AMATH 422/522
Problem set 2

Working together is absolutely encouraged. Please do not refer to previous years' solutions.

For each problem: together with any analysis or explanations, turn in both all code and all relevant plots, labeled and with all line styles, marker sizes etc. adjusted for readability.

Please note: E+G stands for our book, by Ellner and Guckenheimer.

I Iterating Leslie Matrices and the Euler-lotka Formula.

- Consider an age-structured population model as follows: maximum age $A = 3$. Also: $p_0 = 0.5$, $p_1 = .9$, $p_2 = .95$, $f_0 = 0$, $f_1 = 1$, $f_2 = 5$, $f_3 = .5$. Write a python code (feel free to modify any code from class) that simulates the state vector $\mathbf{n}(\mathbf{t})$ of individuals of each age $a = 0, 1, 2, 3$. Start with an initial population consisting of 100 individuals of each age. Plot as functions of time (1) the log of the total population size $N(t) = n_0(t) + n_1(t) + n_2(t) + n_3(t)$ and (2) the fraction of individuals in each age, $w_a(t) = n_a(t)/N(t)$, for $a = 0, 1, 2, 3$. Do this from $t = 1$ to $t = Tmax$, where $Tmax = 50$. Use the numpy `polyfit` function to fit a first order polynomial to the log $N(t)$ and report the growth rate λ. *Turn in the code/notebook you used for this, again OK if just modified from class.*
- Write down the Euler-lotka formula for this example, and solve it numerically or the population growth rate λ. How close are your predictions of λ from the Euler-lotka formulas and from the simulations above? *Turn in the code you used for this.*

II (Taken with modifications from Ellner and Guckenheimer Ex 2.12). According to Lande (1988), females of the northern spotted owl begin breeding at age a=3 and are estimated to have an average of 0.24 female offspring until they die ($f_a = 0.24$ for $a \geq 3$). The survival probability from birth to age 3 is estimated to be 0.0722, and the annual survival probability of adults (p_a for age $a = 3$ to $a = 49$) is 0.942. In our model we will take the maximum age $A = 50$. (These values refer to age-structured conventions, so newborns are age 0).

The owl has been controversial in our region, because of the conflict of interest between the need for old-growth forests as habitat, and the interest of logging companies in harvesting those forests.

- (a) We told you that $I_3 = p_0 p_1 p_2 = 0.0722$ but not the values of the individual p values. That is because any choice of these individual p values with the same product will yield the same long-term population growth rate (λ, from class). Why is this true?
- (b) Construct the projection matrix for the population.
- (c) Compute the long-term growth rate λ for the population.