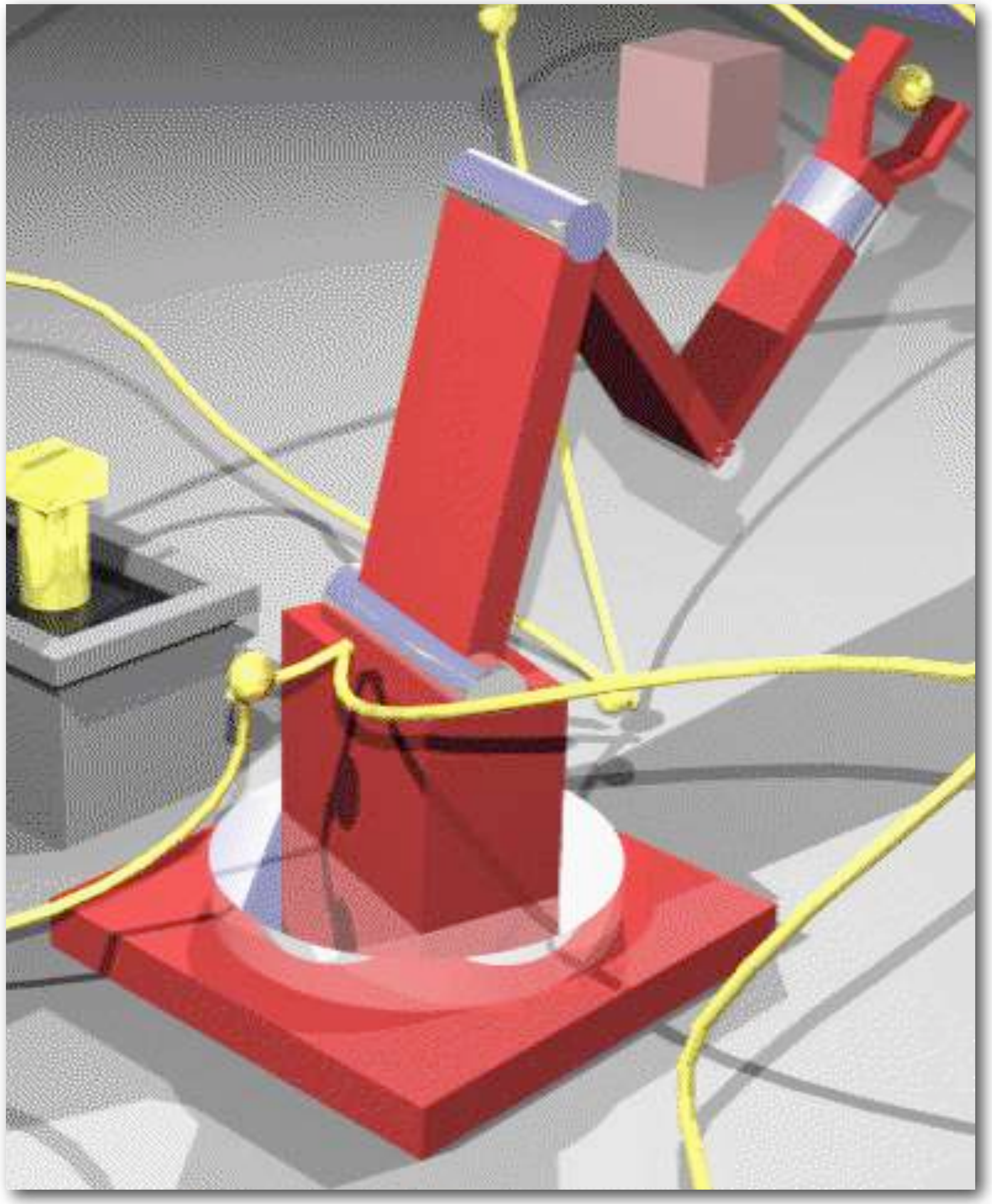
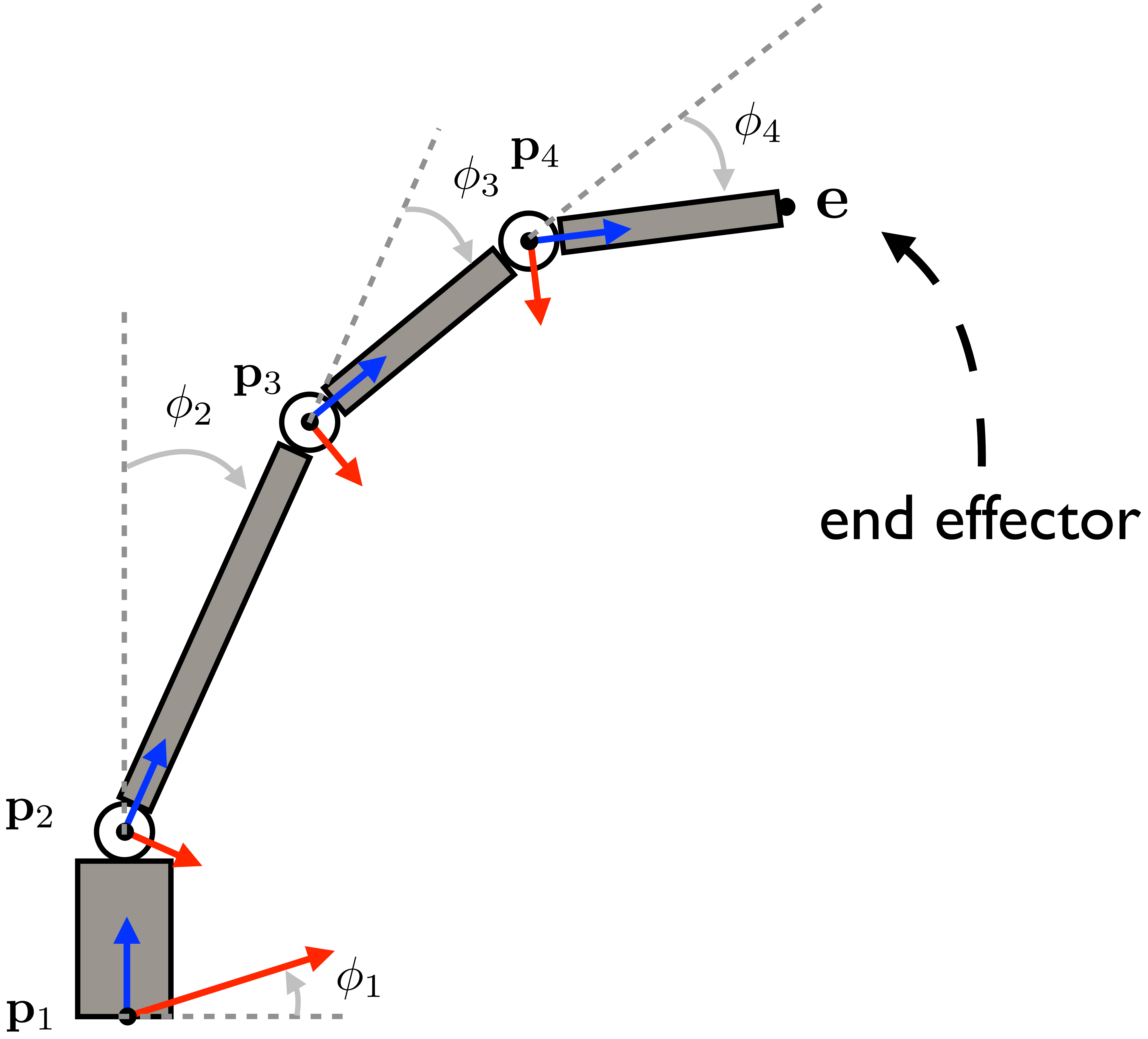
Animating a robot arm: Part 3

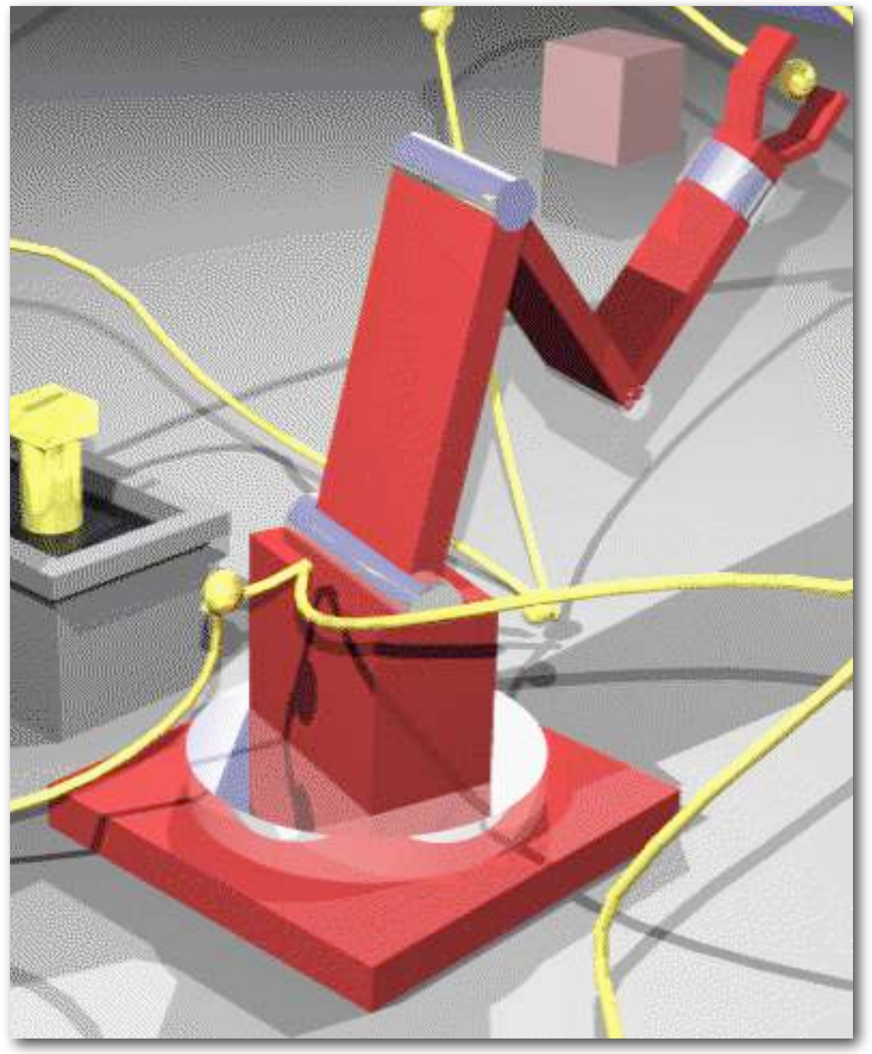


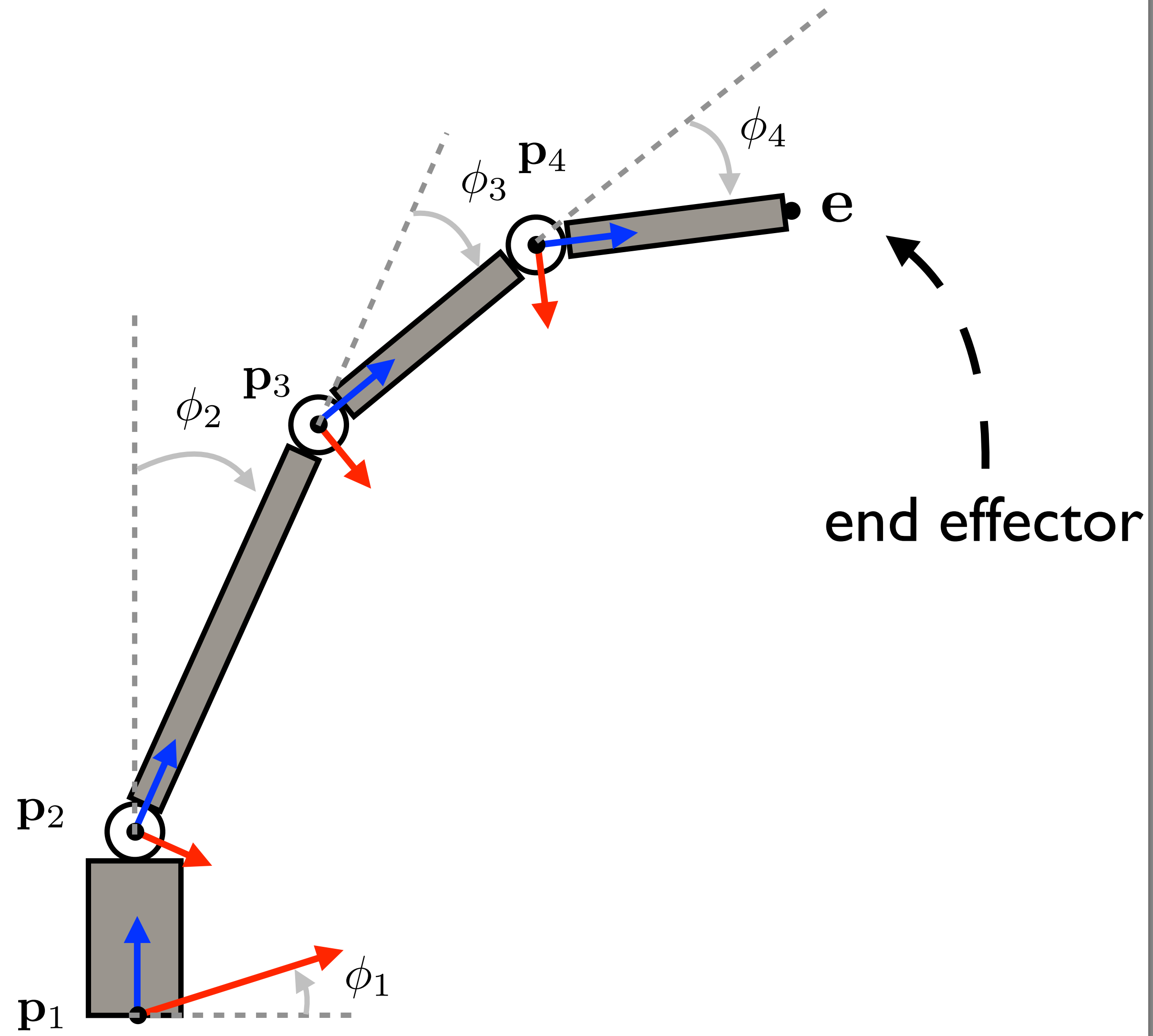


Overview

- Rigid-body transformation and its inverse Case 2: **multiple frames**

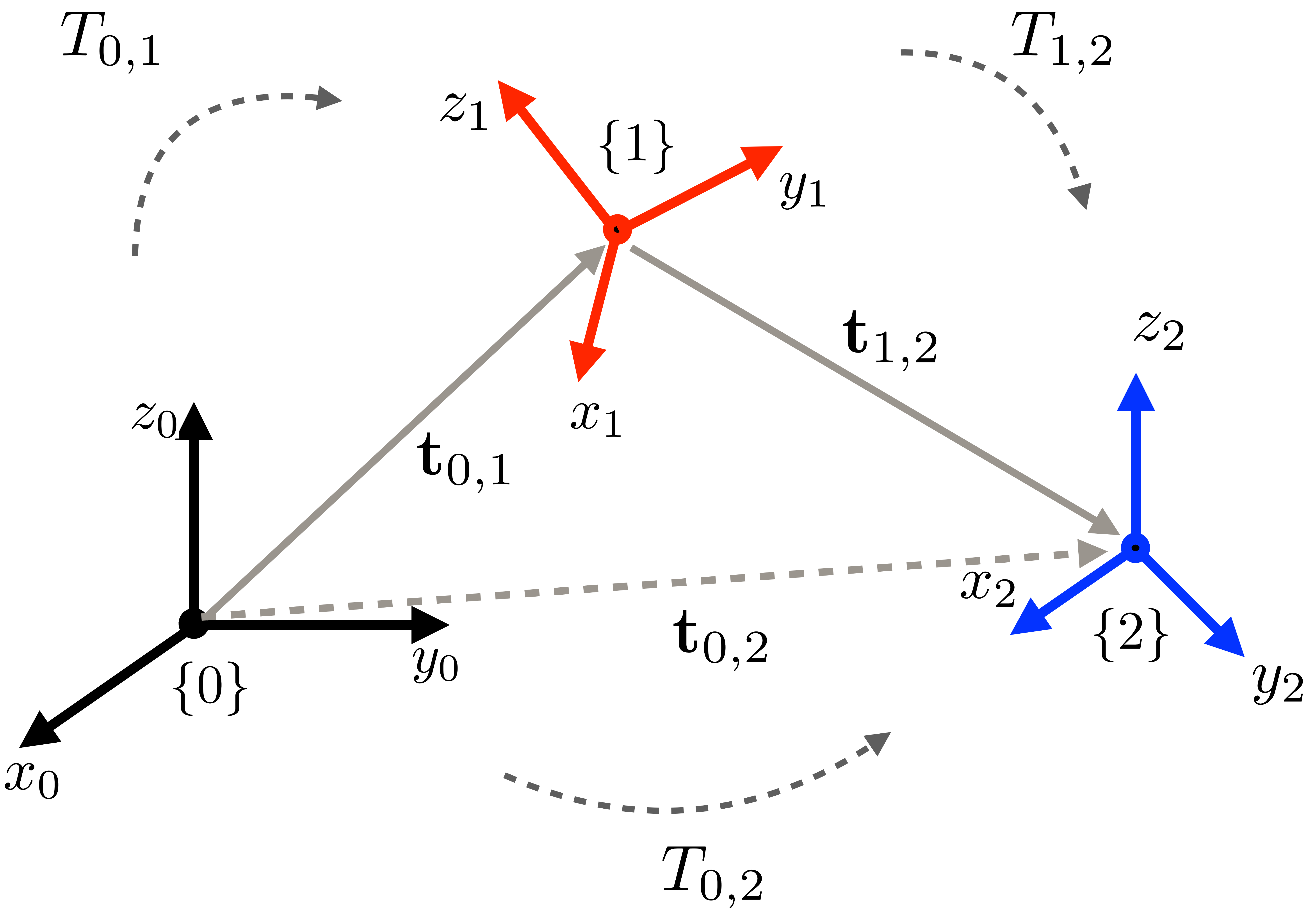
Animating a robot arm





We can extend the modeling from 2 frames to multiple frames.

$$T_{0,2} = T_{0,1}T_{1,2} = \begin{bmatrix} R_{0,1} & \mathbf{t}_{0,1} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} R_{1,2} & \mathbf{t}_{1,2} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} R_{0,2} & \mathbf{t}_{0,2} \\ \mathbf{0} & 1 \end{bmatrix}.$$

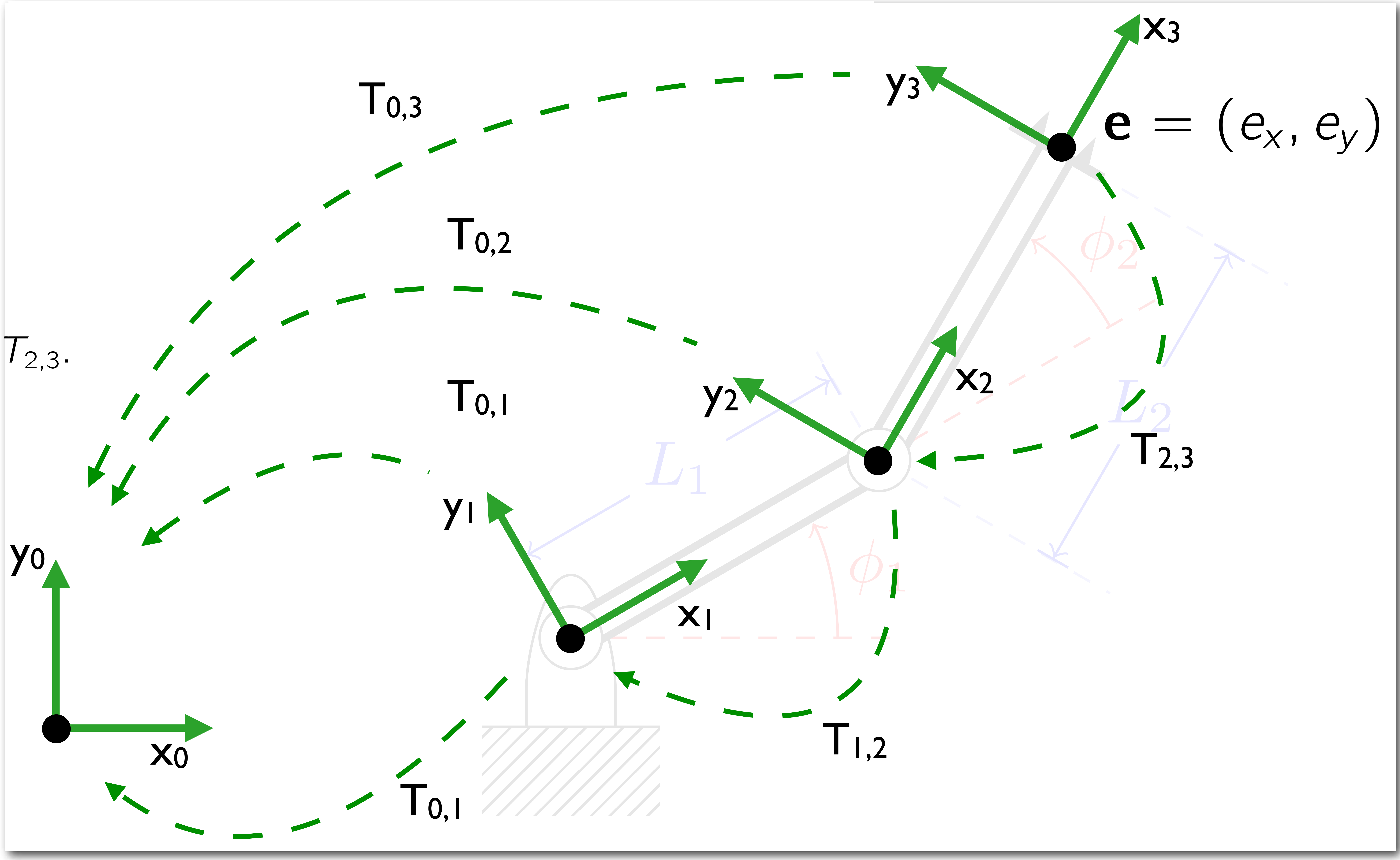


- Going back to the arm model, we can calculate the conversion transformation from local to global by recursive concatenation of the local transformations.

$$T_{0,1} = T_{0,1}.$$

$$T_{0,2} = T_{0,1}T_{1,2}.$$

$$T_{0,3} = T_{0,2}T_{2,3} = T_{0,1}T_{1,2}T_{2,3}.$$



- How to represent a point in a different coordinate system.

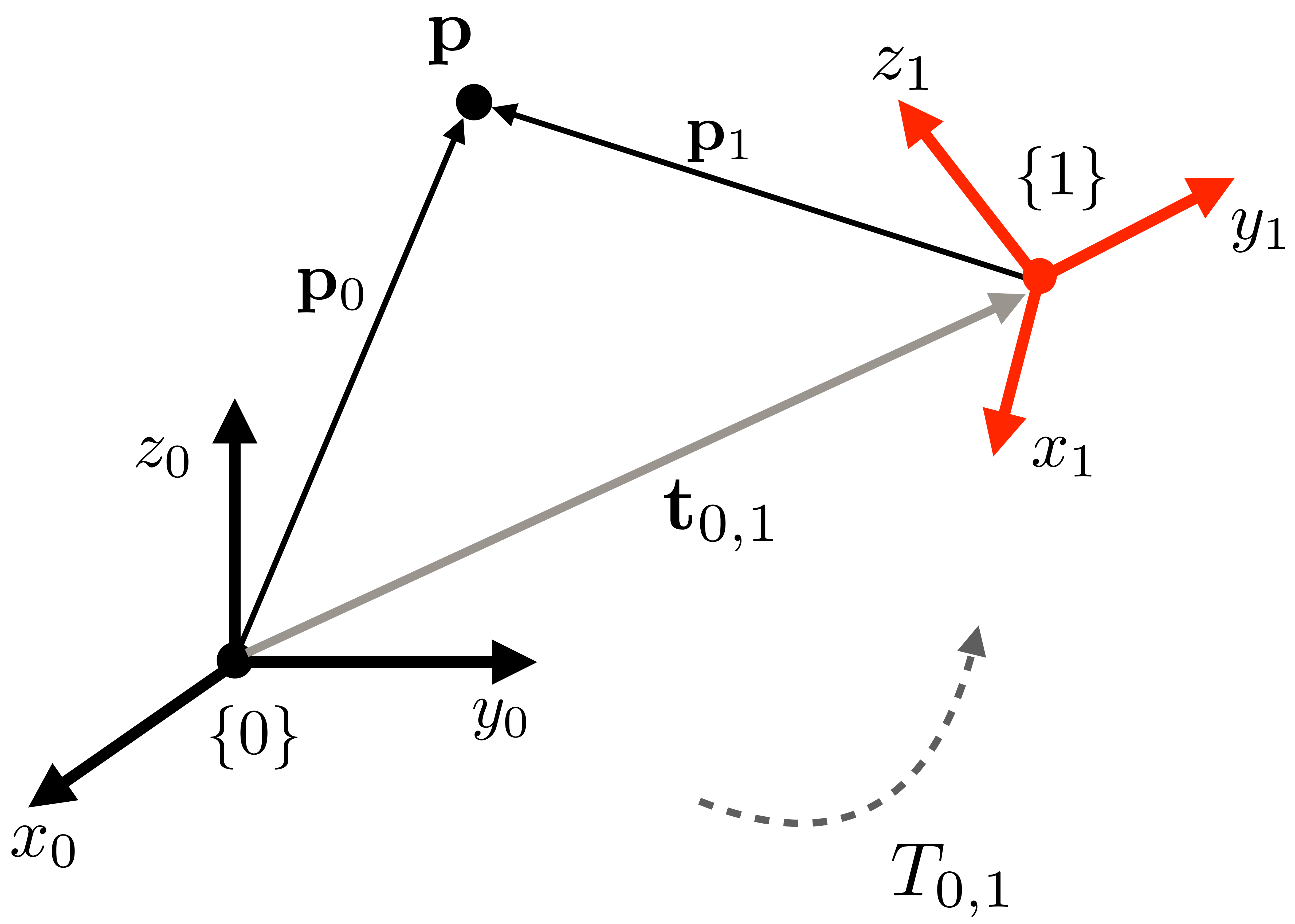


Figure 5: A 3-D point represented with respect to two different frames of reference. Each frame has its own representation of point $\mathbf{p}$. It is represented by vector $\mathbf{p}_0$ with respect to frame 0 and by vector $\mathbf{p}_1$ with respect to frame 1.

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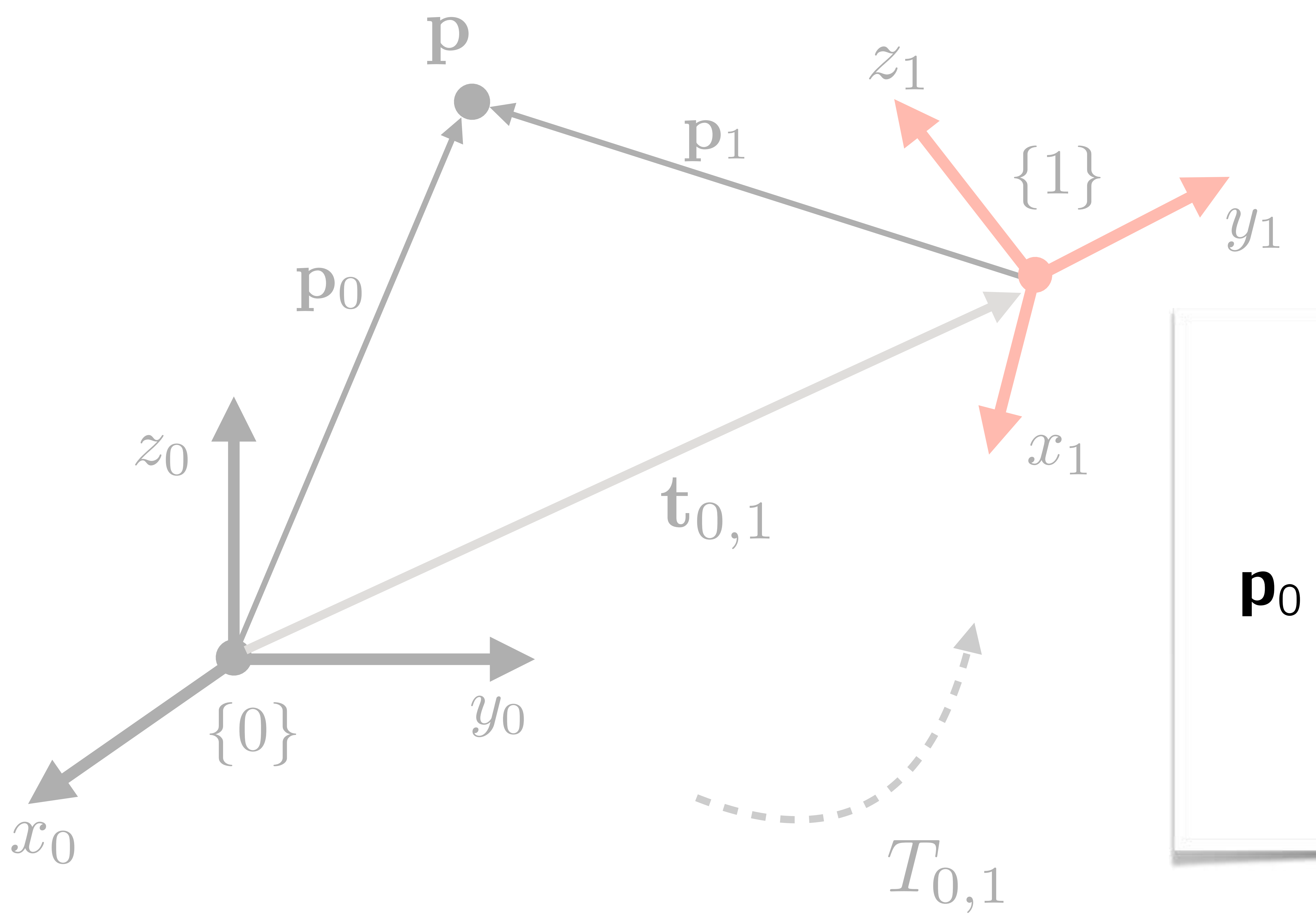


Graphics: If we have p1 in local coordinates, we must convert it to p0 first calling the plotting function

$$\mathbf{p}_0 = T_{0,1}\,\mathbf{p}_1 = \begin{bmatrix} R_{0,1} & \mathbf{t}_{0,1} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}.$$

Figure 5: A 3-D point represented with respect to two different frames of reference. Each frame has its own representation of point $\mathbf{p}$. It is represented by vector $\mathbf{p}_0$ with respect to frame 0 and by vector $\mathbf{p}_1$ with respect to frame 1.

A numerical example of change of coordinate frames in 2-D is show in Figure 6. In this example, we will use homogeneous coordinates. Here, the representation of point **p** with respect to frame 0 is $\mathbf{p}_0 = (1, 1, 1)^\mathsf{T}$. The same point represented with respect to frame 0 is $\mathbf{p}_1 = \left(\sqrt{2}/2, 0, 1\right)^\mathsf{T}$. Frame 1 is rotated by an angle of $\pi/4$ (i.e., 45 degrees) and translated by a vector **t** with respect to the frame 0.

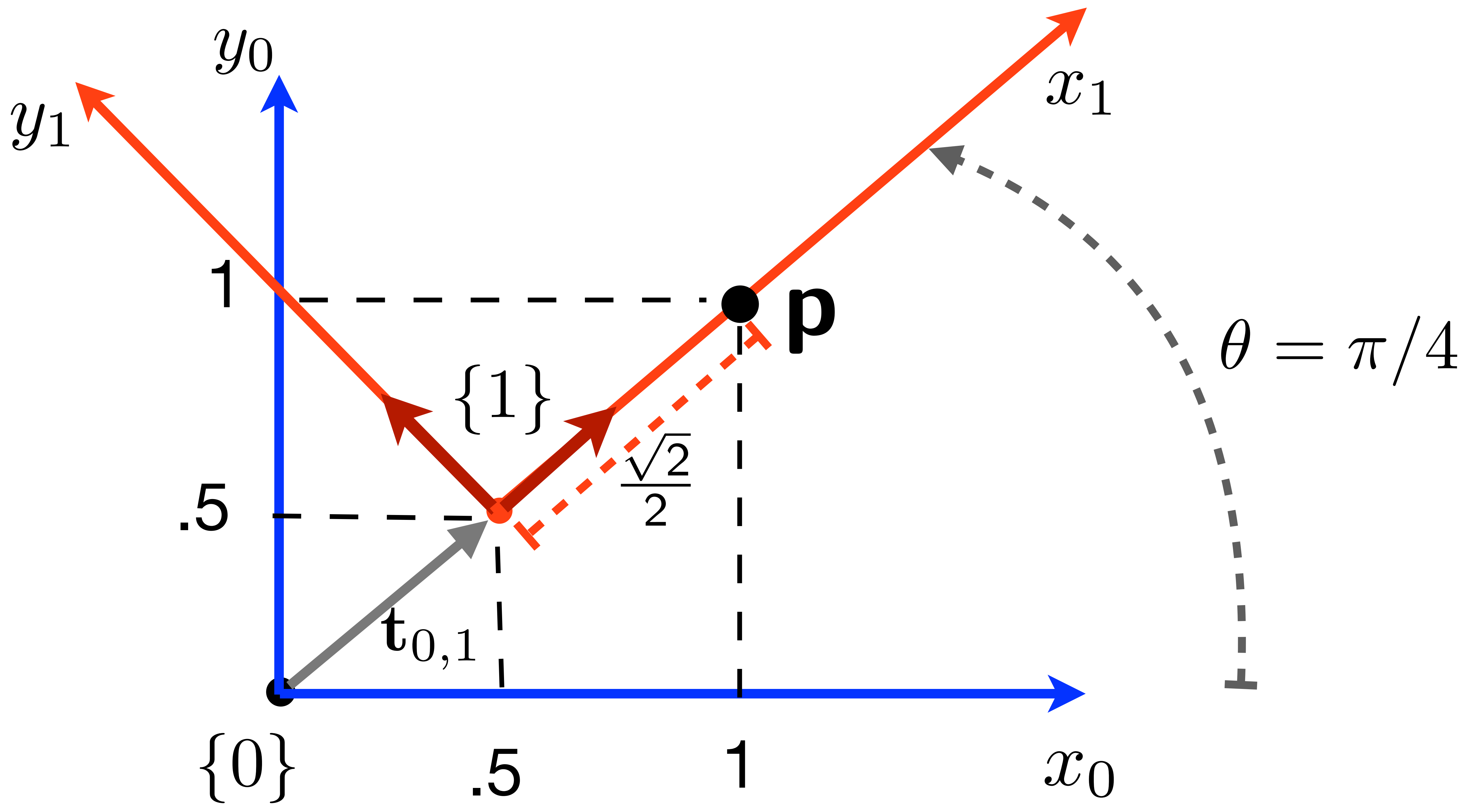
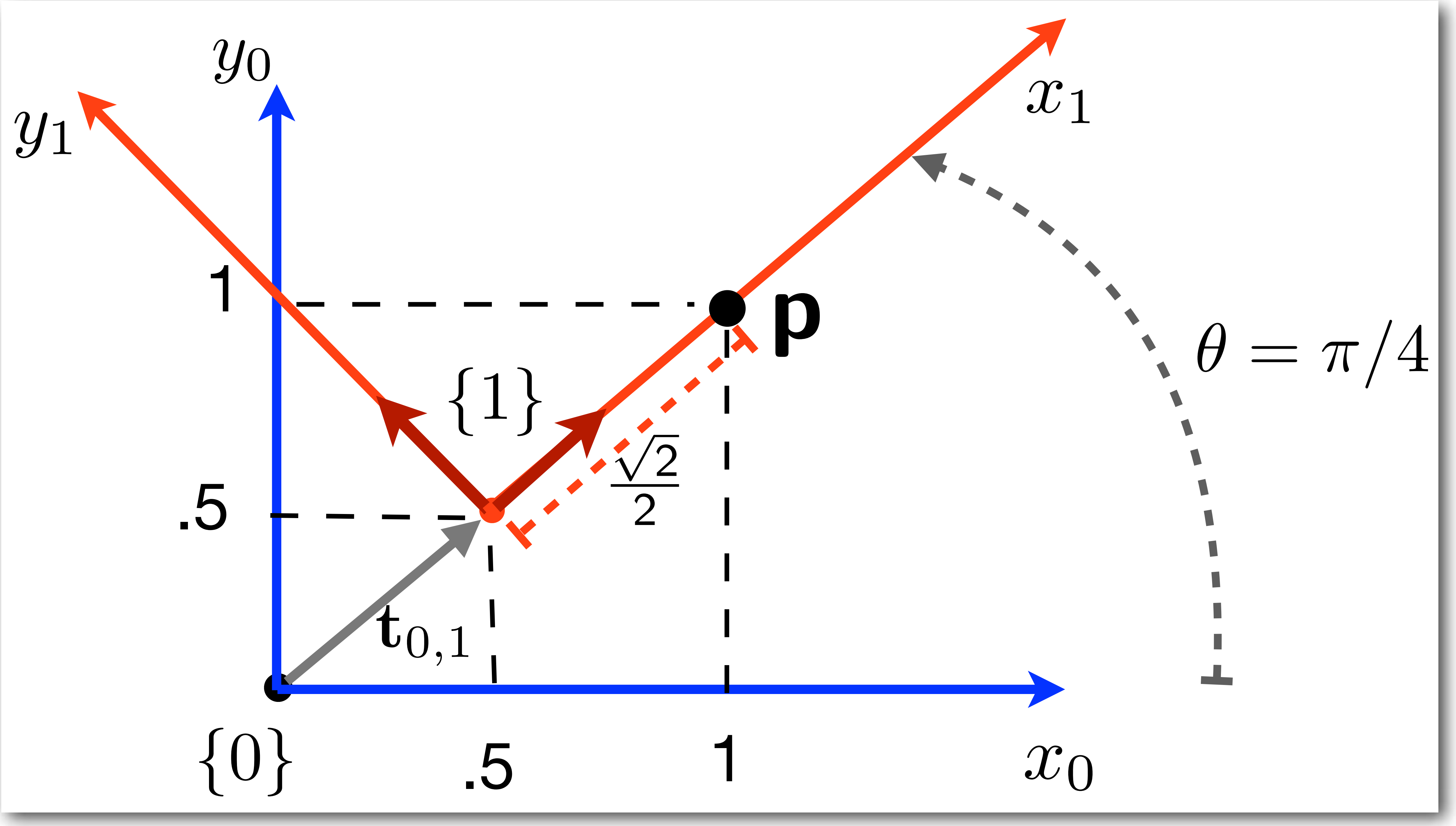


Figure 6: Change of coordinate frames of a point in 2-D.



To convert from $\mathbf{p}_1$ to $\mathbf{p}_0$,

$$\mathbf{p}_0 = T_{0,1}\,\mathbf{p}_1 = \begin{bmatrix} R_{0,1} & \mathbf{t}_{0,1} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 1/2 \\ \sqrt{2}/2 & \sqrt{2}/2 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

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