Article

Transformation of A Point between Two Coordinate Frames

Pascal Xie [1]

[1] China

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Abstract: Represent a point in two coordinate frames.

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1. Introduction

Transformation in 3D space is a fundamental tool that is used by Simultaneous Localization and Mapping (SLAM) in robotics Barfoot (2017). The transformation considers the rotation and translation of a robot. It describes a robot's movement in 3D space. A combination of rotation and translation of the robot refers to a pose. Identifying the pose is what the localization in SLAM does.

Objects that dwell in 3D space and are observed by a robot are used to localize the robot. The object is stable and not moving during observation. After observing an object, points that could shape the object are stored in the memory of the robot. A bunch of points that have been stored by the robot are denoted by a point cloud. Point-cloud alignment is a typical problem of localization. Iterative closest point (ICP) algorithm is a popular method to solve the problem and estimate the pose of the robot.

2. Transformation between Two Frames

Suppose a vehicle that works in a 3D space. We build a coordinate frame that follows the vehicle, which means that there is no transformation between the frame and the vehicle. Denote this frame by $\underrightarrow{\boldsymbol{F}_v}$, in which, v refers to the vehicle. The pose of the vehicle is the pose of the frame, $\underrightarrow{\boldsymbol{F}_v}$. In order to describe the pose, we need a reference, which is also a coordinate frame, as denoted by the reference frame, $\underrightarrow{\boldsymbol{F}_i}$. Usually, we denote the first frame at the same time the vehicle is initiated as the reference frame.

Denote a point from the point cloud that the vehicle has by P. A relationship of the point and two coordinate frames ($\underrightarrow{\boldsymbol{F}_v}$ and $\underrightarrow{\boldsymbol{F}_i}$) is shown below,

$$\underrightarrow{r^{pi}} = \underrightarrow{r^{pv}} + \underrightarrow{r^{vi}} \tag{1}$$

in which, $\underrightarrow{r^{pi}}$ represents the vector from the origin of frame $\underrightarrow{\boldsymbol{F}_i}$ to the point P. $\underrightarrow{r^{pv}}$ represents the vector from the origin of frame $\underrightarrow{\boldsymbol{F}_v}$ to the point P. And $\underrightarrow{r^{vi}}$ represents the vector from the origin of frame $\underrightarrow{\boldsymbol{F}_i}$ to the origin of frame $\underrightarrow{\boldsymbol{F}_v}$, i.e. the translation vector of frame $\underrightarrow{\boldsymbol{F}_v}$ with respect to frame $\underrightarrow{\boldsymbol{F}_i}$. These vectors could be represented with respect to frame $\underrightarrow{\boldsymbol{F}_i}$, as follows,

$$r_i^{pi} = r_i^{pv} + r_i^{vi} \tag{2}$$

in which, r_i^{pv} has a property, as shown below.

$$\underset{\rightarrow}{r}^{pv} = \underset{\rightarrow}{\boldsymbol{F}_i}^T r_i^{pv} = \underset{\rightarrow}{\boldsymbol{F}_v}^T r_v^{pv} \tag{3}$$

Thus, we have,

$$r_i^{pv} = \underset{\rightarrow}{\boldsymbol{F}_i} \cdot \underset{\rightarrow}{\boldsymbol{F}_v}^T r_v^{pv} \tag{4}$$

in which, $\underset{\rightarrow}{\boldsymbol{F}_i} \cdot \underset{\rightarrow}{\boldsymbol{F}_v}^T$ refers to a rotation matrix. We denote this rotation matrix by,

$$\underset{\rightarrow}{\boldsymbol{F}_i} \cdot \underset{\rightarrow}{\boldsymbol{F}_v}^T \equiv C_{iv} \tag{5}$$

According to Equation (4) and (5), we can rewrite Equation (2), as shown below.

$$r_i^{pv} = C_{iv} r_v^{pv} + r_i^{vi} \tag{6}$$

The point that is represented in the coordinate frame $\underset{\rightarrow}{\boldsymbol{F}_v}$ is denoted by,

$$r_v^{pv} = C_{iv}^T (r_i^{pi} - r_i^{vi}) \tag{7}$$

Consider that $C_{iv}^T = C_{vi}$, Equation (7) can be represented below.

$$r_v^{pv} = C_{vi} (r_i^{pi} - r_i^{vi}) \tag{8}$$

References

Barfoot, Timothy D. 2017. *State Estimation for Robotics* (1st ed.). USA: Cambridge University Press.