u_{1\_max} := q_{1\_max} \cdot \Phi_{-1} = \begin{bmatrix} 0.044 \\ 0.065 \\ 0.084 \end{bmatrix} \text{ m}$$

$$s_{-1} := \Gamma(0) \cdot M \cdot \Phi_{-1} = \begin{bmatrix} 1738.168 \\ 1734.912 \\ 8867.068 \end{bmatrix} \text{ kg}$$

$$F_{-1\_max} := s_{-1} \cdot S_{-1} = \begin{bmatrix} 2433.435 \\ 2428.877 \\ 12413.896 \end{bmatrix} \text{ N}$$

$$V_{-1} := \begin{bmatrix} F_{-1\_max}(2) + F_{-1\_max}(1) + F_{-1\_max}(0) \\ F_{-1\_max}(2) + F_{-1\_max}(1) \\ F_{-1\_max}(2) \end{bmatrix} = \begin{bmatrix} 17.276 \\ 14.843 \\ 12.414 \end{bmatrix} \text{ kN}$$

$$u_{2\_max} := q_{2\_max} \cdot \Phi_{-2} = \begin{bmatrix} 0.007 \\ 0.004 \\ -0.002 \end{bmatrix} \text{ m}$$

$$s_{-2} := \Gamma(1) \cdot M \cdot \Phi_{-2} = \begin{bmatrix} 1161.883 \\ 415.053 \\ -932.189 \end{bmatrix} \text{ kg}$$

$$F_{-2\_max} := s_{-2} \cdot S_{-2} = \begin{bmatrix} 4908.958 \\ 1753.597 \\ -3938.497 \end{bmatrix} \text{ N}$$

$$V_{-2} := \begin{bmatrix} F_{-2\_max}(2) + F_{-2\_max}(1) + F_{-2\_max}(0) \\ F_{-2\_max}(2) + F_{-2\_max}(1) \\ F_{-2\_max}(2) \end{bmatrix} = \begin{bmatrix} 2.724 \\ -2.185 \\ -3.938 \end{bmatrix} \text{ kN}$$

$$u_{3\_max} := q_{3\_max} \cdot \Phi_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ m}$$

$$s_3 := \Gamma(2) \cdot M \cdot \Phi_3 = \begin{bmatrix} 99.949 \\ -149.965 \\ 65.12 \end{bmatrix} \text{ kg}$$

$$F_{3\_max} := s_3 \cdot S_3 = \begin{bmatrix} 422.284 \\ -633.601 \\ 275.134 \end{bmatrix} \text{ N}$$

$$V_3 := \begin{bmatrix} F_{3\_max}(2) + F_{3\_max}(1) + F_{3\_max}(0) \\ F_{3\_max}(2) + F_{3\_max}(1) \\ F_{3\_max}(2) \end{bmatrix} = \begin{bmatrix} 0.064 \\ -0.358 \\ 0.275 \end{bmatrix} \text{ kN}$$

$$\left( (V_1)^2 + (V_2)^2 + (V_3)^2 \right)^{\frac{1}{2}} = \begin{bmatrix} 17.49 \\ 15.007 \\ 13.027 \end{bmatrix} \text{ kN}$$

$$\left( (F_{1\_max})^2 + (F_{2\_max})^2 + (F_{3\_max})^2 \right)^{\frac{1}{2}} = \begin{bmatrix} 5.495 \\ 3.062 \\ 13.027 \end{bmatrix} \text{ kN}$$