3.3 Histogram Processing

- Histogram of a digital image with intensity levels in the range [0,L-1] is a discrete function $h(r_k) = n_k$, where r_k is the kth intensity value and n_k is the number of pixels in the image with intensity r_k
- Normalized histogram $p(r_k)=n_k/MN$, for k = 0,1,2....... L-1.
- Histogram manipulation can be used for image enhancement.
- Information inherent in histogram also is quite useful in other image processing applications, such as image compression and segmentation.

3.3 Histogram Processing

Histogram Equalization

- Intensity mapping form

$$s = T(r),\ 0 \le r \le L-1$$ Maps histogram values r to s

Conditions:

a) T(r) is a monotonically increasing function in the interval [0, L-1] and

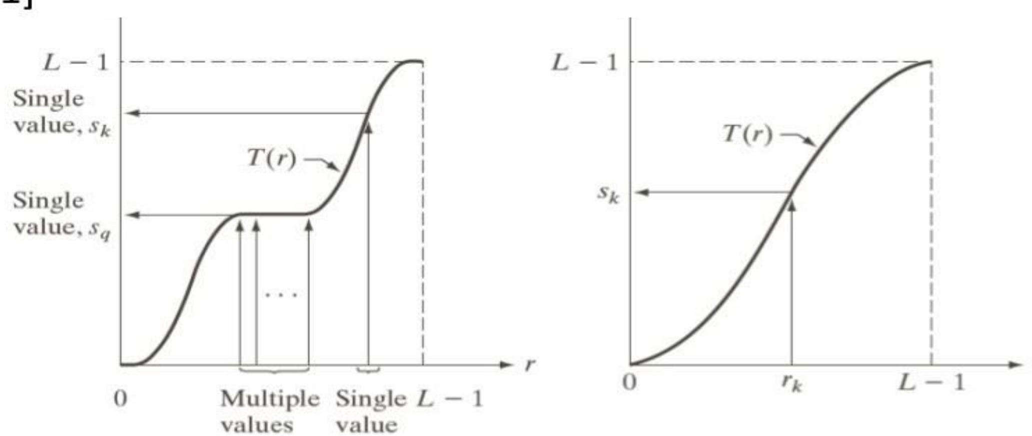
b) $0 \le T(r) \le L-1 \, for \, 0 \le r \le L-1$

In some formulations, we use the inverse in which case

$$r = T^{-1}(s),\ 0 \le s \le 1$$

For transformation to be reversible

a’) T(r) is a strictly monotonically increasing function in the interval [0, L-1]



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

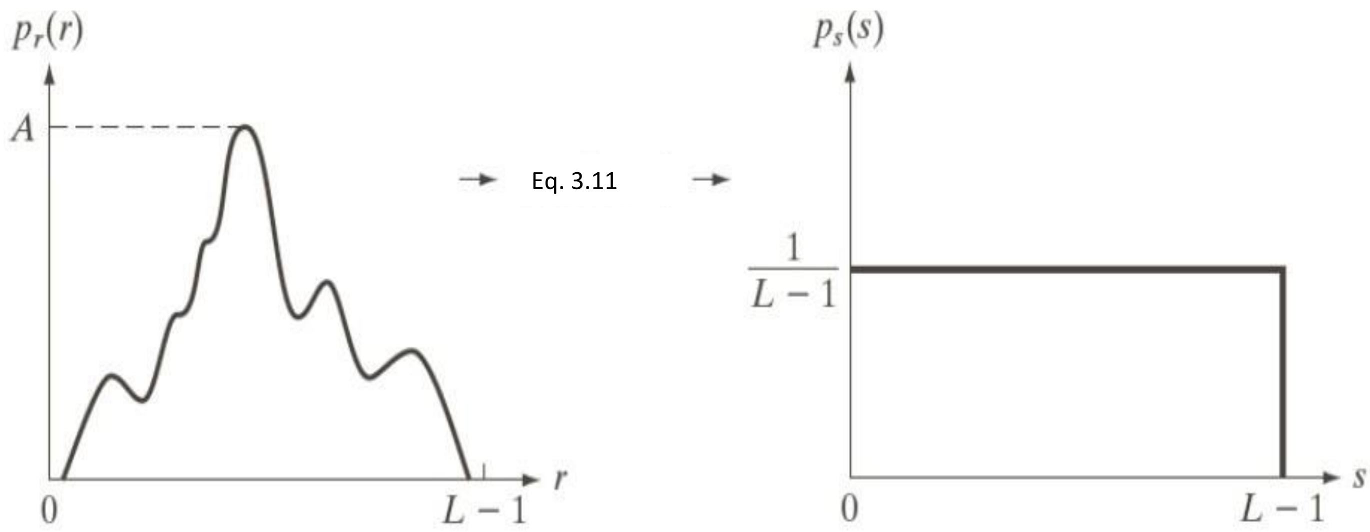
3.3 Histogram Processing

Histogram Equalization

given histogram

desired histogram

- Intensity levels in an image may be viewed as random variables in the interval [0,L-1]
- Fundamental descriptor of a random variable is its probability density function (PDF)
- Let $p_r(r)$ and $p_s(s)$ denote the PDFs of r and s respectively



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. 3.11 to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Equalization

$\int_0^s p_s(w)\mathrm{dw} = \int_0^r p_r(w)\mathrm{dw}$ assumes monotonicity

$p_s(s) = \frac{1}{L-1}$ "equalized" histogram (constant)

$\frac{1}{L-1}\int_0^s \mathrm{dw} = \int_0^r p_r(w)\mathrm{dw}$ substitute "equalized" histogram

$\frac{1}{L-1}\, s = \int_0^r p_r(w)\mathrm{dw}$ integrate constant term

$\mathrm{s} = \mathrm{T}(r) = (\mathrm{L}-1)\boxed{\int_0^r p_r(w)\mathrm{dw}}$ solve for s

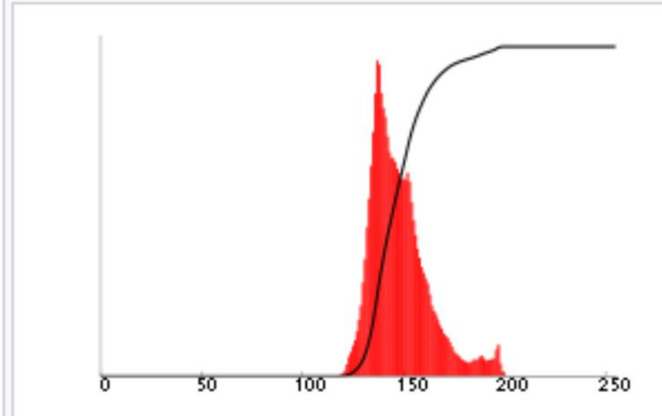
CDF

$\mathrm{s}_\mathrm{k} = \mathrm{T}(r_k) = (L-1)\boxed{\sum_{j=0}^{k} \frac{n_j}{MN}}, \quad k = 0,1,2,\ldots,L-1$ implemented discretely

Histogram Equalization

Calculate CDF from histogram
s = T(r) = (L-1)(CDF)

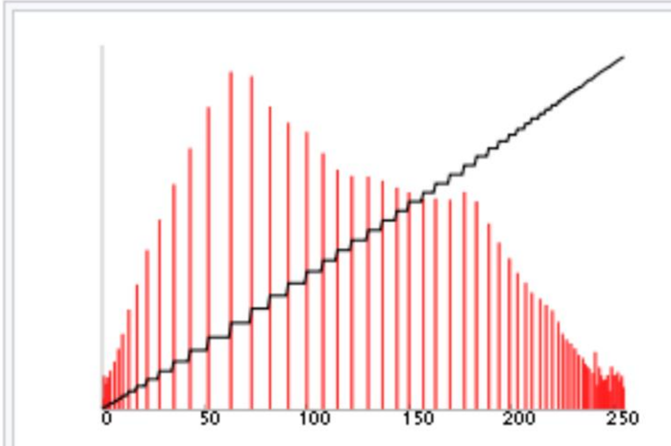
Before Histogram Equalization



Corresponding histogram (red) and cumulative histogram (black)

The CDF of the new image will be “linear” – consequence of a “constant” pdf

After Histogram Equalization



Corresponding histogram (red) and cumulative histogram (black)

3.3 Histogram Processing

Example: Histogram Equalization

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in following table.

Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Transformation of s_i

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7 \times 0.19 = 1.33 \quad \rightarrow 1$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \quad \rightarrow 3$$

$s_2 = (L-1)(n_0 + n_1 + n_2)/MN$

$s_4 = (L-1)(n_0 + n_1 + n_2 + n_3 + n_4)/MN$

$s_2 = 4.55 \rightarrow 5$ $\quad s_3 = 5.67 \rightarrow 6$

$s_4 = 6.23 \rightarrow 6$ $\quad s_5 = 6.65 \rightarrow 7$

$s_6 = 6.86 \rightarrow 7$ $\quad s_7 = 7.00 \rightarrow 7$

3.3 Histogram Processing

Example: Histogram Equalization

Fewer values than original because of quantization

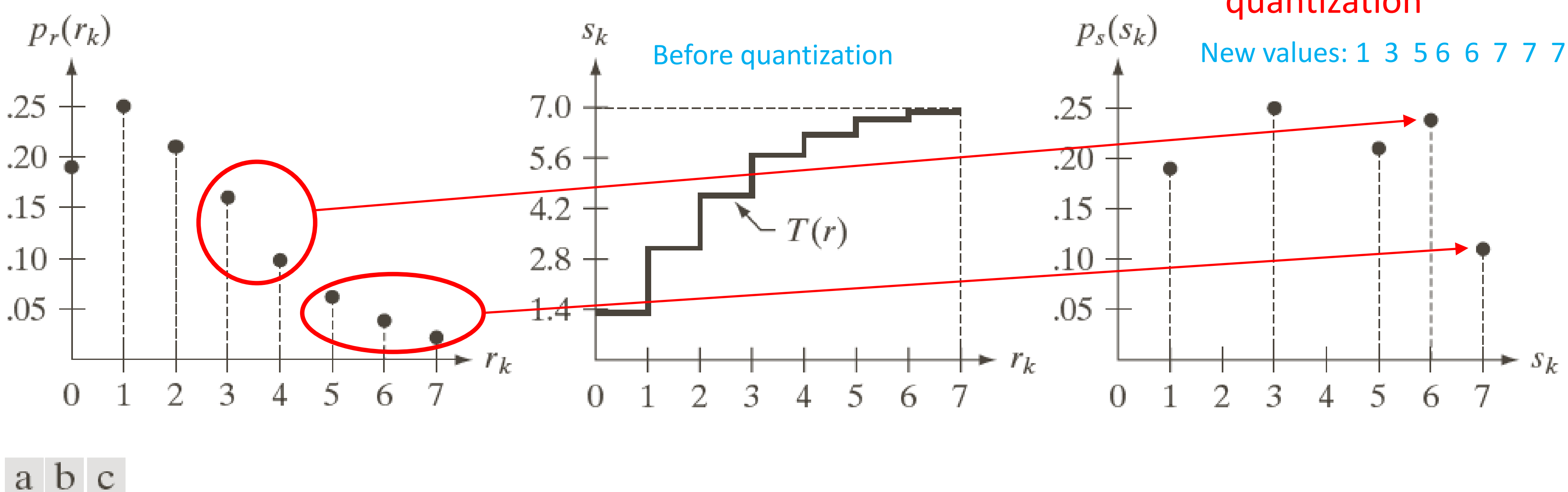


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

3.3 Histogram Processing

- Transformation functions

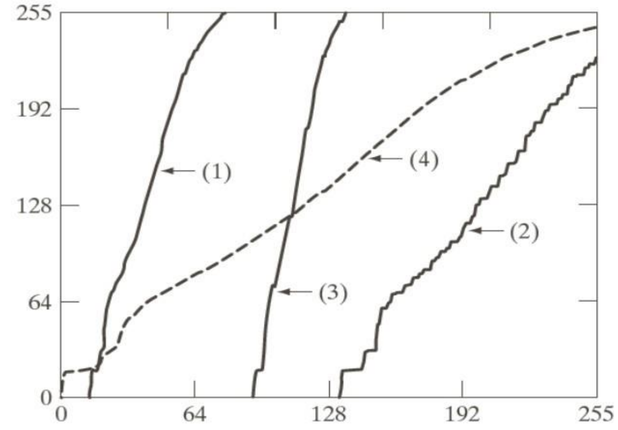


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

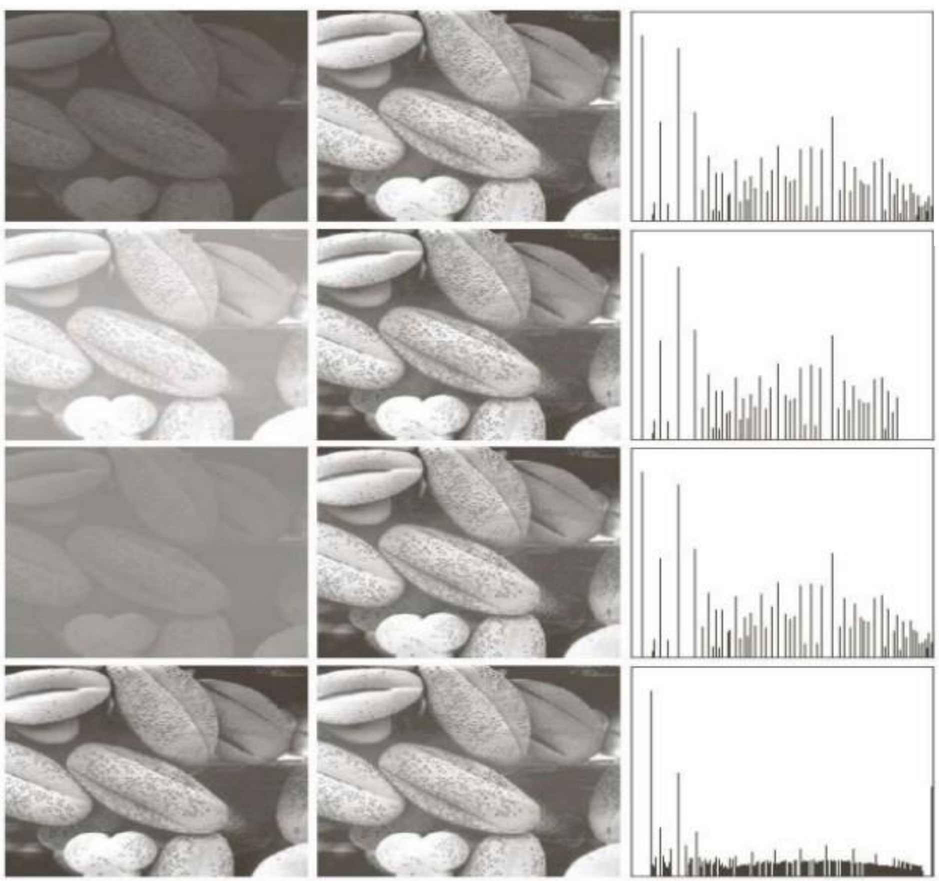


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

3.3 Histogram Processing

Histogram Matching

2
Convert constant PDF to desired PDF

1
Convert exiting PDF to a constant (histogram equalization)

CDF

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s = T(r) = (L-1)\boxed{\int_0^r p_r(w)\,dw}$$

Variables:
r – original image
s – equalized image
z – matched image

Write z as a function of s $\quad z = G^{-1}(s) = G^{-1}[T(r)]$

Implemented discretely from
CDF of specified histogram $\quad z_q = (L-1)\sum_0^q p_z(z_i)$

3.3 Histogram Processing

Histogram Matching

Suppose an image has the intensity PDF $p_r(r) = r$ for $0 < r < L-1$

Find the transformation that will produce an image with PDF $p_z(z) = z^2$ for $0 < z < L-1$

Perform histogram equalization

$s = T(r) = (L-1)\int_0^r p_r(w)\,dw = (L-1)\int_0^r w\,dw = (L-1)\, r^2/2$

Find the histogram equalization transformation to the specified histogram

$G(z) = (L-1)\int_0^z p_z(t)\,dt = (L-1)\int_0^z t^2\,dt = (L-1)\, z^3/3$

Set $G(z) = s$ and solve for z

$(L-1)\, z^3/3 = (L-1)\, r^2/2$

$z = (3/2r^2)^{1/3}$

3.3 Histogram Processing

Example: Histogram Matching

Equalize histogram, then transform to desired histogram

Equalize histogram from (CMF),

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 6,$$

$$s_5 = 7, s_6 = 7, s_7 = 7.$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 7 \times 0 \rightarrow 0$$

$G(z_1) = 0.00 = 7 \times 0 \rightarrow 0$ $G(z_2) = 0.00 = 7 \times 0 \rightarrow 0$

$G(z_3) = 1.05 = 7 \times 0.15 \rightarrow 1$ $G(z_4) = 2.45 = 7(0.20+0.15) \rightarrow 2$

$G(z_5) = 4.55 \rightarrow 5$ $G(z_6) = 5.95 \rightarrow 6$

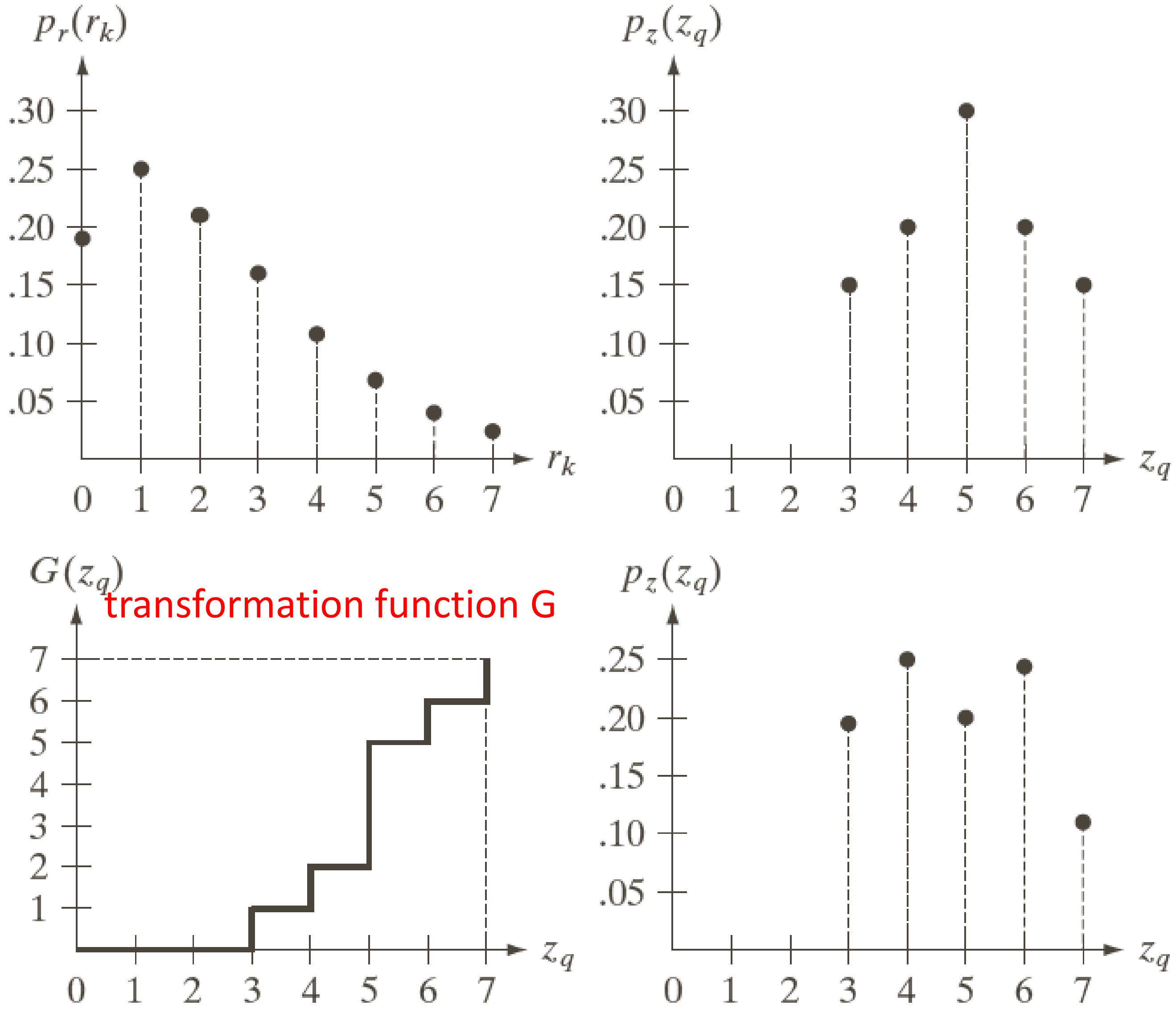
$G(z_7) = 7.00 \rightarrow 7$

Specified cumulative probabilities

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

3.3 Histogram Processing

Example: Histogram Matching

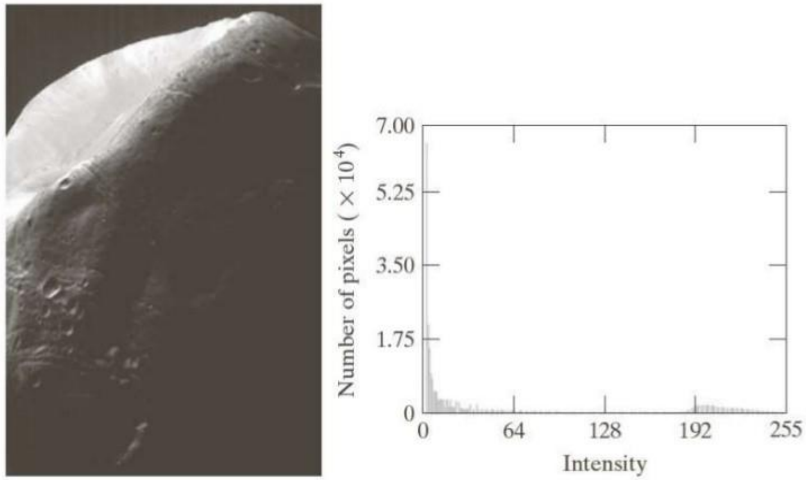


a	b
c	d

FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

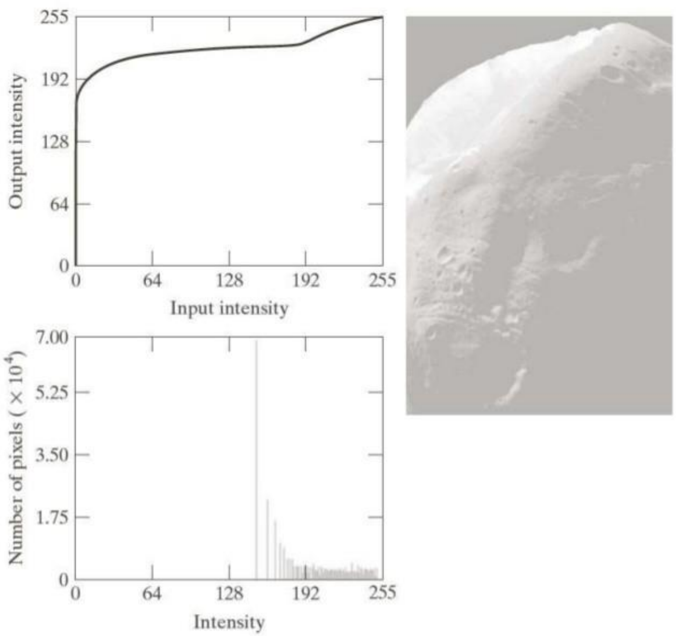
Note that equalized histogram was quantized before final transformation

3.3 Histogram Processing



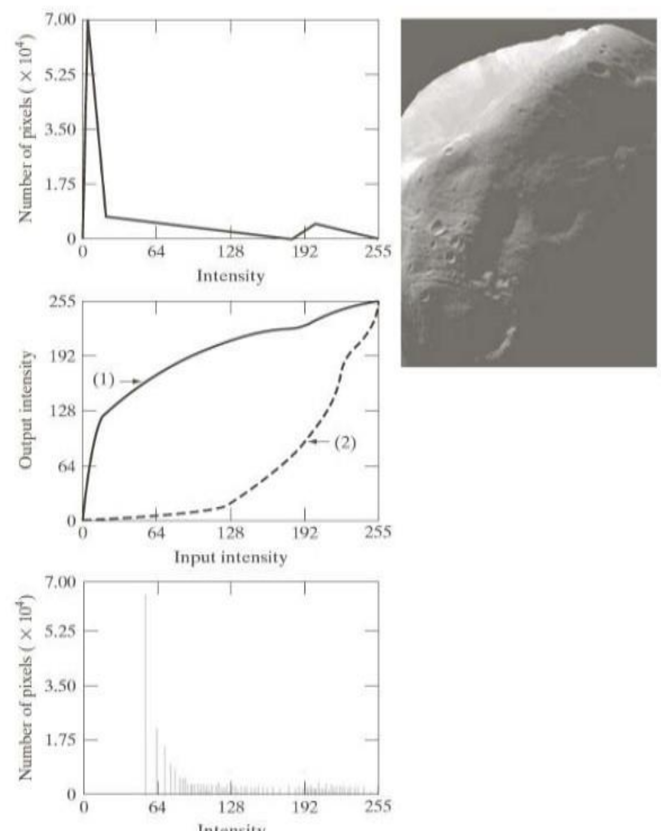
a b

FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)



a b
c

FIGURE 3.24 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).



a c
b
d

FIGURE 3.25 (a) Specified histogram. (b) Transformations. (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).

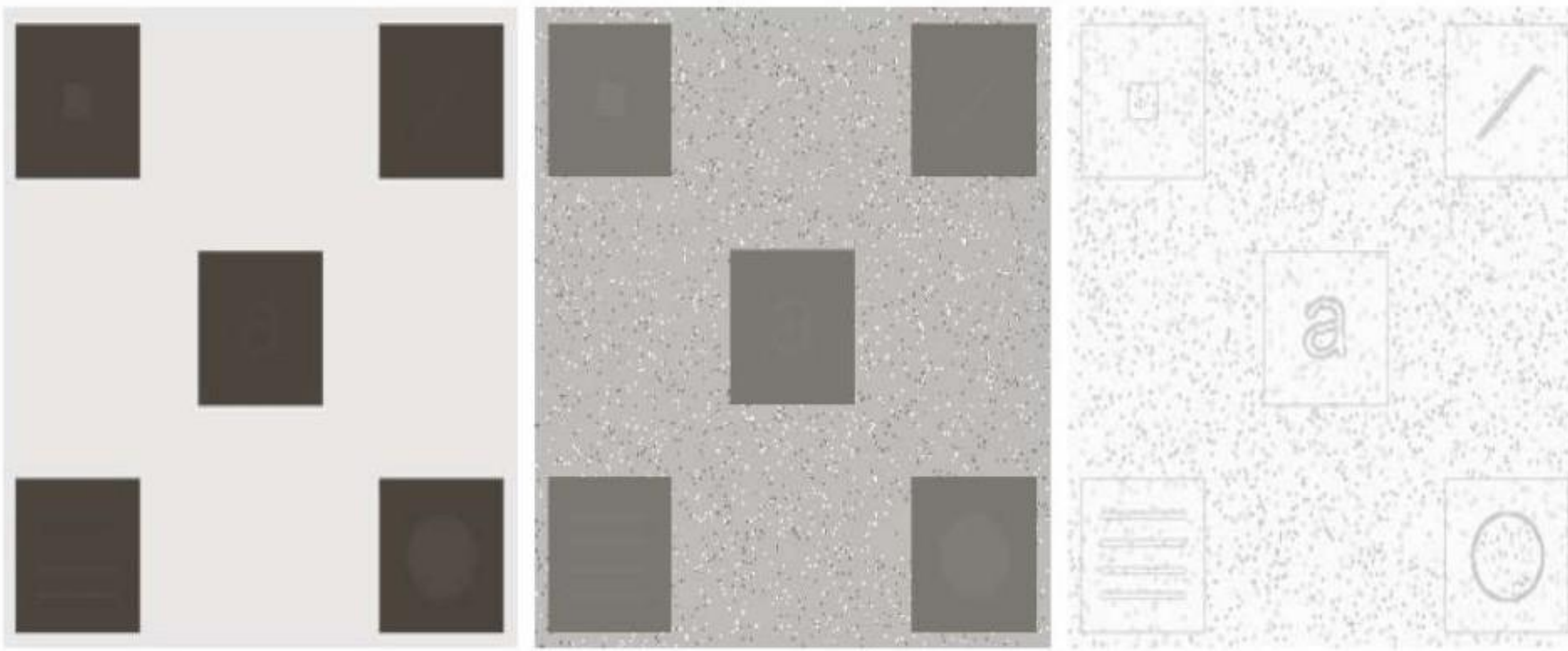
3.3 Histogram Processing

Local Histogram Processing

- Histogram Processing methods discussed in the previous two sections are *Global,* in the sense that pixels are modified by a transformation function based on the intensity distribution of an entire image.
- There are some cases in which it is necessary to enhance detail over small areas in an image.
- This procedure is to define a neighborhood and move its center pixel to pixel.
- At each location, the histogram of the points in the neighborhood is computed and either a histogram equalization or histogram specification transformation function is obtained.
- Map the intensity of the pixel centered in the neighborhood.
- Center of the neighborhood region is then moved to an adjacent pixel location and the procedure is repeated.

3.3 Histogram Processing

Local Histogram Processing



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

3.4 Fundamentals of Spatial Filtering

Spatial Filtering

- Also called spatial masks, kernels, templates, and windows.
- It consists of (1) a neighborhood (typically a small window), and (2) a predefined operation that is performed on the image pixels encompassed by the neighborhood.
- Filtering creates a new pixel with coordinates equal to the center of the neighborhood.
- If operation is linear, then filter is called a *linear spatial filter* otherwise *nonlinear.*

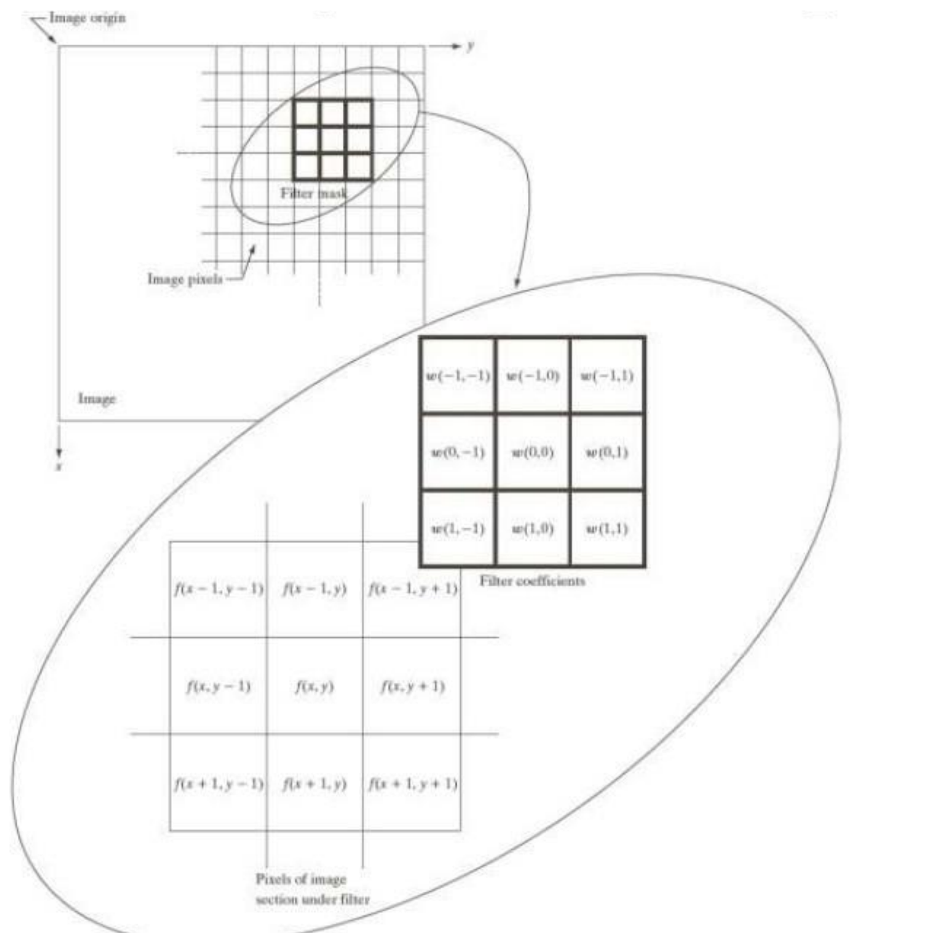


FIGURE 3.28 The mechanics of linear spatial filtering using a 3×3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

3.4 Fundamentals of Spatial Filtering

Spatial Correlation & Convolution

- Correlation is the process of moving a filter mask over the image and computing the sum of the products at each location.
- Convolution process is same except that the filter is first rotated by 180 degree.

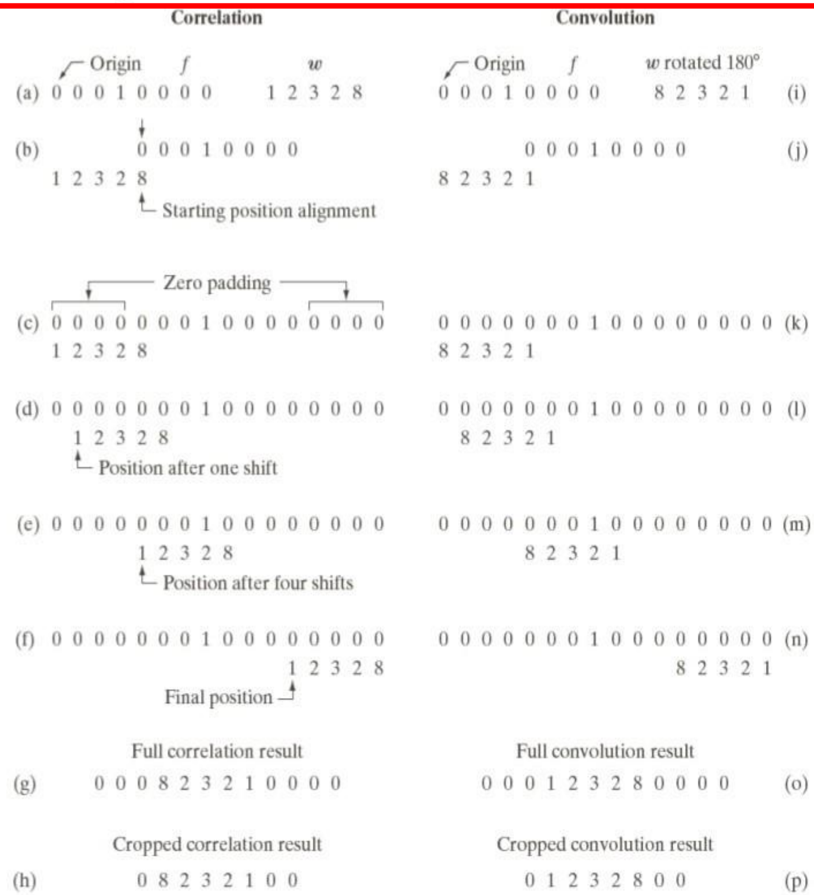


FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

3.4 Fundamentals of Spatial Filtering

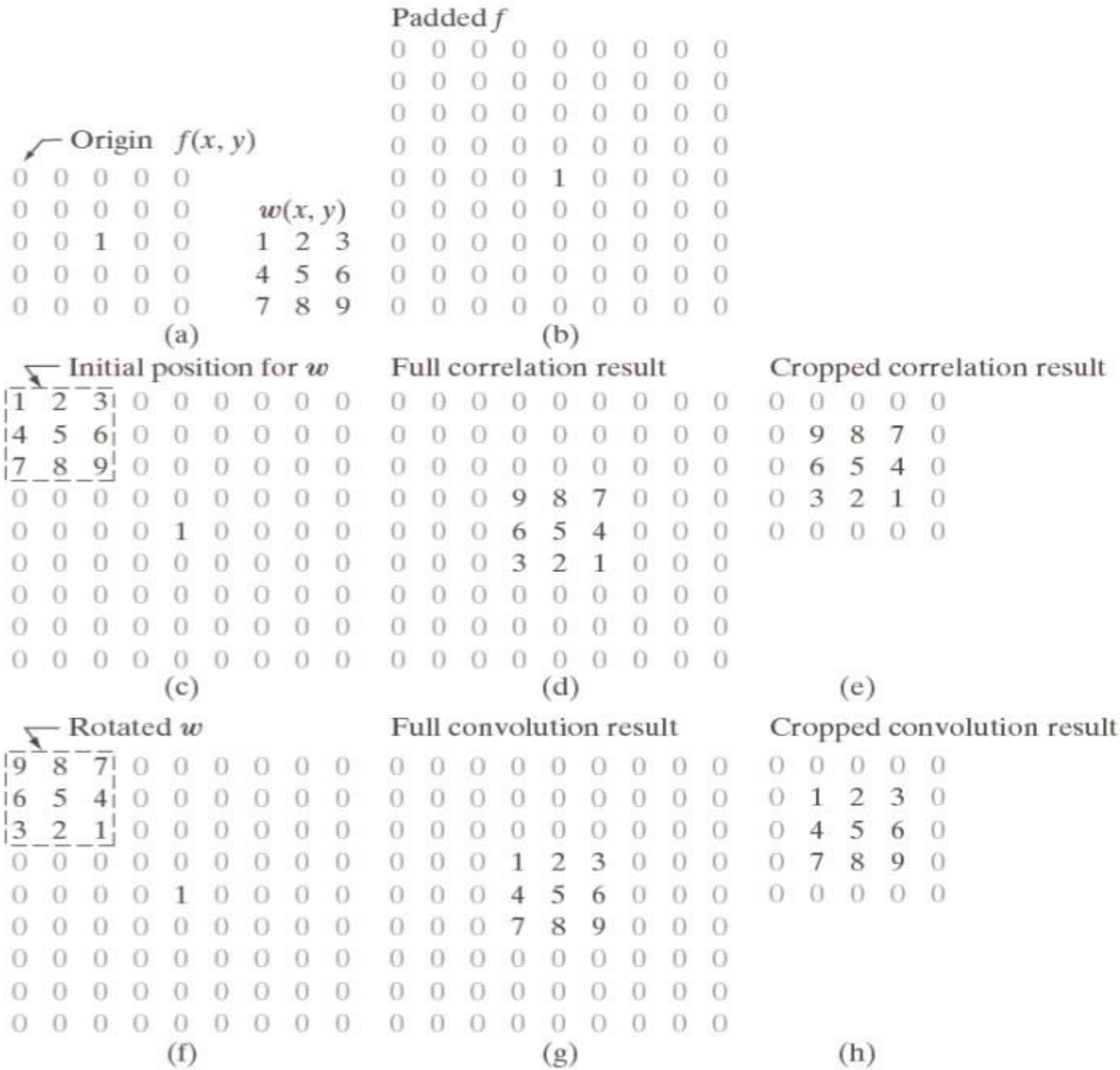


FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

3.5 Smoothing (Lowpass) Spatial Filters

Smoothing Spatial Linear Filters

- Also called *averaging filters* or *Lowpass filter.*
- By replacing the value of every pixel in an image by the average of the intensity levels in the neighborhood defined by the filter mask.
- Reduced "sharp" transition in intensities.

- The other mask is called *weighted average,* terminology used to indicate that pixels are multiplied by different coefficient.
- Center point is more weighted than any other points.
- Strategy behind weighing the center point the highest and then reducing value of the coefficients as a function of increasing distance from the origin is simply an attempt to reduce blurring in the smoothing process.

3.5 Smoothing (Lowpass) Spatial Filters

Smoothing Linear Filter

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

3.5 Smoothing (Lowpass) Spatial Filters

FIGURE 3.33 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

a b
c d
e f