Convergence of Allen-Cahn equations to multi-phase mean curvature flow

Pascal Maurice Steinke

Born 22.05.1999 in Gießen, Germany

09.09.2022

Master's Thesis Mathematics

Advisor: Prof. Dr. Tim Laux

Second Advisor: Prof. Dr. X Y

INSTITUTE FOR APPLIED MATHEMATICS

Contents

1 The Allen–Cahn equation

Let $\Lambda > 0$ and define the flat torus $\mathbb{T} = [0, \Lambda)^d \subset \mathbb{R}^d$, where we as usual assume periodic boundary conditions. Then for $u \colon [0, \infty) \times \mathbb{T} \to \mathbb{R}^N$ and some potential $W \colon \mathbb{R}^N \to [0, \infty)$, the *Allen–Cahn equation* with parameter $\varepsilon > 0$ is given by

$$\partial_t u = \Delta u - \frac{1}{\varepsilon^2} \partial_u W(u). \tag{1.1}$$

To understand this equation better, we consider the *Cahn–Hilliard energy* assigns to u for a fixed time the real number

$$\mathrm{E}_\varepsilon(u) := \int_{\mathbb{T}} \frac{1}{\varepsilon} W(u) + \frac{\varepsilon}{2} \|\nabla u\|^2 \, \mathrm{d}x. \tag{1.2}$$

If everything is nice and smooth, we can compute that under the assumption that u satisfies equation 1.1, we have that

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathrm{E}_\varepsilon(u) = \int stuff$$

Bibliography

[LS16]   Tim Laux and Thilo Simon. "Convergence of the Allen-Cahn Equation to
         Multiphase Mean Curvature Flow". In: *Communications on Pure and Applied
         Mathematics* 71 (June 2016). DOI: 10.1002/cpa.21747.