A01 - PAINT BY NUMBERS

DRAW LINES

drew Compass → CIRCLE WITH RADIUS & GRADES (MANY LINES)

RESOLUTION → HOW MANY LINES TO GET A GOOD CIRCULAR LINE

A02 - BASIC TRANSFORM

T1 → TRANSLATE → LAST COLUMN

R1 → ROTATE → $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (X AXIS)

S1 → SCALING → MAIN DIAGONAL

S2 → NON-UNIFORM SCALING → MAIN DIAGONAL ON SOME ROWS

S3 → MIRROR → SCALING OF -1 OVER THE OTHER AXIS

S4 → FLATTEN → SCALE OF 0 ON AN AXIS

S5 → SHEAR → $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ → $z' = y + z$

A03 - ADVANCED TRANSFORMS

① $T(0,-1,1)^* R_y(15°)^* R_z(45°)^* R_x(60°)^* R_z(45°)^{-1*} R_y(15°)^{-1*} T(0,-1,1)^{-1}$

X AXIS ALIGNED WITH AXIS PASSING THROUGH $(0,1,-1)$

② $R_z(45°)^* S_x(0,5)^* R_z(45°)^{-1}$

ALIGNING X WITH BISECTION OF XY

③ $T(1,1,1)^* R_x(15°)^* S_y(-1)^* R_x(15°)^{-1*} T(1,1,1)^{-1}$

MIRRORING ON XZ AXIS

④ THE INVERSE ORDER IS THE OPPOSITE OF THE SEQUENCE: $R_y(-30°)^* T(0,0,-5)^* S(1/3)$

(IT WORKS, THE CODE ORDER IS DIFFERENT FROM THE EX. TEXT)

A04 - AXONOMETRY

getOrthProjMatrix → MAKES AN ORTHOGONAL PROJECTION MATRIX

① ISOMETRIC → $Orth^* R_x(35,26°)^* R_y(45°)$

② DIMETRIC → $Orth^* R_x(\underset{20°}{\alpha})^* R_y(45°)$

③ TRIMETRIC $\rightarrow Orth * R_x(\overset{-30°}{\alpha}) * R_y(\overset{30°}{\beta})$

④ CAVALIER $\rightarrow Orth * Sh_z(-\cos\alpha, -\sin\alpha)$ $\xrightarrow{\rho = 1}$

⑤ CABINET $\rightarrow Orth * Sh_z(-0{,}5\cos\alpha, -0{,}5\sin\alpha)$ $\xrightarrow{\rho = 0{,}5}$

$$\begin{bmatrix} 1 & 0 & -\rho\cos\alpha & 0 \\ 0 & 1 & -\rho\sin\alpha & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A05 - PERSPECTIVE

getPersMatrix
- IMPLEMENTATION WITH NORMALIZED SCREEN COORDINATES
- IMPLEMENTATION WITH ASPECT RATIO

ALL EXERCISES OVERLOAD SUCH FUNCTIONS

A06 - VIEW

FUNCTIONS THAT COMPUTE LOOK-IN AND LOOK-AT MATRICES FOR THE CAMERA. BY INVERTING THEM WE GET THE VIEW MATRIX

LOOK-IN: $M_c = T(\underbrace{c_x, c_y, c_z}_{\text{POSITION}}) * R_y(\alpha) * R_x(\beta) * R_z(\gamma)$

LOOK-AT: $M_c = \left[\begin{array}{ccc|c} V_x & V_y & V_z & c \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

ALL EXERCISES OVERLOAD SUCH FUNCTIONS

A07 - WORLD MATRIX

yaw $= R_y(\alpha)$ pitch $= R_x(\alpha)$ roll $= R_z(\alpha)$

① $T(0,0,-3) * R_y(90°)$

② $T(0,2,0) * R_x(60°) * S(0,1)$

③ $R_y(30°) * R_z(40°)$

④ $T(2,0,2) * R_y(180°) * S_{NU}(2,1,1)$

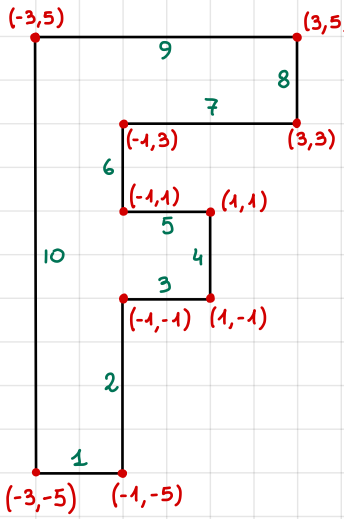
⑤ $T(1,-1,2.5) * R_y(-30°) * R_x(45°) * R_z(-15°) * S_{NU}(0.8, 0.75, 1.2)$

A08 - QUATERNION ROTATION

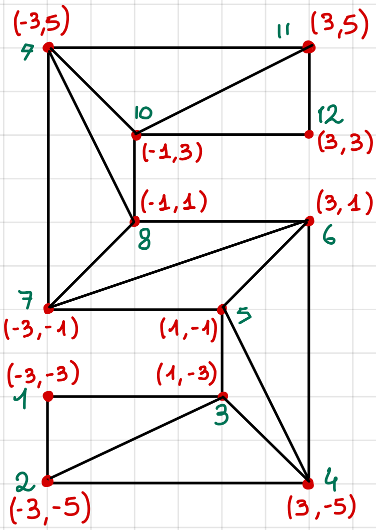
QUATERNION LIBRARY

EULER ANGLES $\xrightarrow{\text{QUATERNION.fromEuler}}$ QUATERNION $\xrightarrow{\text{toMatrix4}}$ WORLD MATRIX

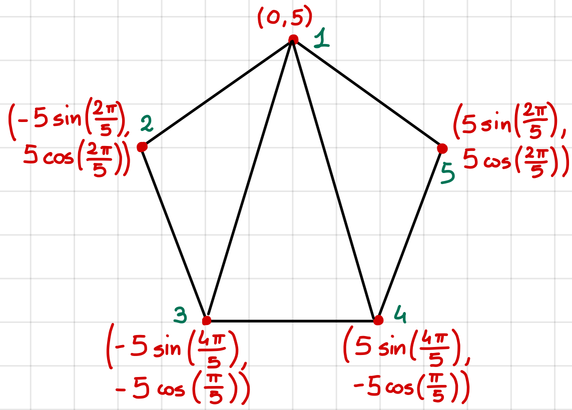
A09 - OUTLINES



LINE LOOP
(NO LAST POINT REPETITION)



TRIANGLE STRIPS
(KEEP LAST TWO POINTS AND ADD THE THIRD ONE)



TRIANGLE FAN
(FIRST POINT IN ALL TRIANGLES + KEEP LAST POINT)

A10 - INDEXED PRIMITIVES

① SINCOS WAVE

PRECISION → n° OF SQUARES IN THE GEOMETRY $(.025 < p < 1)$
SQUARES → LENGTH

ADD VERTICES BY ITERATING NESTED LOOPS (BY PAYING ATTENTION TO POSITIONS)
- X → VALUE OBTAINED BY ITERATION
- y → FUNCTION $\sin(x)\cos(z)$
- z → VALUE OBTAINED BY ITERATION

INDICES ADDED TOO BY NESTED LOOPS, CREATING A SQUARE (6 INDICES) PER ITERATION

② HALF-SPHERE

SIMILAR SETUP TO THE WAVE, BUT:
- SPHERICAL COORDINATES AS VERTICES (TO GET A COMPLETE SPHERE)
- y IS ZERO IF IT IS < 0, OTHERWISE IT FOLLOWS THE SPHERICAL COORDINATES
- SAME INDEXING TECHNIQUE

A11 - GLSL LIGHTS

① SIMPLE

② DIRECTION → $\text{norm}(\vec{lx} - pos)$

③ DIRECTION → $\text{norm}(\vec{lx} - pos)$
LIGHT COLOR → $\text{color} * \text{clamp}\left(\frac{\cos\alpha - out}{in - out}\right)$

④ DIRECTION → $\text{norm}(\vec{lx} - pos)$
LIGHT COLOR → $\text{color} * (\text{target}/\text{len}(\text{lightPos} - \text{vertPos}))^{\text{decay}}$

⑤ DIRECTION → $\text{norm}(\vec{lx} - pos)$
LIGHT COLOR → $\text{color} * (\text{target}/\text{len}(\text{lightPos} - \text{vertPos}))^{\text{decay}} * \text{clamp}\left(\frac{\cos\alpha - out}{in - out}\right)$

⑥ DIRECTION → $\text{norm}(\vec{lx} - pos)$
HEMISPHERIC → $\frac{\text{NORMAL} \cdot \text{UP DIR} + 1}{2}\,\text{lightColorUp} + \frac{1 - \text{NORMAL} \cdot \text{LIGHT DIR}}{2}\,\text{lightColorDown}$

⑦ DIRECTION → $norm(\vec{lx} - pos)$

LIGHT COLOR → $color * clamp\left(\frac{\cos\alpha - out}{in - out}\right)$

SPHERICAL HARMONICS → $SH_{color} + normal.x * \Delta L_{SH,x} + normal.y * \Delta L_{SH,y} + normal.x * \Delta L_{SH,z}$

A12 - GLSL BRDF

① LAMBERT = $clamp(\vec{lx} \cdot n_x) * lightColor$

AMBIENT = $ambientLight * ambientColor$

② LAMBERT = $clamp(\vec{lx} \cdot n_x) * lightColor$

HALF-WAY VECTOR = $norm(\vec{lx} + eyeDir)$;

BLINN = $clamp(n_x \cdot$ HALF-WAY VECTOR$) * lightColor$

③ REFLECTION = $-reflect(\vec{xl}, n_x)$;

PHONG = $clamp(eyeDir \cdot$ REFLECTION$) * lightColor$

AMBIENT = $ambientLight * ambientColor$

④ LAMBERT = $clamp(\vec{lx} \cdot n_x) * lightColor$

REFLECTION = $-reflect(\vec{xl}, n_x)$;

PHONG = $clamp(eyeDir \cdot$ REFLECTION$) * lightColor$

AMBIENT = $ambientLight * ambientColor$

EMISSION = emit

⑤ TOON DIFFUSE → MAX MODELS IF/ELSE BEHAVIOR

TOON SPECULAR ↗

A13 - SMOOTH OBJECTS

1. CUBE

EVERY LINE OF CODE IS A FACE OF THE CUBE

2. SINCOS FUNCTION

$y = \sin(x)\cos(z)$

$$\vec{f}(x,z) = \begin{bmatrix} x \\ \sin(x)\cos(z) \\ z \end{bmatrix} \qquad \frac{\partial \vec{f}}{\partial x} = \begin{bmatrix} 1 \\ \cos(x)\cos(z) \\ 0 \end{bmatrix} \qquad \frac{\partial \vec{f}}{\partial z} = \begin{bmatrix} 0 \\ -\sin(x)\sin(z) \\ 1 \end{bmatrix}$$

$$\frac{\partial \vec{f}}{\partial x} \times \frac{\partial \vec{f}}{\partial z} = \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \cos(x)\cos(z) & 0 \\ 0 & -\sin(x)\sin(z) & 1 \end{bmatrix} = (\cos(x)\cos(z))\,\hat{i} + \hat{j} + (-\sin(x)\sin(z))\,\hat{k}$$

$$\left\| \frac{\partial \vec{f}}{\partial x} \times \frac{\partial \vec{f}}{\partial z} \right\| = \sqrt{\cos^2 x \cos^2 z + 1 + \sin^2 x \sin^2 z} = \text{modulus}$$

$$n_x = \frac{\cos(x)\cos(z)}{\text{modulus}} \qquad n_y = \frac{1}{\text{modulus}} \qquad n_z = -\frac{\sin(x)\sin(z)}{\text{modulus}}$$

NOTE: THE NORMAL VECTORS OBTAINED POINT DOWNWARDS. IN THE ASSIGNMENT THE COMPONENTS n_x, n_y, n_z ARE MULTIPLIED BY -1

3. CYLINDER

STRAIGHT NORMALS FOR TOP AND BOTTOM, CIRCULAR COORDINATES FOR SIDE

4. SPHERE FUNCTION

$$\vec{f}(\phi,\theta)=\begin{bmatrix} r\sin\phi\cos\theta \\ r\sin\phi\sin\theta \\ r\cos\phi \end{bmatrix},\quad r \text{ CONSTANT},\quad 0\leqslant\phi\leqslant\pi,\quad 0\leqslant\theta\leqslant 2\pi$$

$$\frac{\partial\vec{f}}{\partial\phi}=\begin{bmatrix} r\cos\phi\cos\theta \\ r\cos\phi\sin\theta \\ -r\sin\phi \end{bmatrix}\qquad \frac{\partial\vec{f}}{\partial\theta}=\begin{bmatrix} -r\sin\phi\sin\theta \\ r\sin\phi\cos\theta \\ 0 \end{bmatrix}$$

$$\frac{\partial\vec{f}}{\partial\phi}\times\frac{\partial\vec{f}}{\partial\theta}=\det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ r\cos\phi\cos\theta & r\cos\phi\sin\theta & -r\sin\phi \\ -r\sin\phi\sin\theta & r\sin\phi\cos\theta & 0 \end{bmatrix}=(r^2\sin^2\phi\cos\theta)\hat{i}+(r^2\sin^2\phi\sin\theta)\hat{j}+(r^2\cos\phi\sin\phi)\hat{k}$$

$$\left\|\frac{\partial\vec{f}}{\partial x}\times\frac{\partial\vec{f}}{\partial z}\right\|=r^2\sin\phi\sqrt{\sin^2\phi\cos^2\theta+\sin^2\phi\sin^2\theta+\cos^2\phi}=r^2\sin\phi$$

$$n_x=\frac{\cancel{r^2}\sin^{\cancel{2}}\phi\cos\theta}{\cancel{r^2}\cancel{\sin\phi}}=\sin\phi\cos\theta$$

$$n_y=\frac{\cancel{r^2}\sin^{\cancel{2}}\phi\sin\theta}{\cancel{r^2}\cancel{\sin\phi}}=\sin\phi\sin\theta$$

$$n_z=\frac{\cancel{r^2}\cos\phi\cancel{\sin\phi}}{\cancel{r^2}\cancel{\sin\phi}}=\cos\phi$$

A14 - UV

UV COORDINATES COMPUTED FROM PDF

CYLINDER → SIDE OBTAINED BY FRACTIONING THE TEXTURE

A15 - UV ANIMATION

HALF-IMAGE IN THE LEFTMOST POINT

1. FAKE-REPEAT STRATEGY → $u = t \% 0{,}25$
   SCALE TO CENTER THE IMAGE

2. DIFFERENT SOLUTIONS USING DIFFERENT METHODS TO MOVE THE UV COORDS

3. ROTATION MATRIX TO MOVE THE UV COORDINATES

4. KEEP CHANGING THE UV COORDINATES TO SEE EVERY FRAME PICTURE

A16 - SCENE

1. UPDATE LOCAL MATRIX WITH WORLD MATRIX FROM PARENT
2. DRAW ELEMENT
3. IF NOT LEAF, RECURSION TO ALL CHILDREN PASSING ITS UPDATED WORLD MATRIX

A17 - ANIMATION

1. QUATERNIONS FROM STARTING ROTATIONS
2. SLERP FOR ROTATION ANIMATION
3. LERP FOR TRANSLATIONS
4. MULTIPLY FOR WORLD MATRIX