Probing Dark Matter in Dwarf Galaxies with Non-Parametric Mass Models

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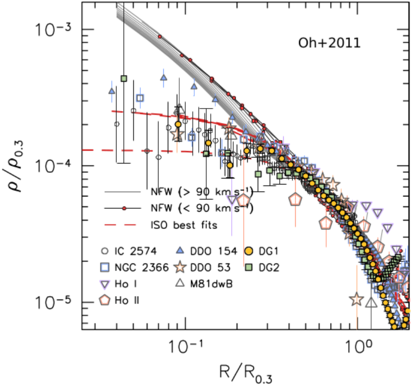
Why Yet Another Mass Modelling Scheme?

Cusp/Core Problem:



- SIDM? Baryons? Others?
- If baryons responsible for a cusp-core transformation, expect core to shrink with decreasing M_*

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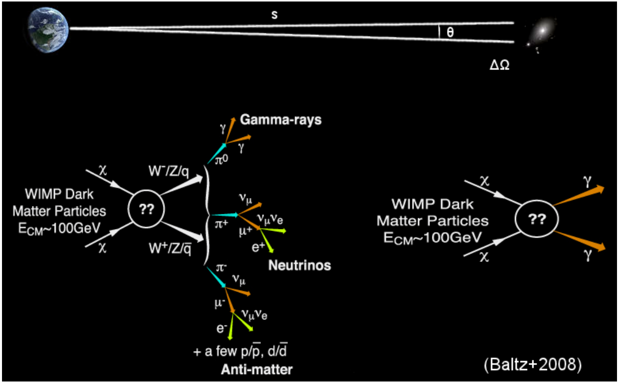


$$\frac{\mathrm{d}\Phi(\Delta\Omega, E_\gamma)}{\mathrm{d}E_\gamma} = \frac{1}{8\pi}\frac{\langle\sigma v\rangle}{m_{DM}^2}\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma} \times J(\Delta\Omega)\Delta\Omega$$

$$J(\Delta\Omega) = \frac{1}{\Delta\Omega}\int_{\Delta\Omega}\mathrm{d}\Omega\int_{\mathrm{LOS}}\mathrm{d}s\,\rho^2(r(s))$$

Why Yet Another Mass Modelling Scheme?

Study	Method	Central Slope
Battaglia 2008	pop splitting	core (low sign.)
Walker, Penarrubia 2011	pop splitting	core (high sign.)
Amorisco, Evans 2012	pop spl.; dist. func	core (high sign.)
Breddels et al. 2011	Schwarzschild	cannot tell
Richardson, Fairbairn 2014	new virial method	cusp
Strigari 2014	pop spl.; dist. func	cusp

Table : Observations: Cusps or Cores?

New Modelling Technique

Our goals:

1. assume non-parametric form for density $\rho(r)$
2. use multiple stellar tracer populations split by e.g. Fe/H

Jeans Modelling

- Stars obey collisionless Boltzmann equation

$$0 = \cancel{\frac{\partial f}{\partial t}} + \vec{\nabla}_{\vec{x}} f \cdot \vec{v} - \vec{\nabla}_{\vec{v}} f \cdot \vec{\nabla}_{\vec{x}} \Phi$$

- assuming spherical symmetry and steady-state: Jeans eq.

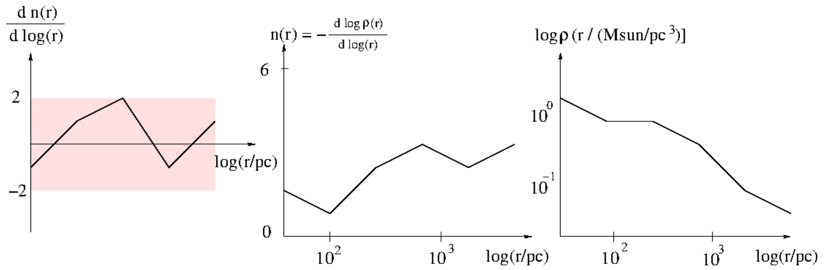
$$\frac{\mathrm{d}(\nu\sigma_r^2)}{\mathrm{d}r} + \frac{2\beta(r)}{r}\nu\sigma_r^2 + \nu\frac{GM(<r)}{r^2} = 0; \qquad \beta(r) = 1 - \frac{\sigma_t^2(r)}{\sigma_r^2(r)}$$

- degeneracy between mass $M(<r)$ and velocity anisotropy $\beta(r)$
- attempted break by using multiple tracer populations with $\beta_1 \neq \beta_2$, but inside the same $M(r)$ (Walker,Penarrubia 2011)

Non-Parametric Jeans Modelling

$$\rho(r) = Ar^{-n(r)}, \qquad n(r) = -\frac{\mathrm{d}\log\rho(r)}{\mathrm{d}\log r}$$

$$\rho(r) = \rho_{1/2}\cdot\exp\left[-\int_{\log r_{1/2}}^{\log r} n(s)\mathrm{d}s\right]$$



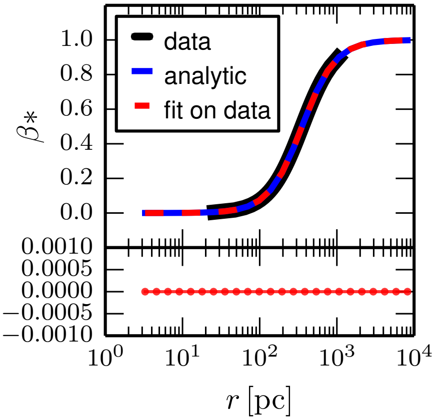
Non-Parametric Jeans Modelling

$$-\infty < \beta(r) \leq 1 \quad \rightarrow \quad -1 \leq \beta^*(r) \leq 1$$

$$\beta^* = \frac{\sigma_r^2 - \sigma_t^2}{\sigma_r^2 + \sigma_t^2} = \frac{\beta}{2-\beta},$$

$$\beta^*(r) = \frac{a_0 - a_\infty}{1 + \kappa \exp(\alpha \ln(r/r_s))} + a_\infty$$

$$\kappa = \frac{a_0 - a_\infty}{\beta^*(r_s) - a_\infty} - 1$$



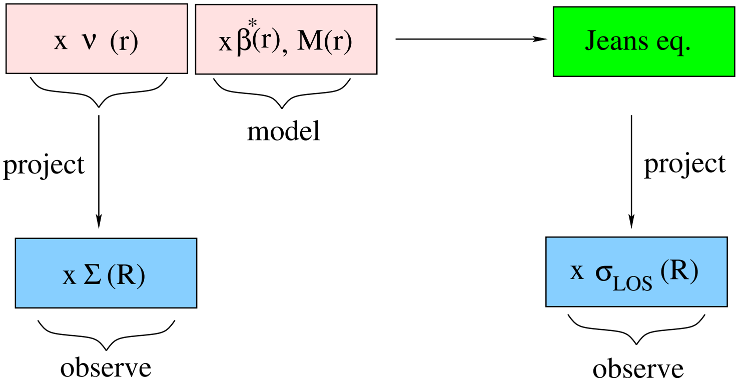
Non-Parametric Jeans Modelling

Numbers of Parameters for x populations:

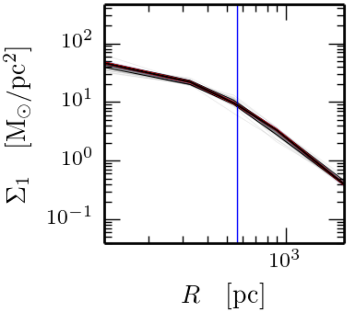
$$\begin{array}{lcl} \beta^* & \rightarrow & 4 \cdot x \\ \nu & \rightarrow & (3 + 3 + N_{\text{bin}} + 3) \cdot x \\ M(r) & \rightarrow & 3 + 3 + N_{\text{bin}} + 3 \end{array}$$

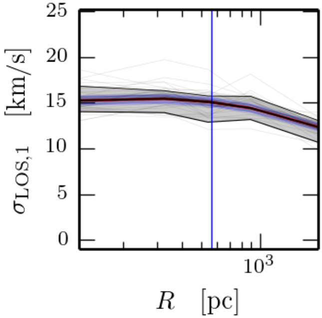
Example: $N_{\text{bin}} = 12, x = 2$ gives 71 parameters.
Need to sample the whole parameter space $\rightarrow$ MultiNest (Feroz+2009, Feroz+2013)

General Procedure



Goodness of Fit



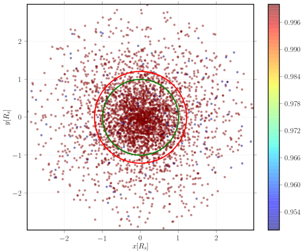


$$\chi^2 = \sum_{i=1}^{N} \chi^2_{\Sigma,i} + \chi^2_{\sigma,i}, \quad (1)$$

$$\chi^2_{\Sigma,i} = \sum_{j=1}^{N_{\mathrm{bin}}} \left(\frac{\Sigma_{\mathrm{data},i}(r_j) - \Sigma_{\mathrm{model},i}(r_j)}{\varepsilon_\Sigma(r_j)} \right)^2 \quad (2)$$

Mock Data

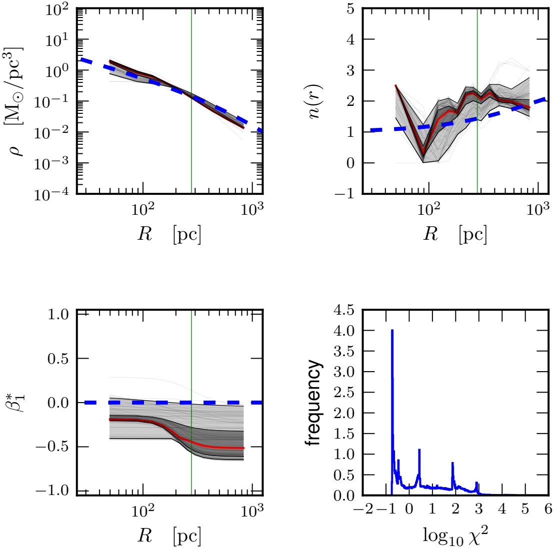
$$
\begin{aligned}
\rho_{\mathrm{DM}} &= \rho_0 \left(\frac{r}{r_{\mathrm{DM}}}\right)^{-\gamma_{\mathrm{DM}}} \left[1 + \left(\frac{r}{r_{\mathrm{DM}}}\right)^{\alpha_{\mathrm{DM}}}\right]^{(\gamma_{\mathrm{DM}}-\beta_{\mathrm{DM}})/\alpha_{\mathrm{DM}}} \\
\nu_*(r) &= \nu_0 \left(\frac{r}{r_*}\right)^{-\gamma_*} \left[1 + \left(\frac{r}{r_*}\right)^{\alpha_*}\right]^{(\gamma_*-\beta_*)/\alpha} \\
\beta(r) &= 1 - \frac{\sigma_\theta^2}{\sigma_r^2} = \frac{r^2}{r^2 + r_a^2}.
\end{aligned}
$$



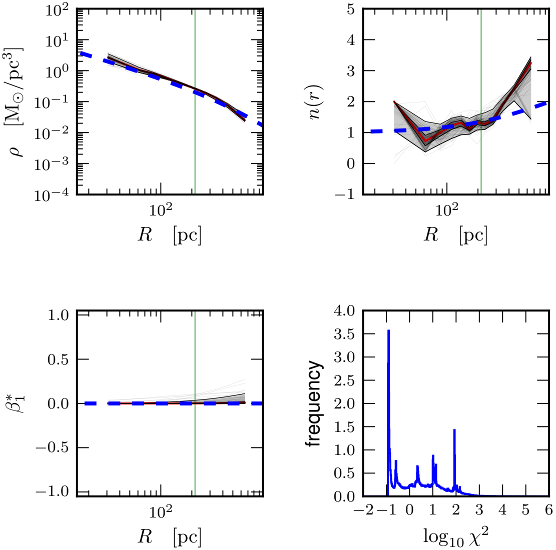
c.f. Walker&Penarrubia 2011;
Gaia Challenge:

http://astrowiki.ph.surrey.ac.uk/dokuwiki/doku.php?id=tests:sphtri

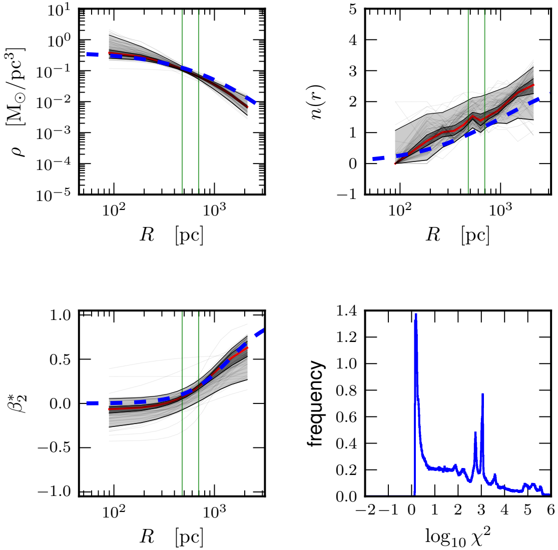
Cusp, 1 Population, No β^* Priors



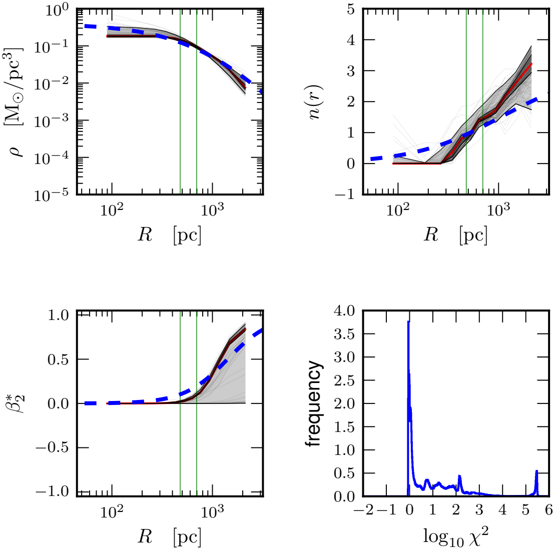
Cusp, 1 Population, β^* Priors



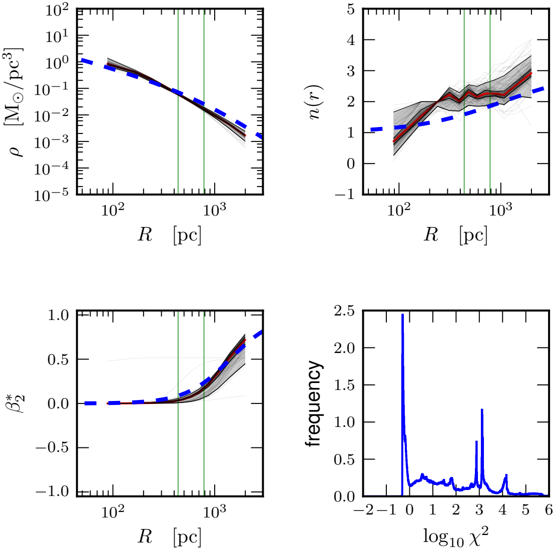
Core, 2 Populations, No β^* Priors



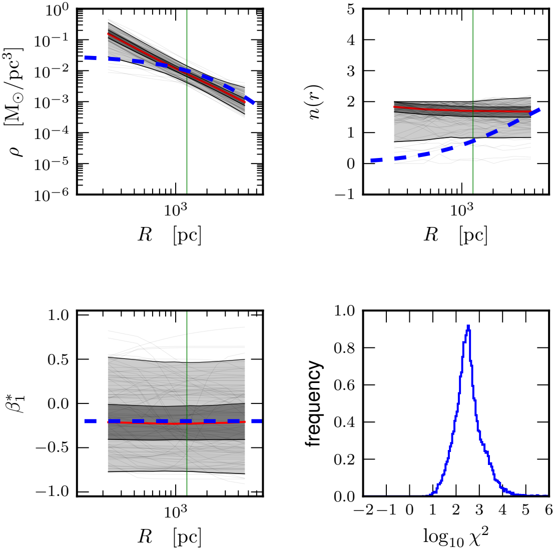
Core, 2 Populations, β^* Priors



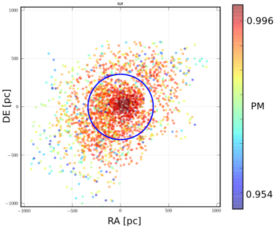
Cusp, 2 Populations, $\beta^* \geq 0$ Prior



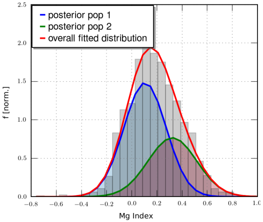
Tangential, 1 Population, No β^* Priors



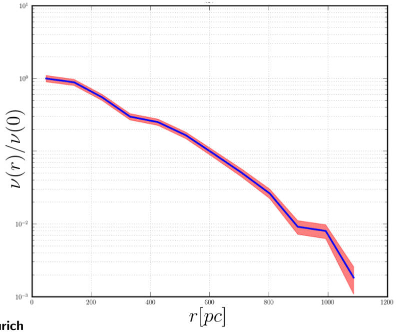
Fornax



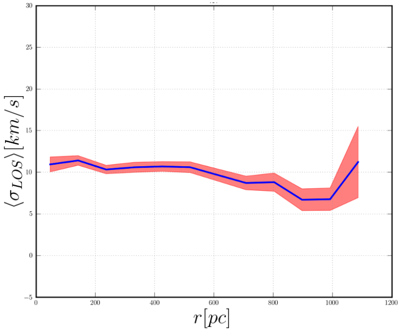
RA [pc]
DE [pc]
PM
0.996
0.954



posterior pop 1
posterior pop 2
overall fitted distribution
f [norm.]
Mg Index

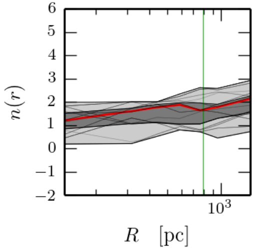


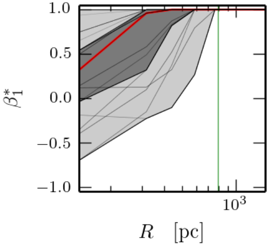
$\nu(r)/\nu(0)$
$r[pc]$

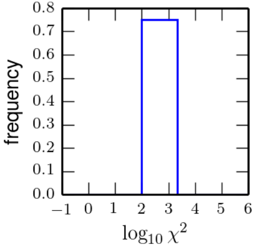


$\langle\sigma_{LOS}\rangle[km/s]$
$r[pc]$

Fornax 1 Pop, $n(r)$ Priors - Preliminary







Summary

- New Modelling Technique with Minimal Assumptions
- Multiple Populations $\rightarrow$ break $M(r), \beta(r)$ degeneracy
- Need 2 pop or priors on $\beta^*(r)$ or $n(r)$
- Cusps and cores get reproduced
- Analysis of observed dSph ongoing

Computation of Velocity Dispersions

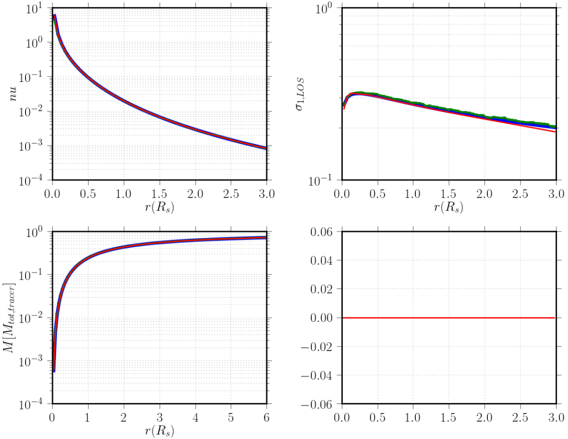
$$\sigma_r^2(R) = \frac{1}{\nu(R)} B(R) \int_R^\infty \frac{GM(r)\nu\sigma_r^2}{r^2} \tilde{B}(R) \mathrm{d}r,$$

$$B(R) = \exp\left[-2\int_{r_{\min}}^R \frac{\beta(s)}{s}\mathrm{d}s\right], \quad \tilde{B}(R) = \exp\left[2\int_{r_{\min}}^r \frac{\beta(s)}{s}\mathrm{d}s\right]$$

$$\sigma_{\mathrm{LOS}}^2(R) = \frac{2}{\Sigma(R)} \int_R^\infty \left[1 - \beta\frac{R^2}{r^2}\right] \frac{\nu(r)\sigma_r^2(r) r}{\sqrt{r^2 - R^2}} \mathrm{d}r$$

Hernquist Model - Integration Check

- check of the integration routine
- using polynomial interpolation
- good fit of σ_{LOS} even at high radii



$n(r < r_{1/2}) < 2$ Prior