AMATH 581: Report 1

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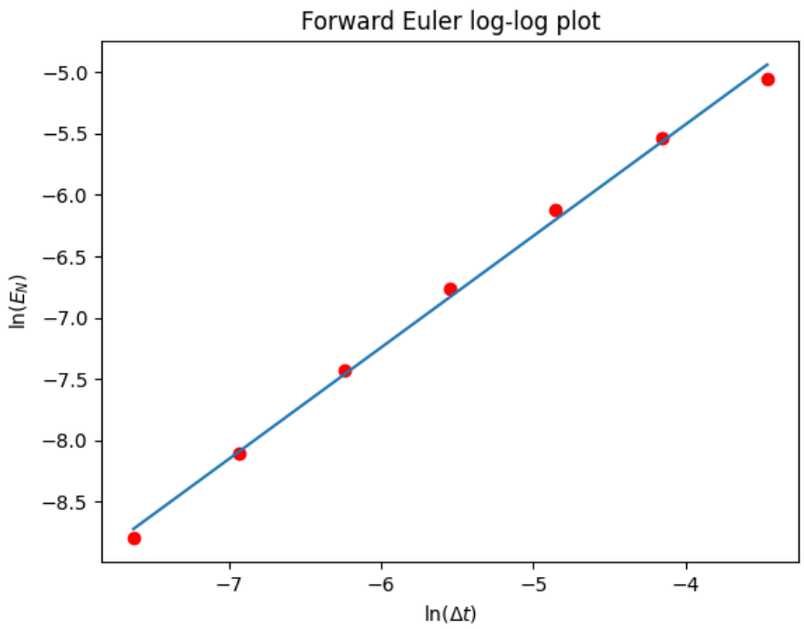
1 Forward Euler

For each Δt value, we get the corresponding E_N value:

Δt	E_N
2^{-5}	0.00637301
2^{-6}	0.00391403
2^{-7}	0.00218312
2^{-8}	0.00115485
2^{-9}	0.00059418
2^{-10}	0.00030141
2^{-11}	0.00015180

Figure 1: Global Error (E_N) for each Δt using Forward Euler

For the log-log plot, we get:



The equation for the best fit line is:

$$\ln(E_N) = 0.9089449553882947 \times \ln(\Delta t) - 1.7908822048968256$$

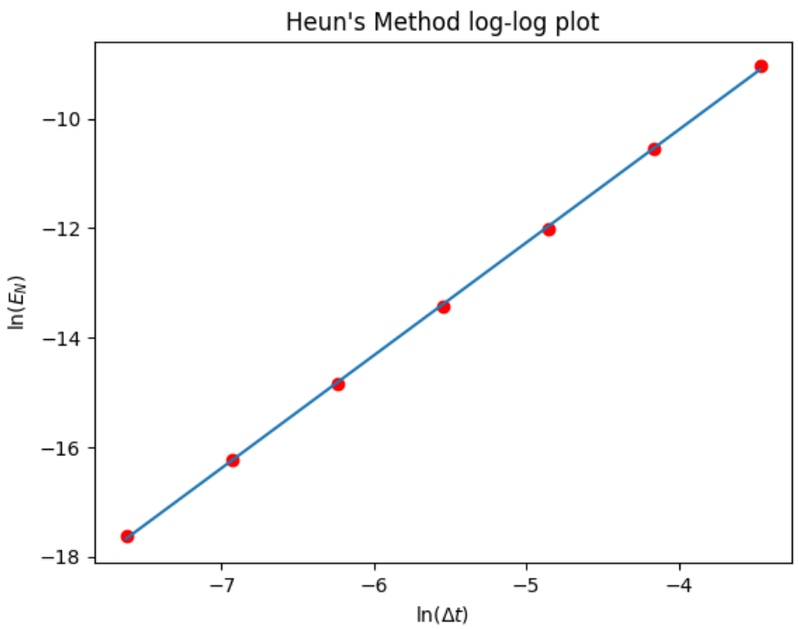
2 Heun's Method

For each Δt value, we get the corresponding E_N value:

Δt	E_N
2^{-5}	$1.19666675 \times 10^{-4}$
2^{-6}	$2.61757562 \times 10^{-5}$
2^{-7}	$6.09014190 \times 10^{-6}$
2^{-8}	$1.46647438 \times 10^{-6}$
2^{-9}	$3.59647006 \times 10^{-7}$
2^{-10}	$8.90423992 \times 10^{-8}$
2^{-11}	$2.21520571 \times 10^{-8}$

Figure 2: Global Error (E_N) for each Δt using Heun's Method

For the log-log plot, we get:



The equation for the best fit line is:

$$\ln(E_N) = 2.05995528141347 \times \ln(\Delta t) - 1.9658392052293192$$

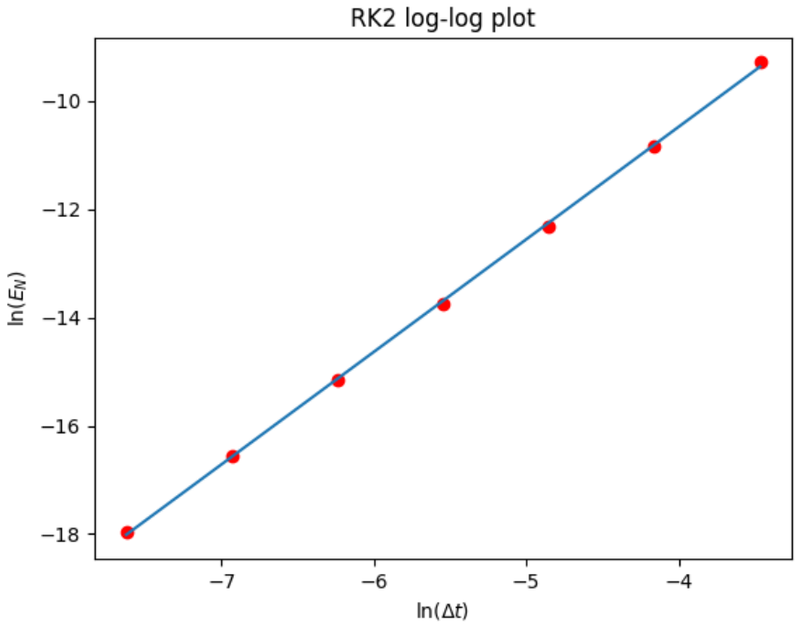
3 RK2

For each Δt value, we get the corresponding E_N value:

Δt	E_N
2^{-5}	$9.36356176 \times 10^{-5}$
2^{-6}	$1.97820656 \times 10^{-5}$
2^{-7}	$4.50174947 \times 10^{-6}$
2^{-8}	$1.07041030 \times 10^{-6}$
2^{-9}	$2.60747828 \times 10^{-7}$
2^{-10}	$6.43314743 \times 10^{-8}$
2^{-11}	$1.59760148 \times 10^{-8}$

Figure 3: Global Error (E_N) for each Δt using RK2

For the log-log plot, we get:



The equation for the best fit line is:

$$\ln(E_N) = 2.0781952218961792 \times \ln(\Delta t) - 2.166960693666943$$

4 Interpretation

The slope of each method is related to the exponent part of the model. Note that the slope and order for each method is the following:

- Forward Euler: slope ≈ 0.909, order $= \mathcal{O}(\Delta t)$
- Heun's Method: slope ≈ 2.06, order $= \mathcal{O}(\Delta t^2)$
- RK2: slope ≈ 2.08, order $= \mathcal{O}(\Delta t^2)$

Hence, the slope is similar to the exponent of the order of each model.
The reason is because we are looking at the logarithms of global error E_N and Δt. For example, consider the RK2 model. We know that the global error of the RK2 model is $\mathcal{O}(\Delta t^2)$, which means:

$$E_N = c\Delta t^2$$

where c is a constant. If we apply logarithms to both sides, we further have:

$$\ln(E_N) = \ln(c) + \ln(\Delta t^2) = 2\ln(\Delta t) + \ln(c)$$

Thus, the slope of the best fit line should match with the exponent part of the order of each method.