$t := 1000\ kg$

$$M := \begin{bmatrix} 6t & 0 & 0 \\ 0 & 4t & 0 \\ 0 & 0 & 4t \end{bmatrix} = \begin{bmatrix} 6000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 4000 \end{bmatrix} kg$$

$$k\_3 := 2 \cdot 12 \frac{1.15 \cdot 10^6\ N \cdot m^2}{(3.75\ m)^3} = 523377.778 \frac{N}{m}$$

$$k\_2 := k\_3 = 523377.778 \frac{kg}{s^2}$$

$$k\_1 := 2 \cdot 12 \cdot \frac{1.15 \cdot 10^6\ N \cdot m^2}{(4.5\ m)^3} = 302880.658 \frac{N}{m}$$

$$K := \begin{bmatrix} k\_1 + k\_2 & -k\_2 & 0 \\ -k\_2 & k\_2 + k\_3 & -k\_3 \\ 0 & -k\_3 & k\_3 \end{bmatrix} = \begin{bmatrix} 826.258 & -523.378 & 0 \\ -523.378 & 1046.756 & -523.378 \\ 0 & -523.378 & 523.378 \end{bmatrix} \frac{kN}{m}$$

$A := det(-\lambda \cdot M + K) = 0$ $\quad A \xrightarrow{expand}$

$\begin{bmatrix} \lambda\_1 \\ \lambda\_2 \\ \lambda\_3 \end{bmatrix} :=$

$\lambda\_1 = 378.309 \frac{1}{s^2}$ $\quad \lambda\_1 := \mathrm{Re}(\lambda\_1) = 378.309 \frac{1}{s^2}$

$\lambda\_2 = 16.92 \frac{1}{s^2}$ $\quad \lambda\_2 := \mathrm{Re}(\lambda\_2) = 16.92 \frac{1}{s^2}$ $\quad \lambda := \begin{bmatrix} \lambda\_1 \\ \lambda\_2 \\ \lambda\_3 \end{bmatrix}$

$\lambda\_3 = 135.014 \frac{1}{s^2}$ $\quad \lambda\_3 := \mathrm{Re}(\lambda\_3) = 135.014 \frac{1}{s^2}$

$\omega\_1 := \sqrt{\lambda\_2} = 4.113 \frac{1}{s}$ $\quad \lambda = \begin{bmatrix} 378.309 \\ 16.92 \\ 135.014 \end{bmatrix} \frac{1}{s^2}$

$\omega\_2 := \sqrt{\lambda\_3} = 11.62 \frac{1}{s}$

$\omega\_3 := \sqrt{\lambda\_1} = 19.45 \frac{1}{s}$ $\quad \omega := \begin{bmatrix} \omega\_1 \\ \omega\_2 \\ \omega\_3 \end{bmatrix} = \begin{bmatrix} 4.113 \\ 11.62 \\ 19.45 \end{bmatrix} \frac{1}{s}$

$$B := K - \omega\_1^2 \cdot M = \begin{bmatrix} 724736.985 & -523377.778 & 0 \\ -523377.778 & 979074.588 & -523377.778 \\ 0 & -523377.778 & 455696.81 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi\_11 := 1 \qquad \Phi\_21 := -\frac{B(0,0)}{B(0,1)} \cdot \Phi\_11 = 1.385$$

$$\Phi\_31 := -\frac{B(1,2)}{B(2,2)} \cdot \Phi\_21 = 1.59$$

$$\Phi\_1 := \begin{bmatrix} \Phi\_11 \\ \Phi\_21 \\ \Phi\_31 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.385 \\ 1.59 \end{bmatrix} \qquad \max(\Phi\_1) = 1.59 \qquad \Phi\_1 := \frac{\Phi\_1}{\max\left(\begin{bmatrix} |\Phi\_11| \\ |\Phi\_21| \\ |\Phi\_31| \end{bmatrix}\right)} = \begin{bmatrix} 0.629 \\ 0.871 \\ 1 \end{bmatrix}$$

$$B \cdot \Phi\_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

$$B := K - \omega\_2^2 \cdot M = \begin{bmatrix} 16177.289 & -523377.778 & 0 \\ -523377.778 & 506701.458 & -523377.778 \\ 0 & -523377.778 & -16676.32 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi\_12 := 1 \qquad \Phi\_22 := -\frac{B(0,0)}{B(0,1)} \cdot \Phi\_12 = 0.031$$

$$\Phi\_32 := -\frac{B(1,2)}{B(2,2)} \cdot \Phi\_22 = -0.97$$

$$\Phi\_2 := \begin{bmatrix} \Phi\_12 \\ \Phi\_22 \\ \Phi\_32 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.031 \\ -0.97 \end{bmatrix} \qquad \max(\Phi\_2) = 1 \qquad \Phi\_2 := \frac{\Phi\_2}{\max\left(\begin{bmatrix} |\Phi\_12| \\ |\Phi\_22| \\ |\Phi\_32| \end{bmatrix}\right)} = \begin{bmatrix} 1 \\ 0.031 \\ -0.97 \end{bmatrix}$$

$$B \cdot \Phi\_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

$$B := K - \omega\_3^2 \cdot M = \begin{bmatrix} -1443597.402 & -523377.778 & 0 \\ -523377.778 & -466481.67 & -523377.778 \\ 0 & -523377.778 & -989859.448 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi\_13 := 1 \qquad \Phi\_23 := -\frac{B(0,0)}{B(0,1)} \cdot \Phi\_13 = -2.758$$

$$\Phi\_33 := -\frac{B(1,2)}{B(2,2)} \cdot \Phi\_23 = 1.458$$

$$\Phi\_3 := \begin{bmatrix} \Phi\_13 \\ \Phi\_23 \\ \Phi\_33 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.758 \\ 1.458 \end{bmatrix} \qquad \max(\Phi\_3) = 1.458 \quad \Phi\_3 := \frac{\Phi\_3}{\max\left(\begin{bmatrix} |\Phi\_13| \\ |\Phi\_23| \\ |\Phi\_33| \end{bmatrix}\right)} = \begin{bmatrix} 0.363 \\ -1 \\ 0.529 \end{bmatrix}$$

$$B \cdot \Phi\_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

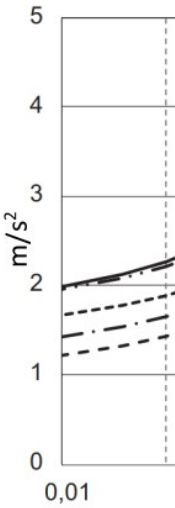
$$\Phi := \text{augment}(\Phi\_1, \Phi\_2, \Phi\_3) = \begin{bmatrix} 0.629 & 1 & 0.363 \\ 0.871 & 0.031 & -1 \\ 1 & -0.97 & 0.529 \end{bmatrix}$$

$$M\_s := \Phi^{\mathrm{T}}\, M \cdot \Phi = \begin{bmatrix} 9404.516 & 0 & 0 \\ 0 & 9768.007 & 0 \\ 0 & 0 & 5906.921 \end{bmatrix} kg$$

$$K\_s := \Phi^{\mathrm{T}}\, K \cdot \Phi = \begin{bmatrix} 159126.682 & 0 & 0 \\ 0 & 1318813.083 & 0 \\ 0 & 0 & 2234643.353 \end{bmatrix} \frac{N}{m}$$

$$T := \frac{2\,\pi}{\omega} = \begin{bmatrix} 1.527 \\ 0.541 \\ 0.323 \end{bmatrix} s \qquad S\_1 := 1.4\ \frac{m}{s^2} \qquad S\_2 := 4.225\ \frac{m}{s^2} \qquad S\_3 := 4.225\ \frac{m}{s^2}$$

$$\Gamma := M\_s^{-1}\, \Phi^{\mathrm{T}} \cdot M\ \text{identity}(3) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.197 \\ 0.23 \\ 0.049 \end{bmatrix} \qquad \omega = \begin{bmatrix} 4.113 \\ 11.62 \\ 19.45 \end{bmatrix} \frac{1}{s}$$



$$q\_1\_max := \Gamma(0) \cdot \frac{1}{\omega\_1^{\,2}} \cdot S\_1 = 0.099\ m$$

$$q\_2\_max := \Gamma(1) \cdot \frac{1}{\omega\_2^{\,2}} \cdot S\_2 = 0.007\ m$$

$$q\_3\_max := \Gamma(2) \cdot \frac{1}{\omega\_3^{\,2}} \cdot S\_3 = 0.001\ m$$

$$\left( (\Phi\_1 \cdot q\_1\_max)^2 + (\Phi\_2 \cdot q\_2\_max)^2 + (\Phi\_2 \cdot q\_2\_max)^2 \right)^{\frac{1}{2}} = \begin{bmatrix} 0.063 \\ 0.086 \\ 0.1 \end{bmatrix} m$$

$$u\_1\_max := q\_1\_max \cdot \Phi\_1 = \begin{bmatrix} 0.062 \\ 0.086 \\ 0.099 \end{bmatrix} m$$

$$s\_1 := \Gamma(0) \cdot M \cdot \Phi\_1 = \begin{bmatrix} 4515.137 \\ 4168.164 \\ 4787.228 \end{bmatrix} kg$$

$$F\_1\_max := s\_1 \cdot S\_1 = \begin{bmatrix} 6321.192 \\ 5835.43 \\ 6702.119 \end{bmatrix} N$$

$$V\_1 := \begin{bmatrix} F\_1\_max(2) + F\_1\_max(1) + F\_1\_max(0) \\ F\_1\_max(2) + F\_1\_max(1) \\ F\_1\_max(2) \end{bmatrix} = \begin{bmatrix} 18.859 \\ 12.538 \\ 6.702 \end{bmatrix} kN$$

$$u\_2\_max := q\_2\_max \cdot \Phi\_2 = \begin{bmatrix} 0.007 \\ 0 \\ -0.007 \end{bmatrix} m$$

$$s\_2 := \Gamma(1) \cdot M \cdot \Phi\_2 = \begin{bmatrix} 1377.969 \\ 28.395 \\ -891.156 \end{bmatrix} kg$$

$$F\_2\_max := s\_2 \cdot S\_2 = \begin{bmatrix} 5821.92 \\ 119.968 \\ -3765.135 \end{bmatrix} N$$

$$V\_2 := \begin{bmatrix} F\_2\_max(2) + F\_2\_max(1) + F\_2\_max(0) \\ F\_2\_max(2) + F\_2\_max(1) \\ F\_2\_max(2) \end{bmatrix} = \begin{bmatrix} 2.177 \\ -3.645 \\ -3.765 \end{bmatrix} kN$$

$$u\_3\_max := q\_3\_max \cdot \Phi\_3 = \begin{bmatrix} 0 \\ -0.001 \\ 0 \end{bmatrix} \; m$$

$$s\_3 := \Gamma(2) \cdot M \cdot \Phi\_3 = \begin{bmatrix} 106.894 \\ -196.559 \\ 103.928 \end{bmatrix} \; kg$$

$$F\_3\_max := s\_3 \cdot S\_3 = \begin{bmatrix} 451.627 \\ -830.461 \\ 439.097 \end{bmatrix} \; N$$

$$V\_3 := \begin{bmatrix} F\_3\_max(2) + F\_3\_max(1) + F\_3\_max(0) \\ F\_3\_max(2) + F\_3\_max(1) \\ F\_3\_max(2) \end{bmatrix} = \begin{bmatrix} 0.06 \\ -0.391 \\ 0.439 \end{bmatrix} \; kN$$

$$\left((V\_1)^2 + (V\_2)^2 + (V\_3)^2\right)^{\frac{1}{2}} = \begin{bmatrix} 18.984 \\ 13.063 \\ 7.7 \end{bmatrix} \; kN$$

$$\left((F\_1\_max)^2 + (F\_2\_max)^2 + (F\_3\_max)^2\right)^{\frac{1}{2}} = \begin{bmatrix} 8.606 \\ 5.895 \\ 7.7 \end{bmatrix} \; kN$$