A Task Allocation Problem Using Genetic Algorithm Method under A Multi-fighter and Multi-bomb Situation

One-strike and One-bomb situation

The utility function using Gaussian distributions

Suppose that two fighters are going to conduct an strike mission. Two types of bombs would be used in this mission. Denote the fighters by $\mathbf{F}=\{f_1,f_2\}$. Denote the bombs by $\mathbf{B}=\{b_1,b_2\}$. Each bomb has the ability to damage the target, as denoted by $\mathbf{D_B}=\{d_{b1},d_{b2}\}$. In this case, the figher f_1 will carry the bomb b_1, and the figher f_2 will carry the bomb b_1 and b_2. Denote the asset by $\mathbf{A}=\{a_1,a_2,a_3\}$. The assets are listed below.

$$\begin{aligned} a_1 &= (f_1,b_1)\\ a_2 &= (f_2,b_1)\\ a_3 &= (f_2,b_2) \end{aligned} \tag{1}$$

Thus, The damage ability of the asstet $\mathbf{A}$ is denoted by $\mathbf{D_A}=\{d_{a1},d_{a2},d_{a3}\}$.

Suppose that two targets are going to be bombed by the fighters $\mathbf{F}$. Denote the targets by $\mathbf{T}=\{t_1,t_2\}$. The target has two properties, the damage point and the value. If the damage that the target received from the bombs were larger than the damage point of this target, the target would be destroied, and the value would be gained by the attacker, which represents the side that commmands the fighters. Denote the damage point by $\mathbf{DP}=\{dp_{t1},dp_{t2}\}$. Denote the values by $\mathbf{V}=\{v_{t1},v_{t2}\}$. The properties of the targets are listed below.

$$\begin{aligned} t_1 &= (dp_{t1},v_{t1})\\ t_2 &= (dp_{t2},v_{t2}) \end{aligned} \tag{2}$$

Denote the strike mission by $\mathbf{M}=\{m_{a1},m_{a2},m_{a3}\}$. In this scenario, the mission would be listed below.

$$\begin{aligned} m_{a_1} &= [m_{a_1,t_1},m_{a_1,t_2}]^T\\ m_{a_2} &= [m_{a_2,t_1},m_{a_2,t_2}]^T\\ m_{a_3} &= [m_{a_3,t_1},m_{a_3,t_2}]^T \end{aligned} \tag{3}$$

In which, $m_{a,t}\in\{0,1\};a\in\mathbf{A},t\in\mathbf{T}$.

In this chapter, we propose a situation that the bomb carrited by each fighter could only strike one target. Thus, the constraint is listed below.

$$\begin{aligned} (m_{a_1,t_1}+m_{a_1,t_2}) &\in \{0,1\}\\ (m_{a_2,t_1}+m_{a_2,t_2}) &\in \{0,1\}\\ (m_{a_3,t_1}+m_{a_3,t_2}) &\in \{0,1\} \end{aligned} \tag{4}$$

Rewrite the strike mission into matrix form, as listed below.

$$\mathbf{M}=[m_{a1},m_{a2},m_{a3}]=\begin{bmatrix} m_{a_1,t_1} & m_{a_2,t_1} & m_{a_3,t_1}\\ m_{a_1,t_2} & m_{a_2,t_2} & m_{a_3,t_2} \end{bmatrix} \tag{5}$$

In order to represent the relation between the missiton and the damage, we propose a damage matrix $\mathbf{D}$, as shown below.

$$\mathbf{D}=\mathbf{Md} \tag{6}$$

In which, $\mathbf{d}$ represents the damage of the asset.

$$\mathbf{d} = [d_{a1}, d_{a2}, d_{a3}]^T = [d_{b1}, d_{b1}, d_{b2}]^T \tag{7}$$

Thus, the damage matrix $\mathbf{D}$ is liseted below.

$$\mathbf{D} = \begin{bmatrix} D_{t1} \\ D_{t2} \end{bmatrix} = \begin{bmatrix} m_{a_1,t_1} & m_{a_2,t_1} & m_{a_3,t_1} \\ m_{a_1,t_2} & m_{a_2,t_2} & m_{a_3,t_2} \end{bmatrix} \begin{bmatrix} d_{a1} \\ d_{a2} \\ d_{a3} \end{bmatrix} \tag{8}$$

In which, D_{t1} and D_{t2} represent the damage recieved by the target t_1 and t_2, respectively. They are listed below.

$$D_t = \sum_{a \in \mathbf{A}} m_{a,t} d_a;\ t \in \mathbf{T} \tag{9}$$

In order to allocate the assets, which are bombs, we proposed an optimization problem, as shown below.

$$\begin{aligned} max_{\mathbf{M}} \quad & u(\mathbf{M}) \\ s.t. \quad & m_{a,t} \in \{0, 1\}; a \in \mathbf{A}, t \in \mathbf{T} \\ & 0 \leq \sum_{t \in \mathbf{T}} m_{a,t} \leq 1; a \in \mathbf{A} \end{aligned} \tag{10}$$

In which, u represents the objective function of the optimization problem. In this objective function, we introduce the Gaussian function. The objective function u is listed below.

$$u(\mathbf{M}) = p_{t1} + p_{t2} \tag{11}$$

In which, p_{t1} and p_{t2} represent the profit of striking target t_1 and t_2, respectively. They are listed below.

$$p_t = v_t Gauss(x = D_t, \mu = dp_t, \sigma = \sigma_t) = \frac{v_t}{\sqrt{2\pi}\sigma_t} e^{-\frac{(D_t - dp_t)^2}{2\sigma_t^2}} = \frac{v_t}{\sqrt{2\pi}\sigma_t} exp(-\frac{(D_t - dp_t)^2}{2\sigma_t^2}); t \in \mathbf{T} \tag{12}$$

Thus, the objective function $u(\mathbf{M})$ is liseted below.

$$\begin{aligned} u(\mathbf{M}) &= \frac{v_{t1}}{\sqrt{2\pi}\sigma_{t1}} exp(-\frac{(\sum_{a \in \mathbf{A}} m_{a,t1} d_a - dp_{t1})^2}{2\sigma_{t1}^2}) + \frac{v_{t2}}{\sqrt{2\pi}\sigma_{t2}} exp(-\frac{(\sum_{a \in \mathbf{A}} m_{a,t2} d_a - dp_{t2})^2}{2\sigma_{t2}^2}) \\ &= \sum_{t \in \mathbf{T}} \frac{v_t}{\sqrt{2\pi}\sigma_t} exp(-\frac{(\sum_{a \in \mathbf{A}} m_{a,t} d_a - dp_t)^2}{2\sigma_t^2}) \end{aligned} \tag{13}$$

Divide the constraints into two parts. The first part represents that any type of the bombs has a minimum number and a maximum number. The number of the bombs that will be used should lies in this range. They are listed below.

$$\begin{aligned} 0 \leq (m_{a_1,t_1} + m_{a_1,t_2}) \leq 1 \\ 0 \leq (m_{a_2,t_1} + m_{a_2,t_2}) \leq 1 \\ 0 \leq (m_{a_3,t_1} + m_{a_3,t_2}) \leq 1 \end{aligned} \tag{14}$$

Rewrite the first part of the constrains into another form.

$$\begin{aligned} m_{a_1,t_1} + m_{a_1,t_2} - 1 \leq 0 \\ -m_{a_1,t_1} - m_{a_1,t_2} \leq 0 \\ m_{a_2,t_1} + m_{a_2,t_2} - 1 \leq 0 \\ -m_{a_2,t_1} - m_{a_2,t_2} \leq 0 \\ m_{a_3,t_1} + m_{a_3,t_2} - 1 \leq 0 \\ -m_{a_3,t_1} - m_{a_3,t_2} \leq 0 \end{aligned} \tag{15}$$

The second part represents the constraint of each assest. They are listed below.

$$
\begin{aligned}
0 \le m_{a_1,t_1} \le 1 \\
0 \le m_{a_2,t_1} \le 1 \\
0 \le m_{a_3,t_1} \le 1 \\
0 \le m_{a_1,t_2} \le 1 \\
0 \le m_{a_2,t_2} \le 1 \\
0 \le m_{a_3,t_2} \le 1
\end{aligned} \tag{16}
$$

The utility function using Half-Gaussian distributions

Repeat that we proposed an optimization problem, as shown below.

$$
\begin{aligned}
max_{\mathbf{M}} \quad & u(\mathbf{M}) \\
s.t. \quad & m_{a,t} \in \{0,1\}; a \in \mathbf{A}, t \in \mathbf{T} \\
& 0 \le \sum_{t \in \mathbf{T}} m_{a,t} \le 1; a \in \mathbf{A}
\end{aligned} \tag{17}
$$

In this chapter, we use the Gaussian distribution and a linear function. The objective function u is listed below.

$$
u(\mathbf{M}) = p_{t1} + p_{t2} \tag{18}
$$

In which, p_{t1} and p_{t2} represent the profit of striking target t_1 and t_2, respectively. They are listed below.

$$
p_t = \begin{cases} v_t Gauss(x = D_t, \mu = dp_t, \sigma = \sigma_t), & x \le dp_t \\ v_t Gauss(x = dp_t, \mu = dp_t, \sigma = \sigma_t), & x > dp_t \end{cases} \tag{19}
$$

One-strike and Multi-bomb situation

In this chapter, we propose a situation that the number of the bombs may lager than 1. Thus, more than one bomb carrited by each fighter could be allocated to one target.. Due to the situation that the size of each bomb carried by the fighter f_1 or f_2 is fixed, the maximum bombs that a figher would use is fixed and is euqal to the size of the bomb.