UNIVERSITY OF TRENTO - Italy

COURSE OF

"DYNAMICS AND CONTROL OF VEHICLE AND ROBOTS"

Tutorial 02

Modelling and Simulation of Single Track Model. Oversteering and Understeering behaviour

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Tutorial description

The tutorial builds a dynamic model of a single track vehicle using the Maple and the CodeGeneration package. The single track model is implemented with a Simulink model where the equations are written inside a function block. The simulation is programmatically setup and run with a Matlab script.

The equations of the single track dynamics in explicit form:

$$\dot{x}(t) = u(t)\cos(\psi(t)) - \sin(\psi(t))v(t) \tag{1}$$

$$\dot{y}(t) = u(t)\sin(\psi(t)) + \cos(\psi(t))v(t) \tag{2}$$

$$\dot{\psi}(t) = \Omega(t) \tag{3}$$

$$\dot{u}(t) = \frac{M\Omega(t)v(t) - k_v u(t)^2 - F_{y_f}(t)\delta(t) + F_{x_r}(t) + F_{x_f}(t)}{M} \tag{4}$$

$$\dot{v}(t) = -\frac{M\Omega(t)u(t) - F_{x_f}(t)\delta(t) - F_{y_f}(t) - F_{y_r}(t)}{M} \tag{5}$$

$$\dot{\Omega}(t) = \frac{L_f F_{x_f}(t)\delta(t) + L_f F_{y_f}(t) - L_r F_{y_r}(t)}{I_z} \tag{6}$$

The input of single track models are the steering angle δ the rear longitudinal force F_{x_r} and the front longitudinal force F_{x_f}. Their definition depend on the type of vehicle we want to simulate. In case of a front axle traction vehicle the rear force can be only negative. The lateral forces can be modelled both for linear case and non linear case.
The side slip angles are:

$$\alpha_r(t) = -\frac{v(t) + \Omega(t)L_f}{u(t)} \tag{7}$$

$$\alpha_f(t) = \delta - \frac{v(t) - \Omega(t)L_r}{u(t)} \tag{8}$$

The side slip angles are linear but their validity is extended almost up to the adherence peak. The steering angle as a function of the froward velocity depends on the steering gradient K_s according to the following if the drag is neglected:

$$\delta_A = \delta(t) + \frac{K_s M {V_G}^2}{RL} \tag{9}$$

or the following if the drag is included (and therefore the longitudinal force):

$$\delta_A = \left(1 + \left(\frac{L_f}{C_r L_r} - \frac{Ks}{L_r}\right) k_v {V_G}^2\right)\delta + \frac{K_s M {V_G}^2}{RL} \tag{10}$$

The steady state solution of the equation (1) can be found by substituting in the equation the linear tyre model and solving for Ω, β, F_{x_f}. The forward velocity u is considered constant and equal to V_G. When we neglect the aerodynamic drag $k_v = 0$ and the effect of longitudinal force F_{x_f} we can solve only for Ω, β and we get the following steady state solution:

$$\frac{\Omega}{\frac{V_G \delta}{L}} = \frac{1}{\frac{1 - KsM{V_G}^2}{L^2}} \tag{11}$$

$$\frac{\beta}{\frac{\delta L_r}{L}} = \frac{1 - \frac{M{V_G}^2 L_f}{L_r C_r}}{1 - \frac{KsM{V_G}^2}{L^2}} \tag{12}$$

where $\frac{V_G\delta}{L} = \Omega_{ss}$ is a steady state yaw rate for low velocities (indeed $\frac{\delta}{L} = \frac{1}{R}$). Similarly $\frac{\delta L_r}{L} = \beta_{ss}$ is the steady state chassis side slip angle. Finally the steady state solutions can be derived including the the drag effect and the longitudinal force that keep the forward velocity constant.

$$F_{x_f} = k_v V_G \tag{13}$$

$$\frac{\Omega}{\frac{V_G\delta}{L}} = \frac{1}{\frac{1-KsMV_G{}^2}{L^2}} \tag{14}$$

$$\frac{\beta}{\frac{\delta L_r}{L}} = \frac{1-\frac{MV_G{}^2L_f}{L_rC_r}}{1-\frac{KsMV_G{}^2}{L^2}} \tag{15}$$

Objective

The focus is analysis of the vehicle dynamic characteristics (oversteer or understeer behaviour) changing the tyre parameters and the vehicle velocity given a constant steering angle $\delta_o = 0.1(rad)$. The analysis is carried out with a numerical simulation using the single track equations (1) and the analytical solution (11). All the reported equations and the related code generation is reported in the Maple file *single_track_equations.mw*.

What you learn with this tutorial

In this tutorial you will learn:

- build simple dynamics system model using Simulink and user function block to define the right hand side of a system of differential equations
- automatically generate the Matlab code from Maple (optimized)
- create a function to calculate the tyre force that can be called in the Simulink model
- create a simple closed loop control on speed
- run programmatically a set of parametric simulations
- understand the effect of tyre parameters on vehicle dynamic characteristics
- understand the importance of many different terms
- understand the validity range of the steady state equations.

Discussion

The simulations shows the curvature radius as a function of the forward velocity or lateral acceleration. The simulation includes the effect of the drag and of the longitudinal force and clearly prove that the model is valid for low velocities and deviates, significantly for the Oversteering vehicle, from the linear behaviour.

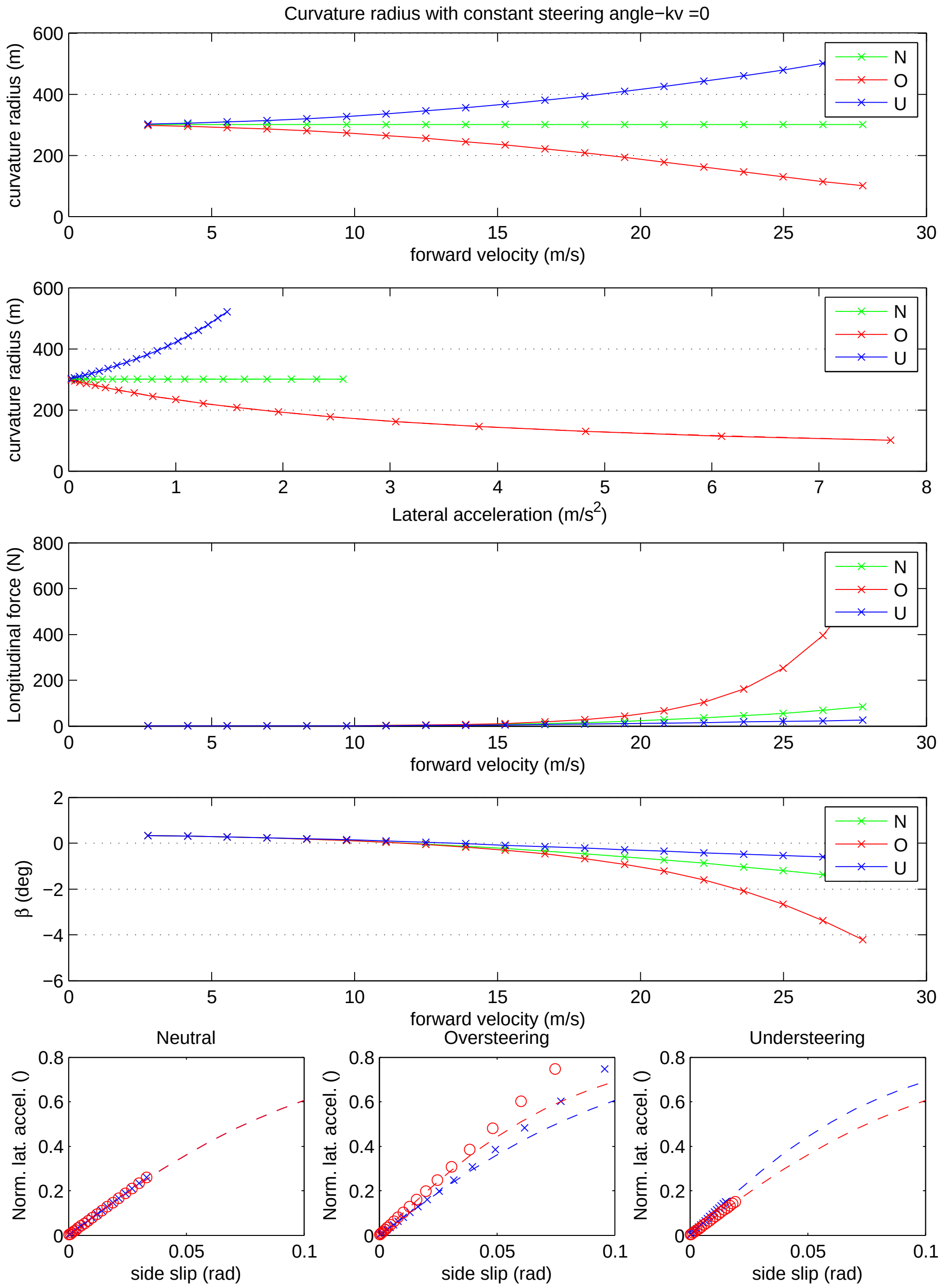

Figure 1: Oversteering, Understeering and Neutral behaviour obtained with the numerical solution of the single track model (1) for the steady state conditions.