Random Variables

Probability and Statistics

(Chapter 05)

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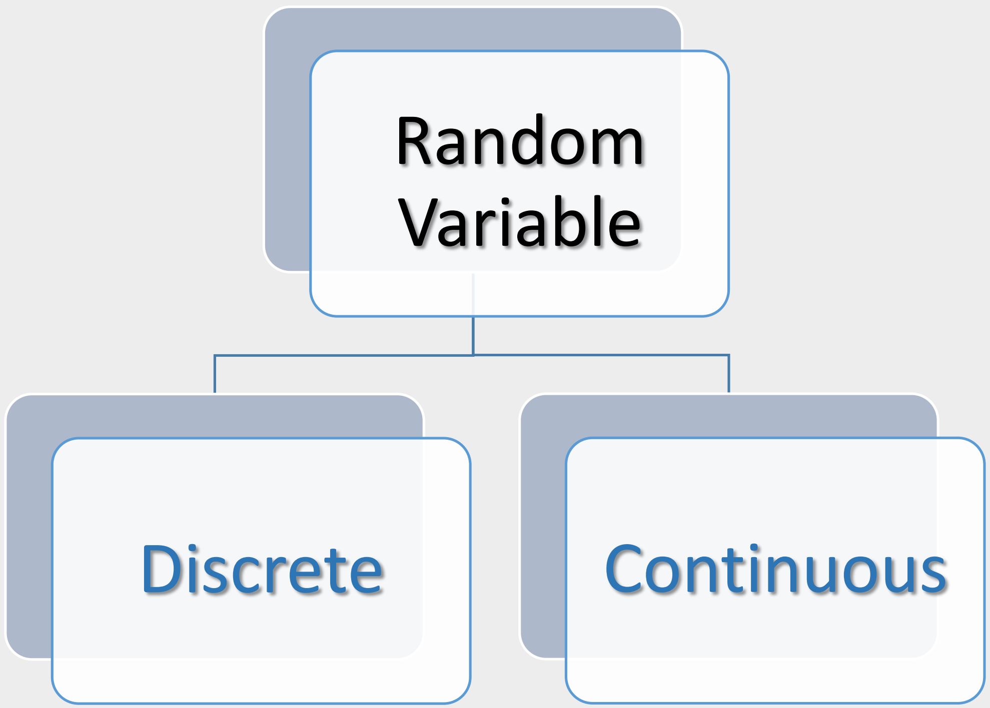
Random variable

- Represents a possible numerical value from an uncertain event.



Discrete Random variable

- Can assume only a countable set of values.

- Ex: Rolling a six-sided fair dice. Let X be the number showing on the dice. Then $P(X)$ is the probability that X getting a certain value.

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

- Set of all possible values a random variable X, together with the associated probability is called **Probability distribution.**

Summery Measures.

Expected Value (Mean):-

$$\mu = E(X) = \sum_{i=1}^{n} x_i P(x_i)$$

Variance

$$\sigma^2 = \sum_{i=1}^{n} \left(x_i - E(X)\right)^2 P(x_i)$$

or

$$s = E(X^2) - \left(E(X)\right)^2$$

Standard deviation

$$\sigma = \sqrt{\sigma^2}$$

Summery Measure

- A Study shows that the number of passengers in a car (including driver) between 7.00 am to 8.00 am in Colombo kandy road is likely to assume the values 1,2,3,4.Let X be the number of passengers in a car and consider the following probability distribution.

X	1	2	3	4	5
P(X)	0.5	0.2	a	0.15	0.05

i) **Find the value of a.**

ii) **Find E(X).**

iii) **Find the variance and the standard deviation.**

iv) **Find** $P(X = 3)$

v) **Find** $P(X \geq 3)$.

Random Variable as a Function.

A random variable is a function defined on the sample space Ω ,

$$X : \Omega \rightarrow \mathrm{R}$$

Where Ω (Sample Space) is the domain and R (Set of real numbers) is Codomain.

Ex:- Consider a toss of an unbalanced coin with P(Head)=1/3 and P(Tail)=2/3.

then, $X : \Omega \rightarrow R$ where,

X = number of heads in the given outcome and Ω={Head,Tail}

And $P(X = 1) = \frac{1}{3}$ and $P(X = 0) = \frac{2}{3}$

Bernoulli Distribution

Bernoulli distribution is a discrete probability distribution which has exactly two outcomes.

X	0	1
P(X)	1-p	p

Examples ………………….???????

Binomial Distribution

If we have n number of Bernoulli trials with probability of success equal to p then the probability of r successes is given by,

$$\boldsymbol{P(X = r) =^{n} C_{r}\, p^{r}(1 - p)^{r}}$$

$n = number\ of\ trials$

$r = number\ of\ successes$

$p = probability\ of\ success$

Binomial Distribution

The probability that Sri Lanka **wins** a one day cricket match with India in a home ground is **0.6**. Suppose a Sri Lanka Vs. India one day series has **7** matches. Assuming that there is no possibility to be drawn a match find,

1) The probability that Sri Lanka wins exactly 4 matches.
2) The Probability that Sri Lanka wins the series.
3) The probability that India wins the series.

Binomial Distribution

The probability of getting **head** in a toss of a biased coin is **0.4**. If the coin is tossed **10** times find the probability that,

1) Getting head exactly 7 times.
2) Getting head more than 7 times.
3) Getting heads in-between 4 and 7.

Binomial Distribution

If X has a binomial distribution with parameters n and p then $E(X) = np$ and $Var(X) = np(1-p)$

Example:

A random variable X has a binomial distribution with parameters $n = 100$ and $P = 0.8$. Find the mean and the variance of X.

Binomial Distribution (Home Work)

A random variable X has a binomial distribution with parameters $n = 6$ and $P = 0.8$. Find the mean and the variance of X.

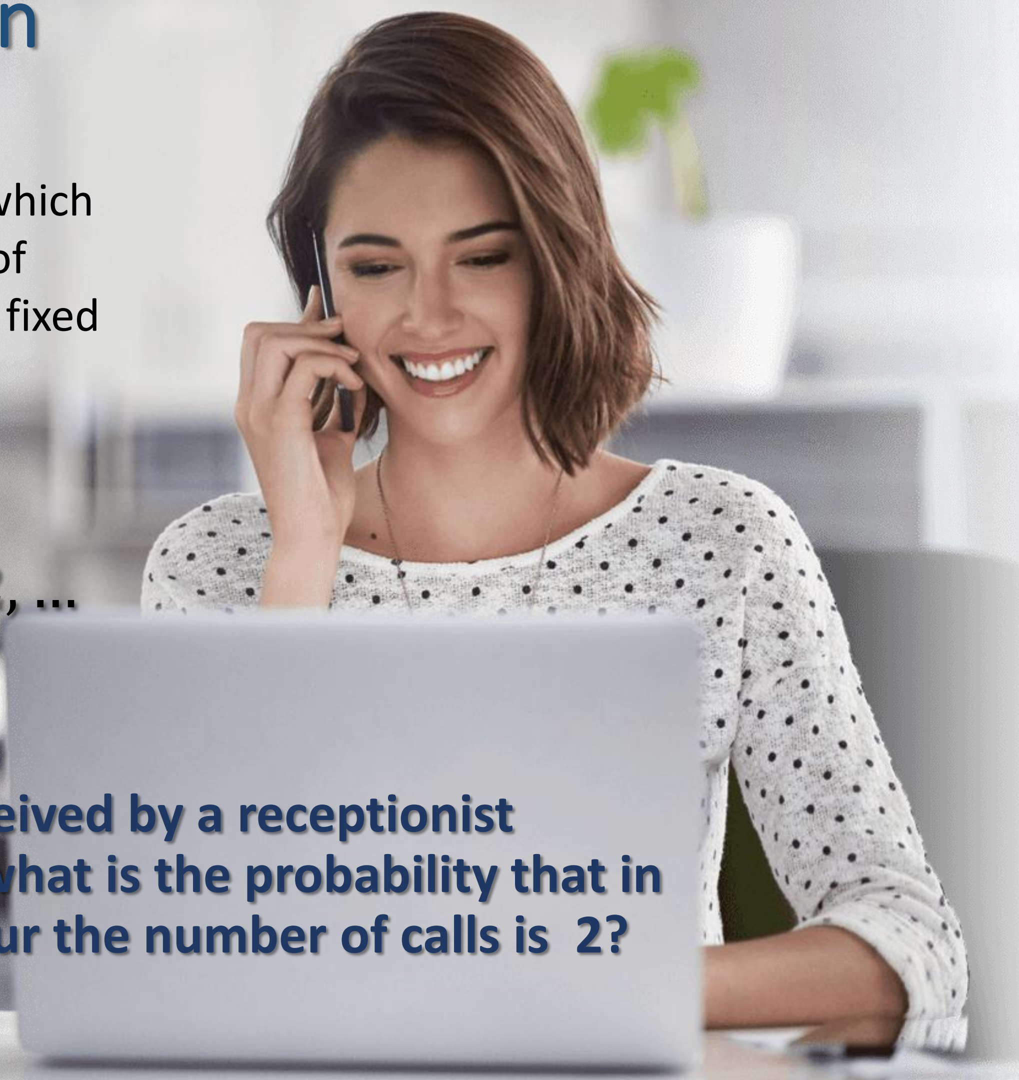
Complete the following table and justify your answer by calculating the mean and the variance using its definition.

X	0	1	2	3	4	5	6
P(X=r)							

Poisson Distribution

A discrete frequency distribution which gives the probability of a number of independent events occurring in a fixed time or space.

$$P(X=r)=\frac{e^{-\lambda}\lambda^{r}}{r!}\quad ; r = 0, 1, 2, \ldots$$



- **The number of calls received by a receptionist averages 1.5 per hour, what is the probability that in a randomly selected hour the number of calls is 2?**

Poisson Distribution

A discrete frequency distribution which gives the probability of a number of independent events occurring in a fixed time or space.

$$P(X=r)=\frac{e^{-\lambda}\lambda^{r}}{r!} \quad ; r = 0,1,2,\ldots$$

Note:-

(i) $E(X) = \lambda$ and $V(X) = \lambda$

(ii) If X is Poisson with mean λ then kX is Poisson with mean $k\lambda$.

(iii) If a binomial distribution with parameters n and p where p is very small such that $(1-p) \approx 1$, X can be approximated by Poisson random variable of mean np.

Poisson Distribution

Question 1: Suppose that in late summer, the Fremantle surf Life saving club makes an average of two surf rescues per day. Using the Poisson probability distribution determine the probability that

a) More than two rescues are made on a particular day.

b) Five surf rescues are made in a 3- day period.

Question 2: The probability that a car has a defective gearbox is 0.02. If I check the gearboxes of 140 cars what is the probability that I find

a) Two defectives.

b) More than five defectives.

c) Fever than 4 defectives.

Properties

Let X & Y be to random variables.

- $E[g(X)] = \sum_x g(x) Pr(X = x)$
- $E(c) = c$.
- $E[g(X) + c] = E[g(X)] + c$.
- $E[cg(X)] = cE[g(X)]$.
- $E(X + Y) = E(X) + E(Y)$.
- $Var[g(X) + c] = Var[g(X)]$.
- $Var[cg(X)] = c^2 Var[g(X)]$.
- $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$.
- If X & Y are independent, $Cov(X, Y) = 0$

Self Study

- Covariance
- Expected Value of the sum of two random variables
- Variance of constant time a variable
- Standard deviation of the sum of two random variables:

THE END

Next: Continuous Random variables.