CO543: Image Processing
Lab 8 – Fourier Transformation

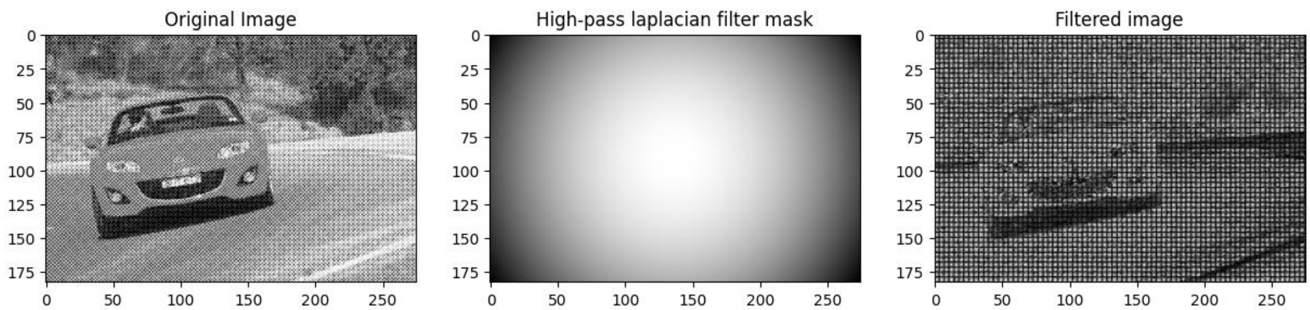
Ranage R.D.P.R. - E/19/310

The following functions are used throughout the lab to plot each figure to show the results.

```python
import cv2
import numpy as np
import matplotlib.pyplot as plt

def show_2_images(image1, image2, title1, title2):
    fig, ax = plt.subplots(1, 2, figsize=(10, 5))

    ax[0].imshow(image1, cmap='gray')
    ax[0].set_title(title1)

    ax[1].imshow( image2, cmap='gray')
    ax[1].set_title(title2)

    plt.show()

def show_3_images(image1, image2, image3, title1, title2, title3):
    fig, ax = plt.subplots(1, 3, figsize=(15, 5))

    ax[0].imshow(image1, cmap='gray')
    ax[0].set_title(title1)

    ax[1].imshow( image2, cmap='gray')
    ax[1].set_title(title2)

    ax[2].imshow( image3, cmap='gray')
    ax[2].set_title(title3)

    plt.show()
```

1. Apply high pass laplacian filter on Car.jpg image.

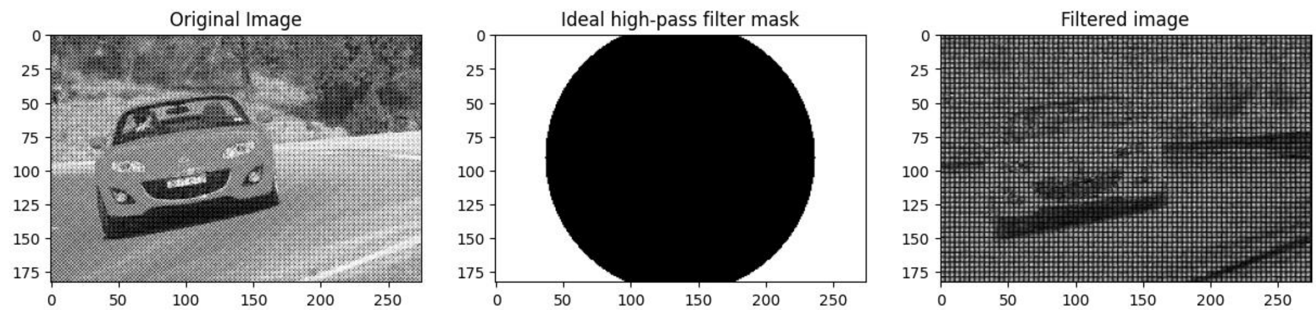
```
# Reading the image
img = plt.imread('car-2.jpg')
gray_img = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)

def high_pass_laplacian_filter(img):
    # Convert image to float32 for DFT processing
    img_float32 = np.float32(img)

    # Compute 2D Fourier transform
    dft = cv2.dft(img_float32, flags=cv2.DFT_COMPLEX_OUTPUT)
    dft_shift = np.fft.fftshift(dft)

    # Create Laplacian high-pass filter mask
    rows, cols = img.shape
    crow, ccol = rows // 2, cols // 2
    mask = np.ones((rows, cols, 2), np.uint8)

    # Create a Laplacian filter in the frequency domain
    X = np.linspace(-ccol, ccol, cols)
    Y = np.linspace(-crow, crow, rows)
    X, Y = np.meshgrid(X, Y)
    laplacian_mask = -4 * (np.pi**2) * (X**2 + Y**2)
    laplacian_mask[crow, ccol] = 1  # Avoiding division by zero

    laplacian_mask = np.dstack((laplacian_mask, laplacian_mask))

    # Apply Laplacian high-pass filter
    fshift = dft_shift * laplacian_mask

    # Inverse shift to bring back to original position
    f_ishift = np.fft.ifftshift(fshift)

    # Inverse DFT to get the filtered image
    img_back = cv2.idft(f_ishift)
    img_back = cv2.magnitude(img_back[:,:,0], img_back[:,:,1])

    # Normalize the filtered image to 0-255
    img_back = cv2.normalize(img_back, None, 0, 255, cv2.NORM_MINMAX, dtype=cv2.CV_8U)

    return img_back, laplacian_mask[:,:,0]

filtered_image, mask = high_pass_laplacian_filter(gray_img)
show_3_images(gray_img, mask, filtered_image, 'Original Image', 'High-pass laplacian filter mask', 'Filtered image')
```



2. Apply ideal high-pass filter on Car.jpg image for D0=100

```
def ideal_high_pass_filter(img, d0=100):
    img_float32 = np.float32(img)

    dft = cv2.dft(img_float32, flags=cv2.DFT_COMPLEX_OUTPUT)
    dft_shift = np.fft.fftshift(dft)

    # Create ideal high-pass filter mask
    rows, cols = img.shape
    crow, ccol = rows // 2, cols // 2
    mask = np.ones((rows, cols, 2), np.uint8)

    # Create a circular mask with radius d0
    center = (ccol, crow)
    for i in range(rows):
        for j in range(cols):
            if (i - crow) ** 2 + (j - ccol) ** 2 <= d0 ** 2:
                mask[i, j] = 0

    fshift = dft_shift * mask

    # Inverse shift to bring back to original position
    f_ishift = np.fft.ifftshift(fshift)

    # Inverse DFT to get the filtered image
    img_back = cv2.idft(f_ishift)
    img_back = cv2.magnitude(img_back[:,:,0], img_back[:,:,1])

    # Normalize the filtered image to 0-255
    img_back = cv2.normalize(img_back, None, 0, 255, cv2.NORM_MINMAX,
dtype=cv2.CV_8U)

    # Convert mask to uint8 for display
    mask_img = np.uint8(mask[:,:,0] * 255)

    return img_back, mask_img
```

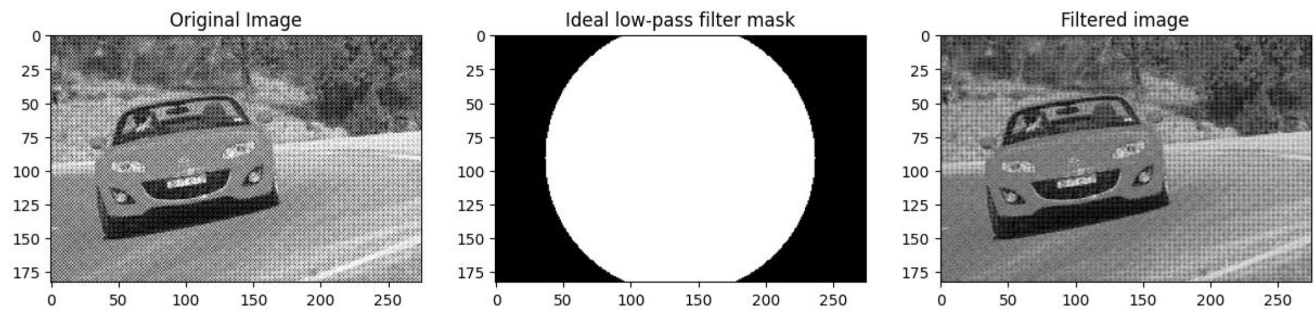
```python
filtered_image, mask = ideal_high_pass_filter(gray_img, 100)
show_3_images(gray_img, mask, filtered_image, 'Original Image', 'Ideal high-pass
filter mask', 'Filtered image')
```



3. Apply ideal low-pass filter on Car.jpg image for D0=100

```python
def ideal_low_pass_filter(img, d0=100):
    # Convert image to float32 for DFT processing
    img_float32 = np.float32(img)

    # Compute 2D Fourier transform
    dft = cv2.dft(img_float32, flags=cv2.DFT_COMPLEX_OUTPUT)
    dft_shift = np.fft.fftshift(dft)

    # Create ideal low-pass filter mask
    rows, cols = img.shape
    crow, ccol = rows // 2, cols // 2
    mask = np.zeros((rows, cols, 2), np.uint8)

    # Create a circular mask with radius d0
    center = (ccol, crow)
    for i in range(rows):
        for j in range(cols):
            if (i - crow) ** 2 + (j - ccol) ** 2 <= d0 ** 2:
                mask[i, j] = 1

    # Apply mask in frequency domain
    fshift = dft_shift * mask

    # Inverse shift to bring back to original position
    f_ishift = np.fft.ifftshift(fshift)

    # Inverse DFT to get the filtered image
    img_back = cv2.idft(f_ishift)
    img_back = cv2.magnitude(img_back[:,:,0], img_back[:,:,1])

    # Normalize the filtered image to 0-255
```

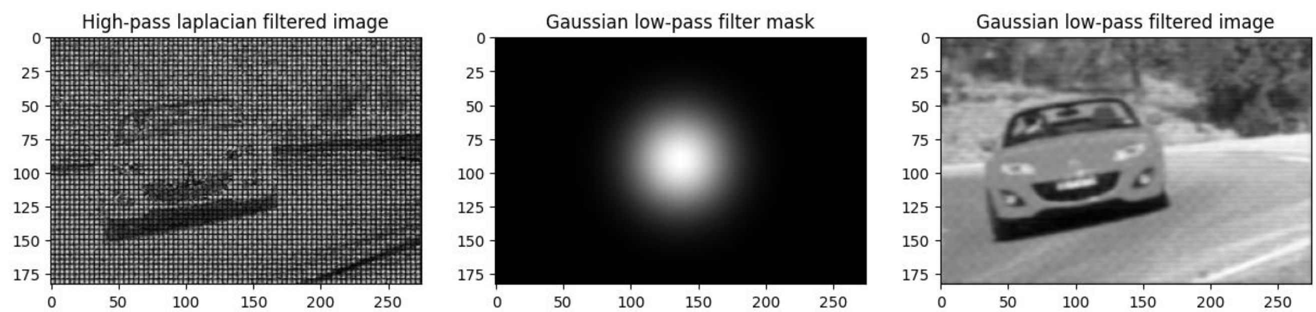
```
    img_back = cv2.normalize(img_back, None, 0, 255, cv2.NORM_MINMAX,
dtype=cv2.CV_8U)

    # Convert mask to uint8 for display
    mask_img = np.uint8(mask[:,:,0] * 255)

    return img_back, mask_img

filtered_image, mask = ideal_low_pass_filter(gray_img, 100)
show_3_images(gray_img, mask, filtered_image, 'Original Image', 'Ideal low-pass
filter mask', 'Filtered image')
```



4. Apply FFT2, IFFT2, low-pass Gaussian filter, and high-pass laplacian filter on Car.jpg image.

```
def low_pass_gaussian_filter(img, sigma=10):
    # Convert image to float32 for DFT processing
    img_float32 = np.float32(img)

    # Compute 2D Fourier transform
    dft = np.fft.fft2(img_float32)
    dft_shift = np.fft.fftshift(dft)

    # Create Gaussian low-pass filter mask
    rows, cols = img.shape
    crow, ccol = rows // 2, cols // 2
    x = np.linspace(-ccol, ccol, cols)
    y = np.linspace(-crow, crow, rows)
    X, Y = np.meshgrid(x, y)
    gaussian_mask = np.exp(- (X**2 + Y**2) / (2 * sigma**2))

    # Apply Gaussian low-pass filter
    low_pass_filtered = dft_shift * gaussian_mask

    # Inverse shift to bring back to original position
    f_ishift = np.fft.ifftshift(low_pass_filtered)

    # Inverse DFT to get the filtered image
```

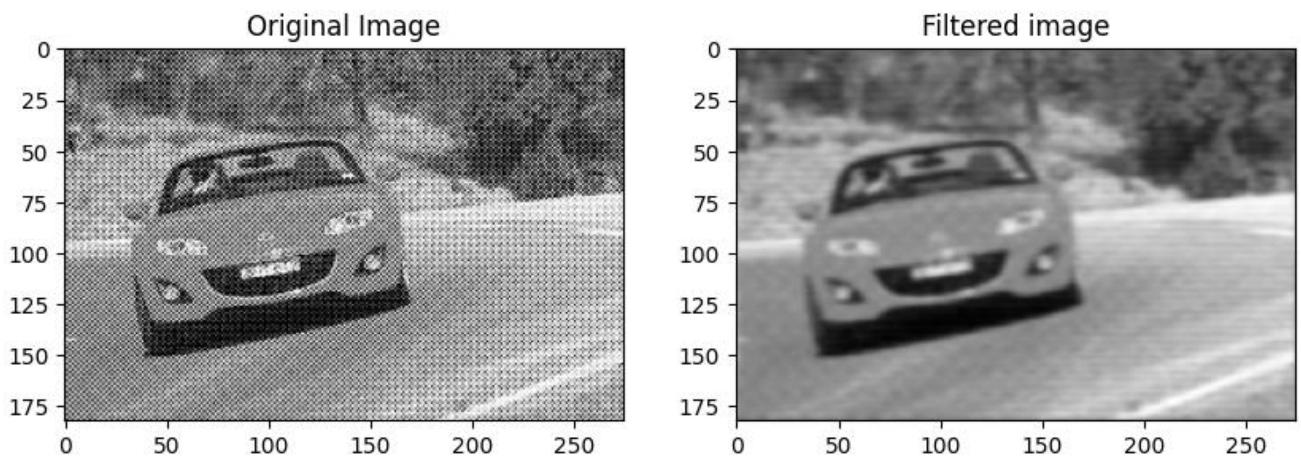
```
    img_back = np.fft.ifft2(f_ishift)
    img_back = np.abs(img_back)

    # Normalize the filtered image to 0-255
    img_back = cv2.normalize(img_back, None, 0, 255, cv2.NORM_MINMAX, dtype=cv2.CV_8U)

    return img_back, gaussian_mask

laplacian_filtered_image, mask = high_pass_laplacian_filter(gray_img)
gaussian_filtered_image, mask = low_pass_gaussian_filter(gray_img, 25)
show_3_images(laplacian_filtered_image, mask, gaussian_filtered_image, 'High-pass laplacian filtered image', 'Gaussian low-pass filter mask', 'Gaussian low-pass filtered image')
```



5. Apply the necessary filter and correct the noise in the image.

```
def gaussian_filter_in_frequency_domain(img, sigma=25):
    img_float32 = np.float32(img)

    # Compute 2D Fourier transform
    transformed_img = np.fft.fft2(img_float32)
    fftshift_img = np.fft.fftshift(transformed_img)

    # Get the height and width of the image
    height, width = img.shape

    # Create a Gaussian filter
    x = np.linspace(-width//2, width//2, width)
    y = np.linspace(-height//2, height//2, height)
    X, Y = np.meshgrid(x, y)
    gaussian_filter = np.exp(-(X**2 + Y**2) / (2 * sigma**2))

    # Apply Gaussian filter in the frequency domain
    filtered_fftshift_img = fftshift_img * gaussian_filter

    # Compute the inverse shift to bring back to original position
    filtered_img = np.fft.ifftshift(filtered_fftshift_img)
```

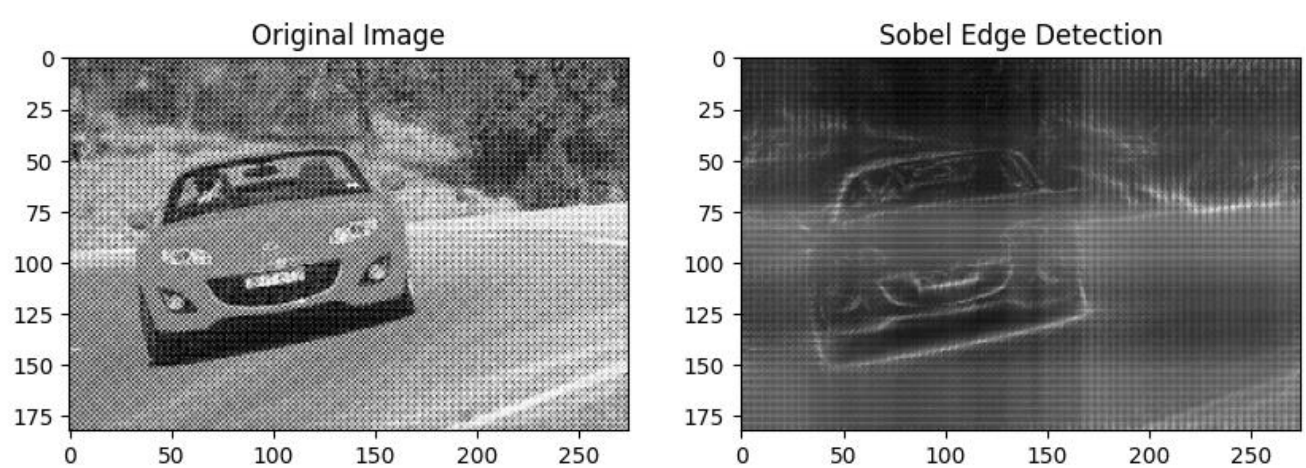
```
    # Compute the inverse DFT to get the filtered image
    filtered_img = np.fft.ifft2(filtered_img)
    filtered_img = np.abs(filtered_img)

    # Normalize the filtered image to 0-255
    filtered_img = cv2.normalize(filtered_img, None, 0, 255, cv2.NORM_MINMAX,
dtype=cv2.CV_8U)

    return filtered_img, gaussian_filter

filtered_image, mask = gaussian_filter_in_frequency_domain(gray_img, 25)
show_2_images(img, filtered_image, 'Original Image', 'Filtered image')
```



6. Apply the sobel operator (filter) on Car.jpg in the Fourier domain to detect edges.

```
def sobel_filter(img):
# Convert image to float32 for DFT processing
    img_float32 = np.float32(img)

    # Compute 2D Fourier transform
    dft = np.fft.fft2(img_float32)
    dft_shift = np.fft.fftshift(dft)

    # Get dimensions of the image
    rows, cols = img.shape
    crow, ccol = rows // 2 , cols // 2

    # Create frequency domain grid
    u = np.fft.fftfreq(cols).reshape(1, -1)
    v = np.fft.fftfreq(rows).reshape(-1, 1)
```

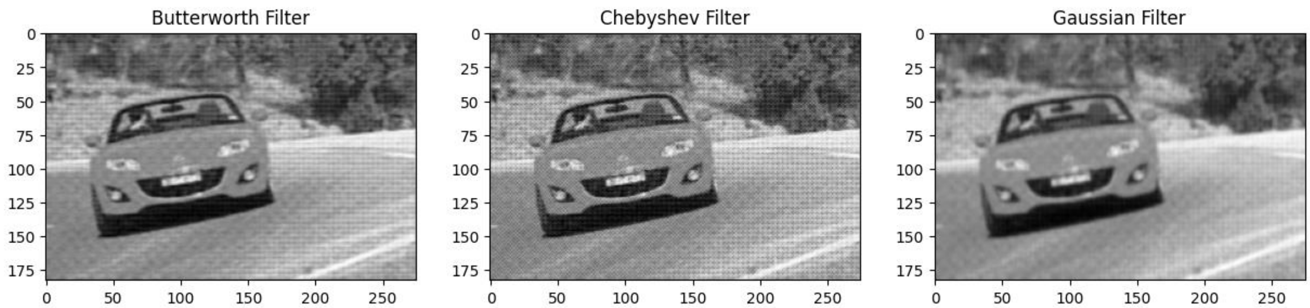
```
    # Sobel filters in the frequency domain
    sobel_x_freq = 1j * 2 * np.pi * u
    sobel_y_freq = 1j * 2 * np.pi * v

    # Apply Sobel filters in the frequency domain
    dft_shift_sobel_x = dft_shift * sobel_x_freq
    dft_shift_sobel_y = dft_shift * sobel_y_freq

    # Inverse shift to bring back to original position
    sobel_x_img = np.fft.ifftshift(dft_shift_sobel_x)
    sobel_y_img = np.fft.ifftshift(dft_shift_sobel_y)

    # Inverse Fourier transform to obtain filtered images
    sobel_x_filtered = np.fft.ifft2(sobel_x_img)
    sobel_y_filtered = np.fft.ifft2(sobel_y_img)

    # Compute magnitude of gradients
    edge_detected_img = np.sqrt(np.abs(sobel_x_filtered)**2 + np.abs(sobel_y_filtered)**2)

    # Normalize the edge-detected image to 0-255
    edge_detected_img = cv2.normalize(np.abs(edge_detected_img), None, 0, 255, cv2.NORM_MINMAX, dtype=cv2.CV_8U)

    return edge_detected_img

sobel_filtered_image = sobel_filter(gray_img)
show_2_images(gray_img, sobel_filtered_image, 'Original Image', 'Sobel Edge Detection')
```



7. Discuss applying Butterworth and Chebyshev filters and compare the output image with the Gaussian Filter image

```
from scipy import fftpack

def butterworth_filter(image, cutoff, n):
    fft_image = fftpack.fft2(image)
    fft_shifted = fftpack.fftshift(fft_image)

    rows, cols = image.shape
    center_row, center_col = rows // 2, cols // 2

    # Create Butterworth Filter
    butterworth_filter = np.zeros((rows, cols))
    for i in range(rows):
        for j in range(cols):
            dist = np.sqrt((i - center_row)**2 + (j - center_col)**2)
            butterworth_filter[i, j] = 1 / (1 + (dist / cutoff)**(2*n))

    # Apply the filter
    filtered = fft_shifted * butterworth_filter
    filtered = fftpack.ifftshift(filtered)
    filtered_image = fftpack.ifft2(filtered).real

    return np.uint8(filtered_image)

def chebyshev_filter(image, cutoff, n, ripple_db):
    fft_image = fftpack.fft2(image)
    fft_shifted = fftpack.fftshift(fft_image)

    rows, cols = image.shape
    center_row, center_col = rows // 2, cols // 2

    # Create Chebyshev Filter
    chebyshev_filter = np.zeros((rows, cols))
    for i in range(rows):
        for j in range(cols):
            dist = np.sqrt((i - center_row)**2 + (j - center_col)**2)
            chebyshev_filter[i, j] = 1 / np.sqrt(1 + (dist / cutoff)**(2*n))

    # Apply the filter
    filtered = fft_shifted * chebyshev_filter
    filtered = fftpack.ifftshift(filtered)
    filtered_image = fftpack.ifft2(filtered).real

    return np.uint8(filtered_image)
```

```python
def gaussian_filter(image, sigma=1.0):
    return cv2.GaussianBlur(image, (0, 0), sigma)

image = cv2.imread('car-2.jpg', cv2.IMREAD_GRAYSCALE)
butterworth_filtered = butterworth_filter(image, 50, 2)
chebyshev_filtered = chebyshev_filter(image, 50, 2, 1)
gaussian_filtered = gaussian_filter(image, 1.2)
show_3_images(butterworth_filtered, chebyshev_filtered, gaussian_filtered,
'Butterworth Filter', 'Chebyshev Filter', 'Gaussian Filter')
```



The Butterworth filter provides a trade-off between preserving sharp edges and smoothing, making it useful for applications that require a certain level of detail preservation.

The Chebyshev filter is designed to achieve exact edge enhancement or detection by providing sharp filtering with controlled passband ripple.

The Gaussian Filter is used to generate a blurred or smoothed image, which is particularly useful for reducing noise and achieving a general-purpose smoothing effect.