$t := 1000\ kg$

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$$M := \begin{bmatrix} 12\ t & 0 & 0 \\ 0 & 12\ t & 0 \\ 0 & 0 & 8\ t \end{bmatrix} = \begin{bmatrix} 12000 & 0 & 0 \\ 0 & 12000 & 0 \\ 0 & 0 & 8000 \end{bmatrix} kg$$

$$k\_3 := 2 \cdot 12\ \frac{1.5 \cdot 10^6\ N \cdot m^2}{(4.25\ m)^3} = 468959.902\ \frac{N}{m}$$

$$k\_2 := k\_3 + 12 \cdot \frac{2.2 \cdot 10^6\ N \cdot m^2}{(4.25\ m)^3} = 812863.831\ \frac{kg}{s^2}$$

$$k\_1 := 12 \cdot \frac{1.5 \cdot 10^6\ N \cdot m^2}{(4.5\ m)^3} + 12 \cdot \frac{2.2 \cdot 10^6\ N \cdot m^2}{(4.25\ m)^3} = 541434.793\ \frac{N}{m}$$

$$K := \begin{bmatrix} k\_1 + k\_2 & -k\_2 & 0 \\ -k\_2 & k\_2 + k\_3 & -k\_3 \\ 0 & -k\_3 & k\_3 \end{bmatrix} = \begin{bmatrix} 1354.299 & -812.864 & 0 \\ -812.864 & 1281.824 & -468.96 \\ 0 & -468.96 & 468.96 \end{bmatrix} \frac{kN}{m}$$

$$A := det(-\lambda \cdot M + K) = 0$$

$$A \xrightarrow{expand}$$

$$\begin{bmatrix} \lambda\_1 \\ \lambda\_2 \\ \lambda\_3 \end{bmatrix} :=$$

$$\lambda\_1 = 186.773\ \frac{1}{s^2} \qquad \lambda\_1 := \mathrm{Re}(\lambda\_1) = 186.773\ \frac{1}{s^2}$$

$$\lambda\_2 = 12.074\ \frac{1}{s^2} \qquad \lambda\_2 := \mathrm{Re}(\lambda\_2) = 12.074\ \frac{1}{s^2} \qquad \lambda := \begin{bmatrix} \lambda\_1 \\ \lambda\_2 \\ \lambda\_3 \end{bmatrix}$$

$$\lambda\_3 = 79.45\ \frac{1}{s^2} \qquad \lambda\_3 := \mathrm{Re}(\lambda\_3) = 79.45\ \frac{1}{s^2}$$

$$\omega\_1 := \sqrt{\lambda\_2} = 3.475\ \frac{1}{s}$$

$$\lambda = \begin{bmatrix} 186.773 \\ 12.074 \\ 79.45 \end{bmatrix} \frac{1}{s^2}$$

$$\omega\_2 := \sqrt{\lambda\_3} = 8.913\ \frac{1}{s}$$

$$\omega\_3 := \sqrt{\lambda\_1} = 13.666\ \frac{1}{s} \qquad \omega := \begin{bmatrix} \omega\_1 \\ \omega\_2 \\ \omega\_3 \end{bmatrix} = \begin{bmatrix} 3.475 \\ 8.913 \\ 13.666 \end{bmatrix} \frac{1}{s}$$

$$B:=K-\omega\_1^2 \cdot M=\begin{bmatrix} 1209414.844 & -812863.831 & 0 \\ -812863.831 & 1136939.954 & -468959.902 \\ 0 & -468959.902 & 372370.716 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi\_11:=1 \qquad \Phi\_21:=-\frac{B(0,0)}{B(0,1)} \cdot \Phi\_11=1.488$$

$$\Phi\_31:=-\frac{B(1,2)}{B(2,2)} \cdot \Phi\_21=1.874$$

$$\Phi\_1:=\begin{bmatrix} \Phi\_11 \\ \Phi\_21 \\ \Phi\_31 \end{bmatrix}=\begin{bmatrix} 1 \\ 1.488 \\ 1.874 \end{bmatrix} \qquad \max(\Phi\_1)=1.874 \qquad \Phi\_1:=\frac{\Phi\_1}{\max\left(\begin{bmatrix} |\Phi\_11| \\ |\Phi\_21| \\ |\Phi\_31| \end{bmatrix}\right)}=\begin{bmatrix} 0.534 \\ 0.794 \\ 1 \end{bmatrix}$$

$$B \cdot \Phi\_1=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

$$B:=K-\omega\_2^2 \cdot M=\begin{bmatrix} 400897.84 & -812863.831 & 0 \\ -812863.831 & 328422.949 & -468959.902 \\ 0 & -468959.902 & -166640.62 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi\_12:=1 \qquad \Phi\_22:=-\frac{B(0,0)}{B(0,1)} \cdot \Phi\_12=0.493$$

$$\Phi\_32:=-\frac{B(1,2)}{B(2,2)} \cdot \Phi\_22=-1.388$$

$$\Phi\_2:=\begin{bmatrix} \Phi\_12 \\ \Phi\_22 \\ \Phi\_32 \end{bmatrix}=\begin{bmatrix} 1 \\ 0.493 \\ -1.388 \end{bmatrix} \qquad \max(\Phi\_2)=1 \qquad \Phi\_2:=\frac{\Phi\_2}{\max\left(\begin{bmatrix} |\Phi\_12| \\ |\Phi\_22| \\ |\Phi\_32| \end{bmatrix}\right)}=\begin{bmatrix} 0.72 \\ 0.355 \\ -1 \end{bmatrix}$$

$$B \cdot \Phi\_2=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

$$B := K - \omega\_3^2 \cdot M = \begin{bmatrix} -886979.024 & -812863.831 & 0 \\ -812863.831 & -959453.914 & -468959.902 \\ 0 & -468959.902 & -1025225.196 \end{bmatrix} \frac{kg}{s^2}$$

$$\Phi\_13 := 1 \qquad \Phi\_23 := -\frac{B(0,0)}{B(0,1)} \cdot \Phi\_13 = -1.091$$

$$\Phi\_33 := -\frac{B(1,2)}{B(2,2)} \cdot \Phi\_23 = 0.499$$

$$\Phi\_3 := \begin{bmatrix} \Phi\_13 \\ \Phi\_23 \\ \Phi\_33 \end{bmatrix} = \begin{bmatrix} 1 \\ -1.091 \\ 0.499 \end{bmatrix} \qquad \max(\Phi\_3) = 1 \qquad \Phi\_3 := \frac{\Phi\_3}{\max\left(\begin{bmatrix} |\Phi\_13| \\ |\Phi\_23| \\ |\Phi\_33| \end{bmatrix}\right)} = \begin{bmatrix} 0.916 \\ -1 \\ 0.457 \end{bmatrix}$$

$$B \cdot \Phi\_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \frac{kg}{s^2}$$

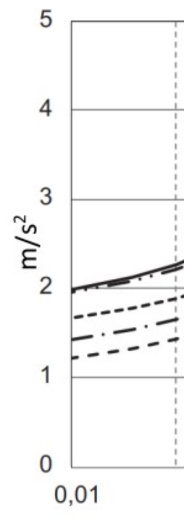
$$\Phi := \text{augment}(\Phi\_1, \Phi\_2, \Phi\_3) = \begin{bmatrix} 0.534 & 0.72 & 0.916 \\ 0.794 & 0.355 & -1 \\ 1 & -1 & 0.457 \end{bmatrix}$$

$$M\_s := \Phi^{\mathrm{T}}\, M \cdot \Phi = \begin{bmatrix} 18983.699 & 0 & 0 \\ 0 & 15744.513 & 0 \\ 0 & 0 & 23752.241 \end{bmatrix} kg$$

$$K\_s := \Phi^{\mathrm{T}}\, K \cdot \Phi = \begin{bmatrix} 229202.499 & 0 & 0 \\ 0 & 1250902.559 & 0 \\ 0 & 0 & 4436280.545 \end{bmatrix} \frac{N}{m}$$

$$T := \frac{2\,\pi}{\omega} = \begin{bmatrix} 1.808 \\ 0.705 \\ 0.46 \end{bmatrix} s \qquad S\_1 := 1.4\ \frac{m}{s^2} \qquad S\_2 := 4.225\ \frac{m}{s^2} \qquad S\_3 := 4.225\ \frac{m}{s^2}$$

$$\Gamma := M\_s^{-1}\ \Phi^{\mathrm{T}} \cdot M\ \text{identity}(3) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.261 \\ 0.312 \\ 0.112 \end{bmatrix} \qquad \omega = \begin{bmatrix} 3.475 \\ 8.913 \\ 13.666 \end{bmatrix} \frac{1}{s}$$



$$q\_1\_max := \Gamma(0) \cdot \frac{1}{\omega\_1^2} \cdot S\_1 = 0.146\ m$$

$$q\_2\_max := \Gamma(1) \cdot \frac{1}{\omega\_2^2} \cdot S\_2 = 0.017\ m$$

$$q\_3\_max := \Gamma(2) \cdot \frac{1}{\omega\_3^2} \cdot S\_3 = 0.003\ m$$

$$\left(\left(\Phi\_1 \cdot q\_1\_max\right)^2 + \left(\Phi\_2 \cdot q\_2\_max\right)^2 + \left(\Phi\_2 \cdot q\_2\_max\right)^2\right)^{\frac{1}{2}} = \begin{bmatrix} 0.08 \\ 0.116 \\ 0.148 \end{bmatrix} m$$

$$u\_1\_max := q\_1\_max \cdot \Phi\_1 = \begin{bmatrix} 0.078 \\ 0.116 \\ 0.146 \end{bmatrix} m$$

$$s\_1 := \Gamma(0) \cdot M \cdot \Phi\_1 = \begin{bmatrix} 8073.701 \\ 12012.411 \\ 10085.539 \end{bmatrix} kg$$

$$F\_1\_max := s\_1 \cdot S\_1 = \begin{bmatrix} 11303.182 \\ 16817.375 \\ 14119.755 \end{bmatrix} N$$

$$V\_1 := \begin{bmatrix} F\_1\_max(2) + F\_1\_max(1) + F\_1\_max(0) \\ F\_1\_max(2) + F\_1\_max(1) \\ F\_1\_max(2) \end{bmatrix} = \begin{bmatrix} 42.24 \\ 30.937 \\ 14.12 \end{bmatrix} kN$$

$$u\_2\_max := q\_2\_max \cdot \Phi\_2 = \begin{bmatrix} 0.012 \\ 0.006 \\ -0.017 \end{bmatrix} m$$

$$s\_2 := \Gamma(1) \cdot M \cdot \Phi\_2 = \begin{bmatrix} 2696.264 \\ 1329.776 \\ -2494.835 \end{bmatrix} kg$$

$$F\_2\_max := s\_2 \cdot S\_2 = \begin{bmatrix} 11391.716 \\ 5618.302 \\ -10540.68 \end{bmatrix} N$$

$$V\_2 := \begin{bmatrix} F\_2\_max(2) + F\_2\_max(1) + F\_2\_max(0) \\ F\_2\_max(2) + F\_2\_max(1) \\ F\_2\_max(2) \end{bmatrix} = \begin{bmatrix} 6.469 \\ -4.922 \\ -10.541 \end{bmatrix} kN$$

$$u\_3\_max := q\_3\_max \cdot \Phi\_3 = \begin{bmatrix} 0.002 \\ -0.003 \\ 0.001 \end{bmatrix} m$$

$$s\_3 := \Gamma(2) \cdot M \cdot \Phi\_3 = \begin{bmatrix} 1230.035 \\ -1342.186 \\ 409.297 \end{bmatrix} kg$$

$$F\_3\_max := s\_3 \cdot S\_3 = \begin{bmatrix} 5196.896 \\ -5670.738 \\ 1729.278 \end{bmatrix} N$$

$$V\_3 := \begin{bmatrix} F\_3\_max(2) + F\_3\_max(1) + F\_3\_max(0) \\ F\_3\_max(2) + F\_3\_max(1) \\ F\_3\_max(2) \end{bmatrix} = \begin{bmatrix} 1.255 \\ -3.941 \\ 1.729 \end{bmatrix} kN$$

$$\left((V\_1)^2 + (V\_2)^2 + (V\_3)^2\right)^{\frac{1}{2}} = \begin{bmatrix} 42.751 \\ 31.573 \\ 17.705 \end{bmatrix} kN$$

$$\left((F\_1\_max)^2 + (F\_2\_max)^2 + (F\_3\_max)^2\right)^{\frac{1}{2}} = \begin{bmatrix} 16.868 \\ 18.616 \\ 17.705 \end{bmatrix} kN$$