Generations in the Bounded Confidence Model

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ABSTRACT
I investigate the evolution and stability of an agent based model for opinions in a group of agents with a finite lifetime.

I answer the following questions: What conditions must be fulfilled to enable a significant change in the consensus of an opinion group? What is the typical timescale before such a change occurs?

Key words: Bounded Confidence Model: Opinion Groups, Evolution – Methods: Numerical

1 MOTIVATION

The bounded confidence model describes the way an ensemble of agents converges on a common opinion, or several final opinions.

An effect that cannot be described with this model is long-time change of opinions, as is e.g. seen in human societies over several generations. A prominent example would be the support for governmental control in the U.S., as e.g. studied by (Coggins+ 2013)[1].

I want to model the evolution of opinions with time, when new agents without bias appear in the system, by adding one additional internal parameter for an agent: its age.

I propose that as old agents disappear and are replaced by young, unbiased agents, noise is introduced and a shift in the overall opinion occurs. The conditions allowing the opinion of a group to change significantly are investigated.

2 MODEL AND RULES

We model agents $i \in [1, N]$ with two internal parameters: an opinion o_i and an age a_i.

Initial conditions are set such that the opinions are uniformly distributed. The creation times of the agents are also unfiormly distributed over the span $[-a_{\max}, 0]$, where $a_{\max}$ is the maximum lifetime of an agent.

We introduce a distance d between two agents measured with a metric

$$d(o_i, o_j, x_i, x_j) = \begin{cases} |o_i - o_j|/\varepsilon, & \text{opinionspace}, \\ |a_i - a_j|/\varepsilon, & \text{temporalspace}, \end{cases} \quad (1)$$

with opinions o_i, age a_i, and fudge parameters α, β. They determine which part of the social distance is considered: Only difference in opinions, only difference in age, or both. For sake of simplicity, I restrict myself to distance in the opinion space, $\alpha = 1, \beta = 0$.

In each timestep, two random agents meet. They interact with interaction strength

$$\Sigma = \frac{1}{1 + \exp\left(\frac{d-\varepsilon}{s\varepsilon}\right)} \quad (2)$$

such that for a distance d smaller than scale ε the interaction reaches maximum strength 1, and falls off to 0 at ε over a distance s.

The interaction is based on the bounded confidence model

$$\begin{aligned} o_i(t+1) &= o_i(t) + \zeta \cdot (o_j(t) - o_i(t)) \quad (3) \\ o_j(t+1) &= o_j(t) + \zeta \cdot (o_i(t) - o_j(t)) \quad (4) \end{aligned}$$

For $\zeta = 0.5$, the agents adjust their opinion to the average of both opinions.

I define groups to be sets of agents with a common opinion, such that

(i) they contain at least two members or 10 percent of the fraction that is theoretically expected to be contained in the maximal influence region $1/(2\varepsilon)$;

(ii) their agents are closer to each other than to any nonmember. This is ensured by a hierarchical clustering algorithm with cut at $\varepsilon/2$. A classical k-means algorithm does not work, as one needs a-priori knowledge about the number of groups one wants to find. This number is not constant in the dynamic evolution we are looking at.

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[1] under review, `www.unc.edu/~mlatkins/docs/OpinionChange_CSAB.pdf`

3 ANALYTIC TREATMENT

3.1 Decay of prevalent consensus

Let us first determine the decay of a prevalent consensus. We start with the following setup: All N agents of a population have converged on a common consensus c, and thus their opinion distribution is given by

$$D_{\rm old}(o) = \delta(o - c) \tag{5}$$

as a function of opinion o, consensus opinion c, and delta function δ, such that the distribution is normalized to

$$\int_{-\infty}^{\infty} D_{\rm old}(o) do = 1 \tag{6}$$

I will assume a uniform age distribution,

$$A(a) = \frac{1}{a_{\max} - 0} = \frac{1}{a_{\max}} \tag{7}$$

with age $0 < a < a_{\max}$ and normalization

$$\tilde{A} = \int_0^{a_{\max}} A(a) da = \frac{1}{a_{\max}} \cdot (a_{\max} - 0) = 1 \tag{8}$$

Now if time evolves, agents with finite lifetime $a_{\max}$ start to disappear, giving a total number of "dead" agents

$$\tilde{\Delta}(t) = \int_{a_{\max}}^{a_{\max}} = \frac{t}{a_{\max}} \tag{9}$$

and a remaining fraction

$$R(t) = \frac{\tilde{A} - \tilde{\Delta}(t)}{\tilde{A}} = 1 - t/a_{\max} \tag{10}$$

To keep things simple, I assume this subtracted fraction to be replaced by newly created agents. These agents do not have a preset opinion, and thus sample uniformly from the available opinion space.

When new agents are randomly introduced and start to interact with an opinion group, the consensus value will be corrected in the direction of the new agents. I want to determine the timespan required for the consensus to evolve significantly from its initial value. I define this to be the smallest ΔT for which

$$|o(T + \Delta T) - o(T)| \geqslant \mathrm{std}(o(T + \Delta T)) \tag{11}$$

with the standard deviation $\mathrm{std}(o(T+\Delta T))$ of all opinions inside the group. The std is not chosen from the first group, as this generally features a high spread in opinions and would never get changed above that level.

To first order, the summed distribution of (remaining agents with fixed opinion)+(new agents with random opinions in $[0, 1]$) will feel significant influence from the new agents as soon as their integrated opinion is of order of the consensus integral, and thus after half the lifetime, $T = a_{\max}/2$. This neglects any temporal change, which we will include in the next part.

3.2 Steady State

Further analytic treatment can be performed for the case of steady state. An opinion group with $N_i(T)$ agents at time T has a mean opinion $O_i(T)$ which evolves according to Brownian motion, if new agents with opinion o_i appear inside its region of attraction $O_i(T) - \varepsilon \leqslant o_i \leqslant O_i(T) + \varepsilon$. In steady state, it is expected that the number of new agents corresponds to the number of removed agents. In this case, a random walk is initiated with mean steps of size $\varepsilon\zeta/(2N_i)$. This gives an expected distance covered by diffusion after ΔT steps of

$$E(|O_i(T + \Delta T) - O_i(T)|) = \sqrt{\frac{2\Delta T}{\pi} \frac{\varepsilon\zeta}{2N_i}} \tag{12}$$

I assumed that $N_i \approx$ const, which is not necessarily the case as agents die, but a reasonable approximation for steady state.

I expect that as soon as this random walker in 1D evolves a distance $\varepsilon/2$ away from its initial state, its agents begin to interact with the adjacent group, and merge over a characteristic timescale of $T_p = (N_i(T)/N_{\rm tot}) \cdot N_{\rm it.pertimestep} \cdot \zeta$. In physics, this would correspond to a phase transition, in the style of droplet coagulation. The timescale from the first detection of a consensus group up to a significant change in opinion can thus be determined from

$$\varepsilon/2 = E(|O_i(T + \Delta T) - O_i(T)|) = \sqrt{\frac{2\Delta T}{\pi} \frac{\varepsilon}{2N_i}}, \tag{13}$$

$$\Delta T \approx \frac{\pi}{4} \frac{N_i \varepsilon}{\zeta} + T_p \tag{14}$$

For a fiducial model with $\varepsilon = 0.25$, $\zeta = 0.1$, $N_i \approx 50 \cdot 2\varepsilon = 25$ this gives a $\Delta T = 40 + 5 = 45$, where 40 timesteps are needed for group evolution, and 5 timesteps are needed for merging.

4 SIMULATION

4.1 Parameter Sweep

Independent parameters are

(i) the scale of the interaction bracket, $\varepsilon + s$. This determines the region of influence and the number of contemporary opinion groups. The region sampled is $[0.1, 0.2, 0.3, 0.4, 0.5]$. From the classical bounded confidence model, we know that floor$(1/2\varepsilon)$ is the expected number of separate opinion groups.

For values $\varepsilon < 0.1$, we need a very high number of agents to track all possible groups. That comes from the fact that each groups is defined to contain at least 2 agents. For values $\varepsilon > 0.5$, we get the same result as for $\varepsilon = 0.4$, namely that a central group forms onto which all agents converge.

(ii) the noise introduced by replacing agents. If, as done in our first setup, agents die and are replaced by agents with a uniformly distributed opinion, this corresponds to nudgeing a randomly selected fraction of opinions around in the box. The introduced noise is given by

$$\text{Noise} = \frac{\text{number replaced agents}}{\text{number all agents}} = \frac{1}{a_{\max}} \qquad (15)$$

We sweep the range $a_{\max} = [20, 40, 60, 80, 100]$ for simulations up to a maximum time of 300. This range is motivated by human societies, and tracks several evolution tracks.

4.2 Fiducial model

I use a fiducial simulation of the evolution of the opinions for a population of 50 agents to illustrate the process. The agents have a maximum age of 40, the simulation is run for 300 timesteps. The interaction is characterized by $\varepsilon = 0.25$, $\zeta = 0.1$, and 100 interactions per timestep. In figure 1 one can see the formation and classification of opinion groups. The figure on the right shows the distribution of observed timespans ΔT.

As soon as the initial groups have formed, there is a timeline associated with that group. It is only seldomly observed that a group disappears. There is no occurence of a split into two groups forward in time. This is only partly given by the setup of the model with introduction of new agents: New agents are likely to be offset from any consensus, but the randomly selected interactions could mean that only part of the group gets diverted by the outsider. Apparently, the interactions between agents inside the group outfactor interactions with outliers significantly.

The vertical line depicts the lifetime of a single agent. The analytically derived approximation for the generation duration lies close-by, at 45. I get time-spans of the same order of magnitude, with the split stemming from discreteness effects: If by chance a group is detected to be disrupted as the number of agents drops below the critical group detection limit, the maximum timescale is shortened. With only two groups over a total of 8 agent lifetimes, this does not give enough statistics to reject the analytic hypothesis derived under the assumption of steady state.

4.3 Array of Simulations

I show representations from the parameter sweep: in figures 2 and 3 the effect of changing the maximum lifespan, and in figures 4 and 5 the effect of changing ε.

Reducing the lifetime to $a_{\max} = 20$ introduces many new shortlived groups. This comes mainly from the fact that a shorter lifetime allows more wiggling in the mean opinion of a group. This in turn comes from a) less old agents to be changed, as they quickly disappear, and b) a higher frequency of new agents, away from the consensus. So many in fact, that the group finder algorithm is identifying clusters of agents away from a consensus. These quickly evolve back to the nearest group, and thus introduce the peak at small ΔT. Two opinion groups still evolve for a long time (90 timesteps), but even that one is shorter compared to the fiducial model, owing to the higher variability of the mean.

Enhancing the lifetime to $a_{\max} = 80$, on the other hand, keeps the outliers relatively stable, and shrinks the spread in each group to smaller values. This then leads to a plethora of consensus changes as defined above, as a difference of two standard deviations is reached readily when the standard deviations are small.

For small $\varepsilon = 0.1$, one observes more interruptions in the group classification step, originating from the discreteness effects as the 50 agents used in the model cannot fully track all possible groups. This means shorter timescales overall, once from the fact that the old groups disappear and restart, and secondly from the fact that there are more groups forming in the regions between the old groups. These groups are highly susceptible to the influence from their neighbors, and thus quickly merge.

The other extreme with high $\varepsilon = 0.4$ shows more stable behavior, as almost all agents immediately converge to the central group, and thus a) keep the mean opinion stable by their high numbers b) dissolve secondary groups before they become real competitors. The resulting distribution shows a very high peak at around 3 times the lifetime, and renders this parameter combination the most stable configuration under study. Short-term fluctuations happen at a little under the lifetime, and are comparable to what we previously analyzed with pen and paper.

5 DISCUSSION

I include an additional internal parameter in the bounded confidence model: the age of an agent.

I confirm the hypothesis that opinions evolve significantly away from the previous consensus value over time if agents are replaced by new agents with random opinions.

The timescale of significant change is found to be comparable to the lifetime of a single agent.

I derive following consequences for a highly connected society (where distance in Euclidean space is not important):

(i) Opinions will not stay in a previously found consensus if new people with random opinions appear (either by birth or exchange of people by means of transport).

(ii) there is only small resilience against quick changes: If people do not discuss with people of moderately different opinions (low ε) there is the possibility to perform continuous small phase space jumps. If the people are long-lived, they will suppress groups of differently-thinking people.

(iii) Big changes appear to happen on the timescales of the order of a lifespan of a single agent. This does **not** originate from the fact that old people die all at once. It comes from the fact that after a series of consistent opinion nudges, the influence from a group of another group with strongly differing opinion is visible.

Further investigations should be carried out to include

(i) spatial distances in the metric to account for the fact that consensus does not have to be found globally, but only locally;

(ii) stubborn agents, i.e. agents that tend to relax back to their initial opinion.

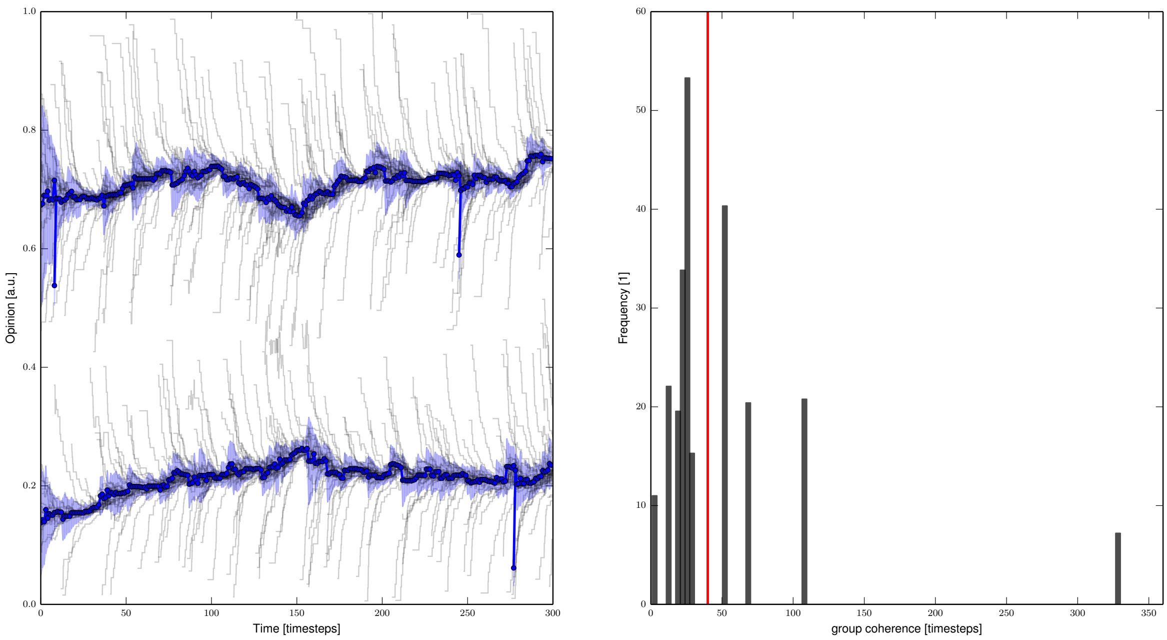

Figure 1. Visualization of the fiducial simulation. Gray timelines show the opinion evolution of single agents. Blue lines indicate groups, with the standard deviation of the contained agents as blue shaded area. See text for definition. Right: ΔT distribution for the groups identified in the fiducial simulation fig. 1. The vertical line shows the lifetime of a single agent.

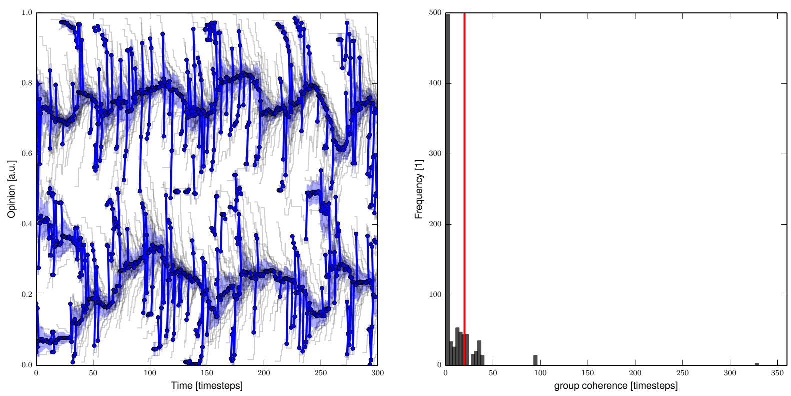

Figure 2. $T_{\max} = 20$

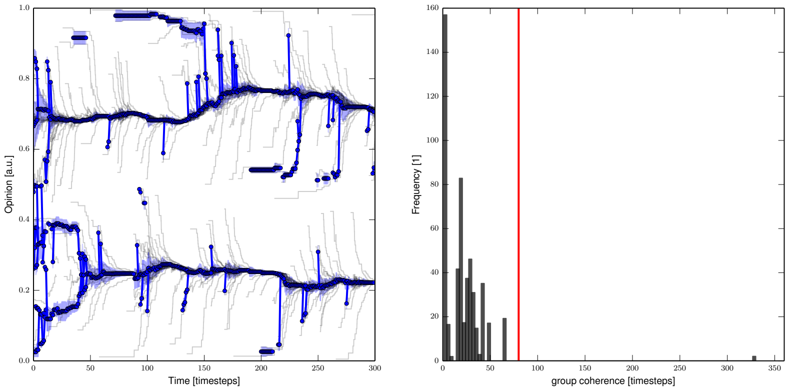

Figure 3. $T_{\max} = 80$

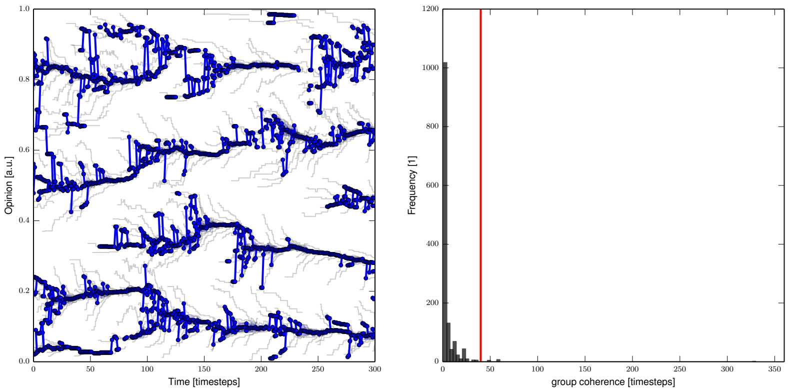

Figure 4. $\varepsilon = 0.1$

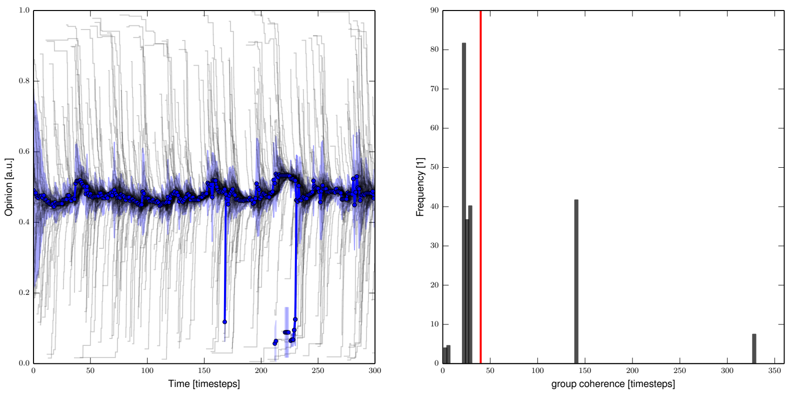

Figure 5. $\varepsilon = 0.4$