3D_reconstruct

October 6, 2021

[ ]:

```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from mpl_toolkits.mplot3d.art3d import Poly3DCollection
```

[ ]:

```python
#Pinhole model
def pinHole(W, A, O, T):
    motionMat = np.hstack((O, T))
    X = np.zeros((W.shape[0], W.shape[1]))
    i = 0
    for w in W:
        w_ = np.vstack((np.reshape(w, (3, 1)), [1]))
        x_ = A @ motionMat @ w_
        x_ = x_/x_[2]
        X[i] = x_.T
        i += 1
    return X

#Plotting function


def plotProj(proj, title='2D proj'):
    proj = np.hstack((proj, np.zeros((proj.shape[0], 1))))
    print(proj.shape)

    x_max = np.amax(proj[:, 0])
    x_min = np.amin(proj[:, 0])

    y_max = np.amax(proj[:, 1])
    y_min = np.amin(proj[:, 1])

    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')

    ax.scatter3D(proj[:, 0], proj[:, 1], proj[:, 2])
    ax.set_xlabel('X')
    ax.set_ylabel('Y')
    ax.set_zlabel('Z')

    padding = 25
    # ax.set_xlim(400, 800)
    # ax.set_xlim(200, 500)
    ax.set_xlim(x_min-padding, x_max+padding)
    ax.set_ylim(y_min-padding, y_max+padding)
    # ax.set_xlim(-1, 1)
    # ax.set_ylim(-1, 1)
    verts = [[proj[0], proj[1], proj[2], proj[3]], [proj[4], proj[5], proj[6],␣
↪proj[7]], [
        proj[0], proj[1], proj[5], proj[4]], [proj[3], proj[2], proj[6],␣
↪proj[7]]]
    #[X[1],X[2],X[6],X[5]], [X[1],X[2],X[5],X[7]]]

    # plot sides
    ax.add_collection3d(Poly3DCollection(
        verts, facecolors='w', linewidths=1, edgecolors='k', alpha=.25))
    ax.set_title(title)
    ax.view_init(90, 90)


def plotReProj(proj, proj_, title='2D proj'):
    proj = np.hstack((proj, np.zeros((proj.shape[0], 1))))
    print(proj.shape)

    proj_ = np.hstack((proj_, np.zeros((proj_.shape[0], 1))))

    x_max = np.amax(proj[:, 0])
    x_min = np.amin(proj[:, 0])

    y_max = np.amax(proj[:, 1])
    y_min = np.amin(proj[:, 1])

    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')

    ax.scatter3D(proj[:, 0], proj[:, 1], proj[:, 2])
    ax.set_xlabel('X')
    ax.set_ylabel('Y')
    ax.set_zlabel('Z')

    padding = 25
    # ax.set_xlim(400, 800)
    # ax.set_xlim(200, 500)
    ax.set_xlim(x_min-padding, x_max+padding)
    ax.set_ylim(y_min-padding, y_max+padding)
    # ax.set_xlim(-1, 1)
    # ax.set_ylim(-1, 1)
    verts = [[proj[0], proj[1], proj[2], proj[3]], [proj[4], proj[5], proj[6],␣
 ↪proj[7]], [
        proj[0], proj[1], proj[5], proj[4]], [proj[3], proj[2], proj[6],␣
 ↪proj[7]]]

    verts_ = [[proj_[0], proj_[1], proj_[2], proj_[3]], [proj_[4], proj_[5],␣
 ↪proj_[6], proj_[7]], [
        proj_[0], proj_[1], proj_[5], proj_[4]], [proj_[3], proj_[2], proj_[6],␣
 ↪proj_[7]]]
    #[X[1],X[2],X[6],X[5]], [X[1],X[2],X[5],X[7]]]

    # plot sides
    ax.add_collection3d(Poly3DCollection(
        verts, facecolors='w', linewidths=1, edgecolors='b', alpha=.25))

    ax.add_collection3d(Poly3DCollection(
        verts_, facecolors='w', linewidths=1, edgecolors='r', alpha=.25))
    ax.set_title(title)
    ax.legend()
    # ax.legend(verts, 'Ground Truth')
    # ax.legend(verts_, 'Estimation')
    ax.view_init(90, 90)

def getPinHoleParams(d,t):
    Int = np.array([[p[0], s, princPtn[0], 0], [
        0, p[1], princPtn[1], 0], [0, 0, 1, 0]])  # Intrinsic parameters
    O = np.array([[cos(t), 0, -sin(t)], [0, 1, 0],  # Rotation matrix
                  [sin(t), 0, cos(t)], [0, 0, 0]])
    T = np.array([[d[0]], [d[1]], [d[2]], [1]])  # Translation matrix
    return Int, O, T


def sin(x): return np.sin(x)
def cos(x): return np.cos(x)
```

[ ]:

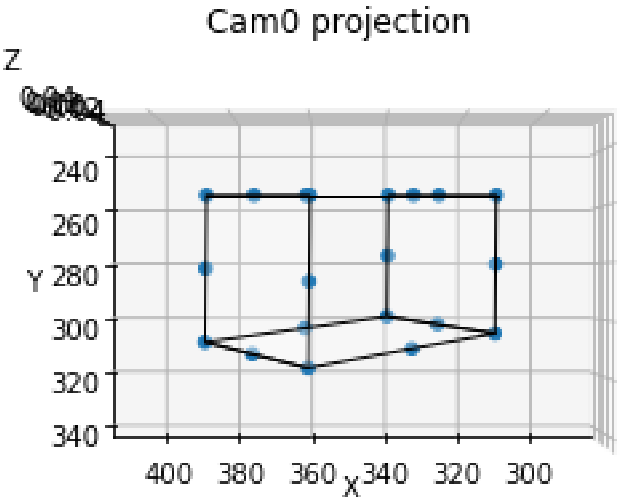
```
# Cube
W = np.array([[0, 0, 200], [50, 0, 200], [50, 50, 200], [0, 50, 200], [
             0, 0, 250], [50, 0, 250], [50, 50, 250], [0, 50, 250], [25, 0,␣
 ↪200], [0, 0, 225], [50, 0, 225], [25, 0, 250], [25, 50, 250], [50, 25, 250],␣
 ↪[0, 25, 250], [0, 50, 225], [0, 25, 200], [25, 50, 200], [50, 25, 200], [50,␣
 ↪50, 225]])

princPtn = np.array([256, 256])  # Principal point
p = np.array([200, 200])  # Focal point
s = 0  # Skew
```

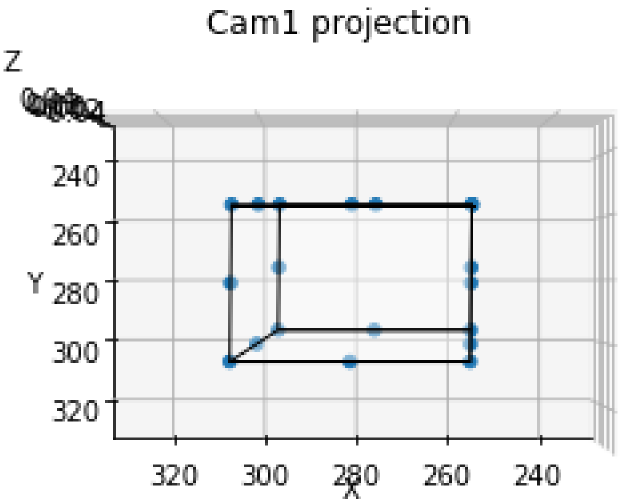
1 Camera rig

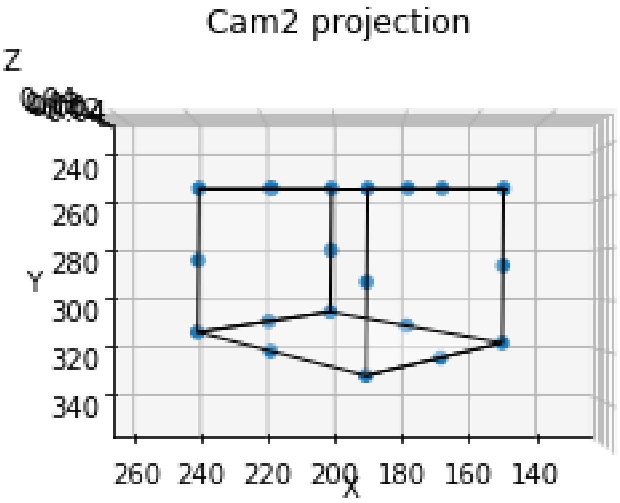
[ ]:

```python
thetas = [np.radians(35), np.radians(0), np.radians(-35)]
D = [np.array([200, 0, 0]), np.array([0, 0, 0]), np.array([-200, 0, 0])]
Om = []
Xs = []
taus = []

i = 0
for theta, d in zip(thetas, D):
    Int, O, T = getPinHoleParams(d, theta)
    Xs.append(pinHole(W, Int, O, T))
    Om.append(O)
    taus.append(T)
    plotProj(Xs[-1][:, :2], title=f'Cam{i} projection')
    i+=1

IntOg = Int
```

```
(20, 3)
(20, 3)
(20, 3)
```







2 3D Reconstruction

```
[ ]: np.set_printoptions(formatter={'float': lambda x: "{0:0.3f}".format(x)})


     def reconstruct3D(X_, Y_, O, T):
```

```
    A = np.zeros((X_.shape[0]*2, X_.shape[0]))
    b = np.zeros((X_.shape[0]*2,))
    W_rec = np.zeros((X_.shape[1], 3))

    for j in range(W_hat.shape[0]):
        i = 0
        for x, y, o, t in zip(X_, Y_, O, T):
            x_j = x[j]
            y_j = y[j]
            A[i] = np.array([(o[2][0]*x_j)-o[0][0], (o[2][1]*x_j) -
                             o[0][1], (o[2][2]*x_j)-o[0][2]])
            A[i+1] = np.array([(o[2][0]*y_j)-o[1][0],
                               (o[2][1]*y_j)-o[1][1], (o[2][2]*y_j)-o[1][2]])

            b[i] = t[0] - t[2]*x_j
            b[i+1] = t[1] - t[2]*y_j

            i += 2
        #Using eq from appendix C.47
        w_hat = np.linalg.inv(A.T @ A) @ (A.T @ b)
        W_rec[j] = w_hat
    return W_rec

def getReconstructParams(Xs, Om, taus, Int):

    O1 = Om[0]
    O2 = Om[1]
    O3 = Om[2]

    T1 = taus[0]
    T2 = taus[1]
    T3 = taus[2]

    X1 = Xs[0]
    X2 = Xs[1]
    X3 = Xs[2]

    X = np.array([[X1[:, 0]], [X2[:, 0]], [X3[:, 0]]]).reshape(3,20)
    Y = np.array([[X1[:, 1]], [X2[:, 1]], [X3[:, 1]]]).reshape(3,20)

    #Normalize projections
    X1_ = (np.linalg.inv(Int[:,:3]) @ X1.T).T
    X2_ = (np.linalg.inv(Int[:, :3]) @ X2.T).T
    X3_ = (np.linalg.inv(Int[:, :3]) @ X3.T).T

    X_ = np.array([[X1_[:, 0]], [X2_[:, 0]], [X3_[:, 0]]]).reshape(3, 20)
    Y_ = np.array([[X1_[:, 1]], [X2_[:, 1]], [X3_[:, 1]]]).reshape(3, 20)
    # X_.shape

    O = [O1[:-1],O2[:-1],O3[:-1]]
    T = np.array([T1, T2, T3]).reshape(3,4)[:, :-1]
    return X_, Y_, O, T


X_, Y_, O, T = getReconstructParams(Xs, Om, taus, Int)
W_hat = np.zeros((X_.shape[1], 3))
W_hat = reconstruct3D(X_, Y_, O, T)

print(f'Estimated w = {W_hat.shape}')
print(f'True w = {W.shape}')
```

```
Estimated w = (20, 3)
True w = (20, 3)
```

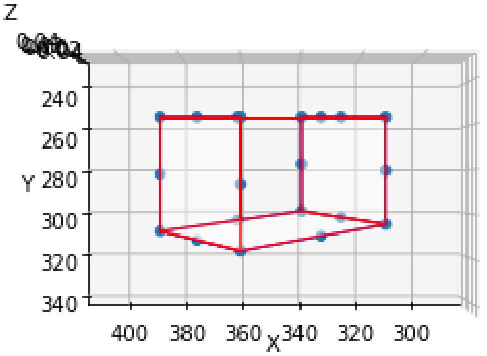
3 2D Projections of Estimated 3D reconstruction

[ ]:

```
def flatten(a):
    return a.reshape((a.shape[0]*a.shape[1]))

Xs_hat = []
i = 0
for theta, d in zip(thetas, D):
    Int, O, T = getPinHoleParams(d, theta)
    x_hat = pinHole(W_hat, Int, O, T)
    Xs_hat.append(x_hat)
    x_true = Xs[i][:, :2]
    x_hat2D = x_hat[:,:2]
    E = np.sum((flatten(x_true)-flatten(x_hat2D))**2)
    plotReProj(x_true, x_hat2D,f'Camera {i} reprojection True(blue) vs␣
 ↪Estimated(red), Error = {E}')
    plt.show()
    i += 1
```

```
No handles with labels found to put in legend.
```

```
(20, 3)
```



Camera 0 reprojection True(blue) vs Estimated(red), Error = 1.2924697071141057e-26

```
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```
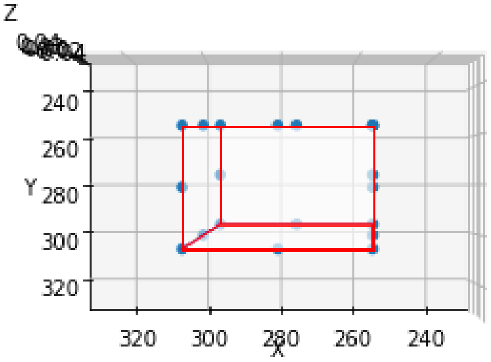
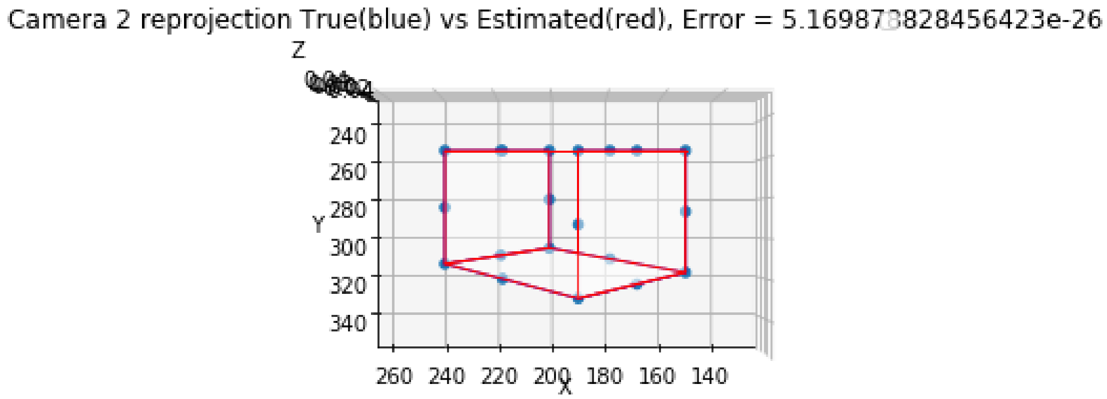

```
(20, 3)
```



Camera 1 reprojection True(blue) vs Estimated(red), Error = 3.5542911694563791e-26

```
No handles with labels found to put in legend.
```

```
(20, 3)
```

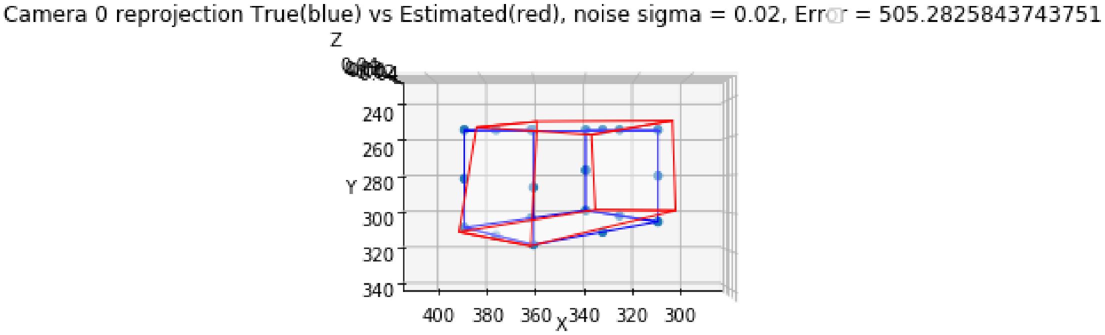


4 Noise sensitivity Analysis

[ ]:
```
# Add noise
mu = 0
sigma = 0.02 #Play with noise sensitivity here
Xs_noisy = []

for x in Xs:
    noise = np.random.normal(mu, sigma, [x.shape[0], x.shape[1]])
    Xs_noisy.append(x + noise)

X_, Y_, O, T = getReconstructParams(Xs_noisy, Om, taus, Int)
W_hat = reconstruct3D(X_, Y_, O, T)

Xs_hat = []
i = 0
for theta, d in zip(thetas, D):
    Int, O, T = getPinHoleParams(d, theta)
    x_hat = pinHole(W_hat, Int, O, T)
    Xs_hat.append(x_hat)
    x_true = Xs[i][:, :2]
    x_hat2D = x_hat[:, :2]
    # print(f'x true shape: {x_true.shape}')
    # print(f'x hat2D shape: {x_hat2D.shape}')
    E = np.sum((flatten(x_true)-flatten(x_hat2D))**2)
    plotReProj(x_true, x_hat2D,
               f'Camera {i} reprojection True(blue) vs Estimated(red), noise␣
 ↪sigma = {sigma}, Error = {E}')
    plt.show()
    i += 1

# print(f'Estimated w = {W_hat.shape}')
# print(f'True w = {W.shape}')
```

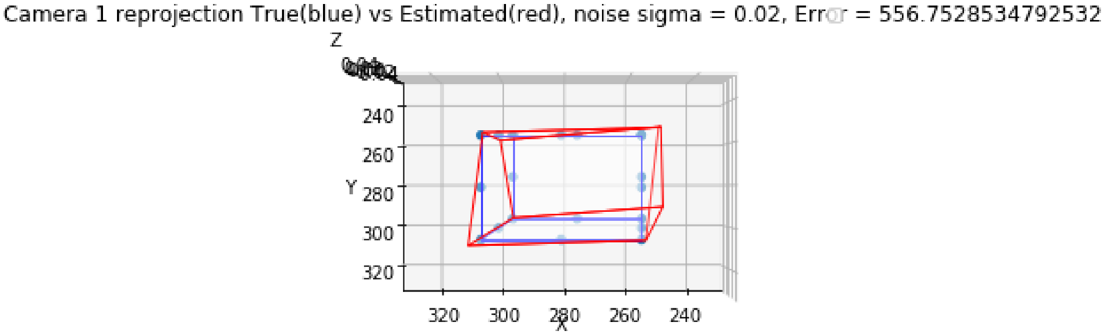
```
No handles with labels found to put in legend.
```

```
(20, 3)
```



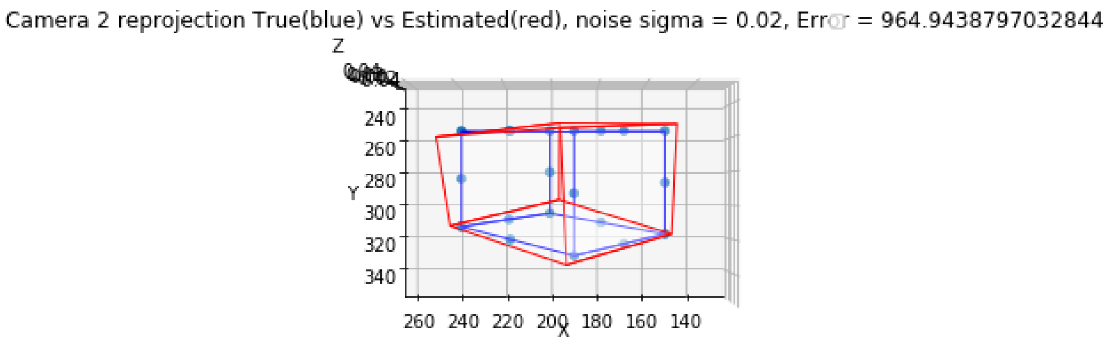
```
(20, 3)
```

```
No handles with labels found to put in legend.
```



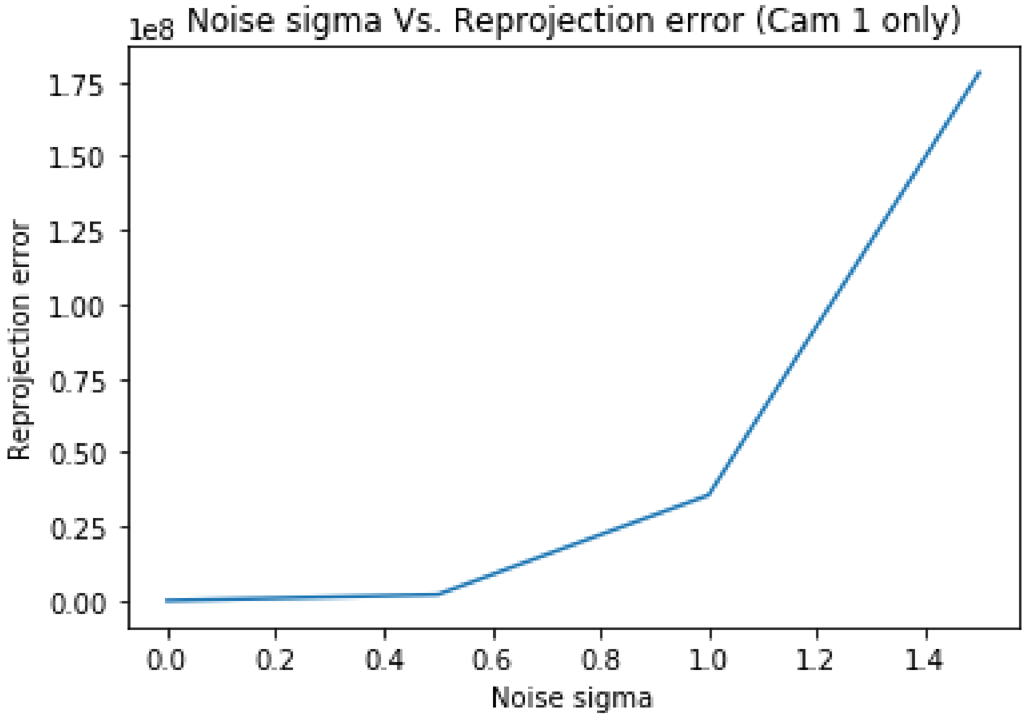
```
(20, 3)
```

```
No handles with labels found to put in legend.
```



4.1 Noise sensitivity plot

```
[ ]: mu = 0
     # sigma = 0.0  # Play with noise sensitivity here
     E = []
     S = []
     for s in np.arange(0, 2, 0.5):
         Xs_noisy = []
         S.append(s)
         for x in Xs:
             noise = np.random.normal(mu, s, [x.shape[0], x.shape[1]])
             Xs_noisy.append(x + noise)

         X_, Y_, O, T = getReconstructParams(Xs_noisy, Om, taus, Int)
         W_hat = reconstruct3D(X_, Y_, O, T)

         Xs_hat = []
         i = 0

         Int, O, T = getPinHoleParams(D[0], thetas[0])
         x_hat = pinHole(W_hat, Int, O, T)
         Xs_hat.append(x_hat)
         x_true = Xs[i][:, :2]
         x_hat2D = x_hat[:, :2]
         E.append(np.sum((flatten(x_true)-flatten(x_hat2D))**2))

     plt.plot(S, E)
     plt.xlabel('Noise sigma')
     plt.ylabel('Reprojection error')
     plt.title("Noise sigma Vs. Reprojection error (Cam 1 only)")
     plt.show()
```


Noise sigma Vs. Reprojection error (Cam 1 only)
1e8
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Reprojection error
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Noise sigma