A13 - NORMAL VECTOR CALCULATIONS

1. SINCOS FUNCTION

$y = \sin(x)\cos(z)$

$$\vec{f}(x,z) = \begin{bmatrix} x \\ \sin(x)\cos(z) \\ z \end{bmatrix} \qquad \frac{\partial \vec{f}}{\partial x} = \begin{bmatrix} 1 \\ \cos(x)\cos(z) \\ 0 \end{bmatrix} \qquad \frac{\partial \vec{f}}{\partial z} = \begin{bmatrix} 0 \\ -\sin(x)\sin(z) \\ 1 \end{bmatrix}$$

$$\frac{\partial \vec{f}}{\partial x} \times \frac{\partial \vec{f}}{\partial z} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \cos(x)\cos(z) & 0 \\ 0 & -\sin(x)\sin(z) & 1 \end{bmatrix} = (\cos(x)\cos(z))\,\hat{i} + \hat{j} + (-\sin(x)\sin(z))\,\hat{k}$$

$$\left\| \frac{\partial \vec{f}}{\partial x} \times \frac{\partial \vec{f}}{\partial z} \right\| = \sqrt{\cos^2 x \cos^2 z + 1 + \sin^2 x \sin^2 z} = \text{modulus}$$

$$n_x = \frac{\cos(x)\cos(z)}{\text{modulus}} \qquad n_y = \frac{1}{\text{modulus}} \qquad n_z = -\frac{\sin(x)\sin(z)}{\text{modulus}}$$

NOTE: THE NORMAL VECTORS OBTAINED POINT DOWNWARDS. IN THE ASSIGNMENT THE COMPONENTS n_x, n_y, n_z ARE MULTIPLIED BY -1

2. SPHERE FUNCTION

$$\vec{f}(\phi,\theta) = \begin{bmatrix} r\sin\phi\cos\theta \\ r\sin\phi\sin\theta \\ r\cos\phi \end{bmatrix}, \quad r \text{ CONSTANT}, \quad 0 \leq \phi \leq \pi, \quad 0 \leq \theta \leq 2\pi$$

$$\frac{\partial \vec{f}}{\partial \phi} = \begin{bmatrix} r\cos\phi\cos\theta \\ r\cos\phi\sin\theta \\ -r\sin\phi \end{bmatrix} \qquad \frac{\partial \vec{f}}{\partial \theta} = \begin{bmatrix} -r\sin\phi\sin\theta \\ r\sin\phi\cos\theta \\ 0 \end{bmatrix}$$

$$\frac{\partial \vec{f}}{\partial \phi} \times \frac{\partial \vec{f}}{\partial \theta} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ r\cos\phi\cos\theta & r\cos\phi\sin\theta & -r\sin\phi \\ -r\sin\phi\sin\theta & r\sin\phi\cos\theta & 0 \end{bmatrix} = (r^2\sin^2\phi\cos\theta)\,\hat{i} + (r^2\sin^2\phi\sin\theta)\,\hat{j} + (r^2\cos\phi\sin\phi)\,\hat{k}$$

$$\left\| \frac{\partial \vec{f}}{\partial x} \times \frac{\partial \vec{f}}{\partial z} \right\| = r^2\sin\phi\sqrt{\sin^2\phi\cos^2\theta + \sin^2\phi\sin^2\theta + \cos^2\phi} = r^2\sin\phi$$

$$n_x = \frac{\cancel{r^2}\sin^{\cancel{2}}\phi\cos\theta}{\cancel{r^2}\,\cancel{\sin\phi}} = \sin\phi\cos\theta$$

$$n_y = \frac{\cancel{r^2}\sin^{\cancel{2}}\phi\sin\theta}{\cancel{r^2}\,\cancel{\sin\phi}} = \sin\phi\sin\theta$$

$$n_z = \frac{\cancel{r^2}\cos\phi\,\cancel{\sin\phi}}{\cancel{r^2}\,\cancel{\sin\phi}} = \cos\phi$$