

COMP 9517 Computer Vision

Feature Representation

Outline

- Need for feature representation
- Major types of features
 - Colour
 - Texture
 - Shape
- Feature descriptors and their use in various computer vision applications

Image Features

- Image features are essentially vectors that are a compact representation of images
- They represent important information shown in an image
- Intuitive examples of image features:
 - Blobs
 - Edges
 - Corners
 - Ridges
 - Circles
 - Ellipses
 - Lines
 - Etc...



Image Features

- We need to represent images as feature vectors for further processing in a more efficient and robust way
- Examples of further processing include:
 - Object detection
 - Image segmentation
 - Image classification
 - Content-based image retrieval
 - Image stitching
 - Object tracking

Object Detection



Segmentation





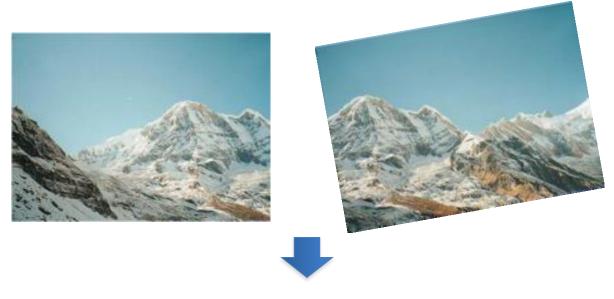
Image Classification



Content-Based Image Retrieval

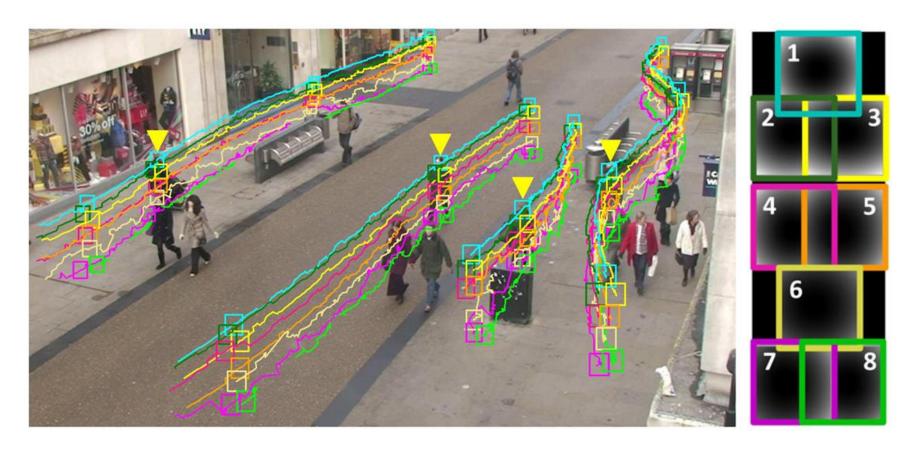


Image Stitching





Object Tracking



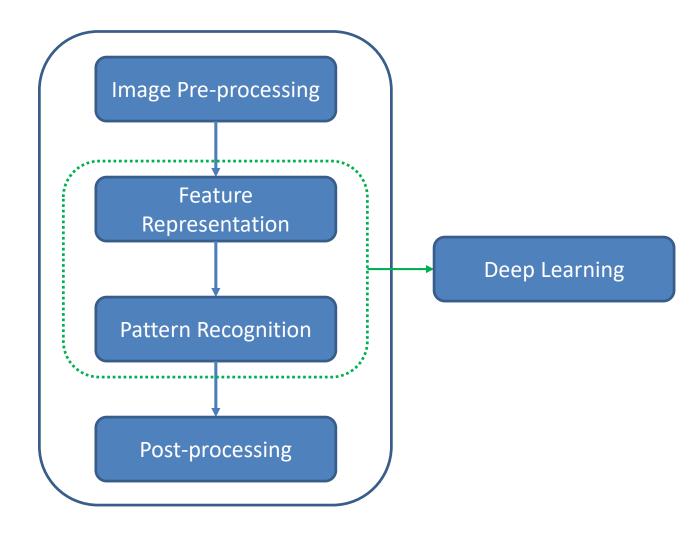
https://heartbeat.fritz.ai/

Properties of Features

- Why not just use pixels values directly?
 - Pixel values change with light intensity, colour and direction
 - They also change with camera orientation
 - And they are highly redundant
- Repeatability (robustness)
 - Should be detectable at the same locations in different images despite changes in illumination and viewpoint
- Saliency (descriptiveness)
 - Same points in different images should have similar features
- Compactness (efficiency)
 - Fewer features
 - Smaller features

General Framework

Object detection
Image segmentation
Image classification
Image retrieval
Image stitching
Object tracking



Feature Types

- Colour features
 - Colour histogram
 - Colour moments
- Texture features
 - Haralick texture features
 - Local binary patterns (LBP)
 - Scale-invariant feature transform (SIFT)
 - Texture feature encoding
- Shape features
 - Basic shape features
 - Shape context
 - Histogram of oriented gradients (HOG)

Colour Features

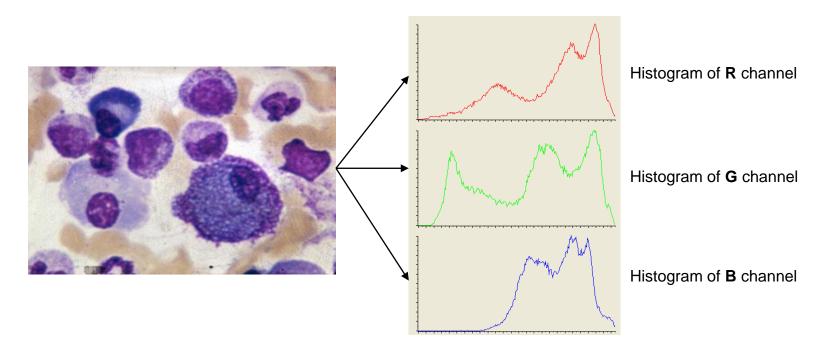
- <u>Colour</u> is the simplest feature to compute, and is invariant to image scaling, translation and rotation.
- Example: colour-based image retrieval



http://labs.tineye.com/multicolr/

Colour Histogram

- Represent the global distribution of pixel colours in an image
 - Step 1: Construct a histogram for each colour channel (R, G, B)
 - Step 2: Concatenate the histograms (vectors) of all channels as the final feature vector



Colour Moments

 f_{ij} is the value of the *i*-th colour component of pixel *j* and *N* is the number of pixels in the image

Another way of representing colour distributions

First-order moment

$$\mu_i = \frac{1}{N} \sum_{j=1}^{N} f_{ij}$$

(mean)

Second-order moment

$$\sigma_i = (\frac{1}{N} \sum_{j=1}^{N} (f_{ij} - \mu_i)^2)^{\frac{1}{2}}$$

(variance)

Third-order moment

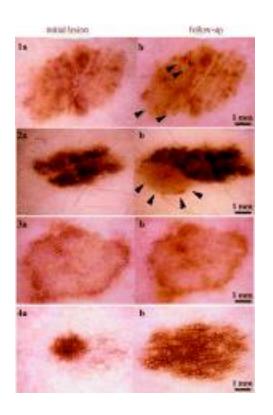
$$s_i = \left(\frac{1}{N} \sum_{j=1}^{N} (f_{ij} - \mu_i)^3\right)^{\frac{1}{3}}$$

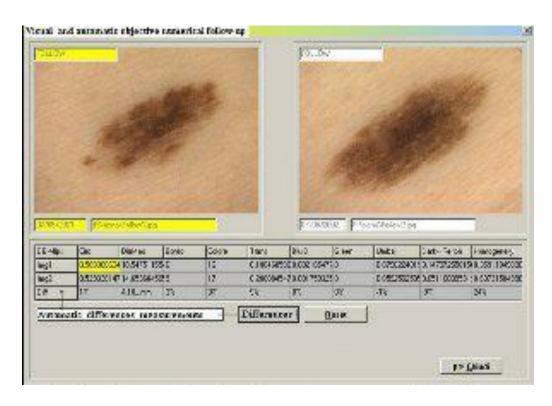
(skewness)

- Moments based representation of colour distributions
 - Gives a feature vector of only 9 elements (for RGB images)
 - Lower representation capability than the colour histogram

Application Example

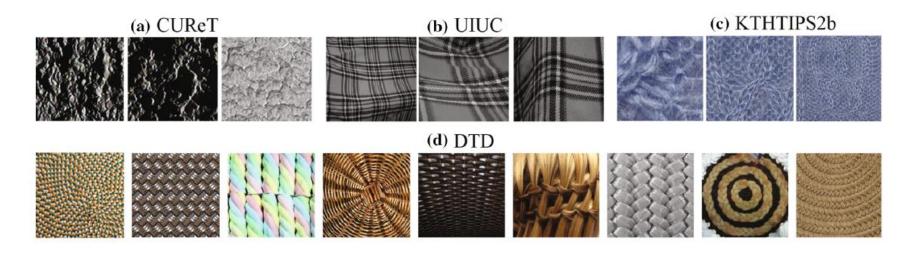
Colour-based image retrieval





Texture Features

- <u>Texture</u> is a powerful discriminating feature for identifying visual patterns with properties of homogeneity that cannot result from the presence of only a single color or intensity
- Example: texture classification



https://arxiv.org/abs/1801.10324

- Haralick features give an array of statistical descriptors of image patterns to capture the spatial relationship between neighbouring pixels, that is, textures
 - Step 1: Construct the gray-level co-occurrence matrix (GLCM)
 - Step 2: Compute the Haralick feature descriptors from the GLCM

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IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS, VOL. SMC-3, NO. 6, NOVEMBER 1973

Textural Features for Image Classification

ROBERT M. HARALICK, K. SHANMUGAM, AND ITS'HAK DINSTEIN

Abstract—Texture is one of the important characteristics used in identifying objects or regions of interest in an image, whether the image be a photomicrograph, an aerial photograph, or a satellite image. This paper describes some easily computable textural features based on graytone spatial dependancies, and illustrates their application in category-

array. If $L_x = \{1, 2, \dots, N_x\}$ and $L_y = \{1, 2, \dots, N_y\}$ are the X and Y spatial domains, then $L_x \times L_y$ is the set of resolution cells and the digital image I is a function which assigns some gray-tone value $G \in \{1, 2, \dots, N_g\}$ to each and every

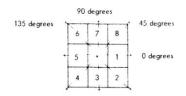


Fig. 1 Resolution cells 1 and 5 are 0° (horizontal) nearest neighbors to resolution cell *; resolution cells 2 and 6 are 135° nearest neighbors; resolution cells 3 and 7 are 90° nearest neighbors; and resolution cells 4 and 8 are 45° nearest neighbors to *. (Note this information is purely spatial, and has nothing to do with gray-tone values.)

Step 1: Construct the GLCMs

- Given a distance d at an orientation angle ϑ , then $p_{(d,\vartheta)}(I_1,I_2)$, being the (I_1,I_2) coefficient of the corresponding matrix $P_{(d,\vartheta)}$, is the co-occurrence count or probability of going from a grey level I_1 to another grey level I_2 with an intersample spacing of d along the axis making an angle ϑ with the x axis.
- If the number of distinct gray levels in the quantized image is L, then the cooccurrence matrix **P** will be of size $L \times L$.

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

$$\mathbf{P}_{(1,0^{\circ})} = \begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 0 \\ 1 & 0 & 6 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\mathbf{P}_{(1,0^{\circ})} = \begin{vmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 0 \\ 1 & 0 & 6 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} \qquad \mathbf{P}_{(1,135^{\circ})} = \begin{vmatrix} 2 & 1 & 3 & 0 \\ 1 & 2 & 1 & 0 \\ 3 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{vmatrix}$$

co-occurrence matrix construction

• Step 1: Construct the GLCMs

- For computational efficiency, the number of gray levels (L) can be reduced by binning (similar to histogram binning), e.g. L = 256/n, with n a constant factor.
- Different co-occurrence matrices can be constructed by using various combinations of distance (d) and angular directions (ϑ).
- On their own, these co-occurrence matrices do not provide any measure of texture that can easily be used as texture descriptors.
- The information in the co-occurrence matrices needs to be further extracted as a set of feature values => Haralick descriptors.

- Step 2: Compute the Haralick descriptors from the GLCMs
 - One set of Haralick descriptors for each GLCM corresponding to a particular distance (d) and angular direction (ϑ)

Angular Second Moment

Contrast

Correlation

Sum of Squares: Variance

Inverse Difference Moment

Sum Average

$$\sum_{i} \sum_{j} p(i,j)^{2}$$

$$\sum_{n=0}^{N_{g}-1} n^{2} \{ \sum_{i=1}^{N_{g}} \sum_{j=1}^{N_{g}} p(i,j) \}, |i-j| = n$$

$$\underline{\sum_{i} \sum_{j} (ij) p(i,j) - \mu_{x} \mu_{y}}$$

where μ_x , μ_y , σ_x , and σ_y are the means and std. deviations of p_x and p_y , the partial probability density functions

$$\sum_{i}\sum_{j}(i-\mu)^{2}p(i,j)$$

$$\sum_{i} \sum_{j} \frac{1}{1+(i-j)^2} p(i,j)$$

$$\sum_{i=2}^{2N_g} i p_{x+y}(i)$$

where x and y are the coordinates (row and column) of an entry in the co-occurrence matrix, and $p_{x+y}(i)$ is the probability of co-occurrence matrix coordinates summing to x + y

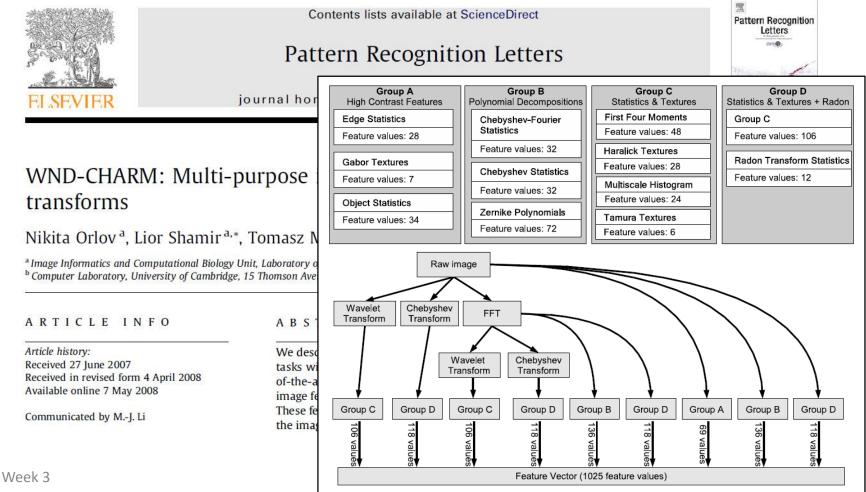
- Step 2: Compute the Haralick descriptors from the GLCMs
 - One set of Haralick descriptors for each GLCM corresponding to a particular distance (d) and angular direction (ϑ)

```
\sum_{i=2}^{2N_g} (i - f_8)^2 p_{x+y}(i)
Sum Variance
                                             -\sum_{i=2}^{2N_g} p_{x+y}(i) \log\{p_{x+y}(i)\} = f_8
Sum Entropy
                                             -\sum_{i}\sum_{j}p(i,j)log(p(i,j))
Entropy
                                             \sum_{i=0}^{N_g-1} i^2 p_{x-y}(i)
Difference Variance
                                             -\sum_{i=0}^{N_g-1} p_{x-y}(i) \log\{p_{x-y}(i)\}\
Difference Entropy
                                             \frac{HXY-HXY1}{\max\{HX,HY\}}
Info. Measure of Correlation 1
                                             (1 - \exp[-2(HXY2 - HXY)])^{\frac{1}{2}}
Info. Measure of Correlation 2
                                              where HXY = -\sum_{i}\sum_{j}p(i,j)\log(p(i,j)), HX,
                                                  HY are the entropies of p_x and p_y, HXY1 =
                                                           -\sum_{i}\sum_{j}p(i,j)\log\{p_x(i)p_y(j)\}HXY2=
                                                                   -\sum_{i}\sum_{j}p_{x}(i)p_{y}(j)\log\{p_{x}(i)p_{y}(j)\}
                                             Square root of the second largest eigenvalue of Q where \mathbf{Q}(i,j) = \sum_{k} \frac{p(i,k)p(j,k)}{p_x(i)p_y(k)}
Max. Correlation Coeff.
```

Application Example

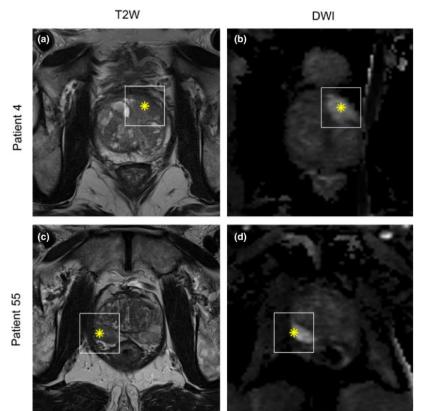
Pattern Recognition Letters 29 (2008) 1684-1693

https://doi.org/10.1016/j.patrec.2008.04.013



Application Example

 Commonly used nowadays in medical imaging studies due to its simplicity and interpretability



C. Jensen et al.

Assessment of prostate cancer prognostic
Gleason grade group using zonal-specific
features extracted from biparametric MRI
using a KNN classifier
Journal of Applied Clinical Medical Physics, 2019
https://doi.org/10.1002/acm2.12542

- 1. Pre-processing
- Extract Haralick, run-length, and histogram features from the region of interest
- 3. Feature selection
- 4. Classification using kNN

- Describe the spatial structure of local image texture
 - Divide the image into cells of $N \times N$ pixels (e.g. N = 16 or 32)
 - Compare each pixel in a cell to each of its 8 neighbouring pixels:
 If the centre pixel's value is greater than the neighbour's value,
 write "0", otherwise write "1"
 - This gives an 8-digit binary pattern per pixel after comparing with all 8 neighbouring pixels, representing a value in the range 0...255

0	0	1	1	
0	0	1	1	
0	2	2	2	
2	2	3	3	

00001111

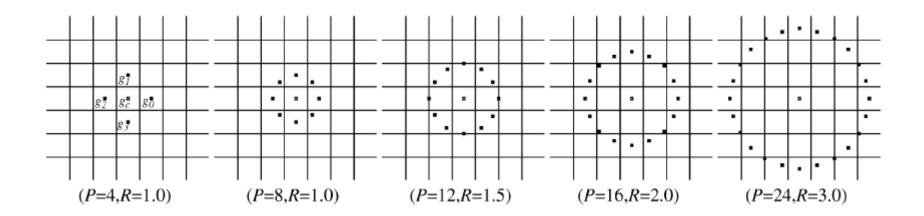
- Describe the spatial structure of local image texture (cont.)
 - Generate the histogram for all pixels in the cell, computing the frequency of each 8-digit binary number occurring in the cell
 - This gives a 256-bin histogram (the LBP feature vector)
 - Combine the histograms of all cells to obtain the image-level
 LBP feature descriptor

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3



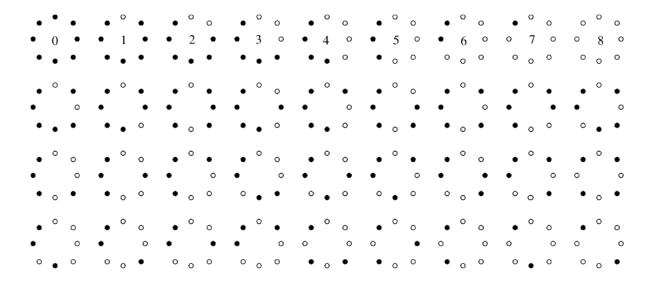
A histogram of 256 elements

- LBP can be multi-resolution and rotation-invariant
 - Multi-resolution: varying the distance between the centre pixel and neighbouring pixels, and the number of neighbouring pixels



T. Ojala, M. Pietikainen, and T. Maenpaa, "Multiresolution gray-scale and rotation invariant texture classification with local binary patterns," IEEE Transactions on Pattern Analysis and Machine Intelligence 24(7):971-987, 2002. https://doi.org/10.1109/TPAMI.2002.1017623

- LBP can be multi-resolution and rotation-invariant
 - Rotation-invariant: varying the way of constructing the 8-digit binary number, e.g. performing bitwise shift to derive the smallest number
 this reduces the LBP feature dimension from 256 to 36

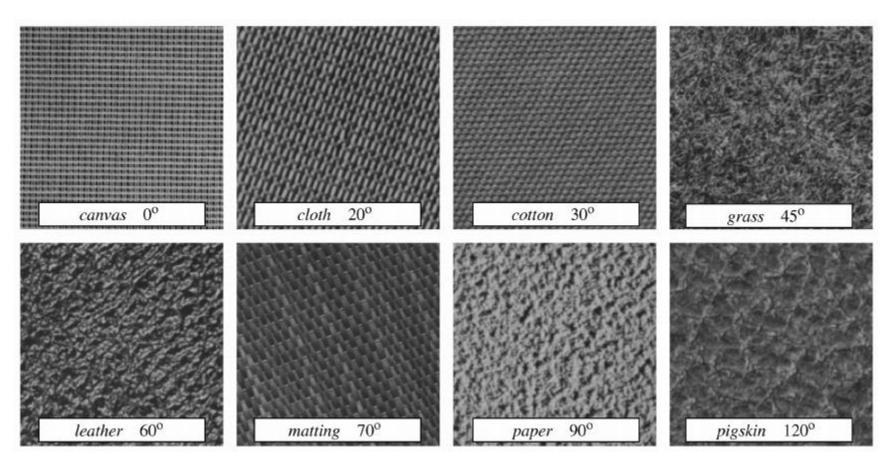


T. Ojala, M. Pietikainen, and T. Maenpaa, "Multiresolution gray-scale and rotation invariant texture classification with local binary patterns," IEEE Transactions on Pattern Analysis and Machine Intelligence 24(7):971-987, 2002. https://doi.org/10.1109/TPAMI.2002.1017623

Application Example

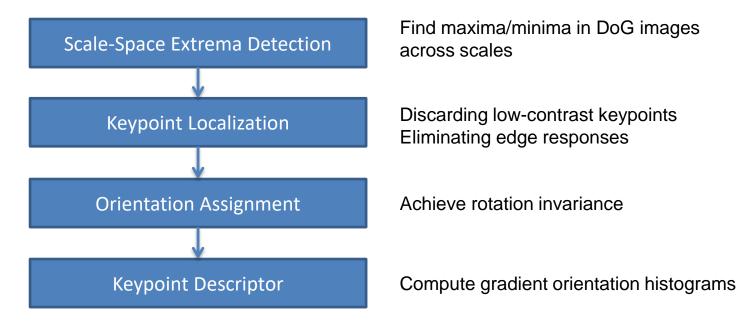
 $LBP_{P,R}$ P,RRESULT BINS 8.1 10 88.2 16,2 98.5 18 24,3 99.1 26 8,1+16,2 10 + 1899.0 8,1+24,3 10 + 2699.6 16,2+24,3 18 + 2699.0 8,1+16,2+24,3 10+18+2699.1

Texture classification



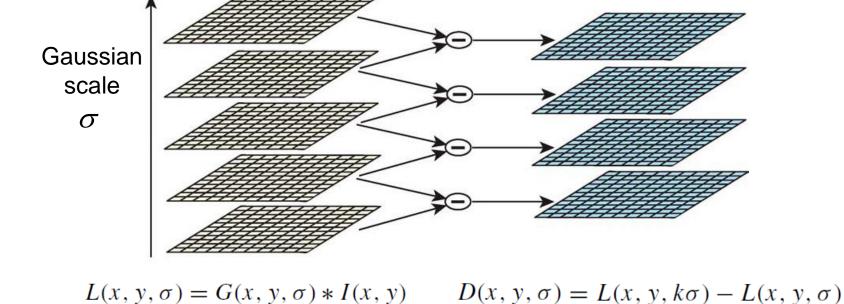
Scale-Invariant Feature Transform

- SIFT feature describes the texture features in a localised region around a keypoint
- SIFT descriptor is invariant to uniform scaling, orientation, and partially invariant to affine distortion and illumination changes



SIFT Extrema Detection

Detect maxima and minima in the scale space of the image



D. G. Lowe, "Distinctive image features from scale-invariant keypoints," Int. J. Comput. Vis. 60(2):91-110, November 2004. https://doi.org/10.1023/B:VISI.0000029664.99615.94

SIFT Keypoint Localization

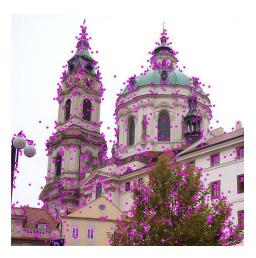
- Improve and reduce the set of found keypoints
 - Use 3D quadratic fitting in scale-space to get subpixel optima
 - Reject low-contrast and edge points using Hessian analysis



Initial keypoints from scale-space optima



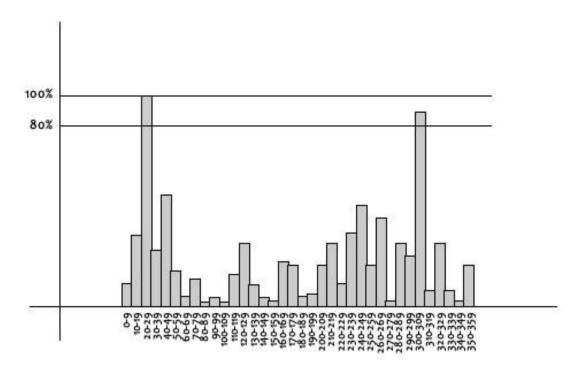
Keypoints after rejecting low-contrast points



Final keypoints after rejecting edge points

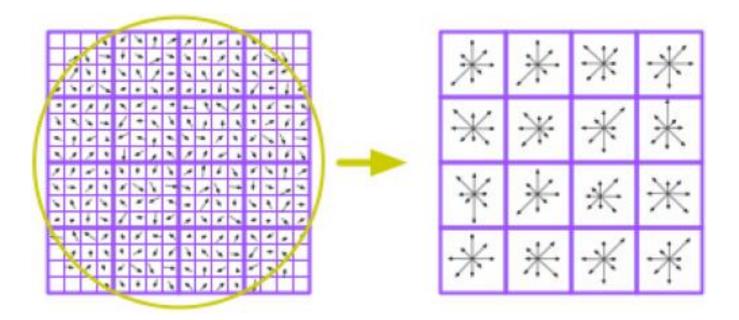
SIFT Orientation Assignment

- Estimate keypoint orientation using local gradient vectors
 - Make an orientation histogram of local gradient vectors
 - Find the dominant orientations from the peaks of the histogram



SIFT Keypoint Descriptor

- 4 x 4 array of gradient histogram weighted by magnitude
- 8 bins in gradient orientation histogram
- Total 8 x 4 x 4 array = 128 dimensions



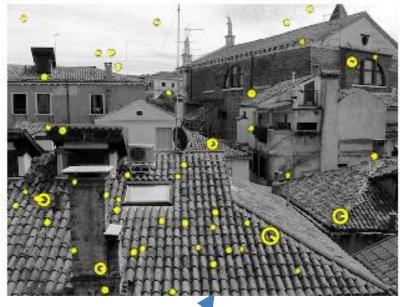
Application Example

Image matching

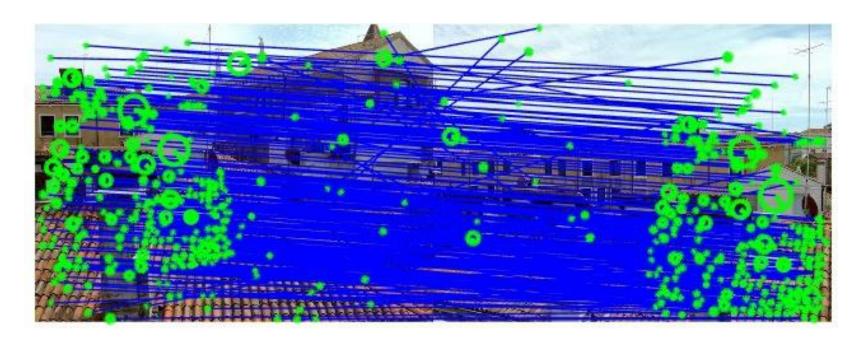


- Image matching
 - Find SIFT keypoints





- Image matching
 - Find best matching between SIFT keypoints



Descriptor Matching

Nearest Neighbour Distance Ratio

$$NNDR = \frac{d_1}{d_2} = \frac{\|D_A - D_B\|}{\|D_A - D_C\|}$$

- d1 is the distance to the first nearest neighbour
- d2 is the distance to the second nearest neighbour
- Neighbours in feature space

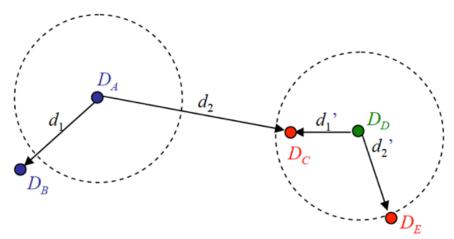
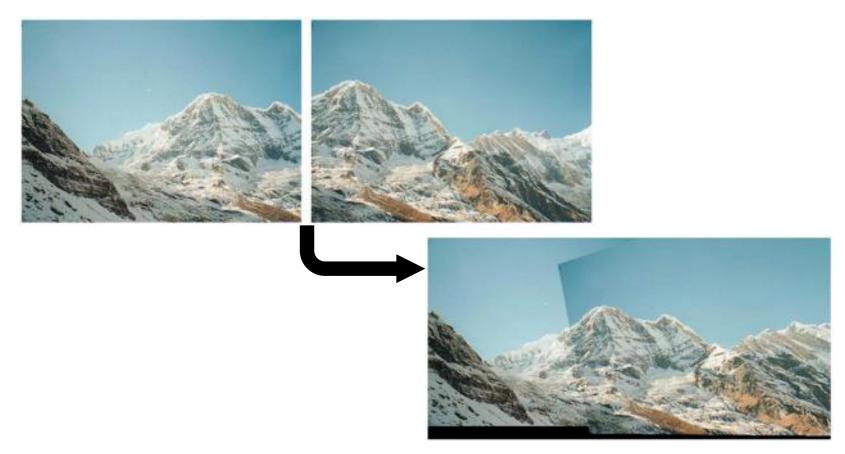
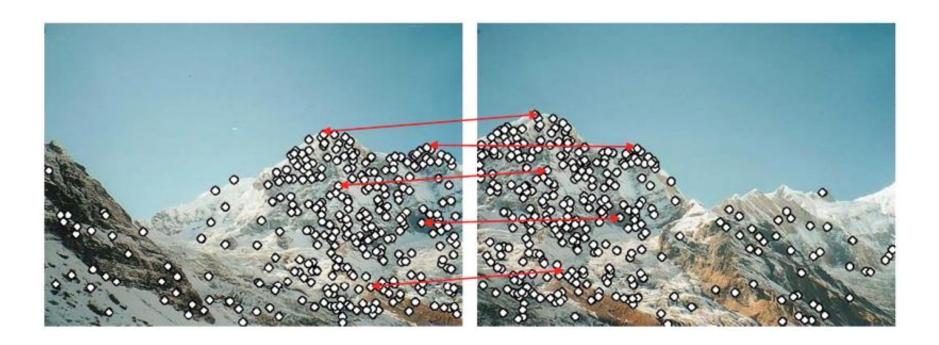


Image stitching



- Image stitching
 - Find SIFT keypoints and feature correspondences



- Image stitching
 - Find the right spatial transformation



Transformations



translation



affine



original



rotation



perspective



scale

Transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha_x \\ \alpha_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Projective

Least-squares (LS) fitting of corresponding keypoints $(\mathbf{x}_i, \mathbf{x}_i)$

$$E_{LS} = \sum_{i} \left\| \mathbf{r}_{i} \right\|^{2} = \sum_{i} \left\| f(\mathbf{x}_{i}; \mathbf{p}) - \mathbf{x}_{i}^{'} \right\|^{2}$$

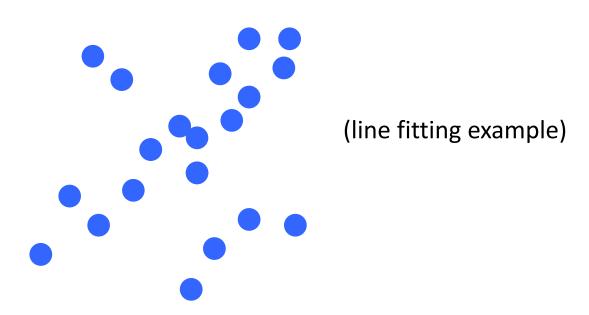
where \mathbf{p} are the parameters of the transformation f

where
$$\mathbf{p}$$
 are the parameters of the transformation f

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \implies \begin{bmatrix} x & y & 0 & 0 & 1 & 0 \\ 0 & 0 & x & y & 0 & 1 \\ & & & \\$$

- RANdom SAmple Consensus (RANSAC) fitting
 - Least-squares fitting is hampered by outliers
 - Some kind of outlier detection and rejection is needed
 - Better use a subset of the data and check inlier agreement
 - RANSAC does this in a iterative way to find the optimum

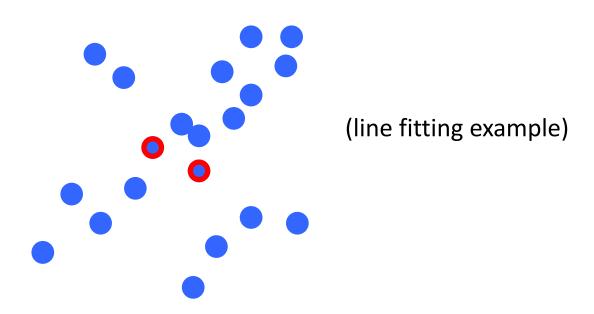
RANSAC



Algorithm:

- Sample (randomly) the number of points required to fit the model
- 2. **Solve** for model parameters using samples
- 3. **Score** by the fraction of inliers within a preset threshold of the model

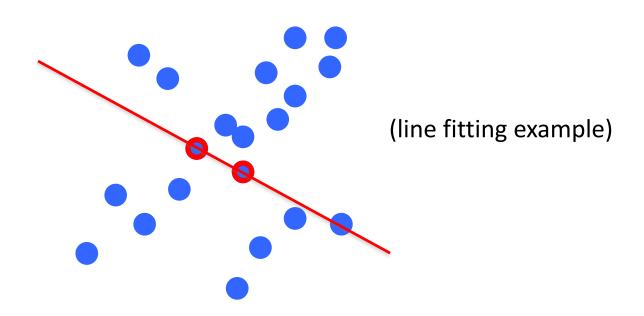
RANSAC



Algorithm:

- 1. Sample (randomly) the number of points required to fit the model
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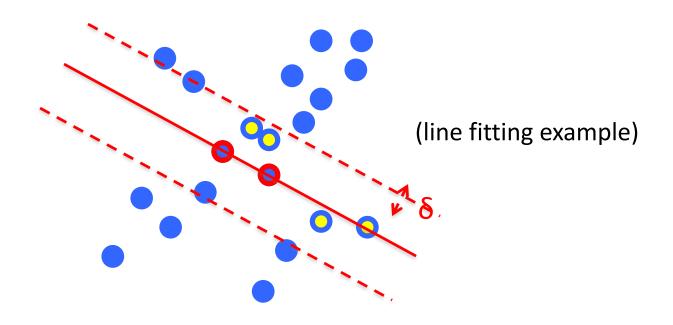
RANSAC



Algorithm:

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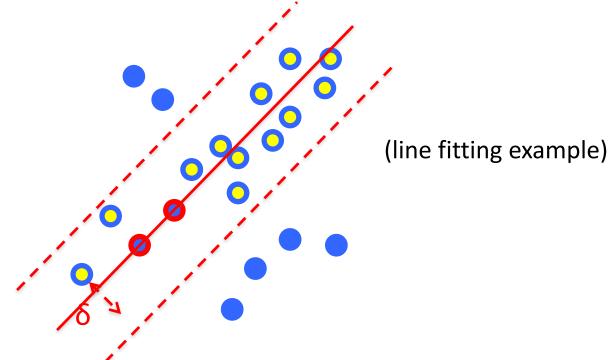
RANSAC



Algorithm:

- Sample (randomly) the number of points required to fit the model
- 2. **Solve** for model parameters using samples
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RANSAC

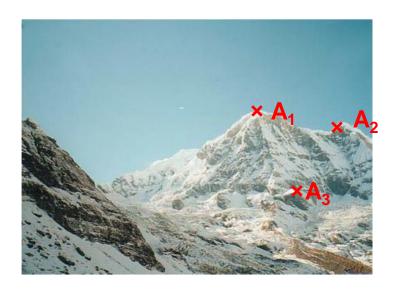


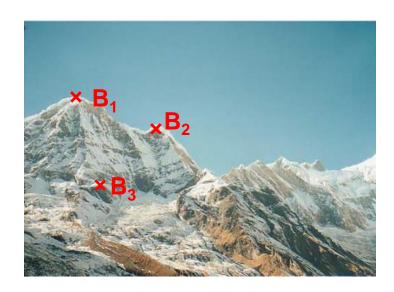
Algorithm:

- Sample (randomly) the number of points required to fit the model
- 2. **Solve** for model parameters using samples
- 3. **Score** by the fraction of inliers within a preset threshold of the model

Given matched points A and B, estimate the translation

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

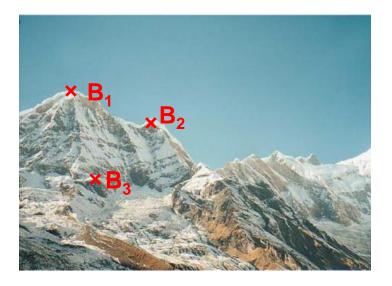




Alignment by Least Squares

- 1. Write down the objective function
- 2. Obtain the analytical solution
 - a) Compute derivative
 - b) Compute solution
- 3. Obtain computational solution
 - a) Write in form Ap = b
 - b) Solve using pseudo-inverse

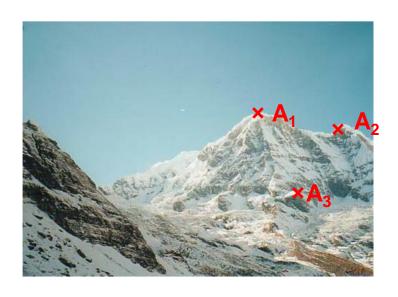
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_1^B - x_1^A \\ y_1^B - y_1^A \\ \vdots \\ x_n^B - x_n^A \\ y_n^B - y_n^A \end{bmatrix}$$

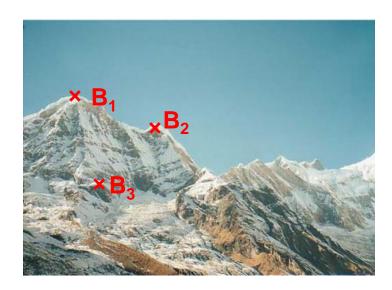


Alignment by RANCAC

- 1. Sample a set of matching points (1 pair)
- 2. Solve for transformation parameters
- 3. Score parameters with number of inliers
- 4. Repeat steps 1-3 N times

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$





SIFT-based texture classification – how to do this?



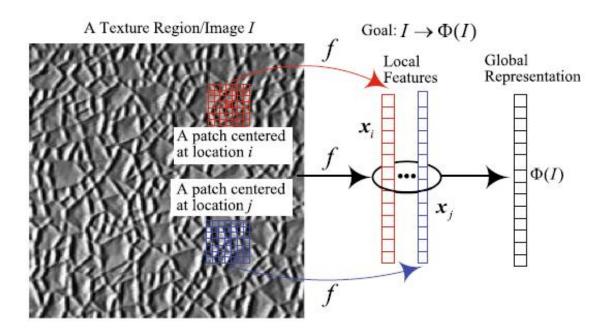




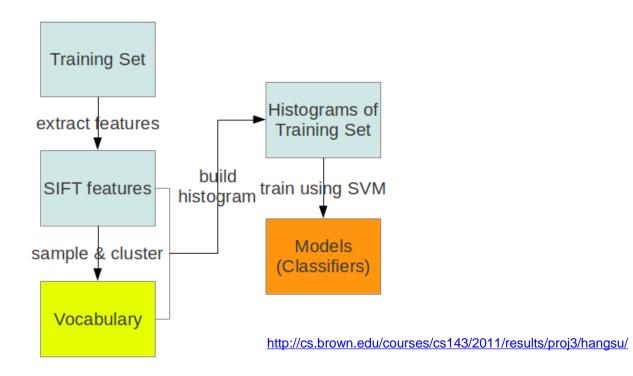
cracker

Problem: the number of SIFT keypoints (and thus the number of SIFT feature descriptors) may vary highly between images

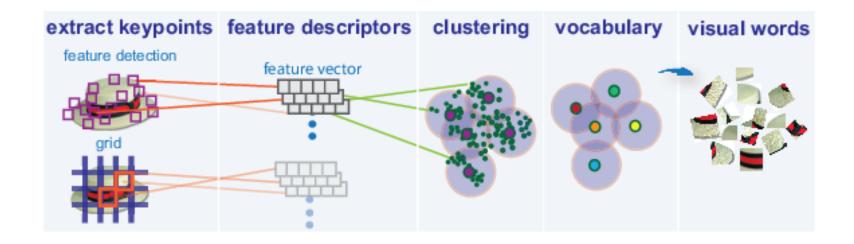
- Global encoding of local SIFT features
 - Integrate the local features (SIFT keypoint descriptors) of an image into a global vector to represent the whole image



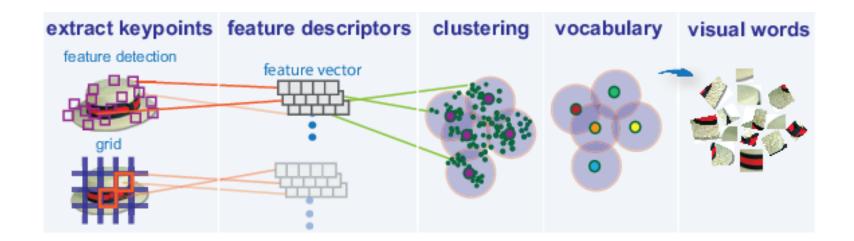
- Most popular method: Bag-of-Words (BoW)
 - The variable number of local image features are encoded into a fixed-dimensional histogram to represent each image



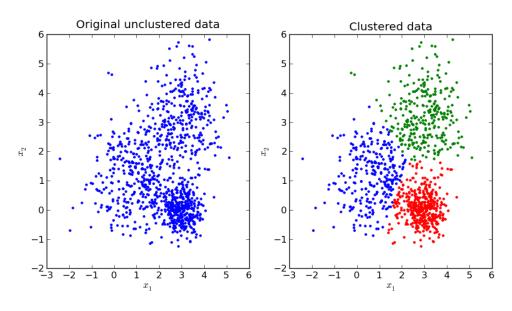
- Bag-of-Words (BoW) step 1
 - Create the vocabulary from the set of local descriptors (SIFT keypoint descriptors) extracted from the training data
 - This vocabulary represents the categories of local descriptors



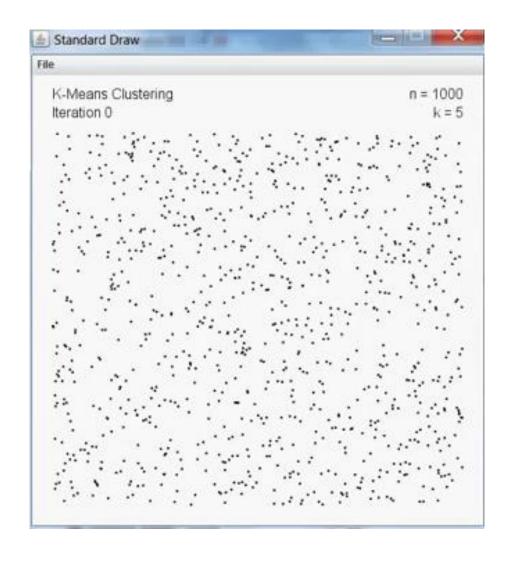
- Bag-of-Words (BoW) step 1
 - Main technique used to create the vocabulary: k-means clustering
 - k-means clustering is one of the simplest and most popular unsupervised learning approaches that perform automatic clustering (partitioning) of the training data into multiple categories



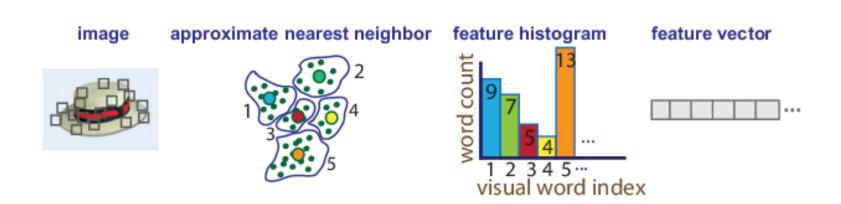
- Bag-of-Words (BoW) step 1
 - K-means clustering:
 - Initialize: k cluster centres, typically randomly
 - Iterate: 1) Assign data (feature vectors) to the closest cluster (Euclidean distance)
 - 2) Update cluster centres as the mean of the data samples in each cluster
 - Terminate: When converged or the number of iterations reaches the maximum



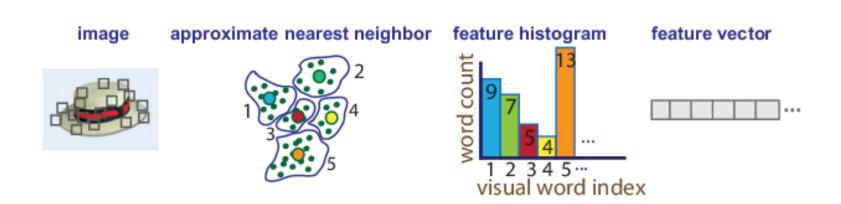
K-Means Clustering



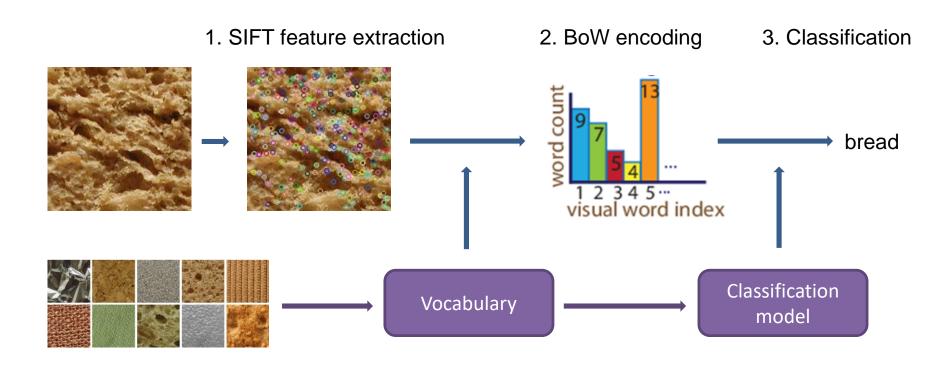
- Bag-of-Words (BoW) step 2
 - The cluster centres are the "visual words" which form the "vocabulary" that is used to represent an image
 - An individual local feature descriptor (e.g. SIFT keypoint descriptor) is assigned to one visual word with the smallest distance



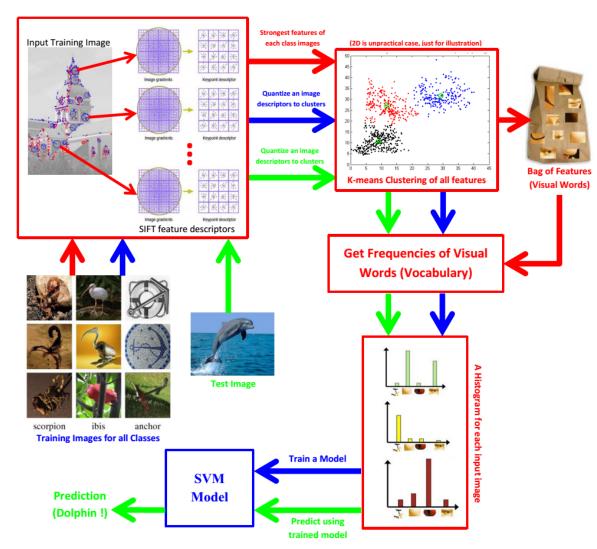
- Bag-of-Words (BoW) step 2
 - For an image the number of local feature descriptors assigned to each visual word is computed
 - The numbers are concatenated into a vector which forms the BoW representation of the image



SIFT-based texture classification



 SIFT-based texture classification



http://heraqi.blogspot.com/2017/03/BoW.html

- Local features can be other types of features, not just SIFT
 - LBP, SURF, BRIEF, ORB
- There are also more advanced techniques than BoW
 - VLAD, Fisher Vector
- A very good source of additional information is VLFeat.org
 - http://www.vlfeat.org/

Summary

- Feature representation is essential in solving almost all types of computer vision problems
- Most commonly used image features:
 - Colour features (Part 1)
 - Colour moments and histogram
 - Texture features (Part 1)
 - Haralick, LBP, SIFT
 - Shape features (Part 2)
 - Basic, shape context, HOG

Summary

- Other techniques described (Part 1)
 - Descriptor matching
 - Feature encoding (Bag-of-Words)
 - k-means clustering
 - Alignment and RANSAC
 - Spatial transformations
- To be discussed (Part 2)
 - Shape features
 - Shape matching
 - Sliding window detection

References and Acknowledgements

- Szeliski, Chapter 4 (in particular Sections 4.1.1 to 4.1.3 and 4.3.2), Chapter 6 (in particular Sections 6.1.1 to 6.1.4)
- Some content are extracted from the above resource, James Hays slides, and slides from Michael A. Wirth
- L. Liu et al., From BoW to CNN: two decades of texture representation for texture classification, International Journal of Computer Vision, 2019
- And other resources as indicated by the hyperlinks