AMATH 581: Report 3

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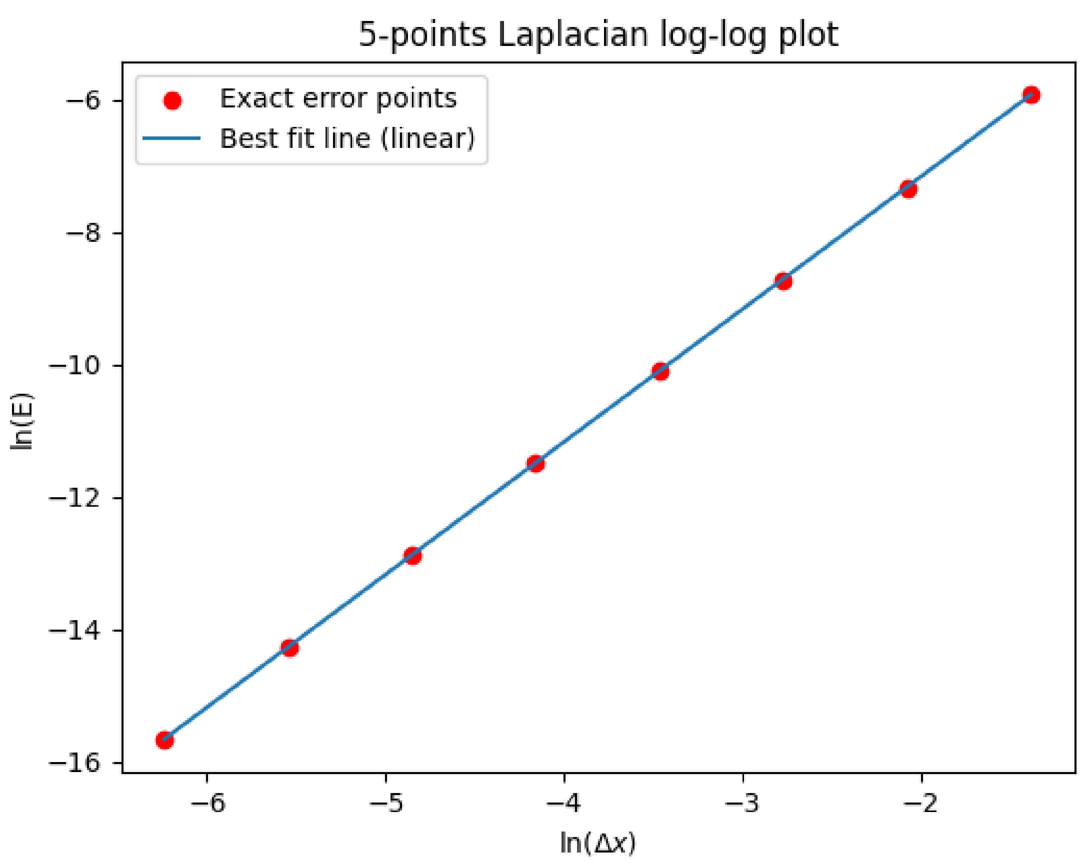
Problem 4 (a)

To determine the order of accuracy for the 5-point Laplacian, we use the numerical method of comparing the logarithms of error (E) and Δx. The error (E) is calculated using 1-norm, so we have:

$$E = \max \left|\text{Approximation} - \text{Exact Solution}\right|$$

For Δx values, we used 8 different values, which are $2^{-2}, 2^{-3}, 2^{-4}, 2^{-5}, 2^{-6}, 2^{-7}, 2^{-8}, 2^{-9}$. For the 5-point Laplacian, we obtain the following log-log plot:



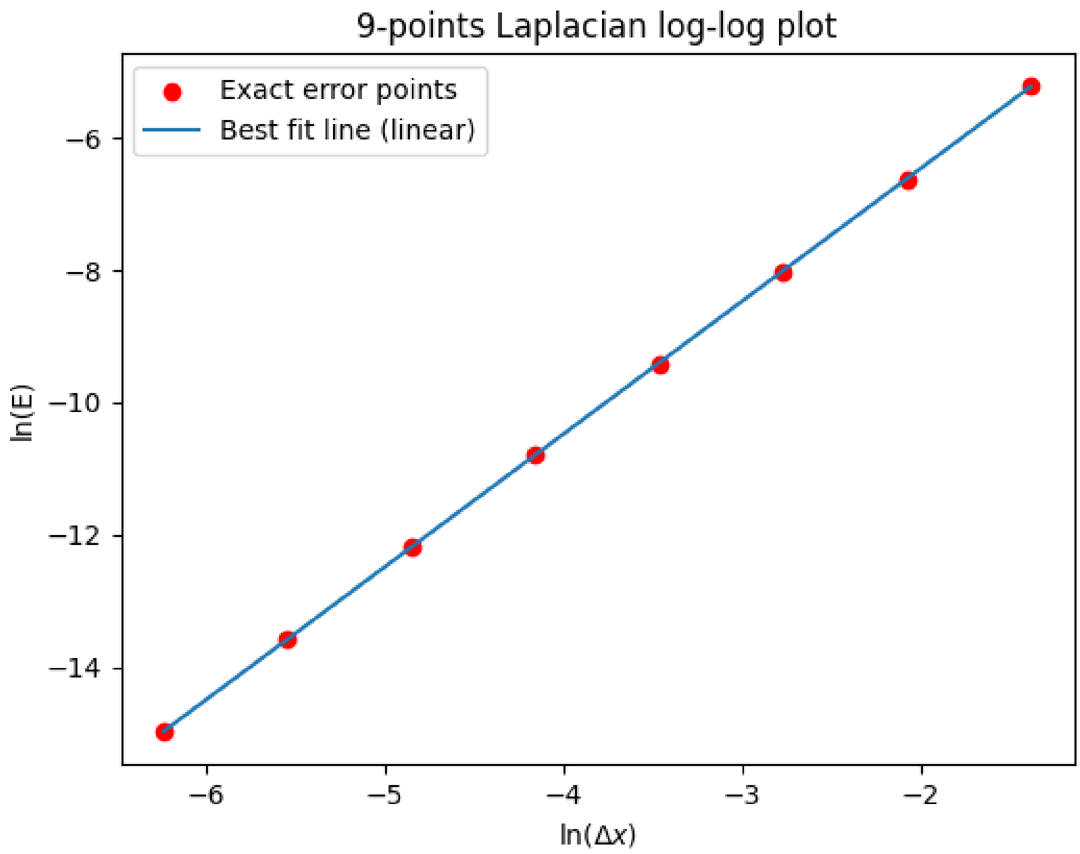
The equation of the best fit line is:

$$\ln(E) = 2.004508788898138 \times \ln(\Delta x) - 3.1556870128339423$$

Therefore, the order of accuracy of the 5-point Laplacian is $\mathcal{O}(\Delta x^2)$.

Problem 4 (b)

We repeat the same process in **Problem 4(a)** (with the same Δx values and 1-norm to calculate the error) to calculate the order of accuracy for the 9-point Laplacian. We obtain the following log-log plot:



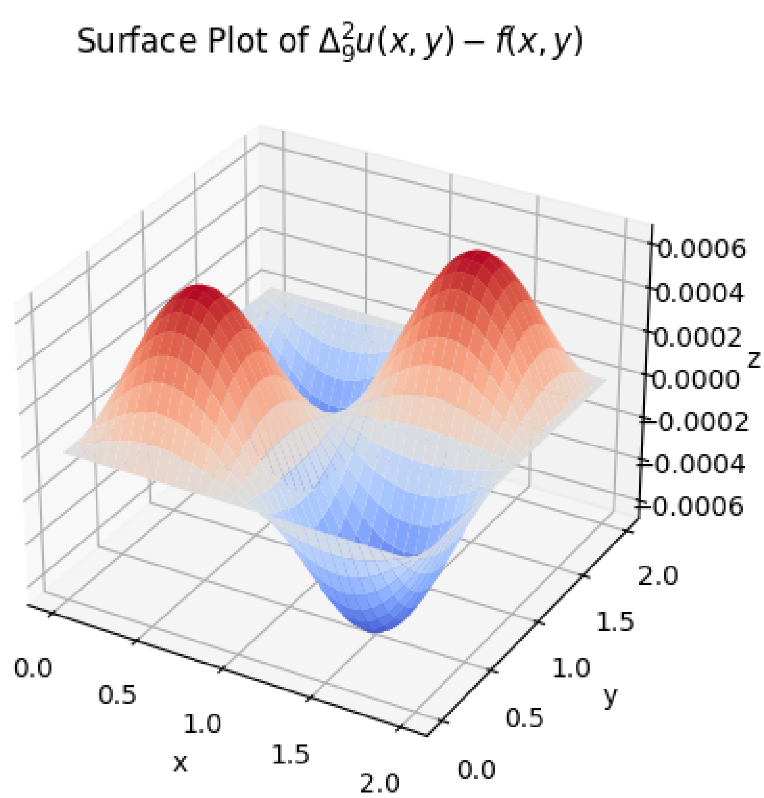
The equation of the best fit line is:

$$\ln(E) = 2.005902589364498 \times \ln(\Delta t) - 2.455615978948617$$

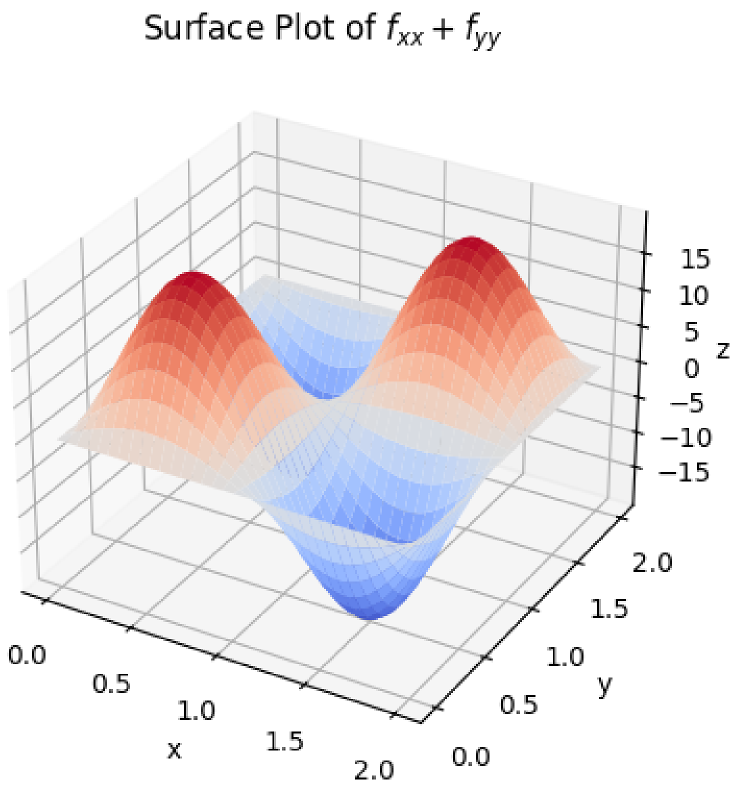
Therefore, the order of accuracy of 9-point Laplacian is also $\mathcal{O}(\Delta x^2)$.

Problem 4 (c)

For the discretization error of the 9-point Laplacian, we get the following 3-dimensional surface plot:



For the function $f_{xx} + f_{yy}$, where $f(x, y) = -\sin(\pi x)\sin(\pi y)$, we get the following 3-dimensional surface plot:



Note that the shape of the two surfaces are very similar to each other. The only difference is the scale in z-axis. Hence, the leading error term of the 9-point Laplacian approximation is proportional to $f_{xx} + f_{yy}$.

From **Problem 4(b)**, we know that the order of accuracy for 9-point Laplacian is $\mathcal{O}(\Delta x^2)$. Then, by subtracting the function $f_{xx} + f_{yy}$ from the approximation, we can eliminate the leading error term which is in order $\mathcal{O}(\Delta x^2)$. As a result, the order of accuracy for 9-point Laplacian will decrease to the next leading error term in the finite difference, which is $\mathcal{O}(\Delta x^4)$.