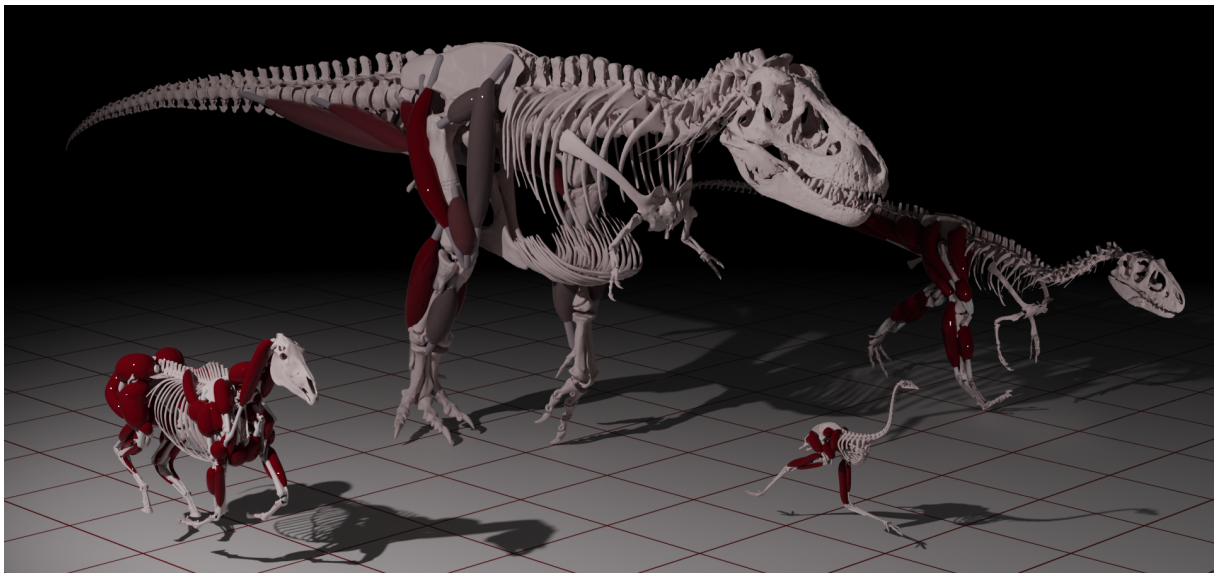
MuSkeMo Manual



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Github Repository: Github
Bug Reports and Feature Requests: Submit an issue on Github

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Preprint: I am preparing a publication to submit for peer-review describing MuSkeMo. Until that is available, please cite the preprint on bioRxiv if you used MuSkeMo for your work.

PA van Bijlert. *MuSkeMo: Open-source software to construct, analyze, and visualize human and animal musculoskeletal models and movements in Blender.* bioRxiv (preprint) 2024.12.10.627828; doi: 10.1101/2024.12.10.627828.

Contents

1 Introduction

MuSkeMo is a tool for musculoskeletal modeling in Blender. MuSkeMo allows you to translate biological 3D scans to useful biomechanical information, in the form of full models or model components. These can then be used for further analysis within Blender, or exported for simulations in other biomechanical software. Simulation trajectories can be imported back into MuSkeMo to make publication-ready stills and videos, using Blender's built-in raytracing rendering. MuSkeMo can also be used for 3D landmarking and for simply calculating inertia tensors from 3D scans.

This MuSkeMo documentation file is meant to complement the video tutorial series on YouTube. This documentation is focused on features specific to MuSkeMo, and where necessary provides suggestions regarding Blender settings and features. Because Blender can be overwhelming at first, new users are recommended to follow the "Donut tutorial" series by BlenderGuru on Youtube. For Blender-specific questions, the reader is referred to the extensive Blender documentation. MuSkeMo currently officially supports Blender 4.0 -4.1, but will offer 4.2+ support in the future.

2 Installation instructions

To download: Go to the releases page on the Github repository, and download the most recent version of "MuSkeMo.zip". The .zip file is used to install the plugin, but also contains a folder with utility functions (e.g., for OpenSim conversion).

To install (Fig. 1): In Blender v4-v4.1, go to Edit → preferences → Add-ons and then click "Install...", and then select your downloaded zip file. Then type in "MuSkeMo" in the search bar of the addon window, and enable the plugin by pressing the check mark. If using Blender 4.2+, you may need to use the legacy installer, but MuSkeMo has currently not been tested above v4.1.

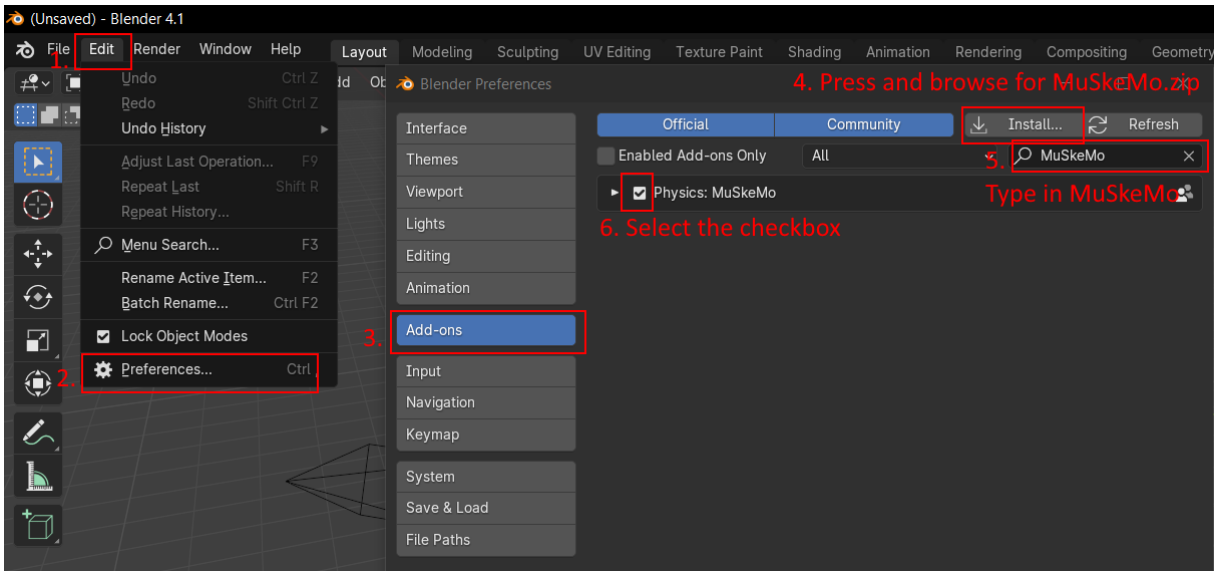

Figure 1: Installing MuSkeMo in Blender versions 4.0-4.1

3 Units

MuSkeMo uses the following units:

- **Spatial distance/position:** metres (m)
- **Mass:** kilogram (kg)
- **Moment of inertia:** kilogram metre squared ($\mathrm{kg\,m^2}$)
- **Force:** Newton (N)

- **Angle:** Degrees (°, only for pennation angle). Internally MuSkeMo uses radians during computations.

WARNING: Blender's default units are in metres. Many of the calculations in MuSkeMo are derived from the spatial position of individual points (vertices) of the 3D meshes. If you change the base units of Blender, these calculations are also implicitly scaled, and the units will no longer be correct (e.g., if you change Blender's base units to centimetres, linear dimensions will be off by a factor of 100, but volumetrics and inertial properties will be off by a factor of 100^3). MuSkeMo currently only officially supports metres as the base unit, correct scaling of the outputs if using other units is up to the user.

4 A Note on Precision

MuSkeMo is built using the Blender Python API, which internally uses double precision numbers (64-bit floating point, $\sim$ 16 significant digits). However, Blender itself stores numbers using single precision (32-bit floating point, $\sim$ 8 significant digits), including all 3D point and mesh data. This means that precision beyond 8 significant digits cannot be expected when using MuSkeMo. Single precision floats nevertheless have a very large range of numbers that can be represented—the smallest number that can be accurately reported is in the order of 1×10^{-38}, approximately 1 million times smaller than the mass of an electron in kilograms.

Because computing inertial properties from 3D vertex data involves successive multiplications of single precision numbers, mass and inertial properties have an expected accuracy of 5-6 digits (see D.1).

5 Orientations

Orientations are represented as XYZ body-fixed Euler angles (see Appendix A) and as unit quaternions (see Appendix B). Internally, MuSkeMo avoids Euler angles where possible because they are prone to gimbal lock, but popular simulators often use Euler angles by default (e.g., OpenSim, SCONE).

6 Using MuSkeMo

MuSkeMo lives in Blender panels (Fig. 2A). Panels can be interactively resized and collapsed, which can be useful depending on your screen size. The panels contain buttons that perform operations on the data (implemented as Blender Operators). When hovering over a button, a tooltip appears that briefly explains the workings of the button in question.

Nearly all functions in MuSkeMo work by selecting the correct objects in the scene, and then pressing a button. If the selected objects are incompatible with the button in question, MuSkeMo will display an error message that explains what went wrong. To make the user aware of what MuSkeMo object (types) are currently selected, each has a live selection display for the relevant object types.

Although Blender does support using a trackpad, it is substantially more intuitive to navigate with a mouse, and a keyboard with a separate numberpad. There are also certain settings that make navigation easier (see Section 6.5).

WARNING: Ensure that 3D meshes (visual geometry and tissue outlines) have all their transformations applied before you start constructing the model. To do this, select the object, press `Control + A`, and select "All transforms". This accounts for the way that Blender stores local transformations. When you move, rotate, and/or scale 3D models in Blender, these transformations are initially stored only locally in the object's data (an extra storage layer in Blender). This means that the object's position has not changed in 3D space, you have defined an extra local transformation that can easily be undone. Some of the functions in MuSkeMo may behave unpredictably unless transformations have first been applied.

6.1 Where MuSkeMo stores model data

All the model components and other user-created objects are stored in Blender Collections (Fig. 2B), which are essentially just folders. The collection names all have sensible defaults in MuSkeMo (e.g., "Bodies" for bodies), but can be changed if desired. The collection names are also used as the default

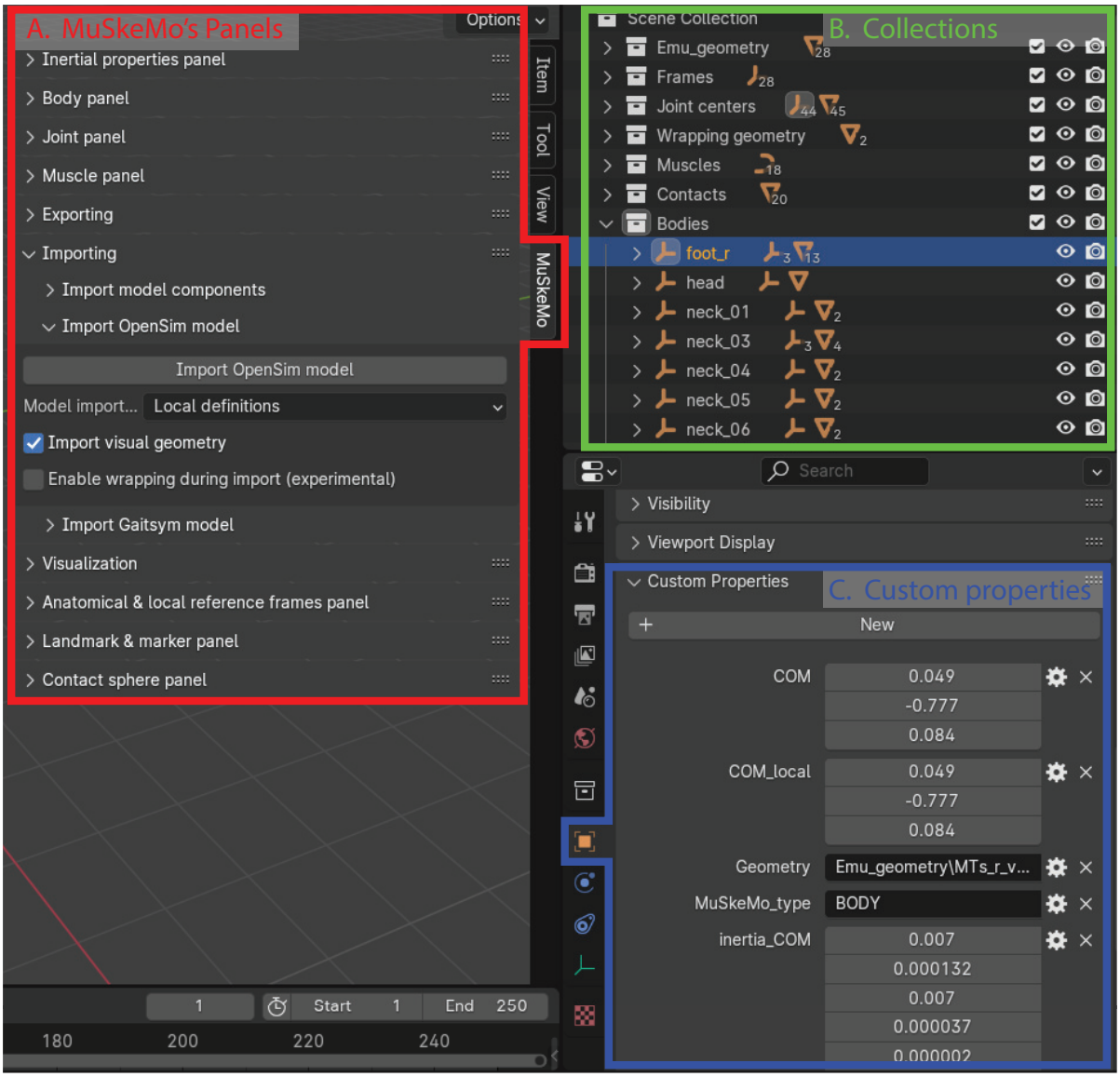

Figure 2: A. All of MuskeMo's functionality can be accessed via these panels. The panels can be interactively resized and collapsed. B. After creating or importing model components, they are stored in Blender collections. If child objects are not visible, turn off object children in the outliner filter (see Section 6.5 C. A component's data are stored as custom properties. Press the yellow square, scroll all the way down, and open the Custom Properties dropdown

filenames during import/export, but this is also customizable. By default, child objects aren't always visible in the collection that they're actually in (see Section 6.5).

For each model component, data created by MuSkeMo are stored in Blender as Custom Properties assigned to each object in question (Fig. 2C, see also Section 7 on MuSkeMo Data Types).

6.2 Model components and modifiers

MuSkeMo creates a new Blender Object for each model component. Blender has many different object types, but MuSkeMo mainly uses "Mesh", "Curve", and "Empty" (see 7 for a full specification of all the used data types). Objects in Blender do not interact with other objects, unless the user somehow defines their inter-relationships. MuSkeMo handles all of this under the hood. For instance, components such as contact spheres need to be parented to specific bodies in the model, and MuSkeMo handles this while also keeping track of these inter-relationships in "Custom properties" so they are exposed to the user (see 6.1).

Combining parenting and Blender's native object types does not provide all of the required functionality. MuSkeMo extensively makes use of Blender's Modifier system for both generating and visualizing components. For instance, individual muscle curve points cannot be attached to bodies using Blender's parenting system, so MuSkeMo achieves this by adding a hook modifier for each curve point (see 7.3).

MuSkeMo uses both regular modifiers (which perform a predefined operation/modification on the object, e.g. the Bevel modifier is used to round the edges of muscle visualizations), but also several custom-made Geometry nodes modifiers.

Geometry Nodes modifiers are a very powerful way perform a customized sequence of operations on a specific object. MuSkeMo uses custom-made geometry nodes modifiers for muscle visualization (7.3.1 and 7.3.2), muscle wrapping 7.3.3, creating parametric wrap geometry 7.9, amongst other features. The most important inputs for most MuSkeMo's custom Geometry nodes are accessible from the modifier panel itself [FIGURE], but advanced Blender users can also dive into the nodes themselves for more in depth modifications to their behavior. Where possible MuSkeMo tries to reuse node groups, which means that it is possible to make global changes to geometry node behavior by changing the parameters of a single node group. For example, the visualization of all the muscles can be changed by modifying the underlying node groups (7.3.1 and 7.3.2).

By default, MuSkeMo color codes each component (see 6.15.3 for details).

6.3 Blender Auto Save

Blender is quite stable, but it is possible for it to crash. Fortunately, by default, Blender has an Auto Save function. After a crash, restart Blender, and go to File, Recover, Auto Save. Remember to save the recovered file.

6.4 Default pose

Constructing a model in Blender provides the user with a lot of freedom to interactively repose model elements during model construction. However, certain computations are only valid in the specific pose they are computed in (e.g., inertial properties in the global frame). To prevent incorrect exports, MuSkeMo keeps track of the 'default_pose', which is the pose in which the relevant computation was performed. If the user tries to perform an operation that can only be performed in the default pose, the operation is cancelled (with an error message). In that situation, you must either recompute the parameters, or reposition the model components back into the default pose.

6.5 MuSkeMo Global Settings Panel

The Blender user interface has several default settings that do not work well with MuSkeMo. By default, Blender forces the Z-axis as the "up" direction in the viewport. The International Society of Biomechanics assumes Y as the up direction. To achieve this, the user should set the view rotation setting to "Trackball", and use "orbit around selection". The "MuSkeMo Global Settings" panel provides a button that does this automatically: "Set recommended Blender settings". This button also toggles off "Object Children" in the outliner. By default, objects are placed under their parents in the outliner, but this would for instance result in a body being placed under the parent joint in the "Joint collection", instead of in the "Body collection". Turning off "Object children" avoids this behavior. However, this is a project file setting, so this needs to be reset with each new project, unlike the navigation settings.

6.6 Inertial properties panel

The main goal of this panel is to compute inertial properties from 3D volumetric meshes, eg. from CT-segmentations or surface scans. Inertial properties are not dynamic, if you move the 3D meshes or would like to change their densities, you must recompute their inertial properties, otherwise COM, mass, and or inertia can be outdated. MuSkeMo warns you if this has occured, by tracking the 'default_pose' of each mesh as a custom property (see 6.4). Density can only be changed by changing the 'Segment density' in the panel and recomputing the object's inertial properties.

To compute the volumetric inertia tensor (with elements in the unit m^5) of a triangular mesh, MuSkeMo implements the solution derived and presented in [5]. This gives an exact solution for the volumetric moments of inertia of a closed, triangulated mesh, based on the Divergence Theorem. The volumetric

tensor is multiplied by density to acquire the inertial tensor elements. This algorithm requires the mesh to be triangulated and watertight to provide meaningful results, and MuSkeMo alerts the user if this is not the case.

It is possible to change the density property of an object, after which you'll have to select the object and rerun "Compute for selected meshes". See D.1 for a validation of the outputs.

6.6.1 Generate minimal convex hulls

Several studies have used convex hulls as the starting point for estimating inertial properties of (extinct) animals directly from full skeletons In this subpanel, you can automatically generate convex hulls around (skeletal) meshes in a collection, and then apply methods from the literature to reconstruct the inertial properties.

You have to designate the "Skeletal mesh collection" where your skeletal meshes are located. This defaults to the "Geometry" collection, but it can also be a separate collection. Each separate object in the collection will receive its own convex hull, so the meshes in the collection should represent functional body segments. See [12, 3, 9] for examples. Convex hulls are placed in a new collection.

6.6.2 Expand convex hulls

Both the expansion panels (arithmetic and logarithmic) work with segment names and corresponding scale factors or logarithmic parameters. If only "whole_body" is typed in as Segment name 1 (with no other segment names defined), all the objects in the target collection will be treated as one - in arithmetic mode, all objects are scaled by a single factor, in logarithmic mode, all object volumes are summed before determining the whole-body scale factor.

If segment 1 is not "whole_body", or if you have more than one segment name defined in the panel, MuSkeMo tries to match whatever segment names you have typed into the panel to the names of the objects in your target collection. E.g., if you've typed "neck" as one of the segment names in the panel, with a corresponding scale factor, all the objects in Convex hull collection that have the case-sensitive word "neck" in their name will be expanded by the scale factor (e.g., "neck_1", "neck_prox.obj", but not "Neck_1" and also not "torso"). The "Expansion Template" dropdown menu gives several presets from the literature, and you can customize them if desired.

MuSkeMo assumes you want bilaterally symmetric models, and therefore when using the expansion functions, MuSkeMo generates bilaterally symmetric expanded hulls for the following segments: "head", "neck", "torso", "tail". Furthermore, MuSkeMo also applies directional scaling. The following segments are scaled about the Y and Z axes (and thus not the X axis): "head", "neck", "torso", "tail", "forearm", "hand", "toe". All other segments are scaled about the X and Z axes. The resultant scaled shapes may not be very biologically realistic. You could add a "Maintain volume" constraint and change the shape in Blender (this was a suggestion by Matt Dempsey). Apply the constraint after you are satisfied.

Under **Expand convex hulls - arithmetic**, you can use arithmetic expansion factors (i.e., linear scale factors) to scale your hulls. If a segment initially has a volume of 1 m^3 and a scale factor of 1.2, the resultant segment will have a volume of 1.2 m^3.

- **Custom:** Type in the segment names, and the desired scale factor.
- **Macaulay 2023 Bird:** The per-segment "Bird" average expansions from Macaulay et al. 2023 [9], supplementary data S6.
- **Macaulay 2023 Non-Avian Sauropsid:** The per-segment "Croc_Lizard" average expansions from Macaulay et al. 2023 [9], supplementary data S6.
- **Macaulay 2023 Average (Bird and Non-Avian Sauropsid):** The per-segment "Av." average expansions from Macaulay et al. 2023 [9], supplementary data S6.
- **Sellers 2012 Large Mammals:** This is a "whole_body" scale factor of 1.206 as described in [12]

Under **Expand convex hulls - logarithmic**, it will be possible to use log-transformed regression equations from the literature to scale your hulls. This functionality is currently disabled.

6.7 Body panel

Create new bodies by typing in a (unique) name, and then pressing the 'Create new body' button. The newly created body will have all 'NaN's as inertial properties, and will be centered at the origin. Once you assign rigid body parameters, the body will be moved to the correct position in the global reference frame. A body's position is always its center of mass in the global frame, and its orientation is always aligned with the global frame.

You can assign rigid body parameters in two ways. The recommend approach is to assign values that were computed for meshes in the inertial properties panel (see 6.6). A single body can represent the combined inertial properties of more than one mesh. MuSkeMo computes the combined inertial properties using the parallel axes theorem [15, 10]. Simply select the target body, and 1+ source objects, and press 'Assign precomputed inertial properties'. MuSkeMo will give you an error message if you selected objects that don't have inertial properties precomputed, and the live selection boxes in the panel can help you with ensuring you have selected the correct (number of) objects. See D.2 for a validation of these features.

It is also possible to assign inertial properties manually. This is not recommended, but it is possible by manually typing the values in the Body's custom properties. If going this route, the COM position can be updated using the button in the "Assign inertial properties manually" subpanel.

Inertial properties are not dynamic, if you move the source objects that the rigid bodies were based on, or change their densities, you must recompute all inertial properties of the body (see 6.4). Otherwise COM, mass, and or inertia can be outdated. MuSkeMo warns you if you attempt to export inertial property data in a different pose than the default pose.

In this panel, you can also attach visualization geometry (eg., bone meshes) to bodies. Select the meshes and the target body, and press the button.

6.8 Joint panel

Define joints, and assign (and remove) parent and child bodies. If you want to change a joint's position or orientation, detach the parent and child bodies first. If a parent or child body has an anatomical (local) reference frame assigned, MuSkeMo automatically computes the relative positions and orientations in these frames as well. Orientations are stored as body-fixed, XYZ-Euler angles and as quaternions. All data that are created are included during export (if local frames are not assigned, these values will be nan).

It is also possible to define coordinate names in the joint panel. After exporting from MuSkeMo, the model conversion scripts (e.g., MuSkeMo_to_OpenSim) will only add DOFs to model if they are named (e.g. hip_angle_r). If no coordinates are named for a joint, the joint is turned into an immobilized joint (e.g., WeldJoint in OpenSim).

In the joint panel (under joint utilities), you can also mirror right side joints, fit geometric primitives (sphere, cylinder, ellipsoid, plane), and match the transformations of a joint to the fitted geometry. By default, MuSkeMo ensures that child objects and parent objects are not transformed with the joint. Instead, only the joint's position or orientation is changed, and related data (e.g., pos_in_child) are recomputed automatically.

6.9 Muscle panel

Define path-point muscles. Muscle points are added to the 3D cursor location, and parented to the selected Body (so you have to define bodies before creating muscles). Muscles points are added to whatever muscle name is currently active in the panel, and muscles are added to the user-designated "Muscles" collection.

6.9.1 Adding muscle points

To add a point to the end of the muscle (or to create a new muscle), type in the muscle name, and select the target body. Press shift + right mouse button to position the 3D cursor at the desired location in 3D space, and then press "Add muscle point". Muscle points are added to the 3D cursor location. You can change the locations of the path points by selecting the muscle in edit mode (select the muscle and

press "TAB", then select the relevant point and press "G" to move it around). If the parent bodies are repositioned, you must delete and redraw the muscles.

6.9.2 Inserting muscle points

It is possible to insert muscle points (instead of adding them at the end). To insert a point, select the point index after which you would like to insert a point (starting at 1), and press "Insert muscle point".

6.9.3 Visualization radius

By default, muscles are visualized using a Geometry node modifier named "musc_name_SimpleMuscleViz" (see section 7.3.1). If this visualization option is used, you can change the visualization radius of all muscles simultaneously by using the "Update visualization radius" button after selecting a desired radius. Alternatively, you can manually tune individual radii of muscles by changing the input in the "SimpleMuscleViz" modifier.

If the volumetric visualization option is used for the muscles, visualization parameters can be tuned in the "VolumetricMuscleViz" modifier (see section 7.3.2 for details).

6.9.4 View length in realtime

This button adds a geometry node modifier named "LiveLengthViewer" to the active muscle. This allows the user to interactively pose the model and see how the muscle lengths change. This can be useful during muscle construction, and for tuning musculotendon parameters. The user can change the placement and the size of the text from the modifier inputs. The node uses a custom node group named "LiveLengthViewerNodeGroup", which is shared between all muscles that have a length viewer modifier added to it. To disable the length viewer, remove the modifier from the muscle's modifier stack.

6.9.5 Assigning wrapping

You can use the "Wrapping" subpanel to perform all the necessary steps to assign wrapping to a muscle. This includes creating wrapping object geometry (currently, only cylinders are supported), assigning parent bodies, and assigning muscles to wrap around an object. The panel also performs the reverse operations.

6.9.6 Parametric wraps

Wrapping geometries can be interactively resized using the inputs in the object's modifier (e.g., the cylinder). To achieve this in Blender, wrapping geometries are also generated using geometry nodes (see 7.9 for details). When a muscle gets assigned a WRAP, it receives a very complex geometry nodes modifier (see 7.3.3 for details). This geometry node modifier, amongst other inputs, requires the wrap object dimensions (e.g., cylinder radius and height) as inputs. In Blender, it is not straightforward to couple the wrap parameters of a muscle to the wrapping geometry, because they are two separate objects. Turning on "Parametric Wraps" before creating (or importing) a wrap achieves this behavior using Blender Drivers. **Having many drivers on simultaneously can reduce performance and cause instability. It is recommended to turn Parametric Wraps ON when constructing muscle paths and modifying them, and then to export the model, and reimport it into a new scene with Parametric Wraps turned OFF for further processing.**

6.9.7 Moment arms

MuSkeMo can compute a muscle's moment arms, about a single degree of freedom. Specify the muscle name in the top of the Muscle panel, and the "Active joint 1" name in the Moment arms panel. You can choose between rotating about local x, y, or z axes, and the range (in degrees) over which the moment arms should be computed. MuSkeMo assumes that 0 degrees rotation is the current model's position. Angle step size determines how fine or coarse the joint range will be sampled.

When a computation is succesful, a Python dictionary named "length_data" is added to the muscles's custom properties. MuSkeMo computes the length of the target muscle over the desired joint range. Moment arms are computed using the principle of virtual work [14, 1], where $r = -dL/d\phi$ (see section D.4 in the appendix for more details and a validation of MuSkeMo's moment arms). Only lengths are

stored, moment arms are computed during plotting or export. If "Generate plot" is selected, moment arms are plotted straight away (see also 6.9.8), which may be useful when constructing muscle paths. It is also possible to plot muscle lengths instead.

If "Export length and moment arm" is selected, a CSV (or other file, see 6.13) is exported during moment arm computation. This requires setting a target export directory.

Warning: While the moment arms computed by MuSkeMo are physically accurate (Fig. 3), they are slightly noisy when using wrapping, in the order of 0.015% of the value of the moment arm, see D.4 for details. Readers intending to perform simulations using the exported moment arms themselves (instead of exporting full models for simulations), may want to smooth the wrapping moment arms before use.

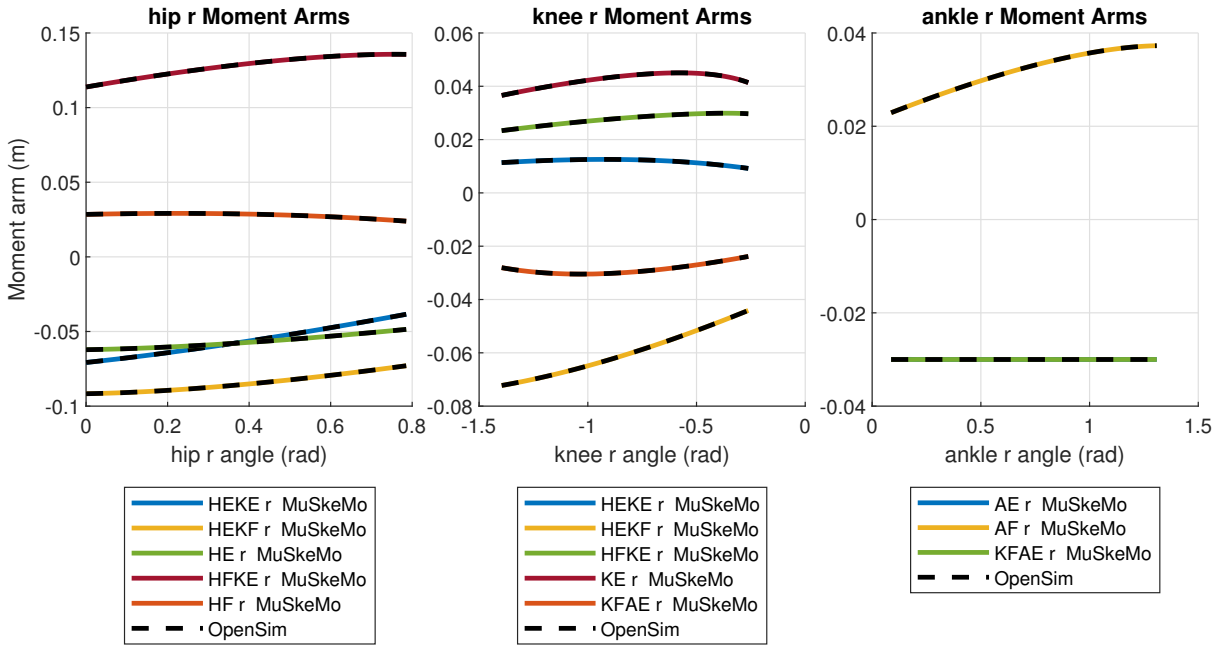

Figure 3: Moment arms of the emu model from [17], computed by MuSkeMo and exported as .csv files, compared to those computed by OpenSim 4.0 using the Matlab API. See section D.4 for details. A python script that performs a moment arm analysis is provided in the Utilities folder 6.17

6.9.8 Plotting

The plotting subpanel provides the user with some flexibility to alter the plots. Change the parameters in the panel, and then press "(Re)generate a muscle plot", which updates the length or moment arm plot of the muscle in question. The plotter can only (re)generate a plot for muscles where "length_data" were previously computed in the moment arms subpanel.

6.10 Anatomical & local reference frames panel

Construct anatomical and local reference frames, by assigning landmarks or markers as the reference points to construct the axes directions. Local reference frames have a position and an orientation with respect to the global reference frame. If the global frame is denoted with the letter $\mathcal{G}$, then the position of an arbitrary frame in the global reference frame can be written as the following vector: $\vec{v}_{\mathcal{G}}$. Internally, orientations are stored through rotation matrices. The rotation matrix that rotates a vector from the local $\mathcal{B}$-frame to the global $\mathcal{G}$-frame can be written as: ${}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}}$.

Orientations are exported as rotation (unit) quaternions (w, x, y, z), and also as body-fixed (intrinsic, active) XYZ-Euler angles (phi_x, phi_y, phi_z, in rad) (see Appendices A & B). The latter is prone to gimbal lock.

If anatomical / local frames are assigned to a body, MuSkeMo also computes inertial properties, joint positions and orientations, contact positions, and muscle path points with respect to these frames. This

requires the transpose of matrix ${}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}}$, namely: ${}^{\mathcal{B}}\mathbf{R}_{\mathcal{G}}$. This rotates a vector from the global $\mathcal{G}$-frame to the local $\mathcal{B}$-frame.

For an arbitrary point p expressed in $\mathcal{G}$, MuSkeMo computes the transformation to $\mathcal{B}$ as follows:

$$\vec{p}_{\mathcal{B}} = {}^{\mathcal{B}}\mathbf{R}_{\mathcal{G}} \, (\vec{p}_{\mathcal{G}} - \vec{v}_{\mathcal{G}}) \tag{1}$$

For an arbitrary matrix $\mathbf{I}$ expressed in $\mathcal{G}$ (e.g., an inertial tensor with respect to the body COM), MuSkeMo computes the transformation to $\mathcal{B}$ as follows [15]:

$$\mathbf{I}_{\mathcal{B}} = {}^{\mathcal{B}}\mathbf{R}_{\mathcal{G}} \, \mathbf{I}_{\mathcal{G}} \, {}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}} \tag{2}$$

Readers familiar with linear algebra will recognize this as a similarity transformation, that defines a change of basis.

6.11 Landmark & marker panel

Similar to muscle points, landmarks are added to the 3D cursor location.

6.12 Contact panel

Similar to muscle points, contacts are added to the 3D cursor location. Contacts can also be assigned a parent body.

6.13 Export panel

You can export all the user-created datatypes via this panel. The individual exporters export all the data types from the user-designated collections (folders) in Blender. It is possible to export all the visual geometry to a subfolder.

MuSkeMo exports all the data with respect to both the global reference frame (origin), and body-fixed local reference frames. Orientations are exported as XYZ-Euler angle decompositions, and as quaternion decompositions.

Under export options, it is possible to configure other text-based filetypes for export (e.g., txt, bat), configure custom delimiters, and choose the number formatting in the exported files.

6.14 Import panel

You can currently import bodies, joints, muscles, frames, and contacts, if they are MuSkeMo-created CSV files.

6.14.1 OpenSim model import

MuSkeMo provides an OpenSim importer. This can import most of the components of an OpenSim model. Although the default behavior is to import models using local definitions, it is also possible to import models using global definitions (i.e., all the component positions are defined as their position in the global frame, and for joints specifically, transformations in parent and child are both set to the global position and orientation). Global definition import is probably only useful for models that were created using MuSkeMo's conversion script, using the "Global definitions" option.

During import, cylinder wrapping is supported (see 7.3.3 and 7.9). MuSkeMo's wrapping definitions are different from OpenSim, and the importer tries to convert wrapping definitions. If the wrapping does not visualize correctly, it may be necessary to change the projection angle (combined with enabling "force wrap"), selecting "flip wrap", or "shortest wrap", depending on the situation. See 7.3.3 for a full description of MuSkeMo's wrapping.

Warning: Blender does not allow *any* object name to be reused. This means that all of your model components need to have unique names in MuSkeMo (so e.g., if a body is named 'shank_r', and it has only one wrapping object, it till cannot be named 'shank_r'. It would need a unique name such as 'shank_wrap_r').

If all components don't have unique names, the OpenSim importer may fail. The importer currently checks whether wrapping objects have non-unique names, but not other model components.

MuSkeMo does not support conditional or moving path points. These are automatically converted to regular path points during model import. For moving path points, the point position is selected that corresponds to the position when the controlling joint coordinate is equal to 0.

OpenSim models with CustomJoints (and perhaps other joint types with a SpatialTransform) can have Transform Axes that are different from the axes defined by the joint orientation. In essence, it is possible to define local frames within a joint, and joint coordinates can be expressed with respect to those local frames. This is currently not possible in MuSkeMo. MuSkeMo imports the joints as if they are regular joints, but stores the transform axes as a Python dictionary in a custom property named "transform_axes". These transform axes are mapped onto the joint rotations themselves when importing trajectories (see 6.15.1).

6.14.2 Gaitsym 2019 model import

MuSkeMo includes a Gaitsym (2019) importer. It currently imports bodies, joints, muscles (DampedSpring elements are treated as muscles), contacts, and markers (as frames). Muscles that include wrapping are currently not supported, but limited wrapping support is planned in a future update. Visual geometry can be imported, but requires the user to type the name of the containing folder in "Gaitsym geometry folder". The geometries must be in a subdirectory of the model directory, and the name of this subdirectory must currently be manually typed into the panel.

It is possible to automatically apply a rotation to the entire model during import. This can be convenient because Gaitsym is generally geared towards the Z-axis being the "up" axis, whereas ISB recommends Y-up. To rotate a model from Z-up to Y-up, apply a -90 °rotation about the x-axis. Points simply get rotated, MOI gets transformed according to eq. 2 (although technically the transformation is now from one global frame to another). The same change-of-basis transformation is also applied to orientions (of joints, frames, etc.). The result is that the old Z-axis becomes the new Y-axis, and the old -Y-axis becomes the new Z-axis.

6.14.3 Other simulators

Future updates will include Hyfydy and MuJoCo model support.

6.15 Visualization panel

Rendering in Blender can be a complicated process. It is capable of professional level video graphics rendering, and there are a lot of settings that the user can modify to achieve this. This panel provides some ease of use functions that preset the rendering settings that the author finds visually appealing, while also providing adequate performance. There exist thousands of video tutorials for creating renders in Blender. Until a MuSkeMo-specific rendering tutorial is recorded, it is recommended that you follow one of the many out there (e.g., the Donut tutorial referenced at the top of this document). **If your computer does not have a powerful graphics card (GPU), it may be necessary to tweak the recommended settings.**

6.15.1 Trajectory import

MuSkeMo enables you to import simulated trajectories back into Blender to create high-quality animations with complex camera movements. Currently, it is only possible to import trajectories using OpenSim .sto files. If you are importing a periodic stride, it is possible to automatically loop these in sequence to create a video using multiple strides, while progressing the (selectable) forward translation coordinate. In this case, you have to define the root joint (default = groundPelvis) and the forward progression coordinate (default = coordinate_Tx).

OpenSim can have Transform Axes that do not align with the joint axes themselves (see 6.14.1). In the case of translations, these are simply added on to the joint position, but in the directions specified by the transform axes. For rotations, MuSkeMo defines three successive axis-angle Rotation matrices, and applies them in XYZ order (see C).

6.15.2 Visualization options

This subpanel includes several convenience tools to aid users who are new to animations in Blender. These are:

- **Convert to volumetric muscles:** adds volumetric muscle visualization to each muscle in the scene. See section 7.3.2 for details.
- **Create a ground plane:** adds a ground plane to the scene
- **Set recommended render settings:** sets the rendering engine to Cycles (see below), the device to GPU, Max samples to 1000, turns on persistent data (under performance), and sets the "Look" to very high contrast (under Color Management). **WARNING:** the Cycles rendering engine is a path tracing (raytracing) rendering engine. If you do not have access to a computer with a powerful GPU, rendering performance will be incredibly poor. In that case, you should switch to the Eevee rendering engine.
- **Set black background gradient for renders:** creates a node setup in the compositor that de-emphasizes the background.

6.15.3 Default colors

In Blender, objects get assigned a color (and other surface rendering properties, such as roughness) by assigning a material. Before importing and/or model component creation, you can define the desired default colors in this panel for muscles, visual geometry (bones), joints, contacts, markers, geometric primitives, and wrapping geometry. If an instance of the object has been already created (e.g., you have already created a joint), you can change the colors by changing the object's material in the properties tab or in the shader editor. Unlike all other object types, each individual muscle in the model gets a unique material. This is to ensure that its activation can be independently animated when importing trajectories. This means that if you want a different default muscle color, you need to define it before model import/creation, or you must change all the muscle-materials individually.

Joints, contacts, wrap geometry, geometric primitives, and markers are provided with a transparent material by default (this is achieved with a mix shader node in the shader editor).

Importantly, the coloration in Blender depends on Viewport Shading mode in Blender. "Solid" mode uses a material's Viewport Display, whereas "Material Preview" and "Rendered" modes actually use the object's Material properties themselves. Eevee does not support transparent materials, so this is only available when selecting "Cycles" as the rendering engine.

6.16 Reflection panel

Bodies, frames, joints, contacts, muscles, and wrapping geometry have a robust reflection approach.

The reflection panel enables symmetric model construction, by only constructing model components on one side, and then reflecting them across the mid-sagittal plane. You can choose what plane defines the midline (default = 'XY'), and change the strings with which you designate the left and right sides (default is '_l' and '_r', respectively, at the end of an object's name, e.g., "thigh_r"). The script then checks for all components whether its other-side counterpart exists, and if not, creates it. The script checks this for both left and right-sided components simultaneously. Each component's reflection function searches within the collection that is designated next to the button.

Warning: All the reflection functions use a case-sensitive string search to check whether the side string is at the end of object's name. Because this can potentially cause conflicts (e.g., the name "dorsal_rib_l" contains both '_l' and '_r'), MuSkeMo only looks for the side string at the end of the object name. Because it is impossible to account for all naming conventions, a workaround can be to place the target component in its own collection and temporarily changing the name(s). In particular with muscles, where each point is parented to a body with a HOOK modifier (see 7.3), conflicts may need to be resolved manually.

Muscle point positions are reflected by multiplying the relevant component by -1, depending on which reflection plane is chosen. All other positional data are reflected by defining reflection matrix (a unit matrix with one diagonal equal to -1). Orientations are reflected by enacting a change of basis with the reflection matrix, using equation 2.

6.17 MuSkeMo utilities

The MuSkeMo.zip release contains a folder named MuSkeMo utlities, which includes several useful functions and scripts.

6.17.1 OpenSim conversion

The Matlab script "MuSkeMo_to_OpenSim.m" converts your MuSkeMo outputs to an OpenSim model. You must have the OpenSim Matlab API installed, and "CreateOpenSimModelFunc.m" must be in the same directory. The script provides a graphical user interface that lets you select MuSkeMo-created csv files of the model components. It is possible to create a model using global model definitions (optional local frames are ignored, and position/orientation in parent and child are both set to the global position and orientation). It also possible to construct a model using local model definitions. This requires the user to assign a `local_frame` to each body.

6.17.2 Fitting muscle lines of action

It can be useful to fit a curve to a 3D mesh of a muscle, for instance, acquired via DICE-CT scanning or surface scanning. "MuscleLineOfActionFitter.py" is a rudimentary fitting tool. It is not currently built into MuSkeMo yet, but the script can be opened in a Blender scene via the script editor. The user must fill in the target muscle name in the script, and ensure that two objects named 'origin' and 'insertion' are present in the scene. You can set the desired resolution.

The fitter works by slicing up the mesh into n-sections, whose heights are determined by the user-input resolution, and the origin-insertion distance. The slices are created by performing a boolean intersection between the target mesh, and a cuboid that progresses in n-steps from the origin position to the insertion position, aligned in this direction. The volumetric centroid is computed for each slice, and these centroids combined with the origin and insertion form the fitted curve.

High resolution results in slices with a smaller height and thus a smoother curve, with very high resolutions approximating thin cross-sections instead of volumetric slices. High resolutions can potentially result in clipping (ignoring) bits of the muscle during the fitting procedure: if due to the shape, sections of the muscle are proximal to the origin, or distal to the insertion, they are not included in the boolean intersection, and their volumes thus not represented in the line of action. It is up to the user to account for this, and this is why the default behavior is to display both the slice-cuboids, and the resultant slices, which can be compared to the input muscle.

Lower resolution gives a less smooth result, but in most cases will include the entire muscle during its computation.

The resultant objects are Blender curves. It is possible to resample these, if desired. By selecting a curve and right-clicking, it is possible to convert the curve to a mesh. This makes it possible to snap to the fitted curve when creating a MuSkeMo muscle based on the fitted curve.

6.17.3 Compute closest point between objects

The Python script "ComputeClosestPointBetweenMeshes.py" can be run in the Blender script editor. It outputs the closest point between two meshes (in meters). This can be useful to compute minimal joint spacing while articulating a skeleton, or for instance in a series of XROMM frames. An example of how to extend it to work with multiple objects, loop through frames, and export distances as a CSV can be seen in "ComputeClosestPointRealExample.py". This script was written for Voeten et al. (in prep). If that paper has come out by when using this script, please consider citing it as the source.

6.17.4 Muscle length script

Run this script in the Blender script editor (after filling in a target muscle) to get its current length. Can be used in more elaborate scripts and functions (e.g., for computing moment arms).

6.17.5 MuSkeMo version updater

The folder also contains an updater (Python) script to update older MuSkeMo scenes to v0.6.3 and up. To run this, open the python script in the Blender script editor and run it. Back up your work first. This script will be removed in a future update.

7 MuSkeMo Data Types

7.1 BODY

A rigid body. Rigid bodies have inertial properties, which can be computed during model creation from 3D scans:

- `mass` (kg)
- `COM` (center of mass (m) in the global reference frame)
- `inertia_COM` (moment of Inertia (kg m^2) about the COM, in the global reference frame)

Inertial tensor elements are listed in the following order: (Ixx, Iyy, Izz, Ixy, Ixz, Iyz).

Bodies can have one or more optional attached `Geometry` meshes for visualization (e.g., 3D meshes of bones). These are delimited with a '`;`' if present, and usually preceded with the name of the subdirectory (default = '`Geometry`') in which the meshes will be exported. For example:

'`Geometry/cranium.obj;Geometry/mandible.obj;`'

COM is always reported in the global reference frame, and MOI is always computed with respect to the COM in the global reference frame and orientation. It is possible to assign a `local_frame` to a body (see section 7.7). This automatically computes the following properties:

- `COM_local` (center of mass (m) in the body's local reference frame)
- `inertia_COM_local` (moment of Inertia (kg m^2) about the COM, in the body's local reference frame)

The underlying object type in Blender is an "Empty".

7.2 GEOMETRY

Visual geometry (e.g., bone meshes, or tissue outline meshes) can be attached to bodies for visualization purposes. In most simulators, these don't have a physical function, but are purely used for visualization purposes and for determining where muscle attachments are relative to the bodies.

WARNING: OpenSim has a visualization bug that prevents models from loading correctly if the total file size of the attached geometry exceeds 50MB. You can decimate the meshes to reduce them to about 50MB or attempt to load the model into OpenSim Creator.

If Decimating the model doesn't work, the visualization error can sometimes also be triggered by meshes that are composites of several different parts, or that have many loose triangles. Using MuSkeMo, first detach the visual geometry via the Geometry panel. Then, select the mesh and press TAB for edit mode, and then press P (for seParate). This separates the model by loose parts. The resulting meshes should be parented to the body individually. This will require you to re-export the geometries, re-export the bodies CSV, and regenerate the OpenSim model. However, it is usually the total filesize that triggers this issue, not composite meshes.

7.3 MUSCLE

A path-point muscle. Each muscle point is automatically parented to a body. Muscles have the following user-definable contractile properties:

- `F_max` (maximal contractile force of the muscle fibers, in Newtons)
- `optimal_fiber_length` (in meters)
- `pennation_angle` (in degrees)
- `tendon_slack_length` (in meters)

The underlying object type in Blender is a poly-curve. Each point in the curve is attached to a body using a hook-modifier.

7.3.1 Simple muscle visualization

Simple muscle visualizations are achieved by adding a simple Geometry node setup to each muscle (curve in Blender). This node setup essentially lofts a curve with the user-specified radius (default = 0.015 m) across the entire length of the curve. The radius can be changed without going into the Geometry nodes setup, by selecting the correct modifier in the modifier stack.

Under "Geometry nodes", with the "SimpleMuscleViz" node of a muscle selected, the node setup is visible. The visualization setup itself is specified by the "SimpleMuscleNode" node group (select it and press 'TAB' to modify, this node group is shared by all muscles and thus modifications are applied to all muscles). The node setup also applies a material to each individual muscle, so that their colors can be animated invidually.

7.3.2 Volumetric muscle visualization

Similar to the simple muscle visualizations, volumetric muscle visualizations are also achieved by adding a Geometry node to each muscle. The node-group itself is shared across muscles, but muscle visualizations are individualized through four parameters, which can be accessed directly in the "VolumetricMuscleViz" modifier. These are:

- MuscleVolume: Computed using each muscle's F_max, optimal_fiber_length, and specific_tension (set in the Visualization panel, see section 6.15). The node setup adjusts the muscle's radius to keep the volume constant irrespective of the length.
- MuscleTendonLengthRatio: This decides the ratio of the muscle belly to the total musculotendon complex length. If set to 1, the muscle belly is stretched across the entire muscle's length.
- TendonMuscleRadiusRatio: This sets the relative ratio of the tendon with respect to the muscle's radius. Set to 0 if you want no tendon visualization.
- ProxToDistMuscleBellyBias: By default, the muscle belly is in the middle of the curve. If MuscleTendonLengthRatio is less than 1, you can shift the muscle belly more proximally or more distally using "ProxToDistMuscleBellyBias".

7.3.3 Muscle wrapping

MuSkeMo implements [6] for cylindric wrapping, using a custom-designed Geometry node. The wrapping node will give a true tangent-curve solution as long as there is maximum of one wrapping object between each pair of subsequent muscle points. Garner et al. [6] propose an iterative root finding method for multi-object wrapping between two curve points, which is currently very challenging to implement using Geometry nodes. **If you add multiple wrapping objects per pair of muscle points, MuSkeMo only provides a true tangent curve solutions if each wrapped section is separated by a curve point.**

Wrapping will not work if any of the muscle curve points clip into the wrapping object.

[Documentation about the wrapping settings to follow]

7.4 JOINT

A joint in MuSkeMo represents the connection between two rigid bodies, allowing them to articulate relative to each other. The joint position and orientation can be expressed in XYZ Euler angles or quaternions.

`parent_body` and `child_body`: these are the two bodies connected by the joint. These must be defined by the user. When defined by the user, `pos_in_global` and `or_in_global` are both computed.

- `pos_in_global`: The position of the joint center in the global reference system (in meters).
- `or_in_global_XYZeuler`: The orientation of the joint in the global reference system (in radians) using an XYZ Euler decomposition.
- `or_in_global_quat`: The orientation of the joint in the global reference system, expressed as a quaternion (w, x, y, z).

These define the joint's position and orientation in the global reference frame. The position is given as a list of three floats (x, y, z) in meters. The orientation is expressed in both XYZ Euler angles (radians) and as a quaternion (w, x, y, z).

If a `parent` or `child` body also has a local `FRAME` defined, MuSkeMo automatically computes the transformations with respect to that frame, resulting in six more paremeters:

- `pos_in_parent_frame`: The position of the joint center in the local reference frame attached to the parent body (in meters).
- `or_in_parent_frame_XYZeuler`: The orientation of the joint in the local reference frame attached to the parent body (in radians) using an XYZ Euler decomposition.
- `or_in_parent_frame_quat`: The orientation of the joint center in the local reference frame attached to the parent body , expressed as a quaternion (w, x, y, z).
- `pos_in_child_frame`: The position of the joint center in the local reference frame attached to the child body (in meters).
- `or_in_child_frame_XYZeuler`: The orientation of the joint in the local reference frame attached to the child body (in radians) using an XYZ Euler decomposition.
- `or_in_child_frame_quat`: The orientation of the joint center in the local reference frame attached to the child body , expressed as a quaternion (w, x, y, z).

Each joint also has six coordinates which can be named. These are meant to represent the generalized coordinates, or degrees of freedom, of the model. If they are named, this coordinate becomes a degree of freedom when using one of the provided scripts to convert a MuSkeMo model to a simulator (e.g., if you want `coordinate_Rz` to be a degree of freedom in your OpenSim model, give `coordinate_Rz` a name such as 'hip_flexion_r').

- `coordinate_Tx`: Translation along the x-axis
- `coordinate_Ty`: Translation along the y-axis
- `coordinate_Tz`: Translation along the z-axis
- `coordinate_Rx`: Rotation along the x-axis
- `coordinate_Ry`: Rotation along the y-axis
- `coordinate_Rz`: Rotation along the z-axis

The underlying object type in Blender is a UV sphere mesh. This object is essentially only for visualization purposes, but a future version of MuSkeMo may add the option to add axes as well.

7.5 LANDMARK

A `LANDMARK` in MuSkeMo refers to a user-specified point on a rigid body or geometry. Landmarks are currently immediately parented to visual geometry, and can be used to help define (anatomical or local) reference frames. Future updates will include support for parenting to bodies, and exporting/importing global and local positions.

The underlying Blender object type is a UV sphere mesh.

7.6 CONTACT

Contact geometry defines areas or regions where external forces might act on a rigid body. These points represent places where a body or object interacts with another object or the environment during simulations. MuSkeMo does not compute any contact forces, but the user can define contact positions if the model will be used for simulations. MuSkeMo currently only supports contact spheres. Contact the developer if you need more geometry types. Like joints, contact spheres can have a `pos_in_global` and a `pos_in_parent_frame`.

The underlying object type in Blender is a UV sphere mesh.

7.7 FRAME

In MuSkeMo, a `FRAME` is a local coordinate system that can be assigned to any rigid body. It defines a reference position and orientation relative to which other properties of the body (such as the center of mass or moment of inertia) can be computed. Each rigid body can be assigned one (optional) local `FRAME`, which can represent an anatomical reference frame. If a local frame is assigned to a body, body segment parameters with respect to that frame are automatically computed, as are the local transformations of any parent and/or child joints. These are removed if the frame is detached from the body.

7.8 GEOM_PRIMITIVE

In MuSkeMo, it is possible to fit geometric primitive shapes (spheres, cylinders, ellipsoids, and planes) to 3D meshes. This can be useful for defining joint centers of rotation using the skeletal geometry. The geometric details of the shapes are stored for reuse. These are:

- Sphere: `sphere_radius`
- Cylinder: `cylinder_height`
- Ellipsoid: `ellipsoid_radii` (x, y, and z components)
- Plane: `plane_dimensions` (x and y components)

The underlying object types in Blender are regular meshes (UV Sphere, cylinder, ellipsoid).

MuSkeMo implements two different sphere fitting algorithms, described in [7, 18]. The cylinder fit algorithm was adapted from [4]. The ellipsoid fit algorithm [13] was provided courtesy of Mark Semple, who converted a Matlab implementation written by Yuri Petrov to Python. I modified it further to ensure right-handed coordinate systems.

7.9 WRAP

Wrapping geometry can be user-created in the Wrapping subpanel of the Muscle panel. Wrapping geometry defined in OpenSim models (but currently not yet Gaitsym models) are included during model import. WRAP objects are not regular meshes like GEOM_PRIMITIVES. To achieve parametric wrap geometry, WRAP geometry are generated using Geometry nodes that parametrically generate the desired object. The wrapping geometry dimensions (e.g., cylinder radius) can be altered in the modifier on the object (press the blue wrench in the interface). Currently, only cylinders are supported.

WRAP geometry are the physical objects that muscles can be assigned to wrap around, but the actual curve wrapping is achieved with modifiers on individual muscles (see **??**).

Just like JOINTS, WRAPS have parent bodies, and orientations and positions with respect to parent frames if those are assigned to the body. Unique to WRAP are the following properties:

- `target_muscles`. A list of all the muscles (delimited with ';') that wrap around this object. Can be 'not_assigned'.
- `wrap_type`. Currently only 'Cylinder'.

A Euler angles

A.1 Conventions used by MuSkeMo

Euler angles define rotation matrices that transform between reference frames. Conceptually, they are relatively straightforward: 3D rotations are defined by three successive rotations about different axes in space. However, Euler angles are ambiguous without explicitly defining what convention is being used. It is possible to choose twelve different sets of axes about which we perform the successive rotations (e.g., XYZ, XYX, etc). It is furthermore possible to define these axes with respect to the rotating body (body-fixed, intrinsic), or the global frame (space-fixed, extrinsic). This determines the order of the application of the individual rotations. The rotations can also be defined to rotate the body itself (active rotations), or rotate the frame around a stationary vector (passive rotations). Active and passive rotation matrices are related by being each others' transpose.

To enable cross-compatibility with OpenSim, MuSkeMo uses the same convention as OpenSim. This convention uses a body-fixed (intrinsic) XYZ-decomposition using active rotations (so the object is being rotated, not the reference frame). OpenSim is built on Simbody, and the Simbody Documentation cites page 423 of [8] as their source.

The full rotation matrix ${}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}}$ that MuSkeMo uses to rotate a vector from a body-fixed frame (the $\mathcal{B}$-frame) to the global reference frame (the $\mathcal{G}$-frame) is presented here to prevent ambiguity:

$$ {}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}} = \begin{bmatrix} \cos\phi_y \cos\phi_z & -\cos\phi_y \sin\phi_z & \sin\phi_y \\ \cos\phi_x \sin\phi_z + \cos\phi_z \sin\phi_x \sin\phi_y & \cos\phi_x \cos\phi_z - \sin\phi_x \sin\phi_y \sin\phi_z & -\cos\phi_y \sin\phi_x \\ \sin\phi_x \sin\phi_z - \cos\phi_x \cos\phi_z \sin\phi_y & \cos\phi_z \sin\phi_x + \cos\phi_x \sin\phi_y \sin\phi_z & \cos\phi_x \cos\phi_y \end{bmatrix} \tag{3} $$

Here, ϕ_x, ϕ_y, and ϕ_z represent rotations about the body-fixed, right-handed x-, y-, and z-axes. Brackets are omitted because each cosine and sine only has one term inside it. A vector ${}^{\mathcal{B}}\vec{v}$ expressed in the $\mathcal{B}$-frame (hence the ${}^{\mathcal{B}}$-prefix), can be rotated to the $\mathcal{G}$-frame by pre-multiplying it with ${}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}}$. In other words:

$$ {}^{\mathcal{G}}\vec{v} = {}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}}{}^{\mathcal{B}}\vec{v} \tag{4} $$

Blender itself also implements Euler angles, but uses the convention of space-fixed (extrinsic) rotations. **As a result, what Blender refers to as ZYX-Euler angles is the same as the body-fixed XYZ-Euler angles used by MuSkeMo, OpenSim, SCONE, etc.**

A.2 Understanding decomposed successive rotations

The different Euler-angle conventions in use were the source of much confusion for the author. A step-by-step explanation is provided in the next sections of this Appendix. For further reading, the reader is referred to [8, 15].

Rotation About the x-axis

To rotate by ϕ_x about the x-axis, the rotation matrix $\mathbf{R}_x$ is given by:

$$ \mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_x & -\sin\phi_x \\ 0 & \sin\phi_x & \cos\phi_x \end{pmatrix} \tag{5} $$

For example, applying this to a unit vector initially pointing in the y-axis direction:

$$ \mathbf{R}_x \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{6} $$

If $\phi_x = \frac{\pi}{4}$, this results in:

$$ \begin{pmatrix} 0 \\ \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \tag{7} $$

Thus, the second and third columns give the new y- and z-axes after rotating by ϕ_x in the yz-plane.

Rotation About the y-axis

To rotate by ϕ_y about the y-axis, the rotation matrix $\mathbf{R}_y$ is:

$$\mathbf{R}_y = \begin{pmatrix} \cos\phi_y & 0 & \sin\phi_y \\ 0 & 1 & 0 \\ -\sin\phi_y & 0 & \cos\phi_y \end{pmatrix} \tag{8}$$

Similarly, applying this to a unit vector initially pointing in the x-axis direction:

$$\mathbf{R}_y \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

If $\phi_y = \frac{\pi}{4}$, this results in:

$$\begin{pmatrix} \cos\frac{\pi}{4} \\ 0 \\ -\sin\frac{\pi}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{10}$$

Thus, the first and third columns of $\mathbf{R}_y$ give the x- and z-axes directions after the rotation.

Rotation About the z-axis

To rotate by ϕ_z about the z-axis, the rotation matrix $\mathbf{R}_z$ is:

$$\mathbf{R}_z = \begin{pmatrix} \cos\phi_z & -\sin\phi_z & 0 \\ \sin\phi_z & \cos\phi_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

Similarly, the first and second columns of $\mathbf{R}_z$ give the x- and y-axes directions after rotating about the z-axis.

Three Successive Rotations about Body-Fixed X, Y, and then Z

If we decompose the full rotation into three successive rotations about the body-fixed X-axis, Y-axis, and Z-axis (denoted as $\mathbf{R}_x$, $\mathbf{R}_y$, and $\mathbf{R}_z$ respectively), the combined rotation matrix is:

$$^{\mathcal{G}}\mathbf{R}_{\mathcal{B}} = \mathbf{R}_x\mathbf{R}_y\mathbf{R}_z$$

We said that we were starting with a rotation about X, but the vector will first be multiplied by Rz Why is this?

Consider we want to rotate a vector $\vec{v}_{\mathcal{B}}$ (expressed in the body-fixed frame $\mathcal{B}$) by arbitrary angles about body-fixed X, Y, and Z-axes, in that order. After the rotations, what will $\vec{v}_{\mathcal{G}}$ (the vector in the global frame $\mathcal{G}$) be?

When the two frames are aligned (i.e., when all rotation angles are zero), $\vec{v}_{\mathcal{B}} = \vec{v}_{\mathcal{G}}$.

We start by rotating about the body-fixed x-axis. Thus, we have:

$$\vec{v}_{\mathcal{G}} = \mathbf{R}_x\vec{v}_{\mathcal{B}}$$

At this point, the vector $\vec{v}_{\mathcal{B}}$ is rotated about the $x_{\mathcal{B}}$-axis, which was coincident with the global $x_{\mathcal{G}}$-axis.

Why Can't We Simply Add the Next Rotation by pre-multiplying the previous equation?

The next rotation becomes less intuitive. It may be tempting to add the rotation about the y-axis as follows:

$$\cancel{\mathbf{R}_y \mathbf{R}_x \vec{v}_{\mathcal{B}}}$$

However, this is incorrect! This formulation would result in a rotation about the global $y_{\mathcal{G}}$-axis, because the first rotation was about the global $x_{\mathcal{G}}$-axis (since $x_{\mathcal{B}}$ and $x_{\mathcal{G}}$ were coincident before the first rotation).

Given that we want to rotate about the local $y_{\mathcal{B}}$-axis (which coincides with $y_{\mathcal{G}}$ before the first rotation), we must instead insert $\mathbf{R}_y$ between $\mathbf{R}_x$ and $\vec{v}_{\mathcal{B}}$, as follows:

$$\vec{v}_{\mathcal{G}} = \mathbf{R}_x \mathbf{R}_y \vec{v}_{\mathcal{B}}$$

Note: This implies that successively rotating about body-fixed x- and y-axes is equivalent to successively rotating about the global y- and x-axes, but the order is reversed depending on the frame of reference! Rotations about body-fixed axes are called *intrinsic*, whereas rotations about global axes are called *extrinsic*.

Adding the Third Rotation

A similar argument applies to the third rotation about the local $z_{\mathcal{B}}$-axis. Therefore, the total equation for the vector in the global frame after all three rotations is:

$$\vec{v}_{\mathcal{G}} = \mathbf{R}_x \mathbf{R}_y \mathbf{R}_z \vec{v}_{\mathcal{B}}$$

Thus, a body-fixed XYZ-sequence of rotations (intrinsic) is identical to a global-frame ZYX-sequence of rotations (extrinsic), with the rotation order reversed!

In body-fixed notation, the successive rotation matrices are multiplied from left to right when rotating from the body-fixed frame $\mathcal{B}$ to the global frame $\mathcal{G}$. In the above example, using XYZ about the body-fixed axes gives:

$${}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}} = \mathbf{R}_x \mathbf{R}_y \mathbf{R}_z$$

The inverse operation, which rotates from the global frame back to the body-fixed frame, is given by:

$${}^{\mathcal{B}}\mathbf{R}_{\mathcal{G}} = ({}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}})^{-1} = ({}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}})^{T} = \mathbf{R}_z^T \mathbf{R}_y^T \mathbf{R}_x^T$$

B Rotation quaternions

To be written. MuSkeMo uses algorithms based on [5].

C Axis angle rotation matrix

To be written. MuSkeMo uses an algorithm adapted from [15]

D Validation tests

Many of MuSkeMo's modeling features rely on correctly computing positions and orientations with respect to local frames. Errors in some of these computations would be quite obvious during import (e.g., if the transformations are incorrectly propagated, resulting in incorrect model component positions and orientations). However, not all potential sources of errors can be detected visually.

This section lists several validation tests that rely on the correct application and propagation of all model construction and analysis steps. Section D.1 demonstrates that mesh inertial properties are computed correctly by MuSkeMo, and D.2 demonstrates these are correctly combined using the parallel axes theorem. Section D.2 also demonstrates internal consistency between rigid body parameters combined from different meshes ("composite body"), and rigid body parameters from combined meshes ("composite mesh"). Section D.3 demonstrates that MuSkeMo correctly re-expresses vectors and matrices in arbitrary

frames. The scripts that construct the rotation matrices are reused by MuSkeMo for all transformations during model construction, import, and export. Section D.4 demonstrates that MuSkeMo can accurately compute moment arms from an imported OpenSim model. This also directly validates MuSkeMo's implementation of cylindric wrapping from [6], also demonstrates joint positions, orientations, and rotations are treated correctly by MuSkeMo, because the moment arms rely on their correct application.

All Blender scenes, files, scripts, and outputs described in this section are provided here: LINK. Where relevant, each Blender scene has a script in the script editor that manually performs all the calculations that are listed in the equations below.

For further validation of MuSkeMo, the reader is referred to [17, 16], where animal models constructed using MuSkeMo were used for dynamic simulations, and simulator outputs were compared to empirical data from the animals in question.

D.1 Mesh inertial properties

To compute inertial properties from arbitrary 3D meshes, MuSkeMo implements pseudo-code from [5] (see 6.6). I will validate the implementation in MuSkeMo by demonstrating that it: 1) accurately computes the inertial tensors for objects with known inertial tensors; 2) inertial properties of a complex mesh acquired from a CT scan match the outputs from Meshlab [2]. We will assume a density of 1000 kgm^{-3} for all computations, although this is arbitrary since all of the underlying computations are volumetric only (see 6.6).

The inertial properties are computed through successive multiplications of the 3D vertex coordinates of the input meshes. Because Blender stores these coordinates as 32 bit single precision digits (see 4), the repeated multiplications required to compute the inertial properties will reduce their precision to approximately 5-6 digits.

Icosahedron and sphere

If the edge lengths l_e are known, the volume of an icosahedron can be computed as [11]:

$$V = l_e^3 \cdot \frac{5}{6} \cdot \tau^2, \tag{12}$$

where $\tau = \frac{\sqrt{5}+1}{2}$ is the golden ratio. Multiplying the volume by the density ρ gives the mass m.

In Blender, an icosahedron with a default radius of 1 has $l_e = 1.05146\,\text{m}$ (Fig. 5). Using the algebraic equation, the computed mass is $2536.13\,\text{kg}$. MuSkeMo computes a mass of $2536.15\,\text{kg}$.

The principal moments of inertia of an icosahedron can be computed as [11]:

$$I = \frac{\tau^2}{10} \cdot l_e^2 \cdot m, \tag{13}$$

where all three principal moments are equal.

Using the algebraic equations, the computed moment of inertia is $734.063\,\text{kgm}^2$. MuSkeMo provides a moment of inertia value of $734.070\,\text{kgm}^2$.

The volume of a sphere with radius r_s can be computed as:

$$V = 4/3 \cdot \pi \cdot r_s^3 \tag{14}$$

Thus, a sphere with $r_s = 1$ m weighs 4188.79 kg. MuSkeMo computes its mass as 4188.65 kg.

The principal moments of inertia of a sphere can be computed as [10]:

$$I = 2/5 \cdot m \cdot r_s^2, \tag{15}$$

again with all three principal moments equal. This gives 1675.52 kgm^2 for our sphere. MuSkeMo computes it as 1675.42 kgm^2.

We will reuse the parameters of the icosahedron and the sphere in D.2.

Complex tissue model from CT scan

3D models acquired from CT scans can be complex and irregular. The algorithm presented by Eberly [5] can deal with such meshes without problems. Coatham et al. [3] provided a dataset of CT scanned animals, with manually segmented tissue (skin) outlines. Fig. 4 shows the torso tissue outline of a cheetah, the "Torso.obj" mesh from the "skin" folder of the *Acinonyx jubatus* dataset. This is a relatively detailed mesh (78 MB), likely an unnecessary level of detail for a rigid body model.

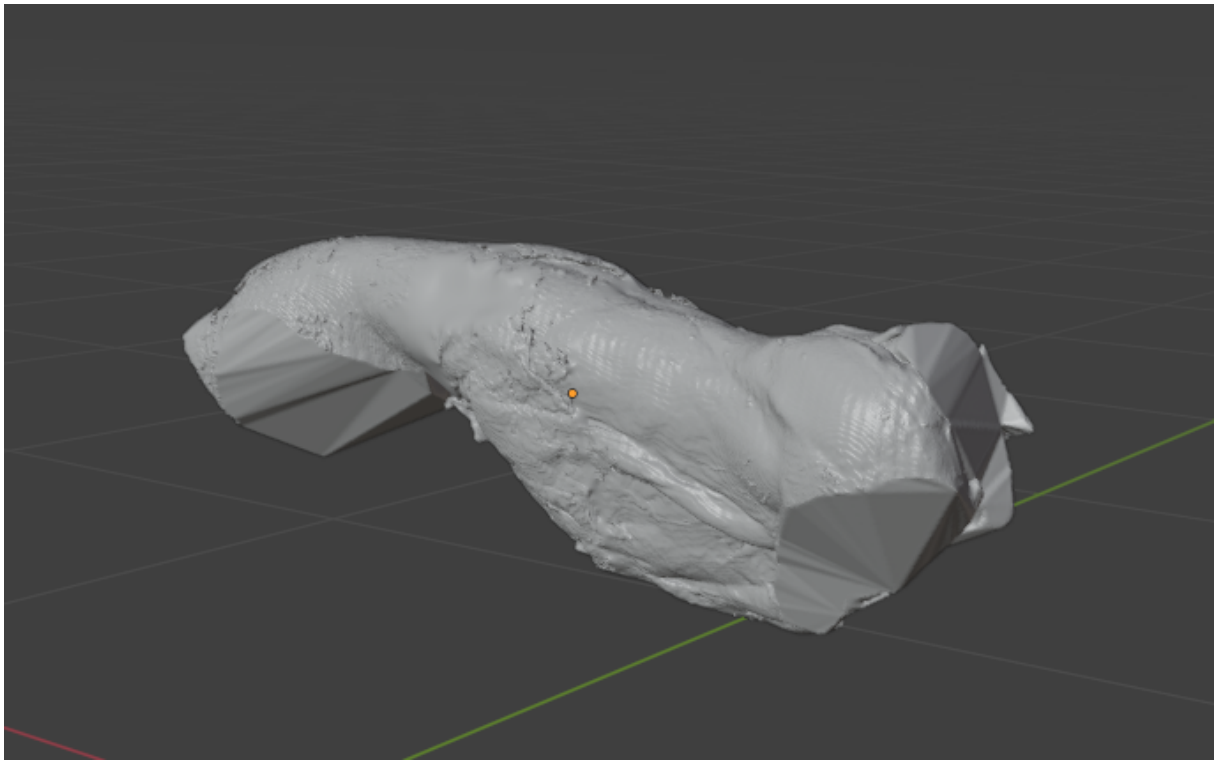

Figure 4: The "Torso.obj" mesh from [3], providing a 3D model of the torso of a cheetah, segmented from a CT-scan.

Using the "Compute Geometric Measures" filter, Meshlab gives: $V = 0.027241$ m^3, center of mass = $(0.461789, -0.002036, 0.555943)$ m, and the volumetric inertia tensor as (0.000177, 0.002123, 0.002045, 0.000060, 0.000113, 0.000011) m^5. MuSkeMo gives the following outputs (setting density at 1000 kgm^{-3}): $m = 27.2405$ kg, center of mass = $(0.461789, -0.002036, 0.555943)$ m, and the mass inertial tensor as (0.176614, 2.12329, 2.0455, 0.060367, 0.113492, 0.010864).

The numerical values are the same, save for a factor of 1000 representing the density.

D.2 Composite bodies and meshes

MuSkeMo can combine the inertial properties of several source objects into a single composite body using the (3D) parallel axes theorem [15, 10]. Here, we will compute the values manually for the situation pictured in Fig. 5, and compare the computations by MuSkeMo.

Consider the icosahedron and sphere from D.1. The icosahedron has mass $m_i = 2536.13$ kg, center of mass $\vec{c_i} = (0, 0, 0)$ m, and its inertia tensor:

$$\mathbf{I}_{i/c,\mathcal{G}} = \begin{pmatrix} 734.063 & 0 & 0 \\ 0 & 734.063 & 0 \\ 0 & 0 & 734.063 \end{pmatrix}. \tag{16}$$

For the inertia tensor, the subscript i denotes the icosahedron, and c signifies the inertia is given with respect to its center of mass. Like in A, $\mathcal{G}$ signifies the tensor is expressed in the global frame.

The sphere has mass $m_s = 4188.79$ kg, center of mass $\vec{c_s}$ = (4,1,-1.5) m, and its inertia tensor:

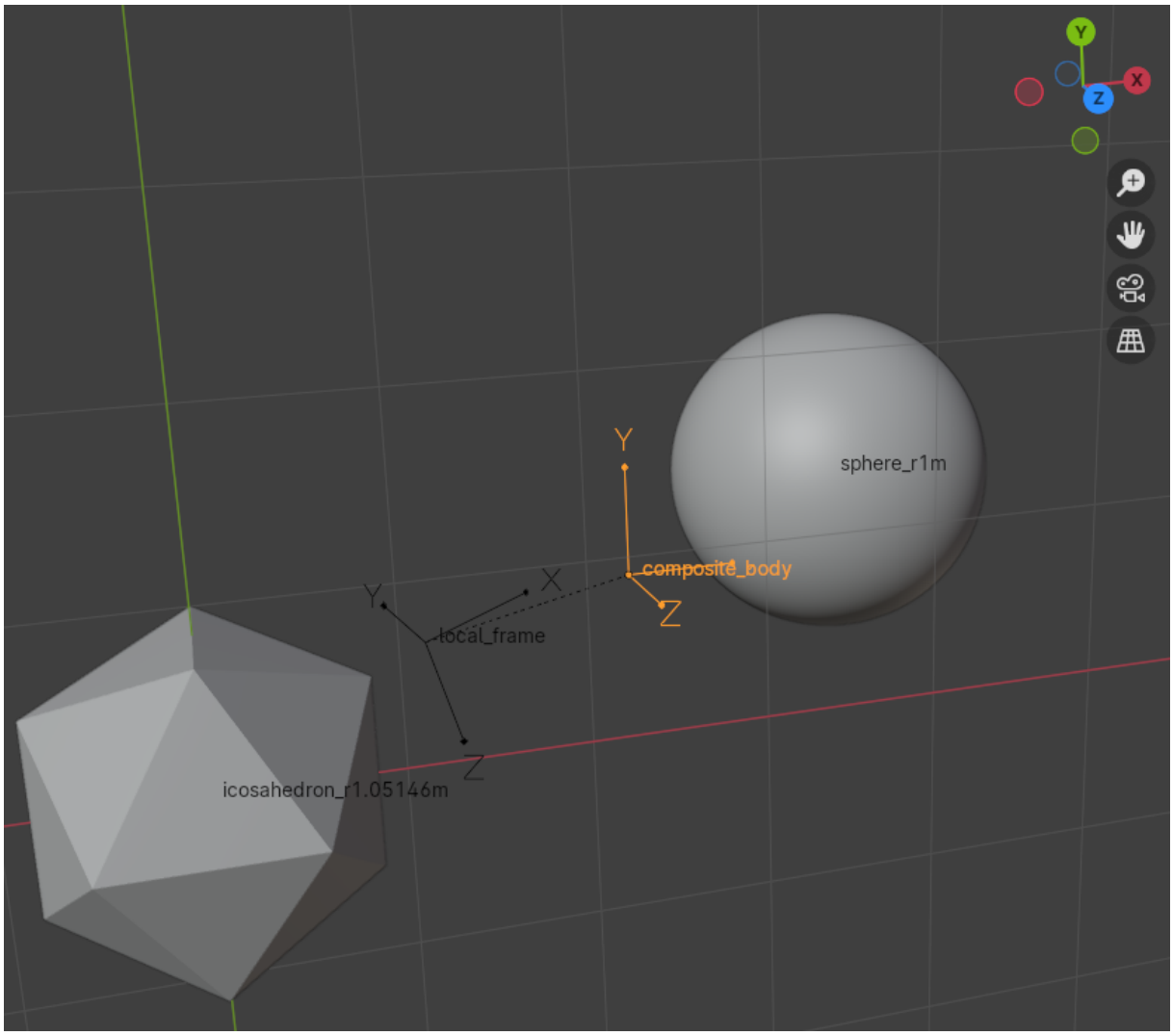

Figure 5: The icosahedron, sphere, composite body, and local frame described in sections D.1, D.2, and D.3

$$\mathbf{I}_{s/c,\mathcal{G}} = \begin{pmatrix} 1675.52 & 0 & 0 \\ 0 & 1675.52 & 0 \\ 0 & 0 & 1675.52 \end{pmatrix} \tag{17}$$

The combined center of mass $\vec{c_c}$ is:

$$\frac{m_i \cdot \vec{c_i} + m_s \cdot \vec{c_s}}{m_i + m_s} = \vec{c_c}, \tag{18}$$

which is at (2.4915, 0.6229, -0.9343) m in the $\mathcal{G}$-frame when we compute it manually. In MuSkeMo, the inertial properties computed for the icosahedron and the sphere in D.1 can be assigned to a single body in the Body panel (see 6.7). When their rigid body parameters are assigned to a BODY named "composite_body", the combined center of mass is computed as (2.4915, 0.6229,-0.9343) m.

The combined inertial parameters can be calculated manually using the 3D parallel axes theorem [15, 10]. To fully write out the equation, it is useful to first define:

$$\vec{d_{i//c}} = \vec{c_i} - \vec{c_c}, \tag{19}$$

and a matrix constructed of its elements:

$$\mathbf{D}_{i//c,\mathcal{G}} = \begin{pmatrix} \vec{d_{i//c,y}}^{\,2} + \vec{d_{i//c,z}}^{\,2} & -\vec{d_{i//c,x}} \cdot \vec{d_{i//c,y}} & -\vec{d_{i//c,x}} \cdot \vec{d_{i//c,z}} \\ -\vec{d_{i//c,x}} \cdot \vec{d_{i//c,y}} & \vec{d_{i//c,x}}^{\,2} + \vec{d_{i//c,z}}^{\,2} & -\vec{d_{i//c,y}} \cdot \vec{d_{i//c,z}} \\ -\vec{d_{i//c,x}} \cdot \vec{d_{i//c,z}} & -\vec{d_{i//c,y}} \cdot \vec{d_{i//c,z}} & \vec{d_{i//c,x}}^{\,2} + \vec{d_{i//c,y}}^{\,2} \end{pmatrix}. \tag{20}$$

The matrix $\mathbf{D}_{s//c,\mathcal{G}}$ is similarly constructed from $\vec{d_{s//c}}$. It is also possible to define $\vec{d_{c//i}}$ and its respective matrix $\mathbf{D}_{c//i,\mathcal{G}}$, but because each term is either squared or multiplied with another term, this gives the same result.

To calculate $\mathbf{I}_{c/c,\mathcal{G}}$, the combined inertial properties with respect to $\vec{c_c}$ in the $\mathcal{G}$-frame, we apply the following:

$$\mathbf{I}_{c/c,\mathcal{G}} = \mathbf{I}_{i/c,\mathcal{G}} + m_i \cdot \mathbf{D}_{i//c,\mathcal{G}} + \mathbf{I}_{s/c,\mathcal{G}} + m_s \cdot \mathbf{D}_{s//c,\mathcal{G}}, \tag{21}$$

which is the 3D parallel axes theorem [15, 10].

Manually calculated, the inertial properties are: (7543.59, 31239.0, 29264.4, -6318.78, 9478.17, 2369.54) kgm^2, using the same element order as defined in 7.1. After applying the parallel axes theorem, MuSkeMo computes the inertial properties as: (7543.47,31238.7,29264.1,-6318.73,9478.10,2369.53) kgm^2.

As an extra internal validation step, it is also possible to combine the meshes from Fig. 5 into a single composite mesh, using Blender's join command. If inertial properties for this composite mesh are computed using MuSkeMo, MuSkeMo treats them as a single (but disjointed) mesh, and uses Eberly's algorithm to compute the combined rigid body parameters. This gives numerically identical results, even though the "composite body" approach uses the parallel axes theorem to combine two mesh inertial properties, whereas the "composite mesh" approach computes the inertial properties as if they were one single mesh.

D.3 Re-expressing matrices and vectors in arbitary frames

When assigning a local frame (see 7.7) to a BODY , MuSkeMo also expresses the parameters of all associated model components with respect to that local frame (see 7). The scripts that compute these matrices are reused throughout MuSkeMo, so we will demonstrate their implementation by expressing the composite rigid body parameters from D.2 with respect to a local frame.

The frame is pictured in 5. The frame's global position is $\vec{r_f}$ = (1,1,1) m. Its Euler angles are 45 ° (ϕ_x), 10 ° (ϕ_y), and 25 ° (ϕ_z), see A for a full treatment on Euler angles in MuSkeMo. Thus, the rotation matrix ${}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}}$, which rotates a vector from the local frame to the global frame, follows from equation A.1 as:

$${}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}} = \begin{pmatrix} 0.892539 & -0.416198 & 0.173648 \\ 0.410120 & 0.588964 & -0.696364 \\ 0.187553 & 0.692749 & 0.696364 \end{pmatrix} \tag{22}$$

To express the combined center of mass $\vec{c_c}$ from D.2 with respect to the local frame, we will start explitcly defining in which frames the vectors are expressed, $\mathcal{G}$ for the global frame, and $\mathcal{B}$ for the local frame (the B stands for body-fixed). To compute the transformation manually, we use:

$$\vec{c}_{c,\mathcal{B}} = {}^{\mathcal{B}}\mathbf{R}_{\mathcal{G}} \cdot (\vec{c}_{c,\mathcal{G}} - \vec{r}_{f,\mathcal{G}}). \tag{23}$$

Calculated manually, this gives: (0.813771, -2.18287, -0.825374) m. When assigning the local frame to the composite body, MuSkeMo computes: (0.813736, -2.18285, -0.825365) m.

To re-express $\mathbf{I}_{c/c,\mathcal{G}}$ in the *mathcalB*-frame (but still with respect to the combined center of mass), we apply equation 2 (as can be found in standard mechanics textbooks [15]):

$$\mathbf{I}_{c/c,\mathcal{B}} = {}^{\mathcal{B}}\mathbf{R}_{\mathcal{G}} \cdot \mathbf{I}_{c/c,\mathcal{G}} \cdot {}^{\mathcal{G}}\mathbf{R}_{\mathcal{B}}. \tag{24}$$

Manual calculations give: (11205.0, 25752.7, 31089.3, 12358.1, 6113.85, -3495.72) kgm^2. MuSkeMo computes: (11204.8, 25752.4, 31089.0, 12358.0, 6113.81, -3495.70) kgm^2.

D.4 Muscle moment arms

Several definitions of muscle moment arms exist [1]. MuSkeMo computes the moment arms using the principle of virtual work [1, 14]. For muscle m crossing a joint with joint angle ϕ_j, the moment arm r_m is determined by changes in the muscle's length l_m as a function of the the joint angle:

$$r_m = -dl_m/d\phi_j \tag{25}$$

The minus sign in eq. D.4 appears due to the convention in musculoskeletal simulators to define muscle contractile force as a positive scalar, even though muscle contraction results in a negative change in length.

Accurate computations of moment arms therefore require correct application of local frames, positions and orientations, require accurate changes in muscle point positions when the joints are rotated, and also require accurate computations of the instantaneous muscle lengths in all of the different joint poses. Fig. 6 compares the moment arms computed by MuSkeMo after importing an OpenSim model, to the moment arms of the same model computed by OpenSim 4.0 using the Matlab api. These are the same data that are combined into three plots in Fig. 3.

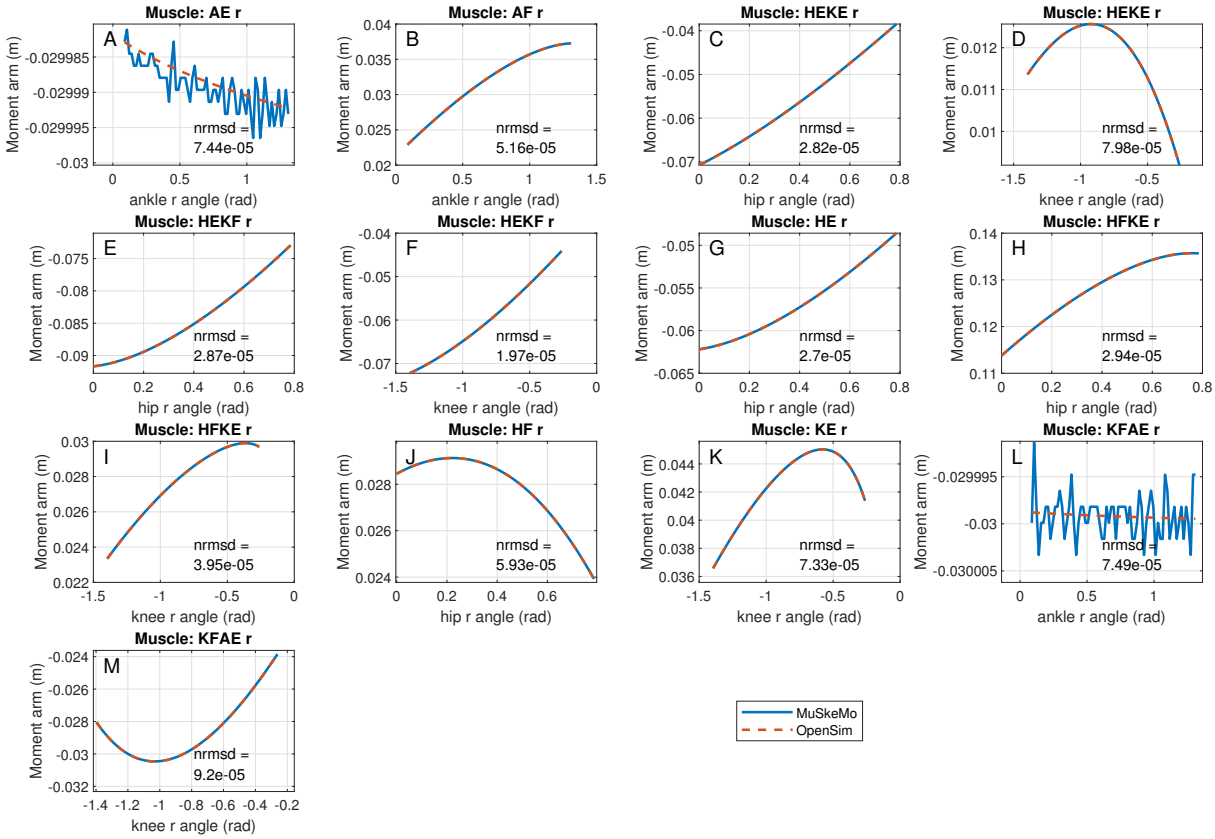

Figure 6: Individual muscle moment arms of the emu model from [17], computed by MuSkeMo and exported as .csv files, compared to those computed by OpenSim 4.0 using the Matlab API. nrmsd stands for "normalized root mean squared differences", rmsd is normalized by the mean value of the moment arm. The average nmrsd over all muscles was 0.0052% of the moment arm magnitudes. Panels A and L show that the moment arms are slightly noisy (in the order of 0.015% the magnitude of the moment arm) when wrapping determines the moment arms in question. As evidenced by similar values of nrmsd, this noise is too small to be relevant in most practical applications (e.g., compare this figure to the same data plotted in Fig. 3, where the noise is not perceptible over a larger range). The python script that performs this moment arm analysis is provided in the Utilities folder 6.17

.

The OpenSim model used for this comparison was the emu model from [17], specifically `Dromaius_model_v4_intermed.osim`. Fig. 6 shows that all the moment arms are numerically identical within expected

precision for Blender, as evidenced by the normalized root mean square differences (nrmsd) not exceeding $9.2 \cdot 10^{-4}$, or 0.0092% of the moment arm magnitudes, with an average of 0.0052% over all the muscles. To compute nrmsd, rmsd was normalized by the mean of the moment arm of each muscle.

Fig. 6 also shows that the moment arms in panels A and L are somewhat noisy. Closer inspection of the vertical axes of those panels reveals that the moment arms are nearly constant for those muscles ($\tilde{=}$0.03 m), because the moment arms are determined by a cylinder with constant diameter radius (0.03 m). Thus, the noise is numerically insignificant (in the order of 0.015% of the moment arm magnitude). The noise is so small in magnitude that it is not visible when plotting multiple muscles in the same plot, with more informative ranges on the y axis (Fig. 3).

While the noise is small enough to make no difference in a practical sense, it is explicitly pointed out here, because readers who want to use MuSkeMo's moment arms directly for their own simulations may want to smooth the moment arms first. The noisy moment arms for wrapping muscles are likely caused by wrapping node (see sec. 7.3.3). A likely possibility is that the many mathematical operations that are performed in the node on single precision digits introduces a floating point error (see A Note on Precision). Another possible culprit may be inaccuracy of the Raycasting operation that geometrically determines intersections between a wrapping geometry and a muscle curve.

The python script (to be run in Blender's script editor) that generates the muscle analysis is provided in MuSkeMo's utilities folder as an example script that calls MuSkeMo's API (sec. 6.17). The Matlab script that generates the plots from Figs. 3 and 6 is also provided.

E Acknowledgements

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