AMATH 422/522
Lecture 1
September 27, 2023

Readings:

- "How to Choose a Good Scientific Problem" Uri Alon (Molecular Cell 2009)
- E & G Chapter 1 (§1.0-1.4, 1.7-1.8)

Dynamic Models:

$\boxed{\text{dynamic}} \leftarrow \boxed{\text{model}}$

dynamic → model describes how "system properties change over time"

model → "simplified representations of some real world entity, in equations or computer code"

capture "essential" behavior of this system/process/etc.

This course: building models of biological processes, analyzing them, and drawing conclusions about these systems from our results (i.e., *interpreting* the model analysis).

Models vary in terms of:

- the area of biology being studied;
- the mathematical properties of the model (discrete v. continuous, deterministic v. stochastic);
- the methods of analysis (mathematical v. computational/numerical, data fitting, etc.);
- the purpose of the model (answering theoretical questions, guiding experimental design, managing ecological systems).

Why dynamic models?

- Dynamic models are mechanistic, as opposed to descriptive (e.g., linear regression)

What processes produce observed results?

– OR –

They explicitly take into account relationships between state variables (the variables that summarize interesting/important/observable properties of the system) via a set of equations that describe how they change over time.

(Good for answering questions about causation ✓)

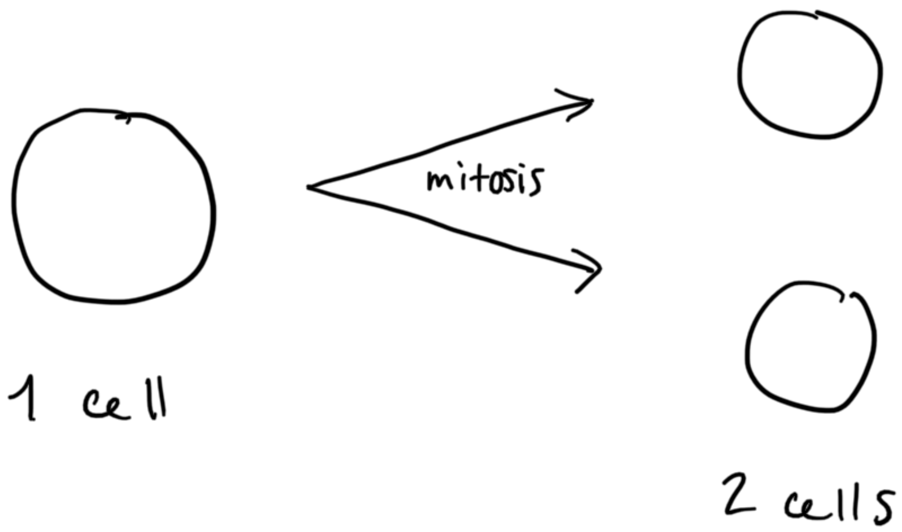
- scientific understanding: model = hypothesis, compare w/ data (→ inform experiments)
- "real-world" interventions: ecol. management, predicting spread of disease, optimize drug treatments

(theoretical v. practical models)

How do we formulate these models?

- build a "conceptual model" of the system at hand: what are the state variables? How do they interact? What are the relevant parameters? (unknown/variable consts.)
- equations developed/derived from "first principles" (+ data, if available) about the system in question: fundamental ideas from physics, chem., and/or bio
- what type of model & how to analyze depends on the purpose of the model (in turn depends on research question: think back to reading by Alon!) and the fundamental properties of the system

An Example: Model of Cell Division



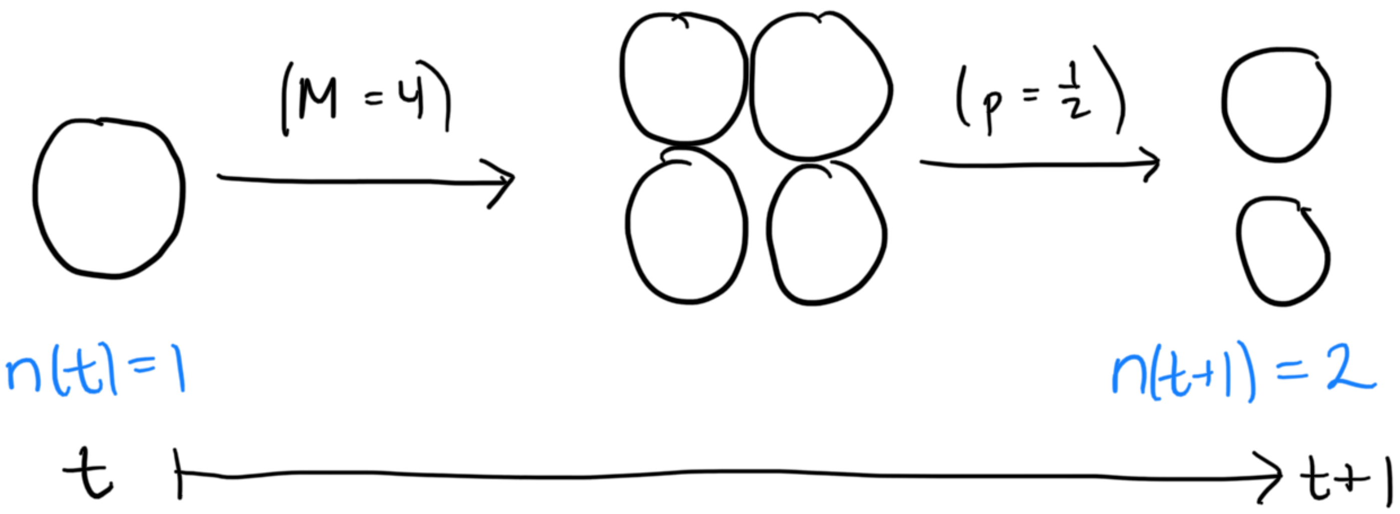
model for $n(t)$ = # cells at time t
(discrete state var.)

stochastic or deterministic?
continuous time or discrete?

Simple model: synchronous division

- instead of keeping track of each individual cell, assume that over 1 generation (discrete time), one cell gives rise to M cells, and some fraction p of these offspring surive to the next round of division

M & p = model parameters



Equations describing this dynamic process:

$$n(t+1) = M \cdot n(t) \cdot p$$

recurrence relation / discrete-time dynamical system

Solution: $\boxed{n(t) = (M \cdot p)^t \, n(0)}$

$$n(0)$$

$$n(1) = Mp \cdot n(0)$$

$$n(2) = Mp \cdot n(1)$$

$$= Mp\,(Kp \cdot n(0))$$

$$= (Mp)^2\, n(0)$$

$$\vdots$$

Demo in Python with $M=4$:

cell_reproduction.ipynb

$p = 0.15, 0.25, 0.95$

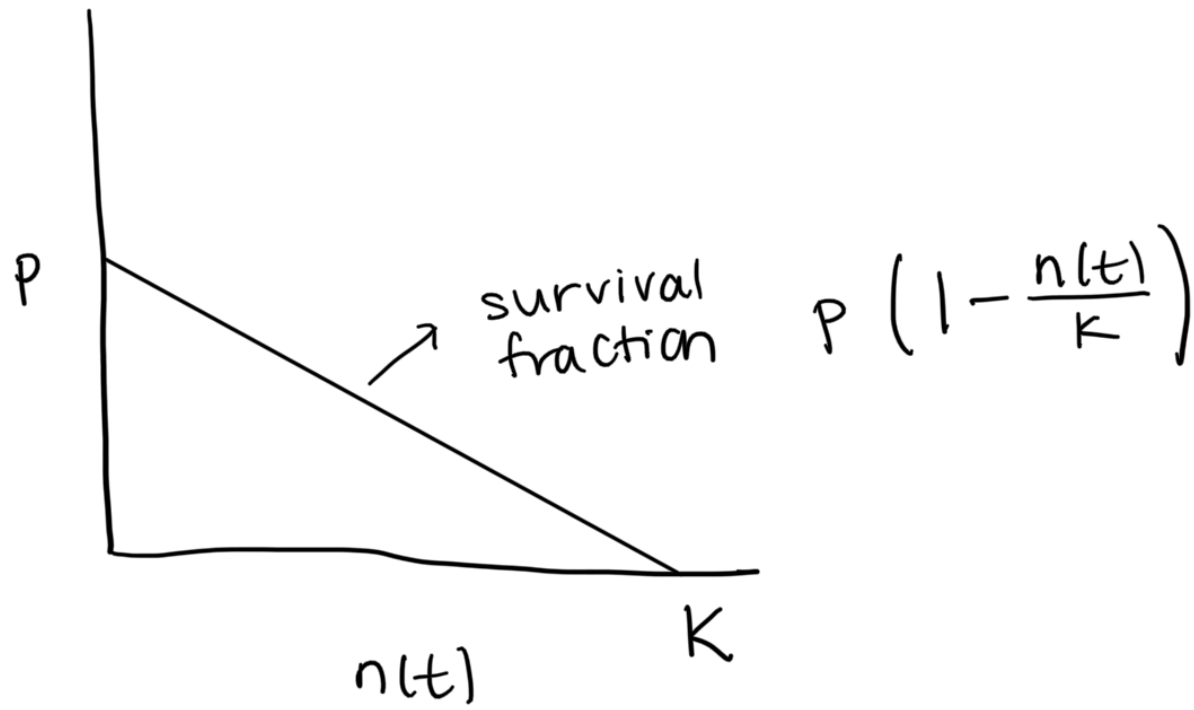
(long-time behavior of $n(t)$ as p varies)

EXAMPLE 1.5: resource-limited cell division

After each synchronous division, fraction

$$p \cdot \left(1 - \frac{n(t)}{K}\right)$$

survives (K = carrying capacity)



How to interpret? competition for resources: as $n(t)$ approaches K, fewer cells survive into next generation.

Updated model:

$$n(t+1) = M\, n(t) \cdot p \left(1 - \frac{n(t)}{K}\right)$$

nonlinear! $(n(t)^2$ term)

$\hookrightarrow$ not as easy to solve, good thing we have a computer to do <u>numerical simulations</u>

(return to cell_reproduction.ipynb)

$M = 4$

$p = 0.15, 0.75, 0.95$

$K = 2000$